Exam Review

Algebra

powers and other operations, polynomials, exponent rules, the quadratic formula, and solving equations.

$$x^4 + x^2 = x^2(x^2+1)$$

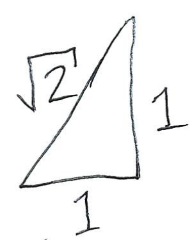
~~There is~~

$x^4 + 3x^3 - 2x^2 + 8x - 57$ } You can't solve this

Geometry

Lines, how to find the equation of a line, slope, ~~para~~ parallel, perpendicular, distances

Pythagorean Theorem.



Functions

What is a function? Graph of a function.

Function composition. Domains of functions.

Limits

Successively better approximations, computing limits by cancelling, limits to infinity, one-sided limits

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$ then the limit exists and is equal to the one-sided limits

Useful for proving that limit doesn't exist, for proving things about continuity, for computing limits involving piece-wise functions and absolute values.

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

Continuity

Intuitive definition (no holes and no jumps)

Definition using limits $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.

Most things are continuous (where defined):

$\sqrt{x}$, $|x|$, polynomials, $\frac{\text{polynomial}}{\text{polynomial}}$

Intermediate Value Theorem (you can't teleport)

- When you use IVT, you should mention IVT, continuity, and the values at the endpoints

Derivative

tangent lines, limit definition $\left(\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\right)$,
derivative can be interpreted as:

- slope of the tangent line
- a limit
- rate of change

differentiable is equivalent to being continuous with no sharp corners or infinite slope

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Differentiation Rules

$$\frac{d}{dx} c = 0 \qquad \frac{d}{dx}\left(f(x) + g(x)\right) = f'(x) + g'(x)$$

$$\frac{d}{dx}\left(c \cdot f(x)\right) = c \cdot \frac{d}{dx} f(x)$$

Power Rule: $\frac{d}{dx} x^n = n \cdot x^{n-1}$

Product Rule: $\frac{d}{dx}\left[f(x) \cdot g(x)\right] = f'(x) g(x) + f(x) g'(x)$

Quotient Rule: $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{\overset{\text{Lo dHi}}{g(x) f'(x)} - \overset{\text{Hi dLow}}{f(x) g'(x)}}{g(x)^2}$

Square the bottom and away we go!

Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

Geometric Interpretations of the First Derivative

Sign charts to find when a function is positive/negative

Critical points are where the derivative is zero or DNE

On sign chart for f', the critical points are the marked points

$f' > 0$, then f is increasing

$f' < 0$, then f is decreasing

We have a relative min/max if the sign changes at a critical point

Geometric Interpretations ~~of the Second Derivative~~

Use sign chart where the marked points are when ~~the~~ $f'' = 0$ or DNE.

$f'' < 0$, then f is concave down

$f'' > 0$, then f is concave up

Inflection points are when the second derivative changes sign. It is a point on the graph, for example $(-1, f(-1)) = (-1, 3)$.

Second derivative Test:

If c is a critical point and $f''(c) > 0$, then $x = c$ is a relative minimum.

Curve Sketching

Vertical Asymptotes

- find by factoring, canceling, and then determing the x values that make the denominator zero.
- This is a line, so you write it as $x = 4$, not as 4

Horizontal Asymptotes

- Find this by $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$
- So if $\lim_{x \to \infty} f(x) = -5$, then the horizontal asymptote is the line $y = -5$
- You can have a maximum of two horizontal asymptotes.

Minor points that you should keep in mind

- Limits should clearly justified. You write as

$$\lim_{x\to 4} \frac{x^2-16}{x-4} = \lim_{x\to 4} \frac{\cancel{(x-4)}(x+4)}{\cancel{x-4}} = \boxed{8}$$

- For limits regarding continuity, piece-wise functions, absolute values, and limits that DNE, you should use one-sided limits.

- For limits to infinity you use the shortcut

- For IVT, you need to specify IVT, continuous, the values at the endpoints, and your conclusion.

- Inflection points are points on the graph, so $(x, f(x))$

- Asymptotes are lines and should be written as $x=5$ or $y=-2$.

Midterm

- Not graded by me
- Justify clearly and ~~legibly~~ make sure it's readable
- When in doubt, cite relevant information.
- You can use a computer, but not other people
- Scan it as a PDF without using seperate pieces of paper. You can use your phone to scan as a PDF.