Math 2551 Worksheet Section 15.7

1. Convert the integral $\displaystyle\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)\,dz\,dx\,dy$ into an integral in cylindrical coordinates, and evaluate the integral.
2. Let D be the right circular cylinder whose base is the circle $r = 2\sin\theta$ in the xy-plane and whose top lies in plane $z = 4 - y$. Recall that $r = 2\sin\theta$ describes a circle centered at $(0, 1)$ with radius 1 in the xy-plane. Using cylindrical coordinates,
   (a) find the volume of the region D.
   (b) find the $\bar{x}$ component of the centroid of the region (hint: use symmetry).
3. Find the volume of the solid that is between the spheres $\rho = \sqrt{2}$ and $\rho = 2$, but outside of the circular cylinder $x^2 + y^2 = 1$.
4. Suppose $a \geq 0$. Find the volume of the region cut from the solid sphere $\rho \leq a$ by the half-planes $\theta = 0$ and $\theta = \pi/6$ in the first octant.