Math 2551 Worksheet Section 14.4

1. Find $\dfrac{dw}{dt}$ when $t = 1$, if $w = 2ye^x - \ln z$, $x = \ln(t^2 + 1)$, $y = \tan^{-1}(t)$, $z = e^t$.
2. Let $w = xy + yz + zx$, where $x = r\cos\theta$, $y = r\sin\theta$, $z = r\theta$.

   Find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$ when $r = 2$ and $\theta = \dfrac{\pi}{2}$.
3. Find $\dfrac{dy}{dx}$ if $\tan^{-1}(x^2y) = x + xy^2$.
4. Suppose that we subsitute polar coordinates $x = r\cos\theta$ and $y = r\sin\theta$ in a differentiable function $w = f(x, y)$.
   (a) Show that
   $$\frac{\partial w}{\partial r} = f_x\cos\theta + f_y\sin\theta \qquad \text{and} \qquad \frac{1}{r}\frac{\partial w}{\partial \theta} = -f_x\sin\theta + f_y\cos\theta$$
   (b) Solve the equations in part (a) to express f_x and f_y in terms of $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$.