Programming Assignment 2
Digital Library of Arendelle

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1 Introduction

The Arendelle's Digital Library is a project developed by the Plan of Democratization of the Education Access (PDEA). This project aims to guarantee all the Arendelle people the right of information access from the Internet. The problem proposed by Elsa, Arendelle's Queen, was to develop an efficient sort algorithm that could be able to sort the huge Arendelle's Digital Library. Since it is a massive amount of books, we need to find an effective quicksort variation that could be able to sort the book fast and, in this paper, we will try to compare many quick sort variations to give the final report to Elsa about which is the most effective one.

2 Implementation

All the quicksort implementations were developed in C++ and compiled by G++ of GNU Compiler Collection

2.1 Data Structure

Most of the QuickSorts were designed using a recursive implementation, except for the last one that we used a **Stack** to simulate the recursive implementation. A recursive implementation is when some function calls itself and it always needs to have a stop condition. After we achieve the stop condition, the last function starts to return to its predecessor and so on. Just like pushing elements to a Stack and then popping it until it gets empty.

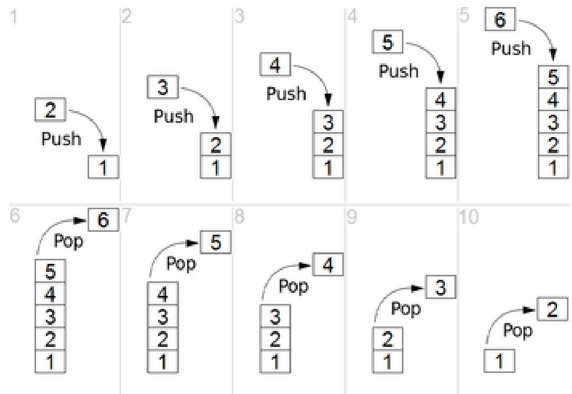
So this is what is happening inside a stack:

In this example of an implementation of a Stack of length equals to 6, we first push all the 6 elements and then, when the stop condition is reached, we start to pop each element out, until the end of the stack.

Ok, but which are these elements that we are pushing and popping out of the stack? These elements, in our case, were the left and right edges of our vector. Considering that our vector represents each of our books.

Since we are using quicksort implementations, we are always dividing our vector in half and, these elements that we are pushing and popping, are the left and right corners of the divided vector.

We will explain more about QuickSort implementations latter on this paper.



Lifo_stack.png

2.2 Classes

Keeping in mind the idea of letting our program more modular, we tried to separate the whole program in some classes. We divided it into two main classes besides our `main.cpp` These two classes are:

2.2.1 Array

In our array class, we tried to develop all of our QuickSort implementations and all our array handling. We initialize this class passing only the size of the array and it auto creates an ascending ordered array and set `moves = 0` and `comparisons = 0`. If we are trying to use and descending ordered or a shuffled array, we only call `desc_array()` or `shuffle_array()` respectively.

Then, when we call any quicksort but non recursive quicksort, we call the `quicksort()` function passing the following parameters: `(unsigned int* A, int esq, int dir, int type, int k)`, where `A` is our Array of elements, `esq` and `dir` are the left and right edges of our array that we are looking for, `type` is the type of our pivot that we are using and k is the percentage of the vector that we will use in case of doing the Insertion QuickSort (look that if that is not the case, we set k=0 and we will never satisfy the if that call the insertion sort).

In the case that we are looking for the non recursive quicksort, we call the `quicksort_non_recursive` that acts pretty similar as the quicksort function that we saw above, **except** that we only need to pass `type` and `k`, since, in this case, we will only have one case (pivot = the middle element and k = 0 - since we are not doing a non recursive insertion sort). And, of course, rather than calling itself, this function has a while that only stops when the stack that it created is empty

The other functions do pretty much what their name is saying:

- `which_pivot()` returns the pivot of the quicksort case that we are trying to do.
- `print_array()` print the array (we use it when -p is passed)
- `get_moves()` and `get_comparisons()` returns the number of moves and comparisons that was calculated during the quicksorts.
- `get_median()` gets the median of the array (we use this when we are getting the median of the sorted time array). See that, when we call this function, we have executed 20 examples and insert the time at the `median_time` array. After this, we sort the `median_time` array and then, we call this function to get the median of this array.

And, of course, all the other functions: `get_qc()`, `get_qm3()`, `get_qpe()`, `get_qi()`, `get_qnr()` calls the quicksort functions with the right parameters.

2.2.2 Stack

The class Stack is much more simple than Array class. It has the minimal functions necessary to create a stack and allow us to use it. Popping and Pushing elements of it.

Our constructor here sets the top of the Stack to 0 and creates a stack of the size that was passed (in our case, the size of elements in the array).

Then we have the `push()` function that pushes the edges to the Stack (the left and the right edges). And sets `top = top + 1`

And the `pop()` function that return theses edges. And sets `top = top - 1`

And last but not least, we have the `is_empty()` function that returns a boolean of whether the Stack is empty or not.

2.3 Quick Sorts

2.3.1 Classic Quick Sort

As we can see from the graphs above, this was the quicksort with the best efficiency and it definitely is the one that Elza is looking for. It uses the central element of each array as its pivot.

2.3.2 Median of 3 Quick Sort

This quicksort uses the median between the first, the middle and the last elements as its pivot. But as we can see from the results, we don't have enough efficiency gain and it is a little slower than the classic one.

2.3.3 First element Quick Sort

Wow.. this is one of the most painful quicksorts in terms of computational cost. Especially when we look at the OrdC or the OrdD times, we can see here that it can take over to 5 minutes ordering one ascending vector of 500,000 elements. Here, of course, we use the first element of an array as the pivot.

2.3.4 Insertion Quick Sort

Here we have sort of a hybrid implementation. Since we use quick sort's implementation until the vector has `k%` elements and then, we call an insertion sort algorithm, we have a little bit of lower efficiency when compared to the classic one. Here we use the middle element as the pivot.

One interesting fact here is that, when we get bigger `k`'s, we have a considerative lower of efficiency. Which take us to the conclusion that the quicksort algorithm is much more efficient than the insertion sort one.

2.3.5 Non-recursive Quick Sort

Here we have an implementation that tries to simulate the Classic Quick Sort and that's because we have such a good result. Therefore, since we are implementing a Stack by hand, our efficiency gets a little bit lower than the Classic one.

3 How to use

To execute the program, you need to compile it first.

3.1 Compile

Since we have a Makefile, to compile the file, you only need to open its main repository and execute the following command:

```
make
```

3.2 Execution

To execute you have two options. The first one is to execute all the tests. In this case, you only need to execute:

```
./main
```

and all the tests will be executed.

If you prefer to test each case separelaty, you can run:

```
./main <quickSort> <type> <size>
```

For example:

```
./main QNR OrdC 250000
```

will execute the non-recursive quicksort implementation of an ordered array of length 250000.

If you want to test print the arrays that were used in your test, you just need to add `-p` at the end.

For example:

```
./main QC Ale 10 -p
```

will execute the common quicksort implementation in a shuffled array of length 10 and print all the 20 arrays that were used in the tests.

3.3 Output

The output will contain the quicksort variation, type, size, the average number of comparisons, the average number of moves and the median of the time (**in microseconds**) of all the 20 tests and, if it contains the `-p` parameter, it will also have all the vectors used in each case.

Example:

```
[Input] ./main QPE OrdC 10
[Output] QPE Ale 10 45 13 2
```

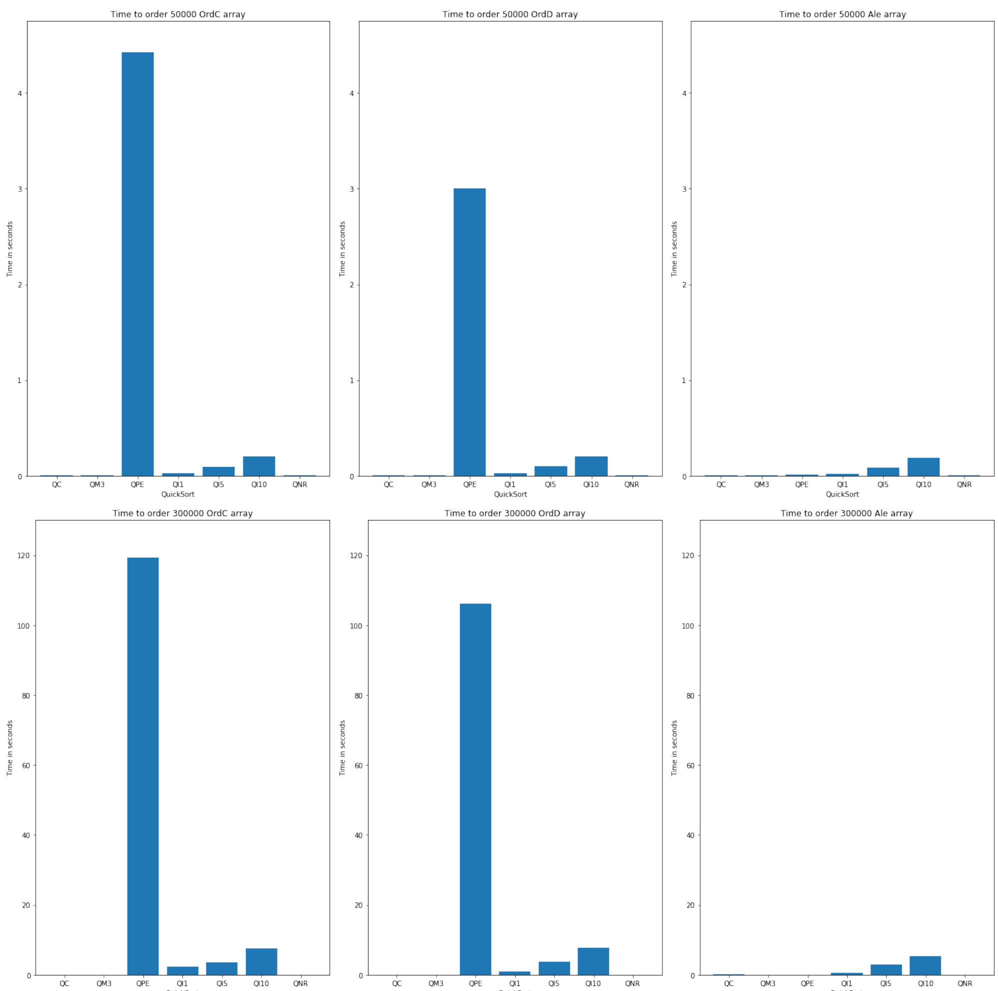
or, with `-p`

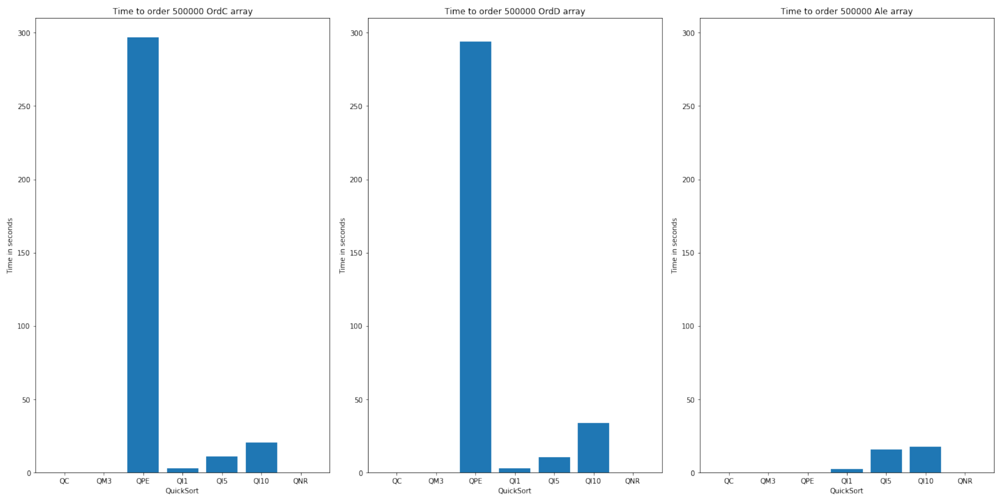
```
[Input] ./main QM3 Ale 10 -p
[Output] QM3 Ale 10 42 13 2
6 6 1 1 7 5 7 1 5 9
1 9 6 2 5 5 0 5 7 9
4 3 2 6 7 6 7 6 4 0
3 1 6 1 4 6 3 2 3 9
6 2 8 1 7 6 1 0 9 6
.
.
.
```

4 Speed analysis

In this session, we will compare all quicksorts and see which one is the best for Elsa to use.

4.1 Timing





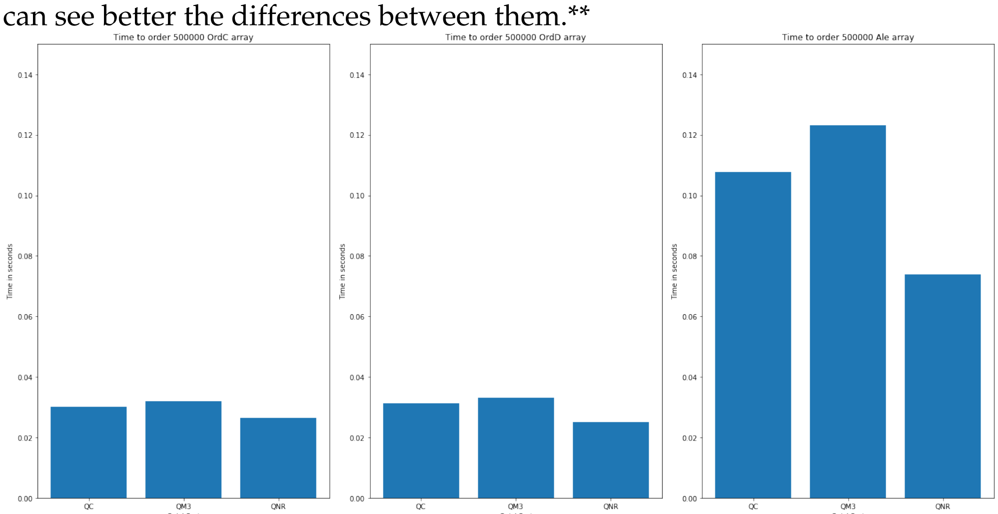
4.1.1 The worst timing scenario

Looking at the graphs above, we just saw that the QPE (First element QuickSort) is the **slowest** one. The worst scenario in this QS is the OrdC one, but we can see here that, when we get bigger vectors, the timing of OrdC and OrdD gets more similar.

It is interesting to look here as well that, when we use QPE in a shuffled array (Ale), our timing is actually pretty ok, compared to the ordered ones. This occurs, because, since we are using the first element as the pivot, in the worst case (ordered array), rather than go until the half of the array and then split, O(n log n), the array is only split when it goes through all the array. So we have an O(n^2).

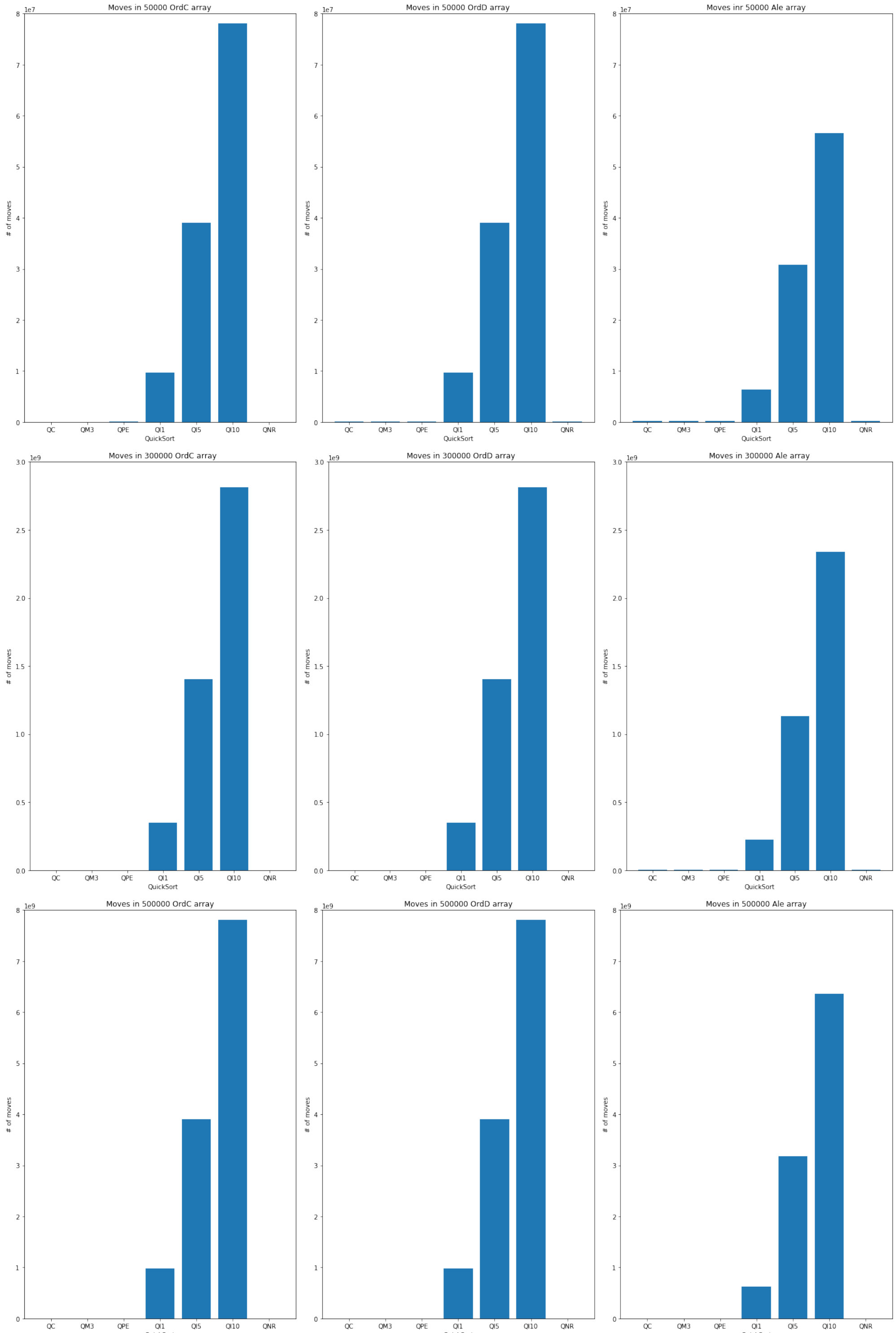
4.1.2 The best timing scenario

According to our graphs, the quicksorts QC, QM3 and QNR are pretty similar. So we will plot only these 3 to compare them. We will plot only the 500000 length array because is the largest and we can see better the differences between them.**



Now we can see that the fastest timing is the QNR (Non-recursive QuickSort) one. This can be explained, maybe, because the recursive function isn't doing only what it should do - since it is only one function for all the recursive quicksort cases.

4.2 Moves



4.2.1 The worst moving scenario

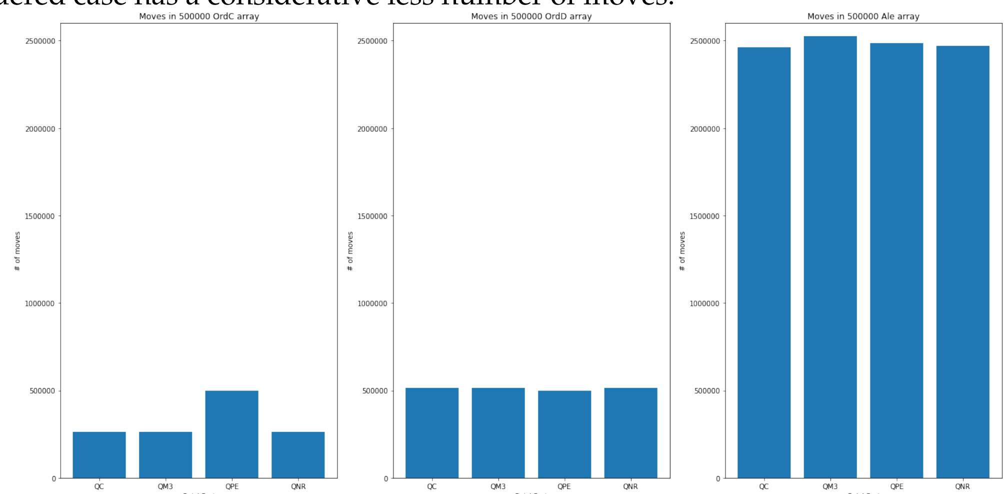
Looking at the graphs above, we just saw that the QI (Insertion Quicksort) is the one with most moves. This was the expected result since the insertion quicksort uses the insertion implementation and it is the most costly in terms of movements. The QI10 is higher, of course, because the QI10 uses more Insertion than the QI5 and QI1.

Here we can see that the ordered ones are the most costly as well, because, in these cases, the worst scenario happens more times than the random one.

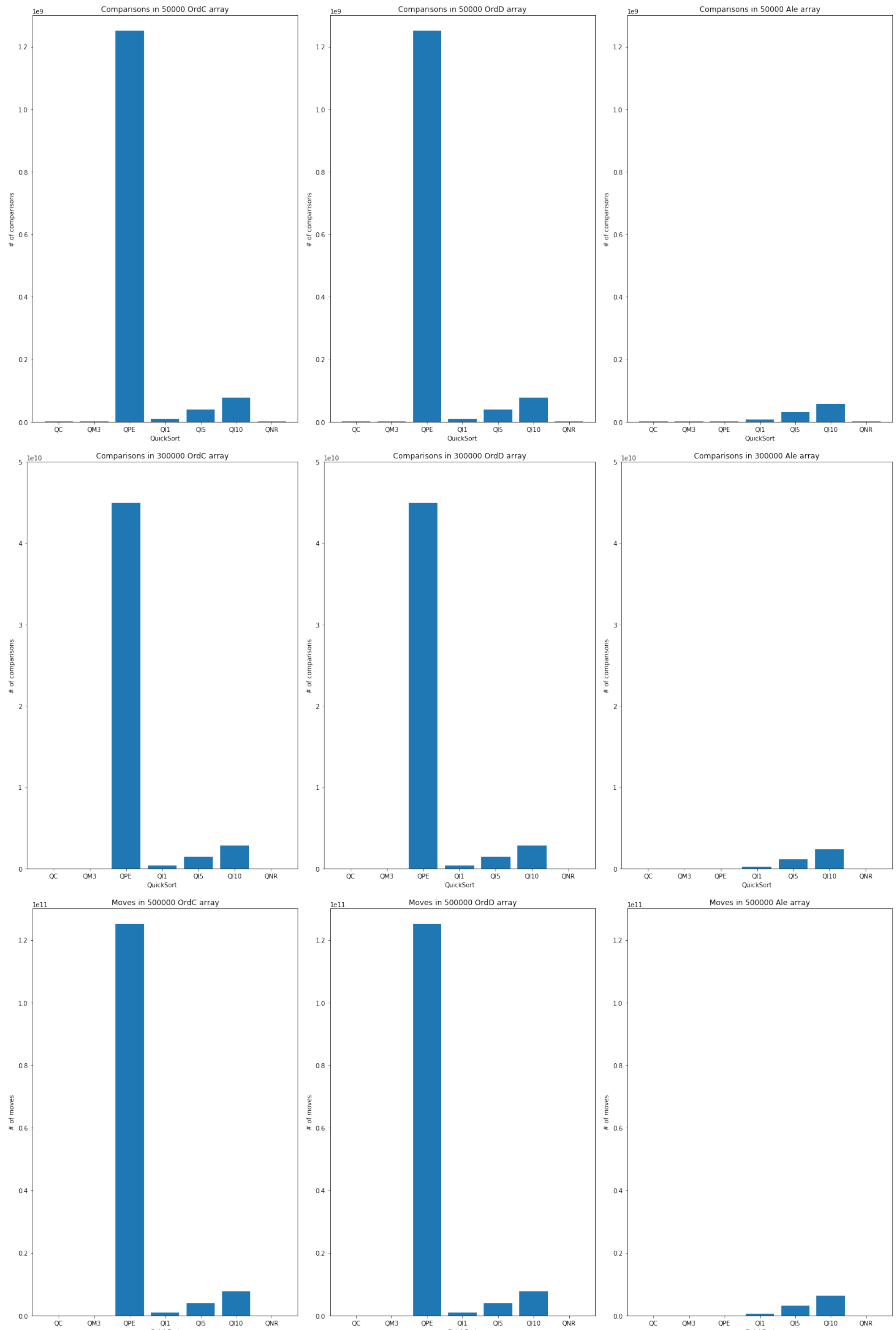
4.2.2 The best moving scenario

According to our graphs, the quicksorts QC, QM3, QPE and QNR are pretty similar. So we will plot only these 4 to compare them. We will plot only the 500000 length array because is the largest and we can see better the differences between them.

Now, we can see that the count of movements of these 4 is pretty similar. And, because of this, we can't guarantee that one is much better than the other. Anyway, we can see here, that now, the ordered case has a considerative less number of moves.



4.3 Comparison



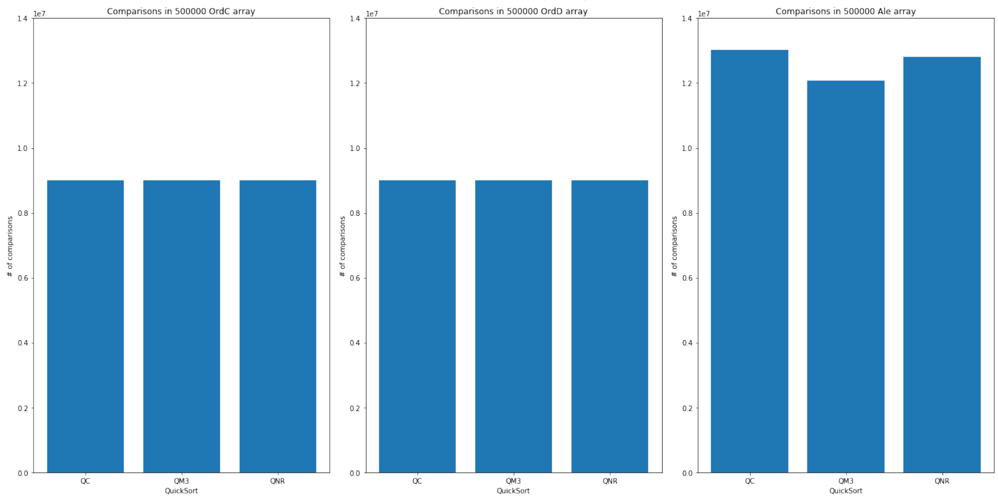
4.3.1 The worst comparison scenario

Here we have exactly what we expected. Since the QPE is using the first element of each vector as the pivot, it will be the one, for sure with most comparisons in the worst case (ordered arrays). In the shuffled one, we have the insertion quicksort as the worst case, because, in the insertion sort, we are always comparing to see when to stop, meanwhile in the QPE, since we have a shuffled array, we will find faster one case the finish the for.

4.3.2 The best comparison scenario

According to our graphs, the quicksorts QC, QM3 and QNR are pretty similar. So we will plot only these 3 to compare them. We will plot only the 500000 length array because is the largest and we can see better the differences between them.

Here we can see that in the ordered array they are pretty similar (since the QM3 will always have its pivot as the array in the middle element). But we can see that the QM3, in the shuffled one has a slightly smaller number of comparisons. This can be explained by the fact that since it gets the median between 3 numbers, we always get the one that is more in the middle.



5 Conclusion

To finish this paper and to give a decent solution to Elsa, we can confirm that the **common quick-Sort** is definitely the better solution for the queen of Arendelle's program. And, just to be clear, when we refer to the common quickSort, we are referring to the QC and the QNR, that uses the same implementation. This because they both have a very good and similar performance when dealing with big numbers in all the scenarios.

Elsa may prefer to use the QC because it's implementation is easier since she doesn't need to create a Stack to simulate the recursive function. And also, when developing a function to deal only with the QC case, she might get a little performance gain.

6 References

Ziviani, N. (2006). *Projetos de Algoritmos com Implementações em PASCAL e C: Capítulo 4: Ordenação*. Editora Cengage.

A Results from my own machine: (used in the plots)

```
QC Ale 50000 1061764 208110 6613
QC Ale 100000 2257713 439520 13727
QC Ale 150000 3534231 679694 29580
QC Ale 200000 4763714 926445 43388
QC Ale 250000 6135936 1173376 59510
QC Ale 300000 7486617 1424746 61941
QC Ale 350000 8895041 1678393 81263
QC Ale 400000 10185034 1940150 76413
QC Ale 450000 11539280 2201062 98855
QC Ale 500000 13011936 2462290 107708
QC OrdC 50000 750015 32767 2673
QC OrdC 100000 1600016 65535 8447
QC OrdC 150000 2456803 84464 8232
QC OrdC 200000 3400017 131071 12601
QC OrdC 250000 4250017 131071 15120
QC OrdC 300000 5213588 168928 17518
QC OrdC 350000 6213588 218928 22737
QC OrdC 400000 7200018 262143 24821
QC OrdC 450000 8100018 262143 29591
QC OrdC 500000 9000018 262143 30017
QC OrdD 50000 750028 57766 2685
QC OrdD 100000 1600030 115534 6003
QC OrdD 150000 2456820 159464 8621
QC OrdD 200000 3400032 231070 12099
QC OrdD 250000 4250032 256070 15147
QC OrdD 300000 5213606 318928 18147
QC OrdD 350000 6213606 393928 24555
QC OrdD 400000 7200034 462142 26481
QC OrdD 450000 8100034 487142 31185
QC OrdD 500000 9000034 512142 31254
QM3 Ale 50000 995656 213407 8974
QM3 Ale 100000 2107078 450502 17558
QM3 Ale 150000 3272236 696788 27081
QM3 Ale 200000 4467416 948803 37274
QM3 Ale 250000 5687475 1204535 46424
QM3 Ale 300000 6882475 1465419 56529
QM3 Ale 350000 8131051 1729295 69968
QM3 Ale 400000 9458076 1990577 79383
QM3 Ale 450000 10716501 2256810 86380
QM3 Ale 500000 12063768 2525702 123199
QM3 OrdC 50000 750015 32767 2663
QM3 OrdC 100000 1600016 65535 6677
QM3 OrdC 150000 2456803 84464 11654
QM3 OrdC 200000 3400017 131071 12559
QM3 OrdC 250000 4250017 131071 15325
QM3 OrdC 300000 5213588 168928 19850
```

```
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QM3 OrdC 450000 8100018 262143 28330
QM3 OrdC 500000 9000018 262143 32064
QM3 OrdD 50000 750028 57766 3003
QM3 OrdD 100000 1600030 115534 7161
QM3 OrdD 150000 2456820 159464 10913
QM3 OrdD 200000 3400032 231070 13681
QM3 OrdD 250000 4250032 256070 16145
QM3 OrdD 300000 5213606 318928 19068
QM3 OrdD 350000 6213606 393928 25621
QM3 OrdD 400000 7200034 462142 31589
QM3 OrdD 450000 8100034 487142 31947
QM3 OrdD 500000 9000034 512142 33198
QPE Ale 50000 1214036 210366 9563
QPE Ale 100000 2585203 444198 18074
QPE Ale 150000 4019063 687680 29607
QPE Ale 200000 5587339 932401 39243
QPE Ale 250000 6981621 1186908 51101
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QPE Ale 350000 10074289 1697778 68256
QPE Ale 400000 11579641 1962326 81491
QPE Ale 450000 13343069 2221223 95672
QPE Ale 500000 14823250 2486175 118951
QPE OrdC 50000 1250074998 49999 4426739
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QI1 Ale 50000 6766551 6407681 22801
QI1 Ale 100000 25989873 25277930 73504
QI1 Ale 150000 57445334 56377200 158007
QI1 Ale 200000 103746915 102299688 283377
```

```
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QI1 OrdC 150000 88716937 87667057 238326
QI1 OrdC 200000 157351712 155951832 512829
QI1 OrdC 250000 245517719 243767839 706614
QI1 OrdC 300000 353214989 351115109 2263584
QI1 OrdC 350000 480443508 477993628 1443056
QI1 OrdC 400000 627203259 624403379 1614785
QI1 OrdC 450000 793494289 790344409 2051418
QI1 OrdC 500000 979316552 975816672 2524155
QI1 OrdD 50000 10041558 9716672 29402
QI1 OrdD 100000 39614212 38964326 133025
QI1 OrdD 150000 88718112 87743226 266413
QI1 OrdD 200000 157353278 156053392 524095
QI1 OrdD 250000 245519676 243894790 671463
QI1 OrdD 300000 353217336 351267450 893259
QI1 OrdD 350000 480446246 478171360 1708751
QI1 OrdD 400000 627206388 624606502 1611973
QI1 OrdD 450000 793497808 790572922 2619654
QI1 OrdD 500000 979320462 976070576 2961240
QI5 Ale 50000 31012535 30781419 86401
QI5 Ale 100000 124376064 123911427 483951
QI5 Ale 150000 288963405 288277778 982244
QI5 Ale 200000 516339670 515396562 1487315
QI5 Ale 250000 770501360 769334561 2637159
QI5 Ale 300000 1133272117 1131818958 3047108
QI5 Ale 350000 1574892788 1573288208 3948376
QI5 Ale 400000 2010752114 2008877341 6760852
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QI5 Ale 500000 3178563385 3176234789 15544073
QI5 OrdC 50000 39239102 38989128 96723
QI5 OrdC 100000 156603161 156103187 385683
QI5 OrdC 150000 352092227 351342253 925021
QI5 OrdC 200000 625706286 624706312 1601094
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QI5 OrdC 450000 3165651602 3163401628 8564268
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QI5 OrdD 100000 156606288 156156310 408366
```

```
QI5 OrdD 150000 352096916 351421938 880309
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QI10 Ale 50000 56739711 56558990 185850
QI10 Ale 100000 251235195 250873270 620260
QI10 Ale 150000 565628949 565073493 1307171
QI10 Ale 200000 1013739586 1012991589 2684356
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QNR Ale 400000 10089666 1944470 61844
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```

```
QNR OrdC 50000 750015 32767 2547
QNR OrdC 100000 1600016 65535 5031
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QNR OrdC 250000 4250017 131071 14773
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QNR OrdD 350000 6213606 393928 17676
QNR OrdD 400000 7200034 462142 20775
QNR OrdD 450000 8100034 487142 21376
QNR OrdD 500000 9000034 512142 25015
```

B Results from Google Cloud Platform Machine:

```
QC Ale 50000 1058884 208268 11124
QC Ale 100000 2264272 439054 47727
QC Ale 150000 3495496 679794 36275
QC Ale 200000 4765539 926320 49501
QC Ale 250000 6170820 1173713 68167
QC Ale 300000 7443548 1427609 113223
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