Global QCQP Solver

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1 Branch and Cut Algorithm

For $x \in \mathbb{R}^n$, we have: $x^T A_i x = A_i \bullet (xx^T)$

$$\begin{aligned}
\text{Maximize} \quad & Q \bullet Y \\
\text{s.t.} \quad & Y - xx^T \succeq 0 \text{ or } \begin{bmatrix} 1 & x^T \\ x & Y \end{bmatrix} \succeq 0 \\
& A_i \bullet Y \ (\leq, =, \geq) \ b_i, \forall i
\end{aligned} \tag{1.1}$$

For MINLP and more specifically for QP, see Belotti et al. (2013), Misener and Floudas (2013). More recently, spacial branch-and-cut method Chen et al. (2017) for QP with complex variables.

Branching node and value selection, namely, select which $x_i, i = 1, ..., N$ and value α, for left and right children, for example:

$$x_i \leq \alpha, x_i \geq \alpha$$

- Audet et al. (2000) using RLT relaxation and LP, literally, $W_{ij} \approx x_i x_j, v_i \approx x_i^2$. The branching is essentially based on $\|W_{ij} - x_i x_j\|, \|v_i - x_i^2\|$, at each node we solve a linear programming relaxation.

- Linderoth (2005) a B-B based on subdividing the feasible region into the Cartesian product of triangles and rectangles.

Branching value:

Some source code to look at:

- Couenne: https://www.coin-or.org/Couenne/
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1.1 Root

The root of the problem can selected from different SDPs as we discussed before. It is a question to answer whether we should use the SDP with the tightest bound.

References

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Appendix