

Math 2551 Worksheet Section 13.4

1. Find \vec{T} , \vec{N} , and κ for

(a) $\vec{r}(t) = (3 \sin t)\hat{i} + (3 \cos t)\hat{j} + 4t\hat{k}$.

(b) $\vec{r}(t) = \langle t, \ln \cos t \rangle$, $-\pi/2 < t < \pi/2$.

2. The graph $y = f(x)$ in the xy -plane automatically has parametrization $x = x$ and $y = f(x)$, and the vector formula $\vec{r}(x) = x\hat{i} + f(x)\hat{j}$. Use this formula to show that if f is a twice-differentiable function of x , then

$$\kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}.$$

3. Find $\kappa(x)$ for

(a) $f(x) = e^x$.

(b) $f(x) = \sin x$.

4. Determine the maximum curvature for $f(x) = \ln x$.

5. Let $\vec{r}(t) = -(t + (1/t))\hat{i} + (2 \ln t)\hat{j}$, $e^{-5} \leq t \leq e^5$.
- (a) Find the radius curvature at $t = 1$.
 - (b) Find \vec{N} at $t = 1$,
 - (c) Find the center of the circle of curvature at $t = 1$.
 - (d) Find the equation for the circle of curvature at $t = 1$.