Global QCQP Solver

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QCQP

Recall QCQP:

$$
\begin{aligned}
\text{Maximize} \quad & x^T Q x + q^T x \\
\text{s.t.} \quad & x^T A_i x + a_i^T x \leq b_i \\
& 0 \leq x \leq e
\end{aligned}
\tag{1}
$$

We apply "Many-Small-Cone" (MSC) relaxation.

EMSC

Recall the decomposition by eigenvalue decomposition,

$$Q = V_0 \Lambda_0 V_0^T, A_i = V_i \Lambda_i V_i^T$$

$$\begin{aligned}
\text{Maximize} \quad & y_0^T \lambda_0 + q^T x \\
\text{s.t.} \quad & V_i z_i = x && i = 0, ..., m && (2) \\
& y_i^T \lambda_i + a_i^T x \leq b_i && i = 1, ..., m && (3) \\
& y_i = z_i \circ z_i && i = 0, ..., m && (4)
\end{aligned}$$

Relax (4) to conic constraint, $\forall j, \ y_{ij} \geq z_{ij}^2$

Related Research

- See Luo et al. 2021, Luo et al. 2017
- This is exactly our previous MSC. (very weak)
- Also proved $v^{\mathrm{Shor}} \geq v^{\mathrm{MSC}}$

$$
\begin{aligned}
\text{Minimize}: \quad & x^T Q^+ x + q^T x - \sum_{i=1}^{r} s_i \\
\text{s.t.} \quad & Cx - t = 0, x \in \mathcal{F}, t \in [l, u], \\
& t_i^2 \leq s_i, \quad s_i \leq (l_i + u_i)\, t_i - l_i u_i, \quad i = 1, \ldots, r \\
& \sum_{i=1}^{r} \frac{s_i}{\hat{\lambda}_i} \leq \bar{u}^T x
\end{aligned}
$$

- Once could see $v^{\mathrm{Shor}} \geq v^{\mathrm{MSC}}$
- ? $v^{\mathrm{Shor}} \geq v^{\mathrm{EMSC}} \geq v^{\mathrm{MSC}}$

EMSC

Rewrite: $x^T Q x + q^T x$

$$\lambda_+^T y_+ + q^T x + (\lambda_{\max} e - \lambda_-)^T y_- - \lambda_{\max} \|z_-\|^2$$

- $y_-^T e = \|z_-\|^2 \leq x^T x \leq x^T e$, which is very weak
- dominated by $\lambda_{\max}$