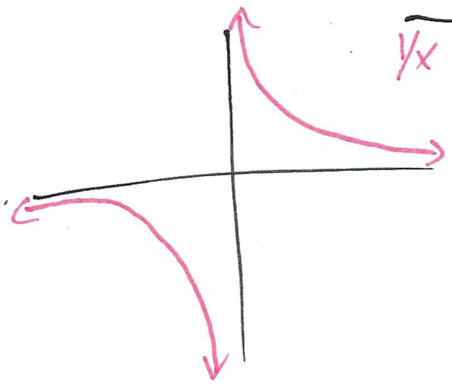
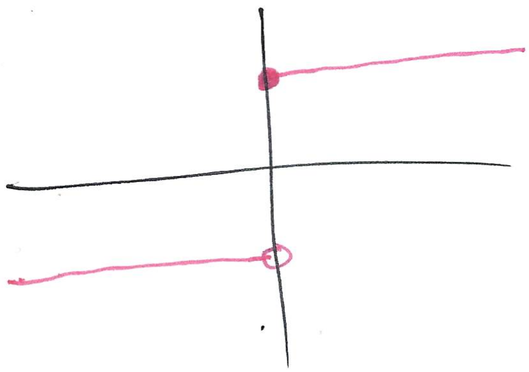
Course Notes

- No office hours tomorrow
- ~~Wk~~ Worksheets and homeworks are out
- When you submit on WebAssign, there's no "You're done" message
- There's a $-1/5$ penalty on multiple-choice.
- Sometimes you need a calculator for WebAssign but nothing else.
- Collab → Online Meetings → Cloud recordings → Click on video → Show password (it's under the video)
- Transfer grades from WebAssign to Collab at the end of the week.

One-sided limits



$\lim_{x\to 0} \frac{1}{x}$ DNE because it goes to infinity



$$f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

$\lim_{x\to 0} f(x)$ if you plug in positive numbers close to zero the limit looks like it should be 1

$$\lim_{x\to 0^+} f(x) = 1$$

only consider numbers > 0

$$\lim_{x\to 0^-} f(x) = -1$$

So in this case $\lim_{x\to 0} f(x)$ DNE.

To solve a limit, there's two approaches:

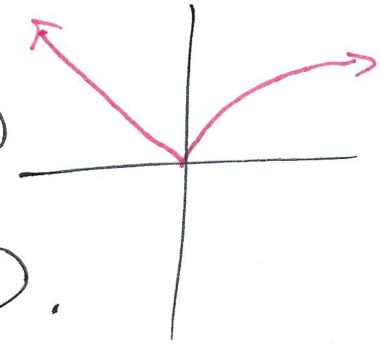
Does exist 1) You can try to use algebra ~~to make~~ so that you can plug the number in.

Doesn't exist 2) Use one-sided limits to show that left and right limits are different, or try to graph the function and see that it blows up.

Ex 1) $g(x) = \begin{cases} -x & x \le 0 \\ \sqrt{x} & x > 0 \end{cases}$. What is $\lim_{x\to 0} g(x)$?

$$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} -x = -0 = 0$$

$$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} \sqrt{x} = \sqrt{0} = 0$$



In this case, $\lim_{x\to 0} g(x) = 0$.

Ex 2) $h(x) = \dfrac{|x+1|}{x+1}$ $\quad \lim_{x\to -1} h(x)$

$$|x| = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$$

This is a positive number

$$\lim_{x\to -1^-} h(x) = \lim_{x\to -1^-} \frac{|x+1|}{x+1}$$

$$= \lim_{x\to -1^-} \frac{-\cancel{(x+1)}}{\cancel{x+1}}$$

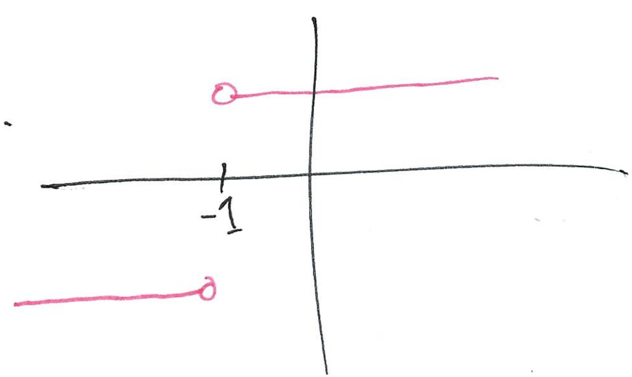
$$= \lim_{x\to -1^-} -1 = -1$$

$$|x+1| = \begin{cases} x+1 & x+1 \ge 0 \\ -(x+1) & x+1 < 0 \end{cases}$$

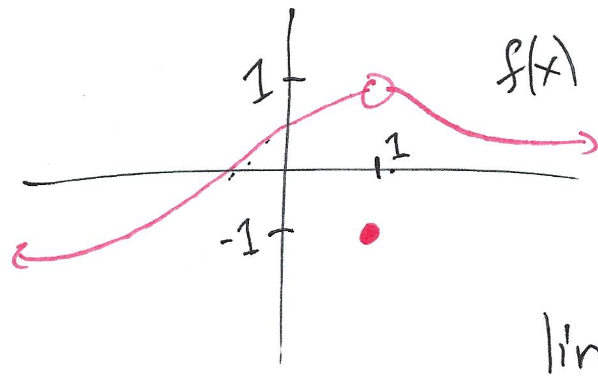
$$= \begin{cases} x+1 & x \ge -1 \\ -x-1 & x < -1 \end{cases}$$

$$\lim_{x\to -1^+} h(x) = \lim_{x\to -1^+} \frac{x+1}{x+1} = \lim_{x\to -1^+} 1 = 1$$

So $\lim_{x\to -1} h(x)$ DNE.



Ex 3



$f(1) = -1$

$$\lim_{x \to 1} f(x) = 1$$

$$\lim_{x \to 1^-} f(x) = 1$$

$$\lim_{x \to 1^+} f(x) = 1$$

~~Ex 4 | What value~~

~~Continuity~~

Continuity

A function f is continuous at a if

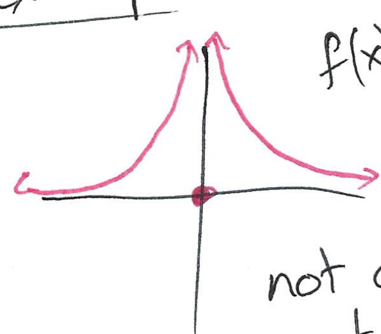
$$\lim_{x \to a} f(x) = f(a).$$

This agrees with our ~~intuition~~ intuitive notion of continuity.

Note: The a must be in the domain f.

Non-examples

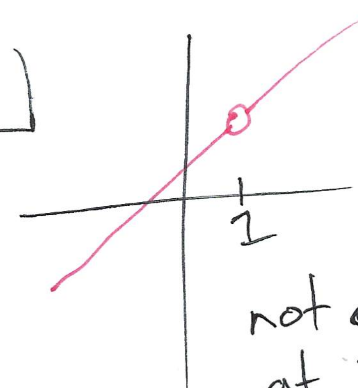
1\]



$$f(x) = \begin{cases} 1/x^2 & x \neq 0 \\ 0 & x = 0 \end{cases}$$

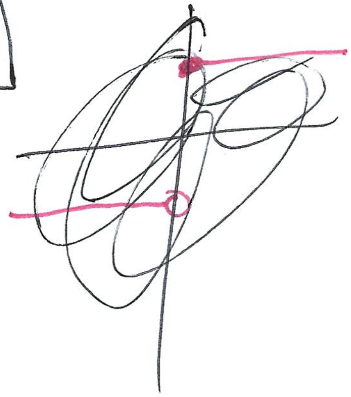
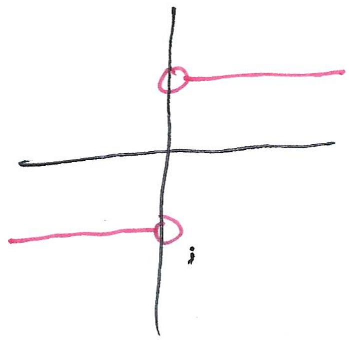
not continuous at 0

2\]



not continuous at 1

but, it's possible to extend the function to be continuous

3)  

Not continuous at zero and we can't extend the function to make it continuous

If you can redefine the function at single point to make it continuous, then that discontinuity is called a hole

If not, it's called a jump.

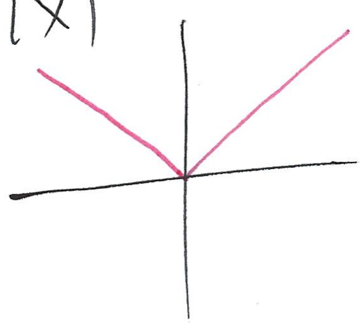
By definition, all discontinuities are either holes or jumps.

Actual Examples

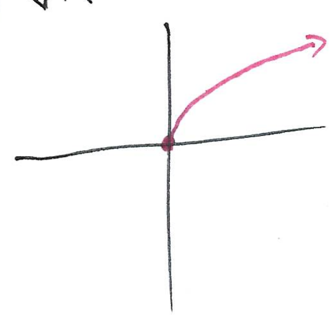
1) Any polynomial is continuous at all points
2) All rational functions $\left(\frac{\text{polynomial}}{\text{polynomial}}\right)$ are continuous everywhere that they're defined

$$\frac{x^2+1}{x^2-x-6} = \frac{x^2+1}{(x-3)(x+2)}$$ is continuous everywhere except $x=3,-2$

3) $|x|$



4) $\sqrt{x}$



continuous where defined

If $f(x)$ and $g(x)$ are continuous, then

$f(x)+g(x)$ $\quad f(x)^n$

$f(x)\cdot g(x)$ $\quad \sqrt{g(x)}$ $\quad$ all continuous

$f(g(x))$ $\quad \frac{f(x)}{g(x)}$ $\quad$ ~~when~~ defined where

$g(f(x))$

So for example, $\sqrt{x^2+1} + \frac{1}{x} + x^{1000}$

is continuous everywhere except 0

What value of k is

$$f(x) = \begin{cases} 5 & x \geq 1 \\ 2x+k & x < 1 \end{cases}$$

continuous everywhere?

Back to our definition,

$$\lim_{x \to 1} f(x) = f(1) = 5$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 2x+k = k+2$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 5 = 5$$

We want $k+2=5$, so $\boxed{k=3.}$

Intermediate Value Theorem

Ex) $p(t)$ is your position walking from your house to your school

So there's an ice cream store between your ~~school~~ house and school

If you're at home at $t=0$, and you're at school at $t=15$, then at some point you had be in front of the ice cream store

Actual definition: If $f(x)$ is continuous on $[a,b]$ and $f(a) \le M \le f(b)$, then there's some c where $f(c)=M$.

Ex 2)

Day of the week	Temperature at noon
Monday	80°F
Tuesday	70°F
Thursday	85°F

At some ~~point~~ time between Monday and Tuesday it was 75°F. At another time between Tuesday and Thursday, it was also 75°F.

Special case: If f is continuous on $[a,b]$, and $f(a)$ and $f(b)$ have different signs, then $f(c)=0$ for some $a \le c \le b$.

<u>Example</u> Is there point x where $x^3 - 8x + 1 = 0$?

x^2+1

$f(x) = x^3 - 8x + 1$

$f(0) = 1$

$f(2) = 8 - 16 + 1 = -7$

We can conclude by IVT that $f(x)=0$ for some x between 0 and 2.

$f(1) = 1 - 8 + 1 = -8$

$f(1/2)$

This is how your calculator finds where a function is zero.

Breakout rooms for Worksheet 2.

Hint: 1(b) and 1(c) do exist, but you'll use algebra.