5.1

MEDIUM AND SHORT LINE APPROXIMATIONS

In this section, we present short and medium-length transmission-line approximations as a means of introducing $ABCD$ parameters. Some readers may prefer to start in Section 5.2, which presents the exact transmission-line equations.

It is convenient to represent a transmission line by the two-port network shown in Figure 5.1, where V_S and I_S are the sending-end voltage and current, and V_R and I_R are the receiving-end voltage and current.

The relation between the sending-end and receiving-end quantities can be written as

$$V_S = AV_R + BI_R \quad \text{volts} \tag{5.1.1}$$

$$I_S = CV_R + DI_R \quad \text{A} \tag{5.1.2}$$

or, in matrix format,

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix} \tag{5.1.3}$$

where A, B, C, and D are parameters that depend on the transmission-line constants R, L, C, and G. The $ABCD$ parameters are, in general, complex numbers. A and D are dimensionless. B has units of ohms, and C has units of siemens. Network theory texts [5] show that $ABCD$ parameters apply to linear, passive, bilateral two-port networks, with the following general relation:

$$AD - BC = 1 \tag{5.1.4}$$

The circuit in Figure 5.2 represents a short transmission line, usually applied to overhead 60-Hz lines less than 80 km long. Only the series resistance and reactance are included. The shunt admittance is neglected. The circuit applies to either single-phase or completely transposed three-phase lines operating under balanced conditions. For a completely transposed

FIGURE 5.1
Representation of two-port network

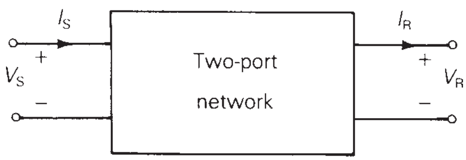
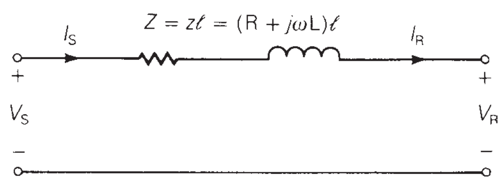


FIGURE 5.2
Short transmission line



three-phase line, Z is the series impedance, V_S and V_R are positive-sequence line-to-neutral voltages, and I_S and I_R are positive-sequence line currents.

To avoid confusion between total series impedance and series impedance per unit length, we use the following notation:

$$z = \mathrm{R} + j\omega\mathrm{L} \quad \Omega/\mathrm{m}, \text{ series impedance per unit length}$$

$$y = \mathrm{G} + j\omega\mathrm{C} \quad \mathrm{S/m}, \text{ shunt admittance per unit length}$$

$$Z = zl \quad \Omega, \text{ total series impedance}$$

$$Y = yl \quad \mathrm{S}, \text{ total shunt admittance}$$

$$l = \text{line length} \quad \mathrm{m}$$

Recall that shunt conductance G is usually neglected for overhead transmission.

The *ABCD* parameters for the short line in Figure 5.2 are easily obtained by writing a KVL and KCL equation as

$$V_S = V_R + ZI_R \tag{5.1.5}$$

$$I_S = I_R \tag{5.1.6}$$

or, in matrix format,

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \left[\begin{array}{c|c} 1 & Z \\ \hline 0 & 1 \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix} \tag{5.1.7}$$

Comparing (5.1.7) and (5.1.3), the *ABCD* parameters for a short line are

$$A = D = 1 \quad \text{per unit} \tag{5.1.8}$$

$$B = Z \quad \Omega \tag{5.1.9}$$

$$C = 0 \quad \mathrm{S} \tag{5.1.10}$$

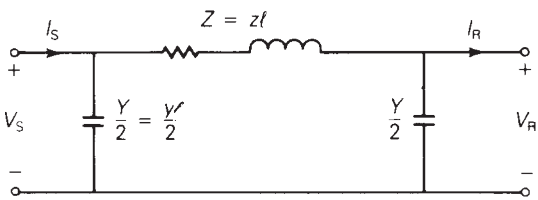
For medium-length lines, typically ranging from 80 to 250 km at 60 Hz, it is common to lump the total shunt capacitance and locate half at each end of the line. Such a circuit, called a *nominal π circuit*, is shown in Figure 5.3.

To obtain the *ABCD* parameters of the nominal π circuit, note first that the current in the series branch in Figure 5.3 equals $I_R + \frac{V_R Y}{2}$. Then, writing a KVL equation,

$$V_S = V_R + Z\left(I_R + \frac{V_R Y}{2}\right)$$

$$= \left(1 + \frac{YZ}{2}\right) V_R + ZI_R \tag{5.1.11}$$

FIGURE 5.3 Medium-length transmission line—nominal π circuit



Also, writing a KCL equation at the sending end,

$$I_S = I_R + \frac{V_R Y}{2} + \frac{V_S Y}{2} \tag{5.1.12}$$

Using (5.1.11) in (5.1.12),

$$\begin{aligned} I_S &= I_R + \frac{V_R Y}{2} + \left[\left(1 + \frac{YZ}{2}\right) V_R + Z I_R\right] \frac{Y}{2} \\ &= Y\left(1 + \frac{YZ}{4}\right) V_R + \left(1 + \frac{YZ}{2}\right) I_R \end{aligned} \tag{5.1.13}$$

Writing (5.1.11) and (5.1.13) in matrix format,

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \left[\begin{array}{c|c} \left(1 + \frac{YZ}{2}\right) & Z \\ \hline Y\left(1 + \frac{YZ}{4}\right) & \left(1 + \frac{YZ}{2}\right) \end{array}\right] \begin{bmatrix} V_R \\ I_R \end{bmatrix} \tag{5.1.14}$$

Thus, comparing (5.1.14) and (5.1.3)

$$A = D = 1 + \frac{YZ}{2} \quad \text{per unit} \tag{5.1.15}$$

$$B = Z \quad \Omega \tag{5.1.16}$$

$$C = Y\left(1 + \frac{YZ}{4}\right) \quad \text{S} \tag{5.1.17}$$

Note that for both the short and medium-length lines, the relation $AD - BC = 1$ is verified. Note also that since the line is the same when viewed from either end, $A = D$.

Figure 5.4 gives the *ABCD* parameters for some common networks, including a series impedance network that approximates a short line and a π circuit that approximates a medium-length line. A medium-length line could also be approximated by the T circuit shown in Figure 5.4, lumping half of the series impedance at each end of the line. Also given are the *ABCD* parameters for networks in series, which are conveniently obtained by multiplying the *ABCD* matrices of the individual networks.

ABCD parameters can be used to describe the variation of line voltage with line loading. *Voltage regulation* is the change in voltage at the receiving end of the line when the load varies from no-load to a specified full load at a specified power factor, while the sending-end voltage is held constant. Expressed in percent of full-load voltage,

$$\text{percent VR} = \frac{|V_{RNL}| - |V_{RFL}|}{|V_{RFL}|} \times 100 \tag{5.1.18}$$

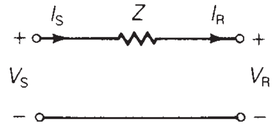
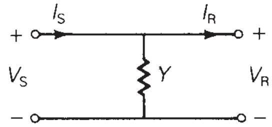
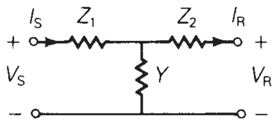
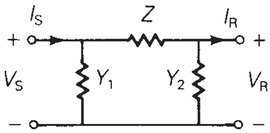
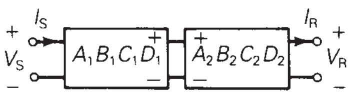
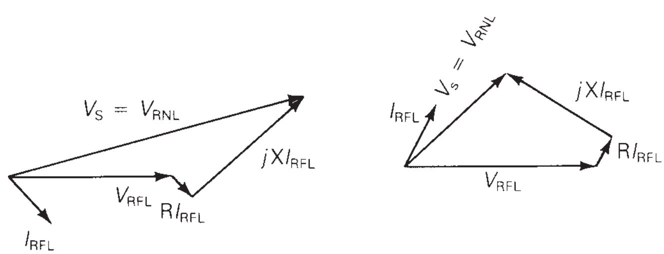
Circuit	*ABCD* Matrix
 Series impedance	$\left[\begin{array}{c\|c} 1 & Z \\ \hline 0 & 1 \end{array}\right]$
 Shunt admittance	$\left[\begin{array}{c\|c} 1 & 0 \\ \hline Y & 1 \end{array}\right]$
 T circuit	$\left[\begin{array}{c\|c} (1+YZ_1) & (Z_1+Z_2+YZ_1Z_2) \\ \hline Y & (1+YZ_2) \end{array}\right]$
 Π circuit	$\left[\begin{array}{c\|c} (1+Y_2Z) & Z \\ \hline (Y_1+Y_2+Y_1Y_2Z) & (1+Y_1Z) \end{array}\right]$
 Series networks	$\left[\begin{array}{c\|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right]\left[\begin{array}{c\|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array}\right]=\left[\begin{array}{c\|c} (A_1A_2+B_1C_2) & (A_1B_2+B_1D_2) \\ \hline (C_1A_2+D_1C_2) & (C_1B_2+D_1D_2) \end{array}\right]$

FIGURE 5.4 *ABCD* parameters of common networks

FIGURE 5.5

Phasor diagrams for a short transmission line



(a) Lagging p.f. load (b) Leading p.f. load where percent VR is the percent voltage regulation, $|V_{RNL}|$ is the magnitude of the no-load receiving-end voltage, and $|V_{RFL}|$ is the magnitude of the full-load receiving-end voltage.

The effect of load power factor on voltage regulation is illustrated by the phasor diagrams in Figure 5.5 for short lines. The phasor diagrams are graphical representations of (5.1.5) for lagging and leading power factor loads. Note that, from (5.1.5) at no-load, $I_{RNL} = 0$ and $V_S = V_{RNL}$ for a short line. As shown, the higher (worse) voltage regulation occurs for the lagging p.f. load, where V_{RNL} exceeds V_{RFL} by the larger amount. A smaller or even negative voltage regulation occurs for the leading p.f. load. In general, the no-load voltage is, from (5.1.1), with $I_{RNL} = 0$,

$$V_{RNL} = \frac{V_S}{A} \tag{5.1.19}$$

which can be used in (5.1.18) to determine voltage regulation.

In practice, transmission-line voltages decrease when heavily loaded and increase when lightly loaded. When voltages on EHV lines are maintained within $\pm 5\%$ of rated voltage, corresponding to about 10% voltage regulation, unusual operating problems are not encountered. Ten percent voltage regulation for lower voltage lines including transformer-voltage drops is also considered good operating practice.

In addition to voltage regulation, line loadability is an important issue. Three major line-loading limits are: (1) the thermal limit, (2) the voltage-drop limit, and (3) the steady-state stability limit.

The maximum temperature of a conductor determines its thermal limit. Conductor temperature affects the conductor sag between towers and the loss of conductor tensile strength due to annealing. If the temperature is too high, prescribed conductor-to-ground clearances may not be met, or the elastic limit of the conductor may be exceeded such that it cannot shrink to its original length when cooled. Conductor temperature depends on the current magnitude and its time duration, as well as on ambient temperature, wind velocity, and conductor surface conditions. Appendix Tables A.3 and A.4 give approximate current-carrying capacities of copper and ACSR conductors. The loadability of short transmission lines (less than 80 km in length for 60-Hz overhead lines) is usually determined by the conductor thermal limit or by ratings of line terminal equipment such as circuit breakers.

For longer line lengths (up to 300 km), line loadability is often determined by the voltage-drop limit. Although more severe voltage drops may be tolerated in some cases, a heavily loaded line with $V_R/V_S \geqslant 0.95$ is usually considered safe operating practice. For line lengths over 300 km, steady-state stability becomes a limiting factor. Stability, discussed in Section 5.4, refers to the ability of synchronous machines on either end of a line to remain in synchronism.

EXAMPLE 5.1 *ABCD* parameters and the nominal π circuit: medium-length line

A three-phase, 60-Hz, completely transposed 345-kV, 200-km line has two 795,000-cmil (403-mm^2) 26/2 ACSR conductors per bundle and the following positive-sequence line constants:

$$z = 0.032 + j0.35 \quad \Omega/\text{km}$$
$$y = j4.2 \times 10^{-6} \quad \text{S/km}$$

Full load at the receiving end of the line is 700 MW at 0.99 p.f. leading and at 95% of rated voltage. Assuming a medium-length line, determine the following:

a. *ABCD* parameters of the nominal π circuit

b. Sending-end voltage V_S, current I_S, and real power P_S

c. Percent voltage regulation

d. Thermal limit, based on the approximate current-carrying capacity listed in Table A.4

e. Transmission-line efficiency at full load

SOLUTION

a. The total series impedance and shunt admittance values are

$$Z = zl = (0.032 + j0.35)(200) = 6.4 + j70 = 70.29\underline{/84.78^\circ} \quad \Omega$$
$$Y = yl = (j4.2 \times 10^{-6})(200) = 8.4 \times 10^{-4}\underline{/90^\circ} \quad \text{S}$$

From (5.1.15)–(5.1.17),

$$A = D = 1 + (8.4 \times 10^{-4}\underline{/90^\circ})(70.29\underline{/84.78^\circ})\left(\tfrac{1}{2}\right)$$
$$= 1 + 0.02952\underline{/174.78^\circ}$$
$$= 0.9706 + j0.00269 = 0.9706\underline{/0.159^\circ} \quad \text{per unit}$$

$$B = Z = 70.29\underline{/84.78^\circ} \quad \Omega$$

$$C = (8.4 \times 10^{-4}\underline{/90^\circ})(1 + 0.01476\underline{/174.78^\circ})$$
$$= (8.4 \times 10^{-4}\underline{/90^\circ})(0.9853 + j0.00134)$$
$$= 8.277 \times 10^{-4}\underline{/90.08^\circ} \quad \text{S}$$

b. The receiving-end voltage and current quantities are

$$\text{V}_R = (0.95)(345) = 327.8 \quad \text{kV}_{LL}$$

$$V_R = \frac{327.8}{\sqrt{3}}\underline{/0^\circ} = 189.2\underline{/0^\circ} \quad \text{kV}_{LN}$$

$$I_R = \frac{700\underline{/\cos^{-1} 0.99}}{(\sqrt{3})(0.95 \times 345)(0.99)} = 1.246\underline{/8.11^\circ} \quad \text{kA}$$

From (5.1.1) and (5.1.2), the sending-end quantities are

$$V_S = (0.9706\underline{/0.159^\circ})(189.2\underline{/0^\circ}) + (70.29\underline{/84.78^\circ})(1.246\underline{/8.11^\circ})$$
$$= 183.6\underline{/0.159^\circ} + 87.55\underline{/92.89^\circ}$$
$$= 179.2 + j87.95 = 199.6\underline{/26.14^\circ} \quad \text{kV}_{LN}$$

$$V_S = 199.6\sqrt{3} = 345.8 \text{ kV}_{LL} \approx 1.00 \quad \text{per unit}$$

$$\begin{aligned} I_S &= (8.277 \times 10^{-4}\underline{/90.08^\circ})(189.2\underline{/0^\circ}) + (0.9706\underline{/0.159^\circ})(1.246\underline{/8.11^\circ}) \\ &= 0.1566\underline{/90.08^\circ} + 1.209\underline{/8.27^\circ} \\ &= 1.196 + j0.331 = 1.241\underline{/15.5^\circ} \quad \text{kA} \end{aligned}$$

and the real power delivered to the sending end is

$$\begin{aligned} P_S &= (\sqrt{3})(345.8)(1.241)\cos(26.14^\circ - 15.5^\circ) \\ &= 730.5 \quad \text{MW} \end{aligned}$$

c. From (5.1.19), the no-load receiving-end voltage is

$$V_{RNL} = \frac{V_S}{A} = \frac{345.8}{0.9706} = 356.3 \quad \text{kV}_{LL}$$

and, from (5.1.18),

$$\text{percent VR} = \frac{356.3 - 327.8}{327.8} \times 100 = 8.7\%$$

d. From Table A.4, the approximate current-carrying capacity of two 795,000-cmil (403-mm^2) 26/2 ACSR conductors is $2 \times 0.9 = 1.8$ kA.

e. The full-load line losses are $P_S - P_R = 730.5 - 700 = 30.5$ MW and the full-load transmission efficiency is

$$\text{percent EFF} = \frac{P_R}{P_S} \times 100 = \frac{700}{730.5} \times 100 = 95.8\%$$

Since $V_S = 1.00$ per unit, the full-load receiving-end voltage of 0.95 per unit corresponds to $V_R/V_S = 0.95$, considered in practice to be about the lowest operating voltage possible without encountering operating problems. Thus, for this 345-kV 200-km uncompensated line, voltage drop limits the full-load current to 1.246 kA at 0.99 p.f. leading, well below the thermal limit of 1.8 kA. ■

5.2 TRANSMISSION-LINE DIFFERENTIAL EQUATIONS

The line constants R, L, and C are derived in Chapter 4 as per-length values having units of Ω/m, H/m, and F/m. They are not lumped, but rather are uniformly distributed along the length of the line. In order to account for the distributed nature of transmission-line constants, consider the circuit shown in Figure 5.6, which represents a line section of length Δx. $V(x)$ and $I(x)$ denote the voltage and current at position x, which is measured in meters from the right, or receiving end of the line. Similarly, $V(x + \Delta x)$ and $I(x + \Delta x)$ denote the voltage and current at position $(x + \Delta x)$. The circuit constants are