1 Newsvendor problem (1-d)

1.0.1 Parameters

- $h = b = 1$
- F Underlying distribution
  - $F_1 \sim N(50, 50)$
  - $F_2 \sim \exp(1/50)$
- $\mathbf{N}, |\mathbf{N}| = N$ is # of realizations

type of distribution set:

- likelihood, see Wang, Zizhuo, Peter W Glynn, and Yinyu Ye. 2016. "Likelihood Robust Optimization for Data-Driven Problems." Computational Management Science 13 (2): 241–61. https://doi.org/10.1007/s10287-015-0240-3.

1.0.2 The Models

Newsvendor primal

minimize the worse-case expected cost:

$$\min_x \max_p \mathbb{E}_p(\mathbf{h})$$

loss function:

$$\mathbf{h}, \quad h_i = b(d_i - x)^+ + h(x - d_i)^+$$

s.t.

$$\mathbf{p} \in \mathscr{D}_d, \quad \text{where } \mathscr{D}_d \text{ is some valid distribution set}$$

Scarf (DRO)

$$x^*_{\text{scar } f} = \hat{\mu} + \frac{\hat{\sigma}}{2}(\sqrt{\frac{b}{h}} - \sqrt{\frac{h}{b}})$$

we discuss the distribution set:

LRO: log-likelihood where

likehood:

$$\sum_{i=0}^{n} N_i \log p_i \geq \gamma$$

$$\sum_{i=0}^{n} p_i = 1, \quad p_i \geq 0, \forall i$$

we have the following:

$$\max_{x,\dots} \theta + \beta\gamma + \beta N + t$$

s.t.

$$(\mathbf{q}, \beta\mathbf{N}, t) \in \mathscr{K}_{\exp}$$

$$\beta \geq 0$$

$$\mathbf{q} \equiv -\mathbf{h} - \theta\mathbf{1} \geq 0$$

$$\mathbf{q} + \theta\mathbf{1} + b \cdot (d - x) \leq 0$$

$$\mathbf{q} + \theta\mathbf{1} + h \cdot (x - d) \leq 0$$

$$x \in D$$

using estimator:

$$\gamma^* = \sum_{i=1}^{n} N_i \log \frac{N_i}{N} - \frac{1}{2}\chi^2_{n-1,1-\alpha}$$

worst-case probability:

$$p^\star = \frac{\beta\mathbf{N}}{q}$$

exact moments where

moments (exact):

$$\sum_{i=0}^{n} d_i p_i = \mu$$

$$\sum_{i=0}^{n} d_i^2 p_i = \mu^2 + \sigma^2, \forall i, \quad \text{can use sample mean/var}$$

$$p^\star = \frac{\beta\mathbf{N}}{h - \theta\mathbf{1} - \alpha d - w(d \bullet d)}$$

$$\max_{x,\dots} \theta + \beta\gamma + \alpha\mu + w(\hat{\mu}^2 + \hat{\sigma}^2) + \beta N + t$$

s.t.

$$(\mathbf{q}, \beta\mathbf{N}, t) \in \mathcal{K}_{\exp}$$

$$\beta \geq 0$$

$$\mathbf{q} \equiv -\mathbf{h} - \theta\mathbf{1} - \dots \geq 0$$

$$\mathbf{q} + \theta\mathbf{1} + \dots + b \cdot (d - x) \leq 0$$

$$\mathbf{q} + \theta\mathbf{1} + \dots + h \cdot (x - d) \leq 0$$

$$x \in D$$

JuMP code

> Remark Julia MathOptInterface uses slight different notation on Cones, refer to MathOptInterface API

```
using JuMP
using Distributions
using StatsBase
using MosekTools
using Plots
using LinearAlgebra
import MathOptInterface
const MOI = MathOptInterface

plotly()

# truncation @[0-200]
int_trunc = x -> round(min(max(x, 0), 200))

# sample object
struct Sample
    h::Float64
    b::Float64
    N::Int32
    n::Int32
    S
    H
    mu::Float64
    sig::Float64
end

# solution object
struct Sol
    x::Float64
```

```
    model::JuMP.Model
    p::Array{Float64,1}
end

h = b = 1
N, n = 1000, 200
S1 = int_trunc.(rand(Normal(50, 50), N))
H1 = fit(Histogram, S1, 0:n)
mu1, sig1 = mean_and_std(S1)
S2 = int_trunc.(rand(Exponential(50), N))
H2 = fit(Histogram, S2, 0:n)
mu2, sig2 = mean_and_std(S2)
d = Array(1:200)

sample1 = Sample(h,b,N,n,S1, H1, mu1, sig1)
sample2 = Sample(h,b,N,n,S2, H2, mu2, sig2)

# simple lambda function
x_scarf = s -> s.mu
x_ro_1 = x_scarf(sample1)
x_ro_2 = x_scarf(sample2)
```

```
48.497
```

```
# solution evaluation
mutable struct Eval
    sol::Sol
    sample::Sample
    d::Array

    # objectives
    obj_worse::Float64
    obj_true::Float64

    function Eval(sol::Sol, sample::Sample, d::Array)
        x = new(sol, sample, d, 0, 0)
        h = max.(
            (d .- sol.x) .* sample.b,
            (sol.x .- d) .* sample.h
        )
        x.obj_true = sum(sample.H.weights .* h) / sample.N
        x.obj_worse = sum(sol.p .* h)
        x
    end
end
```

```julia
# 1. pure lro model
function lro_nv_model(sample)
    h, b, N, n = sample.h, sample.b, sample.N, sample.n
    H = sample.H.weights
    Hs = [i for i in H if i > 0]
    gamma = sum(Hs .* (log.(Hs./N))) - 1/2 * quantile.(Gamma(n-1), [0.95])[1]
    model = JuMP.Model()
    @variable(model, theta)
    @variable(model, beta >= 0)
    @variable(model, q[1:n] >= 0)
    @variable(model, x >= 0)

    @constraint(model, q .+ b * (d .- x) .+ theta .<= 0)
    @constraint(model, q .+ h * (x .- d) .+ theta .<= 0)

    @variable(model, t[1:n])
    @constraint(model, KL_DEV[i=1:n], [t[i], H[i] * beta, q[i]] in
     ↪ MOI.ExponentialCone())
    obj_expr =
    begin
        theta + beta * (gamma + N) + dot(ones(n), t)
    end
    @objective(model, Max, obj_expr)
    set_optimizer(model, Mosek.Optimizer)
    optimize!(model)
    x_sol = value.(x)
    p_sol = value.(beta).*H ./ value.(q)
    return Sol(x_sol, model, p_sol)
end
```

```
lro_nv_model (generic function with 1 method)
```

```julia
lro_sol1 = lro_nv_model(sample1)
# lro_sol2 = lro_nv_model(sample2)
# plot sampling distribution and worse-case


lro_p1 = plot(1:n, [sample1.H.weights lro_sol1.p * N],
    label=reshape(["@true", "@worst-case"], 1, 2),
    title="normal"
)
```

```
Problem
  Name                   :
```

```
  Objective sense        : max
  Type                   : CONIC (conic optimization problem)
  Constraints            : 1000
  Cones                  : 200
  Scalar variables       : 1003
  Matrix variables       : 0
  Integer variables      : 0

Optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Eliminator started.
Freed constraints in eliminator : 0
Eliminator terminated.
Eliminator - tries               : 1              time                   : 0.00
Lin. dep.  - tries               : 1              time                   : 0.00
Lin. dep.  - number                 : 0
Presolve terminated. Time: 0.00
Problem
  Name                   :
  Objective sense        : max
  Type                   : CONIC (conic optimization problem)
  Constraints            : 1000
  Cones                  : 200
  Scalar variables       : 1003
  Matrix variables       : 0
  Integer variables      : 0

Optimizer  - threads                : 12
Optimizer  - solved problem         : the primal
Optimizer  - Constraints            : 560
Optimizer  - Cones                  : 201
Optimizer  - Scalar variables      : 1003           conic                : 602
Optimizer  - Semi-definite variables: 0              scalarized           : 0
Factor     - setup time           : 0.00           dense det. time       : 0.00
Factor     - ML order time        : 0.00           GP order time         : 0.00
Factor     - nonzeros before factor : 1.49e+04      after factor        : 1.52e+04
Factor     - dense dim.           : 4              flops                 : 1.58e+06
ITE PFEAS    DFEAS    GFEAS    PRSTATUS   POBJ          DOBJ          MU     TIME
```

```
0  2.0e+02 2.2e+01 1.8e+02 0.00e+00  -1.835571490e+02 0.000000000e+00  1.0e+00 0.00
1  1.3e+02 1.4e+01 1.5e+02 -1.00e+00 -3.989615636e+02 -2.159883074e+02 6.6e-01 0.01
2  7.2e+01 7.7e+00 1.1e+02 -9.98e-01 -7.824056883e+02 -6.008989258e+02 3.6e-01 0.01
3  2.3e+01 2.5e+00 6.2e+01 -9.94e-01 -3.609534082e+03 -3.435781177e+03 1.2e-01 0.01
4  1.2e+01 1.3e+00 4.3e+01 -9.29e-01 -6.403360798e+03 -6.245779702e+03 6.1e-02 0.01
5  2.8e+00 2.9e-01 1.3e+01 -7.15e-01 -9.520306839e+03 -9.448357156e+03 1.4e-02 0.01
6  6.0e-01 6.5e-02 1.8e+00 3.07e-01  -2.608706123e+03 -2.589949885e+03 3.0e-03 0.01
7  9.2e-02 9.9e-03 1.1e-01 8.07e-01  -4.567125634e+02 -4.534855206e+02 4.6e-04 0.01
8  2.6e-02 2.7e-03 1.5e-02 1.00e+00  -1.768516712e+02 -1.759347962e+02 1.3e-04 0.01
9  8.1e-03 8.7e-04 2.5e-03 1.11e+00  -9.201516567e+01 -9.173400712e+01 4.0e-05 0.01
10 3.4e-03 3.6e-04 6.9e-04 9.65e-01  -6.915641721e+01 -6.903698977e+01 1.7e-05 0.01
11 1.6e-03 1.7e-04 2.1e-04 9.79e-01  -5.895075108e+01 -5.889592367e+01 7.7e-06 0.01
12 7.7e-04 8.2e-05 7.4e-05 1.01e+00  -5.475841261e+01 -5.473144813e+01 3.8e-06 0.02
13 3.3e-04 3.6e-05 2.1e-05 9.96e-01  -5.259406247e+01 -5.258235916e+01 1.7e-06 0.02
14 8.6e-05 9.2e-06 2.8e-06 9.92e-01  -5.134501011e+01 -5.134197255e+01 4.3e-07 0.02
15 8.2e-06 8.8e-07 8.2e-08 9.98e-01  -5.094834719e+01 -5.094805872e+01 4.1e-08 0.02
16 4.9e-07 5.2e-08 1.2e-09 9.99e-01  -5.090962356e+01 -5.090960643e+01 2.4e-09 0.02
17 1.1e-08 1.2e-09 4.2e-12 1.00e+00  -5.090725476e+01 -5.090725436e+01 5.6e-11 0.02
Optimizer terminated. Time: 0.04
```

{"layout":{"annotations":[{"rotation":0,"showarrow":false,"text":"normal","xref":"paper","x":0.522
↪ 0, 0,
↪ 1.000)","size":20,"family":"sans-serif"},"xanchor":"center","y":1}],"height":400,"yaxis":{"zer
↪ 0, 0,
↪ 1.000)","size":15,"family":"sans-serif"},"showticklabels":true,"visible":true,"tickfont":{"col
↪ 0, 0, 1.000)","size":11,"family":"sans-serif"},"zerolinecolor":"rgba(0, 0, 0,
↪ 1.000)","anchor":"x1","tickangle":0,"range":[-5.38671427660452,184.94385683008855],"gridcolor"
↪ 0, 0, 0.100)","tickvals":[0,50,100,150],"title":"","tickcolor":"rgb(0, 0,
↪ 0)","type":"-","linecolor":"rgba(0, 0, 0,
↪ 1.000)","ticks":"inside","tickmode":"array","gridwidth":0.5,"mirror":false},"showlegend":true,
↪ 0, 0,
↪ 1.000)","size":15,"family":"sans-serif"},"showticklabels":true,"visible":true,"tickfont":{"col
↪ 0, 0, 1.000)","size":11,"family":"sans-serif"},"zerolinecolor":"rgba(0, 0, 0,
↪ 1.000)","anchor":"y1","tickangle":0,"range":[-4.97,205.97],"gridcolor":"rgba(0,
↪ 0, 0, 0.100)","tickvals":[0,50,100,150,200],"title":"","tickcolor":"rgb(0, 0,
↪ 0)","type":"-","linecolor":"rgba(0, 0, 0,
↪ 1.000)","ticks":"inside","tickmode":"array","gridwidth":0.5,"mirror":false},"width":600,"margi
↪ 255, 255, 1.000)","plot_bgcolor":"rgba(255, 255, 255,
↪ 1.000)","legend":{"bgcolor":"rgba(255, 255, 255,
↪ 1.000)","x":1,"tracegroupgap":0,"font":{"color":"rgba(0, 0, 0,
↪ 1.000)","size":11,"family":"sans-serif"},"bordercolor":"rgba(0, 0, 0,
↪ 1.000)","y":1}},"data":[{"colorbar":{"title":""},"yaxis":"y1","showlegend":true,"mode":"lines"
↪ 154, 250,
↪ 1.000)","width":1,"shape":"linear","dash":"solid"},"name":"@true","x":[1,2,3,4,5,6,7,8,9,10,11
↪ 111, 71,
↪ 1.000)","width":1,"shape":"linear","dash":"solid"},"name":"@worst-case","x":[1,2,3,4,5,6,7,8,9

```julia
# 2. lro + moments model
function lro_moment_nv_model(sample::Sample)
    h, b, N, n = sample.h, sample.b, sample.N, sample.n
    H, u, sig = sample.H.weights, sample.mu, sample.sig
    Hs = [i for i in H if i > 0]
    gamma = sum(Hs .* (log.(Hs./N))) - 1/2 * quantile.(Gamma(n-1), [0.95])[1]
    model = JuMP.Model()
    @variable(model, theta)
    @variable(model, beta >= 0)
    @variable(model, q[1:n] >= 0)
    @variable(model, x >= 0)
    @variable(model, a)
    @variable(model, w)

    @constraint(model, q .+ b * (d .- x) .+ theta .+ (d .* a) .+ (d .* d .* w) .<= 0)
    @constraint(model, q .+ h * (x .- d) .+ theta .+ (d .* a) .+ (d .* d .* w) .<= 0)
```

```
    @variable(model, t[1:n])
    @constraint(model, KL_DEV[i=1:n], [t[i], H[i] * beta, q[i]] in
     ↪  MOI.ExponentialCone())
    obj_expr =
    begin
        theta + a * u + w * (u^2 + sig^2) + beta * (gamma + N) + dot(ones(n), t)
    end
    @objective(model, Max, obj_expr)
    set_optimizer(model, Mosek.Optimizer)
    optimize!(model)
    x_sol = value.(x)
    p_sol = value.(beta) .* H ./ value.(q)
    return Sol(x_sol, model, p_sol)
end
```

```
lro_moment_nv_model (generic function with 1 method)
```

Wrap up results

```
samples = Dict(
    "normal" => sample1,
    "exp" => sample2
)
models = Dict(
    "lro" => lro_nv_model,
    "lro_mm" => lro_moment_nv_model
)
```

```
Dict{String,Function} with 2 entries:
  "lro_mm" => lro_moment_nv_model
  "lro"    => lro_nv_model
```

```
data = []
results = Dict()
data = [(v = samples[k];
        eval = Eval(models[m](v), v, d);
        results[k, m] = eval;
        a = [eval.sol.x eval.obj_true eval.obj_worse])
    for k in ["normal", "exp"] for m in ["lro", "lro_mm"]]
data = vcat(data...)
```

```
Problem
  Name                   :
  Objective sense        : max
```

```
  Type                   : CONIC (conic optimization problem)
  Constraints            : 1000
  Cones                  : 200
  Scalar variables       : 1003
  Matrix variables       : 0
  Integer variables      : 0

Optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Eliminator started.
Freed constraints in eliminator : 0
Eliminator terminated.
Eliminator - tries               : 1              time                   : 0.00
Lin. dep.  - tries               : 1              time                   : 0.00
Lin. dep.  - number                 : 0
Presolve terminated. Time: 0.00
Problem
  Name                   :
  Objective sense        : max
  Type                   : CONIC (conic optimization problem)
  Constraints            : 1000
  Cones                  : 200
  Scalar variables       : 1003
  Matrix variables       : 0
  Integer variables      : 0

Optimizer  - threads                : 12
Optimizer  - solved problem         : the primal
Optimizer  - Constraints            : 560
Optimizer  - Cones                  : 201
Optimizer  - Scalar variables     : 1003           conic                : 602
Optimizer  - Semi-definite variables: 0              scalarized           : 0
Factor     - setup time           : 0.00           dense det. time      : 0.00
Factor     - ML order time        : 0.00           GP order time        : 0.00
Factor     - nonzeros before factor : 1.49e+04      after factor       : 1.52e+04
Factor     - dense dim.           : 4              flops                : 1.58e+06
ITE PFEAS    DFEAS    GFEAS    PRSTATUS   POBJ              DOBJ             MU       TIME
0  2.0e+02  2.2e+01  1.8e+02  0.00e+00   -1.835571490e+02  0.000000000e+00  1.0e+00  0.00
```

```
1  1.3e+02 1.4e+01 1.5e+02 -1.00e+00 -3.989615636e+02 -2.159883074e+02 6.6e-01 0.00
2  7.2e+01 7.7e+00 1.1e+02 -9.98e-01 -7.824056883e+02 -6.008989258e+02 3.6e-01 0.01
3  2.3e+01 2.5e+00 6.2e+01 -9.94e-01 -3.609534082e+03 -3.435781177e+03 1.2e-01 0.01
4  1.2e+01 1.3e+00 4.3e+01 -9.29e-01 -6.403360798e+03 -6.245779702e+03 6.1e-02 0.01
5  2.8e+00 2.9e-01 1.3e+01 -7.15e-01 -9.520306839e+03 -9.448357156e+03 1.4e-02 0.01
6  6.0e-01 6.5e-02 1.8e+00 3.07e-01  -2.608706123e+03 -2.589949885e+03 3.0e-03 0.01
7  9.2e-02 9.9e-03 1.1e-01 8.07e-01  -4.567125634e+02 -4.534855206e+02 4.6e-04 0.01
8  2.6e-02 2.7e-03 1.5e-02 1.00e+00  -1.768516712e+02 -1.759347962e+02 1.3e-04 0.01
9  8.1e-03 8.7e-04 2.5e-03 1.11e+00  -9.201516567e+01 -9.173400712e+01 4.0e-05 0.01
10 3.4e-03 3.6e-04 6.9e-04 9.65e-01  -6.915641721e+01 -6.903698977e+01 1.7e-05 0.01
11 1.6e-03 1.7e-04 2.1e-04 9.79e-01  -5.895075108e+01 -5.889592367e+01 7.7e-06 0.01
12 7.7e-04 8.2e-05 7.4e-05 1.01e+00  -5.475841261e+01 -5.473144813e+01 3.8e-06 0.01
13 3.3e-04 3.6e-05 2.1e-05 9.96e-01  -5.259406247e+01 -5.258235916e+01 1.7e-06 0.01
14 8.6e-05 9.2e-06 2.8e-06 9.92e-01  -5.134501011e+01 -5.134197255e+01 4.3e-07 0.01
15 8.2e-06 8.8e-07 8.2e-08 9.98e-01  -5.094834719e+01 -5.094805872e+01 4.1e-08 0.01
16 4.9e-07 5.2e-08 1.2e-09 9.99e-01  -5.090962356e+01 -5.090960643e+01 2.4e-09 0.01
17 1.1e-08 1.2e-09 4.2e-12 1.00e+00  -5.090725476e+01 -5.090725436e+01 5.6e-11 0.02
Optimizer terminated. Time: 0.02

Problem
  Name                   :
  Objective sense        : max
  Type                   : CONIC (conic optimization problem)
  Constraints            : 1000
  Cones                  : 200
  Scalar variables       : 1005
  Matrix variables       : 0
  Integer variables      : 0

Optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Eliminator started.
Freed constraints in eliminator : 0
Eliminator terminated.
Eliminator - tries                : 1                 time                   : 0.00
Lin. dep.  - tries                : 1                 time                   : 0.00
Lin. dep.  - number                  : 0
Presolve terminated. Time: 0.00
```

```
Problem
  Name                   :
  Objective sense        : max
  Type                   : CONIC (conic optimization problem)
  Constraints            : 1000
  Cones                  : 200
  Scalar variables       : 1005
  Matrix variables       : 0
  Integer variables      : 0

Optimizer  - threads                : 12
Optimizer  - solved problem         : the primal
Optimizer  - Constraints            : 560
Optimizer  - Cones                  : 201
Optimizer  - Scalar variables      : 1005           conic                : 604
Optimizer  - Semi-definite variables: 0              scalarized           : 0
Factor     - setup time           : 0.00           dense det. time       : 0.00
Factor     - ML order time        : 0.00           GP order time         : 0.00
Factor     - nonzeros before factor : 1.57e+04      after factor        : 1.64e+04
Factor     - dense dim.           : 6              flops                : 1.64e+06
ITE PFEAS    DFEAS    GFEAS    PRSTATUS   POBJ           DOBJ           MU     TIME
0  2.3e+00  4.9e+03  1.8e+02  0.00e+00  -1.835571490e+02 0.000000000e+00  1.0e+00  0.00
1  1.4e+00  3.0e+03  1.4e+02  -8.85e-01 -1.759083628e+02 1.253673382e-01  6.0e-01  0.00
2  5.2e-01  1.1e+03  7.3e+01  -8.12e-01 -1.493340599e+02 8.721907858e-02  2.3e-01  0.01
3  3.2e-01  6.9e+02  4.7e+01  -4.62e-01 -1.241687344e+02 4.918787713e-02  1.4e-01  0.01
4  9.8e-02  2.1e+02  1.3e+01  -1.80e-01 -6.145318972e+01 -4.546293609e-01 4.3e-02  0.01
5  4.9e-02  1.1e+02  5.0e+00  4.30e-01  -3.717216780e+01 -1.689709475e+00 2.1e-02  0.01
6  3.6e-02  7.9e+01  3.3e+00  6.52e-01  -3.036190665e+01 -2.821187711e+00 1.6e-02  0.01
7  1.3e-02  2.7e+01  7.3e-01  7.18e-01  -1.723287775e+01 -6.800197966e+00 5.5e-03  0.01
8  4.9e-03  1.1e+01  1.9e-01  8.55e-01  -1.694069638e+01 -1.262668888e+01 2.2e-03  0.01
9  1.6e-03  3.5e+00  3.8e-02  9.17e-01  -2.487301088e+01 -2.343371479e+01 7.0e-04  0.01
10 5.9e-04  1.3e+00  8.7e-03  9.65e-01  -3.437238333e+01 -3.383444163e+01 2.6e-04  0.01
11 3.3e-04  7.1e-01  3.7e-03  9.13e-01  -3.679711609e+01 -3.648876839e+01 1.4e-04  0.01
12 1.1e-04  2.4e-01  7.7e-04  9.28e-01  -3.906203099e+01 -3.895386696e+01 4.9e-05  0.01
13 4.2e-05  9.0e-02  1.8e-04  9.42e-01  -3.992301273e+01 -3.988173061e+01 1.8e-05  0.01
14 1.7e-05  3.6e-02  4.8e-05  9.38e-01  -4.021501176e+01 -4.019783292e+01 7.4e-06  0.01
15 8.5e-06  1.8e-02  1.8e-05  9.62e-01  -4.031308332e+01 -4.030427480e+01 3.7e-06  0.01
16 5.2e-06  1.1e-02  8.5e-06  9.86e-01  -4.035941179e+01 -4.035402228e+01 2.3e-06  0.02
17 4.4e-07  9.4e-04  2.1e-07  9.95e-01  -4.043109274e+01 -4.043064209e+01 1.9e-07  0.02
18 2.1e-08  5.4e-05  2.9e-09  9.95e-01  -4.043903227e+01 -4.043900654e+01 1.1e-08  0.02
```

```
19 7.1e-10 1.8e-06 1.8e-11 9.96e-01  -4.043952255e+01 -4.043952169e+01 3.6e-10 0.02
20 5.6e-11 1.8e-08 1.7e-14 1.00e+00  -4.043954026e+01 -4.043954025e+01 3.6e-12 0.02
Optimizer terminated. Time: 0.02

Problem
  Name                   :
  Objective sense        : max
  Type                   : CONIC (conic optimization problem)
  Constraints            : 1000
  Cones                  : 200
  Scalar variables       : 1003
  Matrix variables       : 0
  Integer variables      : 0

Optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Eliminator started.
Freed constraints in eliminator : 0
Eliminator terminated.
Eliminator - tries               : 1              time                   : 0.00
Lin. dep.  - tries               : 1              time                   : 0.00
Lin. dep.  - number                 : 0
Presolve terminated. Time: 0.00
Problem
  Name                   :
  Objective sense        : max
  Type                   : CONIC (conic optimization problem)
  Constraints            : 1000
  Cones                  : 200
  Scalar variables       : 1003
  Matrix variables       : 0
  Integer variables      : 0

Optimizer  - threads                : 12
Optimizer  - solved problem         : the primal
Optimizer  - Constraints            : 557
Optimizer  - Cones                  : 201
Optimizer  - Scalar variables      : 1003           conic                : 602
```

```
Optimizer  - Semi-definite variables: 0              scalarized           : 0
Factor     - setup time           : 0.00           dense det. time       : 0.00
Factor     - ML order time        : 0.00           GP order time         : 0.00
Factor     - nonzeros before factor : 1.44e+04      after factor        : 1.47e+04
Factor     - dense dim.           : 4              flops                 : 1.49e+06
ITE PFEAS    DFEAS    GFEAS    PRSTATUS   POBJ              DOBJ             MU       TIME
0  2.0e+02  2.7e+02  3.8e+02  0.00e+00  -3.813651763e+02  0.000000000e+00  1.0e+00  0.00
1  6.1e+01  8.1e+01  2.1e+02  -9.99e-01 -1.575752734e+03  -1.197478623e+03 3.0e-01  0.00
2  1.0e+01  1.3e+01  8.2e+01  -9.87e-01 -1.049503045e+04  -1.015257754e+04 4.9e-02  0.01
3  2.7e+00  3.6e+00  2.4e+01  -6.06e-01 -6.269265671e+03  -6.102343978e+03 1.3e-02  0.01
4  8.1e-01  1.1e+00  4.9e+00  2.50e-01  -2.625761751e+03  -2.561054962e+03 4.0e-03  0.01
5  1.5e-01  2.0e-01  4.2e-01  7.12e-01  -6.382790202e+02  -6.247756165e+02 7.5e-04  0.01
6  3.2e-02  4.3e-02  4.0e-02  9.79e-01  -1.945166950e+02  -1.916269352e+02 1.6e-04  0.01
7  1.0e-02  1.3e-02  7.2e-03  1.06e+00  -1.048601225e+02  -1.039728871e+02 5.0e-05  0.01
8  4.0e-03  5.3e-03  2.0e-03  9.06e-01  -7.478417327e+01  -7.440746021e+01 2.0e-05  0.01
9  1.9e-03  2.5e-03  6.8e-04  8.90e-01  -6.182279303e+01  -6.163627527e+01 9.5e-06  0.01
10 1.0e-03  1.3e-03  2.6e-04  9.74e-01  -5.611241508e+01  -5.601460415e+01 5.0e-06  0.01
11 4.8e-04  6.4e-04  8.6e-05  9.90e-01  -5.291059352e+01  -5.286350318e+01 2.4e-06  0.01
12 1.2e-04  1.7e-04  1.1e-05  9.95e-01  -5.071712829e+01  -5.070490733e+01 6.2e-07  0.01
13 3.5e-05  4.6e-05  1.7e-06  9.98e-01  -5.016046431e+01  -5.015705207e+01 1.7e-07  0.01
14 2.4e-06  3.2e-06  3.0e-08  1.00e+00  -4.995942500e+01  -4.995919178e+01 1.2e-08  0.01
15 1.8e-07  1.1e-07  1.9e-10  1.00e+00  -4.994518262e+01  -4.994517471e+01 4.0e-10  0.01
16 9.7e-08  2.7e-09  7.5e-13  1.00e+00  -4.994467663e+01  -4.994467643e+01 1.0e-11  0.02
17 6.4e-09  1.8e-10  1.3e-14  1.00e+00  -4.994466488e+01  -4.994466487e+01 6.7e-13  0.02
Optimizer terminated. Time: 0.02

Problem
  Name                   :
  Objective sense        : max
  Type                   : CONIC (conic optimization problem)
  Constraints            : 1000
  Cones                  : 200
  Scalar variables       : 1005
  Matrix variables       : 0
  Integer variables      : 0

Optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
```

```
Eliminator started.
Freed constraints in eliminator : 0
Eliminator terminated.
Eliminator - tries                : 1              time                   : 0.00
Lin. dep.  - tries                : 1              time                   : 0.00
Lin. dep.  - number                  : 0
Presolve terminated. Time: 0.00
Problem
  Name                   :
  Objective sense        : max
  Type                   : CONIC (conic optimization problem)
  Constraints            : 1000
  Cones                  : 200
  Scalar variables       : 1005
  Matrix variables       : 0
  Integer variables      : 0

Optimizer  - threads                : 12
Optimizer  - solved problem         : the primal
Optimizer  - Constraints            : 557
Optimizer  - Cones                  : 201
Optimizer  - Scalar variables      : 1005           conic                : 604
Optimizer  - Semi-definite variables: 0              scalarized           : 0
Factor     - setup time           : 0.00           dense det. time       : 0.00
Factor     - ML order time        : 0.00           GP order time         : 0.00
Factor     - nonzeros before factor : 1.52e+04      after factor        : 1.59e+04
Factor     - dense dim.           : 6              flops                 : 1.56e+06
ITE PFEAS    DFEAS    GFEAS    PRSTATUS   POBJ              DOBJ             MU       TIME
0  2.3e+00  4.4e+03  3.8e+02  0.00e+00   -3.813651763e+02  0.000000000e+00  1.0e+00  0.00
1  1.3e+00  2.5e+03  2.7e+02  -8.67e-01  -3.612671151e+02  9.145486696e-02  5.6e-01  0.00
2  5.7e-01  1.1e+03  1.6e+02  -7.72e-01  -3.117447162e+02  9.139371151e-02  2.5e-01  0.01
3  3.5e-01  6.8e+02  1.0e+02  -4.54e-01  -2.612883875e+02  5.589665016e-02  1.5e-01  0.01
4  6.0e-02  1.2e+02  1.5e+01  -1.85e-01  -8.345479093e+01  -5.192001803e-01  2.6e-02  0.01
5  8.1e-03  1.6e+01  8.9e-01  6.33e-01   -1.739752790e+01  -4.162087409e+00  3.5e-03  0.01
6  3.5e-03  6.7e+00  2.5e-01  9.53e-01   -2.149219410e+01  -1.576260441e+01  1.5e-03  0.01
7  1.1e-03  2.2e+00  4.7e-02  9.50e-01   -3.092014255e+01  -2.897653521e+01  5.0e-04  0.01
8  5.7e-04  1.1e+00  1.8e-02  8.81e-01   -3.431858027e+01  -3.328082373e+01  2.5e-04  0.01
9  2.5e-04  4.8e-01  5.5e-03  8.11e-01   -3.619043680e+01  -3.569264375e+01  1.1e-04  0.01
10 1.2e-04  2.3e-01  1.9e-03  8.24e-01   -3.701229705e+01  -3.675630766e+01  5.2e-05  0.01
11 5.8e-05  1.1e-01  6.5e-04  8.95e-01   -3.750361361e+01  -3.737613872e+01  2.5e-05  0.01
```

```
12  2.6e-05  5.0e-02  2.0e-04  9.41e-01  -3.779593643e+01 -3.773809059e+01 1.1e-05 0.02
13  8.9e-06  1.7e-02  4.0e-05  9.84e-01  -3.798025224e+01 -3.796032417e+01 3.9e-06 0.02
14  4.7e-07  9.2e-04  5.0e-07  9.97e-01  -3.807826649e+01 -3.807719987e+01 2.1e-07 0.02
15  1.4e-08  4.0e-05  4.5e-09  9.96e-01  -3.808482943e+01 -3.808478334e+01 8.9e-09 0.02
16  5.0e-09  9.3e-07  1.6e-11  1.00e+00  -3.808514392e+01 -3.808514285e+01 2.1e-10 0.02
17  1.1e-09  8.0e-08  4.1e-13  1.00e+00  -3.808515040e+01 -3.808515030e+01 1.8e-11 0.02
Optimizer terminated. Time: 0.02

4×3 Array{Float64,2}:
 59.8911  36.9653  42.6789
 53.9407  36.5232  40.4394
 47.0     31.793   49.9453
 32.0     30.645   38.0847
```

```
objective_value(results["exp", "lro_mm"].sol.model)
```

```
-38.08515039579801
```

```
using DataFrames
```

```
┌ Info: Precompiling DataFrames [a93c6f00-e57d-5684-b7b6-d8193f3e46c0]
└ @ Base loading.jl:1260
```

```
data
```

```
4×3 Array{Float64,2}:
 59.8911  36.9653  42.6789
 53.9407  36.5232  40.4394
 47.0     31.793   49.9453
 32.0     30.645   38.0847
```

```
df = DataFrame(
    Index=["normal, LRO", "normal, LRO_mm", "Exp, LRO", "Exp, LRO_mm"],
    Sol=data[:, 1],
    True_obj=data[:, 2],
    Worst_obj=data[:, 3],
)
```