# Global QCQP Solver

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September 28, 2021

# QCQP

Suppose we readily have a sparse pattern $G(V, E)$ (and extension $E \subseteq F$) for $Q$
We denote aggregated pattern if $G(V, E)$ stands for all data matrices.
$F = \bigcup_r C_r$, where $C_r$ the maximal cliques can be computed efficiently for chordal graph $F$.
SD constraint $Y \succeq 0$ is equivalent to,

$$Y_{(i,i) \in C_r} \succeq 0, \quad \forall r$$

Can be expressed as follows,

$$\begin{aligned} & E_r \in \mathbf{R}^{|C_r| \times n} \\ & (E_r)_{i,j} = \begin{cases} 1, & \text{if } C_r(i) = j \\ 0, & \text{otherwise} \end{cases} \\ & Y_r \equiv E_r Y E_r^T \succeq 0 \end{aligned} \tag{1}$$

# Sparse-SDP: SSDP

In QCQP, we have the SD constraint:

$$Y - xx^T \succeq 0$$

then any subprinciple matrix is also semidefinite,

$$E_r(Y - xx^T)E_r^T \succeq 0 \tag{2}$$

So we have the QCQP with $r$ smaller SDP constraints.

$$\begin{aligned} \text{Maximize} \quad & Q \bullet Y + q^T x \\ \text{s.t.} \quad & \begin{bmatrix} E_r Y E_r^T & E_r x \\ (E_r x)^T & 1 \end{bmatrix} \succeq 0, \forall r \end{aligned} \tag{3}$$

This formulation incorporates a lot of linear constraints for $C_r \cap C_{r'}$

Consider the case for 2-blocks (2 cliques), let $D_1 = C_1 \bigcap C_2$

$$Y = \sum_{r=1}^{2} E_r^T Y_r Er - E(D_1)^T Y_{D_1} E(D_1) \tag{4}$$

$$\begin{aligned} \text{Maximize} \quad & Q \bullet Y + q^T x \\ \text{s.t.} \quad & \begin{bmatrix} E_r Y E_r^T & E_r x \\ (E_r x)^T & 1 \end{bmatrix} \succeq 0, \forall r \end{aligned} \tag{5}$$

We see the above is equivalent to (3), if there is no intersections for blocks, i.e., $D_1 = \emptyset$, it can be further simplified by letting $Q_r = E_r Q E_r^T$

$$\begin{aligned} \text{Maximize} \quad & \sum_{r=1}^{L} Q_r \bullet Y_r \\ \text{s.t.} \quad & \begin{bmatrix} Y_r & E_r x \\ (E_r x)^T & 1 \end{bmatrix} \succeq 0, \forall r \\ & \text{diag}(Y_r) \leq E_r x \end{aligned} \tag{6}$$

The same for any similarity transformations