Homework 8

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August 3, 2020

1. (a) Not Reflexive, Not Symmetric, Transitive, Not Anti-Symmetric
   (b) Reflexive, Symmetric, Transitive, Not Anti-Symmetric
   (c) Not Reflexive, Symmetric, Not Transitive, Not Anti-Symmetric
   (d) Not Reflexive, Not Symmetric, Not Transitive, Anti-Symmetric
   (e) Reflexive, Symmetric, Transitive, Anti-Symmetric
   (f) Not Reflexive, Not Symmetric, Not Transitive, Not Anti-Symmetric

2. (a) {(1,1), (1,2), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (3,4)}
   (b) {(1,2), (2,3), (3,4)}
   (c) $\emptyset$
   (d) {(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)}

3. *Proof.*

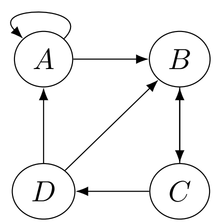
$$\begin{aligned}
&\text{I.}\\
&\text{R is reflexive: } (a,a) \in R\\
&\therefore (a,a) \notin \overline{R}\\
&\therefore \overline{R} \text{ is irreflexive by definition}\\
&\text{II.}\\
&\overline{R} \text{ is irreflexive: } (a,a) \notin \overline{R}\\
&\therefore (a,a) \in R\\
&\therefore \text{R is reflexive by definition}
\end{aligned}$$

$\square$

4. {(6,1,1,1), (1,6,1,1), (1,1,6,1), (1,1,1,6), (3, 2, 1, 1), (2, 3, 1, 1) (3, 1, 1, 2), (3, 1, 2, 1), (1, 3, 2, 1), (1, 3, 1, 2), (1, 2, 3, 1), (2, 1, 3, 1), (1, 1, 3, 2), (2, 1, 1, 3), (1, 2, 1, 3), (1, 1, 2, 3)}

5. (a) {(1,1), (3,1), (2,2), (1,3), (3,3)}
   (b) {(1,2), (2,2), (3,2)}
   (c) {(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)}

6. (a) 500,500
   (b) 1998
   (c) 999
   (d) 500,500
   (e) 1,000,000

7. 

8. If all nodes don't have an edge with themselves

9.

$$
\begin{aligned}
&R \cup \Delta = R \cup \{(a,a) | a \in A\} \\
&\{(a,b) | a \neq b\} \cup \{(a,b) | a = b\} \\
&\{(a,b) | a, b \in \mathbb{Z}\} \\
&= \mathbb{Z} \times \mathbb{Z}
\end{aligned}
$$

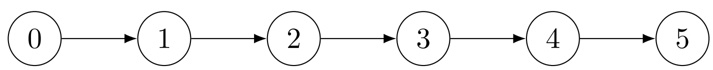
10. (a) {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)}
    (b) {(2,1), (2,3), (2,4), (3,1), (3,3), (3,4), (4,1), (4,3), (4,4)}
    (c) {(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)}
    (d) {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)}

11. (a) Equivalence Relation
    (b) Isn't Transitive
    (c) Isn't reflexive, symmetric, or transitive
    (d) Equivalence Relation

(e) Isn't reflexive or transitive

12. (a) $\{y|y \equiv 2(mod5)\} = \{\dots, -8, -3, 2, 7, 12, \dots\}$
    (b) $\{\dots, -7, -2, 3, 8, 13, \dots\} = \{y|y \equiv 3(mod5)\}$
    (c) $\{\dots, -9, -4, 1, 6, 11, \dots\} = \{y|y \equiv 6(mod5)\}$
    (d) $\{\dots, -8, -3, 2, 7, 12, \dots\} = \{y|y \equiv -3(mod5)\} = \{y|y \equiv 2(mod5)\}$

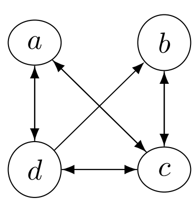
13. (a) Poset
    (b) Not a Poset
    (c) Poset
    (d) Not a Poset

14. 

15. (a) Simple Graph
    (b) Multigraph
    (c) Pseudograph

16. No, the sum all the degrees of all vertices is 15 * 5 = 75. This number must be twice the number of edge so 75 = 2n which no integer satisfies

17. $\frac{\sum_{n=1}^{v}(n-1)}{2} - c$

18. 

19. The two sets are not isomorphic because although they have the same number of vertices and edges, the point u_5 in the first set does not have a point with similar connectivity in the second set

20. *Proof.*

    G is the connected graph of the union $G_1 \cup G_2$
    By definition of a connected graph there must be an edge that connects $v_1 \in G_1$ to $v_2 \in G_1$
    $\therefore$ There must be a common vertex between G_1 and G_2 $\square$