

Math 2551 Worksheet Section 16.3 and 16.4

1. Find a potential function f for

$$\vec{F}(x, y, z) = 2xy\hat{i} + (x^2 - z^2)\hat{j} - 2yz\hat{k}.$$

Evaluate

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is any path from $(0, 0, 0)$ to $(1, 2, 3)$.

2. Let C be the ellipse

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1.$$

- (a) Parametrize this ellipse to give it a positive orientation.
- (b) Let $\vec{F}(t) = 2x\hat{i} + 2y\hat{j}$. Use Green's theorem to find the circulation of \vec{F} around C and its flux across C .
3. Let R be the region in the xy -plane bounded above by the curve $y = 3 - x^2$ and below by the curve $y = x^4 + 1$. Orient this boundary positively. Let

$$\vec{F}(x, y) = (y + e^x \ln y)\hat{i} + (e^x/y)\hat{j}.$$

Use Green's theorem to find the circulation of \vec{F} around C . What happens when you try to use Green's theorem to evaluate the flux across C .

4. Let a, b, c, d, e be real numbers and

$$M(x, y, z) = 3x + 7y + 2z;$$

$$N(x, y, z) = ax + by + 4z;$$

$$P(x, y, z) = cx + dy + ez.$$

For which values of the constants a, b, c, d, e is $\vec{F} = M\hat{i} + N\hat{j} + P\hat{k}$ a conservative vector field?