Brennen Green
CS275
Homework Assignment #1

1. **Ch 1.1 Exercise #2**
   a. Not a proposition
   b. Not a proposition
   c. Is a proposition with truth value F
   d. Not a proposition
   e. Is a proposition with truth value F
   f. Not a proposition
2. **Ch 1.1 Exercise #27**

   a. $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

   b. $p \vee \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	T
F	T	T

   c. $(p \vee \neg q) \to q$

p	q	$\neg q$	$(p \vee \neg q) \to q$
T	T	F	T
F	F	T	F
T	F	T	F
F	T	F	T

   d. $(p \vee q) \to (p \wedge q)$

p	q	$(p \vee q) \to (p \wedge q)$
T	T	T
F	F	T
T	F	F
F	T	F

   e. $(p \to q) \leftrightarrow (\neg q \to \neg p)$

p	q	$p \to q$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
T	T	T	T	T
F	F	T	T	T
T	F	F	F	T
F	T	T	T	T

   f. $(p \to q) \to (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
F	F	T	T	T
T	F	F	T	T
F	T	T	F	F

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3. **Ch 1.2 Exercise #5**

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

p	q	r	$p \wedge (q \vee r)$
T	T	T	T
F	F	F	F
T	F	T	T
F	T	F	F
T	T	F	T
F	F	T	F

p	q	r	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T
F	F	F	F
T	F	T	T
F	T	F	F
T	T	F	T
F	F	T	F

4. **Ch 1.2 Exercise #30**

$$(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$$

p	q	r	$\neg p$	$(p \vee q) \wedge (\neg p \vee r)$	$q \vee r$	$(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$
T	T	T	F	T	T	T
F	F	F	T	F	F	T
T	F	T	F	T	T	T
F	T	F	T	T	T	T
T	T	F	F	F	T	T
F	F	T	T	F	T	T

5. **Ch 1.3 Exercise #3**
   a. T
   b. F
   c. T
   d. F

6. **Ch 1.3 Exercise #15**
   a. T
   b. T
   c. T
   d. F

7. **Ch 1.4 Exercise #15** $L(x, y) = x\ loves\ y : x, y \in all\ humans$
   a. $\forall x L(x,\ Jerry)$
   b. $\forall x \exists y L(x, y)$
   c. $\exists x \forall y L(x, y)$
   d. $\neg \exists x \forall y L(x, y)$
   e. $\exists y \neg L(Lydia, y)$
   f. $\forall x \exists y \neg L(x, y)$
   g. $\exists x \forall y (L(x, y) \leftrightarrow x = y)$

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8. **Ch 1.4 Exercise #24**
    a. For all values of x and y such that if x is greater than or equal to zero and y is less than zero then the sum of y subtracted from x is greater than zero.
    b. This exists some value x and y such that x and y are less than or equal to zero and the result of y subtracted from x is greater than zero.
    c. For all values of x and y such that x and y are not equal to zero if and only if the product of x and y is not equal to zero.
9. **Ch 1.5 Exercise #5**

p = “Randy works hard” q = “Randy is a dull boy” r = “Randy will not get the job”

$$p \to q$$
$$q \to r$$
$$\therefore p \to r$$
$$(p \to q) \land (q \to r) \to r$$

By hypothetical syllogism

10. **Ch 1.5 Exercise #33**

$$Premise: (p \lor q) \land (\neg p \lor q) \land (p \lor \neg q) \land (\neg p \lor \neg q) \equiv F$$

$$(p \lor q) \land (\neg p \lor q) \land (p \lor \neg q) \land (\neg p \lor \neg q)$$
$$q \land (p \lor \neg q) \land (\neg p \lor \neg q)$$
$$q \land \neg q$$
$$F \quad \blacksquare$$

11. **Ch 1.6 Exercise #1**

The sum of two odd integers is an even integer

Odd integer: $2i + 1$

Even Integer: $2j$

$$x + y = z$$
$$(2i + 1) + (2i + 1) = z$$
$$4i + 2 = z$$
$$2(2i + 1) = z \quad \blacksquare$$

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12. **Ch 1.6 Exercise #18**

If n is an integer and 3n + 2 is even, then n is even

$$\boldsymbol{p = n \in \mathbb{Z} \quad q = 3n + 2\ is\ even \quad r = n\ is\ even}$$
$$\boldsymbol{p \land q \to r}$$
$$\boldsymbol{Odd\ integer = 2k + 1\ Even\ Integer = 2k}$$

a. $\neg(p \land q) \to \neg r \quad (n\ is\ an\ integer)$

$$\neg q \to \neg r$$
$$n = 2k + 1$$
$$3n + 2 = 3(2k + 1) + 2$$
$$3(2k + 1) = 6k + 5$$
$$6k + 5 = 2(3k + 2) + 1 \quad \blacksquare$$

b. $p \land q \to \neg r \quad (n\ is\ an\ integer)$

$$q \to \neg r$$
$$n = 2k + 1$$
$$3n + 2 = 3(2k + 1) + 2$$
$$3(2k + 1) + 2 = 6k + 5$$
$$6k + 5 = 2(3k + 2) + 1$$
$$\therefore q \to r \quad \blacksquare$$

13. **Ch 1.7 Exercise #6**

$$Claim: \exists n \left(\sum_{1}^{n} i = n \right)$$
$$n = \frac{n(n+1)}{2}$$
$$0 = \frac{n(n+1)}{2} - n$$
$$n^2 - n = 0$$
$$n(n-1) = 0$$
$$n = 0, 1$$
$$n = 1 = 0 + 1 = 1 \quad \blacksquare$$

This proof is constructive since we found some value for which the claim is true

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14. **Ch 1.7 Exercise #16**

If r is an irrational number, there is a unique integer n such that the distance between r and n is less than ½

$$Claim: \exists n(|r-n| < 0.5)$$
$$Given\ two\ integers\ x, y\ such\ that\ |r-x| < 0.5\ and\ |r-y| < 0.5$$
$$|x-y| \leq |r-x| + |r-y| < 0.5 + 0.5$$
$$|x-y| \leq |r-x| + |r-y| < 1$$
$$However, since\ x\ and\ y\ are\ two\ distinct\ integers\ we\ know\ that\ |n-m| \geq 1$$
$$\therefore We\ have\ proved\ uniqueness\ by\ contradiction\ \blacksquare$$

15. **Ch 1.7 Exercise #27**

$$Claim: \neg\exists n(n^2 + n^3 = 100)$$
$$n^3 \geq n^2$$
$$We\ have\ to\ test\ all\ values\ integers\ n^3 + n^2 \leq 100$$
$$n = 0 \quad (0)^2 + (0)^3 = 0$$
$$n = 1 \quad (1)^2 + (1)^3 = 1$$
$$n = 3 \quad (3)^2 + (3)^3 = 36$$
$$n = 4 \quad (4)^2 + (4)^3 = 80$$
$$n = 5 \quad (5)^2 + (5)^3 = 150$$
$$\therefore There\ is\ no\ value\ of\ n\ for\ which\ this\ claim\ is\ false \quad \blacksquare$$

This is an example of an exhaustive proof