

The following formulation, notation repeats from, Desrosiers and Lübbecke (2005).

0.1 Capacitated Shortest path

For graph $\mathcal{G}(V, A)$

$$\begin{aligned}
 z^* &:= \min \sum_{(i,j) \in A} c_{ij} x_{ij} \\
 \text{s.t.} \\
 \sum_{j: (s,j) \in A} x_{sj} &= 1 \\
 \sum_{j: (i,j) \in A} x_{ij} - \sum_{j: (j,i) \in A} x_{ji} &= 0 \quad i \in I \setminus \{s, t\} \\
 \sum_{i: (i,t) \in A} x_{it} &= 1 \\
 \sum_{(i,j) \in A} t_{ij} x_{ij} &\leq C \\
 x_{ij} &= 0 \text{ or } 1 \quad (i, j) \in A
 \end{aligned}$$

with $p \in P$ as set of feasible s-t paths, Dantzig-Wolfe reformulation as follows:

$$\begin{aligned}
 z^* &= \min \sum_{p \in P} \left(\sum_{(i,j) \in A} c_{ij} x_{pij} \right) \lambda_p \\
 \text{s.t.} \\
 \sum_{p \in P} \left(\sum_{(i,j) \in A} t_{ij} x_{pij} \right) \lambda_p &\leq C \\
 \sum_{p \in P} \lambda_p &= 1 \\
 \lambda_p &\geq 0 \quad p \in P \\
 \sum_{p \in P} x_{pij} \lambda_p &= x_{ij} \quad (i, j) \in A \\
 x_{ij} &= 0 \text{ or } 1 \quad (i, j) \in A
 \end{aligned}$$

Desrosiers J, Lübbecke ME (2005) A primer in column generation. *Column generation*. (Springer), 1–32.