Principles of Mathematical Analysis
Chapter 2

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Exercise 2.1. Prove that the empty set is a subset of every set.

Proof. I will do this in a bit of an unconventional way, as the normal way is too easy. One can see from Remarks (2.11) that $A \cap B \subset A$ and $A \cap \varnothing = \varnothing$. If one lets $B = \varnothing$, then this gives us $A \cap \varnothing = \varnothing \subset A$. $\square$

Exercise 2.4. Is the set of all irrational real numbers countable?

Proof. No. Suppose the set of all irrational real numbers were countable. Then, by Thm (2.12), $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q}) = \mathbb{R}$ would be countable. However, this is a contradiction, as $\mathbb{R}$ is not countable by Collorary of Thm (2.43). $\square$