The Effective Power of Military Coalitions: A Unified Theoretical and Empirical Model

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Abstract

We argue that the effective power of a coalition depends not only on the raw capabilities of its members, but also how much effort they choose to exert. Free-riding may make a coalition amount to less than the sum of its parts. We develop a unified model of crisis bargaining and war fighting between military coalitions. Using data on the escalation and outcomes of international disputes, we structurally estimate the model parameters, allowing us to identify determinants of countries' force multipliers and the degree of prior uncertainty. We find that demographic and economic characteristics are the most important determinants of military effectiveness, with regime type and geopolitical considerations playing lesser roles. The structural model also allows us to simulate counterfactual outcomes in disputes between coalitions. This allows us to test historical claims like that earlier American involvement would have made a substantial difference in both World Wars, even in the face of strategic free-riding by its allies.

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The effective power of a military coalition is not merely the product of raw materials—it depends largely on strategic choices by its constituent states. This is true for individual states as well (Sullivan 2007), but coalitions add significant complications. The most important strategic consideration is the free-rider problem. Weaker members of a coalition have an incentive to contribute even less to collective military efforts than they would if they had to fight on their own, leaving the most powerful partners to take on the bulk of the effort (Olson and Zeckhauser 1966). The likelihood of free-riding within a coalition may have other strategic knock-on effects in crisis bargaining. Bringing in more coalition partners might intimidate an opponent into making concessions at the negotiating table, but only if the new partners' efforts are not expected to be offset by other members' free-riding. And if the crisis escalates into war, each side's choice of how much to spend trying to win will depend on how it expects the other side to behave, which is in part a function of free-riding incentives.

We propose a new theoretical model of crisis bargaining between coalitions that accounts for all of these strategic dynamics, allowing anticipated free-riding by coalition partners to shape strategic decisions in both negotiating and fighting. We then estimate the structural parameters of the model using data on militarized disputes from 1816 to 2010. This approach allows us to simultaneously quantify the determinants of *individual* states' military capabilities and how these are *aggregated* into effective power in crises between coalitions. Situating the analysis within a game-theoretical model of crises also allows us to answer counterfactual questions about strategic behavior, such as how historical crises would have turned out if different coalition partners had joined or dropped out.

Using our theoretical framework, we decompose a coalition's effective military power into the product of two distinct factors. The first of these is the *effort* each state exerts—the resources used or the consumption foregone in order to increase their side's chance of success in the conflict. Effort is an endogenous, strategic choice. This is where free-riding enters our picture of effective power, when states decrease the effort they would have otherwise exerted in anticipation of their coalition partners' efforts. By contrast, the second determinant of effective power is a fixed, exogenous factor, which we call the *force multiplier*. This represents the "bang for the buck" states get from their military resources; the higher a state's force multiplier, the more its side benefits militarily from the same exertion of effort. This force multiplier may depend on military technology, political organization, economic strength, or a host of other factors. The endogenous equilibrium levels of effort are functions of participants' force multipliers, but we take the multipliers themselves to be exogenous.

Our game-theoretical model links the equilibrium chance of war and the probability of victory by each side (in case war occurs) to the *individual* force multipliers of the participants on each side. Our empirical analysis exploits this linkage, using the Militarized Interstate Dispute data to estimate the structural determinants of individual countries' force multipliers. This exercise allows us to characterize the strongest influences on the effectiveness of countries' individual efforts. By calculating the force multipliers for countries at particular points in time, we can answer counterfactual questions about conflict contributions and outcomes—such as how much less likely a coalition would be to win a war if one of its partners had dropped out. Using equilibrium choices of effort in these counterfactuals, we account for free-riding behavior and other strategic incentives in a way that would be impossible if we relied on traditional statistical modeling.

We find that demographic and economic factors are the strongest determinants of individual countries' force multipliers. According to our estimates, states with large, dispersed populations and rich economies have the greatest effectiveness per unit of military resources. Political and geopolitical factors have smaller influences, though there is a discernible democratic advantage in the employment of military resources. For example, despite China's demographic advantage and rough economic parity, we estimate a larger force multiplier for the United States as of 2010 due to its more democratic political regime. Altogether, the most valuable coalition partners—and those whose inclusion are most likely to cause others to free-ride—are rich, populous democracies.

Once we have calculated force multipliers for countries at a particular point in time using our model estimates, we can use the unique equilibrium of the underlying bargaining model to simulate individual contributions and likely outcomes in modified or hypothetical coalitional disputes. We take this approach to analyze individual contributions to the most important conflicts between coalitions in our sample, the world wars. We find that earlier American intervention in the First World War would have raised the Allied Powers' chance of victory by about 13 percentage points. We also find that the Central Powers' chance of victory would have increased by about 12 percentage points if France had been knocked out of the conflict in 1914, partially validating the military philosophy of the Schlieffen Plan. For the Second World War, we find that earlier Soviet participation would have made a moderate difference in the European theater, but only if the United States had continued to stay out. Similarly, in the Pacific theater as of 1941, Soviet participation would have made almost no difference, being offset by British and Chinese free-riding.

By breaking power into the endogenous selection of effort and the exogenous effectiveness of that effort, we follow other scholars who have recognized the distinction between the raw resources a state possesses and its ability or willingness to use those resources (Sullivan 2007; Beckley 2010). We innovate on these earlier efforts in important ways. Most importantly, our analysis encompasses military coalitions as well as individual states. We show how the problem of translating potential power into real military effects is even more complicated in coalitions. In addition, we provide an equilibrium model of the relationship between raw force multipliers and strategic choices of effort. This allows us to predict explicitly how the efforts of individual states—and thus their fortunes in crisis bargaining—depend on the military potentials of their friends and foes.

1 Coalitions, Power, and War: Existing Literature

The canonical bargaining model of war is purely dyadic, treating both crisis negotiations and war itself as two-player affairs (Fearon 1995). Our theoretical model fits into a growing body of work extending the bargaining model to allow coalition relationships among the players. Fang, Johnson and Leeds (2014) show how public recommendations by coalition partners can lead states to accept a wider range of demands in crisis bargaining. Wolford (2014*b*) demonstrates how the need to maintain coalition support can complicate costly signaling, thereby increasing the likelihood of war. Trager (2015) finds that coalitions open up possibilities for cheap talk signaling to adversaries that would otherwise lack credibility, while Smith (forthcoming) shows how free-riding incentives might complicate such signaling.

Our innovation relative to this literature is to endogenize individual states' choices of military effort—and thus the effective power of military coalitions in times of war. In all of the models noted above, aggregation of power is treated in reduced form: the power of one side increases by a fixed amount when a new partner joins.[1] Consequently, these theories do not speak to *how* coalitions aggregate power, nor to which partners would most increase their side's effective power. Because military efforts are not modeled, the only way for a state to free-ride in these models is to stay out of the conflict altogether. Our model, by contrast, produces sharp empirical implications about how individual states' capabilities translate into collective power, depending on which states they are partnered with and fighting against. We also introduce a richer conception of free-riding, quantifying the difference one state's participation makes to its partners' efforts. The formal model closest to ours is by Garfinkel (2004*a*,*b*), who models conflict as an N-player contest with coalitions (see also Skaperdas 1998; Krainin 2014).[2] We simplify Garfinkel's model by treating the coalition structure as exogenous, while expanding on it by modeling pre-conflict negotiations.

[1] Wolford (2014*b*) allows endogenous military mobilization, but only by one state in the interaction (and before it knows whether its partner will participate in the conflict).

[2] Krainin and Wiseman (2016) also model N-player coalitional war, though their dynamic modeling framework is otherwise quite distinct from ours.

We also contribute to the empirical literature on disputes and wars between coalitions of states. Despite the classic theoretical findings demonstrating incentives to free-ride in alliances (Olson and Zeckhauser 1966), empirical studies of coalitions typically measure the power of a coalition as the sum of its members' capabilities (e.g., Grant 2013; Fang, Johnson and Leeds 2014; Wolford 2014*a*, 2017; Morey 2016; Chiba and Johnson 2019). By contrast, our calculations of coalitions' effective power are driven by theory—namely, the equilibrium allocations of effort in our formal model of war fighting.

Our work is also part of a recent tradition breaking away from purely dyadic research designs (see Poast 2010; Dorff and Ward 2013). Cranmer and Desmarais (2016) critique the bias toward "dyadic theory" and "dyadic analysis" in studies of conflict. Theoretically, we bridge between dyadic and k-adic approaches by modeling crisis bargaining as occur between a pair of coalitions, each of which may consist of many individual states. We carry this hybrid conceptualization into our empirical work by directly estimating the parameters of our formal model. We do not artificially treat a conflict between coalitions as a loose collection of dyadic conflicts; our unit of analysis is the dispute. Yet, by using a structural model, we are still able to estimate state-level parameters, such as how demographic and economic variables shape individual states' force multipliers.

By structurally estimating a model of negotiations and conflict, we also advance the burgeoning literature empirically studying the bargaining model of war (see Gartzke and Poast 2017). Building on prior work relating power and uncertainty to the likelihood and outcome of conflict (Reed 2000; Clark and Reed 2003; Reed et al. 2008), we allow for coalitions and introduce the concept of effective power as the product of (exogenous) force multipliers and (endogenous) effort. We also innovate methodologically by structurally estimating a game-theoretical model and by using its equilibrium to perform counterfactual simulations. Structural estimation has been a staple of industrial organization research and has seen increasing use in political science as scholars seek to answer counterfactual questions about complex strategic interactions (e.g., Carter 2010; Whang, McLean and Kuberski 2013; Crisman-Cox and Gibilisco 2018; Ascencio and Rueda 2019; Abramson and Montero forthcoming). Our work thus fits within the tradition in international relations using formal models to reason through counterfactual outcomes in real-world conflicts (see Levy 2008). Whereas such work has traditionally analyzed counterfactuals through qualitiative case studies, structural estimation allows us to scale this up to the analysis of quantitative data.

Finally, our work has some notable methodological antecedents in political science and economics. Our estimation of force multipliers is similar to a Bradley–Terry model (Bradley and Terry 1952), and thus to the recent work by Renshon and Spirling (2015) using such models to estimate the determinants of military effectiveness. We extend this technique by allowing for competitions between coalitions. The war-fighting portion of our model is also similar to work by Kang (2016), who estimates a game-theoretic model of lobbying as a contest. Meanwhile, our estimation of optimal offers and the probability of bargaining breakdown in an ultimatum game is similar to the estimator by Ramsay and Signorino (2009). We extend this technique by allowing for coalitions and by endogenizing the reservation values through a model of war.

2 A Model of Crisis Bargaining between Coalitions

We model crisis bargaining in the shadow of conflict between two coalitions, each comprised of one or more distinct states. In the model, the military contribution of each state depends on how much effort it chooses to exert, as well as an exogenous force multiplier describing its technological effectiveness. Bargaining takes place between representatives of each coalition who anticipate the likely outcome of conflict and calibrate their negotiating stances accordingly. As in the workhorse model of conflict (Fearon 1995), bargaining breakdown occurs due to incomplete information.[3]

[3] This is in line with the focus on informational explanations in formal theories of war between coalitions (Fang, Johnson and Leeds 2014; Wolford 2014*b*; Trager 2015; Smith forthcoming). The development and estimation of a general model of war due to commitment problems between coalitions would be a considerable project in its own right—and thus one that we leave for future research.

2.1 Setup

The coalitions are labeled A and B, consisting of I_A and I_B countries respectively. In the crisis, the sequence of play is as follows. First, a representative of side A makes an offer $x \in \mathbb{R}$. A representative of side B may accept or reject the offer. Each representative's objective is to maximize the total payoff to her side.[4]

If the offer x is accepted, then the game ends. Side A receives x to divide among its constituent countries, while side B receives $1 - x$. Additionally in this case, each side $K \in \{A, B\}$ pays an "audience cost" $\alpha_K \in \mathbb{R}$ for resolving the crisis without fighting. We allow for negative audience costs, $\alpha_K < 0$, reflecting the possibility that dovish domestic audiences in the coalition's constituent states may reward leaders for avoiding war rather than punishing them for backing down (Kertzer and Brutger 2016; Schultz 2001*b*; Snyder and Borghard 2011).[5] Each α_K is drawn independently according to a cumulative distribution function F_K; the realized values of α_K are private information, but the prior distributions are common knowledge. We assume each F_K has full support on $\mathbb{R}$ and is continuously differentiable with an associated density function f_K.

If war occurs, each country involved in the crisis selects a level of effort $e_i \geq 0$. Denote the vector of each country's effort by $e = (e_1, \ldots, e_I)$, where $I = I_A + I_B$, the total number of states involved in the crisis. The postwar distribution of spoils depends on how much effort each country exerts, as well as their force multiplier (or military effectiveness), an exogenous parameter $m_i > 0$. The probability that side A wins is

$$p_A(e) = \frac{\sum_{i \in A} m_i e_i}{\sum_{i \in A} m_i e_i + \sum_{j \in B} m_j e_j},$$

with $p_B(e) = 1 - p_A(e)$. Spoils of size 1 are divided among the countries on the winning side in proportion to their effectiveness-weighted contribution. The expected payoff from the

[4]We do not model the distribution of spoils across states in case of peace, as it is immaterial to the equilibrium.

[5]This approach allows the data to tell us when audience costs or belligerence costs arise in specific circumstances.

contest to country i on side K is

$$\begin{aligned}\pi_i(e) &= p_K(e)\frac{m_i e_i}{\sum_{j \in K} m_j e_j} - c_i e_i \\ &= \frac{m_i e_i}{\sum_{j=1}^{I} m_j e_j} - c_i e_i,\end{aligned}$$

where $c_i > 0$ is the country's marginal cost of effort. Each country's force multiplier and marginal cost of effort are common knowledge. As the equilibrium would be identical if we replaced each country's force multiplier with m_i/c_i and normalized their marginal cost of effort to 1, we will call the ratio m_i/c_i the *effective force multiplier* for country i. Without loss of generality, assume countries are labeled in order of their effective force multipliers: $m_1/c_1 \geq m_2/c_2 \geq \cdots \geq m_I/c_I$.

Taking the effort contributions in case of war as given, each side's payoff from bargaining is

$$U_A = \begin{cases} x - \alpha_A & \text{if } B \text{ accepts,} \\ p_A(e) - \sum_{i \in A} c_i e_i & \text{if } B \text{ rejects;} \end{cases}$$

$$U_B = \begin{cases} 1 - x - \alpha_B & \text{if } B \text{ accepts,} \\ p_B(e) - \sum_{i \in B} c_i e_i & \text{if } B \text{ rejects.} \end{cases}$$

Our assumption that coalition membership is fixed from the outset is, of course, a simplification. It is also a departure from models that treat coalition wars as resulting from the expansion of bilateral crises (Wolford 2014*a*; Fang, Johnson and Leeds 2014, e.g.,). This assumption provides important tractability in estimating the model,[6] but we do not see it as a major limitation of the analysis. If states have rational expectations about who will ultimately enter the crisis, their behavior at the bargaining stage should reflect those expectations—even if those states are not literally present at the table when bargaining

[6] Otherwise, for each dispute in our data, we would need to model hundreds of states' decisions whether to become involved.

takes place. Moreover, a critical parameter in most theories of war expansion is the military difference made by each new coalition partner. By modeling and estimating how power is aggregated in military coalitions, our analysis provides a necessary first step toward a richer empirical model of how those coalitions form in the first place.

2.2 Equilibrium

We solve for a perfect Bayesian equilibrium. First, we derive the equilibrium choices of effort in case of war. Equilibrium effort is greatest among the players that have the highest effective force multipliers. Players with low ratios, m_i/c_i, may opt out of exerting any effort at all. At least two players exert some effort, however, and all players exert effort in equilibrium if the effective force multipliers do not vary much among players. The following proposition formalizes the equilibrium of the war subgame.[7]

Proposition 1. *The war subgame has a unique Nash equilibrium e^*, in which the number of players who exert positive effort is*

$$J = \max \left\{ j \in A \cup B : \sum_{i=1}^{j} \frac{c_i}{m_i} > (j-1)\frac{c_j}{m_j} \right\}.$$

Equilibrium effort allocations are

$$e_i^* = \begin{cases} \frac{1}{c_i} q_i (1 - q_i) & i \leq J, \\ 0 & i > J, \end{cases}$$

where each q_i is defined as

$$q_i = \frac{(J-1)\frac{c_i}{m_i}}{\sum_{j=1}^{J} \frac{c_j}{m_j}}.$$

Figure 1 illustrates the equilibrium quantities defined in the proposition for a war subgame involving three players. Each player's effort decreases as their marginal cost of effort

[7] See Stein (2002) for the proof.

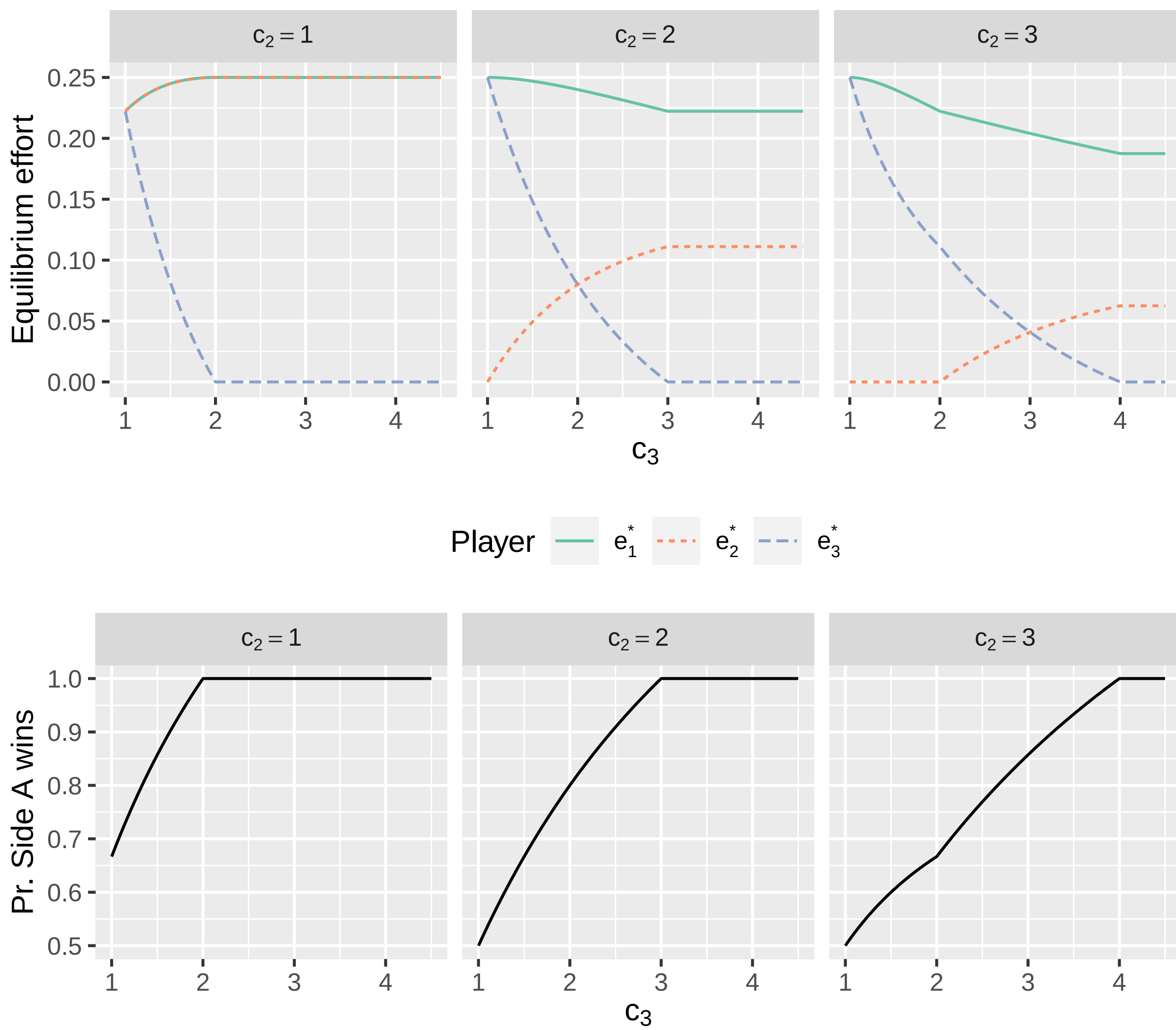

Figure 1. Equilibrium outcomes in the war subgame with three players, $A = \{1, 2\}$ and $B = \{3\}$. ($c_1 = 1$ and $m_1 = m_2 = m_3 = 1$.)

increases. When one player's effective force multiplier is much lower than the others', that player drops out of the contest altogether. Short of that, however, a decrease in one player's effort is not always associated with lower efforts by the others. For example, in the leftmost panel of Figure 1, even as player 3's effort decreases with its marginal cost of effort, the other two players' efforts increase.

Moving up to the bargaining stage, each side's optimal strategy depends on its war payoff. Let $\pi_A^* = \sum_{i \in A} \pi_i(e^*)$ denote side A's expected utility from fighting, and define π_B^* analogously. Each side has private information about its audience cost α_K, so the peace payoffs are partly private information. The following proposition characterizes the bargaining equilibrium.

Proposition 2. *Assume F_B is log-concave. In any perfect Bayesian equilibrium of the bargaining game:*

(a) Each type α_A of side A makes the unique offer that solves

$$x^*(\alpha_A) = \alpha_A + \pi_A^* + \frac{F_B(1 - x^*(\alpha_A) - \pi_B^*)}{f_B(1 - x^*(\alpha_A) - \pi_B^*)}. \tag{1}$$

(b) Each type α_B of side B accepts $x < 1 - \pi_B^* - \alpha_B$ and rejects $x > 1 - \pi_B^* - \alpha_B$.

(c) Strategies in all war subgames are given by Proposition 1.

Proof. As war payoffs do not depend on x or α_K, statement (c) is immediate from Proposition 1. Statements (a) and (b) then follow from Fearon (1995, Claim 2). □

We observe the usual risk-reward tradeoff in the bargaining equilibrium. The *ex ante* (i.e., before types are realized) probability of war in equilibrium is

$$\Pr(\text{war}) = \int [1 - F_B(1 - x^*(\alpha_A) - \pi_B^*)] \, dF_A(\alpha_A). \tag{2}$$

This probability reflects side A's incentives to take risks to increase its settlement payoff. This risk-taking incentive increases with A's expected utility from war and the magnitude of its audience cost for peace. It also depends, perhaps non-monotonically, on the extent of uncertainty about B's audience cost.

The comparative statics of our model are, in many ways, similar to the comparative statics of other bargaining models of war, which have been widely tested (e.g., Reiter and Stam 1998*a*; Reed 2000; Schultz 2001*a*; Reed et al. 2008; Tomz 2007; Kurizaki and Whang 2015; Crisman-Cox and Gibilisco 2018). Increasing audience costs increases the probability of war; if democracies are more effective at fighting, then coalitions with democracies should get better settlements; and if there are costs for being belligerent or backing down, then those states facing higher costs of war are less likely to fight. In a situation with many models supported by the same empirical results, what is next? Our answer is to quantify our model.

The goal of our analysis is, therefore, different from a standard test of comparative statics and proceeds in two stages. In what follows we ask: if we require our model to explain the data on international militarized disputes, what does that does that imply about the unobserved theoretical parameters of crisis bargaining between coalitions? The answer is our quantitative estimates. Then, taking these real world implications for our theoretical model, we turn to making predictions and exploring counterfactuals that are theoretically driven and empirically informed.

3 Quantifying the Bargaining Model

Our goal is to quantify the major components of the bargaining model of war laid out in the previous section. First, we want to identify which real-world variables contribute most to the key parameters: military effectiveness (m_i), marginal costs of effort (c_i), and the prior distributions of audience costs (the mean and variance of each α_K). We model these parameters as functions of the relevant variables, then structurally estimate their weights from data on interstate crises. The structural estimation procedure identifies the parametric form of the model that best corresponds to what we observe empirically. Second, with our structural estimates in hand, we wish to quantify the features of crisis bargaining that would otherwise be difficult to measure systematically for particular countries, in particular crises, or as a function of directly observable variables. These include the individual effort contributions of coalition partners and the extent of free-riding in conflicts with more than two actors.

We observe data on N crises. The n'th crisis involves I_n states, with $I_{nA} \geq 1$ on side A and $I_{nB} \geq 1$ on side B. The variable $Y_n^{\text{war}} \in \{0, 1\}$ denotes whether the n'th crisis resulted in war, and $Y_n^{\text{win}} \in \{A, B\}$ denotes its winner.[8] The other endogenous components of the game—offers and effort—are not observed. We estimate the parameters of the model as a function of observable characteristics:

- State-level characteristics affecting military effectiveness, X_{ni}.[9]
- State-level characteristics affecting the marginal cost of effort, Z_{ni}.
- Coalition- and dispute-level characteristics affecting the average audience cost, L_{nA} and L_{nB}.
- Coalition- and dispute-level characteristics affecting the variance of audience costs, S_{nA} and S_{nB}.

Consider the i'th state in the n'th crisis. We assume its military effectiveness and cost of effort are log-linear functions of state-level characteristics:

$$m_{ni}(\beta) = \exp(X_{ni}^{\top}\beta);$$
$$c_{ni}(\gamma) = \exp(Z_{ni}^{\top}\gamma).$$

[8]If the n'th crisis does not end in war, we write $Y_n^{\text{win}} = \emptyset$.
[9]We treat all vectors as column vectors.

This gives the coefficients the natural interpretation of the effort multiplier effect. We treat β and γ as unknown parameters to be estimated. For the n'th observation, let $p^*_{nA}(\beta, \gamma)$ denote the equilibrium probability of victory by side A under the given parameters, and let p^*_{nB}, π^*_{nA}, and π^*_{nB} be defined analogously.

For statistical identification, there must be no overlap between the variables included in the cost and effectiveness equations. When this restriction seems substantively insensible, we may consider the parameter estimates as net effects—effectiveness minus cost for X variables, and vice versa for Z variables.

In the bargaining stage, we assume each side K's audience cost in the n'th crisis is drawn from a logistic distribution with location parameter μ_{nK} and scale parameter σ_{nK}.[10] Structural models of bargaining with a logistic shock to reservation values have previously been studied by Ramsay and Signorino (2009). We use a linear model for location and a log-linear model for scale:

$$\mu_{nK}(\eta) = L_{nK}^\top \eta,$$
$$\sigma_{nK}(\theta) = \exp(S_{nK}^\top \theta),$$

where η and θ are unknown parameters to be estimated. The equilibrium probability of war is increasing in both α_A and α_B,[11] so a positive coefficient in η implies that the corresponding variable increases the probability of war. The effects of the scale parameters, σ_A and σ_B, are ambiguous in general, though we can calculate marginal effects for any given dispute in the data.

We estimate the parameters $(\beta, \gamma, \eta, \theta)$ by maximum simulated likelihood.[12] The log-

[10]Note that $\mathbb{E}[\alpha_K] = \mu_{nK}$ and $V[\alpha_K] = \sigma^2_{nK}\pi^2/3$.
[11]See Proposition 5 in the Appendix.
[12]See the Appendix for details.

likelihood function is

$$\log \ell(\beta, \gamma, \eta, \theta) = \sum_{n:Y_n^{\text{war}}=0} \log(1 - r_n^*(\beta, \gamma, \eta, \theta)) + \sum_{n:Y_n^{\text{war}}=1} \log r_n^*(\beta, \gamma, \eta, \theta) \\ + \sum_{n:Y_n^{\text{win}}=A} \log p_{nA}^*(\beta, \gamma) + \sum_{n:Y_n^{\text{win}}=B} \log p_{nB}^*(\beta, \gamma), \tag{3}$$

where $r_n^*(\beta, \gamma, \eta, \theta)$ denotes the probability of war for the n'th crisis as a function of the unknown parameters, calculated using Equation 2. We use multiple imputation to correct for missingness in state-level variables, and we calculate inferential statistics via the jackknife.[13]

4 Data and Specification

We use historical data on disputes to identify how political, economic, and technological characteristics shape states' military effectiveness, marginal costs of effort, and audience costs. The key assumption is that the data we observe is generated according to equilibrium play of the model. Because the model accommodates purely bilateral as well as coalitional disputes, our sample includes both types.

4.1 Sample and Outcomes

The unit of observation is the interstate dispute, and our sample consists of Militarized Interstate Disputes (MIDs) between 1816 and 2010.[14] The sample size is $N = 2{,}101$. There are 284 disputes with multiple states on one or both sides, which we call *coalitional disputes*. These involve about 4.7 disputants each, giving us 4,962 distinct dispute participants in the data.[15]

Following the MID codings, we treat the initiating side as side A, which makes the offer in our bargaining model. We code a dispute as war, $Y_n^{\text{war}} = 1$, if it resulted in more than 25

[13] Kinks in the likelihood function preclude ordinary gradient-based estimation of the standard errors.

[14] See the Appendix for details on all data sources.

[15] Six states are coded as being on different sides of World War II at different times. We include them each only on the side on which they spent the most time.

fatalities. In case of war, we code side A as the winner, $Y_n^{\text{win}} = A$, if the MID data records victory by side A or yield by side B as the outcome. In all other cases we code side B as the winner, $Y_n^{\text{win}} = B$.[16] Table 1 reports the distribution of dispute outcomes in the sample.

Outcome	Frequency
$Y_n^{\text{war}} = 0$	1851
$Y_n^{\text{war}} = 1, Y_n^{\text{win}} = A$	65
$Y_n^{\text{war}} = 1, Y_n^{\text{win}} = B$	185

Table 1. Distribution of dispute outcomes.

4.2 State Characteristics

Even though our dependent variables are the collective outcomes of whether war occurred and who won if so, our structural approach allows us to estimate the determinants of *individual* states' effective force multipliers. We model these as functions of individual state characteristics, dividing the underlying state-level variables into demographic, economic, political, and geopolitical characteristics. Table 2 summarizes all of the variables included in the model.

Demographics. Indices of national power, including the popular CINC score, typically include population measures. We therefore include **Total Population** and **Urban Population** in the equation for m_i, raw military effectiveness. We use the measures from the Correlates of War project's National Material Capabilities data.

Theoretically, large states might enjoy greater military effectiveness because of their ability to field mass armies (Posen 1993). This is particularly relevant in our time period, which starts just after the Napoleonic Wars. Conversely, the costs of governing and extracting taxes from large populations might inhibit military effectiveness (Beckley 2018).

[16] This entails assuming that a stalemate favors side B, which we see as natural given that side A is the initiator of the dispute.

Variable	Units	Mean	Std. Dev.
State-Level: Demographic			
Total Population	Thousands of persons‡	86,494	192,649
Urban Population	Thousands of persons‡	18,250	44,268
State-Level: Economic			
Energy Consumption	Thousands of coal-tons‡	223,027	581,944
GDP	Millions USD, 2011†	654,983	1,848,001
Import Percentage	Percentage‡	22.09	17.33
Iron and Steel Production	Thousands of tons‡	13,556	39,598
State-Level: Political			
Democracy	Polity IV score	−0.21	7.22
State-Level: Geopolitical			
Distance to Dispute	Miles‡	1,098	1,871
Nuclear Weapons	Binary	0.14	0.35
Dispute-Level			
Democracy, Side A	Polity IV score	−1.06	6.88
Democracy, Side B	Polity IV score	0.00	7.01
Major Power, Side A	Binary	0.32	0.46
Major Power, Side B	Binary	0.21	0.41
Participants, Side A	Count†	1.19	1.31
Participants, Side B	Count†	1.17	1.19
Contiguity	Binary	0.66	0.45
Interest Similarity	S-score	0.55	0.40
Peace Years	Years‡	11.75	23.36

Table 2. State- and dispute-level characteristics included in the empirical model. †: log-transformed in estimation. ‡: $\log(1+x)$-transformed in estimation.

Economics. Economic development is a key component of military effectiveness (Wright and Wright 1983; Singer et al. 1972). Richer states have more resources available for military use and may also have access to more effective coercive technologies. We include three measures in the military effectiveness equation, m_i, to capture economic influences on the force multiplier: **GDP** from the Penn World Tables, as well as **Energy Consumption** and **Iron and Steel Production** from the National Material Capabilities data as additional proxies.[17]

The last economic component we include in the effective force multiplier is **Import Percentage**, the ratio of the state's total imports to its GDP, measured by the World Development Indicators. We include this in the equation for c_i, the marginal cost of effort. If states must acquire raw materials or other resources via international markets in times of war, it might be more costly to field the same level of military effort than if those resources were available domestically (Gartzke 2007).

Political. We include **Democracy**, the net score (Democracy minus Autocracy) from the Polity IV project, as a component of c_i, the marginal cost of effort. Citizens of democracies can more easily resist extractive or economically inefficient military production than can the subjects of authoritarian regimes (Bennett and Stam 1996). Scholars have also observed, however, that the political and cultural features of democratic societies may lend themselves to battlefield success (Reiter and Stam 2002). Our statistical model does not allow us to separately identifying the force-multiplying and cost-increasing effects of democracy. If the force-multiplying effect of democratic politics is stronger, that would manifest itself as a negative coefficient on Democracy in the cost equation.

Geopolitical. We include two geopolitical determinants of a state's effective force multiplier in the equation for m_i, raw military effectiveness. The first is **Nuclear Weapons**, an

[17] The remaining CINC components, Military Expenditures and Military Personnel, are endogenous state choices more so than structural components of the force multiplier, making them inappropriate for inclusion.

indicator for whether a state has a deliverable nuclear weapon. Even when nuclear weapons are not used, they may serve as a force multiplier by increasing the effectiveness of states' coercive threats (Kroenig 2013). Alternatively, nuclear weapons might reduce a state's force multiplier, as the increased risk of nuclear disaster impels states to show restraint in their deployment of conventional capabilities (Asal and Beardsley 2007; Sechser and Fuhrmann 2017).

The other geopolitical factor is the **Distance to Dispute** for each participant state. This variable measures the distance from each state's capital to the location of the dispute as coded in the MIDLOC project.[18] We expect distance to act as a negative force multiplier, as states' ability to project power declines over long distances (Boulding 1962).

4.3 Coalition and Dispute Characteristics

In the bargaining stage, we treat the mean and variance of each side's audience cost, α_K, as a function of coalition- and dispute-level variables. First, we model a coalition's average audience cost as a function of **Democracy**, the average Polity IV score among its constituent members. Theories of coercive diplomacy have traditionally posited that audience costs are greater for democracies than for autocracies due to greater public accountability (Fearon 1994; Schultz 2001*a*). However, more recent empirical studies have identified wide variation in audience costs across regime types (Crisman-Cox and Gibilisco 2018) and have suggested that political leaders may be punished more for belligerence than for backing down (Kertzer and Brutger 2016).

In the equation for the average audience cost, we also include an indicator for whether the coalition contains a **Major Power**, taken from the Correlates of War project's State System Membership data. We thus account for the possibility that a major power would find it more costly to back down from a crisis than a smaller state would.

Finally, in modeling the mean audience cost, we include three dispute-level characteristics

[18]We recode Distance to Dispute as 0 for disputes occurring on or within a state's borders.

that might influence states' willingness to pursue peace rather than violence. We include **Contiguity**, the proportion of pairs of disputants that have a land border or are separated by less than 150 miles of water. Contiguity is a well-known predictor of international conflict (Bremer 1992), as states may be particularly unwilling to back down from disputes over issues with their neighbors. We also include the S-score (Signorino and Ritter 1999) of states' alliance portfolios (again, averaged over each pair of disputants) as a proxy for **Interest Similarity** (Gowa 2011), using the measure provided by the Alliance Treaty Obligations and Provisions project. Finally, we include **Peace Years**, measuring the average time since the last interstate dispute between pairs of disputants.

We model the variance of a coalition's privately known audience costs as a function of the **Number of Participants** in the coalition. This allows us to test whether uncertainty is systematically greater in crises involving many participants.

5 Structural Estimation Results

5.1 Force Multipliers

We can write a dispute participant's effective force multiplier, m_i/c_i, as the product of its four substantive components:

$$\text{Total Multiplier}_i = \text{Demographic}_i \times \text{Economic}_i \times \text{Political}_i \times \text{Geopolitical}_i.$$

For example, $\text{Economic}_i > 1$ would indicate that a country's economic variables enhance its effectiveness per unit of military force, whereas $\text{Economic}_i < 1$ would indicate that its economic position is a net drain on its martial effectiveness. The components of the multipliers are calculated using the raw model estimates (Table 6 below), with terms in the effectiveness equation m_i entering positively and those in the cost equation c_i entering negatively. For example, the demographic component is computed as

$$\text{Demographic}_i = \exp(0.100 \times \text{Total Population}_i - 0.077 \times \text{Urban Population}_i).$$

	Country	Year	MID#	Multiplier	Demographic	Economic	Political	Geopolitical
1	India	1971	2099	3.09	1.64	1.64	1.15	1.00
2	India	1969	2098	3.08	1.66	1.62	1.15	1.00
3	India	1969	2633	3.08	1.66	1.62	1.15	1.00
4	India	1969	2634	3.08	1.66	1.62	1.15	1.00
5	India	1969	2635	3.08	1.66	1.62	1.15	1.00
6	United States	1919	2185	3.03	1.45	1.79	1.17	1.00
7	United States	1918	2184	2.99	1.45	1.76	1.17	1.00
8	India	1965	623	2.95	1.65	1.56	1.15	1.00
9	India	1965	1463	2.95	1.65	1.56	1.15	1.00
10	India	1967	1715	2.94	1.65	1.55	1.15	1.00
4953	Bahrain	1991	3957	1.00	1.28	0.95	0.86	0.96
4954	Jordan	1967	1067	0.99	1.34	0.87	0.87	0.97
4955	Jordan	1967	1035	0.99	1.34	0.87	0.87	0.97
4956	Bahrain	1986	2572	0.98	1.27	0.95	0.86	0.95
4957	Jordan	1962	1018	0.98	1.37	0.85	0.87	0.97
4958	Jordan	1959	3231	0.98	1.40	0.83	0.87	0.97
4959	Jordan	1963	1019	0.97	1.37	0.84	0.87	0.97
4960	Jordan	1966	3412	0.97	1.34	0.86	0.87	0.97
4961	Jordan	1961	122	0.96	1.38	0.84	0.87	0.95
4962	Jordan	1962	1108	0.96	1.37	0.85	0.87	0.95

Table 3. Greatest and lowest estimated force multipliers for dispute participants in the analysis data.

Demographic and economic characteristics have the strongest influence on countries' effective force multipliers. As an illustration, Table 3 reports the ten highest and lowest values for dispute participants in our data. Our estimates range from a maximum of 3.09 for India in its 1971 dispute with China, to a minimum of 0.69 for Jordan in its 1962 dispute with Egypt and Yemen. The top spots are occupied by large democracies involved in disputes on or within their own borders—India in the 1960s–70s and the United States in the 1910s. By contrast, the lowest force multipliers belong to small autocracies involved (in most cases) some distance from their home territory. While the top and bottom countries differ along many dimensions, it is apparent that the quantitative difference in their estimated force multipliers is due largely to the demographic and economic components.

In a more systematic analysis of the full data, Table 4 shows that the demographic and economic components of the force multiplier account for the most variation across disputants. If we were to remove either of these components, the remaining variation in force multipliers would decrease by more than half. Interestingly, the geopolitical component tends to suppress variation in the total force multiplier across countries. Countries in the strongest demographic and economic positions tend to have relatively unfavorable geopolitical multipliers—they are relatively likely to be involved in disputes far from their homeland, reducing their ability to project power.

Component	SD	%Explained	Correlation
Total	0.37	100.0	1.00
Demographic	0.17	55.8	0.33
Economic	0.24	53.1	0.57
Political	0.11	29.3	0.50
Geopolitical	0.11	−55.6	−0.08

Table 4. Determinants of force multipliers in the empirical model. SD = standard deviation. %Explained = $100 \times \frac{V[\text{total}]-V[\text{total/component}]}{V[\text{total}]}$. (Values do not sum to 100 due to correlations between components.) Correlation = Spearman correlation between total and component.

The primacy of demographic and economic factors remains apparent when we look exclusively at major powers. Figure 2 plots the estimated force multiplier for China, Japan, Russia, the United Kingdom, and the United States throughout our sample period.[19] Both within each country's time series and when comparing different countries at the same point in time, we see the most variation in the demographic and economic components.[20] Nevertheless, political factors have a nontrivial influence too—for example, the United States' democratic political system pushes its force multiplier above China's in 2010, despite the latter's demographic advantage.

[19]We assume zero distance to dispute in this figure, which raises the overall force multiplier and slightly decreases variation in the geopolitical component.

[20]The negative trend in the demographic multiplier time series is because urban population growth has outpaced total population growth, and we find a moderate negative coefficient on urban population in the equation for m_i.

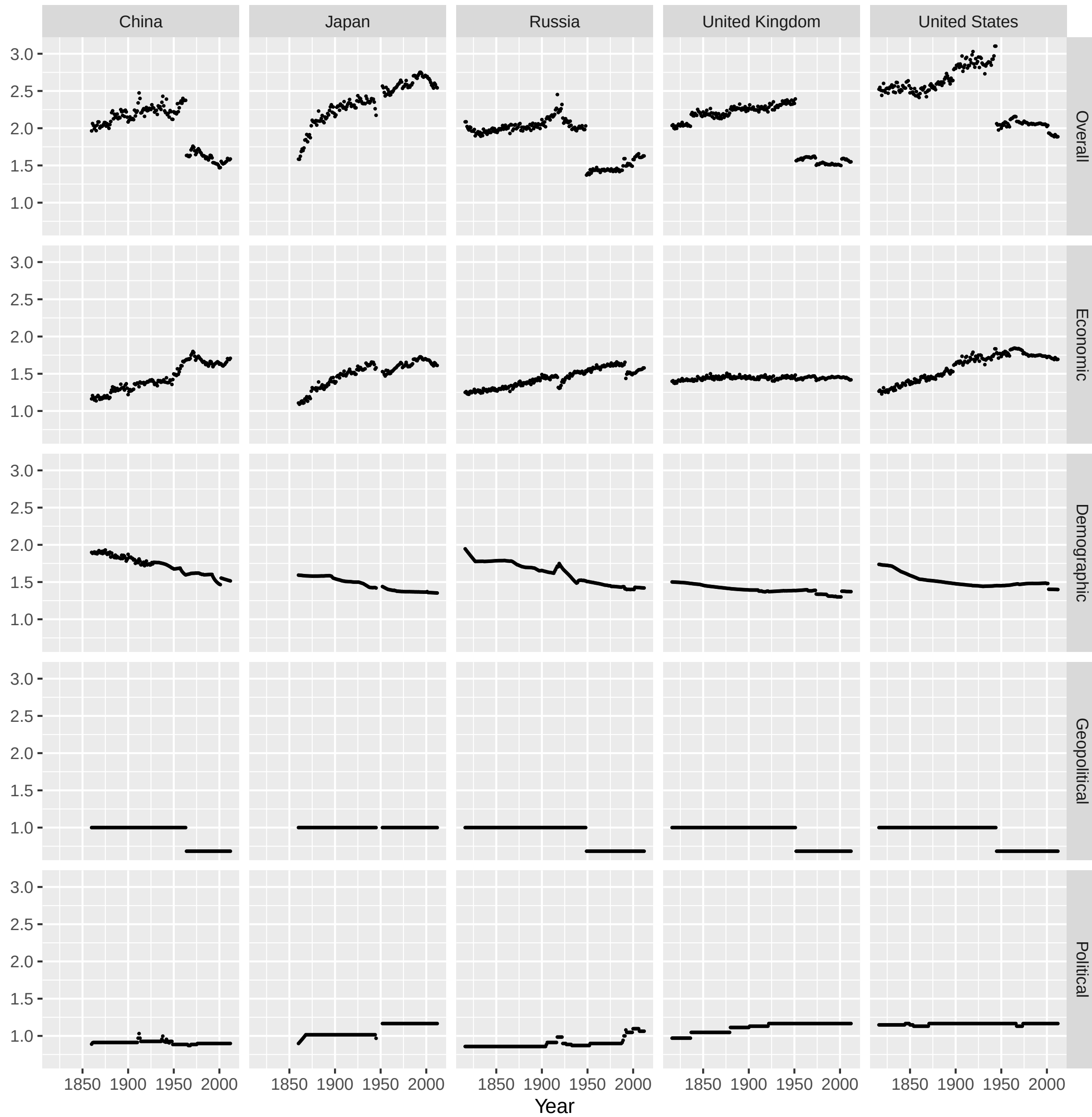

Figure 2. Estimated force multipliers for major powers over time.

We observe a downward secular trend in the major powers' force multipliers. A common trend like this has little effect on likely war outcomes, as equilibrium probabilities of victory are a function of the relative force multipliers at any point in time. Still, the downward trends imply that it takes more effort for a major power to achieve a given military objective now than at their mid–20th century peaks. For example, we observe this in how the United States spent similar amounts in real terms on World War II and on the Iraq War (Stiglitz and Blimes 2008), despite accomplishing far less militarily in the latter.

Table 5 breaks down each component of the total multiplier into its constituent variables, displaying the dispute participants with the highest and lowest values of each component. The constituent variables are given as their contribution to the force multiplier, so that each component multiplier equals the product of the constituent variable multipliers.

We see the highest demographic multipliers for countries with moderately sized, totally rural populations. This reflects the countervailing effects of Total Population and Urban Population in the structural model. Total Population has the largest positive effect among the variables we consider, with a 1% population increase being associated with a 0.1% increase in military effectiveness. As our sample covers the post-Napoleonic world, this finding is consistent with the idea that large national states possess a military advantage through their ability to field mass armies (Posen 1993). We find a partially offsetting force multiplication effect of Urban Population, with a 1% increase in urbanites associated with a 0.07% decline in the force multiplier.

Each economic development variable contributes to greater military effectiveness, though none is statistically significant on its own. Meanwhile, import dependence reduces effective force multipliers, as a percentage point increase in Import Percentage is associated with a 0.1% increase in the marginal cost of effort. Altogether, we find the greatest economic multipliers for large economies with relatively low import dependence, such as the United States in the 1960s and China in the 1970s. Small, open economies have the lowest economic multipliers.

	Country	Year	MID#	Demographic	Total Pop. $\beta = 0.10$	Urban Pop. $\beta = -0.08$
1	Ethiopia	1931	407	2.56	2.56	1.00
2	Ethiopia	1923	1669	2.51	2.51	1.00
3	Afghanistan	1934	3159	2.44	2.44	1.00
4	Afghanistan	1925	1781	2.42	2.42	1.00
5	Papua New Guinea	2008	4589	2.41	2.41	1.00
4958	Qatar	1986	2572	1.20	1.82	0.66
4959	Singapore	1987	2785	1.20	2.19	0.55
4960	Singapore	1987	2789	1.20	2.19	0.55
4961	Qatar	1984	3617	1.19	1.79	0.66
4962	Bahamas	1984	3038	1.17	1.72	0.68

(a) Greatest and lowest demographic force multipliers.

	Country	Year	MID#	Economic	GDP $\beta = 0.02$	Energy $\beta = 0.02$	Iron and Steel $\beta = 0.01$	Imports $\gamma = 0.10$
1	United States	1965	2916	1.84	1.42	1.33	1.16	0.85
2	United States	1965	2929	1.84	1.42	1.33	1.16	0.85
3	United States	1964	611	1.84	1.42	1.33	1.16	0.85
4	United States	1964	1213	1.84	1.42	1.33	1.16	0.85
5	United States	1964	2220	1.84	1.42	1.33	1.16	0.85
4958	Grenada	1983	3058	0.80	1.14	1.04	1.00	0.67
4959	Dominica	1983	3058	0.80	1.15	1.02	1.00	0.68
4960	Maldives	1987	2790	0.79	1.17	1.04	1.00	0.65
4961	St. Vincent	1983	3058	0.79	1.15	1.04	1.00	0.66
4962	Palau	2000	4243	0.77	1.16	1.06	1.00	0.62

(b) Greatest and lowest economic force multipliers.

	Country	Year	MID#	Geopolitical	Distance $\beta = -0.01$	Nuclear $\beta = -0.38$
4958	United States	1975	356	0.635	0.930	0.683
4959	Russia	1979	2224	0.635	0.930	0.683
4960	United States	1968	3300	0.635	0.930	0.683
4961	Russia	1977	3122	0.635	0.930	0.683
4962	United States	2003	4460	0.635	0.929	0.683

(c) Lowest geopolitical force multipliers.

Table 5. Estimated force multiplier components by category for dispute participants in the analysis data.

The geopolitical multiplier is a function of a dispute participant's nuclear status and distance to the location of the dispute.[21] Perhaps surprisingly, we find that nuclear weapons reduce a country's effective force multiplier: the military effectiveness of a nuclear state is only about 68% that of an otherwise equivalent non-nuclear state. This provides at least suggestive evidence that the possession of nuclear weapons inhibits the deployment of conventional force (Asal and Beardsley 2007). In the shadow of nuclear weapons, states may have extra incentives to limit their efforts so as to forestall devastating nuclear conflict (Schelling 1966). In line with usual expectations (Boulding 1962), Distance from Dispute erodes military effectiveness. Combining these findings, we find the lowest geopolitical force multipliers for nuclear powers involved in disputes far from their homeland—the United States and Russia during the Cold War, as well as the United States in a 2003 dispute with Indonesia.

The political component is omitted from Table 5, as it depends solely on a country's Polity IV score. Democracies have lower marginal costs of effort ($\gamma = -0.015$), and thus a greater effective force multiplier: the marginal cost of effort for a total autocracy is about 35% greater than that for an otherwise identical full democracy. This apparently runs contrary to theoretical expectations that political accountability makes war-fighting more costly for democratic leaders (Bueno de Mesquita et al. 1999; Jackson and Morelli 2007). However, because we cannot separately identify the influence of the same variable on m_i and c_i, our estimates of γ represent the *net* effect of each variable on the overall cost-to-effectiveness ratio. Consequently, it is possible that political accountability increases the costs of effort, but that these costs are outweighed by democratic advantages in raw military effectiveness (see Reiter and Stam 1998*a*,*b*, 2002).

[21] Table 5 does not display the greatest geopolitical force multipliers, as all 1,896 non-nuclear states with zero distance to dispute attain the maximum geopolitical multiplier of 1.0.

5.2 Audience Costs

We estimate how coalition- and dispute-specific factors shape the prior distribution of audience costs, α_K. Remember that μ_K denotes the average audience cost, and σ_K denotes the scale parameter (proportional to the standard deviation). An increase in μ_K is associated with a greater probability of war. Table 6 reports the full estimated model.

Beginning with the coalition-level influences on the mean, we see that Democracy has a statistically significant negative effect on the average audience cost. This means more democratic coalitions will tend to make more restrained demands at the bargaining table, and militarized disputes will be less likely to escalate to war as disputant coalitions become more democratic. This is consistent with Reed's (2000) finding that joint democracy reduces the probability of escalation, though our specification allows for monadic effects as well. On the other hand, the estimate is inconsistent with the body of theory predicting particularly large audience costs among democracies (Fearon 1994; Schultz 2001*a*). We interpret the finding as a dovish bias (Snyder and Borghard 2011), or high belligerence cost (Kertzer and Brutger 2016), among democracies that outweighs any potentially greater audience cost.

The other coalition-level determinant of the average audience cost, Major Power, is statistically insignificant. The point estimate indicates larger audience costs among coalitions containing a major power, meaning such coalitions are less likely to reach peaceful settlements.

At the dispute level, Peace Years has a statistically significant and negative impact on the average audience cost. It is therefore less politically costly for a country or coalition to reach a peaceful settlement with an adversary it has recently had peaceful relations with. Meanwhile, Contiguity and Interest Similarity are associated with greater audience costs, though neither relationship is statistically significant.

Finally, we turn to the sole influence on variance in audience costs that we include in our model, the Number of Participants. In line with numerous studies suggesting that coalitions increase uncertainty in crises (Wolford 2014*a*; Bas and Schub 2016; Vasquez and Rundlett

Term	Estimate	Std. Err.	Z	p
Force Multiplier: Demographic				
m_i: Total Population	$0.100^{\dagger}$	0.056	1.798	0.072
m_i: Urban Population	-0.077^{*}	0.036	-2.113	0.035
Force Multiplier: Economic				
m_i: Energy Consumption	0.020	0.032	0.614	0.539
m_i: GDP	0.023	0.039	0.585	0.559
m_i: Iron and Steel Production	0.012	0.024	0.518	0.604
c_i: Import Percentage	0.100	0.132	0.758	0.449
Force Multiplier: Political				
c_i: Democracy	-0.015^{*}	0.006	-2.531	0.011
Force Multiplier: Geopolitical				
m_i: Distance to Dispute	-0.008	0.019	-0.428	0.669
m_i: Nuclear Weapons	-0.382^{*}	0.100	-3.814	<0.001
Audience Cost Distribution				
μ_K: Intercept	-2.100	0.615	-3.417	0.001
μ_K: Contiguity	0.055	0.225	0.242	0.809
μ_K: Democracy	-0.080^{*}	0.022	-3.568	<0.001
μ_K: Interest Similarity	0.289	0.186	1.551	0.121
μ_K: Major Power	0.382	0.361	1.059	0.290
μ_K: Peace Years	-0.442^{*}	0.170	-2.603	0.009
σ_K: Intercept	-0.846	0.315	-2.684	0.007
σ_K: Coalition Size	1.240^{*}	0.198	6.272	<0.001
Disputes	2101			
Participants	4962			
Log-likelihood	-875.104			
AIC	1784			
BIC	1880			

Table 6. Structural estimation results. †: $p < 0.1$. *: $p < 0.05$.

2016; Smith forthcoming), we identify a statistically significant and positive effect of coalition size on the degree of uncertainty. According to our structural model estimates, going from a single state to a two-state coalition increases the standard deviation of α_K by about 54%. As we show in a counterfactual analysis below, these increases in uncertainty can significantly impact the likelihood of conflict, even when the additional coalition members have little effect on the underlying military balance.

5.3 Joiners and Bargaining

In our baseline analysis, we calculate expected war payoffs using all participants in the crisis, even those that joined after the onset of the dispute. This reflects a rational expectations assumption that the countries at the bargaining table correctly anticipate which other countries will participate in case of war. To assess whether the data support this assumption, we estimate an alternative specification in which the countries at the bargaining table assume war would take place only among the originators of the dispute.[22] The probability of war in the modified model is based on the expected war payoffs from a war only among the originators, as well as covariates pertaining only to the originators. After estimating the alternative model, we perform a Vuong (1989) non-nested model test comparing it to the baseline model to assess which better fits the data. We find that the baseline model fits significantly better ($\text{LR} = 44.0, Z = 4.2, p < 0.001$), favoring the idea that states form rational expectations about likely coalition joiners.

6 Counterfactual Analyses

One of the primary virtues of the structural estimation approach is that it allows us to calculate counterfactual quantities of substantive interest. We use the following procedure to calculate counterfactual estimates:

[22]See the Appendix for details and estimates.

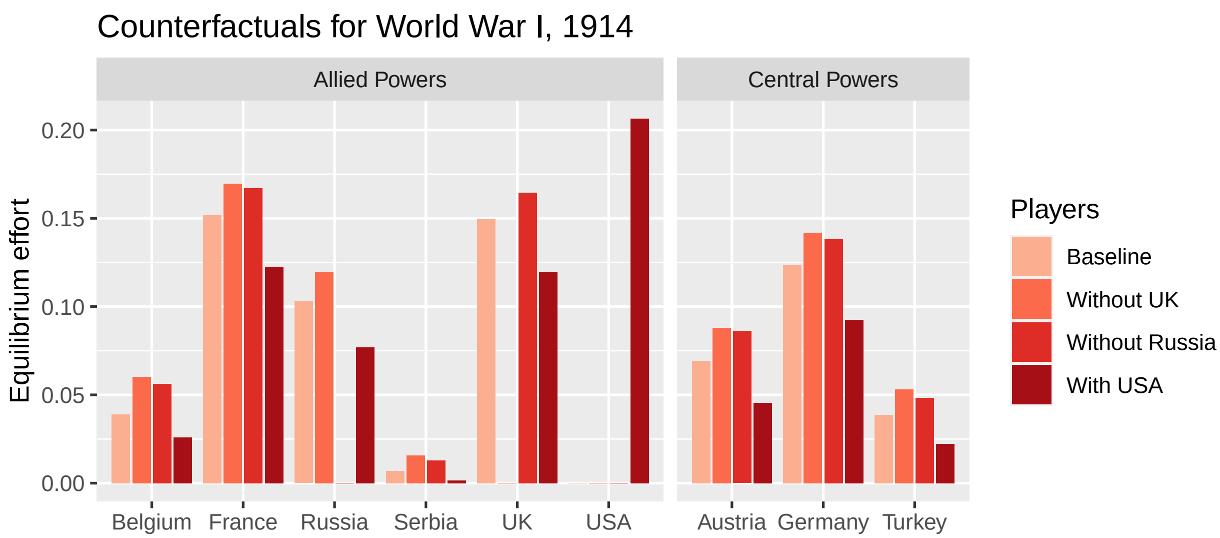

Figure 3. Equilibrium choices of effort in World War I, 1914.

1. Given two (perhaps hypothetical) coalitions of countries at a particular time, use the estimates from Table 6 to calculate the military effectiveness of each participant (m_i), the marginal cost of effort of each participant (c_i), and the prior distribution of each side's audience costs (means μ_A and μ_B, scale parameters σ_A and σ_B).

2. Use Proposition 1 to calculate each individual country's effort and each side's probability of winning in case of war, according to the equilibrium of the war subgame.

3. Use Proposition 2 to calculate the equilibrium offer and the probability of war.

6.1 Contributions to Coalitions

Perhaps the most novel feature of our estimation approach is that it allows us to estimate the contributions of individual countries or groups of countries to the outcomes of coalitional conflicts. To illustrate this capability, we analyze counterfactual war outcomes as a function of the set of participants in a few prominent historical conflicts.

We begin by analyzing the contributions of three participants on the Allied side of World War I—Russia, the United Kingdom, and the United States.[23] We model each country's

[23] Given our assumption that informational problems cause war, one might worry about the validity of

equilibrium effort, the probability of Allied victory, and the *ex ante* probability of war as of 1914. We vary the set of participants on the Allied side while holding fixed participation among the Central Powers. Equilibrium choices of war effort under each scenario are plotted in Figure 3, and Table 7 reports the dispute-level equilibrium quantities.

Counterfactual	Pr(Allied Powers Win)
Baseline	0.65
Without Russia	0.57
Without UK	0.55
With USA	0.78

Table 7. Counterfactual equilibrium quantities in World War I, 1914.

Our results suggest that Russia and the United Kingdom both made meaningful contributions to the war effort, but that these contributions were at least somewhat offset by free-riding. In the baseline equilibrium, Russia contributes 26% of the Allied Powers' effective effort (effort weighted by military effectiveness, $m_i e_i$), and the United Kingdom contributes about 32%. Yet most of each country's contribution would have been made up by other Allied Powers had it not participated in the conflict, meaning their observed efforts far outweigh their effects. Meanwhile, American entry at the outset of the dispute would have made an even bigger difference than British entry did. We estimate that the Allied Powers' chance of winning would have increased 13 percentage points to 78% had the United States participated in 1914, with the United States contributing about 40% of this side's effective efforts. This is because the United States has a much larger effective force multiplier than the UK, with most of the difference owing to the United States' larger population and lower import dependence.

Still in the context of World War I, we can also evaluate the fortunes of the Central Powers as of 1915 if the "Schlieffen Plan" had succeeded. We calculate equilibrium military

these estimates if the war were instead caused by commitment problems. However, the bulk of the identifying power for our force multiplier estimates—which drive these counterfactuals—comes from the contest subgame after bargaining has broken down. Consequently, we expect our estimates of optimal contributions and the overall military balance would not change much if we modeled a distinct bargaining process, as long as the war subgame followed the same contest model.

outcomes as of 1915 if France, Belgium, or both had been knocked out of the war in the prior year, while holding fixed the rest of the participants on each side. Would the French and Belgian military efforts just have been made up by Russia and the United Kingdom?

Counterfactual	Pr(Allied Powers Win)
Baseline	0.68
Without Belgium	0.66
Without France	0.56
Without both	0.52

Table 8. Counterfactual equilibrium quantities in World War I, 1915.

Table 8 presents the results of the counterfactual analysis of the "Schlieffen Plan" scenario. Our model suggests that the plan's success would indeed have made a substantial difference. In the baseline scenario, our model projects a 68% chance of victory by the Allied Powers as of 1915. Knocking out Belgium alone would have made a small difference of about 2 percentage points, while eliminating France would have reduced the Allied Powers' chances of victory by 12 percentage points. The effect of knocking out both would be 16 percentage points, greater than the combined effect of eliminating either alone—this is because Belgium would have made up some of the lost effort had only France been eliminated, and vice versa. Overall, the success of the Schlieffen Plan would have changed the likely outcome as of 1915 from favoring the Allied Powers to being nearly a toss-up.

We analyze similar counterfactuals for World War II. First, in the European theater, we analyze equilibrium war efforts and the probability of victory if the Soviet Union, the United States, or both had contributed to the Allies from the war's outset in 1939. Figure 4 plots the predicted contributions, and Table 9 reports the dispute-level equilibrium quantities for each scenario.

Soviet contribution to the Allies in 1939 would have had a moderate effect, raising the Allied probability of victory by 6 percentage points. Meanwhile, the United States would have made a substantial difference, a 14 percentage point increase, by joining the war effort two years earlier. Conditional on the United States joining, the participation of the Soviet

Counterfactual	Pr(Allies Win)
Baseline	0.78
With USSR	0.84
With USA	0.92
With both	0.94

Table 9. Counterfactual equilibrium quantities in the European theater of World War II, 1939.

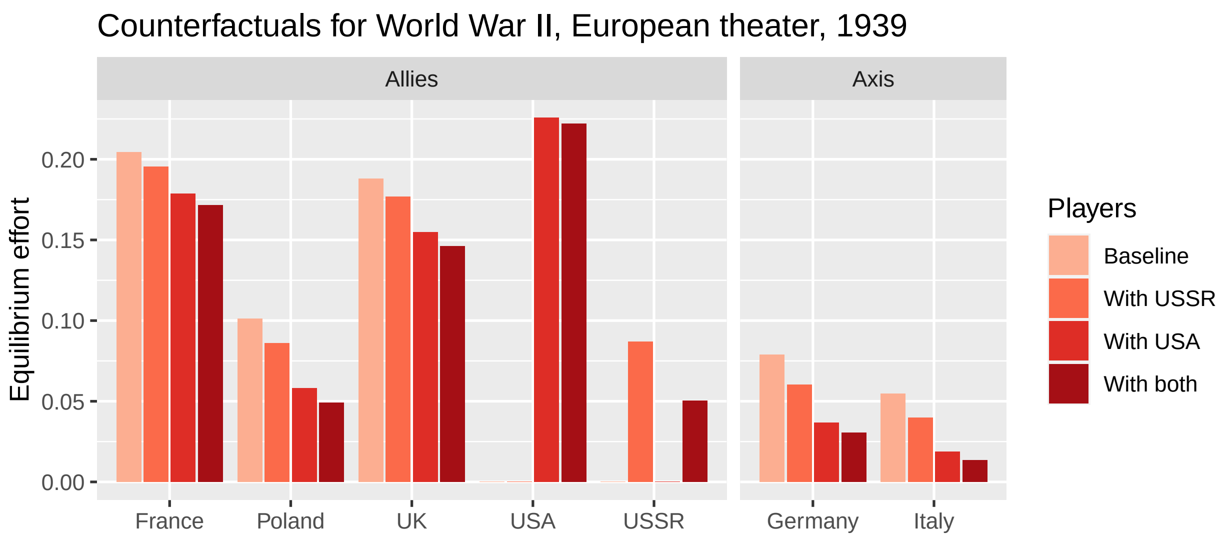

Figure 4. Equilibrium choices of effort in the European theater of World War II, 1939.

Union would have been even less pivotal, making only a 2 percentage point difference in the Allies' chances of winning. In this way, American participation would have created a strong incentive for the Soviet Union to free-ride—as well as the other powers, as seen in Figure 4. As before, the United States has a larger effective force multiplier than its prospective allies. It compares favorably to the UK and France in terms of total population and import dependence, while its advantage over the Soviet Union is largely due to its democratic political system.

Turning to the Pacific theater, we find that the participation of the Soviet Union would not have made much difference as of 1941. As shown in Table 10, Soviet involvement would have increased the probability of victory in the Pacific by just 2 percentage points. We predict that the Soviet Union would have contributed relatively little effort and that even

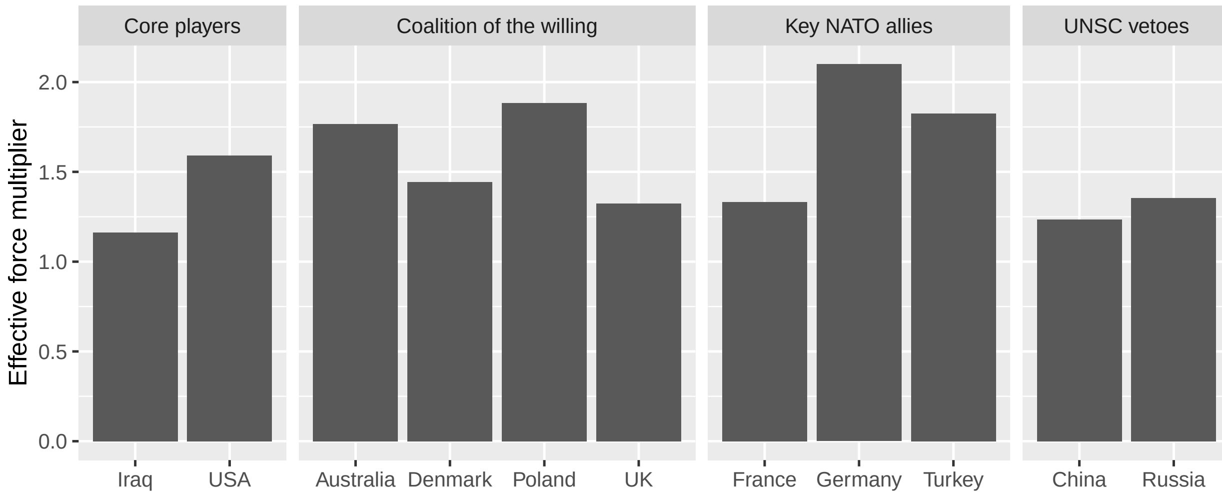

Figure 5. Estimated force multipliers for a dispute in Iraq, 2003.

this small contribution would have been largely offset by decreases in British and Chinese contributions.

Counterfactual	Pr(Allies Win)
Baseline	0.73
With USSR	0.75

Table 10. Counterfactual equilibrium quantities in the Pacific theater of World War II, 1941.

6.2 Crisis Bargaining Behavior

Our model also allows us to study how changes in coalitions would affect bargaining behavior. For example, we examine how the 2003 Iraq War may have played out differently if the United States had garnered additional multilateral support.

We consider three counterfactual scenarios for the Iraq War. Our baseline is the actual conflict between the "coalition of the willing" and Iraq.[24] To quantify the effect that coalition had on the military balance and the likelihood of bargaining breakdown, we compare to a

[24]We only include participants in the initial invasion. The results do not change substantially if we include Spain and the Netherlands, which supported the war politically but did not contribute troops to the invasion.

hypothetical bilateral conflict solely between the United States and Iraq. We then consider the effects of hypothetical expansions of the coalition—first to include key NATO allies that did not support the war (France, Germany, and Turkey) and then to include the remaining veto holders on the United Nations Security Council (China and Russia).

Figure 5 plots the effective force multiplier (m_i/c_i) for each country. There are no extreme differences in military effectiveness among the actual or counterfactual participants, though Iraq does have the lowest multiplier of all. The United States' multiplier is only about average among its allies, due to the penalty for nuclear weapons and its relatively high distance to the dispute location.[25]

Table 11 reports the equilibrium quantities and outcomes for the bargaining game under each counterfactual scenario. We find that the coalition of the willing made more difference to the battlefield outcome than the bargaining outcome. We estimate about a 40 percentage point increase in the United States' probability of winning, with victory virtually guaranteed for the coalition of the willing. The coalition has countervailing effects on the distribution of audience costs: the average cost for both sides decreases, thereby reducing the probability of war, but this is offset by the additional uncertainty on the coalition side due to the increase in participants. The net effect is a small increase in the probability of war.

Coalition	p_A	μ_A	σ_A	μ_B	σ_B	Pr(War)
USA only	0.57	−2.6	0.4	−2.1	0.4	0.11
+ Coalition of the willing	0.97	−4.0	3.3	−3.5	0.4	0.16
+ Key NATO allies	0.99	−3.7	5.9	−3.3	0.4	0.28
+ Russia and China	0.99	−3.7	7.7	−3.4	0.4	0.33

Table 11. Bargaining quantities and outcomes for counterfactual anti-Iraq coalitions in 2003. A = coalition, B = Iraq.

The inclusion of additional partners beyond the coalition of the willing would only have served to increase the probability of conflict. Participation by France, Germany, and Turkey would have had only a marginal effect on the military balance, increasing the coalition's

[25]Using distance to the closest military base would reduce our estimate of the military effect of the coalition of the willing, but would not substantively change other results here.

probability of victory from 97% to 99%. Meanwhile, their participation would have slightly decreased the average audience cost for each side, while substantially increasing uncertainty on the coalition side due to the additional number of participants. The net effect is a 12 percentage point increase in the probability of war. Essentially, the additional partners are doing nothing to shape the military balance—the war was already as good as won—but are increasing the incentive for the coalition to make extreme demands on Iraq. At the same time, their participation slightly reduces Iraq's willingness to back down, though Iraq's audience cost is still lower here than if the United States were acting alone.

Participation by the remaining Security Council members would have had similar consequences, though not of the same magnitude. Russian and Chinese involvement would have left the military balance and average audience costs essentially unchanged, while further increasing the variance on the coalition size. Consequently, we see small further increase in the probability of bargaining breakdown.

7 Conclusion

We have developed a new model of crisis bargaining between coalitions and structurally estimated its parameters using data on interstate disputes. The model allows us to estimate the determinants of individual states' effective force multipliers and of the precision of prior beliefs in crises. We find that states' force multipliers are primarily functions of demographic and economic characteristics, with political and geopolitical features playing a less important role. We have also shown how the model allows us to estimate optimal contributions by individual states and the extent of free-riding in actual and counterfactual conflicts.

An important lesson of both our formal model and our empirical results is that we cannot simply look to how much a country contributed to a war effort to determine the importance of its contribution to the outcome. The key counterfactual question is how much of the country's contribution would have been made up by its allies if that country had not been involved. Our theory-driven approach allows us to give quantifiable answers to this difficult counterfactual question.

On a broader methodological note, our approach and findings illustrate the utility of the structural estimation of game-theoretical models. Debates about these techniques in political science have largely revolved around whether structural estimation is necessary to test comparative statics predictions without bias (Carrubba, Yuen and Zorn 2007; Signorino 2007). But we see the marriage of formal theory and statistical estimation as more than just a tool for bias reduction. Structuring our statistical analysis around a formal model led us to develop a theory-driven alternative to problematic dyadic research designs (Cranmer and Desmarais 2016). More importantly, it directed our attention toward latent variables (audience costs in crisis bargaining) and counterfactual questions (changes in contributions with different coalition structures) that would have been much harder to uncover in a traditional analysis. We encourage political scientists to think of structural estimation as a tool to expand the range of substantive questions that we can answer with data, not merely as a methodological fix for particular statistical problems.

References

Abramson, Scott F and Sergio Montero. forthcoming. "Learning about Growth and Democracy." *American Political Science Review* .

Asal, Victor and Kyle Beardsley. 2007. "Proliferation and International Crisis Behavior." *Journal of Peace Research* 44(2):139–155.

Ascencio, Sergio J. and Miguel R. Rueda. 2019. "Partisan Poll Watchers and Electoral Manipulation." *American Political Science Review* 113(3):727–742.

Bas, Muhammet and Robert Schub. 2016. "Mutual Optimism as a Cause of Conflict: Secret Alliances and Conflict Onset." *International Studies Quarterly* 60(3):552–564.

Beckley, Michael. 2010. "Economic Development and Military Effectiveness." *The Journal of Strategic Studies* 33(1):43–79.

Beckley, Michael. 2018. "The Power of Nations: Measuring What Matters." *International Security* 43(2):7–44.

Bennett, D Scott and Allan C Stam. 1996. "The Duration of Interstate Wars, 1816–1985." *American Political Science Review* 90(2):239–257.

Boulding, Kenneth. 1962. *Conflict and Defense: A General Theory*. Harper and Row.

Bradley, Ralph Allan and Milton E Terry. 1952. "Rank Analysis of Incomplete Block Designs: I. The Method of Paired Comparisons." *Biometrika* 39(3/4):324.

Bremer, Stuart A. 1992. "Dangerous Dyads: Conditions Affecting the Likelihood of Interstate War, 1816-1965." *Journal of Conflict Resolution* 36(2):309–341.

Bueno de Mesquita, Bruce, James D. Morrow, Randolph M. Siverson and Alastair Smith. 1999. "An Institutional Explanation of the Democratic Peace." *American Political Science Review* 93(4):791–807.

Carrubba, Clifford J., Amy Yuen and Christopher Zorn. 2007. "In Defense of Comparative Statics: Specifying Empirical Tests of Models of Strategic Interaction." *Political Analysis* pp. 465–482.

Carter, David B. 2010. "The Strategy of Territorial Conflict." *American Journal of Political Science* 54(4):969–987.

Chiba, Daina and Jesse C. Johnson. 2019. "Military Coalitions and Crisis Duration." *The Journal of Politics* 81(4):1466–1479.

Clark, David H. and William Reed. 2003. "A Unified Model of War Onset and Outcome." *Journal of Politics* 65(1):69–91.

Cranmer, Skyler J. and Bruce A. Desmarais. 2016. "A Critique of Dyadic Design." *International Studies Quarterly* 60(2):355–362.

Crisman-Cox, Casey and Michael Gibilisco. 2018. "Audience Costs and the Dynamics of War and Peace." *American Journal of Political Science* 62(3):566–580.

Dorff, Cassy and Michael D. Ward. 2013. "Networks, Dyads, and the Social Relations Model." *Political Science Research and Methods* 1(2):159.

Fang, Songying, Jesse C. Johnson and Brett Ashley Leeds. 2014. "To Concede or to Resist? The Restraining Effect of Military Alliances." *International Organization* 68(4):775–809.

Fearon, James D. 1994. "Domestic Political Audiences and the Escalation of International Disputes." *American Political Science Review* pp. 577–592.

Fearon, James D. 1995. "Rationalist Explanations for War." *International Organization* 49(3):379–414.

Garfinkel, Michelle R. 2004*a*. "On the Stability of Group Formation: Managing the Conflict Within." *Conflict Management and Peace Science* 21(1):43–68.

Garfinkel, Michelle R. 2004*b*. "Stable Alliance Formation in Distributional Conflict." *European Journal of Political Economy* 20(4):829–852.

Gartzke, Erik. 2007. "The capitalist peace." *American journal of political science* 51(1):166–191.

Gartzke, Erik A and Paul Poast. 2017. Empirically Assessing the Bargaining Theory of War: Potential and Challenges. In *Oxford Research Encyclopedia of Politics*.

Gowa, Joanne. 2011. *Ballots and bullets: The elusive democratic peace.* Princeton University Press.

Grant, Keith A. 2013. "Outsourcing Security: Alliance Portfolio Size, Capability, and Reliability." *International Studies Quarterly* 57(2):418–429.

Jackson, Matthew O. and Massimo Morelli. 2007. "Political Bias and War." *American Economic Review* 97(4):1353–1373.

Kang, Karam. 2016. "Policy Influence and Private Returns from Lobbying in the Energy Sector." *The Review of Economic Studies* 83(1):269–305.

Kertzer, Joshua D. and Ryan Brutger. 2016. "Decomposing Audience Costs: Bringing the Audience Back into Audience Cost Theory." *American Journal of Political Science* 60(1):234–249.

Krainin, Colin. 2014. "Multilateral War Bargaining." *Quarterly Journal of Political Science* 9(4):407–439.

Krainin, Colin and Thomas Wiseman. 2016. "War and Stability in Dynamic International Systems." *Journal of Politics* 78(4):1139–1152.

Kroenig, Matthew. 2013. "Nuclear Superiority and the Balance of Resolve: Explaining Nuclear Crisis Outcomes." *International Organization* 67(1):141–171.

Kurizaki, Shuhei and Taehee Whang. 2015. "Detecting Audience Costs in International Disputes." *International Organization* 69(4):949–980. Publisher: Cambridge University Press.

Levy, Jack. 2008. Counterfactuals and Case Studies. In *The Oxford Handbook of Political Methodology*. Oxford University Press.

Morey, Daniel S. 2016. "Military Coalitions and the Outcome of Interstate Wars." *Foreign Policy Analysis* 12(4):533–551.

Olson, Mancur and Richard Zeckhauser. 1966. "An Economic Theory of Alliances." *The Review of Economics and Statistics* pp. 266–279.

Poast, Paul. 2010. "(Mis)Using Dyadic Data to Analyze Multilateral Events." *Political Analysis* 18(4):403–425.

Posen, Barry R. 1993. "Nationalism, the Mass Army, and Military Power." *International Security* 18(2):80–124.

Ramsay, Kristopher W. and Curtis S. Signorino. 2009. "A Statistical Model of the Ultimatum Game.". Typescript, Princeton University.
URL: https://scholar.princeton.edu/sites/default/files/logitbarg_0.pdf

Reed, William. 2000. "A Unified Statistical Model of Conflict Onset and Escalation." *American Journal of Political Science* pp. 84–93.

Reed, William, David H. Clark, Timothy Nordstrom and Wonjae Hwang. 2008. "War, Power, and Bargaining." *Journal of Politics* 70(4):1203–1216.

Reiter, Dan and Allan C. Stam. 1998*a*. "Democracy and Battlefield Military Effectiveness." *Journal of Conflict Resolution* 42(3):259–277.

Reiter, Dan and Allan C. Stam. 1998*b*. "Democracy, War Initiation, and Victory." *The American Political Science Review* 92(2):377.

Reiter, Dan and Allan C. Stam. 2002. *Democracies at War.* Princeton, N.J.: Princeton University Press.

Renshon, Jonathan and Arthur Spirling. 2015. "Modeling 'Effectiveness' in International Relations." *Journal of Conflict Resolution* 59(2):207–238.

Schelling, Thomas C. 1966. *Arms and Influence.* Yale University Press.

Schultz, Kenneth A. 2001*a*. *Democracy and Coercive Diplomacy.* Cambridge University Press.

Schultz, Kenneth A. 2001*b*. “Looking for Audience Costs.” *The Journal of Conflict Resolution* 45(1):32–60. Publisher: Sage Publications, Inc.

Sechser, Todd S and Matthew Fuhrmann. 2017. *Nuclear weapons and coercive diplomacy*. Cambridge University Press.

Signorino, Curtis S. 2007. “On Formal Theory and Statistical Methods: A Response to Carrubba, Yuen, and Zorn.” *Political Analysis* pp. 483–501.

Signorino, Curtis S and Jeffrey M Ritter. 1999. “Tau-b or not tau-b: measuring the similarity of foreign policy positions.” *International Studies Quarterly* 43(1):115–144.

Singer, J David, Stuart Bremer, John Stuckey et al. 1972. Capability distribution, uncertainty, and major power war, 1820-1965. In *Peace, war, and numbers*, ed. Bruce Russett. Beverly Hills: Sage pp. 19–48.

Skaperdas, Stergios. 1998. “On the Formation of Alliances in Conflict and Contests.” *Public Choice* 96(1-2):25–42.

Smith, Bradley C. forthcoming. “Military Coalitions and the Politics of Information.” *Journal of Politics* .

Snyder, Jack and Erica D. Borghard. 2011. “The Cost of Empty Threats: A Penny, Not a Pound.” *American Political Science Review* 105(3):437–456. Publisher: Cambridge University Press.

Stein, William E. 2002. “Asymmetric Rent-Seeking with More than Two Contestants.” *Public Choice* 113(3/4):325–336.

Stiglitz, Joseph E. and Linda J. Blimes. 2008. *The Three Trillion Dollar War*. Norton.

Sullivan, Patricia L. 2007. “War Aims and War Outcomes: Why Powerful States Lose Limited Wars.” *Journal of Conflict Resolution* 51(3):496–524.

Tomz, Michael. 2007. “Domestic Audience Costs in International Relations: An Experimental Approach.” *International Organization* 61(4):821–840. Publisher: Cambridge University Press.

Trager, Robert F. 2015. “Diplomatic Signaling among Multiple States.” *The Journal of Politics* 77(3):635–647.

Vasquez, John A. and Ashlea Rundlett. 2016. “Alliances as a Necessary Condition of Multiparty Wars.” *Journal of Conflict Resolution* 60(8):1395–1418.

Vuong, Quang H. 1989. “Likelihood Ratio Tests for Model Selection and Non-Nested Hypotheses.” *Econometrica* pp. 307–333.

Whang, Taehee, Elena V. McLean and Douglas W. Kuberski. 2013. “Coercion, Information, and the Success of Sanction Threats.” *American Journal of Political Science* 57(1):65–81.

Wolford, Scott. 2014*a*. “Power, Preferences, and Balancing: The Durability of Coalitions and the Expansion of Conflict.” *International Studies Quarterly* 58(1):146–157.

Wolford, Scott. 2014*b*. “Showing Restraint, Signaling Resolve: Coalitions, Cooperation, and Crisis Bargaining.” *American Journal of Political Science* 58(1):144–156.

Wolford, Scott. 2017. “The Problem of Shared Victory: War-Winning Coalitions and Postwar Peace.” *The Journal of Politics* 79(2):702–716.

Wright, Quincy and Louise Leonard Wright. 1983. *A study of war.* University of Chicago Press.

"The Effective Power of Military Coalitions": Online Appendix

A Additional Formal Results

In our first additional result, we derive the equilibrium shares of the prize and expected utilities in the war subgame.

Proposition 3. *In the unique equilibrium of the war subgame, country i's expected share of the prize is*

$$p_i^* = \begin{cases} 1 - q_i & i \leq J, \\ 0 & i > J, \end{cases}$$

and its expected utility is $\pi_i^* = \pi_i(e^*) = (p_i^*)^2$.

Proof. For $i > J$, we have $e_i^* = 0$ and therefore $p_i^* = \pi_i^* = 0$, as claimed. For $i \leq J$, we have

$$p_i^* = \frac{m_i e_i^*}{\sum_{j=1}^{I} m_j e_j^*} = 1 - q_i$$

and

$$\pi_i^* = p_i^* - c_i e_i^* = (1 - q_i)^2 = (p_i^*)^2,$$

as claimed. □

In our second additional result, we derive an expression for the optimal offer under the assumption that B's audience costs are drawn from a logistic distribution.

Proposition 4. *If $\alpha_B \sim \mathit{Logistic}(\mu_B, \sigma_B)$, then*

$$x^*(\alpha_A) = \alpha_A + \pi_A^* + \sigma_B \left[1 + \mathcal{W}\left(\exp\left(\frac{1 - \pi_A^* - \pi_B^* - \mu_B - \alpha_A}{\sigma_B} - 1 \right) \right) \right], \tag{4}$$

where $\mathcal{W}$ is the Lambert W function.[26]

Proof. We work with the following distribution functions:

$$F_B(\alpha_B) = \frac{1}{1 + \exp\left(\frac{\mu_B - \alpha_B}{\sigma_B}\right)},$$
$$f_B(\alpha_B) = \frac{\exp\left(\frac{\mu_B - \alpha_B}{\sigma_B}\right)}{\sigma_B \left[1 + \exp\left(\frac{\mu_B - \alpha_B}{\sigma_B}\right)\right]^2}.$$

Observe that

$$\frac{F_B(\alpha_B)}{f_B(\alpha_B)} = \frac{\sigma_B(1 + \exp\left(\frac{\mu_B - \alpha_B}{\sigma_B}\right))}{\exp\left(\frac{\mu_B - \alpha_B}{\sigma_B}\right)} = \sigma_B \left[1 + \exp\left(\frac{\alpha_B - \mu_B}{\sigma_B}\right)\right].$$

Substituting into Equation 1 gives

$$x^*(\alpha_A) = \alpha_A + \pi_A^* + \sigma_B \left[1 + \exp\left(\frac{1 - x^*(\alpha_A) - \pi_B^* - \mu_B}{\sigma_B}\right)\right] \quad (5)$$
$$= \alpha_A + \pi_A^* + \sigma_B + \sigma_B \exp\left(\frac{1 - \pi_B^* - \mu_B}{\sigma_B}\right) \exp\left(-\frac{1}{\sigma_B} x^*(\alpha_A)\right). \quad (6)$$

The claim in the proposition then follows from the fact that $x = a + b\exp(cx)$ if and only if $x = a - \frac{1}{c}\mathcal{W}\left(-bc\exp(ac)\right)$ (Corless et al. 1996). □

We now derive comparative statics on the probability of war.

Proposition 5. *If each $\alpha_K \sim$ Logistic(μ_K, σ_K), then the* ex ante *probability of war is increasing in μ_A and μ_B.*

Proof. Recall that Equation 2 gives the *ex ante* probability of war. To prove the result for μ_A, let $\mu_A' < \mu_A''$, and let F_A' and F_A'' denote the corresponding CDFs, each with scale σ_A.

[26] $\mathcal{W} : \mathbb{R}_+ \to \mathbb{R}_+$ defined implicitly by $x = \mathcal{W}(x)\exp[\mathcal{W}(x)]$. See Ramsay and Signorino (2009).

F_A'' first-order stochastically dominates F_A', as

$$\frac{\partial F_K(\alpha_K)}{\partial \mu_K} = \frac{-\exp\left(\frac{\mu_K - \alpha_K}{\sigma_K}\right)}{\sigma_K \left[1 + \exp\left(\frac{\mu_K - \alpha_K}{\sigma_K}\right)\right]^2} = -f_K(\alpha_K) < 0. \tag{7}$$

Meanwhile, implicit differentiation of Equation 5 gives

$$\frac{dx^*(\alpha_A)}{d\alpha_A} = 1 - \exp\left(\frac{1 - x^*(\alpha_A) - \pi_B^* - \mu_B}{\sigma_B}\right) \frac{dx^*(\alpha_A)}{d\alpha_A},$$

which implies

$$\frac{dx^*(\alpha_A)}{d\alpha_A} = \frac{1}{1 + \exp\left(\frac{1 - x^*(\alpha_A) - \pi_B^* - \mu_B}{\sigma_B}\right)} > 0.$$

Because F_A'' is FOSD over F_A' and x^* is increasing, we have

$$\int \left[1 - F_B(1 - x^*(\alpha_A) - \pi_B^*)\right] \, dF_A''(\alpha_A) > \int \left[1 - F_B(1 - x^*(\alpha_A) - \pi_B^*)\right] \, dF_A'(\alpha_A).$$

To prove the result for μ_B, it will suffice to prove that the integrand in Equation 2 is strictly increasing in μ_B for all α_A. Implicit differentiation of Equation 5 gives

$$\frac{\partial x^*(\alpha_A)}{\partial \mu_B} = -\exp\left(\frac{1 - x^*(\alpha_A) - \pi_B^* - \mu_B}{\sigma_B}\right) \left[\frac{\partial x^*(\alpha_A)}{\partial \mu_B} + 1\right],$$

which implies

$$\frac{\partial x^*(\alpha_A)}{\partial \mu_B} = \frac{-\exp\left(\frac{1 - x^*(\alpha_A) - \pi_B^* - \mu_B}{\sigma_B}\right)}{1 + \exp\left(\frac{1 - x^*(\alpha_A) - \pi_B^* - \mu_B}{\sigma_B}\right)} = -F_B(1 - x^*(\alpha_A) - \pi_B^*).$$

Recall that Equation 7 above gives the derivative of F_B with respect to μ_B. Therefore, differentiating the integrand of Equation 2 with respect to μ_B gives

$$\begin{aligned}
&\frac{\partial}{\partial \mu_B}\left[1 - F_B(1 - x^*(\alpha_A) - \pi_B^*)\right] \\
&\quad = -f_B(1 - x^*(\alpha_A) - \pi_B^*)\left[-\frac{\partial x^*(\alpha_A)}{\partial \mu_B}\right] - \frac{\partial F_B(1 - x^*(\alpha_A) - \pi_B^*)}{\partial \mu_B} \\
&\quad = f_B(1 - x^*(\alpha_A) - \pi_B^*)\left[-F_B(1 - x^*(\alpha_A) - \pi_B^*) + 1\right] \\
&\quad > 0. \qquad \square
\end{aligned}$$

In our final supplemental result, we verify that there is always a positive-measure set of parameters at which the log-likelihood is finite.

Proposition 6. *The log-likelihood is finite in a neighborhood of any* $(\beta, \gamma, \eta, \theta)$ *such that* $\beta = 0$ *and* $\gamma = 0$.

Proof. Consider the n'th crisis. By Proposition 3, $p_{ni}^*(0,0) = 1/I_n$ for each country i. Consequently, $p_{nA}^*(0,0) = I_{nA}/I_n$, so each side has a positive probability of winning in case of war. Meanwhile, because each α_K has full support on $\mathbb{R}$, we always have $0 < \Pr(\text{war}) < 1$. Therefore, $\log \ell(0,0,\eta,\theta) > -\infty$ for all η and θ. Because the equilibrium probabilities of victory in case of war are continuous in the structural model parameters, the same is true in a neighborhood of $(0,0,\eta,\theta)$. $\square$

B Estimation Details

B.1 Multiple Imputation

We have missingness in five of our state-level variables and two of our dispute/coalition-level variables, as documented in Table 12. We impute missing values using Amelia II (Honaker, King and Blackwell 2011). To perform the imputation, we construct a dataset of all state-years from 1816 to 2012, not just those that were involved in disputes in our final analysis

Variable	Pct. Missing
State-Level	
Import Percentage	51.6
GDP	42.1
Democracy	2.4
Urban Population	0.9
Energy Consumption	0.6
Distance to Dispute	0.0
Iron and Steel Production	0.0
Nuclear Weapons	0.0
Total Population	0.0
Dispute-Level	
Democracy, Side B	3.3
Democracy, Side A	1.6
Contiguity	0.0
Major Power, Side A	0.0
Major Power, Side B	0.0
Peace Years	0.0
Interest Similarity	0.0

Table 12. Percentage of each covariate missing in the raw data.

data. The imputation dataset contains all state-level variables used in the model except for Distance to Dispute. We include additional GDP and population measures from the Penn World Tables, the Maddison project, and the World Development Indicators data; we also include the military personnel, military expenditures, and CINC score measures from the National Material Capabilities data. To best fit Amelia II's joint normal model, we log-transform all strictly positive variables and use an inverse hyperbolic sine transformation for all variables with a minimum of 0.

We create 10 imputations of the state-year dataset. We treat the imputation dataset as time-series cross-section data using the methods described in Honaker and King (2010). The model includes a linear time control and its interaction with each state identifier. To reduce variance given the large number of parameters in the imputation model, we include a ridge penalty equal to 1% of the number of observations, following the best practices outlined by Honaker, King and Blackwell (2011).

We use the imputed state-level data to create 10 imputed versions of the participant-level data for each dispute. Additionally, we create 10 corresponding imputations of the coalition/dispute-level data, using the state-level imputed values to calculate each side's average Democracy. (All other coalition/dispute-level variables have zero missingness.) We estimate the model separately with each of the 10 imputed datasets to obtain a point estimate, and we calculate a separate variance matrix for each imputation using the leave-one-out jackknife. We finally calculate overall point estimates and standard errors using Equations 2 and 3 of King et al. (2001).

B.2 Simulated Likelihood

As there is no closed-form expression for the equilibrium probability of war, Equation 2, we estimate the model by maximum simulated likelihood. Each time we evaluate the log-likelihood function during optimization, we use a numerical approximation to calculate the equilibrium probability of war for each dispute.

Let $\upsilon_1, \ldots, \upsilon_G$ be independent draws from a uniform distribution on the unit interval. In practice, we use $G = 1024$ draws from the low-discrepancy Halton sequence, as optimization would never converge if we redrew these values at each iteration. We take these values to represent cumulative probabilities of coalition A's audience cost in the n'th dispute, α_{nA}. We calculate the inverse CDF at each υ_g to obtain the corresponding draw from the prior distribution of α_{nA}. Now writing α_{nA} as a function of the cumulative probability υ_g, we have

$$\alpha_{nA}(\upsilon_g) = F_{nA}^{-1}(\upsilon_g) = \mu_{nA} + \sigma_{nA} \log\left(\frac{\upsilon_g}{1 - \upsilon_g}\right).$$

We use Equation 4 to calculate the optimal offer for each type drawn in the simulation. We calculate the probability of war for each type,

$$\begin{aligned}\Pr(\text{war} \mid \alpha_{nA} = \alpha_{nA}(\upsilon_g)) &= 1 - F_{nB}(1 - x^*(\alpha_{nA}(\upsilon_g)) - \pi^*_{nB}) \\ &= 1 - \frac{1}{1 + \exp\left(\frac{\mu_{nB} + x^*(\alpha_{nA}(\upsilon_g)) + \pi^*_{nB} - 1}{\sigma_{nB}}\right)}.\end{aligned}$$

Finally, we average over the simulated types to approximate equilibrium war probabilities, substituting the following into the log-likelihood (Equation 3):

$$r^*_n \approx \frac{1}{G} \sum_{g=1}^{G} \Pr\left(\text{war} \mid \alpha_{nA} = \alpha_{nA}(\upsilon_g)\right).$$

We need not use such an approximation for the victory probabilities, p^*_{nA} and p^*_{nB}, as these have the closed-form solution given in Proposition 1.

C Data Sources and Versions

Militarized Interstate Disputes. We use version 2.1.1 of the Gibler–Miller–Little MID data (Gibler, Miller and Little 2016).

National Material Capabilities. We use version 5.0 of the National Material Capabilities data (Singer, Bremer and Stuckey 1972).

GDP. We use version 9.0 of the Penn World Tables (Feenstra, Inklaar and Timmer 2015). We use the $RGDP^0$ measure, the one best suited for comparisons of "productive capacity across countries and across years." To estimate missing values of GDP as accurately as possible, we also include GDP measures from the Maddison Project, version 2018 (Bolt et al. 2018) and the World Development Indicators dataset (The World Bank 2019) (accessed 2019-02-28) in the multiple imputation model.

Import Percentage. We use the World Development Indicators data (The World Bank 2019) as of 2019-02-28.

Democracy. We use version 2017 of the Polity IV project (Marshall, Gurr and Jaggers 2014).

Nuclear Weapons. We use the data from Gartzke and Kroenig (2009).

Distance to Dispute. We use version 2.0 of the MIDLOC dataset (Braithwaite 2010; Bezerra and Braithwaite 2019) for dispute locations. Capital locations are taken from version 3.212 of EUGene (Bennett and Stam 2000).

Major Power. We use version 2016 of the State System Membership data (Correlates of War Project 2016).

Contiguity. We use version 3.20 of the Correlates of War project's Direct Contiguity Data (Stinnett et al. 2002).

Interest Similarity. We use the S-score measure provided by the Alliance Treaty Obligations and Provisions project, version 4.0 (Chiba, Johnson and Leeds 2015).

Peace Years. For dyads that existed in 1816, we use the corrected measure developed by Werner (2000).

D Originators Model

We run an alternative model to examine whether we could better explain the patterns in the data by excluding "joiners"—countries that were not involved at the outset of the dispute—from the bargaining stage. For each country i in a dispute, let $o_i \in \{0, 1\}$ be an indicator for whether i is an "originator," involved in the dispute from the outset. Those with $o_i = 0$ are joiners. In our data analysis, we use the MID data codings of originators. Each side always has at least one originator: $\sum_{i \in A} o_i \geq 1$ and $\sum_{i \in B} o_i \geq 1$.

In the bargaining stage, we now calculate optimal offers and the probability of war using only originators.[27] Let $\tilde{e}^*$ denote the equilibrium effort allocations from a contest only among the originators, calculated according to Proposition 1. Let $\tilde{\pi}_A^*$ and $\tilde{\pi}_B^*$ be the coalitions' expected utilities from such a contest: $\tilde{\pi}_K^* = \sum_{i \in K} o_i \pi_i(\tilde{e}^*)$ for $K \in \{A, B\}$. Let $\tilde{x}^*(\alpha_A)$ be the optimal offer for each type of A, which we calculate by replacing each π_K with the corresponding $\tilde{\pi}_K$ in Equation 1. Substituting the new values into Equation 2, the probability of war is now

$$\Pr(\text{war}) = \int [1 - F_B(1 - \tilde{x}^*(\alpha_A) - \tilde{\pi}_B^*)] \, dF_A(\alpha_A). \tag{8}$$

To estimate the originators model, we modify the log-likelihood from the baseline model by calculating the probability of war using only the originators. For those disputes that actually proceed to war, however, we still calculate the probability of victory for each side using both originators and joiners. Letting $\tilde{r}_n^*$ denote the probability of war calculated using Equation 8, the modified log-likelihood is

$$\begin{aligned} \log \tilde{\ell}(\beta, \gamma, \eta, \theta) = & \sum_{n: Y_n^{\text{war}} = 0} \log(1 - \tilde{r}_n^*(\beta, \gamma, \eta, \theta)) + \sum_{n: Y_n^{\text{war}} = 1} \log \tilde{r}_n^*(\beta, \gamma, \eta, \theta) \\ & + \sum_{n: Y_n^{\text{win}} = A} \log p_{nA}^*(\beta, \gamma) + \sum_{n: Y_n^{\text{win}} = B} \log p_{nB}^*(\beta, \gamma), \end{aligned}$$

We maximize this function to obtain estimates for the originators model. We follow the same process of multiple imputation for missing data, simulated likelihood for approximation of $\tilde{r}_n$, and jackknife for standard errors as in the baseline model.

Table 13 reports the structural estimations for the originators model. For the state-level determinants of m_i and c_i, the point estimates are broadly similar to the baseline model,

[27] We also recalculate the coalition- and dispute-level variables to reflect the values only for originators. For example, "Democracy, Side A" is now the average Polity score of originators on side A.

Term	Estimate	Std. Err.	Z	p
Force Multiplier: Demographic				
m_i: Total Population	0.106	0.119	0.896	0.370
m_i: Urban Population	−0.086	0.074	−1.162	0.245
Force Multiplier: Economic				
m_i: Energy Consumption	0.021	0.062	0.337	0.736
m_i: GDP	0.026	0.124	0.209	0.835
m_i: Iron and Steel Production	0.016	0.056	0.281	0.779
c_i: Import Percentage	0.150	0.188	0.797	0.425
Force Multiplier: Political				
c_i: Democracy	−0.013	0.011	−1.187	0.235
Force Multiplier: Geopolitical				
m_i: Distance to Dispute	−0.013	0.029	−0.456	0.648
m_i: Nuclear Weapons	−0.387	0.386	−1.002	0.316
Audience Cost Distribution				
μ_K: Intercept	−68.163	406.644	−0.168	0.867
μ_K: Contiguity	−5.674	35.261	−0.161	0.872
μ_K: Democracy	−3.200	19.113	−0.167	0.867
μ_K: Interest Similarity	3.551	24.483	0.145	0.885
μ_K: Major Power	−2.507	20.239	−0.124	0.901
μ_K: Peace Years	−6.229	37.598	−0.166	0.868
σ_K: Intercept	2.544	6.794	0.374	0.708
σ_K: Coalition Size	−0.367	0.702	−0.524	0.601
Disputes	2101			
Participants	4962			
Log-likelihood	−919.128			
AIC	1872			
BIC	1968			

Table 13. Structural estimation results for the originators model.

though estimated less precisely. The dispute- and coalition-level determinants of the audience cost distribution sometimes differ from the baseline model estimates, but are estimated with considerably less precision. The model clearly fits worse overall, with a lower log-likelihood and greater AIC and BIC than the baseline model. As noted in the main text, a non-nested model test confirms that the baseline model fits the data significantly better.

Additional References

Bennett, D. Scott and Allan C. Stam. 2000. "EUGene: A Conceptual Manual." *International Interactions* 26(2):179–204.

Bezerra, Paul and Alex Braithwaite. 2019. "Codebook for the Militarized Interstate Dispute Location (MIDLOC-A/I) Dataset, v2.1." Online dataset.
URL: https://correlatesofwar.org/data-sets/MIDLOC

Bolt, Jutta, Robert Inklaar, Herman de Jong and Jan Luiten. 2018. "Rebasing 'Maddison': New Income Comparisons and the Shape of Long-Run Economic Development.". Maddison Project Working Paper 10.
URL: https://www.rug.nl/ggdc/html_publications/memorandum/gd174.pdf

Braithwaite, Alex. 2010. "MIDLOC: Introducing the Militarized Interstate Dispute Location Dataset." *Journal of Peace Research* 47(1):91–98.

Chiba, Daina, Jesse C. Johnson and Brett Ashley Leeds. 2015. "Careful Commitments: Democratic States and Alliance Design." *Journal of Politics* 77(4):968–982.

Corless, Robert M, Gaston H Gonnet, David EG Hare, David J Jeffrey and Donald E Knuth. 1996. "On the Lambert W function." *Advances in Computational Mathematics* 5(1):329–359.

Correlates of War Project. 2016. "State System Membership List, v2016." Online dataset. **URL:** http://correlatesofwar.org

Feenstra, Robert C., Robert Inklaar and Marcel P. Timmer. 2015. "The Next Generation of the Penn World Table." *The American Economic Review* 105(10):3150–82.

Gartzke, Erik and Matthew Kroenig. 2009. "A Strategic Approach to Nuclear Proliferation." *Journal of Conflict Resolution* 53(2):151–160.

Gibler, Douglas M., Steven V. Miller and Erin K. Little. 2016. "An Analysis of the Militarized Interstate Dispute (MID) Dataset, 1816–2001." *International Studies Quarterly* 60(4):719–730.

Honaker, James and Gary King. 2010. "What to Do about Missing Values in Time-Series Cross-Section Data." *American Journal of Political Science* 54(2):561–581.

Honaker, James, Gary King and Matthew Blackwell. 2011. "Amelia II: A Program for Missing Data." *Journal of Statistical Software* 45(7):1–47.

King, Gary, James Honaker, Anne Joseph and Kenneth Scheve. 2001. "Analyzing incomplete political science data: An alternative algorithm for multiple imputation." *American political science review* pp. 49–69.

Marshall, Monty G, Ted Robert Gurr and Keith Jaggers. 2014. Polity IV Project: Political Regime Characteristics and Transitions, 1800–2013. Technical report.

Singer, David J, Stuart A Bremer and John Stuckey. 1972. Capability Distribution, Uncertainty, and Major Power War, 1820–1965. In *Peace, War, and Numbers*, ed. Bruce M Russett. Beverly Hills: pp. 19–48.

Stinnett, Douglas M., Jaroslav Tir, Philip Schafer, Paul F. Diehl and Charles Gochman. 2002. "The Correlates of War Project Direct Contiguity Data, Version 3." *Conflict Management and Peace Science* 19:58–66.

The World Bank. 2019. "World Development Indicators." Online dataset.

URL: https://datacatalog.worldbank.org/dataset/world-development-indicators

Werner, Suzanne. 2000. "The effects of political similarity on the onset of militarized disputes, 1816-1985." *Political Research Quarterly* 53(2):343–374.