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Non-linear superposition models of blast vibration

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Abstract

Current waveform models of blast vibration typically consider a linear superposition of characteristic (seed) waveforms. However, blasting produces large strains in the surrounding medium, which, in turn, implies a non-linear response of the material. It is therefore questionable to use a linear superposition scheme to add the contributions of individual charge masses. These individual charge masses could be the elements of charge within a single blasthole, or the separate charge masses associated with each blasthole within a full-scale blast. Within a single blasthole, each charge element is necessarily in the very-near-field of its neighbour. In this case, a simple non-linear superposition scheme can be devised such that the vibration due to the entire explosive column is consistent with charge weight scaling. If A is the traditional scaling constant, then non-linear superposition predicts that the vibration is less than that obtained using linear superposition provided $A<1$. However, if $A>1$, then non-linear superposition predicts a vibration greater than that predicted by linear superposition, which is in conflict with accepted theories of large strain non-linear attenuation. For a full-scale blast, each charge mass is clearly separated, which complicates the situation. In this case, two non-linear superposition approaches are given—one based solely on charge weight scaling, the other based on the distinct notion of blast damage. In principle, the damage model is more realistic than the charge weight scaling model, and this is also consistent with the predictions based on these two schemes when compared with measured results.
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1. Introduction

Models of linear superposition to predict the vibration waveform radiated from a single column of explosive have been reported by Blair and Minchinton [1,2]. In these models, the explosive column is divided into a number, M, of elements and the vibration contribution from each element, of mass w_e, is summed in a simple linear manner to produce the total vibration waveform at any desired point in a surrounding uniform medium. However, such linear summing raises an immediate problem with regard to traditional charge weight scaling laws. For example, if the entire column of charge is considered as a total mass $W(=Mwe)$, then the traditional charge weight scaling law predicts that the peak vibration due to the explosive column is proportional to $(Mw_e)^A$, where A is a site constant with a typical, approximate value of 0.7 [3]. Thus the charge weight scaling law implies a non-linearity with respect to W. Although linear superposition models employ charge weight scaling for each single element of charge, such that each element has a peak vibration proportional to $(w_e)^A$, they use a simple linear sum of all M elemental waveforms to predict the total peak level. Thus, these models predict that the far-field peak vibration due to an entire column of explosive with infinite velocity of detonation (infinite V_D) is proportional to $M(w_e)^A$, which can be significantly different than the scaling law prediction of $(Mw_e)^A$.

There are advantages and disadvantages in the charge weight scaling technique and the linear superposition technique as applied to a single column of explosive. In particular, the scaling law implies a non-linear influence of the total charge weight, Mw_e, which is, at least, consistent with expectations for materials under large strain. However, this approach ignores any effect due to a finite V_D. Linear superposition, on the other hand, implies a linear sum of elemental charge weights, w_e, which is unrealistic, but does give a realistic account of finite V_D.

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Thus the first aim of the present work is to present a non-linear scheme for the superposition of waveforms from each element of charge within a single blasthole. It will also be required that this non-linear scheme reduce to the charge weight scaling law when the entire charge is detonated instantaneously (i.e., V_D infinite) and measured in the far-field. Such a scheme will then have the advantages of both techniques and be free of their disadvantages.

To mimic a full-scale blast, linear superposition models have also been used to predict the total vibration due to a collection of blastholes [4–7]. However, it is well known that blasting induces a highly non-linear response in the near-field of the material under consideration, and thus non-linear superposition might be appropriate. Nevertheless, unlike the elements of charge within a single column of explosive, each charge mass of a blast is clearly separated, making the problem much more complicated. In this regard, there are two approaches that could be considered—one based on the principle of charge weight scaling and another based on the distinct notion of blast damage.

According to charge weight scaling, blastholes that initiate close in space *and time* will superpose non-linearly, similar to the elements within a single blasthole. On the other hand, it is also reasonable to assume that blastholes spaced a sufficient distance apart will superpose linearly, irrespective of time. This aspect of linearity versus non-linearity was originally raised in [8], and considered more recently with greater detail in [9]; however, both these articles only gave a qualitative description of the inherent problems. Thus the second aim of the present work is to derive a charge weight scaling model that gives a smooth transition from non-linear superposition to linear superposition as the distance increases between blastholes.

However, the charge weight scaling law does not embody any notion of time, and this creates a problem. For example, if all the blastholes are close in space yet initiate with markedly different times, then, as discussed in [9], the scaling law cannot strictly be used to select the governing charge weight. Thus the third aim of the present work is to present an alternative non-linear superposition scheme based directly on the notion of blast damage. In this scheme, two mechanisms are proposed to describe the reduction in vibration from each blasthole at the time of its initiation. The first mechanism is attenuation due to the changing rock mass condition as the blast proceeds, and this is a global damage phenomenon. The second mechanism is attenuation due to vibration screening whereby the severe and localised damage zone of some of the previously initiated holes lies in the direct path between the initiating hole and the vibration monitor. Such vibration screening has already been given in a previous model [7], which, incidentally, transforms it into a non-linear superposition model. However, an improved screening model is given in the present work.

The non-linear case appropriate to a single column of explosive (single blasthole) is discussed first and is followed by the non-linear schemes appropriate to a collection of blastholes (i.e., a production blast). The particular production blast considered is near the wall of an open pit, which, itself, can be considered as a large structure. The present work deals with linear and non-linear superposition with regard to vibration in general, which can measured as displacement, velocity or acceleration. In the field of blast damage around a single blasthole, it is traditional to use particle velocities [1,3]. Thus in this section, peak particle velocities, only, are simulated. However, in the field of structural dynamics, it is traditional to use particle accelerations. One obvious reason is that this measure has a natural yardstick insofar as an acceleration exceeding 1 g can cause particle "lift-off". There is no such natural yardstick for velocity. Thus in the second section, particle accelerations are measured and simulated for analysing the influence of a full-scale blast on the pit wall. Both these examples are also chosen in order to illustrate the generality of the superposition techniques, which can be applied to any form of vibration measurement (displacement, velocity or acceleration).

2. Non-linear superposition for a single column of explosive

A model is now developed that uses a non-linear superposition scheme to predict the total horizontal, $v_r(r,z,t)$, and vertical, $v_z(r,z,t)$, components of the particle velocity at any monitoring point, $P(r,z)$, due to an explosive column of length L in a hole of infinite length and radius a, embedded in a uniform medium of infinite extent. The scaled Heelan equations for a single blasthole given in Blair and Minchinton [2] are taken as the basis of the non-linear model for which the geometry of Fig. 1 applies. These equations, which will not be repeated here, are relevant only for a base-primed blasthole and involve a summation over all m elements with associated angles ϕ_m and distances R_m. Fig. 1a shows five such representative elements including the first and last (Mth) elements as well as the general (mth) element. The full waveform solution at P is obtained by an appropriate non-linear summation of the waveform contributions from all M elements, each having a length $\delta L(= L/M)$. An odd number of elements (typically 41) is chosen to preserve symmetry about the centre of the explosive column.

A hallmark of non-linear superposition is that the total contribution from the entire column does not equal the sum of the separate contributions from each element if fired in isolation. In order to remain consistent with the non-linearity implied by charge weight scaling, the first element of charge must always produce a peak vibration governed by $(w_e)^A$, while all subsequent elements produce a progressively weaker (if $A<1.0$) or stronger (if $A>1.0$) contribution such that initiation of any M- element column produces a total contribution of $(Mw_e)^A$. This can be achieved if each elemental contribution, E_m, is given by the

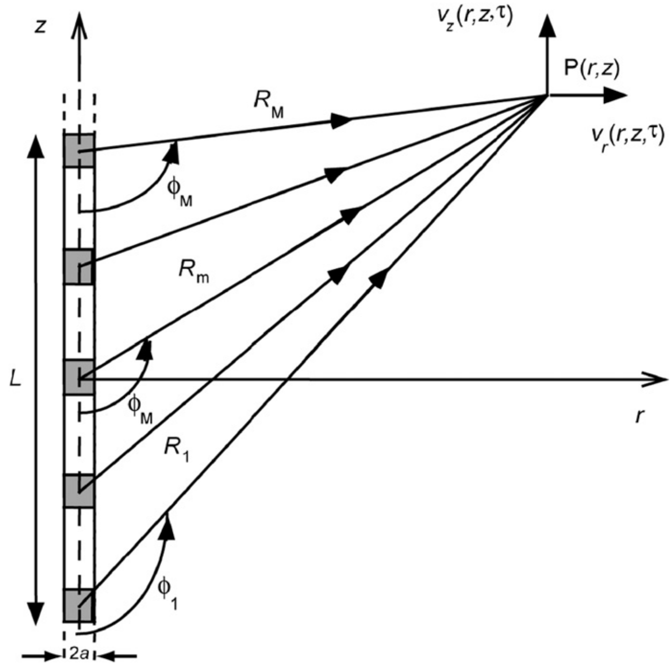

Fig. 1. The blasthole geometry.

following non-linear function of m:

$$E_m = [mw_e]^A - [(m-1)w_e]^A. \tag{1}$$

Thus $E_1 = (w_e)^A$, as required. Furthermore, the total contribution, E_T, is given by

$$E_T = \sum_{m=1}^{M} E_m = \sum_{m=1}^{M} \{[mw_e]^A - [(m-1)w_e]^A\} = [Mw_e]^A, \tag{2}$$

which is also required. Using this scheme, the scaled Heelan solution reported in [2] can be altered to account for the non-linear dependence on charge weight. For example, in the linear superposition model, the solution depends on a factor $\gamma_n K w_e^A R_m^{-B}$, where γ_n is a specifically defined constant, and K and B are the other site constants appropriate to the particle velocities produced by each element of charge [2]. In the present non-linear scheme, this factor is replaced by

$$\gamma_n K E_m R_m^{-B} = \gamma_n K\{[mw_e]^A - [(m-1)w_e]^A\} R_m^{-B}. \tag{3}$$

This is the only change required to ensure non-linear superposition within the scaled Heelan model. For the standard Heelan, full-field and dynamic finite element (DFEM) models also employed in [2], non-linear superposition can easily be achieved by multiplying the pressure load on the mth element by the factor $E_m(w_e)^{-A}$.

Fig. 2 shows the scaled Heelan predictions for the vector peak particle velocity, *vppv*, (in mm/s) from a 5 m column of charge assuming linear superposition with $M = 41$ elements [2]. Fig. 3 shows the results using the non-linear superposition model with $A = 0.7$. Other than the non-linearity implied by Eq. (3) all remaining model parameters

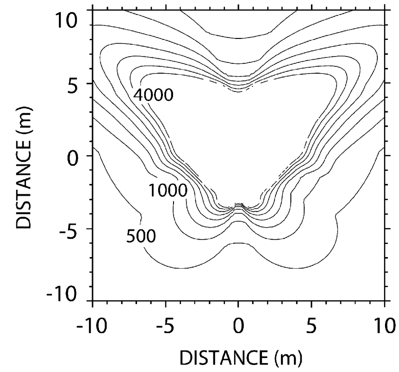

Fig. 2. The Scaled Heelan predictions for *vppv* (mm/s) from a 5 m column of charge; linear superposition is assumed (after [2]).

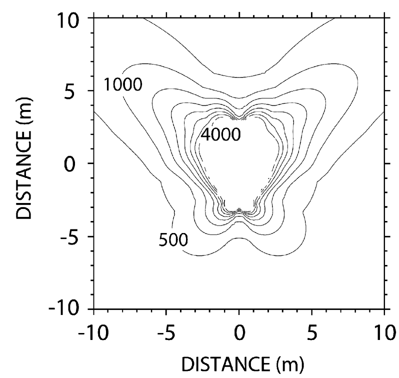

Fig. 3. The Scaled Heelan predictions for *vppv* (mm/s) from a 5 m column of charge; non-linear superposition is assumed.

remain unaltered from those given in [2]. These figures show that the non-linear model predicts a general reduction in *vppv* while retaining the relative strength of the SV-wave radiation pattern. Provided M is sufficiently large ($\geqslant$21, approximately) the results are independent of M because the total mass, Mw_e, must be conserved, and so E_T of Eq. (2) is independent of M. There appears to be no experimental data available with sufficient coverage to make comparisons with these predictions of linear and non-linear models. However, it is reasonable to assume that the non-linear model is more realistic simply because it is consistent with accepted ideas on the large strain, non-linear attenuation expected in media close to blastholes.

The present models are limited insofar as they ignore the influence of any nearby free surfaces. In this regard, Blair and Minchinton [1] did a dynamic finite element model (DFEM) of an extended column of explosive beneath a free surface and showed an increase in near-surface vibration due to the reflected waves. Despite the absence of free surfaces in the present analytical models, they do at least illustrate the radiation pattern for waves emitted by columns of explosive, which will have a significant influence on near-field damage. The analytical treatment so far has been given only for a base-primed blasthole. However, the case of a top-primed hole can be readily obtained by simply rotating the contour plots of Figs. 2 and 3 through 180°. Furthermore, the elemental contributions defined by Eq. (1) can also be generalised to account for initiation at any point within the explosive column. This point is given some consideration, later, within the discussion.

3. A superposition scheme for full-scale blasts

If the vibration waveform (typically velocity or acceleration) from representative, single blastholes (seeds) is measured, then it is possible to construct a viable model for the vibration waves produced by a full-scale blast using a linear superposition of the seeds. In this process, each seed waveform representing each single blasthole is displaced in time according to the delay sequence and appropriately added. The complete description of this model is given in [7] and will not be repeated here except for that detail relevant to the present work. Under linear superposition, each blasthole contributes to the total vibration independently of any other hole. In other words the general structure of such models is given by

$$V(t) = \sum_{n=1}^{N} K d_n^{-B} w_n^A s_n(t - \delta_n), \tag{4}$$

where $V(t)$ is the total vibration waveform at time, t, predicted at some monitoring site, due to the initiation of N blastholes each of charge weight w_n and at a distance d_n; $s_n(t-\delta_n)$ represents the waveform produced by the nth blasthole, which is delayed by a total amount δ_n due to the delay initiation sequence and wave travel time differences in the local ground. As noted in the Introduction, it is the particle accelerations that are to be modelled within this section, and thus the site constants, K, B and A are now those obtained for acceleration measurements of entire columns of charge (rather than for one element of charge within a single blasthole), and are determined by single-blasthole trials. However, because of the inherent non-linearities, an alternative non-linear superposition scheme is sought to replace Eq. (4). As previously noted, two separate models will be considered—one based purely on charge weight scaling, the other based on the notion of blast damage. All these models are three-dimensional insofar as all distances are based on the (x, y, z) coordinates of all charges and all monitors. Furthermore, other mechanisms to be considered later (such as the screening influence) are also three-dimensional.

3.1. The charge weight scaling model

It is clear from Eq. (4) that the scaled mass of explosive, ${w_n}^A$, in each blasthole contributes to the total vibration in a linear fashion. However, in order to examine the validity of this equation for the superposition of a collection of spatially separated blastholes, it is worthwhile considering two distinct and extreme situations. In both situations all N blastholes have equal charge weight w, are initiated instantaneously in uniform, undamaged, identical ground and are monitored in the far-field; thus all distances are assumed equal and given by $d = d_n$ for all n. Furthermore, the ground is assumed to have an infinite wave speed so that travel time differences in the radiation from each blasthole can be neglected. All holes would thus independently produce identical waveforms, $s(t)$, perfectly overlaid in time.

The first situation is that in which all the blastholes are separated to such an extent that they have no influence on each other. In other words, each hole is unaware of its neighbours and will remain so even after the blast. In this case, each blasthole contributes to the total vibration in a linear fashion, and from Eq. (4), the total monitored vibration is given by

$$V(t) = kNw^A s(t), \tag{5}$$

where $k = Kd^{-B}$. Thus, the peak vibration will be determined by $W_\lambda = Nw^A$, where W_λ is defined as the linear scaled mass, because it is relevant to linear superposition.

The second situation is that in which all the blastholes are so close that each contributes non-linearly and collectively act as a single charge of mass $W_\mathrm{T} = Nw$. Although in this case it is not possible to use Eq. (4), some type of waveform superposition must still occur. According to the charge weight scaling law, the peak vibration will be determined by W_T, and thus the general form of the total monitored vibration that is consistent with charge weight scaling will be given by

$$V(t) = kW_\mathrm{T}^A s(t) = k(Nw)^A s(t). \tag{6}$$

Although the waveform, $s(t)$, in (6) could well have a different shape to that in (5), this has no bearing on the issue under consideration. The salient point here is that the peak vibration in (6) will be determined by $W_v = (Nw)^A$, where W_v is defined as the non-linear scaled mass, because it is relevant to non-linear superposition. It should be appreciated that this non-linear scaled mass is also the basis of the non-linear superposition scheme previously used to model the single column of charge.

It is also important to note that the concept of the scaled masses, W_λ and W_v, is based purely on an attempt to reconcile charge weight scaling laws for holes close together and holes far apart, i.e., the concept is not directly based on any notion of blast damage.

The linear superposition Eq. (5) is relevant to widely-spaced blastholes, whereas the non-linear superposition Eq. (6), is relevant to blastholes that are all virtually coalesced. In reality, the blasthole separation will lie somewhere between the two extremes, thus neither (5) nor (6) can be a valid description of the superposition required. It is thus worthwhile constructing a superposition scheme, based on the concept of linear and non-linear scaled masses, in which the peak vibration is determined by a scaled mass that approaches W_v as the blasthole separation decreases and approaches W_λ as the separation increases.

The discussion so far has assumed that all blastholes will initiate instantaneously as supposed by Eqs. (5) and (6). However, in reality this is clearly not the case, and this issue must now be considered. Under the assumption of linear superposition it is reasonable to use a linear superposition scheme for time-delayed blastholes as given in Eq. (4), and so it is also reasonable to use the concept of a linear scaled mass, W_λ. However, the assumption of non-linear superposition raises a conceptual problem simply because charge weight scaling laws, themselves, do not embody any notion of time, and this aspect has been considered by Blair [9] in some detail. For example, if all the blastholes are close together yet initiate with different times (as in a typical blast), it is not clear at all which value to use for the effective total charge weight, W_T, in Eq. (6). In fact, as noted in [9], in a strict sense, charge weight scaling laws cannot be used in this case.

Nevertheless, it is still worthwhile proceeding under the assumption that $W_T = W_v = (Nw)^A$ for time-delayed non-linear superposition based on the charge weight scaling model alone, and then to explore the ramifications of such an assumption. For time-delayed blastholes, each blasthole must be considered in turn, and its contribution to the scaled mass added to the total in some consistent manner. In this regard, it is useful to consider the simple example of 12 blastholes as illustrated in Fig. 4, in which the shaded holes have initiated and the nth hole is about to initiate. The distance to the closest blasthole that has previously initiated is shown as h_n. In this scheme, the value of h_1 is considered infinite simply because the first blasthole to initiate in the current blast is assumed to be far removed from any other blasthole that may have initiated elsewhere in a prior blast.

A radial zone for non-linear response produced by the nth hole can be defined in terms of the mean hole spacing, s_P, and assumed to be proportional to the charge weight, w_n. Thus the radius of this zone can be defined as $Ds_P(w_n/\bar{w})$, where $\bar{w}$ is the mean charge weight of all

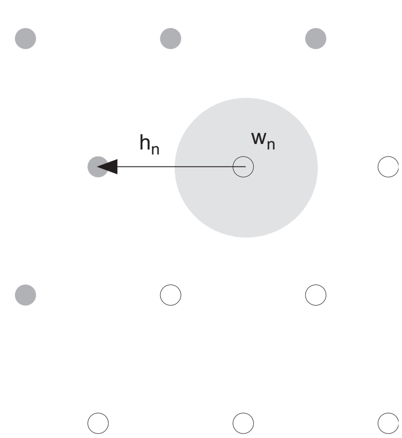

Fig. 4. A simple example in which the shaded holes have initiated and hole n is about to initiate and create a radially non-linear zone.

blastholes, and D is a constant factor. This zone is also illustrated in Fig. 4. A corresponding dimensionless range, r_n, to describe a general transition, increasing from non-linear to linear response for the nth blasthole, may then be defined as

$$r_n = [h_n/(Ds_P)](\bar{w}/w_n). \tag{7}$$

The range r_1 is also considered to be infinite, which is consistent with the notion that the first blasthole to initiate does not involve a non-linear interaction, and so will contribute a factor of w_1^A to the total vibration. This is equivalent to the role of E_1 in Eq. (2), i.e. the superposition role played by the first element within a single column of charge.

In a strict sense, the value of D in Eq. (7) would need to be determined by a numerical model that could account for realistic non-linear behaviour of materials under blast loads. Unfortunately, such models have not been developed. However, it is still worthwhile proceeding under the assumption that Ds_P is given approximately by the blasthole spacing, and then investigating the ramifications of alternative values. Thus, $D = 1$, unless stated otherwise.

The next stage in the development of a superposition scheme is to consider all blastholes up to and including the nth hole to initiate, and give a function, F_n, that describes a smooth change from a non-linear scaled mass (i.e., non-linear superposition) to a linear scaled mass (i.e. linear superposition) as the range, r_n, increases. A suitable function in this regard is given by

$$F_n = W_v + \frac{r_n^P}{1 + r_n^P}(W_\lambda - W_v), \tag{8}$$

where P is a positive integer, and the scaled masses now become

$$W_v = \left(\sum_{j=1}^{n} w_j\right)^A; \; W_\lambda = \sum_{j=1}^{n}(w_j^A). \tag{9}$$

As r_n increases from zero to infinity, the function ${r_n}^P/(1+{r_n}^P)$ increases smoothly from zero to 1.0, with a half-amplitude point at $r_n = 1$. For small P, the function change is slow about the point $r_n = 1$, while for large P, the change is rapid. Thus the integer P can be used to vary the rate of change from non-linear to linear scaled weights with increasing range. These phenomena are illustrated later, with reference to Fig. 5.

If C_n represents the contribution made by the nth hole itself, then $C_n = F_n - F_{n-1}$. Thus, from Eqs. (8) and (9):

$$C_n = \left(\sum_{j=1}^{n} w_j\right)^A - \left(\sum_{j=1}^{n-1} w_j\right)^A + \frac{r_n^P}{1 + r_n^P}\left[w_n^A + \left(\sum_{j=1}^{n-1} w_j\right)^A - \left(\sum_{j=1}^{n} w_j\right)^A\right]. \tag{10}$$

Many blast designs use a nominally constant charge weight, w, in all blastholes, and under this condition a

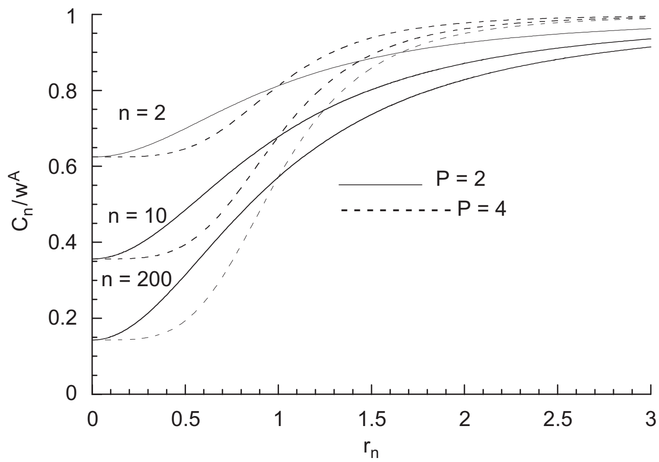

Fig. 5. C_n/w^A as a function of the range, r_n.

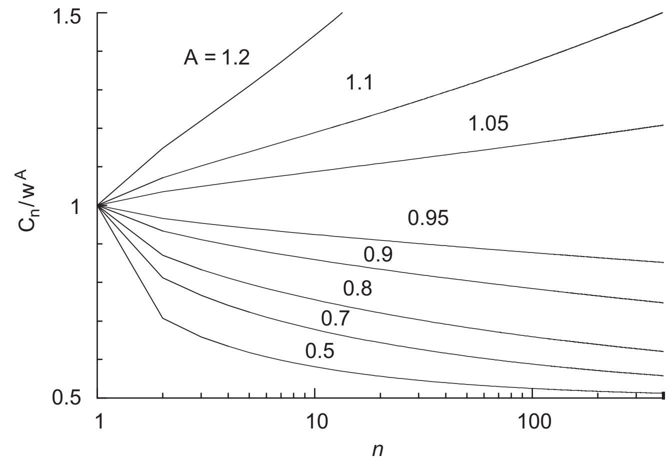

Fig. 6. C_n/w^A as a function of the number of blastholes; $r_n = 1.0$.

useful insight can be given. For example, Eq. (10) then becomes

$$C_n = w^A\{n^A - (n-1)^A + \frac{r_n^P}{1+r_n^P}[1+(n-1)^A - n^A]\}. \quad (11)$$

The first two terms on the right-hand side of Eq. (11) represent the contribution due to non-linear superposition and thus have a form identical to Eq. (1) for the equivalent case of a single column of charge. Fig. 5 shows a plot of Eq. (11) in the form of C_n/w^A as a function of r_n for $n = 2$, 10, 200 and $P = 2, 4$ with A fixed at 0.7. Many blast designs also use a nominally constant blasthole separation that is reasonably close to the mean spacing. Under this further condition, ${r_n}^P \to 1$ for all P, and then $C_n/w^A \to 0.5*[1+n^A-(n-1)^A]$. Fig. 6 shows a plot of this function for various values of the site constant A, for which $C_n \to w^A/2$ as $n \geqslant \infty$. If $A>1$, the present model predicts that the total influence of multiple holes is greater than the sum of the individual holes. However, this makes no physical sense especially if the non-linear zone is viewed as a damage zone. Furthermore, the large strain, non-linear response of realistic media under blasting implies that $A<1$ always.

Provided $A<1$, Fig. 6 also shows a phenomenon similar to one noted for the single column of explosive insofar as each blasthole produces a progressively weaker contribution to the total. This is due to the component of non-linear superposition that still operates at the range $r_n = 1.0$. For much larger ranges, $r_n \to \infty$, Eq. (10) directly yields $C_n/w^A = 1.0$ (assuming identical charge masses), and this is the hallmark of linear superposition. In other words, this nth charge contributes to the total superposition as if it was a single charge within a standard charge weight scaling law, i.e., it acts independent of the initiation of any other charge.

If $A \leqslant 0.7$, approximately, then Fig. 6 shows that the major influence of non-linear superposition is to reduce the total vibration in such a way that the majority of blastholes have their vibration almost halved. It is only the first 10 or so blastholes that have a significant rate of change in their

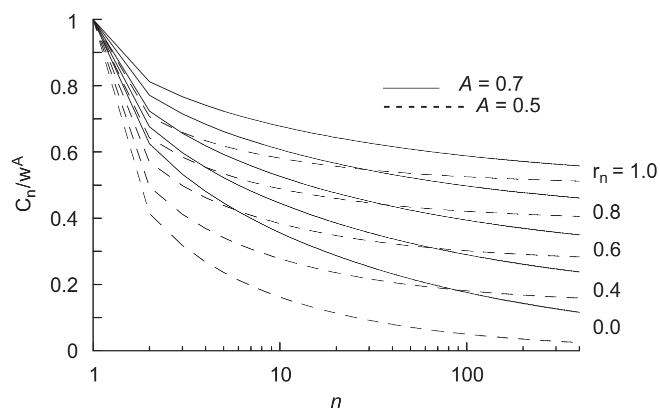

Fig. 7. C_n/w^A as a function of the number of blastholes for various r_n.

dependence on the number of previously initiated blastholes.

Thus far, $D = 1$ has been assumed in Eq. (7) and the consequences of assuming other values are now considered. If the previous simplifications, $h_n = s_P$, and $w_n = \bar{w}$ for all n, are made, then Eq. (7) yields $r_n = 1/D$. Thus blastholes producing a large zone of non-linear response will have $r_n<1$. Fig. 7 shows a plot of C_n/w^A as a function of n for various values of the parameter $r_n \leqslant 1$, and for two selected values of A. The particular case $r_n = 0$ is equivalent to each blasthole having an infinite non-linear zone, which is obviously unrealistic. Nevertheless, this particular case is analysed simply because it shows the limit for complete non-linearity.

According to Fig. 7, low values of r_n (i.e., large values of D) imply that as the blast progresses the vibration produced by successive holes is dramatically reduced, so much so that even moderately sized blasts would be characterised by a marked decrease in vibration with time. However, this is certainly not consistent with observations in a general sense, and so sets a natural upper limit for realistic values of the parameter D. In this regard it seems prudent to set $D = 1$ to avoid this problem.

Returning now to Eq. (10), it should be noted that the function C_n is a scaled mass term that accounts for both linear and non-linear superposition, and thus plays the same role as the term ${w_n}^A$ in Eq. (4) for the case of linear superposition alone. Hence, an equation that combines both linear and non-linear superposition (dependent on the blasthole separation) is given by

$$V(t) = \sum_{n=1}^{N} K d_n^{-B} C_n s_n(t - \delta_n). \tag{12}$$

If the blasthole separation is large (i.e., r_m is large), then $C_n \to {w_n}^A$, and Eq. (12) reduces to Eq. (4) as expected. If the blasthole separation is small, then

$$V(t) = \sum_{n=1}^{N} K d_n^{-B} \left[\left(\sum_{j=1}^{n} w_j \right)^A - \left(\sum_{j=1}^{n-1} w_j \right)^A \right] s_n(t - \delta_n). \tag{13}$$

Furthermore, if the total vibration, $V(t)$, is measured in the far-field, then $d = d_n$ for all n; also if all holes are identically charged and instantaneously initiated, then all the waveform signatures are identical. Under these conditions Eq. (13) reduces to Eq. (6), as expected.

Unlike Eq. (4), some waveform linear superposition techniques [4,7] employ only two site constants, a and b, in a model having the form

$$V(t) = \sum_{n=1}^{N} a \left(\frac{\sqrt{W_n}}{d_n} \right)^b s_n(t - \delta_n). \tag{14}$$

Eqs. (4) and (14) are equivalent, provided that $A = b/2$, $B = b$ and $K = a$. Eq. (12) can then be re-cast as

$$V(t) = \sum_{n=1}^{N} a d_n^{-b} C_n s_n(t - \delta_n) \tag{15}$$

and C_n evaluated with $b/2$ replacing A in Eq. (10).

In summary, Eq. (15) shows the hallmark of non-linear superposition because the vibration due to the nth blasthole depends on the behaviour of previous blastholes via the term C_n. For linear superposition, however, the behaviour of the nth blasthole depends simply on w_n (Eq. (4)) and so is independent of the behaviour of any other blasthole.

3.2. The blast damage model

As noted in the Introduction, two mechanisms are proposed to describe the reduction in vibration from each blasthole at the time of its initiation. The first mechanism is attenuation due to global damage in which there is a general deterioration in the rock mass as the blast proceeds. The second mechanism is attenuation due to vibration screening by the highly localised damage zone created by those previously initiated holes that lie along a path from the initiating hole to the vibration monitor.

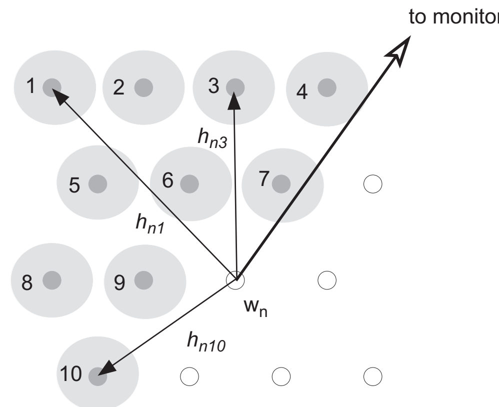

Fig. 8. Illustrating the cumulative damage model in which ten (shaded) holes have initiated and hole $n = 11$ is about to initiate.

The cumulative damage model is illustrated in Fig. 8 for the case of 16 blastholes in which the first 10 (in time) have initiated and a hole of charge weight w_n is about to initiate. The distance from the initiating hole to any previously initiated hole is given by h_{nj}, $(j<n)$ and some of these distances are also illustrated. In this example, all 10 initiated holes will contribute to the cumulative (global) damage faced by the hole about to initiate. A simple, dimensionless and intuitive estimate for the vibration attenuation factor due to such damage can be given by

$$D_n = 1 + \eta \sum_{j=1}^{n-1} \frac{w_j}{\bar{w}} \left(\frac{s_P}{h_{nj}} \right)^3, \tag{16}$$

where η is a dimensionless constant to be estimated, and $D_n = 1$ implies no damage. This particular form is chosen because the damage zone around each blasthole is a volume measure, and is therefore assumed to decay as an inverse cube of distance. The charge weight term $w_j/\bar{w}$ is included because it is reasonable to expect that the degree of damage is also dependent on the mass of explosive within any previously initiated hole. A term such as $(w_j/\bar{w})^C$, for any C could also be used, however, in view of the inherently approximate nature of Eq. (16), a simple linear form ($C = 1$) is assumed.

Eq. (16) implies that all holes of equal charge weight at a distance equal to the mean spacing (s_P) from the current (nth) hole will contribute an equal amount, η, to the damage surrounding this hole at the time of its initiation. An approximate method to estimate η can be obtained by noting that linear superposition models typically overestimate the true vibration by a factor of 2 or more [9]. Thus as a first guess it is reasonable to put $D_n = 2$, and then consider a current (nth) hole, at the centroid of a simple staggered pattern (such as that of Fig. 8) with all holes having an equal charge weight. Further, assume that the current hole initiates half way through the time sequence and is surrounded by an approximately equal number of initiated and uninitiated holes. Thus $n \sim N/2$. The mean spacing, s_P, bears a fixed relationship with each of the h_{nj} values, and it is a simple matter to calculate the sum,

G_S, for all blastholes (excluding the nth), given by

$$G_S = \sum_{j=1}^{N/2-1} \left(\frac{s_P}{h_{nj}}\right)^3 + \sum_{j=N/2+1}^{N} \left(\frac{s_P}{h_{nj}}\right)^3. \tag{17}$$

This sum is a purely geometric factor, and converges rapidly for increasing distances, h_{nj}, from the central (assumed nth) hole. For example, assuming an 11×11 blasthole staggered pattern ($N = 121$) yields $G_S = 9.9$, whereas for a 21×21 pattern ($N = 441$), $G_S = 10.6$. Thus for most cases of practical interest, the first sum in Eq. (17) is approximately $G_S/2 = 5$, which is considered an approximation to the sum in Eq. (16). Hence as a first guess, $\eta = 0.2$. Although in practice η will be an adjustable parameter used to calibrate the model against observed results, it should be appreciated that it is based on a proposed damage mechanism, and could, in principle, be determined from separate experiments.

According to Fig. 8, holes 4 and 7 would be the most effective in screening the vibration produced by Hole 11 along a path to the monitor. Fig. 9 shows the particular geometry used to evaluate this screening, where O represents the hole about to initiate and B represents any of the previously initiated holes. OM is the line from the current hole to the monitor, and BP is the perpendicular to this line; OM is described by the radius vector $\underline{\boldsymbol{a}}$, and OB by the radius vector $\underline{\boldsymbol{b}}$ these are three-dimensional vectors.

It is quite obvious that any previously initiated hole located close to the line OM will provide the maximum amount of screening, whereas those off to one side will provide less-effective screens. A model to estimate this effect in three dimensions can be given by the shape of a Gaussian Bell, also shown in Fig. 9, which is defined (later) in terms of the mean spacing, s_P. In this particular example, a hole located at B would provide a relative screen that is only 0.3 times as effective as that due to a hole lying along the direct path OM.

Experimental data on vibration screening for holes located along the line OM has been reported in [7], and a simple two-dimensional model was used to calculate the screening effect in which all holes lying within a specified rectangular area contributed equally to the screening. This model is now upgraded to estimate screening in three dimensions and also to account for off-axis holes (using the Gaussian Bell).

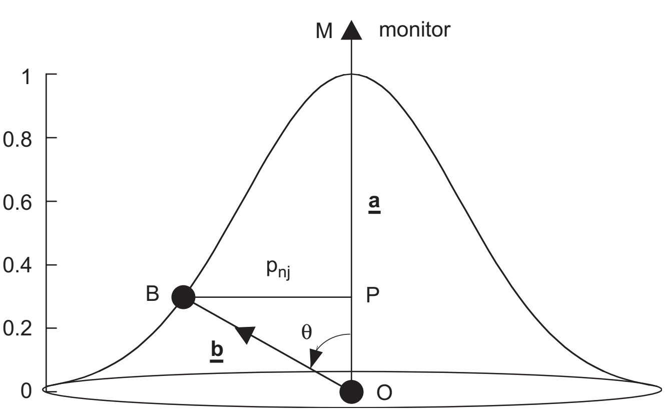

Fig. 9. Geometry used to calculate the vibration screening effect.

The off-axis (perpendicular) distance associated with the current (nth) hole and any previously initiated (jth) hole is defined as p_{nj} and is given by the length of BP. Using standard vector geometry

$$\mathrm{BP} = p_{nj} = |\underline{\boldsymbol{b}}| \sin\theta = |\underline{\boldsymbol{b}}|(1-\cos^2\theta)^{1/2}, \tag{18}$$

where $\cos\theta = \underline{\boldsymbol{a}}\,.\,\underline{\boldsymbol{b}}/(|\underline{\boldsymbol{a}}||\boldsymbol{b}|)$, and $\underline{\boldsymbol{a}}\,.\,\underline{\boldsymbol{b}}$ is the dot product of the two vectors. The screening contribution from the off-axis hole located at B can then be given by a Gaussian Bell factor, $\exp(-\kappa_{nj}{}^2)$, where $\kappa = p_{nj}/s_P$.

The treatment on blasthole screening reported in [7] yielded the following screening function, S_n, to describe the degree of screening of vibration from the nth hole along a direct path to the monitor:

$$S_n = \left(1 + g d_n^{-2}\sqrt{N_S}\right)^{-1}, \tag{19}$$

where g is a constant (= 23337) and N_S is the number of screening blastholes. However, the experimental data used to derive this function were only applicable to screening blastholes with $\theta = 0$, i.e., all such holes were on the line OM and provided maximum screening. A more realistic screening function (to account for off-axis holes) is given by

$$S_n = \left[1 + g d_n^{-2}\sqrt{\sum_{s=1}^{N_S} \frac{w_s}{\bar{w}} \exp -\left(\frac{p_{ns}}{s_P}\right)^2}\,\right]^{-1}, \tag{20}$$

where the summation over s applies only to screening holes, which are defined by $-\pi/2 < \theta < \pi/2$, i.e. those holes above the "horizontal" plane containing the point O in Fig. 9. Thus, according to Eq. (18), N_S is the subset of the previously initiated ($n-1$) holes for which $\underline{\boldsymbol{a}}\,.\,\underline{\boldsymbol{b}} > 0$.

The charge weight term $w_S/\bar{w}$ is included because it is reasonable to expect that the degree of screening is also dependent on the mass of explosive within any screening hole, and, as with the damage model (Eq. (16)), this is the simplest form. The total attenuation, E_n, of vibration produced by the nth hole is then given by the influence of screening superimposed on the cumulative damage, thus $E_n = S_n D_n{}^{-1}$. The non-linear superposition model that accounts for both cumulative damage and screening is then given by Eqs. (14), (16) and (20) as

$$V(t) = \sum_{n=1}^{N} a\left(\frac{\sqrt{W_n}}{d_n}\right)^b \left\{\left[1 + \eta\sum_{j=1}^{n-1}\frac{w_j}{\bar{w}}\left(\frac{s_P}{h_{nj}}\right)^3\right] \times \left[1 + \frac{g}{d_n^2}\sqrt{\sum_{s=1}^{N_S}\frac{w_s}{\bar{w}}\exp -\left(\frac{p_{ns}}{s_P}\right)^2}\,\right]\right\}^{-1} s_n(t-\delta_n). \tag{21}$$

If there is no cumulative damage, $\eta = 0$; furthermore, if there are no screening holes, $N_S = 0$, and under these conditions, Eq. (21) reduces to Eq. (14) as expected. Although the mechanisms of damage and screening are assumed to be linearly dependent on the charge mass (via Eqs. (16) and (20), respectively) the entire superposition (Eq. (21)) is non-linear because each blasthole contribution necessarily depends on the behaviour of all previous holes, as given by the cumulative sums within the major sum from $n = 1$ to N blastholes.

4. Application of models to observed vibration

It should be appreciated that the waveforms, $s_n(t)$, given in all superposition models are generic. In practice there will be three such waveforms from each triaxial array, and the total vibration at each monitor station can then be obtained for either the measurements or the models.

Fig. 10 shows the plan view of a centre-lift blast within a large open pit. This blast consisted of 320 holes, all of 165 mm diameter, depth 11.5 m and charge weight of 200 kg. The first initiated (lead-in) hole of the blast is shown, together with initiation times (in ms) for selected blastholes; these times were due to the use of mostly 65 ms surface delays between each of the East–West rows and 17 ms between holes in each row. The initiation clearly proceeds North and South from the lead-in hole for 733 ms, and thereafter proceeds South only, with the last hole initiating at 1911 ms. Also shown are the locations of six triaxial accelerometer arrays bonded directly to the pit wall as well as the locations of four individual (seed) blastholes, charged similarly to each hole in the main blast. The seed holes were fired after the blast, in the order shown, with a separation of 1 s between each hole, and were used to construct a waveform superposition model of the blast.

As noted in the Introduction, the pit wall region of interest is considered to be a large structure and it is required to measure and model its blast-induced accelerations. The acceleration waveforms recorded at each of the six sites have been reported in [10] along with the results of a linear superposition model. However, of particular interest in the present work is the average amplitude of acceleration along the monitored wall region and non-linear superposition models of such. In this regard, the acceleration amplitude as a function of time (i.e., envelope function) was evaluated

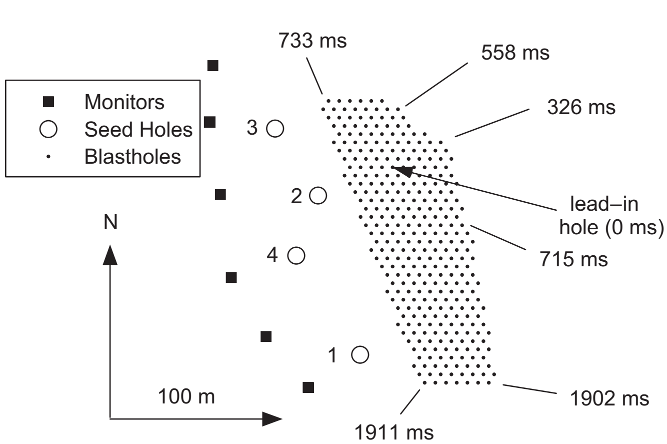

Fig. 10. The location of monitors, seed holes and the blast.

from each monitor recording using fast Hilbert Transform techniques as described in [11]. Fig. 11 shows the mean acceleration amplitude (in g) as a function of time, averaged for all monitors. The dominant contribution during the first 740 ms (approximately) is due to the fact that holes are initiating North and South, producing a large number of holes per unit time. Thereafter, this number drops as holes initiate only South. The standard linear superposition model [7] was then used to predict this mean envelope function as outlined in [11], and the results are shown in Fig. 12. Although there is a reasonable agreement in the shape between observed and modelled waveforms, the modelled amplitudes are overestimated by a factor of 3, approximately, which is consistent with previous comments made with regard to linear superposition models.

The envelope function was then predicted assuming non-linear superposition based on the charge weight scaling model (Eqs. (7), (10) and (12)) for which D was fixed at infinity (i.e., $r_n = 0$ in Eq. (7)), and is also shown in Fig. 12. Although $D = 1$ was considered a reasonable value, it produced a waveform approximately half-way between the two shown and so still resulted in a significant over-estimation compared to the observed results. Although this particular non-linear model gives an improved prediction of the peak amplitude, it gives a much inferior prediction of the shape, especially of the dominant contribution over the first 0.74 s or so. Furthermore, the almost-linear decrease in amplitude with time is unrealistic. This is a consequence of the fact, noted previously, that the charge weight scaling approach implies that subsequent holes will always give a progressively weaker contribution to the total vibration. Such a prediction is not consistent with observations.

Fig. 13 shows the predictions based on the damage model, including the influences of cumulative damage and screening (Eq. (21)). The results are given for the screening parameter, g, fixed according to the experimental results shown in [7], and the damage parameter, η, equal to 0.0 and 0.3, the latter value providing a reasonable fit to the observed peak amplitude, which is sufficiently close to the first guess value of 0.2. Comparing Figs. 12 and 13, it is interesting to note that the screening mechanism emphasises the dominant contribution over the first 0.74 s as seen in the observed results (Fig. 11) and this is direct evidence for a realistic mechanism of vibration screening. Furthermore, the shape of the waveform for the damage model is closer to the observed shape than that predicted by the charge weight model.

Fig. 14 shows the observed and modelled amplitude spectra. Again, the damage model provides the better fit to observations. The large contribution at approximately 46 Hz is due to a dominance of 17 ms (nominal) surface delays in addition to travel time delays and signal tube delays, which are approximately 1 and 4 ms, respectively.

5. Discussion and conclusions

Considering a single column of explosive it has been shown that the assumption of non-linear superposition

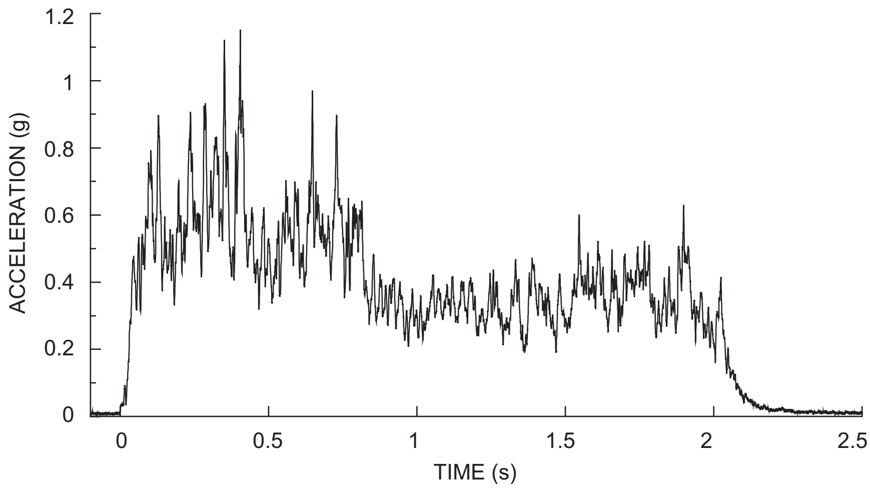

Fig. 11. The mean vibration envelope for the monitored wall region.

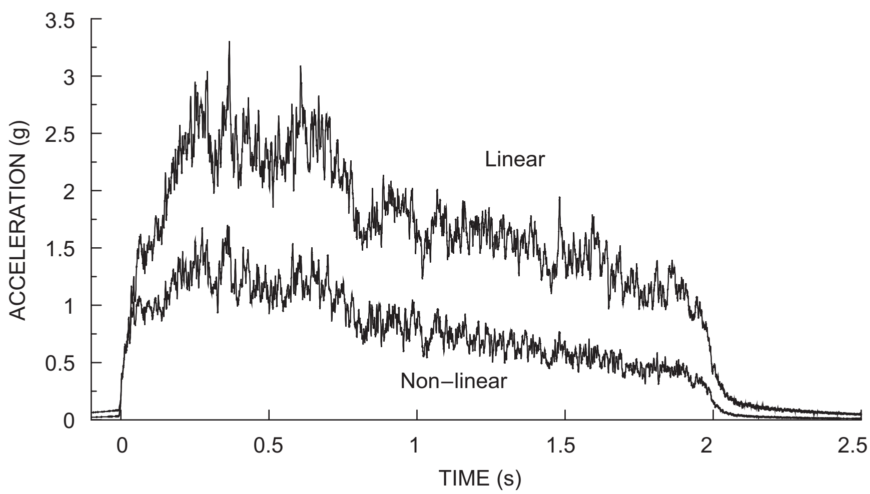

Fig. 12. Predictions for non-linear (charge weight) and linear superposition.

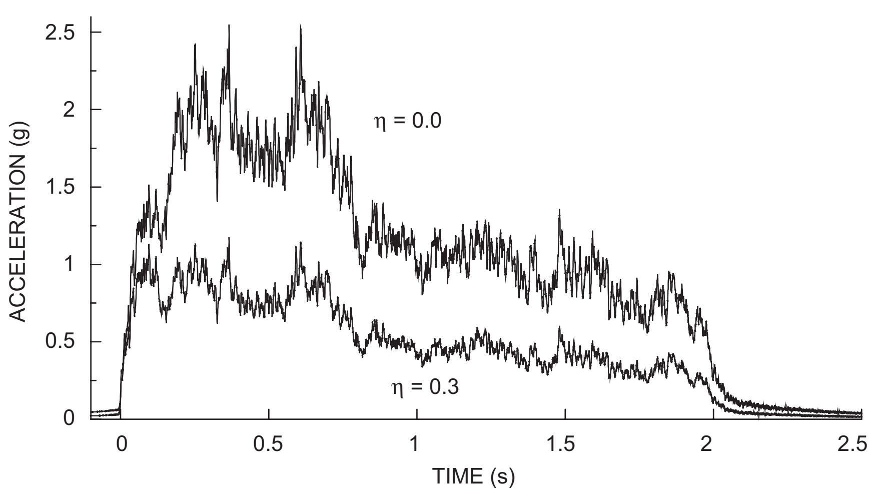

Fig. 13. Non-linear superposition predictions using the damage model.

results in a vibration lower than that predicted by linear superposition. However, it is very difficult to perform measurements close to an explosive column with the detail and sensitivity required to map the peak levels of vibration. Nevertheless, although there is no experimental data that could be used to verify the predictions, the assumption of

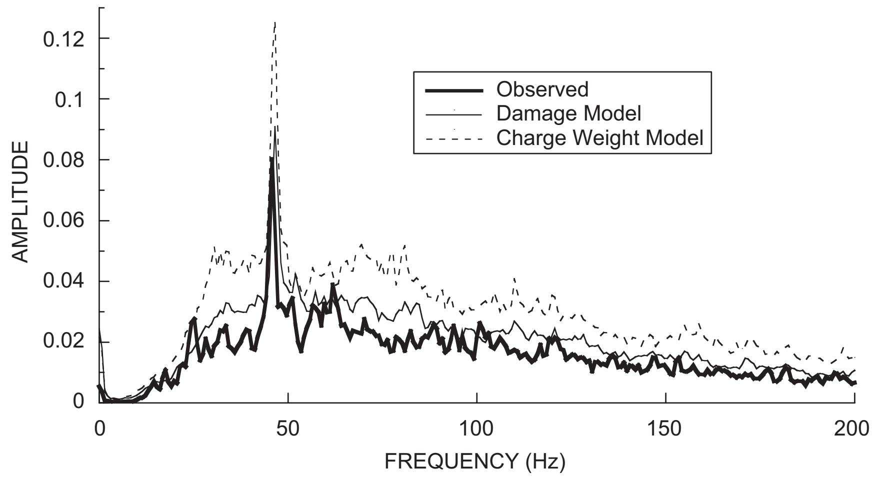

Fig. 14. Observed and modelled amplitude spectra.

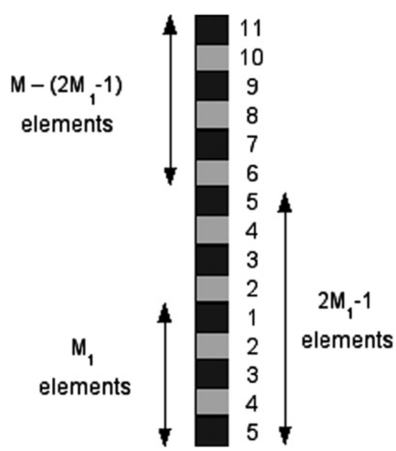

Fig. 15. Illustrating the initiation of elements within a blasthole primed at Location 1.

non-linear superposition is, at least, consistent with accepted theories on large strain behaviour of materials.

The non-linear contribution of each element given by Eq. (1) is only valid for a base-primed blasthole, and was derived in order to make comparisons with the base-primed model of Blair and Minchinton [1]. However, it is possible to apply a similar non-linear technique to a primer located at any point along the charge length, and in this case there will be initiation fronts moving in opposite directions. Fig. 15 illustrates a blasthole having $M(=15)$ elements in which the primer is located $M_1(=5)$ elements above the base. The numbers beside each element refer to the element initiation times, j, in numerical order. In this model, the first initiation is that of the primer element. This is then followed by (M_1-1) occurrences of the simultaneous initiation of two elements symmetrically either side of the primer until one end of the column has been consumed. After this time there will be $[M-(2M_1-1)]$ occurrences of single-element initiation until the entire column is consumed. The element non-linear contributions, E_j, at each initiation time, j, are now determined.

As in the case of the base-primed hole, the first (primer) element will still have an elemental contribution given by $E_1=(w_e)^A$. However, the next two elements to initiate (simultaneously) produce a total contribution given by $E_2=(3w_e)^A-(w_e)^A$, and each of these elements can then be assigned the contribution $E_2/2$. The next two elements in the sequence initiate with a total contribution given by $E_3=(5w_e)^A-(3w_e)^A$, and again, each of these elements can be assigned the contribution $E_3/2$. This process can be continued up to the stage where one end of the explosive column has been consumed. In the present example, this occurs at initiation time $j=5$, and the two elements initiating then produce a total contribution $E_5=(9w_e)^A-(7w_e)^A$. The next contribution is due to a single element and is given by $E_6=(10w_e)^A-(9w_e)^A$, and so on up to the last element to initiate with its contribution given by $E_{11}=(15w_e)^A-(14w_e)^A$. In this manner, the contribution from each element can be determined. The sum of the contributions over all initiation times, j, is then given by

$$E_T = w_e^A + \sum_{j=2}^{M_1}\{[(2j-1)w_e]^A - [(2j-3)w_e]^A\} + \sum_{j=M_1+1}^{M-M_1+1}\{[(j+M_1-1)w_e]^A - [(j+M_1-2)w_e]^A\}. \tag{22}$$

Evaluating this total sum yields $E_T=(Mw_e)^A$, as required. Furthermore, for a base-primed blasthole, $M_1=1$, and then the second term on the right-hand side of Eq. (22) vanishes. Under this condition Eq. (22) reduces to Eq. (2), as expected.

Unlike a single column of explosive in which each charge element is necessarily in the very-near-field of its neighbour, the behaviour of spatially separated charges is much more complex. In this regard two models were considered, one based on the inherent non-linearity of the charge weight scaling law, the other based on direct damage criteria. There are significant conceptual problems with the charge weight model, particularly for blastholes separated in time, simply because charge weight scaling, itself, embodies no notion of time. Thus it is not surprising that this model makes predictions that are quite unrealistic. For example, this model predicts that as the blast progresses, the vibration produced by successive holes is dramatically reduced, so much so that even moderately sized blasts would be characterised by a marked decrease in vibration with time as shown by the results for the non-linear, charge weight model in Fig. 12.

Due to the inherent problems of the charge weight model, an alternative model was developed based on direct notions of blast damage, in particular vibration screening and cumulative damage. Unlike the predictions of the charge weight model, the predictions of the damage model provide a reasonable agreement with observations, especially in shape content. Nevertheless, both models do, at least, employ waveform superposition techniques, which has allowed a rational assessment of their performance.

Non-waveform models, on the other hand, use only distances and charge weights to predict just a single point (the peak level) on an entire vibration waveform, and so do not allow any realistic assessment of their performance. In this regard, it is always possible to devise some measure of blasthole charge weights and hole–monitor distances that might be relevant to predicting the peak level. Such measures could include the mean scaled distance, minimum scaled distance, weighted scaled distance, or some combination of charge weights and distances for all those charge weights initiating within a prescribed time window [9]. However, none of these measures can give any information on the vibration behaviour with time, and so are of little use in attempting to understand the nature of blast vibration. For example, a dominant contribution in the observed vibration occurs over the first 0.8 s with a dip at approximately 0.5 s (Fig. 11). Obviously simple charge weight/distance measures give no insight to these phenomena. The waveform superposition models, however, clearly show that they are due to the particular delay sequence used.

Waveform superposition models can take a considerable amount of computing time, especially if the mean vibration properties are required over many Monte Carlo simulations. For example, the particular form used in [7] involves five convolutions for each simulation. However, it is also possible to construct simple waveform models based on either linear or non-linear superposition that require insignificant computing time. In these models, the vibration from each blasthole is represented by a waveform, $s_n(t-\delta_n)$, of a specific analytical shape. The peak value of this waveform is given by an appropriate charge weight scaling law for single blastholes. In fact it is also possible to use two waveforms to represent the vibration from each blasthole—one for P-waves, the other for S-waves, with associated time delays, δ_{nP} and δ_{nS}, respectively, to account for the different travel time component. If τ is a dimensionless time, then an example of the analytical shape, $g(\tau)$, that could be used to represent each P or S waveform is [2]

$$g(\tau) = H(\tau)e^{-\tau}(\tau^6 - 12\tau^5 + 30\tau^4), \tag{23}$$

where $H(\tau)$ is the Heaviside unit step function. The total waveform from each blasthole has the form

$$s_n(t-\delta_n) = g_{nP}(\tau_{nP} - \delta_{nP}) + g_{nS}(\tau_{nS} - \delta_{nS}). \tag{24}$$

This particular form for $s_n(t-\delta_n)$ could then be used within any of the previous models to yield a rapid solution. Only a single simulation is done for each monitoring location because random fluctuations within the model are ignored (i.e., seed wave fluctuations and delay scatter are ignored), and only a single waveform, $s_n(t-\delta_n)$, is used instead of the three usual components. Nevertheless, such a model still includes important features such as waveform superposition and delay sequence, and so is a useful tool for blast design.

Fig. 16 shows a contour map of the peak acceleration surrounding the blast of present interest. These results are obtained for the damage model described by Eq. (21), with $s_n(t-\delta_n)$ given by Eq. (24). The contour values are also in reasonable agreement with the observed peak levels at the monitor stations, which were 1.8, 1.2, 1.7, 2.6, 3.2 and 1.4 g for the monitor locations going North (Fig. 10).

The total influence of cumulative damage and screening is given by the function $S_nD_n^{-1}$, which, in turn, is given by Eqs. (16) and (20). However, these equations remain only suggestions for models to describe the mechanisms. Although the factor g in the screening mechanism was determined from experiments [7], the parameters for the Gaussian Bell and the damage Eq. (16) are merely reasonable guesses. Nevertheless it is also reasonable to suppose that S_n will be a function of w_s, s_P and p_{ns}, and D_n will be a function of w_j, s_P and h_{nj}. If these functional dependencies can be determined from separate experiments then the new function $S_n(w_s, s_P, p_{ns})D_n^{-1}(w_j, s_P, h_{nj})$ can be inserted directly into Eq. (14) to yield the damage model.

In the present treatment, non-linearity has been modelled by simply applying weighting functions (such as C_n of Eq. (10) or $S_nD_n^{-1}$ implied by Eq. (21)) to each waveform

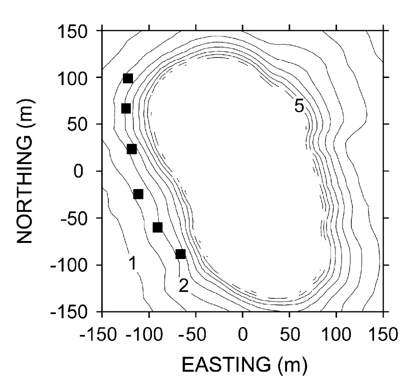

Fig. 16. Contours of peak acceleration (g) surrounding the blast; the six monitor locations are shown.

$s_n(t-\delta_n)$, without altering the waveform shape itself. However, the true nature of non-linear superposition is somewhat more complex insofar as the waveforms, themselves, may superpose linearly in time regions where both have low amplitudes and non-linearly in time regions where at least one has high amplitude, for example. This would change the shape of the resultant waveform, which then could not be described by any weighted versions of original wave shapes. Although such a non-linear superposition scheme is beyond the present scope, it might be possible to model the situation using a point-by-point amplitude criterion on the cumulative envelope function in order to decide which time portions of vibration from the currently initiating blasthole add linearly and which portions add non-linearly. Obviously, the rules for any non-linear "addition" must be specified, and the determination of such rules might be possible from appropriate modelling/experiments under the regime of large strain, where non-linear attenuation is expected to occur.

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