Homework 5

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1. `Ch5.1 Exercise #2`

   There are 27 * 37 or 999 offices in the building.

2. `Ch5.1 Exercise #8`

   There are 26*25*24 or 15600 possible initials

3. `Ch5.1 Exercise #23`

   (a) $10^3 - 10 = 990$
   (b) $5 * 10 * 10 = 500$
   (c) $1 * 1 * 9 * 3 = 27$

4. `Ch5.1 Exercise #31`

   (a) $21^8 = 37,822,859,361$
   (b) $21 * 20 * 19 * 18 * 17 * 16 * 15 * 14 = 8,204,716,800$
   (c) $5 * 26^7 = 40,159,050,880$
   (d) $5 * 25 * 24 * 23 * 22 * 21 * 20 * 19 = 12,113,640,000$
   (e) $26^8 - 21^8 = 171,004,205,215$
   (f) $8 * 5 * 21^7 = 72,043,541,640$
   (g) $26^7 - 21^7 = 6,230,721,635$
   (h) $1 * 25 * 24 * 23 * 22 * 21 * 20 * 19 - 1 * 20 * 19 * 18 * 17 * 16 * 15 * 14 = 2,032,027,200$

5. `Ch5.2 Exercise #4`

   (a) 5 balls
   (b) 13 balls

6. `Ch5.2 Exercise #9`

   $99(students) * 50(states) + 1(student) = 4951\ students$

7. `Ch5.2 Exercise #9`

   (a) *Proof.* The list of integers which add up to 11 are:

   $$(1, 10), (2, 9), (3, 8), (4, 7), (5, 6)$$

   Looking at the list you can see it is split into the first and second half of the 10 integers. In otherwords, the first and last groupings of 5 integers.

   1. If we pick 5 integers, there is no gaurantee any will add to 11
   2. Adding 2 integers will give at-least 2 combinations equal to 11
   3. This is an effective example of the Pigeonhole Principle □

   (b) There would not be two combinations but there would be at-least one combination that equals 11 if you choose six integers.

8. `Ch5.2 Exercise #21`

   $\{4, 3, 2, 1, 8, 7, 6, 5, 12, 11, 10, 9, 16, 15, 14, 13\}$

9. `Ch5.3 Exercise #2`

   5040

10. `Ch5.3 Exercise #5`

   (a) $P(6, 3) = 120$
   (b) $P(6, 5) = 720$
   (c) $P(8, 1) = 8$
   (d) $P(8, 5) = 6,720$
   (e) $P(8, 8) = 40,320$
   (f) $P(10, 9) = 3,628,800$

11. `Ch5.3 Exercise #11`

   (a) $10nCr4 = 210$ strings
   (b) $1 + C(10, 1) + C(10, 2) + C(10 = 3) + C(10, 4) = 386$ strings
   (c) $C(10, 4) + C(10, 5) + C(10, 6) + C(10, 7) + C(10, 8) + C(10, 9) + C(10, 10) = 848$ strings
   (d) $C(10, 5) = 252$

12. `Ch5.3 Exercise  #28`

    $C(40,17) = 88,732,378,800$

13. `Ch5.4 Exercise #3`

    $(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

14. `Ch5.4 Exercise #6`

    $\binom{11}{7}x^7 = 330$

15. `Ch5.4 Exercise #23`

    Proof. Show that $\forall n,k \in \mathbb{Z}^+(\binom{n+1}{k} = (n+1)\binom{n}{k-1}/k)$

$$\begin{aligned}
\binom{n+1}{k} &= \frac{(n+1)!}{k!((n+1)-k)!} \\
&= \frac{(n+1)*n!}{k!((n+1)-k)!} \\
&= \frac{(n+1)*n!}{k*(k-1)!*(n+1-k)!} \\
&= \frac{n+1}{k} * \frac{n!}{(k-1)!*(n+1-k))!} \\
&= \frac{n+1}{k} * \frac{n!}{(k-1)!*(n-(k-1))!} \\
&= \frac{n+1}{k} * \binom{n}{k-1} \\
\therefore \binom{n+1}{k} &= (n+1)\binom{n}{k-1}/k
\end{aligned}$$

□

16. `Ch5.5 Exercise  #9`

    (a) $C(n+r-1,r) = C(13,6) = 1,716$
    (b) $C(n+r-1,r) = C(19,12) = 50,388$
    (c) $C(n+r-1,r) = C(31,24) = 2,629,575$
    (d) $C(n+r-1,r) = C(11,4) = 330$
    (e) $C(15,9) + C(14,8) + C(13,7) = 9,724$

17. `Ch5.5 Exercise #14`

    $C(n+r-1,r) = C(20,4) = \frac{20!}{4!(16!)} = 4,845$

18. `Ch5.5 Exercise #30`

    $\frac{11!}{1!*4!*4!*2!} = 34,650$

19. `Ch5.6 Exercise #2`

    156423, 156423, 165432, 165432, 231456, 231456, 234561, 234561, 435612, 543216, 543216

20. `Ch5.6 Exercise  #4`

    (a) 1423
    (b) 51234
    (c) 13254
    (d) 612354
    (e) 1623574
    (f) 23587461