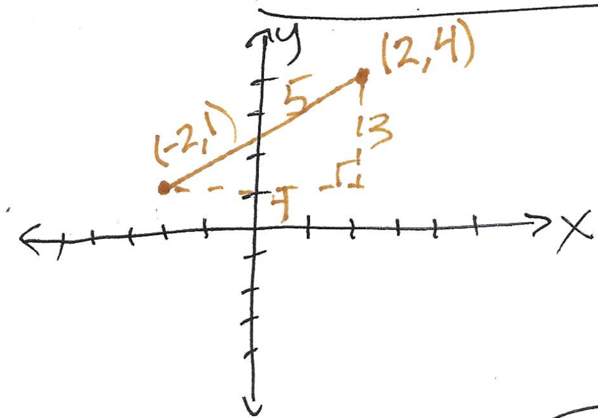
The Coordinate System x^2



What is the distance between $(-2,1)$ and $(2,4)$?

Use Pythagorean Theorem

$$a^2+b^2=c^2. \quad \sqrt{a^2+b^2}=c$$

The distance is $\sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5.$

The general formula is $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ (x_1, y_1) (x_2, y_2)

Lines

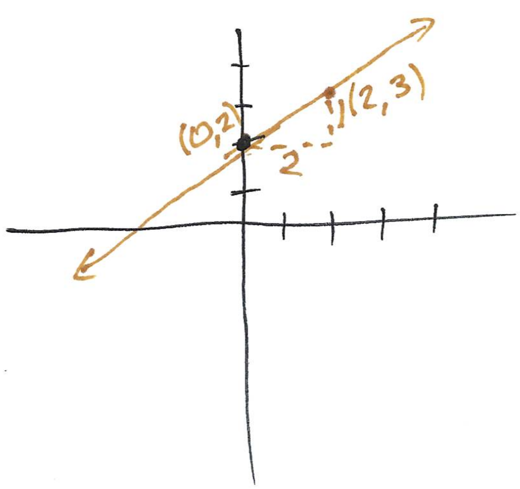
Slope is how "vertical" a line is.

Lines that look like ⤢ have positive.

Lines like ↕ hase large positive slope.

Calculating slope: Pick two points on a line (x_1, y_1) and (x_2, y_2) then the slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \begin{array}{l} \leftarrow \text{the change in } y \\ \leftarrow \text{the change in } x \end{array} \quad \frac{\text{rise}}{\text{run}}$$



The slope of this line is

$$m = \frac{3-2}{2-0} = \frac{1}{2}$$

You can any points you want.

Finding a function of a line

Any non-vertical can represented as a function.

$y = mx + b \qquad y(x) = \underset{\text{slope}}{m}x + b$

Ex 1) Give an equation of a line with slope $1/5$ that passes through $(4,2)$.

We already know that $m = 1/5$. $y = 1/5\,x + b$

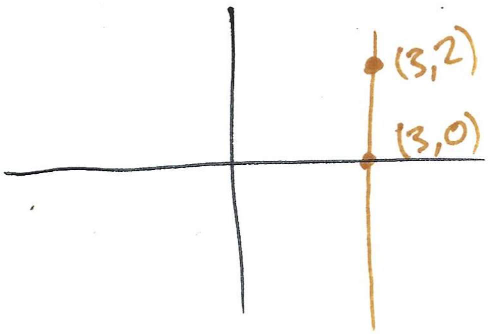
$2 = 1/5 \cdot 4 + b \Rightarrow b = 2 - 4/5 = 6/5$.

So the line is $y = 1/5\,x + 6/5$.

Ex 2) Give an equation of a line passing through $(-1,1)$ and $(4,-1)$.

The slope is $\frac{-1-1}{4-(-1)} = \frac{-2}{5}$. $y = \frac{-2}{5}x + b$

$1 = \frac{2}{5} + b \Rightarrow b = \frac{3}{5}$. So $\boxed{y = \frac{-2}{5}x + \frac{3}{5}}$

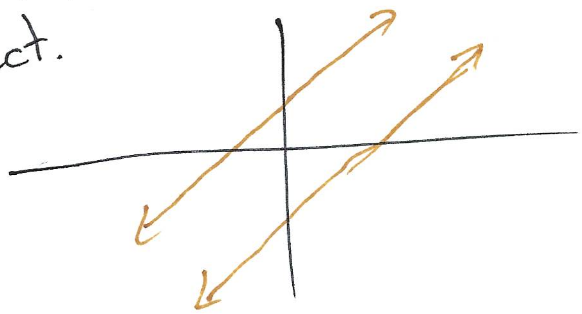


The slope is

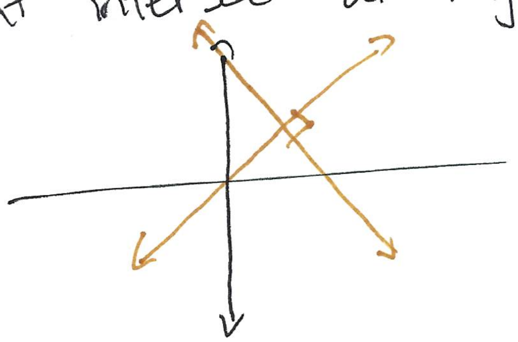
$$m = \frac{2-0}{3-3} = \cancel{\frac{2}{0}} \quad \text{"undefined"}$$

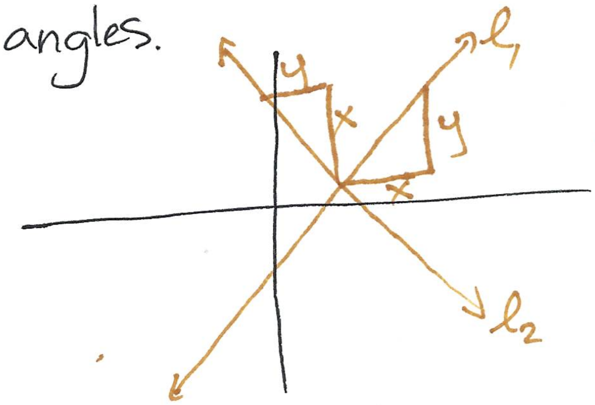
Vertical lines have "undefined" slope

Two lines are <u>parallel</u> if they have the same slope (or both lines have undefined slope). Parallel lines never intersect.



On the other hand, <u>perpendicular</u> lines are lines that intersect at right-angles.





$$m_1 = \frac{y}{x}$$

$$m_2 = \frac{y}{-x} = -\frac{1}{m_1}$$

If a line has slope m, any perpendicular line has slope $-\frac{1}{m}$.

Functions

- A function is an assignment or rule that takes a number (the input) and returns a number (the output).
- Sometimes the allowed inputs are specified. If not specified, the allow inputs are anything that makes sense.
- Possible inputs are called the domain and possible outputs are called the range.

Examples

1) $f(x) = 1 + \sqrt{x}$

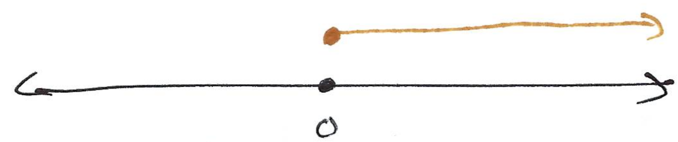
$f(0) = 1$

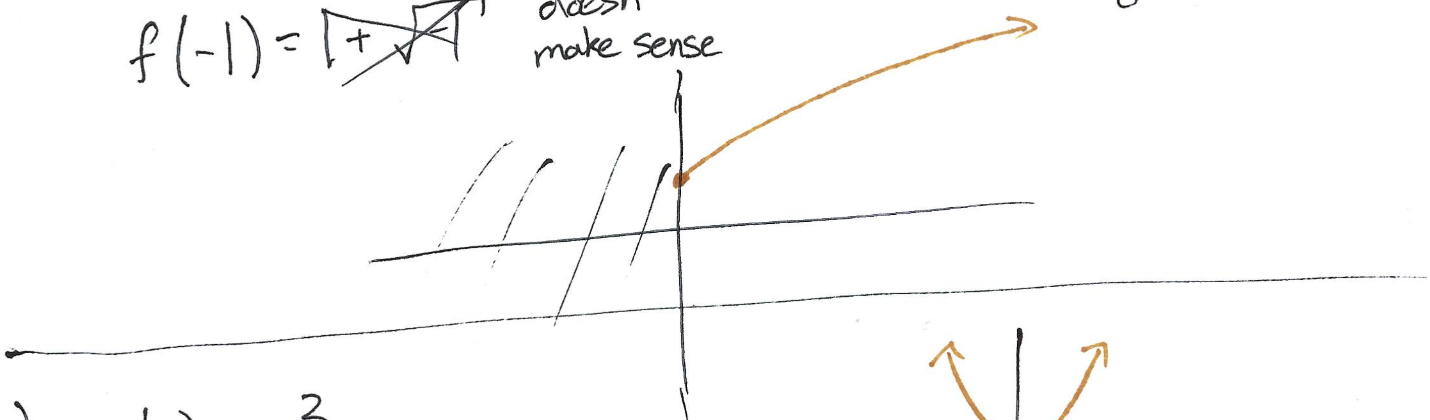
$f(4) = 1 + \sqrt{4} = 3$

$f(-1) = \cancel{1 + \sqrt{-1}}$ doesn't make sense

Domain of $f(x)$: $x \geq 0$

$[0, \infty)$

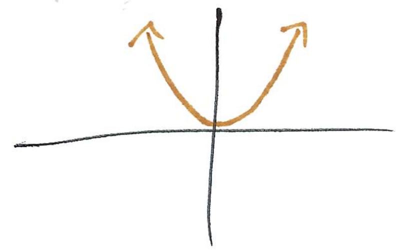




2) $g(x) = x^2$

Domain: All numbers

$(-\infty, \infty)$



3) $h(x) = \sqrt{3x+1}$

$h(1) = \sqrt{4} = 2$

$h(-1) = \cancel{\sqrt{-2}}$ doesn't make sense

Domain? $3x + 1 \geq 0$

$3x \geq -1$

$x \geq -1/3$

$[-1/3, \infty)$

4) $f(x) = \dfrac{1}{x^2-x-6}$

Domain? $x^2-x-6 \neq 0$

$$(x-3)(x+2) \neq 0$$

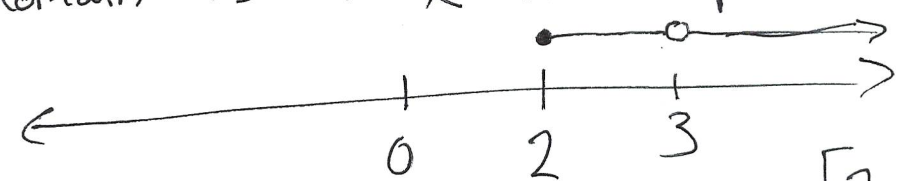
$$x \neq 3, -2$$

Using interval notation: $(-\infty,-2) \cup (-2,3) \cup (3,\infty)$

5) $f(x) = \sqrt{x^2-x-6}$ $\qquad g(x) = \dfrac{\sqrt{x-2}}{x^2-x-6}$

Domain of $g(x)$: $x-2 \geq 0$ and $x^2-x-6 \neq 0$ so $x \geq 2$, $(x-3)(x+2) \neq 0$

The domain is all $x \geq 2$ except for $x = 3, -2$



$[2,3) \cup (3,\infty)$

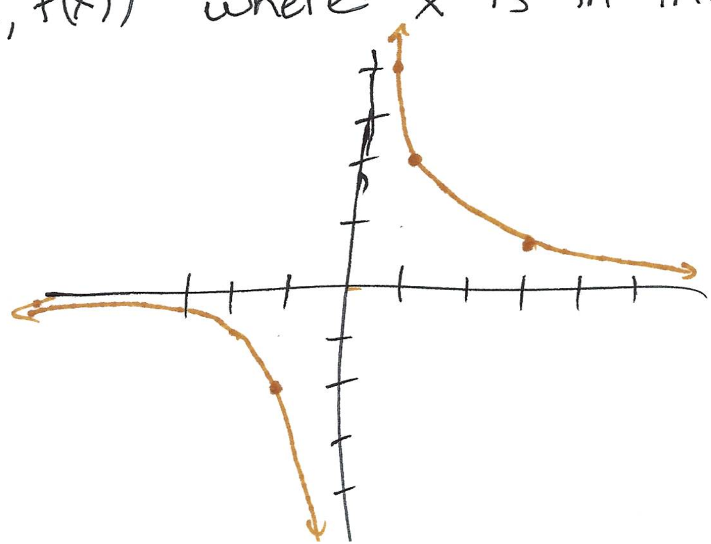
Graph of a function

A graph of a function $f(x)$ is all points on the plane $(x, f(x))$ where x is in the domain of $f(x)$

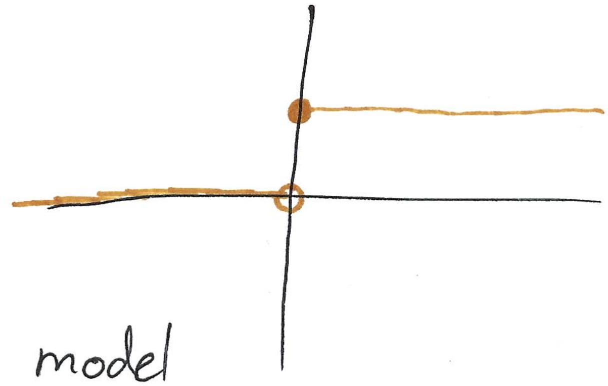
Examples

1) $f(x) = \dfrac{2}{x}$

x	$f(x)$
1	2
0	undefined
-1	-2
3	2/3
1/2	4

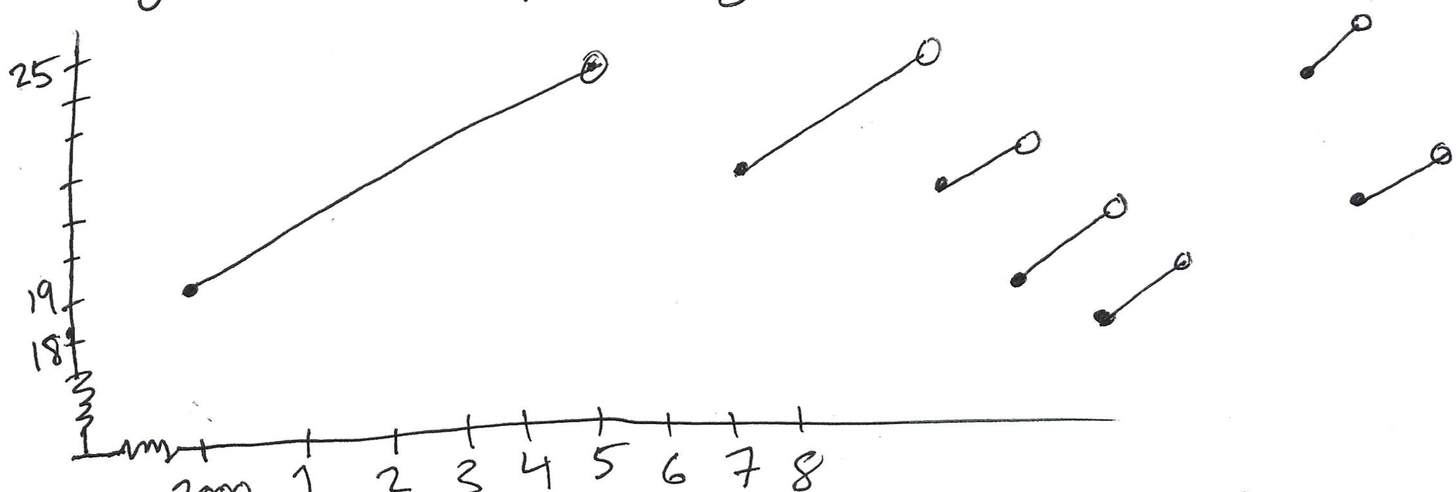


2) $f(x) = \begin{cases} 0, & x<0 \\ 1, & x \geq 0 \end{cases}$



This could mathematical model of a light switch.

3) Age of Dicaprio's girlfriend

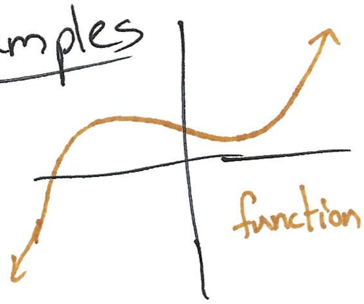


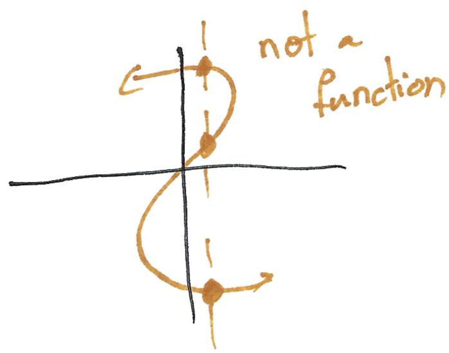
Conclusion: DiCaprio changes girlfriends often.
It seems there's a cap at 25.

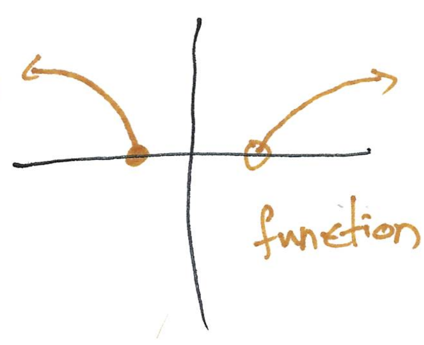
When is a curve a graph?

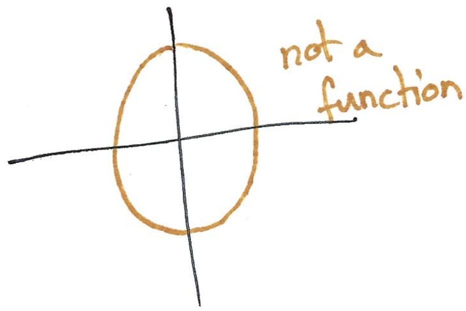
Answer: Vertical Line Test - a ~~graph~~ curve is a graph of a function if every vertical line intersects the curve at most once.

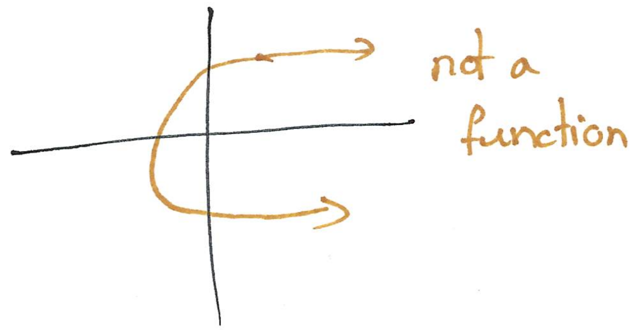
Examples











Combining Functions

We can take the sum, difference, product, and quotient of functions to obtain new functions

$$f(x) + g(x) = h(x) = x + \sqrt{x} + 1 \qquad f(x) = x + 1$$

$$\frac{f(x)}{g(x)} = h(x) = \frac{x+1}{\sqrt{x}} \qquad g(x) = \sqrt{x}$$

domain $x > 0$

For sums, products, and differences, the domain of the new function is all the numbers in the domain of the parts.

For quotients, you also need the bottom to not be zero

Function composition

You can compose functions, where the output of one function is the input of another function.

$f(x)$, $g(x)$ compose to get $f(g(x))$

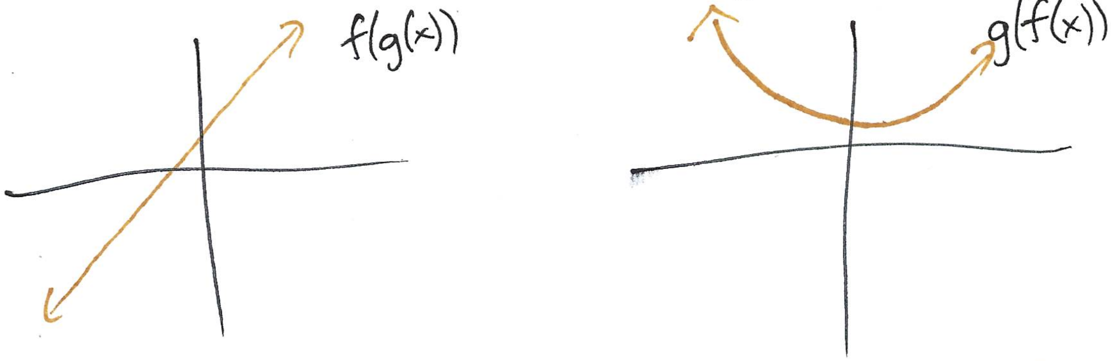
$g(f(x))$

<u>Examples</u> $f(x) = x^2+1$ $\qquad g(x) = \sqrt{x}$

$$f(g(x)) = f(\sqrt{x}) = \sqrt{x}^{\,2}+1 = x+1 \leftarrow \text{domain everything}$$

$$g(f(x)) = g(x^2+1) = \sqrt{x^2+1} \leftarrow \text{domain everything}$$

Note: The order definitely matters



What is the domain of the new function?

For this class, evaluate composition and then find the domain of that.

End of Review

- Homework 1 on WebAssign is due tonight
- Worksheet 1 is due on Collab tonight