Problem Set 9
Graph Theory

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Ex 3.3.2 Exhibit a maximum matching in the graph below, and use a result in this section to a give a short proof that it has no larger matching.

Proof. The matching below is of size 8. Thus, to prove that it's optimal, we need only to prove that there is no perfect matching of the graph. To accomplish this, a vertex set S is marked where $o(G - S) > |S|$, which by Tutte's condition, means there's no perfect matching. Thus, the matching below is optimal. □

Ex 3.3.4 Let G be a k-regular bipartite graph. Prove that G can be decomposed into r-factors if and only if r divides k.

Ex 3.3.6 Prove that a tree T has a perfect matching if and only if $o(T - v) = 1$ for every $v \in V(T)$.

Ex 3.3.8 Prove that if a graph G decomposes into 1-factors, then G has no cut-vertex. Draw a connected 3-regular simple graph that has a 1-factor and has a cut-vertex.

Ex 3.3.10 For every graph G, prove that $\beta(G) \leq 2\alpha'(G)$. For each $k \in \mathbb{N}$, construct a simple graph G with $\alpha'(G) = k$ and $\beta(G) = 2k$.