Assignment 1

1. Differential Equation: A mathematical equation for an unknown function of one or several variables that relates the values of the function itself and of its derivatives of various orders

Ordinary Differential Equation: A differential equation that does not contain any partial derivatives of its function

Order of a differential equation: The order of the highest derivative that appear in the function

Linear and Nonlinear Differential Equations: The ordinary equation $F(x, y, y', \dots, y^{(n)}) = 0$ is said to be linear if F is a linear function of the variables $y, y', \dots, y^{(n)}$. Otherwise, the equation is non-linear

Solution of a differential equation: Any function $f: I \to \mathbb{R}$ such that $F(x, f(x), f'(x), \dots, f^{(n)}(x)) = 0$ and I is an interval in $\mathbb{R}$, for all $x \in I$.

Initial Value Problem: An ordinary differential equation together with a specified value (initial condition), of the unknown function at a given point in the domain of the solution.

Partial Differential Equation: A type of differential equation that contain partial derivatives of its function

2. Equation	Linear	Order	Ind. var	Dep. Var	Name
$4x^2y'' + 3xy' + 2y = 0$	yes	2	x	y	Euler
$\frac{dx}{dt} + (t^2+1)x = (t^3-1)x^2$	yes	1	t	x	?
$\frac{dP}{dt} = kP(1-\frac{P}{L}), k, L \in \mathbb{R}$	yes	1	t	P	logistic
$\frac{dy}{dt}y = t+1$	no	1	t	y	Brennan
$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$	yes	2	x	y	Chebyshev
$(1-u^2)\frac{d^2v}{du^2} - 2u\frac{dv}{du} + 20v = 0$	yes	2	u	v	?

3. $f(x) = \ln \frac{x}{x+1}$ $\quad f'(x) = \frac{1}{x} - \frac{1}{x+1}$

$= \ln(x) - \ln(x+1)$

$\Rightarrow = x^2 y' - e^y = 0$

$= x^2\left(\frac{1}{x} - \frac{1}{x+1}\right) - e^{\ln\left(\frac{x}{x+1}\right)}$ (substituting)

$= x - \frac{x^2}{x+1} - \frac{x}{x+1}$

$= \frac{x(x+1) - x^2 - x}{x+1}$

$= \frac{x^2 + x - x^2 - x}{x+1}$

$= \frac{0}{x+1}$

$= 0$

Make sure $\frac{x}{x+1} > 0$

if $x > 0 \Rightarrow \frac{x}{x+1} > 0$

if $x < -1 \Rightarrow \frac{x}{x+1} > 0$

if $-1 < x < 0 \Rightarrow \frac{x}{x+1} < 0$

$\therefore x \in (-\infty, -1) \cup (0, \infty)$

Since $I = (0, \infty)$ is captured in the domain of x, $f(x)$ is a solution. However $J = (-1, 0)$ is not captured in the domain so $f(x)$ is not a solution the

4. $f(x) = c_1 e^{-3t} + c_2 e^{t}$

$f'(x) = -3c_1 e^{-3t} + c_2 e^{t}$

$y(0) = 2: \quad 2 = c_1 e^{-3(0)} + c_2 e^{0} = c_1 + c_2$

$y'(0) = -1: \quad -1 = -3c_1 e^{-3(0)} + c_2 e^{0} = -3c_1 + c_2$

$c_1 + c_2 = 2$
$-3c_1 + c_2 = -1$
$\Rightarrow 4c_1 = 3 \Rightarrow c_1 = 3/4$
$c_2 = 5/4$

The solution is the function $f(t) = 3/4 e^{-3t} + 5/4 e^{t}$