Math 2551 Worksheet Section 15.8

1. Let R be the parallelogram in the first quadrant bounded by the lines

$$y = -2x + 4; \quad y = -2x + 7; \quad y = x - 2; \quad y = x + 1.$$

   Evaluate

$$I = \int\int_R 2x^2 - xy - y^2 \ dx \ dy$$

   by using the transformation $u = x - y$ and $v = 2x + y$ in the following steps:

   (a) Sketch R.

   (b) Solve for x, y in terms of u, v.

   (c) Describe the region in the uv-plane that corresponds to R.

   (d) Find the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$.

   (e) Use the substitution rule for double integrals to evaluate I.

2. Use the transformation $x = u^2 - v^2$ and $y = 2uv$ to evaluate the integral

$$\int_0^1 \int_0^{2\sqrt{1-x}} \sqrt{x^2 + y^2} \ dy \ dx.$$

3. Let R be the region in the first quadrant of the xy-plane bounded by

$$xy = 4; \quad xy = 16; \quad y = x; \quad y = 4x.$$

   Use the transformation $x = u/v$ and $y = uv$ with $u > 0$ and $v > 0$ to evaluate

$$\int\int_R \left(\frac{y}{x}\right)^2 + \frac{1}{xy} \ dx \ dy.$$