# Homework 7

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1. (a) 2, 12, 72, 432, 2592, 15552
   (b) 2, 4, 16, 256, 65536, 4294967296
   (c) 1, 2, 5, 11, 26, 59
   (d) 1, 1, 6, 27, 204, 1695
   (e) 1, 2, 0, 1, 3, 3

2. (a) -3(0) + 4(0) = 0 = $a_n$
   (b) -3(1) + 4(1) = 1 = $a_n$
   (c) $-3(-4^{n-1})+4(-4^{n-2}) = -3(-4)(-4^{n-2})+4(-4^{n-2}) = (-4)^{n-2} * (-4)^2 = -4^n$
   (d) $-3(2(-4)^{n-1}+3)+4(2(-4)^{n-2}+3)$
   $= -6(-4)^{n-1} - 9 + 4(2(-4)^{n-2}+3)$
   $= (-4)^{n-2}(-6*(-4)+8)+3$
   $= (-4)^{n-2}(16)(2)+3$
   $= 2(-4)^n + 3 = a_n$

3. (a) $a_n = a_{n-1} * 1.09$ $a_0 = 1000$
   (b) $1.09^n * 1000$
   (c) $\$5,529,041$

4. (a) $a_n = a_{n-1} + a_{n-2} + 2^{n-2}$
   (b) $a_0 = 0$ $a_1 = 0$
   (c) $a_7 = a_6 + a_5 + 2^5 = 43 + 19 + 32 = 94$

5. (a) Linear Homogenous Relationship with degree of 2
   (b) Not Homogenous
   (c) Not Linear
   (d) Linear Homogenous Relationship with degree of 3
   (e) Not Linear
   (f) Not Homogenous
   (g) Linear Homogenous Relationship with degree of 7

6. (a) $P_n = 1.2P_{n-1} + 0.45P_{n-2}$ $P_0 = \$100,000$ $P_1 = \$120,000$
   (b) $P_n = (\frac{250000}{3}) * (\frac{3}{2})^n + (50000) * (-\frac{3}{10})^n$

7. $a_n = 5 + 3(-2)^n - 3^n$

8. $a_n = \alpha_1 * (1)^n + \alpha_2 * (-2)^n + \alpha_3 * (3)^n + \alpha_4 * (-4)^n$

9.
$$\begin{aligned} f(64) &= f(32) + 2 \\ &= f(16) + 2 + 2 \\ &= f(8) + 2 + 2 + 2 \\ &= f(4) + 2 + 2 + 2 + 2 \\ &= f(2) + 2 + 2 + 2 + 2 + 2 \\ &= f(1) + 2 + 2 + 2 + 2 + 2 + 2 \\ &= 2 + 12 = 14 \text{ comparisons} \end{aligned}$$

10. (a) $f(3) = f(1) + 1 = 1 + 1 = 2$
    (b) $f(27) = f(9) + 1 = f(3) + 1 + 1 = f(1) + 1 + 1 + 1 = 4$
    (c) $f(729) = f(243) + 1 = f(81) + 2 = f(27) + 3 = f(9) + 4$
    $= f(3) + 5 = f(1) + 6 = 7$

11.

$$
\begin{aligned}
&f(n) = 2f(\frac{n}{3}) + 4 \\
&f(3^k) = 2f(3^{k-1}) + 4 \\
&\cdots \\
&= 2^k * f(1) + 4(2^k - 1) \\
&= 2^k + 4(2^k - 1) \\
&= 2^k + 2^{k+2} - 4
\end{aligned}
$$

12. *Proof.*

$f(n) = C_1 * n^d + C_2 * n^{\log_b a}$
$C_1 = \frac{b^d * c}{b^d - 1}$ $C_2 = \frac{b^d * c}{a - b^d}$
$a = 5$ $b = 4$ $c = 6$ $d = 1$ (Divide and Conquer Recurrence Relation)
$C_1 = -24$ $C_2 = 25$
$f(n) = -24n + 25n^{\log_4 5}$
$f(4^k) = -24(4^k) + 25(4^k)^{\log_4 5}$
$= 3 * 2^{2k+3} + 25(4^k)^{\log_4 5}$
$= 3 * 2^{2k+3} + 5^{k+2}$ □

13. (a) 1024
    (b) 11
    (c) 66
    (d) 292,864
    (e) 20,412

14. $a_k = 3 * a_{k-1} + 2 = 2 * 3^k - 1$

15. (a) 30
    (b) 29
    (c) 24
    (d) 18

16. $(650,000 + 1,250,000) - 1,450,000 = 450,000$

17. (a) 10,000
    (b) 11,100
    (c) 11,095

18. 68 of the integers not exceeding 100 are divisible by 5 or 7