A QCQP Solver

Strengthen SOCP relaxation

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November 21, 2021

QCQP

Recall QCQP:

$$\begin{aligned} \text{Maximize} \quad & x^T Q x + q^T x \\ \text{s.t.} \quad & x^T A_i x + a_i^T x \, (\leq, =, \geq) \; b_i \\ & 0 \leq x \leq e \end{aligned} \tag{1}$$

- Q, A_i maybe indefinite

Related Literature

S-free sets$P = \{x : x \in R - int(S), S \text{ convex}\}$

- ✗ Gonzalo Muñoz, '20, '21, IPCO, Maximal quadratic free sets by LP (extreme rays)
- Bienstock, '14, SIOPT, Cutting planes for optimization over convex function over nonconvex sets. **also lifting**

SOCPr and aggregation (SOCP-representable) For convex inequalities $\{f_i \leq 0\}$, consider aggregation $\{\sum_i \lambda_i f_i \leq 0\}$

- Burer '17, Modaresi, Vielma' 17 two quadratic functions
- Dey' 20, quadratic and polytope. app. for bilinear: Dey et al' 19, New SOCP relaxation and branching rule for bipartite bilinear programs
- Dey et al' 21, 3 quadratic intersection.

Linear disjunction

Burer '17

For intersection with a paraboloid and quadratic function (indefinite)

$$\mathcal{Y} = \left\{ y = \begin{pmatrix} \tilde{y} \\ y_n \end{pmatrix} \in R^n : \begin{array}{c} \tilde{y}^T \tilde{y} \leq y_n \\ \tilde{y}^T \tilde{Q} \tilde{y} + 2g^T y + f \leq 0 \end{array} \right\}$$

Homogenize: $x = [\tilde{y}, y_n, x_{n+1}]$

$$A_0 := \begin{bmatrix} I & 0 & 0 \\ 0^T & 0 & -\frac{1}{2} \\ 0^T & -\frac{1}{2} & 0 \end{bmatrix}, \quad A_1 := \begin{bmatrix} \tilde{Q} & 0 & \tilde{g} \\ 0^T & 0 & g_n \\ \tilde{g}^T & g_n & f \end{bmatrix}, \quad H^1 := \{x : x_{n+1} = 1\}$$

$$\text{Def} \quad A_t := \begin{bmatrix} (1-t)I + t\tilde{Q} & 0 & t\tilde{g} \\ 0^T & 0 & (1-t)\left(-\frac{1}{2}\right) + tg_n \\ t\tilde{g}^T & (1-t)\left(-\frac{1}{2}\right) + tg_n & tf \end{bmatrix}$$

$$t = 1/(1-\lambda), \lambda = \min(\lambda(Q)) < 0$$

$$\mathcal{Y}_t = \left\{ y : \tilde{y}^T \tilde{y} \leq y_n, x^T A_t x \leq 0 \right\}$$

$$\Rightarrow \quad \mathcal{Y}_t = \text{cl.convex.hull}(\mathcal{Y})$$

Cutting planes for QCQP

- use SOCPr to improve QCQP relaxations
- consider cases for box constraint (so as to incorporate Branch and Cut)