Exam One

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1.

p	q	¬ (p → q)	p ∧ ¬ q
T	T	F	F
T	F	T	T
F	T	F	F
F	F	F	F

2.

1. $\neg a \wedge b$ (Original Premise)
2. $\neg a$ (Simplication: $\neg a \wedge b \rightarrow \neg a$)
3. $c \rightarrow a$
4. $\neg c$ (Modus Tollens: $\neg a \wedge (c \rightarrow a) \rightarrow \neg c$)
5. $\neg c \rightarrow d$
6. d (Modus Ponens: $\neg c \wedge (\neg c \rightarrow d) \rightarrow d$)
7. $d \rightarrow e$

7\. $\therefore e$ (Modus Ponens: $d \wedge (d \rightarrow e) \rightarrow e$)

3. Prove: $a, b \in \mathbb{Z} a^2 + b^2$ is even, then $a + b$ is also even

Proof.

$$
\begin{aligned}
&\text{Even Integer} = 2i \\
&a = 2i \; b = 2j \\
&a^2 + b^2 = 4i^2 + 4j^2 = 2(2(i^2 + j^2)) \text{ (Even Form)} \\
&a + b = 2i + 2j = 2(i + j) \text{ (Even Form)} \\
&\therefore a, b \in \mathbb{Z} a^2 + b^2 \text{is even, then } a + b \text{ is also even}
\end{aligned}
$$

□

4. Prove: $\nexists x, y \in \mathbb{Z}(4x^2 - y^2 = 1)$

Proof.

$$
\begin{aligned}
&\exists x, y \in \mathbb{Z}(4x^2 - y^2 = 1) \ldots (1) \\
&x = 2k \;\; y = 2k + 1 \\
&4(2k)^2 - (2k + 1)^2 = 1 \\
&16k^2 - 4k^2 - 4k - 1 = 1 \\
&12k^2 - 4k - 2 = 0 \\
&\nexists k \in \mathbb{Z}(12k^2 - 4k - 2 = 0) \qquad \text{(Quadratic Equation)} \\
&\therefore \nexists x, y \in \mathbb{Z}(4x^2 - y^2 = 1) \text{ by contradiction}
\end{aligned}
$$

□

5. Prove: $\forall n > 2$(if n is prime then n is odd)

Proof.

Contrapositive: $\forall n > 2$(if n is not prime then n is even)
If n is not prime then its divisible by more than 1 and itself
If n is even then$n = 2k$
$2|2k \wedge 1|2k \wedge k|2k \wedge 2k|2k$
$\therefore \forall n > 2$(if n is not prime then n is even)
$\therefore \forall n > 2$(if n is prime then n is odd) by contraposition

□

6. (a) $M \cap A$
   (b) $F - M$
   (c) $R \cap (A \cap F)$
   (d) $(R - Y) \cap (F \cap M) - A$

7. Premise:$(B - A) \cup (C - A) = (B \cup C) - A$

   $(B - A) = \{x|x \in B \wedge x \notin A\}$
   $(C - A) = \{x|x \in B \wedge x \notin A\}$
   $(B - A) \cup (C - A) = \{x|(x \in B \wedge x \in C) \wedge x \notin A\}$

   $(B \cup C) = \{x|x \in B \wedge x \in C\}$
   $(B \cup C) - A = \{x|(x \in B \wedge x \in C) \wedge x \notin A\}$

   $\therefore$ The premise has been shown

8. No the function $g(x) = 2x - 1$ is not onto because there are elements in the set of integers that the function $g(x)$ does not map to given the domain. As an example, there is no integer such that g(x) = 2. In fact, no even integer is mapped by this function because it is actually just a different representation of an odd integer.

9. If $f(x) = \frac{x^2}{1-x}$ and $g(x) = |x^2 - x|$ then $f \circ g = f(g(x)) = \frac{x^2-x}{1-|x^2-x|}$

10. $\sum_{n=0}^{142}(7n)$

11. *Proof.*

$$\text{Basis}: \prod_{i=2}^{2} 1 - \frac{1}{i^2} = 1 - \frac{1}{4} = \frac{2+1}{2*2} = \frac{3}{4}$$

$$\text{Assume}: \prod_{i=2}^{k} (1 - \frac{1}{i^2}) = \frac{k+1}{2k}$$

$$\text{Inductive Step}: \prod_{i=2}^{k+1} (1 - \frac{1}{i^2}) = \frac{k+2}{2k+2}$$

$$\frac{k+1}{2k} * (1 - \frac{1}{k^2}) = \frac{k+2}{2k+2}$$

$$\frac{k+2}{2(k+1)} = \frac{k+2}{2k+2}$$

$$\frac{k+2}{2k+2} = \frac{k+2}{2k+2}$$

$\therefore$ the premise put forth in the question is true by induction

□

12. The smallest amount is 42 such that every integer $n \geq 42$ can be formed with 7 and 8 cent stamps

Proof.

Basis : $P(42) = 7*6 + 8*0 = 42$ $P(43) = 7*5 + 8 = 43$

Assume : $P(42), P(43), \ldots, P(k) = 7x + 8y$

Inductive Step :

In order to achieve P(k+1) just replace one

7 cent stamp with one 8 cent stamp

Alternatively, if P(k) can be made with six 8-cent stamps

then P(k+1) can be made with seven 7-cent stamps

$\therefore$ P(n) is true for all $n \geq 42$ by strong induction

□

13. $a_{i+1} = (i+1)^2 + 7 = i^2 + 2i + 1 + 7$ $a_0 = 7$