Homework 6

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1. $18/36 = 1/2$ *or* 50%
2. $\frac{\binom{50}{3}}{\binom{52}{5}} = \frac{19{,}600}{2{,}598{,}960} \approx 0.00754$
3. $\frac{\binom{13}{5} * 4^5}{\binom{52}{5}} = \frac{1{,}317{,}888}{2{,}598{,}960} \approx 0.5071$
4. $\frac{3^6}{6^6} = 0.015625$
5. $\frac{1}{\binom{100}{8}} = \frac{1}{186{,}087{,}894{,}300} \approx 0.000000000005374$
6. $P(1) = P(2) = P(4) = P(5) = P(6) = \frac{1}{7}$ $P(3) = \frac{2}{7}$
7. $P(7) = \frac{9}{49}$ By adding the probabilities of all 6 possible outcomes that equal 7
8. (a) 0.5
   (b) 0.5
   (c) $\frac{1}{n}$
   (d) $0.5 * 0.5 = 0.25$
   (e) $\frac{1}{3}$
9. *Proof.*

$$
\begin{aligned}
&P(\overline{E} \cap \overline{F}) = 1 - P(E \cup F) \\
&1 - P(E \cup F) = 1 - P(E) - P(F) + P(E \cap F) \\
&= (1 - P(E))(1 - P(F)) \\
&= P(\overline{E})P(\overline{F}) \text{ definition of independence} \\
&\therefore \overline{E} \ \& \ \overline{F} \text{ are independent}
\end{aligned}
$$

□

10. (a) E and F are not independent
    (b) E and F are not independent
    (c) E and F are not independent

11. $P(F|E) = \frac{3}{5}$

12. P(Frida picked from box 1 | she picked a blue ball) = 0.75

13. P(Ann picked from box 2 | she picked an orange ball) = $\frac{35}{68}$

14. P(does not use opium | tests negative) = 0.999
    P(uses opium | tests positive) = 0.3242

15. $P(F_j|E) = \frac{P(E|F_1)P(F_1)}{\sum_{j=1}^{3} P(E|F_j)P(F_j)} = \frac{3}{17} \approx 0.1765$

16. $\sum_{n=0}^{10} \binom{10}{n}(5/6)^{10-n}(1/6)^n = 1.667$

17. $3.5 + 3.5 + 3.5 = 10.5$ Since each dice has an expected value of 3.5

18. 6 times

19. *Proof.*

    X = Number of First Die Z = Number of Second Die
    $E(Z) = E(X) = 3.5$
    $E(Y) = 3.5 + 3.5 = 7$
    $E(XY) = E(X(X + Z)) = E(X^2) + E(X)E(Z)$
    $E(X^2) = 1^2/6 + 2^2/6 + 3^2/6 + 4^2/6 + 5^2/6 + 6^2/6 = 15.1667$
    $E(XY) = 15.1667 + (3.5)(3.5) = 27.41667$
    $E(X)E(Y) = 3.5 * 7 = 24.5$
    $\therefore E(XY) \neq E(X)E(Y)$ □

20. $np(1 - p) = 10 * (.5)(.5) = 2.5$