Global QCQP Solver

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1 Branch and Cut Algorithm

for $x \in \mathbb{R}^n$, we have: $x^T A_i x = A_i \bullet (xx^T)$

$$\begin{aligned} \text{Maximize} \quad & Q \bullet Y \\ \text{s.t.} \quad & Y - xx^T \succeq 0 \text{ or } \begin{bmatrix} 1 & x^T \\ x & Y \end{bmatrix} \succeq 0 \\ & A_i \bullet Y \ (\leq, =, \geq) \ b_i, \forall i \end{aligned} \tag{1.1}$$

Possible ways to do this.

Audet et al. (2000) using RLT relaxation and LP, literally, $W_{ij} \approx x_i x_j, v_i \approx x_i^2$. The branching is essentially based on $\|W_{ij} - x_i x_j\|, \|v_i - x_i^2\|$, at each node we solve a linear programming relaxation.

For MINLP and more specifically for QP, see Belotti et al. (2013), Misener and Floudas (2013).

More recently, spacial branch-and-cut method Chen et al. (2017).

Some source code to look at:

- Couenne: https://www.coin-or.org/Couenne/
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1.1 Root

The root of the problem can selected from different SDPs as we discussed before. It is a question to answer whether we should use the SDP with the tightest bound.

References

Audet C, Hansen P, Jaumard B, Savard G (2000) A branch and cut algorithm for nonconvex quadratically constrained quadratic programming. *Mathematical Programming* 87(1):131–152, publisher: Springer.

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Misener R, Floudas CA (2013) GloMIQO: Global mixed-integer quadratic optimizer. *Journal of Global Optimization* 57(1):3–50, publisher: Springer.

Appendix