Homework 3

Brennen Green

June 27, 2020

1. `Ch3.1 Exercise #5`

   Start at the second element and sequentially iterate over the list of elements and comparing every element to the one before it. If an element is equal to the one before it, remove that element from the list.

2. `Ch3.1  Exercise  #23`

   A function is onto if and only if $f(a) = f(b) \rightarrow a = b$ so in order to check if a our two sets of integers and function are onto, we must ensure there is no case where $f(a) = f(b)$ is true yet $a \neq b$

   So our algorithm must iterate over the image set b_i, at every element we must iterate over the pre-image a_j until we find an element where $a_j = b_i$. If there is an element b_i that does not have some matching element a_j then we return our algorithm as false. If our algorithm iterates over the entire image set with no interuption, the algorithm returns true.

3. `Ch3.2 Exercise #15`

   If some function is $\mathcal{O}(1)$ it is the same as saying that, in the worst case, a function will always produce some constant and never grow.

4. `Ch3.2 Exercise #30`

   If some function is $\Omega(1)$ it is the same as saying that, in the best case, a function will always produce some constant and never grow.

5. `Ch3.2 Exercise #31`

   If some function is $\Theta(1)$ it is the same as saying that, in the best case, a function will always produce some constant and never grow. Or put in other words, the function is both $\Omega(1)$ AND $\mathcal{O}(1)$

6. `Ch3.3  Exercise  #28`

   i Since bubble sort makes $\frac{n(n-1)}{2}$ (or) comparisons with input n, we can plug 2n into this equation to find that with input 2n bubble sort makes $2n^2 - n$ comparisons. Dividing the amount of comparisons at 2n by the number of comparisons at n that this change in the number of comparisons was more than double and definitely significant.

   ii We have an identical process to compare the growth of comparisons for insertion sort. Since instertion sort also makes $\frac{n(n-1)}{2}$ comparisons with an input of n, plugging 2n into this will show us that with an input of 2n insertion sort will need $2n^2 - n$ comparisons. Which is more than a doubling of comparisons and significant growth.

   These two results make sense as both functions have extremely similar computational complexities.

7. `Ch3.4 Exercise #5`

   Proof.

$$
\begin{aligned}
& a|b \wedge b|a \rightarrow (a = b) \vee (a = -b) \\
& a|b \rightarrow ac = b \wedge b|a \rightarrow bc = a && \text{(definition of division)} \\
& (bc)c = b \\
& bc^2 = b \\
& c = \pm 1 \\
& a(1) = ba(-1) = b \\
& a = b \vee a = -b && \square
\end{aligned}
$$

8. `Ch3.4 Exercise #16`

   a 1

   b 4

   c 3

   d 9

9. `Ch3.4 Exercise #24`

   Proof.

   n is an odd integer $\rightarrow n^2 \equiv 1(mod8)$
   $n = 2k + 1$
   $8|n^2 - 1$
   $8|(2k + 1)^2 - 1$
   $8|4k^2 + 4k$
   $8|4k(k + 1)$
   $8|4(2x)$ (k(k+1) is even since k or k+1 is even)
   $8|8x$
   $\therefore 8|n^2 - 1$
   $\therefore n^2 \equiv 1(mod8)$ (definition of congruency) $\square$

10. `Ch3.5 Exercise #4`

   a $3 * 13$

   b $3 * 3 * 3 * 3$

   c 101

   d $13 * 11$

   e $17 * 17$

   f $31 * 29$

11. `Ch3.5 Exercise #31`

Proof.

$\forall k \in \mathbb{Z}(6|k*(k+1)*(k+2))$

1. $2|k*(k+1)*(k+2)$ (at least one number will be even)
2. $3|k*(k+1)*(k+2)$ (every 3rd integer is divisible by 3)
3. $2*3|k*(k+1)*(k+2)$ ((a | c) and (b | c) then ab | c)
4. $\therefore 6|k*(k+1)*(k+2)$ $\square$

12. `Ch3.6 Exercise #1`

a 1110 0111

b 10 0011 0110 100

c 1 0111 1101 0110 1100

13. `Ch3.6 Exercise #3`

a 31

b 513

c 341

d 26896

14. `Ch3.6 Exercise #3`

a 2062

b 79275

c 43962

d 233811181

15. `Ch3.6 Exercise #24`

a 1

b 1

c 1

d 139

e 1

f 1

16. `Ch3.8 Exercise #2`

a $\mathbf{A} + \mathbf{B} = \begin{bmatrix} 0 & 3 & 9 \\ 1 & 4 & -1 \\ 2 & -5 & -3 \end{bmatrix}$

b $\mathbf{A} + \mathbf{B} = \begin{bmatrix} -4 & 9 & 2 & 10 \\ -4 & -5 & 4 & 0 \end{bmatrix}$

17. `Ch3.8 Exercise #3`

a $\mathbf{AB} = \begin{bmatrix} 1 & 11 \\ 2 & 18 \end{bmatrix}$

b $\mathbf{AB} = \begin{bmatrix} 2 & -2 & -3 \\ 1 & 0 & 2 \\ 9 & -4 & 4 \end{bmatrix}$

c $\mathbf{AB} = \begin{bmatrix} -4 & 15 & -4 & 1 \\ -3 & 10 & 2 & -3 \\ 0 & 2 & -8 & 6 \\ 1 & -8 & 18 & -13 \end{bmatrix}$

18. `Ch3.8 Exercise #5`

$\mathbf{A} = \begin{bmatrix} \frac{9}{5} & -\frac{6}{5} \\ -\frac{1}{5} & \frac{4}{5} \end{bmatrix}$

19. `Ch3.8 Exercise #10`

a 3x5

b Undefined

c 3x4

d Undefined

e Undefined

f 4x5

20. `Ch3.8 Exercise #18`

If a matrix A^{-1} is the inverse of matrix A then the product of the two is the identity matrix. Since the product of

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{bmatrix} * \begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which is a variation of the identity matrix then it can be said that the latter matrix is the inverse of the former.