STUDENT NAME: ______________________________

DUE DATE: July 27th, 2020 at midnight

Please sign the pledge:
On my honor as a student, I have neither given nor received aid on this exam.

Directions

Answer each question in the space provided. Show all of your work; your work must justify your answers. Clearly identify your final answers. Please write neatly and organize your work well. Well-organized work containing errors is more likely to earn partial credit than disorganized work containing errors.

You may use the internet and your notes to complete this exam. However, you may not collaborate or compare answers with others. You may use a computer/calculator to graph and verify your answers; however, you can not use this information to justify your answer. All computations are computable by hand. You must simplify results of function evaluations when it is possible to do so. For example, $4^{3/2}$ should be evaluated (replaced by 8).

Keep in mind that I will not be the one grading this midterm. Thus, you should take extra care to clearly justify your answer. You should write all of your answers on this document and refrain from using a separate piece of paper. You should scan this document as a PDF file and upload it to Collab under "Assignments." You can find many phone applications that can use your phone's camera to scan documents as a PDF.

For instructor use only

Page	Points	Score
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1. [7 pts] Not satisfied with his salary as a Calculus instructor, Dr. Strange decides to invest in the stock market. After careful consideration, he decides to invest in a company called "Soda Pop Corporation" (SPC). Let $f(t)$ represent the number of SPC shares that Dr. Strange owns at time t. Let $g(t)$ be the price per share of stock of SPC at time t. For both functions, t is measured in months after Dr. Strange first starts investing.

   a) What does the function $h(t) = f(t)g(t)$ represent?

   b) Assume that f and g are both differentiable at $t = 4$ and that $f(4) = 3000$, $f'(4) = 400$, $g(4) = 20$, and $g'(4) = 2$. Find $h'(4)$.

   c) What does $h'(4)$ represent?

2. [9 pts] Compute the following limits. Be sure to clearly justify your result.

a) $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{\sqrt{x - 3}}$

b) $\lim\limits_{x \to 0} \dfrac{x + |x|}{x}$

c) $\lim\limits_{x \to 0} \dfrac{x + 2}{g(x)}$, where $g(x) = \begin{cases} -x + 1 & x \leq 0 \\ \sqrt{x} + 1 & x > 0 \end{cases}$

3. [4 pts] Let f be the function $f(x) = \frac{1}{15}(x^3 - 23)^{5/2}$. Compute $f'(3)$. Your final answer should be in the form of an integer.

4. [2 pts] Differentiate the function $f(x) = x^4 + 3x^2 - x + \pi^2 + 1$.

5. [4 pts] Answer the following True/False Questions. Circle your response.

   (a) **True** **False** If f is continuous at 0, then f is differentiable at 0.

   (b) **True** **False** If the graph of f passes through $(3, 2)$ and $f'(3) = 5$, then $\lim_{x \to 3} f(x) = 2$.

   (c) **True** **False** If $f''(a) = 0$, then $(a, f(a))$ is an inflection point on the graph of f.

   (d) **True** **False** If $f'''(x) > 0$ for all numbers x, then f can not have any inflection points.

6. [2 pts] For three minutes the temperature of a feverish person has had a positive first derivative and a negative second derivative. What conclusion can you interpret from this information? *Circle one answer.*

   (a) The temperature rose in the last minute more than it rose in the minute before.

   (b) The temperature rose in the last minute, but less than it rose in the minute before.

   (c) The temperature fell in the last minute, but less than it fell in the minute before.

   (d) The temperature rose for the first two minutes, but fell in the last minute.

7. [4 pts] Which of the following is a valid argument as to why $g(x) = 1 + \sqrt{3-x}$ has no roots in the interval $[-1, 2]$? *Circle all that apply.*

   (a) $g(x)$ is continuous on the interval $[-1, 2]$ and its values at the endpoints, $g(-1) = 3$ and $g(2) = 2$, are both positive. Therefore, by the Intermediate Value Theorem, $g(x)$ cannot have a root in the interval $[-1, 2]$.

   (b) Over the interval $[-1, 2]$, g is differentiable and its derivative $g'(x) = -\frac{1}{2\sqrt{3-x}}$ is negative. This means that g must be decreasing on $[-1, 2]$. As $g(2) = 2$, we can conclude that $g(x) \geq 2$ over the interval $[-1, 2]$. Thus, $g(x)$ cannot have a root in the interval $[-1, 2]$.

   (c) When defined, the square-root function always outputs non-negative numbers, so we can conclude that $g(x) \geq 1$ for all x in the domain of g. This means that g doesn't have any roots anywhere. More specifically, g does not have any roots in $[-1, 2]$.

   (d) The limit $\lim_{x\to 0} 1 + \sqrt{3-x}$ is not zero. Therefore the derivative exists and is continuous on $[-1, 2]$. This means there cannot possibly be any roots in $[-1, 2]$.

8. [4 pts] Consider the following function:

$$f(x) = \begin{cases} 2x^2 - 1 & x \leq -1 \\ 4x - 11 & -1 < x < 3 \\ \frac{\sqrt{x+1}}{x-1} & x \geq 3 \end{cases}$$

   Is $f(x)$ continuous at $x = 3$? Justify your answer using one-sided limits and the definition of continuity.

9. [4 pts] Let $f(x) = -x^2 + ax + b$. Find constants a and b such that $f(x)$ has a relative maximum at $x = 1$ and that the value of $f(x)$ at this maximum is 8.

10. [5 pts] A Nuclear Regulatory Commission scientist projects that, for a certain reactor subjected to intermittent coolant-flow interruption, the temperature of the reactor core t hours after the initial interruption, for $0 \leq t \leq 3$, is given by

$$f(t) = 3t^2 - t^3 + t + 1,$$

where $f(t)$ is in thousands of degrees Celsius. The temperature at which a core meltdown occurs is 2.9 thousand degrees Celsius. Show that there exists a time t in the modeling interval $[0, 3]$ at which the temperature of the reactor core equals the meltdown temperature. Carefully justify your answer.

11. [6 pts] Suppose $f(x) = \frac{x^2-9}{2x^2-8x+6}$.

a) Using limits, find any/all horizontal asymptotes of $f(x)$.

b) Find any/all vertical asymptotes of the graph of $f(x)$.

12. [5 pts] Use a sign chart to find the relative maximums/minimums of the function $f(x) = (x^3 - 8)^4$.

13. [9 pts] Consider the function $f(x) = \frac{1}{20}x^5 - \frac{1}{12}x^4 - 2x + 1$.

(*a*) Find the interval(s) on which the graph of $f(x)$ is concave up and the interval(s) on which the graph is concave down.

(*b*) Record the points of inflection on the graph of $f(x)$, if any exist.

(*c*) Show that $f(x)$ has a relative maximum between $x = -2$ and $x = -1$.
(Hint: You have to use the Second Derivative Test.)