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Limitations of electronic delays for the control of blast vibration and fragmentation

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ABSTRACT: Delay accuracy and timing are typically not the major variables that govern blast vibration and fragmentation. The most significant variables are usually the explosive charge weight and the local geology. It is shown that there is only a restricted distance range over which electronic delays can be used to control the frequency (spectral) content of ground vibration. Furthermore, spectral control does not imply reduced levels of peak vibration. An analysis of peak vibration levels due to quarry blasting indicates that electronic delay blasts performed no better than pyrotechnic delay blasts in reducing ground vibration. It is often assumed that electronic delays will ensure stress wave superposition (addition) at specific locations in a rock mass and so promote fragmentation. However, even when using electronic delays, there remains considerable uncertainty in the stress collision point. Irrespective of this uncertainty, the results from a new analytical model of wave propagation do not support the assumption that stress wave superposition promotes fragmentation. Regarding optimum delay intervals for fragmentation, there is, unfortunately, a considerable uncertainty in the published experimental data, which makes it difficult to determine the dependence of fragmentation on delay interval. As an associated issue, data from 36 electronic delay blasts and 36 pyrotechnic delay blasts show no statistical difference in shovel dig rates. The electronic delay timing was designed on the basis of stress wave superposition, with the assumption that an improvement in muckpile fragmentation would yield an improvement in shovel dig rates.

1 INTRODUCTION

Delay accuracy and timing are just two of the many variables controlling blast vibration and fragmentation, and are typically not the major variables, especially in surface mining. In this regard, the most significant parameters are generally variations in local geology and the explosive charge weight. Other significant influences on vibration include the distance to the monitor and its local conditions (i.e. soil layer over bedrock etc), the detector coupling and the local geometry (i.e. the top of a highwall, the crest of a berm, etc). Although there might be less variables controlling fragmentation, the local geology is a very dominant influence, probably more so than for the case of vibration. The undoubted significance of these variables should add a cautionary note to any claim that controlling the delay sequence and its accuracy will guarantee reduced vibration and/or optimum fragmentation. In fact Blair & Armstrong (1999) have already shown that blast vibration is unpredictable in the presence of highly variable geology, irrespective of delay accuracy. Furthermore, they note that simply replacing a standard pyrotechnic delay sequence by the equivalent electronic delay sequence will certainly not guarantee an overall reduction in blast vibration. Nevertheless, vibration, at least, is relatively simple to measure in-situ, and given a sufficient number of blasts it would be straightforward to experimentally determine the influence of delay sequence and accuracy for a particular set of circumstances. Fragmentation, on the other hand, is difficult and expensive to measure in-situ, primarily because the three-dimensional attributes (such as sieve-passing) of all fragments of the entire muckpile should be assessed for each blast. It is because of such difficulties that small-scale explosive tests are often conducted on uniform samples, with the added advantage of eliminating major influences due to geological variations. However, the stress wave produced by the charge in each small blasthole within the sample will reflect off the nearby free faces, and this mechanism, which plagues most small-scale studies, could seriously confound the measured fragmentation. Of course it is possible to confine each face with energy-absorbing material and this could reduce the influence of reflected energy, but this is a relatively rare situation and further complicates the experimental arrangement.

Johansson et al. (2008) used cylindrical samples 0.14 m in diameter and 0.28 m in length with a single explosive source placed in a central hole, and claimed that the cylindrical specimen minimised the influence of geometry in the testing procedure. However, this claim could be questioned because a surrounding cylindrical surface is expected to be a reasonably efficient and coherent reflector of wave energy back into the central portions of the sample. In realistic field measurements, a significant portion of the stress wave energy propagates away from the fragmented zone, whereas in most small-scale tests much of this energy is either trapped within the finite volume (to promote fragmentation) or transformed into kinetic energy of fragments.

As an example of unconfined free faces, Katsabanis et al. (2006) conducted a series of small-scale tests on granodiorite blocks of dimensions 0.92 m × 0.36 m × 0.21 m, with 23 blastholes per block, and with specified uniform delay intervals ranging from 0 μs to 1000 μs. However, each of the 23 outgoing compressive stress waves separated by the specified primary delay interval will also have a sequence of 6 secondary reflected tensile stress waves, one off each face. Unfortunately, this secondary sequence will have a highly non-uniform delay interval dependent on the location of each blasthole to each free face. According to their measured p-wave velocity of 3900 m/s for the intact material, the time delay between the primary and reflected wave could lie within the approximate range of 15 μs to 225 μs. Thus fragmentation within the block is unavoidably influenced by events at non-uniform delay intervals, and this reflection mechanism, alone, suggests that the resulting fragmentation could be insensitive to the delay interval between blasthole initiations. Hence caution should be exercised when inferring optimum delay sequences for full-scale fragmentation based on such tests.

Irrespective of any concerns regarding wave reflections, the experimental data from small-scale tests that purport to show a trend in fragmentation with delay interval are often scant (due to the difficulty of conducting such experiments), and also have a significant scatter with regard to the trend being inferred. This is certainly the case for most of the results shown in Tidman (1991), who summarised data from 10 major studies for which it would seem imperative that acceptable statistical methods be used to make any inferences. Unfortunately, it appears that no such analysis has been conducted. Although the total data set was relatively large (some 50 data points), the overall scatter was significant, and the number of data points from individual tests was small. A small number of these tests showed an increase in fragmentation with decreasing delay interval, however the majority did not. Consequently, when taken as a whole, it is difficult to be convinced that there is any realistic and consistent trend in the data. As an example in this regard, it is worthwhile conducting a statistical assessment of some more recent data given in Figure 6 of Katsabanis et al. (2006). This figure shows only 7 data points for each of two given sieve passing sizes (80%, 50%) as a function of delay interval, and the authors claimed that the results showed a meaningful trend (i.e. they implied a decrease in fragmentation with increasing delay interval). This data for the 80% passing size is reproduced in Figure 1, and shows that the datum at zero delay interval lies significantly above all other data points. If this single point is removed then an approximate insight to the remaining (but very scant) data can be obtained using standard statistical methods (see for example, Davies & Goldsmith 1972). Firstly, very conservative (70%) upper and lower bounds can be placed on the mean (regression) as illustrated, and implies that a perfectly horizontal line (i.e. no dependence of fragmentation on delay interval) is also consistent with these bounds. Secondly, Hypothesis testing under standard linear regression analysis can be applied in which a probability, P, is calculated based on the F-statistic. In this case the Null Hypothesis is that there is no meaningful relationship between delay interval and fragmentation, and current wisdom is to reject the Null Hypothesis only if $P < 0.05$. The set of those 6 data points for a non-zero delay interval yields $P = 0.85$ for the 80% sieve passing fraction and $P = 0.56$ for the 50% passing fraction (data not shown) and thus there is no evidence whatsoever to reject the Null Hypothesis. Of course, in reaching these statistical conclusions the data point at zero-delay was discarded from each set. There are three comments to make in this regard. Firstly, an argument could be made that these points ought to be rejected anyway because they represent the case of instantaneous initiation, which is physically different to all other cases. Secondly, the inclusion of a single extra point, lying well above all others, makes it quite difficult (if not impossible) to select an underlying model of the data. In this regard, linear regression is probably not a reasonable model for all the data shown in Figure 1. Thirdly, and most importantly, even if some underlying model could be found, it would be difficult to accept that the inclusion of a single extra point at the zero-delay interval could significantly alter the previous statistical assessment of the 6-member set. Based on this statistical evidence, one would be perfectly entitled to claim that the data shown in Figure 6 (and Figure 1, for that matter) of Katsabanis et al. (2006) does not support the assumption that fragmentation is a function of delay interval. Their Figure 1 also shows a curve fit that is quite unconvincing in view of the scatter in the data.

The scant data of Katsabanis et al. (2006) suggests that fragmentation is coarsest for a zero delay

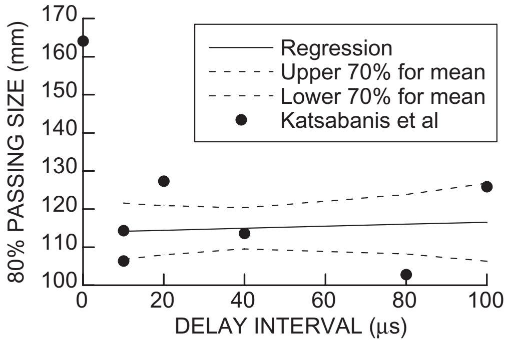


Figure 1. Statistical analysis of the data from Katsabanis et al. (2006).

interval, however this is not verified by all the data sets presented in Tidman (1991). Although it seems reasonable to assume poor fragmentation for zero delay interval, this assumption should also be tested by further experimental data and appropriate statistical analysis.

The modelling of fragmentation is probably even more difficult than its in-situ measurement. In this regard there are two important requirements for any model. Firstly, the model must incorporate mechanisms such as cracking and extension of cracks by gas loading. Although there are two-dimensional models that meet this requirement (see, for example, Minchinton & Lynch 1996), there are, as yet, no satisfactory three-dimensional models.

Secondly, the model must be able to resolve the high-frequency stress waves propagated from the blasthole. The explosive pressure load can be classified by P_{VN} (the von Neumann borehole pressure at the detonation front), t_{CJ} and P_{CJ}, where t_{CJ} is the time taken for the pressure to decay to the CJ (Chapman-Jouguet) value, P_{CJ}. As an example, for ANFO in an 89 mm diameter blasthole, $P_{VN} = 14.2$ GPa, $t_{CJ} = 4.2$ μs and $P_{CJ} = 5.7$ GPa. Using the explosive source function given by Blair (2007) it can be shown that the rise time of this pressure load is 2.9 μs, and its total duration is 21 μs. Current models of wave radiation from a blasthole simply do not use source functions with such short rise times and durations. For example, Rossmanith & Kouzniak (2004) used an unrealistic triangular pulse shape with a rise time of 83 μs and duration of 913 μs. Although the models of Blair & Minchinton (1996, 2006) used a more acceptable pulse shape, its rise time and duration were unrealistically large at 280 μs, and 2028 μs, respectively. Hence all these models will grossly overestimate the regions of wave overlap/superposition near an explosive charge. This is a significant limitation especially if wave superposition is assumed to promote fragmentation.

Unfortunately, in the marketing of electronic delays for the control of fragmentation, in particular, science is too often replaced by claims based on anecdotal evidence, unrealistic modelling, insufficient data or questionable inferences drawn from data that has significant scatter. In light of all previous discussion, there are some very sound reasons to suggest that electronic delays might only have a limited role, at best, in controlling blast vibration and fragmentation. The main aim of the present work is to investigate these limitations. As a part of this aim, results will be presented from a new analytical model that has a realistic pressure load, and these results will be discussed with regard to electronic delays and fragmentation.

2 BLAST VIBRATION CONTROL

There are two quite distinct aspects to controlling blast vibration. The first aspect is spectral control, i.e. the control of the frequency content, and is relevant to vibrations in (resonant) structures. The second aspect is peak level control, i.e. the control of the peak level of vibration (usually the vector peak particle velocity, vppv), and is more relevant to the requirements of regulatory compliance. In this case the monitoring region could be either non-resonant (such as flat open ground in uniform geology) or resonant (such as an urban dwelling or a soil layer over base rock). Unfortunately, there can be no simple relationship between spectral control and peak level control. In other words there can be no simple relationship between frequency content of a vibration waveform and its vppv (Blair 2004). Nevertheless, when monitoring vibrations in resonant regions, it is reasonable to expect that if the spectral output of blast vibration excites a particular resonance, then the measured vppv could increase. In this case, spectral control could imply a degree of peak level control. However, compliance monitoring is often conducted well away from structures, and in such cases spectral control would not imply peak level control. Both these aspects are now considered in detail.

2.1 *Spectral control*

Although Blair (1993) and Blair & Armstrong (1999) have discussed the spectral control of ground vibration in much detail, they did not emphasise the inherent (ideal geology) limitations of spectral control to any great extent. These inherent limitations are now considered, and are probably best introduced by considering a popular misconception, which can be loosely paraphrased as: Use accurate delays and reduce the delay intervals; as the delay intervals become smaller, the frequencies become higher and will be more readily attenuated in the ground, thus reducing vibrations. Another version of this misconception is listed on a product website which claims that the use of electronic delays will allow vibrations to be typically shifted to higher, less damaging frequencies.

It is interesting to note that the ultimate in fast firing is to have a zero delay between blastholes, which, according to the misconception, would imply infinite frequencies. Although this simple example is quite sufficient to expose the misconception, it is worthwhile examining the physics in some detail. In this regard Figure 2 shows the total vibration waveform from 10 blastholes, uniformly delayed by 50, 20, 10, 5 and 2 ms.

All blastholes are charged identically and assumed to be equidistant from the monitor and located in uniform, non-attenuating geology. Furthermore, each blasthole is assumed to produce a single vibration waveform given by the impulse response of a Butterworth filter that is band-limited over the range 4 Hz to 100 Hz. The lower response is selected because it mimics the typical response of most geophones and the upper response is selected because it approximates the upper frequency limit of amplitudes generally observed in compliance monitoring (repeats of the first 50 ms of this filter response are shown in the lower plot). Figure 2 clearly shows that there can be a significant increase in the low-frequency content for small delay intervals, which, in fact, is precisely opposite to the claim implied by the misconception.

Figure 3 shows the amplitude spectra for the delayed blasts, in which a larger range of delay intervals has been modelled. There are 9 spectral amplitude plots, all offset, and for uniform delay blasting of 100 ms down to 0.2 ms. The dashed line on the lower plot shows N times the amplitude of the single hole response, where N is the number of blastholes. In other words, the dashed spectrum is that expected if all N holes were initiated simultaneously. The spectral banding for the larger delay intervals is quite obvious, with dominant contributions spaced according to the delay interval between blastholes. A detailed analysis of this banding for a given delay interval has been previously given (for example, see Blair 1993) and will not be repeated here.

The important insight to gain from Figure 3 is that as the delay interval decreases, all frequencies (including the very low frequencies) are stretched out along the horizontal axis, whilst all amplitudes are constrained to remain under the dashed outline. It is also clear that as the delay interval approaches zero, the amplitude response approaches the case for instantaneous initiation (dashed line), as expected. This spectral plot clearly shows that there is a significant increase in low-frequency content as the delay interval decreases, which is consistent with the time

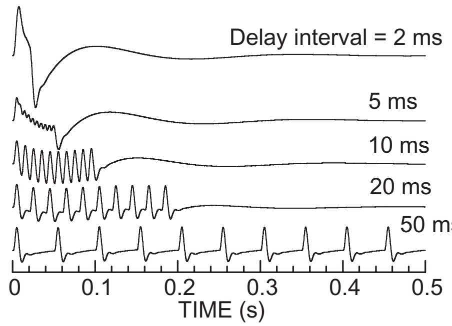


Figure 2. A synthetic vibration waveform for 10 blastholes uniformly delayed.

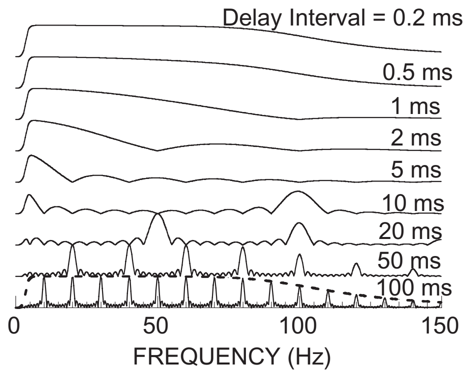


Figure 3. Amplitude spectra of the synthetic vibration waveforms.

waveforms shown in Figure 2. The spectral plot of Figure 3 also explains some of the apparent anomalies seen on vibration records of electronic short-delay blasts in which there can be a significant level of low frequency energy that seems to appear from nowhere. This is due to the limited frequency response of geophones in particular, which are incapable of detecting amplitudes accurately below about 4 Hz or so in a slow-fired blast. However, when firing fast, the amplitudes at ultra-low (un-detected) frequencies are now shifted higher into the low-frequency range, which can be measured by the detectors. Figure 3 shows that this low-frequency intrusion begins to occur for delay intervals of 10 ms or lower.

Siskind et al. (1980) measured the vibration response a large variety of residential structures and determined resonant frequencies in the range 4 Hz to 28 Hz. According to the simple model illustrated by the spectral response in Figure 3, delay intervals would need to be less than 50 ms and somewhat greater than 10 ms in order to avoid exciting these structural resonances. This limitation on delay intervals implies a degradation of the spectral control available even when using ideal (no scatter) delays.

The models discussed so far have involved blastholes of equal charge weights, and equal distances from the monitor. It is now shown that the spectral control is further degraded if the distances or charge weights are unequal (i.e. the scaled distances are unequal), as is typical in many blasts, especially for measurements in the near-field. In order to illustrate this point, Figure 4 shows 10 blastholes, each separated by 5 m, charged identically, initiated from west to east and detected by 5 monitors placed on the circumference of a semicircle of radius 27.5 m. Each monitor location is specified by a rotation angle, θ, defined positive anticlockwise from East. The figure also shows the modelled vibration envelope function recorded by each monitor assuming ideal (no-scatter) delays and uniform geology. These envelope functions are constructed from the measured triaxial components of velocity from a single blasthole (seed) firing in massive sandstone. It is clear that each station receives varying responses from each blasthole, and that monitors located at $\theta = 0$ or 180 degrees have the largest non-uniformity in responses and the monitor at $\theta = 90$ degrees has the most uniform responses.

At this stage it is instructive to introduce the notion of a space-frequency plot, which shows how the amplitude spectrum changes as a function of rotation angle, θ, i.e. as a function of monitor location with respect to the blast. In this case the monitoring space traversed is the circumference of a semicircle, and is specified by the circle radius and the rotation angle. Figure 5a shows the space-frequency results for a monitoring radius of 27.5 m (as illustrated in Figure 3) and Figure 5b shows the results for a monitoring radius of 100 m.

It is obvious from Figure 5a that the spectral banding is least smeared for $\theta = 90$ degrees, i.e. at that monitoring station for which each blasthole vibration is most uniform. Clear spectral banding implies good spectral control. Thus there is poor spectral control (i.e. a significant smearing of spectral components) for $\theta = 0$ or 180 degrees and this is due to the non-uniform responses from each blasthole recorded at these monitors. As expected, this spectral smearing is significantly reduced when monitors are placed at a much greater radius (Figure 5b), because each hole-monitor distance is then approximately equal and so the vibration response from each hole is similar for all rotation angles.

The single row blast shown in Figure 4 is useful in highlighting the influence due to varying blasthole distances to a given monitor, however, it is hardly a practical design. Figure 6 shows a small blast of 96 holes, with a 20 ms delay between each row (on the vertical zip line) and 100 ms between each hole within a (horizontal) row. The monitors are spaced along the circumference of a circle of prescribed radius. Figure 7a shows the space-frequency diagram for monitors placed on a radius of 60 m and Figure 7b shows the results when the monitoring radius is increased to 200 m. Figure 7c shows the results for a monitoring radius of 200 m in variable geology that is modelled by introducing a 20% random fluctuation on each waveform signature, and even this mild degree of variability smears out the spectral control. For interest, Figure 7d shows the results for blasting with pyrotechnic delays (scatter assumed to be 1%) in uniform geology. Thus reduced accuracy of delays will also degrade the spectral control.

Assuming uniform geology and accurate delays, the present results imply that the spectral control improves as the blast-monitor distance increases. However, if the distance is too great, the vibration waveform from each blasthole broadens significantly due to ground attenuation, and this, itself, also reduces the spectral control that can

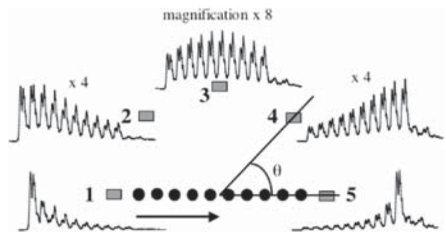
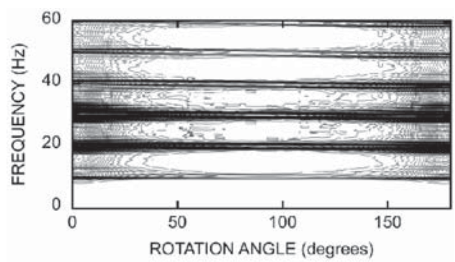
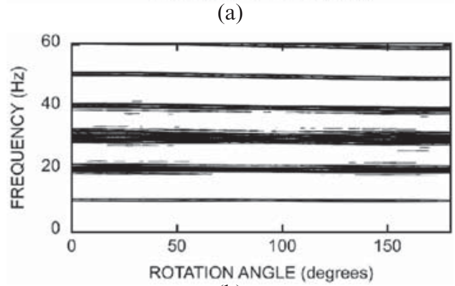


Figure 4. Vibration from10 blastholes detected at 5 locations on a semicircle.



(a)



(b)

Figure 5. Space-frequency plots, ideal delays, ideal geology; (a) monitoring radius 27.5 m; (b) monitoring radius 100 m.

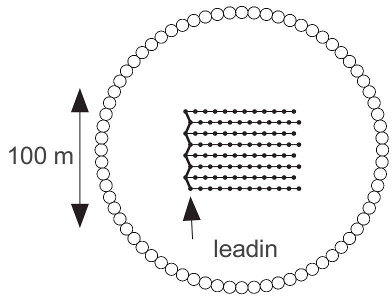


Figure 6. 96-hole blast, monitors on circle.

be achieved with the delay initiation sequence. In order to illustrate this effect, Figure 8a shows the normalised envelope of vibration from a single blasthole fired in coal overburden material and monitored at various distances. The waveform broadening with distance is quite apparent. The spectral content of these waveforms is shown in Figure 8b as the mean of the triaxial (L, T, V) components, and it is obvious that the waveform broadening is equivalent to a decrease in frequency content as is expected. In fact for distances ≥500 m, there is little contribution from each blasthole for frequencies above 10 Hz. Thus any production blast that is comprised of such blastholes will also not produce energy above 10 Hz, irrespective of the delay sequence and its accuracy. In other words, there is no possibility of channelling energy into higher frequencies using smaller delay intervals. Thus vibration monitoring at large distances places a severe restriction on the degree of spectral control available. It is worthwhile to recollect at this stage that vibration monitoring at small distances was also shown to place a severe restriction on spectral control, even in ideal geology, and is due to the non-uniform response of each blasthole (Figure 4). Thus there is only a restricted distance range, somewhere in the medium-field, for which accurate delays will be an aid in controlling the spectral output of vibration.

2.2 *Peak level control*

As noted previously, spectral control does not imply peak level control. However, a lack of spectral control (i.e. for blasts comprised of broad waveforms) could well imply a lack of peak level control. In this regard it is instructive to analyse a 10-hole blast having 100 ms delay between holes (similar to the design shown in Figure 4) and having those seed functions used to construct Figure 8a. Figure 9 shows the normalised vibration envelope assuming ideal delays, and it is obvious that the individual holes can be detected only for a monitoring distance of 200 m; for distances of 500 m or

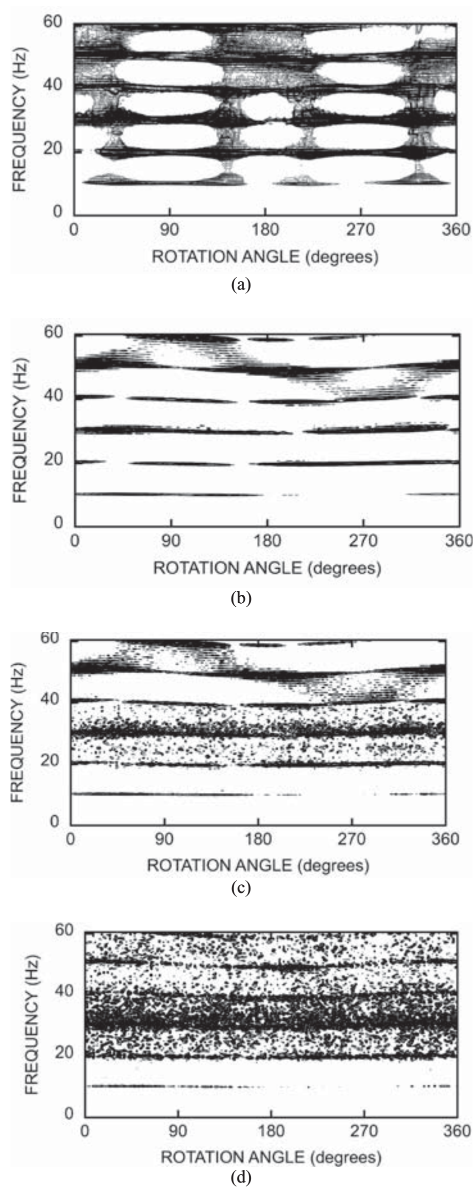


Figure 7. Space frequency plots. (a) ideal delays, ideal geology, monitoring radius 60 m; (b) ideal delays, ideal geology, monitoring radius 200 m; (c) variable geology, ideal delays, monitoring radius 200 m; (d) uniform geology, pyrotechnic delays, monitoring radius 200 m.

greater the individual holes are smeared throughout the broad nature of the waveforms which should be insensitive to any scatter in delay time intervals. This viewpoint is entirely consistent with the previous viewpoint regarding the lack of spectral control.

In order to demonstrate the insensitivity of far-field vibration to delay scatter, a Monte Carlo wave superposition model was constructed for the 10 blastholes with a nominal delay interval of 100 ms. Figure 10 shows the normalised envelope of vibration for monitoring distances of 500 m and 4000 m, predicted using electronic delays and pyrotechnic delays. These envelope functions represent the peak amplitude predicted during 100 simulations of each delay type, i.e. they are peak-detection envelopes, which show the maximum vibration amplitude at any particular time. These results clearly demonstrate that the peak vibration level is very insensitive to delay accuracy for this simple blast. However, it is worthwhile considering a more realistic blast having 96 blastholes in the drill pattern shown in Figure 6, and monitored at 500 m. However, the zip line now goes North up the centre of the pattern, and two different delay sequences were modelled. Sequence 1 consisted of a 42 ms delay between rows and 17 ms between holes in each row initiating to the East. For holes in each row initiating to the West there was one offset delay of 25 ms followed by the remainder at 17 ms. Sequence 2 was a time-expanded version of Sequence 1 and consisted of an inter-row delay of 100 ms, an inter-hole delay of 42 ms and an offset delay of 65 ms. These types of blast designs are quite realistic for quarry and even open pit operations.

Figure 11 shows the peak envelope results, all normalised by the same factor in order to compare the amplitudes for both sequences and both delay types (pyrotechnic, electronic). As expected, the time-expanded Sequence 2 gives lower amplitudes simply because it reduces the crowding of blasthole waveforms.

Again, it is quite obvious that the peak vibration level is very insensitive to delay accuracy for these realistic blast designs. It is also very pertinent to note here that expanded pyrotechnic delay sequences can be just as effective as equivalent electronic delay sequences in reducing wave crowding, which, in turn, reduces vibration. Yet again, for these large enough monitoring distances (500 m), electronic delays provide no clear advantage over pyrotechnic delays for the control of peak levels of ground vibration. Furthermore, in highly attenuating geology the vibration waveforms will be broadened at even moderate distances, which implies that pyrotechnic delays will also compete well with electronic delays for these distances

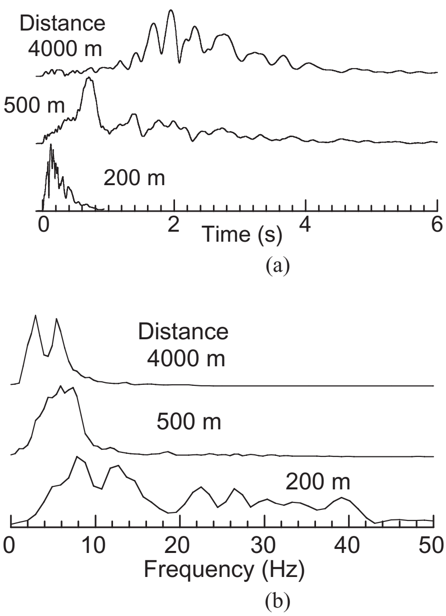


Figure 8. (a) vibration envelopes from single blastholes; (b) the mean amplitude spectra.

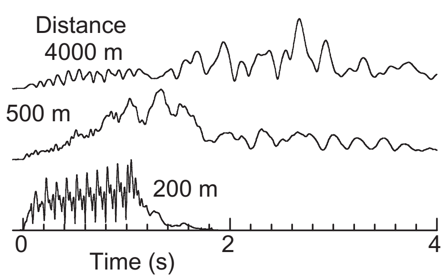


Figure 9. Normalised vibration envelopes, 10 hole blast, 100 ms uniform delay interval.

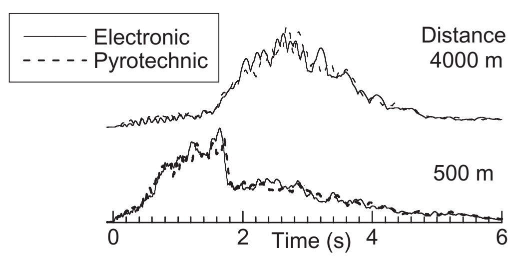


Figure 10. Peak vibration envelopes.

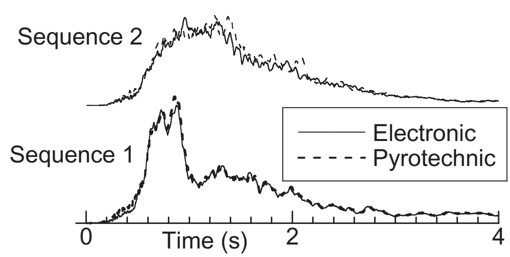


Figure 11. Peak vibration envelopes for the two sequences.

In a practical sense it is expected that influences due to geology, blast-monitor distance and rotation angle, θ, will most likely be present to some extent in any blast. In this regard, Figure 12 shows a plot of log(vppv) versus log(scaled distance) obtained from 446 pyrotechnic delay blasts and 258 electronic delay blasts fired in the same set of quarries; the solid curves are linear regression fits to the data. Figure 13 shows a probit plot of the residuals and the non-linearity of this plot, especially the deviation in the tails, visually indicates that the scatter does not obey a normal distribution (Miller 1998). The degree of normality can be formally quantified by evaluating the Shapiro-Wilk W statistic and its associated probability, P_W, for the data set in question (see for example, Royston 1982). For both data sets it was found that $P_W < 0.005$ and so the assumption of normality must be rejected (Miller 1998). Thus the standard statistical tests, based on least squares regression (Davies & Goldsmith 1972), cannot be used to make inferences on vibration differences between the pyrotechnic and electronic delay blasts. In this case, nonparametric (distribution free) statistics should be applied. The slope and intercept of each plot were estimated using Theil-Sen statistics (Sprent 1989), and bootstrap techniques (Efron & Tibshirani 1993) were used to determine confidence intervals and associated probabilities, P_B, for these statistics. It was found that $P_B < 0.005$ for the differences in slope as well as intercept, and so the assumption of statistical similarity between electronic and pyrotechnic data sets must be rejected. However, although the differences are statistically significant they are not practically significant. There is certainly nothing to suggest that the electronic delay blasts produced a lower vibration than that of the pyrotechnic delay blasts, especially for high vibration levels, which are pertinent to both compliance and structural damage.

The significant scatter in the results of Figure 12 is due to one or more of the major influences discussed previously, and strongly suggests that a large number of blasts would generally be required in order to detect statistically meaningful differences in vibration between pyrotechnic and electronic delays at any specific site. In view of all previous analysis and discussion it is not surprising that delay accuracy plays only a very minor role in controlling blast vibration. In fact, the vibration screening effect due to damage from prior blastholes in a sequence (Blair 2008) has a far greater influence than delay accuracy or flexibility on the reduction of ground vibration. Furthermore, sequences designed to take advantage of this screening effect can be implemented using either electronic or pyrotechnic delay sequences.

Naturally there are situations (not necessarily related to vibration or fragmentation) where electronic delays can be superior to pyrotechnic delays,

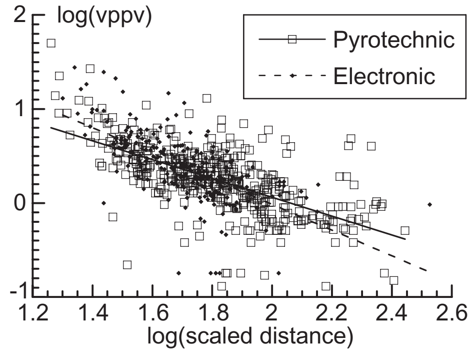


Figure 12. Peak vibration for pyrotechnic and electronic delay sequences fired in the same set of quarries.

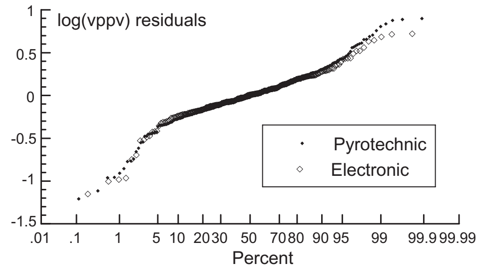


Figure 13. A probit plot of the residuals in log(vppv).

such as in underground mass blasts and large cast shots in surface coalmines. However, in most other cases, pyrotechnic delays perform as satisfactorily as electronic delays with regard to vibration provided reasonable initiation sequences are employed. Of course there are some delay sequences that could be considered unreasonable with regard to vibration. For example, in underground development blasts, long-period (LP) electric detonators are sometimes used. These are pyrotechnic delay devices with a rather restricted range in selectable times, which invariably means that there can be a set of blastholes (sometimes >5) close in space and initiating at the same nominal delay time. Although the pyrotechnic delay scatter ensures that near-instantaneous initiation within a set would be highly improbable, the vibration waves from these holes could still crowd in time to produce an excessive vibration. In this situation a carefully delayed sequence of electronic detonators would significantly reduce the average vibration, not because of the delay accuracy *per se*, but rather because of the flexibility to choose a sequence with a reasonable delay time between all blastholes. However, it is also pertinent to note that there are alternative pyrotechnic delay systems for development blasts that have an improved flexibility over the LP series. In a practical sense, these alternative systems should perform just as well as electronic delay systems for the control of vibration due to development blasts, especially at distant monitoring stations.

3 FRAGMENTATION CONTROL

There are at least three major assumptions implied in most arguments to support the use of electronic delays to optimise fragmentation. Firstly, there is some known delay interval, or range of delay intervals, that will produce optimum fragmentation. Secondly, the scatter in electronic delay times is negligible. Thirdly, superposition of stress waves from individual blastholes will promote overall fragmentation within an extended volume of rock.

The first assumption was discussed in detail in the Introduction where evidence was presented to demonstrate that this assumption is unsound, and much more experimental work and appropriate statistical analysis is required in order to obtain definitive conclusions. With regard to the second assumption it should be noted that it is impossible for any timing device to have perfect accuracy. Electronic delays are clock-based systems and so they must have a scatter in times that is proportional to the desired delay time; thus it makes no sense to claim an absolute accuracy, such as ± 1 ms, for these devices, as is often done. Furthermore, accuracies quoted in this form are statistically imprecise. The scatter in delay times for any detonator within a batch is properly classified by a coefficient of variation, which is defined as σ/μ, where σ is the standard deviation in initiation times and μ is the mean (or nominal) initiation time of the batch. In this regard σ/μ for pyrotechnic delays generally lies in the range of 1% to 4%, whereas σ/μ for electronic delays generally lies in the range of 0.02% to 0.05%. Laboratory tests on either type of delay would probably yield values closer to the lower limit whereas field implementations probably yield values closer to the upper limit, especially in high-temperature environments. Thus if $\sigma/\mu = 0.04\%$ for an electronic delay of nominal time $\mu = 3$ s (as might be required during a moderate-sized blast) then its standard deviation would be 1.2 ms. Assuming that the scatter in delay times is given by a normal distribution, then there is a 31% chance that a 3 s detonator will fire 1.2 ms either above or below its nominal value. This simple example shows that even electronic delays are not accurate enough to maintain a uniform delay interval of the order of milliseconds throughout many blasts. Clearly, this has implications for fast firing in any attempt to optimise fragmentation. This issue is discussed later in more detail.

With regard to the third assumption (superposition of stress waves) there are further points to consider. For example, in modelling and conceptual ideas, it is often assumed that all blastholes produce the same stress waveshape and superpose in an ideal manner to create a zone of high stress via constructive interference. However, it is well known that even identically charged blastholes in uniform geology can produce waveforms of markedly different shape, and that these waveforms will also suffer significant changes as they travel between interacting blastholes in ground dynamically altering as the blast progresses (Blair 1993). Thus it is difficult to accept that ideal stress wave superposition will occur to any great degree. However, even if ideal stress wave superposition did occur, significant problems still remain with the idea that such superposition promotes fragmentation in an extended volume of rock. These problems are now discussed with reference to the results from a new analytical model of ideal stress wave propagation from a blasthole. This model is based on the method described in Blair (2007) that has been altered to include solutions to the full stress tensor under the influence of an extended charge with a finite velocity of detonation (VoD) within a viscoelastic rock mass. The dynamic stresses can be calculated by evaluating the appropriate displacement derivatives in the (r, z) directions of a cylindrical coordinate system for which u_r and u_z are the only displacements of interest. In particular, the maximum shear stress, σ_m, in a rock of shear modulus, μ, is defined as:

$$\sigma_m(r,z,t) = 2\mu\sqrt{A_1^2 + A_2^2} \tag{1}$$

where

$$A_1 = \frac{\partial u_r(r,z,t)}{\partial z} + \frac{\partial u_z(r,z,t)}{\partial r} \tag{2}$$

$$A_2 = \frac{\partial u_r(r,z,t)}{\partial r} - \frac{\partial u_z(r,z,t)}{\partial z} \tag{3}$$

The maximum shear stress is chosen as the wave attribute of interest because it is a stress invariant (i.e. it does not depend upon the rotation angle at any particular point in the rock mass), and, furthermore, can be associated with the strength of materials (Timoshenko & Goodier 1971). Figure 14 shows contour plots of the dimensionless maximum shear stress, $\sigma_m\ (r, z, t)/2\mu$, due to a base primer located at (x, y) coordinates $(0, -3)$ in a 6 m vertical blasthole. Figure 15 shows the results for the simultaneous initiation of a base and top primer. The elastic properties of the rock mass are $\mu = 16.9$ GPa and Poisson's ratio, ν, is 0.25; this gives a p-wave velocity, V_P, of 4.5 km/s. The rock is also assumed to have a seismic Q of 100, typical of the viscoelastic response for a competent material. In each figure, the stress wave propagation is shown at the four time instants when the head (leading) wave is at 2 m, 4 m, 6 m and 7 m from the base of the blasthole. These results are calculated for an ANFO explosive in an 89 mm diameter blasthole for which the VoD, V_D, is assumed to be 4.5 km/s = V_P; all other explosive properties have been given previously. The wave contours are plotted over a space of 4 m × 8 m using 161 × 321 grid points. All contours are plotted from 0.005 to 1.0 in increments of 0.005, and thus the stress level is proportional to the darkness shown in the contours. A contour value of 1.0 occurs where the stress wave amplitude in the medium equals the material shear modulus.

The fastest moving (head) wave in this example is a spherically spreading p-wave, and this produces a Mach cone due to the slower shear-vertical (SV) wave in the surrounding medium. The half-angle of this S-wave Mach cone is given by $\phi = \arcsin(V_S/V_D)$. For the Mach cones in Figures 14 and 15, $\phi = \arcsin(V_S/V_P) = \arcsin(1/\sqrt{3}) = 35.3$ degrees.

The travelling ring load ceases when it has consumed the explosive and from then onwards the wavefronts weaken with distance due to geometric spreading. Figure 14 shows that the single primer produces the dominant stress because it is able to generate the SV-Mach wave over a 6 m column length. However, for the dual-priming case shown in Figure 15, each oppositely directed ring load travels only 3 m before exhausting all the explosive at the point of wave collision ($y = 0$). From then onwards the propagating stress waves reduce rapidly in strength primarily due to geometric spreading, thus the SV Mach cone is much weaker above the original charge than it was for the case of single base priming. Hence such dual priming will produce reduced fragmentation in regions above the central plane of the blasthole. Although the stress wave propagation downwards might give improved fragmentation for those regions below the charge centre, it is reasonable to suspect that the overall volume fragmentation will be worse than it would have been for the case of base priming alone, where a high amplitude SV wave propagates upwards and away from the blasthole. Any surface reflection of this wave would also promote further fragmentation in travelling back down through the medium.

The model used by Rossmanith & Kouzniak (2004) is only valid for infinite charge lengths for which the p- and SV-Mach waves are always fully developed, radiating out through the entire medium and maintaining constant amplitude as they progress in space along any one blasthole. However, the present model shows the more realistic situation in which there is a build-up of these waves as the detonation progresses away from the primer, producing a maximum volume influence at the instant all the charge is consumed.

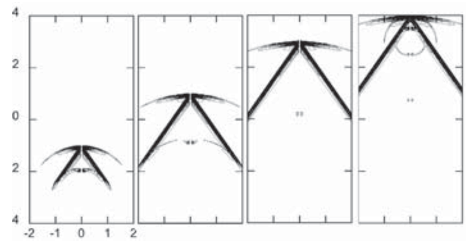


Figure 14. Four stages of wavefront progression due to a single base primer.

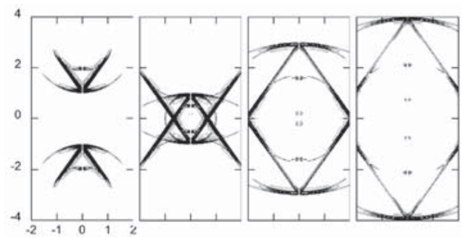


Figure 15. Wavefront progression due to a top and base primer simultaneously initiated.

From then onwards all stress waves decrease due to geometric spreading. The model of Rossmanith & Kouzniak (2004) cannot account for these effects and so it significantly overestimates the influence of stress wave interaction in this regard.

The stages of stress wave progression in Figure 14 can also be used to investigate stress wave interaction between neighbouring holes. For example, neighbouring holes space-delayed by 2 m (i.e. time-delayed by 0.44 ms in this case) would not have their SV-Mach cones intersecting in the first two frames of Figure 14 simply because the waves in the first frame have not had time to radiate far from the blasthole. However, in the Rossmanith-Kouzniak model the SV-Mach waves from two neighbouring holes will always intersect somewhere, and this, again, shows how this model overestimates the degree of stress wave interaction.

The wavefront contours in Figures 14 and 15 also show that each stress wave is highly localised in space (approximately 0.2 m in width) and this implies a time width of 44 μs at any particular location due to the p-wave velocity of 4500 m/s. Thus even the best of the electronic delays are not accurate enough to guarantee waveform collision at precise locations. In fact, as shown previously, there is a 31% chance that a dual-primed electronic delay blasthole in moderate-sized blasts will fire with a time difference greater than 1.2 ms between top and base primers that are planned to fire simultaneously. During this time difference the stress wave from the first initiated primer will have travelled at least 5.4 m and so the stress wave collision point can never be guaranteed, even in an approximate sense. This same argument applies equally well to stress wave interaction between neighbouring holes.

The upshot of all this detail is that the stress waves are not similar in shape (due to variations in local geology and dynamic alteration of the ground as the blast proceeds) and even if they were similar, the collision region will be varying by random and significant amounts as the blast progresses. This occurs even using electronic delays, for either top/base initiation in a single blasthole or planned delays between neighbouring holes. As another point, even under ideal conditions the amplitude of the interacting stress waves will vary significantly, depending on the consumed charge lengths at the time and place of collision. Finally, even if the regions of wave collision could be predicted, they would be highly localised and so comprise only a small fraction of the total volume of rock to be fragmented. All this evidence shows that stress wave collision certainly plays no predictable role in rock fragmentation by blasting, and most likely plays no significant role either. Furthermore, amongst all the complexities and variables involved, there is no evidence to suggest that replacing pyrotechnic delays with electronic delays will provide any consistent improvement in fragmentation.

From a viewpoint of mine productivity, it is often assumed that an improvement in fragmentation will yield an improvement in shovel dig rates. In this regard, a large open pit mine kindly supplied their data from a trial of 72 blasts, 36 fired with pyrotechnic standard delay sequences, and 36 fired with electronic delay sequences designed on the basis of stress wave superposition. Each blast pair (pyrotechnic, electronic) was fired in the same pit region, however they were not necessarily side-by-side and so cannot be considered as matched pairs. The blasts were dug with 4 shovels, two that were old and two that were new. Furthermore, each blast was entirely dug with either a new shovel or an old shovel in order to reduce shovel bias. A preliminary analysis showed that the dig rate of each old shovel was essentially the same and significantly less than the dig rate of each new shovel (which was also essentially the same). Thus the total data was divided into 4 sets: Pyrotechnic 1—pyrotechnic delay blast, dug with old shovels; Pyrotechnic 2—pyrotechnic delay blast, dug with new shovels; Electronic 1—electronic delay blast, dug with old shovels; Electronic 2—electronic delay blast, dug with new shovels. Figure 16 shows the probit plots for these four sets of dig rate data, in bench cubic metres per hour (BCM/hr).

The large vertical offset of the upper plots demonstrates the significantly higher dig rate of the new shovels. However, the parameter of interest here is the difference in dig rates between the pyrotechnic and electronic delay blasts for the same type of shovels. All these probit plots are more linear, over their entire range, than those shown in Figure 13 for the vibration. Thus, visually at least, the variations within each separate set appear to be normally distributed. However, to be more definitive, Table 1 shows the probability, P_W, associated with the Shapiro-Wilk statistic for each set, and is a measure of the strength of the Null Hypothesis (i.e. the variations within each set follow a random normal distribution). Accepted statistical wisdom decrees if $P_W > 0.1$, then there is no evidence against the Null Hypothesis. In fact P_W is much larger than 0.1 in all cases, and thus the statistical evidence strongly suggests that the variations are normally distributed. Inferences based on the t-test can then be made regarding differences in the average dig rates for each set. This is formally done using a two-sample t-test assuming unpaired data with unequal variance (it is quite clear from Figure 16 that Pyrotechnic 1 has a larger variance than Electronic 1). In this case the Null Hypothesis is that there is no difference in the average dig rates between the pairs of sets considered, and P_t is a measure of the strength of this Null Hypothesis. The results are listed in Table 2, and show that there is no statistical difference ($P_t > 0.1$) in the average dig rates between blasts fired with electronic delays and those fired with pyrotechnic delays when both blast types are dug with the same type of shovels. When different types of shovels dig each blast then there is a significant difference in all dig rates ($P_t < 0.005$), which is due solely to the difference in dig rates between shovel types.

All these results show that electronic delay sequences, based on waveform superposition theory, did not alter muckpile dig rates, and hence, by association, did not alter fragmentation. As a further disadvantage, blasts designed on the basis of stress wave collision also generally involve abnormally short delay intervals (1 ms or so) between certain blastholes. Such a design will certainly increase vibration due to wave crowding (see Figure 11 and associated discussion) and so is a poor design with regard to wall stability in open pits.

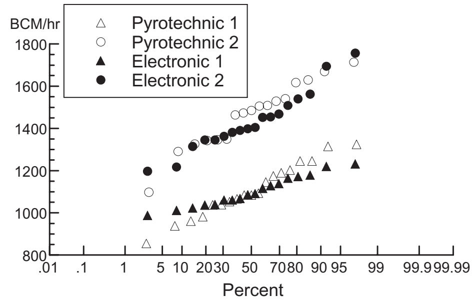


Figure 16. Probit plots of dig rates; 4 data sets from 72 blasts.

4 DISCUSSION AND CONCLUSIONS

4.1 *Vibration control*

It has been shown that there are natural limits to controlling blast vibration at any monitoring station. These limits are independent of the delay accuracy and arise due to geologic influences as well as the relative distance between the monitoring station and the blast. However, if the geology is relatively uniform and the blast-monitor distance lies in the medium-field then there is potential, at least, to control the spectral output of blast vibration using electronic delays. For example, if a structure in the medium-field has a dominant resonant frequency at f_R (Hz) then a sufficiently accurate delay interval, δt (s), given by $\delta t_R = 1/f_R$ will excite resonances at nf_R, where n is a +ve integer (see the spectral banding in Figure 3, for example). So it might seem reasonable to set a delay interval $\delta t < 1/f_R$ to avoid exciting the fundamental resonance. However, most blast designs involve more than a single delay element and this could produce extra delay intervals related to both sums and differences of these elemental delay times. Furthermore, many structures have multiple resonant frequencies. The upshot of this complexity is that when using accurate delays with a number of delay elements it could be difficult to avoid exciting one or more structural resonances. Even when using pyrotechnic delays it is possible to get a degree of spectral banding, especially at low frequencies, where delay accuracy plays a lesser role. Thus it might be advantageous to purposely "randomise" the delay sequence in a precise manner to guarantee no spectral banding over the frequency range of interest. Clearly, this technique cannot be implemented using pyrotechnic delays because it requires the flexibility and accuracy of electronic delays. The randomised sequence should be based on sub-random numbers that maximally avoid each other, rather than the usual pseudo-random numbers, which are known to cluster. It may also be desirable that the sequence maintains the same average delay intervals as the standard (un-randomised) sequence. The optimum sequence can be implemented using a gated function whose mark-space ratio is varied in order to control the strength of the randomness and act as a further aid to avoid clustering. These types of delay sequences have been used for some time now at a particular mine site and demonstrate a significant reduction in the vibration spectral peaks recorded at surface monitoring stations. Nevertheless, whilst such a randomising technique might prove beneficial to controlling peak vibration levels in resonant structures, it does not imply a reduction in peak vibration levels at non-resonant monitoring stations.

Most stations are probably non-resonant, especially for compliance monitoring. For such stations it has been shown that reasonable pyrotechnic delay sequences can control vibrations just as well as electronic delay sequences, especially for monitoring stations at typical distances ≥500 m (Figure 10). One dominant cause of excessive vibration at non-resonant stations is due to wave crowding, whereby there are too many blastholes initiating within a certain time interval. Wave crowding depends mainly on the delay interval and not its scatter, and for reasonable blast designs it has been demonstrated (Figure 11) that pyrotechnic delays can work just as well as electronic delays in reducing this crowding effect.

However, in certain cases the flexibility of electronic delays could provide superior designs, especially for large blasts with numerous holes. Nevertheless, if a vibration problem exists with a particular pyrotechnic delay sequence, then the present work suggests that a first attempt should be made to solve the problem with alternative pyrotechnic sequences before considering electronic sequences with their associated high cost.

In view of the natural limitations to vibration control as well as the fact that pyrotechnic sequences can compete with electronic sequences in many cases, it is not surprising that field measurements of a large number of quarry blasts showed no significant difference in performance between these types of sequences (Figure 12).

Table 1. Shapiro-Wilk probabilities for each dig rate set.

Set	Arrangement	P_W
Pyrotechnic 1	Pyrotechnic, old shovel	0.92
Pyrotechnic 2	Pyrotechnic, new shovel	0.63
Electronic 1	Electronic, old shovel	0.67
Electronic 2	Electronic, new shovel	0.47

Table 2. Pair-wise comparisons of the 4 data sets.

Pair-wise comparisons	P_t
Pyrotechnic 1 vs Electronic 1	0.86
Pyrotechnic 2 vs Electronic 2	0.56
Pyrotechnic 1 vs Pyrotechnic 2	<0.005
Electronic 1 vs Electronic 2	<0.005
Pyrotechnic 1 vs Electronic 2	<0.005
Pyrotechnic 2 vs Electronic 1	<0.005

4.2 *Fragmentation control*

The measurement and modelling of fragmentation are both undoubtedly very difficult problems. Most arguments raised in support of fragmentation control assume that there is some known dependence of fragmentation on delay interval and that stress wave interaction promotes fragmentation. Unfortunately, there is little evidence from experiments or modelling that gives support to either argument.

Fragmentation measurements in small-scale tests are significantly influenced by nearby free faces, and measurements for full-scale blasts are inordinately time-consuming and thus expensive. These are the main reasons why there is scant data available worldwide, and much of that data has significant scatter, making it difficult to draw any convincing conclusions. Although it is conceivable that there could be a measurable dependence of fragmentation on delay interval for a particular site, it would be difficult to predict. It is even more difficult to predict exactly what role, if any, the delay accuracy plays in fragmentation. In fact many detailed trials, coupled with sound statistical analysis, would be required to establish any dependence of fragmentation on delay interval and its scatter.

It could be assumed from past data that fragmentation is most coarse for a zero delay interval. However, this assumption would also need to be verified more soundly in a statistical sense. If this assumption can be verified, then it is quite possible that most of the variation for in-situ fragmentation might well occur for extremely short delay intervals. For example, the data of Katsabanis et al. (2006), reproduced in Figure 1, is consistent with the assumption that this interval is less than 10 μs for their test sample with a burden of approximately 0.09 m. This delay interval would scale up to be approximately 0.1 ms per metre of burden for in-situ blasting. It has already been demonstrated that even electronic delays will not be accurate enough to ensure consistency of such a fine interval throughout a blast.

A new analytical model that considers ideal stress wave propagation shows that any superposition of stress waves from single-primed neighbouring blastholes or from dual-primed single blastholes plays little or no role in promoting fragmentation, especially throughout an extended volume of rock. Furthermore, the model predicts that dual priming within a single blasthole sends out two relatively weak stress waves whereas single priming sends out a single dominant stress wave. This, alone, suggests that dual priming will probably have an adverse influence on fragmentation. In this regard there is an associated point that appears to have been overlooked by proponents of the stress wave collision theory of fragmentation. If stress wave superposition does occur to any meaningful degree within a localised region of rock, then the interacting stress waves must have similar amplitudes and be in phase over that particular region. On the other hand, probability alone dictates that there would also be many other regions where the stress waves would never have similar amplitudes and/or the desired phase relations. This reasoning is also supported by results from the new analytical model as previously discussed. Furthermore, if stress wave interaction was a dominant influence then the overall fragmentation ought to be highly variable, even in uniform geology. There is no experimental evidence to support such an outcome.

In view of these natural limitations to fragmentation control, it is not surprising that dig rate measurements of a large number of open pit blasts showed no difference between blasts fired with pyrotechnic delays and those fired with electronic delays (Table 2). Furthermore, the probit plots of Figure 16 show that even when digging with the same shovel types (eg. Pyrotechnic 1 vs Electronic 1) there can be large differences (over 20%) in dig rates between delay types when comparing individual blasts. Thus it requires many blasts to get a meaningful comparison. Unfortunately, typical trials are conducted using a small number of blasts, and the results of such can be completely misleading.

Although the dig rate results are site-specific, they are consistent with the notion expressed earlier that stress wave collision probably plays no significant role in rock fragmentation by blasting. Nevertheless, experimental results for other sites can only help resolve this complex issue, provided a statistically meaningful number of controlled measurement trials are conducted, and this is not a simple task. However, even if other measurements do show a dependence of fragmentation on the delay sequence/type for a given site, then it might prove difficult to isolate a particular mechanism that is responsible for such dependence. Nevertheless, in some circumstances it could be reasonable to suspect certain mechanisms. For example, in geology with dominant fracture planes, the fragmentation might be influenced by a ratio such as $\delta t/t_f$, where δt is the delay time interval and t_f is some average travel time for stress waves to propagate between the fracture planes. As noted previously, if the required δt is of the order of milliseconds then even electronic delays would not be able to maintain this value accurately throughout most blasts.

It is generally accepted that fragmentation is fundamentally influenced by stress wave propagation from a blasthole and the subsequent interaction of explosive gasses with the stressed rock. Thus there are two requirements for any model of fragmentation. Firstly it must be able to correctly describe the stress wave propagation and secondly it must be able to describe the subsequent influence of explosive gasses on the altered rock. It has been shown that current models are lacking on both accounts when considering the volume (i.e. 3D) assessment of fragmentation.

If an ultimate model of fragmentation was available and computable, it would be possible to directly demonstrate what role, if any, is played by stress wave superposition. Such an ultimate model would need to be numerical rather than analytical, and would probably use a dynamic finite element method (DFEM) or a dynamic finite difference method (DFDM) to describe the stress wave and crack propagation component, coupled with a gas-flow model to describe overall fragmentation. Thus it would be a code similar to the two-dimensional coupled DFEM-gas flow approach reported by Minchinton & Lynch (1996), but made fully three-dimensional and capable of resolving the high frequency stress wave produced by a realistic pressure load. It is instructive to estimate the computing task for such a code. The number of finite elements required can be estimated from the previous low-resolution DFEM component reported in Blair & Minchinton (2006). In order to resolve the high-frequency stress waves shown in Figures 14 and 15, the DFEM would require a minimum of 1000×2000 elements to cover the 4 m × 8 m zone. In three-dimensions the solution would require at least 1000 elements along the third direction, giving a total of 2 billion 3D elements. Calculations for interacting blastholes would require a larger volume with even more elements. Clearly, the required DFEM calculations, alone, are not yet feasible, even using the fastest supercomputers available.

Of course there are alternative (and perhaps faster) numerical codes, such as the Hybrid Stress Blast Model (based on the work of Cundall et al. 1996), that are being used to predict fragmentation. However these alternative codes use small rigid spheres as basic elements and so, unlike DFEM/DFDM codes, they do not have realistic elasticity or viscoelasticity built in at the fundamental level. Because of this significant limitation, as far as I am aware, such numerical codes are not capable of correctly predicting the stress wave propagation characteristics in a viscoelastic material, such as those shown in Figures 14 and 15; the dominant SV-Mach propagation, in particular, has a major influence on fragmentation. Thus, irrespective of any mechanisms that these alternative codes may have to account for cracking and gas flow, it is reasonable to question their capability to realistically model fragmentation.

ACKNOWLEDGEMENTS

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