The Effective Power of Military Coalitions: A Unified Theoretical and Empirical Model

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Abstract

We argue that the effective power of a coalition depends not only on its members' raw capabilities, but also how much effort they exert. Free-riding and competition between partners mean a coalition's power is not the sum of its members' power. We develop a unified model of crisis bargaining and war fighting between coalitions. Using data on the escalation and outcomes of international disputes, we structurally estimate the model parameters to identify determinants of countries' force multipliers and audience costs. We find that demographic and economic characteristics are the most important determinants of military effectiveness, with regime type and geopolitical considerations playing lesser roles. The structural model also allows us to simulate counterfactual outcomes in disputes between coalitions. This allows us to test historical claims like that earlier American involvement would have made a substantial difference in both World Wars, even in the face of strategic free-riding by its allies.

The effective power of a military coalition is not merely the product of raw materials—it depends largely on strategic choices by its constituent states. This is true for individual states as well (Sullivan 2007), but coalitions add significant complications. One key strategic consideration is the free-rider problem. Weaker members of a coalition have an incentive to contribute even less to collective military efforts than they would if they had to fight on their own, leaving the most powerful partners to take on the bulk of the effort (Olson and Zeckhauser 1966). Alternatively, countries might calibrate their efforts with an eye toward maximizing influence within the war-winning coalition (Phillips and Wolford 2021). These strategic considerations within coalitions then affect behavior in bargaining between adversaries. Bringing in more coalition partners might intimidate an opponent into making concessions at the negotiating table, but only if the new partners' efforts are not expected to be offset by other members' free-riding. And if the crisis escalates into war, each side's choice of how much to spend trying to win will depend on how it expects the other side to behave, which depends on the free-riding and competitive incentives within coalitions.

We propose a new theoretical model of crisis bargaining between coalitions that accounts for all of these strategic dynamics, allowing anticipated military contributions by coalition partners to shape strategic decisions in both negotiating and fighting. We then estimate the structural parameters of the model using data on militarized disputes from 1816 to 2010. This approach allows us to simultaneously quantify the determinants of *individual* states' military capabilities and how these are *aggregated* into effective power in crises between coalitions. Situating the analysis within a game-theoretical model of crises also allows us to answer counterfactual questions about strategic behavior, such as how historical crises would have turned out if different coalition partners had joined or dropped out.

Using our theoretical framework, we decompose a coalition's effective military power into the product of two distinct factors. The first of these is the *effort* each state exerts—the resources used or the consumption foregone in order to increase their side's chance of success in the conflict. Effort is an endogenous, strategic choice. This is where free-riding and intra-coalition competition enter our picture of effective power, when states make different contributions than they would have otherwise because of what they expect their partners to contribute. By contrast, the second determinant of effective power is a fixed, exogenous factor, which we call the *force multiplier*. This represents the "bang for the buck" states get from their military resources—the higher a state's force multiplier, the more its side benefits militarily from the same exertion of effort. This force multiplier may depend on military technology, political organization, economic strength, or other factors. In our framework, effective power is the product of endogenous effort and exogenous force multipliers.

Our game-theoretical model links the equilibrium chance of war and the probability of victory by each side (in case war occurs) to the *individual* force multipliers of the participants on each side. Our empirical analysis exploits this linkage, using the Militarized Interstate Dispute data to estimate the structural determinants of individual countries' force multipliers. This exercise allows us to characterize the strongest influences on the effectiveness of countries' individual efforts. By calculating the force multipliers for countries at particular points in time, we can answer counterfactual questions about conflict contributions and outcomes, such as how much less likely a coalition would be to win a war if one of its partners had dropped out. Using equilibrium choices of effort in these counterfactuals, we account for free-riding behavior and other strategic incentives in a way that would be impossible if we relied on reduced-form statistical modeling.

We find that demographic and economic factors are the strongest determinants of individual countries' force multipliers. According to our estimates, states with large, dispersed populations and rich economies have the greatest effectiveness per unit of military resources. Political and geopolitical factors have smaller influences, though there is a discernible democratic advantage in the employment of military resources. For example, despite China's demographic advantage and rough economic parity, we estimate a larger force multiplier for the United States as of 2010 due to its more democratic political regime. Altogether, the coalition partners whose inclusion shifts the balance of effective power most are rich, populous democracies.

Once we have calculated force multipliers for countries at a particular point in time using our model estimates, we use the equilibrium of the underlying bargaining model to simulate individual contributions and likely outcomes in modified or hypothetical coalitional disputes. We take this approach to analyze individual contributions to the most important conflicts between coalitions in our sample, the world wars. We find that earlier American intervention in the First World War would have raised the Allied Powers' chance of victory by about 13 percentage points. We also find that the Central Powers' chance of victory would have increased by about 12 percentage points if France had been knocked out of the conflict in 1914, partially validating the military philosophy of the Schlieffen Plan. For the Second World War, we find that earlier Soviet participation would have made a moderate difference in the European theater, but only if the United States had continued to stay out. Similarly, in the Pacific theater as of 1941, Soviet participation would have made almost no difference, being offset by British and Chinese free-riding.

By breaking power into the endogenous selection of effort and the exogenous effectiveness of that effort, we follow other scholars who have recognized the distinction between the raw resources a state possesses and its ability or willingness to use those resources (Sullivan 2007; Beckley 2010). We innovate on these earlier efforts in important ways. Most importantly, our analysis encompasses military coalitions as well as individual states. We show how the problem of translating potential power into real military effects is even more complicated in coalitions. In addition, we provide an equilibrium model of the relationship between raw force multipliers and strategic choices of effort. This allows us to predict explicitly how the efforts of individual states—and thus their fortunes in crisis bargaining—depend on the military potentials of their friends and foes.

Unlike most quantitative analyses of war, our work does not create a horse race between different theories or even between a specific pair of alternative and null hypotheses. More like a climate or macroeconomic model, our approach uses observable data on crises to infer values of unobserved theoretical parameters. We then use these estimates, with the data and the model, to make predictions about the outcomes of crises under different circumstances. To get at the validity and value of our model, instead of significance tests, we assess which wars our model predicts well and which wars it fits poorly. We find that the major wars with coalitions are well explained with bargaining, incomplete information, and coalitions. Bilateral wars in the periphery are not. While by no means definitive, this assessment provides a starting point to think about how to compare general models of war.

1 Coalitions, Power, and War: Existing Literature

The canonical bargaining model of war is purely dyadic, treating both crisis negotiations and war itself as two-player affairs (Fearon 1995). Our theoretical model fits into a growing body of work extending the bargaining model to allow coalition relationships among the players. Fang, Johnson and Leeds (2014) show how public recommendations by coalition partners can lead states to accept a wider range of demands in crisis bargaining. Wolford (2014*b*) demonstrates how the need to maintain coalition support can complicate costly signaling, thereby increasing the likelihood of war. Trager (2015) finds that coalitions open up possibilities for cheap talk signaling to adversaries that would otherwise lack credibility, while Smith (2021) shows how free-riding incentives might complicate such signaling.

We innovate by endogenizing individual states' choices of military effort—and thus the effective power of military coalitions in times of war. Existing models make a reduced-form assumption: the power of one side increases by a fixed amount when a new partner joins.[1] Consequently, these theories do not speak to *how* coalitions aggregate power, nor to which partners would most increase their side's effective power. The only way for a state to free-ride in these models is to stay out of the conflict altogether. Our theory, by contrast, produces sharp empirical implications about how individual states' capabilities translate into collective power, depending on which states they are partnered with and fighting against. We also introduce a richer conception of how individual contributions translate into overall effective power, quantifying the difference one state's participation makes to its partners' efforts. The formal model closest to ours is by Garfinkel (2004*a*,*b*), who models conflict as a N-player contest with coalitions (see also Skaperdas 1998; Krainin 2014). We simplify Garfinkel's model by treating the coalition structure as exogenous, while expanding on it by modeling pre-conflict negotiations.

[1] Wolford (2014*b*) allows endogenous military mobilization, but only by one state.

We also contribute to the empirical literature on disputes and wars between coalitions of states. Despite the classic theoretical findings demonstrating incentives to free-ride in alliances (Olson and Zeckhauser 1966), empirical studies of coalitions typically measure the power of a coalition as the sum of its members' capabilities (e.g., Grant 2013; Fang, Johnson and Leeds 2014; Wolford 2014*a*, 2017; Morey 2016; Chiba and Johnson 2019). By contrast, our calculations of coalitions' effective power are driven by theory—namely, the equilibrium allocations of effort in our formal model of war fighting.

Our work is also part of a recent tradition breaking away from purely dyadic research designs (see Poast 2010; Dorff and Ward 2013; Cranmer and Desmarais 2016). Theoretically, we bridge between dyadic and *k*-adic approaches by modeling crisis bargaining between a pair of coalitions, each consisting of individual states. We carry this hybrid conceptualization into our empirical work by estimating the formal model's parameters. We do not artificially treat a conflict between coalitions as a loose collection of dyadic conflicts; our unit of analysis is the dispute. Nonetheless, we can still estimate individual states' force multipliers.

We also advance the burgeoning literature empirically studying the bargaining model of war (see Gartzke and Poast 2017). Building on prior work relating power and uncertainty to the likelihood and outcome of conflict (Reed 2000; Clark and Reed 2003; Reed et al. 2008), we allow for coalitions and introduce the concept of effective power as the product of (exogenous) force multipliers and (endogenous) effort. We also innovate methodologically by structurally estimating a game-theoretical model and by using its equilibrium to perform counterfactual simulations. Structural estimation has seen increasing use in political science as scholars seek to answer counterfactual questions about complex strategic interactions (e.g., Carter 2010; Whang, McLean and Kuberski 2013; Crisman-Cox and Gibilisco 2018; Ascencio and Rueda 2019; Abramson and Montero 2020). Our work thus fits within the tradition in international relations using formal models to reason through counterfactual outcomes in real-world conflicts (see Levy 2008). Whereas such work has traditionally analyzed counterfactuals through qualitative case studies, structural estimation allows us to scale this up to the analysis of quantitative data.

Finally, our work has some notable methodological antecedents in political science and economics. Our estimation of force multipliers is similar to a Bradley–Terry model (Bradley and Terry 1952), and thus to the recent work by Renshon and Spirling (2015) using such models to estimate the determinants of military effectiveness. We extend this technique by allowing for competitions between coalitions. Meanwhile, our estimation of optimal offers and the probability of bargaining breakdown in an ultimatum game is similar to the estimator by Ramsay and Signorino (2009). We extend this technique by allowing for coalitions and by endogenizing the reservation values through a model of war.

2 A Model of Crisis Bargaining between Coalitions

We model crisis bargaining in the shadow of conflict between two coalitions. Bargaining takes place between representatives of each coalition who anticipate the likely outcome of conflict and calibrate their negotiating stances accordingly. As in the workhorse model of conflict (Fearon 1995) and in previous theories of war between coalitions (Fang, Johnson and Leeds 2014; Wolford 2014*b*; Trager 2015; Smith 2021), bargaining breakdown occurs due to incomplete information.[2]

2.1 Setup

The coalitions are labeled A and B, consisting of I_A and I_B countries, respectively. In the crisis, the sequence of play is as follows. First, a representative of side A makes an offer $x \in \mathbb{R}$. A representative of side B may accept or reject the offer. Each representative's objective is to maximize the total payoff to her side.[3]

If the offer x is accepted, then the game ends. Side A receives x to divide among its constituent countries, while side B receives $1 - x$. Additionally in this case, each side $K \in \{A, B\}$ pays an "audience cost" $\alpha_K \in \mathbb{R}$ for resolving the crisis without fighting. Each α_K is drawn independently according to a cumulative distribution function F_K; the realized values of α_K are private informa-

[2] The development and estimation of a general model of war due to commitment problems between coalitions would be a considerable project in its own right—and thus one that we leave for future research.

[3] We do not model the distribution of spoils in the case of peace, as it is immaterial to equilibrium behavior.

tion, but the prior distributions are common knowledge. We assume each F_K has full support on $\mathbb{R}$ and is continuously differentiable with an associated density function f_K.

If war occurs, each country involved in the crisis selects a level of effort $e_i \geq 0$. Denote the vector of each country's effort by $e = (e_1, \ldots, e_I)$, where $I = I_A + I_B$, the total number of states involved in the crisis. The postwar distribution of spoils depends on how much effort each country exerts, as well as their force multiplier (or military effectiveness), an exogenous parameter $m_i > 0$. The probability that side A wins is

$$p_A(e) = \frac{\sum_{i \in A} m_i e_i}{\sum_{i \in A} m_i e_i + \sum_{j \in B} m_j e_j},$$

with $p_B(e) = 1 - p_A(e)$. Spoils of size 1 are divided among the countries on the winning side in proportion to their effectiveness-weighted contribution.[4] The expected payoff from the contest to country i on side K is

$$\pi_i(e) = p_K(e) \frac{m_i e_i}{\sum_{j \in K} m_j e_j} - c_i e_i = \frac{m_i e_i}{\sum_{j=1}^{I} m_j e_j} - c_i e_i, \tag{1}$$

where $c_i > 0$ is the country's marginal cost of effort. As the equilibrium would be identical if we replaced each country's force multiplier with m_i/c_i and normalized their marginal cost of effort to 1,[5] we call the ratio m_i/c_i the *effective force multiplier* for country i. Without loss of generality, label countries by their effective force multipliers: $m_1/c_1 \geq m_2/c_2 \geq \cdots \geq m_I/c_I$. Taking the effort contributions in case of war as given, each side's payoff from bargaining is

$$U_A = \begin{cases} x - \alpha_A & \text{if } B \text{ accepts}, \\ p_A(e) - \sum_{i \in A} c_i e_i & \text{if } B \text{ rejects}; \end{cases} \qquad U_B = \begin{cases} 1 - x - \alpha_B & \text{if } B \text{ accepts}, \\ p_B(e) - \sum_{i \in B} c_i e_i & \text{if } B \text{ rejects}. \end{cases}$$

[4]This can be motivated as the reduced form of bargaining among the winning coalition to divide the spoils; see Appendix B.

[5]See Appendix C.1.

2.2 Notes on the Model

Force multipliers. The effective force multiplier, $\frac{m_i}{c_i}$, plays a key role in our formal model and our structural estimator. Variation in force multipliers across players simply means that different countries get a different bang-for-the-buck from their military deployments and resources. If we were to hold the total outlays fixed, an increase in a country's force multiplier would shift the military balance in favor of that country's coalition. Additionally, countries with higher effective force multipliers have a greater incentive to exert effort in the first place, due to the greater benefit (or lower cost) per unit. Therefore, in equilibrium we will see a compounding effect, where countries with higher force multipliers both commit more resources to the conflict and get proportionally more from what they spend.

War payoffs. The war payoff function, Equation 1, reflects a balance of free-riding incentives and intra-coalition competition. Free-riding arises in the contest between coalitions. If we held fixed the division of the prize in case of victory, each country's incentive to contribute would strictly decrease with its partners' efforts (Olson and Zeckhauser 1966). In our model, however, this incentive is offset by intra-coalition jockeying for the greatest share of the prize.[6] Partners' contributions, insofar as they make one's own side more likely to win, raise a state's expected benefit from partaking in the intra-coalition contest for shares of the prize.

Coalition membership. Our assumption of fixed coalition membership is a simplification—and a departure from models that treat coalition wars as the expansion of bilateral crises (e.g., Wolford 2014*a*; Fang, Johnson and Leeds 2014). This provides important tractability in estimating the model,[7] but we do not see it as a major limitation of the analysis. If states have rational expectations about who will ultimately enter the crisis, their behavior at the bargaining stage should reflect those expectations, even if those states are not literally present at the table when bargaining

[6] The assumption that the prize is a pure private good is a necessary simplification for structural estimation. Without reliable measures of individual efforts in wars, we cannot empirically identify parameters concerning the proportion of the prize that is nonrival among the winning coalition, such as in Garfinkel (2004*a*,*b*). Our counterfactual estimates thus provide a lower bound on the extent of free-riding within coalitions: we would find even greater reductions in effort due to the addition of coalition partners if we assumed a nonrival component to victory.

[7] Otherwise, for each dispute in our data, we would need to model hundreds of states' decisions whether to become involved.

takes place. Moreover, a critical parameter in most theories of war expansion is the military difference made by each new coalition partner. By modeling and estimating how power is aggregated in military coalitions, our analysis provides a necessary first step toward a richer empirical model of how those coalitions form in the first place.

Audience costs. The audience cost parameter α_K represents influences on a coalition's bargaining position besides the prospects of victory and expected costs of effort in wartime. We allow for negative audience costs, $\alpha_K < 0$, reflecting the possibility that dovish domestic audiences in the coalition's constituent states may reward leaders for avoiding war rather than punishing them for backing down (Kertzer and Brutger 2016; Schultz 2001*b*; Snyder and Borghard 2011).[8]

Our model slightly differs from other structural analyses of conflict with audience costs (e.g., Kurizaki and Whang 2015; Crisman-Cox and Gibilisco 2018) in that we assume the costs accrue to both sides in case a bargain is struck, not just to the side that backed down. In part, this reflects a data limitation—as detailed below, we do not have data on offer values or the status quo in our data, meaning we cannot empirically identify which side backed down in any particular crisis. More broadly, once we allow the stakes to be divisible through bargaining, there may be no clear loser or winner; the typical peaceful settlement will involve both sides getting only some of what they wanted.

2.3 Equilibrium

We solve for a perfect Bayesian equilibrium. First, we derive equilibrium efforts in case of war. Equilibrium effort is greatest among the players that have the highest effective force multipliers. At least two players exert positive effort, and all players do so if the effective force multipliers do not vary much:[9]

Proposition 1. *The war subgame has a unique Nash equilibrium e^*, in which the number of players who exert positive effort is $J = \max\left\{j \in A \cup B : \sum_{i=1}^{j} \frac{c_i}{m_i} > (j-1)\frac{c_j}{m_j}\right\}$. Each player $i \leq J$*

[8] This allows the data to tell us when audience costs or belligerence costs arise in specific circumstances.

[9] See Stein (2002) for the proof.

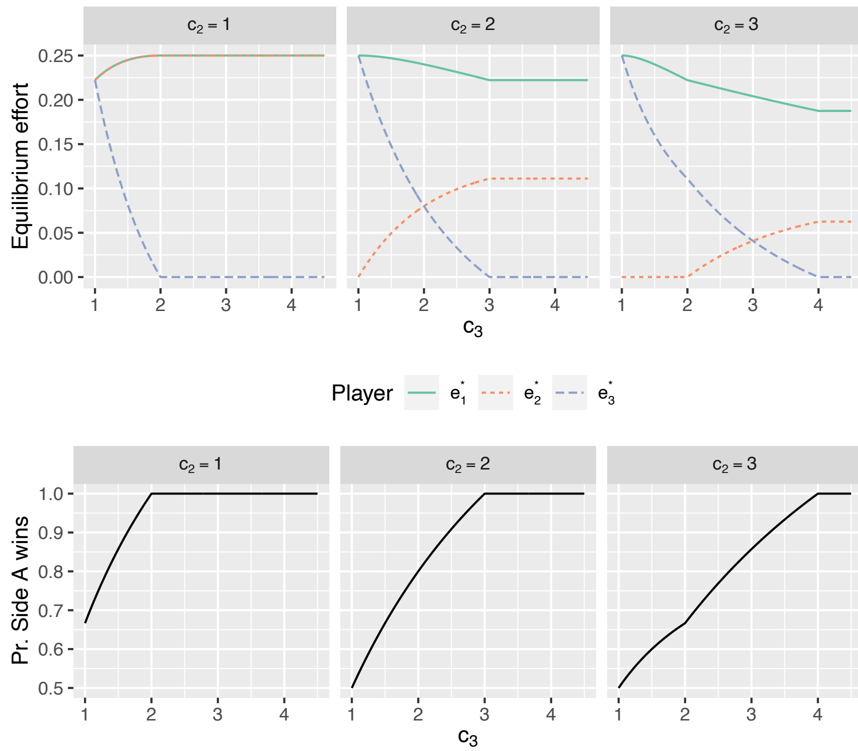

Figure 1. Equilibrium outcomes in the war subgame with three players, $A = \{1, 2\}$ and $B = \{3\}$. ($c_1 = 1$ and $m_1 = m_2 = m_3 = 1$.)

exerts effort $e_i^* = \frac{1}{c_i} q_i(1 - q_i)$, *where* $q_i = \frac{(J-1)\frac{c_i}{m_i}}{\sum_{j=1}^{J} \frac{c_j}{m_j}}$.

Figure 1 illustrates the equilibrium quantities defined in the proposition for a war subgame involving three players. Each player's effort decreases as their marginal cost of effort increases. When one player's effective force multiplier is much lower than the others', that player drops out of the contest altogether. Within coalitions, we see the combination of free-riding and competitive incentives noted above. When partners have similar effective force multipliers (left panel), a reduction in effort by their shared enemy leads to a slight intensification of intra-coalition competition for the prize. The strategic considerations are more subtle when one coalition partner is much stronger than the other (middle and right panels). When the enemy (3) is relatively strong, the weaker partner (2) contributes little or nothing. Its efforts make little difference to the coalition's probability of victory, and that probability is too low to justify spending for the intra-coalition competition. As the enemy becomes weaker, the stronger partner (1) decreases its efforts, as a high probability of victory is attainable at low cost. The increase in the coalition's chance of victory and the stronger partner's reduction in effort both raise the marginal benefit of effort for the weaker partner, whose chief concern now is the intra-coalition contest over the division of spoils. Thus we see a partial convergence of efforts by unequal partners as their shared enemy becomes weaker.

Moving up to the bargaining stage, each side's optimal strategy depends on its war payoff. Let $\pi_A^* = \sum_{i \in A} \pi_i(e^*)$ denote side A's expected utility from fighting, and define π_B^* analogously. Each side has private information about its audience cost α_K, so the peace payoffs are partly private information. The following proposition characterizes the bargaining equilibrium.

Proposition 2. *Assume F_B is log-concave. In any perfect Bayesian equilibrium:*

(a) Each type α_A of side A makes the unique offer that solves

$$x^*(\alpha_A) = \alpha_A + \pi_A^* + \frac{F_B(1 - x^*(\alpha_A) - \pi_B^*)}{f_B(1 - x^*(\alpha_A) - \pi_B^*)}. \tag{2}$$

(b) Each type α_B of side B accepts $x < 1 - \pi_B^* - \alpha_B$ and rejects $x > 1 - \pi_B^* - \alpha_B$.

(c) Strategies in all war subgames are given by Proposition 1.

Proof. As war payoffs do not depend on x or α_K, statement (c) is immediate from Proposition 1. Statements (a) and (b) then follow from Fearon (1995, Claim 2). □

We observe the usual risk-reward tradeoff in the bargaining equilibrium. The *ex ante* (i.e., before types are realized) probability of war in equilibrium is

$$\Pr(\text{war}) = \int \left[1 - F_B(1 - x^*(\alpha_A) - \pi_B^*)\right] dF_A(\alpha_A). \tag{3}$$

This probability reflects side A's incentives to take risks to increase its settlement payoff. This risk-taking incentive increases with A's expected utility from war and the magnitude of its audience cost for peace. It also depends, perhaps non-monotonically, on the extent of uncertainty about B's audience cost. Unlike in the canonical bargaining model, in which the probability of war due to incomplete information is invariant to changes in power (Fearon 1995), here the parameters affecting the effective balance of power, m_i and c_i, may affect the likelihood of conflict. This is because these parameters affect the equilibrium costs of fighting in addition to the probability of victory, so an increase in one side's war payoff is not exactly offset by a decrease in the other's (see Benson, Meirowitz and Ramsay 2016).

The comparative statics of our model are, in many ways, similar to the comparative statics of other bargaining models of war, which have been widely tested (e.g., Reiter and Stam 1998*a*; Reed 2000; Schultz 2001*a*; Reed et al. 2008; Tomz 2007; Kurizaki and Whang 2015; Crisman-Cox and Gibilisco 2018). Audience costs increase the probability of war; if democracies are more effective at fighting, then coalitions with democracies should get better settlements; and if there are costs for being belligerent or backing down, then those states facing higher costs of war are less likely to fight. In a situation with many models supported by the same empirical results, what is next? Our answer is to quantify our model.

The goal of our analysis is therefore different from a standard test of comparative statics and proceeds in two stages. In what follows, we ask: if we require our model to explain the data on international militarized disputes, what does that imply about the unobserved theoretical parameters of crisis bargaining between coalitions? The answer is our quantitative estimates. Then, taking these real-world implications from our theoretical model, we turn to making predictions and exploring counterfactuals that are theoretically driven and empirically informed.

3 Quantifying the Bargaining Model

Our goal is to quantify the major components of our bargaining model. First, we want to identify which real-world variables contribute most to the key parameters: the effective force multiplier ($\frac{m_i}{c_i}$) and the prior distributions of audience costs (the mean and variance of each α_K). We model these parameters as functions of the relevant variables, then structurally estimate their weights from data on interstate crises. The structural estimation procedure selects the parameter values of our model that make its predicted outcomes as close as possible to what we observe empirically. Second, with our structural estimates in hand, we wish to quantify the features of crisis bargaining that would otherwise be difficult to measure systematically for particular countries, in particular crises, or as a function of directly observable variables. These include the individual effort contributions of coalition partners and how they would differ if the coalition composition changed.

Our approach is typical for studies that employ structural estimation. Instead of taking the parameters of the formal model as given, as we would do in an ordinary analysis, we initially treat them as unknown. To capture how key parameters like the force multiplier or audience costs vary across countries and disputes, we write them as functions of observed covariates. Our statistical task is then to estimate how much weight to place on each variable in the calculation of each underlying model parameter. To accomplish this, we use data on the outcomes of historical disputes and wars. Different coefficients imply different equilibrium probabilities of war, and of victory by each side in case war occurs. We estimate the model by finding coefficients that make the equilibrium probabilities most closely mirror what we observe in our data. Thus, the equilibrium conditions allow us to statistically determine which combinations of coefficients are most compatible with observed outcomes.

We observe data on N crises. The n'th crisis involves I_n states, with $I_{nA} \geq 1$ on side A and $I_{nB} \geq 1$ on side B. The variable $Y_n^{\text{war}} \in \{0, 1\}$ denotes whether the n'th crisis resulted in war, and $Y_n^{\text{win}} \in \{A, B\}$ denotes its winner.[10] The other endogenous components of the game—offers and effort—are not observed. We estimate the parameters of the model as a function of observable characteristics. These factors can affect three elements of the model: state-level characteristics affecting military effectiveness and/or the marginal cost of effort (X_{ni}),[11] coalition- and dispute-level characteristics affecting the average audience cost (L_{nA} and L_{nB}), and coalition- and dispute-level characteristics affecting the variance of audience costs (S_{nA} and S_{nB}).

Consider the i'th state in the n'th crisis. We model its effective force multiplier—$\frac{m_i}{c_i}$, the ratio of military effectiveness to the marginal cost of effort—as a log-linear function of state-level

[10]If the n'th crisis does not end in war, we write $Y_n^{\text{win}} = \emptyset$.
[11]We treat all vectors as column vectors.

characteristics.[12] Formally, we write $\frac{m_{ni}}{c_{ni}}(\beta) = \exp(X_{ni}^{\top}\beta)$, where β is an unknown parameter to be estimated.[13] This gives the coefficients the natural interpretation of the effort multiplier effect. For the n'th observation, let $p_{nA}^{*}(\beta)$ denote the equilibrium probability of victory by side A under the given parameters, and let p_{nB}^{*}, π_{nA}^{*}, and π_{nB}^{*} be defined analogously.

In the bargaining stage, we assume side K's audience cost in the n'th crisis is drawn from a logistic distribution with location μ_{nK} and scale σ_{nK}. Structural models of bargaining with a logistic shock to reservation values have previously been studied by Ramsay and Signorino (2009). We use a linear model for location, $\mu_{nK}(\eta) = L_{nK}^{\top}\eta$, and a log-linear model for scale, $\sigma_{nK}(\theta) = \exp(S_{nK}^{\top}\theta)$, where η and θ are unknown parameters to be estimated. The equilibrium probability of war is increasing in α_A and α_B,[14] so a positive coefficient in η means the corresponding variable increases the probability of war. The effects of the scale parameters are ambiguous in general, though we can calculate marginal effects for any given dispute in the data.

We estimate the parameters (β, η, θ) by maximum simulated likelihood.[15] We use multiple imputation to correct for missingness in state-level variables, and we calculate inferential statistics via the nonparametric bootstrap.

4 Data and Specification

We use historical data on disputes to identify how political, economic, and technological characteristics shape states' force multipliers and audience costs. Our estimator assumes that the data are generated according to equilibrium play of our model. This assumption allows us to estimate the relationship between real-world variables and the theoretical parameters of our formal model. The assumption of equilibrium play does not impose any *a priori* restrictions on what these relationships are. For example, if there were no conditional correlation between the participants' GDPs

[12]We cannot separately identify the effects of the same variables on m_i and c_i, as equilibrium behavior depends solely on their ratio. See Appendix C.1.

[13]For identification reasons, we cannot estimate a constant in the equation for the effective force multiplier. See Appendix C.2.

[14]See Proposition 5 in the Appendix.

[15]See Appendix D for details.

and which side won wars, then we would estimate no effect of GDP on the force multiplier.

4.1 *Sample and Outcomes*

The unit of observation is the dispute, and our sample consists of Militarized Interstate Disputes (MIDs) between 1816 and 2010.[16] The sample size is $N = 2{,}101$. There are 284 disputes with multiple states on one or both sides, which we call *coalitional disputes*. These involve about 4.7 disputants each, giving us 4,962 distinct dispute participants in the data.[17]

Following the MID codings, we treat the initiating side as side A, which makes the offer in our bargaining model. We code a dispute as war, $Y_n^{\text{war}} = 1$, if it resulted in more than 25 fatalities. In case of war, we code side A as the winner, $Y_n^{\text{win}} = A$, if the MID data records victory by side A or yield by side B as the outcome. In all other cases we code side B as the winner, $Y_n^{\text{win}} = B$.[18]. In our sample, 1851 MIDs end peacefully, 65 go to war with a victory by the initiating side and 185 go to war with victory by the targets.

4.2 *State Characteristics*

Even though our dependent variables are the collective outcomes of whether war occurred and who won if so, our structural approach allows us to estimate the determinants of *individual* states' effective force multipliers. We model these as functions of individual state characteristics, dividing the underlying state-level variables into demographic, economic, political, and geopolitical characteristics. Table A.2 in the appendix summarizes all the variables included in the model.

Demographics. Indices of national power, including the popular CINC score, typically include population measures. We therefore include **Total Population** and **Urban Population** in the equation for the effective force multiplier. We use the measures from the Correlates of War project's National Material Capabilities data.

[16]See the Appendix for details on all data sources.

[17]The six states originally coded on both sides of World War II are only included on the side on which they spent the most time.

[18]This entails assuming that a stalemate favors side B, which we see as natural given that side A is the initiator of the dispute.

Theoretically, large states might enjoy greater military effectiveness because of their ability to field mass armies (Posen 1993). This is particularly relevant in our time period, which starts just after the Napoleonic Wars. Conversely, the costs of governing and extracting taxes from large populations might inhibit military effectiveness (Beckley 2018).

Economics. Development is a key component of military effectiveness (Wright and Wright 1983; Singer et al. 1972). Richer states have more resources available for military use and may also have access to more effective coercive technologies. To capture these economic influences on the force multiplier, we include **GDP** from the Penn World Tables, as well as **Energy Consumption** and **Iron and Steel Production** from the National Material Capabilities data as additional proxies.[19] We also include **Import Percentage**, the ratio of the state's total imports to its GDP, measured by the World Development Indicators. We see import dependence as a potential influence on the marginal cost of effort. If states must acquire raw materials or other resources via international markets in wartime, it may be more costly to field the same level of military effort than if those resources were available domestically (Gartzke 2007).

Political. We include **Democracy**, the net score (Democracy minus Autocracy) from the Polity IV project, as a component of the effective force multiplier. Citizens of democracies can more easily resist extractive or economically inefficient military production than can the subjects of authoritarian regimes (Bennett and Stam 1996). Scholars have also observed, however, that the political and cultural features of democratic societies may lend themselves to battlefield success (Reiter and Stam 2002). Our statistical model does not allow us to separately identify the force-multiplying and cost-increasing effects of democracy. However, we can estimate which effect is stronger: a positive coefficient in the equation for $\frac{m_i}{c_i}$ would indicate that the battlefield advantages predominate.

Geopolitical. We include two geopolitical determinants of a state's effective force multiplier. The first is **Nuclear Weapons**, an indicator for whether a state has a deliverable nuclear weapon. Even when nuclear weapons are not used, they may serve as a force multiplier by increasing the

[19] The remaining CINC components, Military Expenditures and Military Personnel, are endogenous state choices more so than structural components of the force multiplier, making them inappropriate for inclusion.

effectiveness of states' coercive threats (Kroenig 2013). Alternatively, nuclear weapons might reduce a state's force multiplier, as the increased risk of nuclear disaster impels states to show restraint in their deployment of conventional capabilities (Asal and Beardsley 2007; Sechser and Fuhrmann 2017).

The other geopolitical factor is the **Distance to Dispute** for each participant state. This variable measures the distance from each state's capital to the location of the dispute, as coded in the MIDLOC project.[20] We expect distance to act as a negative force multiplier, as states' ability to project power declines over long distances (Boulding 1962).

4.3 Coalition and Dispute Characteristics

In the bargaining stage, we treat the mean and variance of each side's audience cost, α_K, as a function of coalition- and dispute-level variables. First, we model a coalition's average audience cost as a function of **Democracy**, the average Polity IV score among its constituent members. Theories of coercive diplomacy have traditionally posited that audience costs are greater for democracies than for autocracies due to greater public accountability (Fearon 1994; Schultz 2001*a*). However, more recent empirical studies have identified wide variation in audience costs across regime types (Crisman-Cox and Gibilisco 2018) and have suggested that political leaders may be punished more for belligerence than for backing down (Kertzer and Brutger 2016). We also include an indicator for whether the coalition contains a **Major Power**, as measured by the Correlates of War project.

Finally, in modeling the mean audience cost, we include three dispute-level characteristics that might influence states' willingness to pursue peace rather than violence. We include **Contiguity**, the proportion of pairs of disputants that have a land border or are separated by less than 150 miles of water. Contiguity is a well-known predictor of international conflict (Bremer 1992), as states may be particularly unwilling to back down from disputes over issues with their neighbors. We also include the S-score (Signorino and Ritter 1999) of states' alliance portfolios (again, averaged over each pair of disputants) as a proxy for **Interest Similarity** (Gowa 2011), using the measure

[20]We recode Distance to Dispute as 0 for disputes occurring on or within a state's borders.

provided by the Alliance Treaty Obligations and Provisions project. Finally, we include **Peace Years**, measuring the average time since the last interstate dispute between pairs of disputants.

We model the variance of a coalition's privately known audience costs as a function of the **Number of Participants** in the coalition. This allows us to test whether uncertainty is systematically greater in crises involving many participants.

5 Structural Estimation Results

5.1 Force Multipliers

We can write a dispute participant's effective force multiplier, m_i/c_i, as the product of its four substantive components:

$$\text{Total Multiplier}_i = \text{Demographic}_i \times \text{Economic}_i \times \text{Political}_i \times \text{Geopolitical}_i.$$

The components of the multipliers are calculated using the raw model estimates (Table 3 below). For example, the demographic component is computed as $\text{Demographic}_i = \exp(0.10 \times \text{Total Population}_i - 0.08 \times \text{Urban Population}_i)$.

Demographic and economic characteristics have the strongest influence on countries' effective force multipliers. To illustrate, Table A.3 in the Appendix reports the ten highest and lowest values for dispute participants in our data. The top spots are occupied by large democracies in disputes on or within their own borders—India in the 1960s–70s and the United States in the 1910s. By contrast, the lowest multipliers belong to small autocracies involved (in most cases) some distance from their home territory. While the top and bottom countries differ along many dimensions, it is apparent that the quantitative difference in their estimated force multipliers is due largely to the demographic and economic components.

In a more systematic analysis of the full data, Table 1 shows that the demographic and economic components of the force multiplier account for the most variation across disputants. If we were to remove either of these components, the remaining variation in force multipliers would decrease by at least half. Interestingly, the geopolitical component tends to suppress variation in the total force multiplier across countries. Countries in the strongest demographic and economic positions tend to have unfavorable geopolitical multipliers—they are relatively likely to be involved in disputes far from their homeland, reducing their ability to project power. The primacy of demographic and economic factors remains apparent when we look exclusively at major powers; see Figure A.1 in the Appendix.

Component	SD	%Explained	Correlation
Total	0.36	100.0	1.00
Demographic	0.18	57.8	0.34
Economic	0.22	50.0	0.57
Political	0.11	29.0	0.50
Geopolitical	0.11	−54.2	−0.08

Table 1. Determinants of force multipliers in the empirical model. SD = standard deviation. %Explained = $100 \times \frac{V[\text{total}]-V[\text{total/component}]}{V[\text{total}]}$. (Values do not sum to 100 due to correlations between components.) Correlation = Spearman correlation between total and component.

Table 2 breaks down each component of the total multiplier into its constituent variables, displaying the dispute participants with the highest and lowest values of each component. The constituent variables are given as their contribution to the force multiplier, so that each component multiplier equals the product of the constituent variable multipliers.

We see the highest demographic multipliers for countries with moderately sized, totally rural populations. This reflects the countervailing effects of Total Population and Urban Population in the structural model. Total Population has the largest positive effect among the variables we consider, with a 1% population increase being associated with a 0.1% increase in the effective force multiplier. As our sample covers the post-Napoleonic world, this finding is consistent with the idea that large national states possess a military advantage through their ability to field mass armies (Posen 1993). We find a partially offsetting force multiplication effect of Urban Population, with a 1% increase in urbanites associated with a 0.08% decline in the force multiplier.

Each economic development variable contributes to greater military effectiveness. Meanwhile, import dependence reduces effective force multipliers. Altogether, we find the greatest economic

	Country	Year	MID#	Demographic	Total Pop. $\beta = 0.10$	Urban Pop. $\beta = -0.08$
1	Ethiopia	1931	407	2.60	2.60	1.00
2	Ethiopia	1923	1669	2.55	2.55	1.00
3	Afghanistan	1934	3159	2.48	2.48	1.00
4	Afghanistan	1925	1781	2.45	2.45	1.00
5	Papua New Guinea	2008	4589	2.45	2.45	1.00
4958	Singapore	1987	2789	1.21	2.22	0.55
4959	Qatar	1995	4293	1.21	1.90	0.64
4960	Qatar	1986	2572	1.21	1.83	0.66
4961	Qatar	1984	3617	1.20	1.81	0.66
4962	Bahamas	1984	3038	1.18	1.74	0.68

(a) Greatest and lowest demographic force multipliers.

	Country	Year	MID#	Economic	GDP $\beta = 0.02$	Energy $\beta = 0.02$	Iron and Steel $\beta = 0.02$	Imports $\beta = -0.10$
1	United States	1965	2916	1.76	1.33	1.29	1.20	0.85
2	United States	1965	2929	1.76	1.33	1.29	1.20	0.85
3	United States	1964	611	1.76	1.33	1.29	1.20	0.86
4	United States	1964	1213	1.76	1.33	1.29	1.20	0.86
5	United States	1964	2220	1.76	1.33	1.29	1.20	0.86
4958	Jordan	1949	3161	0.79	1.16	1.00	1.00	0.68
4959	Antigua and Barbuda	1983	3058	0.79	1.13	1.04	1.00	0.67
4960	Maldives	1987	2790	0.78	1.14	1.03	1.00	0.67
4961	St. Vincent	1983	3058	0.78	1.12	1.04	1.00	0.67
4962	Palau	2000	4243	0.76	1.13	1.05	1.00	0.64

(b) Greatest and lowest economic force multipliers.

	Country	Year	MID#	Geopolitical	Distance $\beta = -0.01$	Nuclear $\beta = -0.38$
4958	United States	1975	356	0.638	0.931	0.685
4959	Russia	1979	2224	0.638	0.931	0.685
4960	United States	1968	3300	0.638	0.931	0.685
4961	Russia	1977	3122	0.638	0.931	0.685
4962	United States	2003	4460	0.637	0.930	0.685

(c) Lowest geopolitical force multipliers.

Table 2. Estimated force multiplier components by category for dispute participants in the analysis data.

multipliers for large economies with relatively low import dependence, such as the United States in the 1960s and China in the 1970s. Small, open economies have the lowest economic multipliers.

The geopolitical multiplier is a function of a dispute participant's nuclear status and distance to the location of the dispute.[21] Perhaps surprisingly, we find that nuclear weapons reduce a country's effective force multiplier: the military effectiveness of a nuclear state is only about 68% that of an otherwise equivalent non-nuclear state.[22] This suggests that the possession of nuclear weapons inhibits the deployment of conventional force (Asal and Beardsley 2007). In the shadow of nuclear weapons, states may have extra incentives to limit their efforts to forestall devastating nuclear conflict (Schelling 1966). In line with usual expectations (Boulding 1962), Distance from Dispute erodes military effectiveness. Combining these findings, we find the lowest geopolitical force multipliers for nuclear powers involved in disputes far from their homeland—the United States and Russia during the Cold War, as well as the United States in a 2003 dispute with Indonesia.

The political component is omitted from Table 2, as it depends solely on a country's Polity IV score. We find a democratic advantage in war-fighting: the effective force multiplier for a full democracy is about 35% greater than that of an otherwise identical full autocracy. This apparently runs contrary to theoretical expectations that political accountability makes war-fighting more costly for democratic leaders (Bueno de Mesquita et al. 1999; Jackson and Morelli 2007). Political accountability may increase the costs of effort, but these costs are outweighed by democratic advantages in raw military effectiveness (see Reiter and Stam 1998*a*,*b*, 2002).

5.2 Audience Costs

We estimate how coalition- and dispute-specific factors shape the prior distribution of audience costs, α_K. Remember that μ_K denotes the average audience cost, and σ_K denotes the scale parameter (proportional to the standard deviation). An increase in μ_K is associated with a greater probability of war. Table 3 reports the full estimated model.

[21] Table 2 does not display the greatest geopolitical force multipliers, as all 1,896 non-nuclear states with zero distance to the dispute attain the maximum geopolitical multiplier of 1.0.

[22] The negative coefficient on nuclear weapons is not an artifact of our sample's inclusion of pre-1945 data. We obtain a nearly identical estimate when we run the model on data from 1946–2010; see Table A.9 in the Appendix.

Term	Estimate	Std. Err.	Conf. Int.
Force Multiplier m_i/c_i: Demographic			
Total Population	0.10	0.14	[−0.19, 0.39]
Urban Population	−0.08	0.10	[−0.24, 0.15]
Force Multiplier m_i/c_i: Economic			
Energy Consumption	0.02	0.08	[−0.16, 0.16]
GDP	0.02	0.13	[−0.24, 0.29]
Import Percentage	−0.10	0.19	[−0.44, 0.32]
Iron and Steel Production	0.02	0.06	[−0.09, 0.14]
Force Multiplier m_i/c_i: Political			
Democracy	0.02	0.02	[−0.01, 0.05]
Force Multiplier m_i/c_i: Geopolitical			
Distance to Dispute	−0.01	0.03	[−0.06, 0.07]
Nuclear Weapons	−0.38	0.37	[−1.32, 0.20]
Audience Cost Mean μ_K			
Intercept	−2.04	6.72	[−3.64, −0.99]
Contiguity	−0.06	0.79	[−0.88, 0.80]
Democracy	−0.08	0.22	[−0.17, 0.06]
Interest Similarity	0.03	1.82	[−0.90, 0.66]
Major Power	0.00	1.05	[−0.82, 0.69]
Peace Years	−0.60	1.13	[−1.22, −0.53]
Audience Cost Scale σ_K			
Intercept	−0.78	0.53	[−0.95, 0.54]
Coalition Size	1.24	0.19	[0.82, 1.29]
Disputes	2101		
Participants	4962		
Log-likelihood	−875.93		

Table 3. Structural estimation results with 95% empirical bootstrap intervals.

Beginning with the coalition-level influences on the mean, we see that Democracy has a negative effect on the average audience cost. This means more democratic coalitions will tend to make more restrained demands at the bargaining table, and militarized disputes will be less likely to escalate to war as disputant coalitions become more democratic. This is consistent with Reed's (2000) finding that joint democracy reduces the probability of escalation, though our specification allows for monadic effects as well. On the other hand, the estimate is inconsistent with the body of theory predicting particularly large audience costs among democracies (Fearon 1994; Schultz 2001*a*). We interpret the finding as a dovish bias (Snyder and Borghard 2011), or high belligerence cost (Kertzer and Brutger 2016), among democracies that outweighs any potentially greater audience cost.

The other coalition-level determinant of the average audience cost, Major Power, has a positive sign. This indicates larger audience costs among coalitions containing a major power, meaning such coalitions are less likely to reach peaceful settlements.

At the dispute level, Peace Years reduces the average audience cost. It is therefore less politically costly for a country or coalition to reach a peaceful settlement with an adversary it has recently had peaceful relations with. Meanwhile, Interest Similarity is associated with slightly greater audience costs, while Contiguity is associated with slightly lower costs.

Finally, we turn to the sole influence on variance in audience costs that we include in our model, the Number of Participants. In line with numerous studies suggesting that coalitions increase uncertainty in crises (Wolford 2014*a*; Bas and Schub 2016; Vasquez and Rundlett 2016; Smith 2021), we identify a strong positive effect of coalition size on the degree of uncertainty. According to our structural model estimates, going from a single state to a two-state coalition more than doubles the standard deviation of α_K. As we show in a counterfactual analysis below, these increases in uncertainty can significantly impact the likelihood of conflict, even when the additional coalition members have little effect on the underlying military balance.

6 Model Fit

Our goal is not to test a particular causal hypothesis, but rather to estimate an overall model of the data-generating process in disputes between coalitions. As is evident from Table 3, most of the individual coefficients in our model are not statistically significant at the conventional level.[23] However, tests of null hypotheses about individual parameters do not tell us how well our overall model of competition and free-riding captures the behavior of coalitions in military disputes.[24]

[23] The relatively large standard errors are in part a product of the data. Between the relatively small number of wars and the amount of data that we multiply impute due to missingness, the statistical power of any model with this data is limited. Indeed, the more conventional logistic regressions that we report in Table A.8 in the Appendix have similarly large standard errors.

[24] We do reject the joint null that all non-intercept coefficients equal 0 (Wald = 47.1, df = 15, $p < 0.001$).

Rank	Description	Year	Pr(War)	Rank	Description	Year	Pr(War)
1	Gulf War	1990	0.75	246	Acre War	1902	0.07
2	Korean War	1950	0.71	247	Football War	1969	0.07
3	World War I	1914	0.66	248	French Invasion of Amapá	1895	0.07
4	Vietnam War	1964	0.65	249	Bloody Christmas	1963	0.06
5	World War II	1939	0.61	250	Portugual/Germany in Angola	1914	0.06

Table 4. Greatest and least equilibrium probability of war for disputes where $Y_n^{\text{war}} = 1$.

To more accurately assess model fit in light of our substantive aims, we undertake two types of exercise. First, we perform a face validity check by examining the predicted probability of conflict for disputes that actually escalated to war in our sample. Second and more formally, we follow Clarke's (2007) admonition to compare our empirical model's fit to other specifications. From the perspective of cumulative scientific process, the relevant question is not how well our model fits in absolute terms, but instead whether it out-performs reasonable alternative models.

6.1 Face Validity: War Probabilities

Our model assumes bargaining breaks down due to private information, while existing research argues that different models better explain major wars (e.g., Powell 2006). How well does our informational model predict the escalation of disputes between coalitions into wars? If the alternative models were quantified and applied to the same data, we could run a horse race to see which one best predicts outcomes. Unfortunately, quantified versions of competing bargaining models do not yet exist. As an alternative, we ask: which wars do our model "predict well" and for which wars it is a bad fit?

To answer this question, we plug our structural estimates of the model parameters into the equilibrium probability of war (Equation 3) for each dispute in our data. Table 4 reports the highest and lowest estimated probabilities among disputes that resulted in war. Our model fits major wars quite well, with both world wars estimated as having at least a 60% chance of bargaining failure.[25] The cases where our model fits worst are bilateral conflicts in the periphery.

[25]Note that because we can accommodate wars between coalitions in our model, the improved fit for large multilateral wars is not the result of a proliferation of bilateral observations related to those MIDs.

This exercise is by no means the final word on whether incomplete information, commitment problems, or something else best explains any particular case, but it demonstrates that our informational model does well explaining historically significant conflicts between coalitions. It also tells us where other theories might find fruitful ground for explanation and where the bar is high to improve upon the crisis bargaining model with coalitions and incomplete information.[26]

6.2 *Model Comparison: Capability Aggregation in Wars*

Our model assumes that capabilities are aggregated according to states' equilibrium choices of effort in a contest. To assess this assumption, we compare the model fit to two alternative models with different capability aggregation rules. In the first alternative, we treat a coalition's force multiplier as the sum of all of its members' capabilities, reflecting an assumption of no free-riding. The second alternative instead considers maximal free-riding, treating each coalition's multiplier as a function solely of the capabilities of the member with the greatest CINC score. Finally, as a baseline, we fit a model only on the bilateral wars in the data.

Our equilibrium model does a better job predicting the outcomes of wars between coalitions than do the models with alternative capability aggregation rules, as reported in Table A.8 in the Appendix. For these wars, the model with maximal free-riding performs slightly better than the one with no free-riding, but ours performs substantially better than either. On the other hand, our model predicts the purely bilateral war outcomes worse than the alternatives. This suggests that there may be differences between bilateral and coalition wars that the model does not capture. Identifying the source of these differences is a fruitful direction for future work.

6.3 *Model Comparison: Joiners in Bargaining*

In our baseline analysis, we calculate expected war payoffs using all participants in the crisis, even those that joined after the onset of the dispute. This reflects a rational expectations assumption that the countries at the bargaining table correctly anticipate which other countries will participate

[26]As a different metric, one can rank MIDs by their outcome's likelihood and get a sense of which specific MIDs, and not just wars, are well described by the asymmetric information model and which are not.

in case of war. To assess whether the data support this assumption, we estimate an alternative specification in which the countries at the bargaining table assume war would take place only among the originators of the dispute.[27] The probability of war in the modified model is based on the expected war payoffs from a war only among the originators, as well as covariates pertaining only to the originators. After estimating the alternative model, we perform a Vuong (1989) non-nested model test comparing it to the baseline model to assess which better fits the data. We find that the baseline model fits significantly better ($\text{LR} = 43.2, Z = 4.1, p < 0.001$), favoring the idea that states form rational expectations about likely coalition joiners.

7 Counterfactual Analyses

One of the primary virtues of the structural estimation approach is that it allows us to calculate counterfactual quantities of substantive interest. We use the following procedure to calculate counterfactual estimates:

1. Given two (perhaps hypothetical) coalitions of countries at a particular time, calculate each state's effective force multiplier ($\frac{m_i}{c_i}$) and the prior distribution of each side's audience costs (means μ_A and μ_B, scale parameters σ_A and σ_B).

2. Use Proposition 1 to calculate each individual country's effort and each side's probability of winning in case of war, according to the equilibrium of the war subgame.

3. Use Proposition 2 to calculate the equilibrium offer and the probability of war.

We compute all counterfactuals using the point estimates from Table 3. Because most of the individual coefficients are statistically insignificant at the conventional level, we focus on counterfactual scenarios involving changes in overall coalition membership, rather than on hypothetical changes in individual country variables (e.g., what would have happened in WWII if the Soviet Union were democratic). Even so, given the imprecision in our statistical estimates, these results should be interpreted with caution. That said, in the absence of a model that better predicts the

[27] See Appendix G for details and estimates.

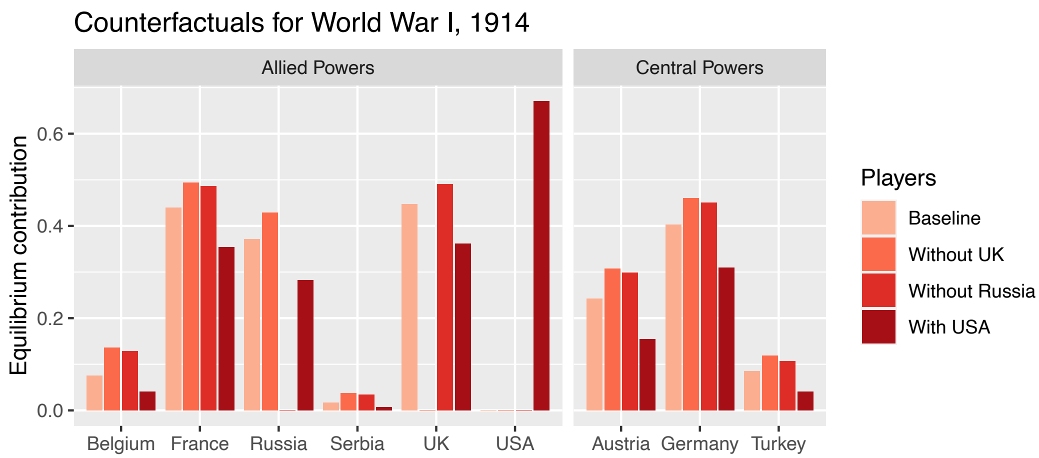

Figure 2. Counterfactual choices of war effort in World War I, 1914.

outcomes of coalition disputes (see subsection 6.2 above), these represent the best estimates we can make from the available data.

7.1 Contributions to Coalitions

Perhaps the most novel feature of our estimation approach is that it allows us to estimate the contributions of individual countries or groups of countries to the outcomes of coalitional conflicts. To illustrate this capability, we analyze counterfactual war outcomes as a function of the set of participants in a few prominent historical conflicts.

We begin by analyzing the contributions of three participants on the Allied side of World War I—Russia, the United Kingdom, and the United States.[28] We model each country's equilibrium effort, the probability of Allied victory, and the *ex ante* probability of war as of 1914. We vary the set of participants on the Allied side while holding fixed participation among the Central Powers. Equilibrium levels of effectiveness-weighted war effort ($m_i e_i^*$) under each scenario are plotted in Figure 2, and Table 5(a) reports the dispute-level equilibrium quantities.

[28]Given our assumption that informational problems cause war, one might worry about the validity of these estimates if the war were instead caused by commitment problems. However, the bulk of the identifying power for our force multiplier estimates—which drive these counterfactuals—comes from the contest subgame after bargaining has broken down. Consequently, we expect our estimates of optimal contributions and the overall military balance would not change much if we modeled a distinct bargaining process, as long as the war subgame followed the same contest model.

Counterfactual	Pr(Allied Powers Win)
Baseline	0.65
Without Russia	0.57
Without UK	0.56
With USA	0.77

(a) 1914.

Counterfactual	Pr(Allied Powers Win)
Baseline	0.67
Without Belgium	0.65
Without France	0.56
Without both	0.52
Without both, with USA	0.72

(b) 1915.

Table 5. Counterfactual equilibrium quantities in World War I.

Our results suggest that Russia and the United Kingdom both made meaningful contributions to the Allied chance of victory, but that these contributions were at least somewhat offset by free-riding. In the baseline equilibrium, Russia contributes 26% of the Allied Powers' effective effort (effort weighted by military effectiveness, $m_i e_i$), and the United Kingdom contributes about 32%. Yet most of each country's contribution would have been made up by other Allied Powers had it not participated in the conflict, meaning their observed efforts far outweigh their effects. Meanwhile, American entry at the outset of the dispute would have made an even bigger difference than British entry did. We estimate that the Allied Powers' chance of winning would have increased 12 percentage points to 77% had the United States participated in 1914, with the United States contributing about 40% of this side's effective efforts. This is because the United States has a much larger effective force multiplier than the UK, with most of the difference owing to the United States' larger population and lower import dependence.

Still in the context of World War I, we can also evaluate the fortunes of the Central Powers as of 1915 if the "Schlieffen Plan" had succeeded. We calculate equilibrium military outcomes as of 1915 if France, Belgium, or both had been knocked out of the war in the prior year, while holding fixed the rest of the participants on each side. Would the French and Belgian military efforts just have been made up by Russia and the United Kingdom?

Table 5(b) presents the results of the counterfactual analysis of the "Schlieffen Plan" scenario. Our model suggests that the plan's success would indeed have made a substantial difference. In the baseline scenario, our model projects a 67% chance of victory by the Allied Powers as of

1915. Knocking out Belgium alone would have made a small difference of about 2 percentage points, while eliminating France would have reduced the Allied Powers' chances of victory by 11 percentage points. The effect of knocking out both would be 15 percentage points, greater than the combined effect of eliminating either alone—this is because Belgium would have made up some of the lost effort had only France been eliminated, and vice versa. Viewed in isolation, the success of the Schlieffen Plan would have changed the likely outcome as of 1915 from favoring the Allied Powers to being nearly a toss-up.

On the other hand, given that the fall of France loomed large in the United States' decision to enter World War II (Braumoeller 2010), one might wonder if a successful Schlieffen Plan would have hastened American entry in the prior war. In fact, our model provides a revealed-preference foundation for this argument. We calculate that the United States' expected utility from entering in 1915 if France and Belgium had been knocked out exceeds its actual payoff from entering in 1917, accounting for the difference in war participants as of the latter date.[29] This is both because the United States would have faced weaker opposition and because it could more easily monopolize the Allied Powers' interests in postwar bargaining. In this counterfactual scenario, where the United States enters after France and Belgium exit, a victorious Schlieffen Plan appears counterproductive, as the Allied Powers' probability of victory increases to 72%.

In Appendix H.1, we perform a similar analysis for World War II. We find that earlier American involvement in the European theater would have increased the chance of Allied victory by 13 percentage points. Soviet participation, however, would have made a much smaller difference in either theater.

7.2 Crisis Bargaining Behavior

Our model also allows us to study how changes in coalitions would affect bargaining behavior. For example, we examine how the 2003 Iraq War may have played out differently if the United States

[29] This argument implicitly assumes that the value of the stakes was roughly equivalent at both dates. The United States' hypothetical 1915 payoff would exceed its actual 1917 payoff as long as the value of the stakes did not decrease by 25%.

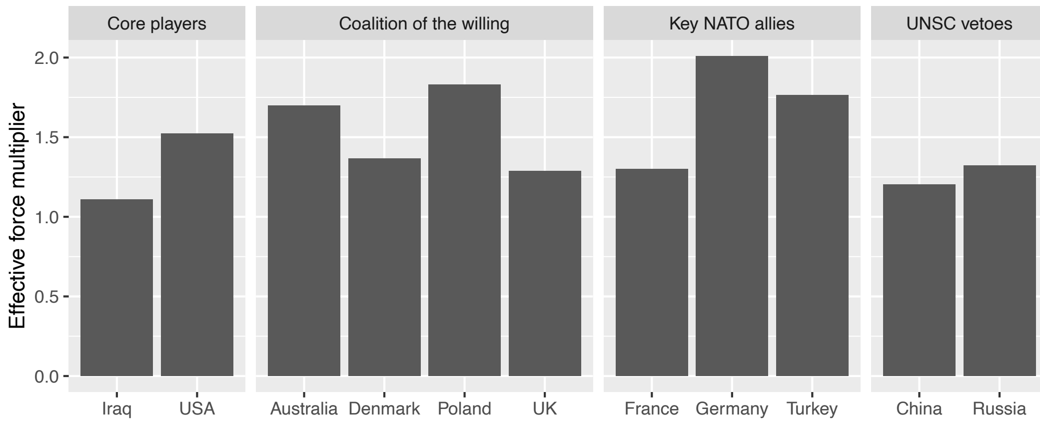

Figure 3. Estimated force multipliers for a dispute in Iraq, 2003.

had garnered additional multilateral support.

We consider three counterfactual scenarios for the Iraq War. Our baseline is the actual conflict between the "coalition of the willing" and Iraq.[30] To quantify the effect that coalition had on the military balance and the likelihood of bargaining breakdown, we compare to a hypothetical bilateral conflict solely between the United States and Iraq. We then consider the effects of hypothetical expansions of the coalition—first to include key NATO allies that did not support the war (France, Germany, and Turkey) and then to include the remaining veto holders on the United Nations Security Council (China and Russia).

Figure 3 plots the effective force multiplier (m_i/c_i) for each country. There are no extreme differences in military effectiveness among the actual or counterfactual participants, though Iraq does have the lowest multiplier of all. The United States' multiplier is only about average among its allies, due to the penalty for nuclear weapons and its relatively high distance to the dispute location.[31]

Table 6 reports the equilibrium quantities and outcomes for the bargaining game under each counterfactual scenario. We find that the coalition of the willing made more difference to the battlefield outcome than the bargaining outcome. We estimate about a 40 percentage point increase

[30]We only include participants in the initial invasion. The results do not change substantially if we include Spain and the Netherlands, which supported the war politically but did not contribute troops to the invasion.

[31]Using distance to the closest military base would reduce our estimate of the military effect of the coalition of the willing, but would not substantively change other results here.

in the probability of defeating Iraq, with victory virtually guaranteed for the coalition of the willing. The coalition has countervailing effects on the distribution of audience costs: the average cost for both sides decreases, thereby reducing the probability of war, but this is offset by the additional uncertainty on the coalition side due to the increase in participants. The net effect is a small increase in the probability of war.

Coalition	p_A	μ_A	σ_A	μ_B	σ_B	Pr(War)
USA only	0.57	−2.8	0.5	−1.9	0.5	0.12
+ Coalition of the willing	0.97	−4.8	3.7	−4.0	0.5	0.16
+ Key NATO allies	0.99	−4.6	6.7	−3.7	0.5	0.28
+ Russia and China	0.99	−4.5	8.8	−3.8	0.5	0.33

Table 6. Bargaining quantities and outcomes for counterfactual anti-Iraq coalitions in 2003. A = coalition, B = Iraq.

The inclusion of additional partners beyond the coalition of the willing would only have served to increase the probability of conflict. Participation by France, Germany, and Turkey would have had only a marginal effect on the military balance, increasing the coalition's probability of victory from 97% to 99%. Meanwhile, their participation would have slightly decreased the average audience cost for each side, while substantially increasing uncertainty on the coalition side due to the additional number of participants. The net effect is a 12 percentage point increase in the probability of war. Essentially, the additional partners are doing nothing to shape the military balance—the war was already as good as won—but are increasing the incentive for the coalition to make extreme demands on Iraq. At the same time, their participation slightly reduces Iraq's willingness to back down, though Iraq's audience cost is still lower here than if the United States were acting alone.

Participation by the remaining Security Council members would have had similar consequences, though not of the same magnitude. Russian and Chinese involvement would have left the military balance and average audience costs essentially unchanged, while further increasing the variance on the coalition size. Consequently, we see a small further increase in the probability of bargaining breakdown.

In Appendix H.2, we undertake a similar exercise for the 1954 Taiwan Strait crisis between

Taiwan and China. We estimate a 17 percentage point effect of American intervention on Taiwan's probability of winning a war in the Strait, even in the face of substantial free-riding by Taiwan. However, we find weak support for moral hazard: American involvement only slightly increases Taiwan's optimal demand and the probability of war.

8 Conclusion

We have developed a new model of crisis bargaining between coalitions, and we have structurally estimated its parameters using data on interstate disputes. The model allows us to estimate the determinants of individual states' force multipliers and of audience costs in crises. We find that states' force multipliers are primarily functions of demographic and economic characteristics, with political and geopolitical features playing a less important role. We have also shown how the model allows us to estimate optimal contributions by individual states and the extent of free-riding in actual and counterfactual conflicts.

An important lesson of both our formal model and our empirical results is that we cannot simply look to how much a country contributed to a war effort to determine the importance of its contribution to the outcome. The key counterfactual question is how much of the country's contribution would have been made up by its allies if that country had not been involved. Our theory-driven approach allows us to give quantifiable answers to this difficult counterfactual question.

In terms of testing, structural models take a different empirical approach than standard approaches. These models make theoretically grounded predictions, in and out of sample, that are most fruitfully tested against a competing theory, rather than an atheoretical null-hypothesis. But even absent a horse race, if one chooses a criterion by which to assess the quality of a model, we can always ask how well the model hits that benchmark. Above, we focused on the ability to predict war in sample and, for the biggest and most important wars, our model did very well with limited data. This should not be surprising, the bargaining model of war contains important insights into the forces that lead to war, and using such a theory as a foundation, a model should do reasonably well at predicting important wars.

On a broader methodological note, our approach and findings illustrate the utility of the structural estimation of game-theoretical models. Debates about these techniques in political science have largely revolved around whether structural estimation is necessary to test comparative statics predictions without bias (Carrubba, Yuen and Zorn 2007; Signorino 2007). But we see the marriage of formal theory and statistical estimation as more than just a tool for bias reduction. Structuring our statistical analysis around a formal model led us to develop a theory-driven alternative to problematic dyadic research designs (Cranmer and Desmarais 2016). More importantly, it directed our attention toward latent variables (audience costs in crisis bargaining) and counterfactual questions (changes in contributions with different coalition structures) that would have been much harder to uncover in a traditional analysis. We encourage political scientists to think of structural estimation as a tool to expand the range of substantive questions that we can answer with data, not merely as a methodological fix for particular statistical problems.

A promising direction for future work is to consider how technological specialization affects coalitions' aggregation of capabilities and the extent of free-riding in crises. States may have a greater incentive to exert effort when their partners possess complementary capabilities than when their areas of specialization are redundant. Theoretically, our formal model could be extended to allow for technological complementarity in each coalition's production of military force. Recent advances in data on states' specific technological capabilities (Gannon 2021) would make it possible to structurally estimate the magnitude of these complementarities and their effects on free-riding within coalitions.

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“The Effective Power of Military Coalitions”: Online Appendix

Contents

A Additional Formal Results

In our first additional result, we derive the equilibrium shares of the prize and expected utilities in the war subgame.

Proposition 3. *In the unique equilibrium of the war subgame, country i's expected share of the prize is*

$$p_i^* = \begin{cases} 1 - q_i & i \leq J, \\ 0 & i > J, \end{cases}$$

and its expected utility is $\pi_i^* = \pi_i(e^*) = (p_i^*)^2$.

Proof. For $i > J$, we have $e_i^* = 0$ and therefore $p_i^* = \pi_i^* = 0$, as claimed. For $i \leq J$, we have

$$p_i^* = \frac{m_i e_i^*}{\sum_{j=1}^{I} m_j e_j^*} = 1 - q_i$$

and

$$\pi_i^* = p_i^* - c_i e_i^* = (1 - q_i)^2 = (p_i^*)^2,$$

as claimed. □

In our second additional result, we derive an expression for the optimal offer under the assumption that B's audience costs are drawn from a logistic distribution.

Proposition 4. *If $\alpha_B \sim Logistic(\mu_B, \sigma_B)$, then*

$$x^*(\alpha_A) = \alpha_A + \pi_A^* + \sigma_B \left[1 + \mathcal{W}\left(\exp\left(\frac{1 - \pi_A^* - \pi_B^* - \mu_B - \alpha_A}{\sigma_B} - 1 \right) \right) \right], \tag{4}$$

where $\mathcal{W}$ is the Lambert W function.[32]

Proof. We work with the following distribution functions:

$$F_B(\alpha_B) = \frac{1}{1 + \exp\left(\frac{\mu_B - \alpha_B}{\sigma_B}\right)},$$
$$f_B(\alpha_B) = \frac{\exp\left(\frac{\mu_B - \alpha_B}{\sigma_B}\right)}{\sigma_B \left[1 + \exp\left(\frac{\mu_B - \alpha_B}{\sigma_B}\right)\right]^2}.$$

Observe that

$$\frac{F_B(\alpha_B)}{f_B(\alpha_B)} = \frac{\sigma_B(1 + \exp\left(\frac{\mu_B - \alpha_B}{\sigma_B}\right))}{\exp\left(\frac{\mu_B - \alpha_B}{\sigma_B}\right)} = \sigma_B \left[1 + \exp\left(\frac{\alpha_B - \mu_B}{\sigma_B} \right) \right].$$

Substituting into Equation 2 gives

$$x^*(\alpha_A) = \alpha_A + \pi_A^* + \sigma_B \left[1 + \exp\left(\frac{1 - x^*(\alpha_A) - \pi_B^* - \mu_B}{\sigma_B} \right) \right] \tag{5}$$
$$= \alpha_A + \pi_A^* + \sigma_B + \sigma_B \exp\left(\frac{1 - \pi_B^* - \mu_B}{\sigma_B} \right) \exp\left(-\frac{1}{\sigma_B} x^*(\alpha_A) \right). \tag{6}$$

The claim in the proposition then follows from the fact that $x = a + b\exp(cx)$ if and only if $x = a - \frac{1}{c}\mathcal{W}(-bc\exp(ac))$ (Corless et al. 1996). □

We now derive comparative statics on the probability of war.

Proposition 5. *If each $\alpha_K \sim Logistic(\mu_K, \sigma_K)$, then the* ex ante *probability of war is increasing in μ_A and μ_B.*

[32] $\mathcal{W} : \mathbb{R}_+ \to \mathbb{R}_+$ defined implicitly by $x = \mathcal{W}(x)\exp[\mathcal{W}(x)]$. See Ramsay and Signorino (2009).

Proof. Recall that Equation 3 gives the *ex ante* probability of war. To prove the result for μ_A, let $\mu_A' < \mu_A''$, and let F_A' and F_A'' denote the corresponding CDFs, each with scale σ_A. F_A'' first-order stochastically dominates F_A', as

$$\frac{\partial F_K(\alpha_K)}{\partial \mu_K} = \frac{-\exp\left(\frac{\mu_K-\alpha_K}{\sigma_K}\right)}{\sigma_K\left[1+\exp\left(\frac{\mu_K-\alpha_K}{\sigma_K}\right)\right]^2} = -f_K(\alpha_K) < 0. \tag{7}$$

Meanwhile, implicit differentiation of Equation 5 gives

$$\frac{dx^*(\alpha_A)}{d\alpha_A} = 1 - \exp\left(\frac{1-x^*(\alpha_A)-\pi_B^*-\mu_B}{\sigma_B}\right)\frac{dx^*(\alpha_A)}{d\alpha_A},$$

which implies

$$\frac{dx^*(\alpha_A)}{d\alpha_A} = \frac{1}{1+\exp\left(\frac{1-x^*(\alpha_A)-\pi_B^*-\mu_B}{\sigma_B}\right)} > 0.$$

Because F_A'' is FOSD over F_A' and x^* is increasing, we have

$$\int \left[1 - F_B(1-x^*(\alpha_A)-\pi_B^*)\right] dF_A''(\alpha_A) > \int \left[1 - F_B(1-x^*(\alpha_A)-\pi_B^*)\right] dF_A'(\alpha_A).$$

To prove the result for μ_B, it will suffice to prove that the integrand in Equation 3 is strictly increasing in μ_B for all α_A. Implicit differentiation of Equation 5 gives

$$\frac{\partial x^*(\alpha_A)}{\partial \mu_B} = -\exp\left(\frac{1-x^*(\alpha_A)-\pi_B^*-\mu_B}{\sigma_B}\right)\left[\frac{\partial x^*(\alpha_A)}{\partial \mu_B}+1\right],$$

which implies

$$\frac{\partial x^*(\alpha_A)}{\partial \mu_B} = \frac{-\exp\left(\frac{1-x^*(\alpha_A)-\pi_B^*-\mu_B}{\sigma_B}\right)}{1+\exp\left(\frac{1-x^*(\alpha_A)-\pi_B^*-\mu_B}{\sigma_B}\right)} = -F_B(1-x^*(\alpha_A)-\pi_B^*).$$

Recall that Equation 7 above gives the derivative of F_B with respect to μ_B. Therefore, differentiating the integrand of Equation 3 with respect to μ_B gives

$$\begin{aligned}
&\frac{\partial}{\partial \mu_B}\left[1 - F_B(1-x^*(\alpha_A)-\pi_B^*)\right] \\
&\quad = -f_B(1-x^*(\alpha_A)-\pi_B^*)\left[-\frac{\partial x^*(\alpha_A)}{\partial \mu_B}\right] - \frac{\partial F_B(1-x^*(\alpha_A)-\pi_B^*)}{\partial \mu_B} \\
&\quad = f_B(1-x^*(\alpha_A)-\pi_B^*)\left[-F_B(1-x^*(\alpha_A)-\pi_B^*)+1\right] \\
&\quad > 0.
\end{aligned}$$

□

In our final supplemental result, we verify that there is always a positive-measure set of parameters at which the log-likelihood is finite.

Proposition 6. *The log-likelihood is finite in a neighborhood of any* (β, η, θ) *such that* $\beta = 0$.

Proof. Consider the n'th crisis. By Proposition 3, $p^*_{ni}(0) = 1/I_n$ for each country i. Consequently, $p^*_{nA}(0) = I_{nA}/I_n$, so each side has a positive probability of winning in case of war. Meanwhile, because each α_K has full support on $\mathbb{R}$, we always have $0 < \Pr(\text{war}) < 1$. Therefore, $\log \ell(0, \eta, \theta) > -\infty$ for all η and θ. Because the equilibrium probabilities of victory in case of war are continuous in the structural model parameters, the same is true in a neighborhood of $(0, \eta, \theta)$. □

B Motivating the Postwar Division of Spoils

Our model assumes that the spoils of war are divided among the victors in proportion to their effectiveness-weighted efforts (Equation 1). Specifically, if the winning side consists of countries $i = 1, \ldots, N$, then each member's share of the spoils is ω_i, defined as

$$\omega_i = \frac{m_i e_i}{\sum_{j=1}^N m_j e_j}.$$

Our goal now is to show that this functional form for the division of spoils can be thought of as the reduced form of a postwar bargaining game among the victorious side.

To fix ideas, it will be helpful to think of the spoils of conflict as a piece of territory (though this interpretation is not strictly necessary for the model). These spoils have total value normalized to 1. We assume each member of the winning coalition initially possesses a share of the territory proportional to the resources they mobilized for conflict, $m_i e_i$. Until a bargain is reached among the winning coalition, each member derives consumption value from the territory they possess. Meanwhile, disagreement has an ongoing cost $C > 0$, which is distributed equally across the members of the coalition. This can be thought of as a political cost incurred due to public disagreement among the coalition, or as a material loss from governing the territory gained in the war without a long-term settlement in place.

A long-run division of the spoils of war may be reached through bargaining among the winning coalition. Bargaining takes place over an infinite sequence of discrete periods, $t = 0, 1, \ldots$, and the countries have a common discount factor $\delta \in (0, 1)$. The sequence of play within each period is as follows:

1. Nature randomly selects one country to have proposal power this period. These selections are i.i.d. across periods, with each country having an equal $\frac{1}{N}$ probability of selection.

2. The proposer offers a division of the spoils, $x^t \in \mathcal{X} \equiv \{x \in \mathbb{R}^N \mid \sum_{i=1}^N x_i = 1\}$.

3. Each country simultaneously decides whether to accept or reject the offer.[33]

4. The bargaining outcome is realized:

   - If every country accepted the offer, the game ends. Each country receives a payoff of x_i^t in this and all subsequent periods.
   - If any country rejected the offer, the game moves to the next period. Each country receives its status quo value of $\omega_i - \frac{C}{N}$ in the current period.

Each country's objective is to maximize the per-period average of its discounted stream of payoffs, $(1-\delta)\sum_{t=0}^{\infty}\delta^t u_i^t$.

Proposition 7. *The postwar bargaining game has a stationary no-delay equilibrium in which:*

- *In any period in which country i is the proposer, they offer x^i, where $x_i^i = \omega_i + \frac{(N-1)(1-\delta)C}{N}$ and $x_j^i = \omega_j - \frac{(1-\delta)C}{N}$ for all $j \neq i$.*
- *Regardless of who is the proposer, each country j accepts an offer $x \in \mathcal{X}$ if and only if $x_j \geq \omega_j - \frac{(1-\delta)C}{N}$.*

All stationary no-delay equilibria are payoff equivalent to this one.

Proof. Existence. Under the proposed strategy profile, country j's continuation value from rejecting any offer is

$$\begin{aligned}
V_j &= (1-\delta)\left(\omega_j - \frac{C}{N}\right) + \delta\left[\sum_{i=1}^{N}\frac{1}{N}x_j^i\right] \\
&= (1-\delta)\left(\omega_j - \frac{C}{N}\right) + \delta\left[\frac{1}{N}\left(\omega_j + \frac{(N-1)(1-\delta)C}{N}\right) + \frac{N-1}{N}\left(\omega_j - \frac{(1-\delta)C}{N}\right)\right] \\
&= (1-\delta)\left(\omega_j - \frac{C}{N}\right) + \delta\omega_j \\
&= \omega_j - \frac{(1-\delta)C}{N}.
\end{aligned}$$

This means the strategies for responding to offers are equivalent to accepting if and only if $x_j \geq V_j$, so they are sequentially rational. Additionally, for each potential proposer i we have $x_j^i = V_j$ for all $j \neq i$ and $x_i^i = 1 - \sum_{j \neq i} V_j$. No other offer could yield more for i while satisfying all other countries' acceptance condition, so the proposed offers are sequentially rational as well.

[33] This process could instead be sequential. As our focus is on stationary no-delay equilibria, the difference is immaterial.

Uniqueness. Take any stationary no-delay equilibrium, and let $V = (V_1, \dots, V_N)$ be the associated continuation values. Because the optimal offer for any proposer must hold every other country exactly to its continuation value, we have

$$V_i = (1-\delta)\left(\omega_i - \frac{C}{N}\right) + \delta\left[\frac{1}{N}\left(1 - \sum_{j\neq i} V_j\right) + \frac{N-1}{N}V_i\right]$$

for all $i = 1, \dots, N$. This condition is equivalent to

$$[\delta + N(1-\delta)]V_i + \delta \sum_{j\neq i} V_j = (1-\delta)[N\omega_i - C] + \delta.$$

Stacking the condition for each country $i = 1, \dots, N$ gives us the linear system

$$[\delta\mathbf{O} + N(1-\delta)\mathbf{I}]V = (1-\delta)[N\omega - C] + \delta$$

where $\mathbf{O}$ is the $N\times N$ matrix of all ones and $\mathbf{I}$ is the $N\times N$ identity matrix. Following from the fact that $\mathbf{O}^2 = N\mathbf{O}$, we can show that $[\delta\mathbf{O} + N(1-\delta)\mathbf{I}]^{-1} = \frac{1}{N(1-\delta)}[-\frac{\delta}{N}\mathbf{O} + \mathbf{I}]$:

$$\begin{aligned}
\frac{1}{N(1-\delta)}[\delta\mathbf{O} + N(1-\delta)\mathbf{I}]\left[\frac{-\delta}{N}\mathbf{O} + \mathbf{I}\right] &= \frac{1}{N(1-\delta)}\left[\frac{-\delta^2}{N}\mathbf{O}^2 + \delta\mathbf{O} - \delta(1-\delta)\mathbf{O} + N(1-\delta)\mathbf{I}\right] \\
&= \frac{-\delta^2 + \delta - \delta(1-\delta)}{N(1-\delta)}\mathbf{O} + \mathbf{I} \\
&= \mathbf{I}.
\end{aligned}$$

This implies

$$V = \frac{1}{N(1-\delta)}\left[-\frac{\delta}{N}\mathbf{O} + \mathbf{I}\right]\left[(1-\delta)[N\omega - C] + \delta\right]$$

and thus each

$$\begin{aligned}
V_i &= \frac{1}{N(1-\delta)}\left[(1-\delta)(N\omega_i - C) + \delta - \frac{\delta}{N}\sum_{j=1}^{N}\left[(1-\delta)(N\omega_j - C) + \delta\right]\right] \\
&= \frac{1}{N(1-\delta)}\left[(1-\delta)(N\omega_i - C) + \delta - \delta(1-\delta)(1-C) - \delta^2\right] \\
&= \omega_i - \frac{(1-\delta)C}{N}.
\end{aligned}$$

These are identical to the continuation values derived for the claimed equilibrium, so all stationary no-delay equilibria are payoff equivalent. □

As players become arbitrarily patient, the players' expected utilities from this bargaining game converge to the functional form employed in our main model:

$$\frac{m_i e_i}{\sum_{j=1}^{N} m_j e_j}.$$

In this way, the postwar stage of the main model may be thought of as a reduced form representation of bargaining among the winning coalition.

C Structural Model Identification

C.1 Effectiveness vs. Cost of Effort

If we have data on the observed outcomes of conflict but not on individual effort allocations, we cannot separately identify the multiplier m_i and the marginal cost c_i. To see why, consider the contest stage of a single interaction, and let e be a vector of effort decisions. Now consider a transformed variant of the model where $\tilde{m}_i = \frac{m_i}{c_i}$ and $\tilde{c}_i = 1$ for each $i = 1, \ldots, I$. The transformed effort vector $\tilde{e} = (c_i e_i)_{i=1,\ldots,I}$ yields an identical distribution of conflict outcomes and payoffs to those under e in the original model. In particular, because each $\tilde{m}_i \tilde{e}_i = m_i e_i$, the probability that side A wins is

$$\tilde{p}_A(\tilde{e}) = \frac{\sum_{i \in A} \tilde{m}_i \tilde{e}_i}{\sum_{i \in A} \tilde{m}_i \tilde{e}_i + \sum_{i \in B} \tilde{m}_i \tilde{e}_i} = \frac{\sum_{i \in A} m_i e_i}{\sum_{i \in A} m_i e_i + \sum_{i \in B} m_i e_i} = p_A(e),$$

and the payoff to country i on side K is

$$\tilde{\pi}_i(\tilde{e}) = \tilde{p}_K(\tilde{e}) \frac{\tilde{m}_i \tilde{e}_i}{\sum_{j \in K} \tilde{m}_j \tilde{e}_j} - \tilde{c}_i \tilde{e}_i = p_K(e) \frac{m_i e_i}{\sum_{j \in K} m_j e_j} - c_i e_i = \pi_i(e).$$

Therefore, the equilibria of the original game and the transformed game are identical in terms of payoffs and the distribution of conflict outcomes. The model with $m_i = m_i'$ and $c_i = c_i'$ is thus observationally equivalent to one with $m_i = \frac{m_i'}{c_i'}$ and $c_i = 1$.

One way to deal with this issue in structural estimation is to model m_i and c_i as functions of distinct sets of variables. Instead, to simplify the interpretation of our results, we directly model the effective force multiplier as a log-linear function of model parameters: $\frac{m_i}{c_i} = \exp(X_i^\top \beta)$. While the equilibrium values of raw effort e_i are sensitive to the choice of normalization for c_i, the equilibrium values of effectiveness-weighted effort $m_i e_i$ are not.

C.2 *Intercept in the Force Multiplier*

The last identification concern is that the equation for the effective force multiplier cannot contain a constant. Per the preceding discussion, we model the effectiveness-cost ratio as the log-linear $\frac{m_i}{c_i} = \exp(X_i^\top \beta)$ and normalize $c_i = 1$, giving us the following utility function in the contest stage:

$$\pi_i(e) = \frac{\exp(X_i^\top \beta) e_i}{\sum_{j=1}^{I} \exp(X_j^\top \beta) e_j} - e_i.$$

Including a constant in β does not change the utility from any outcome; i.e., for all $\beta_0 \in \mathbb{R}$, we have

$$\begin{aligned}
\pi_i(e) &= \frac{\exp(\beta_0 + X_i^\top \beta) e_i}{\sum_{j=1}^{I} \exp(\beta_0 + X_j^\top \beta) e_j} - e_i \\
&= \frac{\exp(\beta_0) \exp(X_i^\top \beta) e_i}{\exp(\beta_0) \sum_{j=1}^{I} \exp(X_j^\top \beta) e_j} - e_i \\
&= \frac{\exp(X_i^\top \beta) e_i}{\sum_{j=1}^{I} \exp(X_j^\top \beta) e_j} - e_i.
\end{aligned}$$

Different values of the intercept give us identical utility functions for all players and therefore are observationally equivalent. Consequently, we can only identify the effective force multiplier $\frac{m_i}{c_i}$ up to a scale factor.

D Estimation Details

D.1 *Multiple Imputation*

We have missingness in five of our state-level variables and two of our dispute/coalition-level variables, as documented in Table A.1. We impute missing values using Amelia II (Honaker, King and Blackwell 2011). To perform the imputation, we construct a dataset of all state-years from 1816 to 2012, not just those that were involved in disputes in our final analysis data. The imputation dataset contains all state-level variables used in the model except for Distance to Dispute. We include additional GDP and population measures from the Penn World Tables, the Maddison project, and the World Development Indicators data; we also include the military personnel, military expenditures, and CINC score measures from the National Material Capabilities data. To best fit Amelia II's joint normal model, we log-transform all strictly positive variables and use an inverse hyperbolic sine transformation for all variables with a minimum of 0.

We create 10 imputations of the state-year dataset. We treat the imputation dataset as time-series cross-section data using the methods described in Honaker and King (2010). The model

Variable	Pct. Missing
State-Level	
Import Percentage	51.6
GDP	42.1
Democracy	2.4
Urban Population	0.9
Energy Consumption	0.6
Distance to Dispute	0.0
Iron and Steel Production	0.0
Nuclear Weapons	0.0
Total Population	0.0
Dispute-Level	
Democracy, Side *B*	3.3
Democracy, Side *A*	1.6
Contiguity	0.0
Major Power, Side *A*	0.0
Major Power, Side *B*	0.0
Peace Years	0.0
Interest Similarity	0.0

Table A.1. Percentage of each covariate missing in the raw data.

includes a linear time control and its interaction with each state identifier. To reduce variance given the large number of parameters in the imputation model, we include a ridge penalty equal to 1% of the number of observations, following the best practices outlined by Honaker, King and Blackwell (2011).

We use the imputed state-level data to create 10 imputed versions of the participant-level data for each dispute. Additionally, we create 10 corresponding imputations of the coalition/dispute-level data, using the state-level imputed values to calculate each side's average Democracy. (All other coalition/dispute-level variables have zero missingness.) To calculate inferential statistics, we perform a nonparametric bootstrap. Specifically, for each of the 10 imputed datasets, we resample at the dispute level 100 times and refit the model to each resampled dataset. This gives us a matrix of 1,000 bootstrap estimates; we compute standard errors by taking sample standard deviations of these estimates.

D.2 Simulated Likelihood

The log-likelihood function is

$$\begin{aligned}\log \ell(\beta, \eta, \theta) = &\sum_{n : Y_n^{\text{war}} = 0} \log(1 - r_n^*(\beta, \eta, \theta)) + \sum_{n : Y_n^{\text{war}} = 1} \log r_n^*(\beta, \eta, \theta) \\ &+ \sum_{n : Y_n^{\text{win}} = A} \log p_{nA}^*(\beta) + \sum_{n : Y_n^{\text{win}} = B} \log p_{nB}^*(\beta),\end{aligned} \tag{8}$$

where $r_n^*(\beta, \eta, \theta)$ denotes the probability of war for the n'th crisis as a function of the unknown parameters, calculated using Equation 3. As there is no closed-form expression for the equilibrium probability of war, Equation 3, we estimate the model by maximum simulated likelihood. Each time we evaluate the log-likelihood function during optimization, we use a numerical approximation to calculate the equilibrium probability of war for each dispute.

Let $\upsilon_1, \ldots, \upsilon_G$ be independent draws from a uniform distribution on the unit interval. In practice, we use $G = 1024$ draws from the low-discrepancy Halton sequence, as optimization would never converge if we redrew these values at each iteration. We take these values to represent cumulative probabilities of coalition A's audience cost in the n'th dispute, α_{nA}. We calculate the inverse CDF at each υ_g to obtain the corresponding draw from the prior distribution of α_{nA}. Now writing α_{nA} as a function of the cumulative probability υ_g, we have

$$\alpha_{nA}(\upsilon_g) = F_{nA}^{-1}(\upsilon_g) = \mu_{nA} + \sigma_{nA} \log\left(\frac{\upsilon_g}{1 - \upsilon_g}\right).$$

We use Equation 4 to calculate the optimal offer for each type drawn in the simulation. We calculate the probability of war for each type,

$$\begin{aligned}\Pr(\text{war} \mid \alpha_{nA} = \alpha_{nA}(\upsilon_g)) &= 1 - F_{nB}(1 - x^*(\alpha_{nA}(\upsilon_g)) - \pi_{nB}^*) \\ &= 1 - \frac{1}{1 + \exp\left(\frac{\mu_{nB} + x^*(\alpha_{nA}(\upsilon_g)) + \pi_{nB}^* - 1}{\sigma_{nB}}\right)}.\end{aligned}$$

Finally, we average over the simulated types to approximate equilibrium war probabilities, substituting the following into the log-likelihood (Equation 8):

$$r_n^* \approx \frac{1}{G} \sum_{g=1}^{G} \Pr\left(\text{war} \mid \alpha_{nA} = \alpha_{nA}(\upsilon_g)\right).$$

We need not use such an approximation for the victory probabilities, p_{nA}^* and p_{nB}^*, as these have the closed-form solution given in Proposition 1.

Variable	Units	Mean	Std. Dev.
State-Level: Demographic			
Total Population	Thousands of persons‡	86,494	192,649
Urban Population	Thousands of persons‡	18,250	44,268
State-Level: Economic			
Energy Consumption	Thousands of coal-tons‡	223,027	581,944
GDP	Millions USD, 2011†	654,983	1,848,001
Import Percentage	Percentage‡	22.09	17.33
Iron and Steel Production	Thousands of tons‡	13,556	39,598
State-Level: Political			
Democracy	Polity IV score	−0.21	7.22
State-Level: Geopolitical			
Distance to Dispute	Miles‡	1,098	1,871
Nuclear Weapons	Binary	0.14	0.35
Dispute-Level			
Democracy, Side *A*	Polity IV score	−1.06	6.88
Democracy, Side *B*	Polity IV score	0.00	7.01
Major Power, Side *A*	Binary	0.32	0.46
Major Power, Side *B*	Binary	0.21	0.41
Participants, Side *A*	Count†	1.19	1.31
Participants, Side *B*	Count†	1.17	1.19
Contiguity	Binary	0.66	0.45
Interest Similarity	S-score	0.55	0.40
Peace Years	Years‡	11.75	23.36

Table A.2. State- and dispute-level characteristics included in the empirical model. †: log-transformed in estimation. ‡: $\log(1 + x)$-transformed in estimation.

E Data Sources and Versions

Militarized Interstate Disputes. We use version 2.1.1 of the Gibler–Miller–Little MID data (Gibler, Miller and Little 2016).

National Material Capabilities. We use version 5.0 of the National Material Capabilities data (Singer, Bremer and Stuckey 1972).

GDP. We use version 9.0 of the Penn World Tables (Feenstra, Inklaar and Timmer 2015). We use the $RGDP^0$ measure, the one best suited for comparisons of "productive capacity across countries and across years." To estimate missing values of GDP as accurately as possible, we also include GDP measures from the Maddison Project, version 2018 (Bolt et al. 2018) and the World Development Indicators dataset (The World Bank 2019) (accessed 2019-02-28) in the multiple imputation model.

Import Percentage. We use the World Development Indicators data (The World Bank 2019) as of 2019-02-28.

Democracy. We use version 2017 of the Polity IV project (Marshall, Gurr and Jaggers 2014).

Nuclear Weapons. We use the data from Gartzke and Kroenig (2009).

Distance to Dispute. We use version 2.0 of the MIDLOC dataset (Braithwaite 2010; Bezerra and Braithwaite 2019) for dispute locations. Capital locations are taken from version 3.212 of EUGene (Bennett and Stam 2000).

Major Power. We use version 2016 of the State System Membership data (Correlates of War Project 2016).

Contiguity. We use version 3.20 of the Correlates of War project's Direct Contiguity Data (Stinnett et al. 2002).

Interest Similarity. We use the S-score measure provided by the Alliance Treaty Obligations and Provisions project, version 4.0 (Chiba, Johnson and Leeds 2015).

Peace Years. For dyads that existed in 1816, we use the corrected measure developed by Werner (2000).

F Multiplier Estimates

Figure A.1 plots the estimated force multiplier for China, Japan, Russia, the United Kingdom, and the United States throughout our sample period.[34] Both within each country's time series and when comparing different countries at the same point in time, we see the most variation in the demographic and economic components.[35] Nevertheless, political factors have a nontrivial influence

[34] We assume zero distance to dispute in this figure, which raises the overall force multiplier and slightly decreases variation in the geopolitical component.

[35] The negative trend in the demographic multiplier time series is because urban population growth has outpaced total population growth, and we find a moderate negative coefficient on urban population in the equation for m_i.

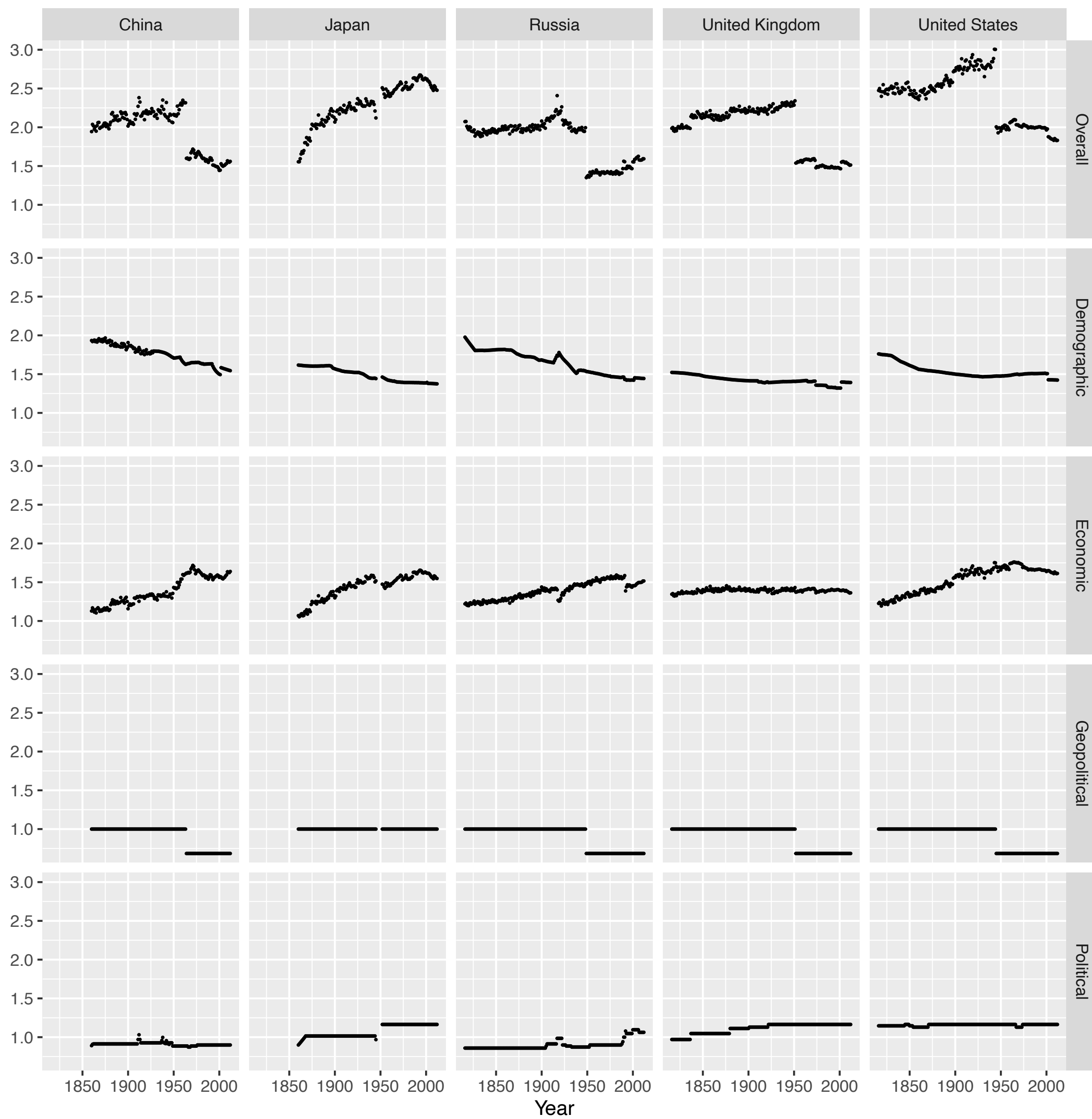


Figure A.1. Estimated force multipliers for major powers over time.

	Country	Year	MID#	Multiplier	Demographic	Economic	Political	Geopolitical
1	India	1971	2099	3.00	1.67	1.56	1.15	1.00
2	India	1969	2098	3.00	1.69	1.55	1.15	1.00
3	India	1969	2633	3.00	1.69	1.55	1.15	1.00
4	India	1969	2634	3.00	1.69	1.55	1.15	1.00
5	India	1969	2635	3.00	1.69	1.55	1.15	1.00
6	United States	1919	2185	2.94	1.48	1.71	1.16	1.00
7	United States	1918	2184	2.90	1.48	1.69	1.16	1.00
8	India	1965	623	2.88	1.68	1.50	1.15	1.00
9	India	1965	1463	2.88	1.68	1.50	1.15	1.00
10	India	1967	1715	2.87	1.68	1.49	1.15	1.00
4953	Bahrain	1986	2565	0.98	1.29	0.92	0.86	0.97
4954	Jordan	1967	1035	0.98	1.35	0.85	0.87	0.97
4955	Jordan	1962	1018	0.97	1.39	0.83	0.87	0.97
4956	Jordan	1959	3231	0.97	1.41	0.81	0.87	0.97
4957	Bahrain	1991	3957	0.97	1.30	0.91	0.86	0.96
4958	Jordan	1963	1019	0.97	1.38	0.83	0.87	0.97
4959	Jordan	1966	3412	0.96	1.36	0.84	0.87	0.97
4960	Bahrain	1986	2572	0.96	1.29	0.92	0.86	0.95
4961	Jordan	1961	122	0.96	1.39	0.83	0.87	0.95
4962	Jordan	1962	1108	0.95	1.39	0.83	0.87	0.95

Table A.3. Greatest and lowest estimated force multipliers for dispute participants in the analysis data.

too—for example, the United States' democratic political system pushes its force multiplier above China's in 2010, despite the latter's demographic advantage.

We observe a downward secular trend in the major powers' force multipliers. A common trend like this has little effect on likely war outcomes, as equilibrium probabilities of victory are a function of the relative force multipliers at any point in time. Still, the downward trends imply that it takes more effort for a major power to achieve a given military objective now than at their mid–20th century peaks. For example, we observe this in how the United States spent similar amounts in real terms on World War II and on the Iraq War (Stiglitz and Blimes 2008), despite accomplishing far less militarily in the latter.

G Originators Model

We run an alternative model to examine whether we could better explain the patterns in the data by excluding "joiners"—countries that were not involved at the outset of the dispute—from the bargaining stage. For each country i in a dispute, let $o_i \in \{0, 1\}$ be an indicator for whether i is an "originator," involved in the dispute from the outset. Those with $o_i = 0$ are joiners. In our data analysis, we use the MID data codings of originators. Each side always has at least one originator: $\sum_{i \in A} o_i \geq 1$ and $\sum_{i \in B} o_i \geq 1$.

In the bargaining stage, we now calculate optimal offers and the probability of war using only originators.[36] Let $\tilde{e}^*$ denote the equilibrium effort allocations from a contest only among the originators, calculated according to Proposition 1. Let $\tilde{\pi}_A^*$ and $\tilde{\pi}_B^*$ be the coalitions' expected utilities from such a contest: $\tilde{\pi}_K^* = \sum_{i \in K} o_i \pi_i(\tilde{e}^*)$ for $K \in \{A, B\}$. Let $\tilde{x}^*(\alpha_A)$ be the optimal offer for each type of A, which we calculate by replacing each π_K with the corresponding $\tilde{\pi}_K$ in Equation 2. Substituting the new values into Equation 3, the probability of war is now

$$\Pr(\text{war}) = \int [1 - F_B(1 - \tilde{x}^*(\alpha_A) - \tilde{\pi}_B^*)] \, dF_A(\alpha_A). \tag{9}$$

To estimate the originators model, we modify the log-likelihood from the baseline model by calculating the probability of war using only the originators. For those disputes that actually proceed to war, however, we still calculate the probability of victory for each side using both originators and joiners. Letting $\tilde{r}_n^*$ denote the probability of war calculated using Equation 9, the modified log-likelihood is

$$\begin{aligned} \log \tilde{\ell}(\beta, \eta, \theta) = & \sum_{n: Y_n^{\text{war}}=0} \log(1 - \tilde{r}_n^*(\beta, \eta, \theta)) + \sum_{n: Y_n^{\text{war}}=1} \log \tilde{r}_n^*(\beta, \eta, \theta) \\ & + \sum_{n: Y_n^{\text{win}}=A} \log p_{nA}^*(\beta) + \sum_{n: Y_n^{\text{win}}=B} \log p_{nB}^*(\beta), \end{aligned}$$

We maximize this function to obtain estimates for the originators model. We follow the same process of multiple imputation for missing data, simulated likelihood for approximation of $\tilde{r}_n$, and bootstrap for standard errors as in the baseline model. The only difference, due to computational limitations, is that we only draw 25 resamples per imputed dataset, leaving us with 250 total bootstrap iterations.

Table A.4 reports the structural estimations for the originators model. For the state-level determinants of m_i and c_i, the point estimates are broadly similar to the baseline model, though estimated less precisely. The dispute- and coalition-level determinants of the audience cost distribution sometimes differ from the baseline model estimates, but are estimated with considerably less precision. The model clearly fits worse overall, with a lower log-likelihood and greater AIC and BIC than the baseline model. As noted in the main text, a non-nested model test confirms that the baseline model fits the data significantly better.

[36] We also recalculate the coalition- and dispute-level variables to reflect the values only for originators. For example, "Democracy, Side A" is now the average Polity score of originators on side A.

Term	Estimate	Std. Err.	Conf. Int.
Force Multiplier m_i/c_i: Demographic			
Total Population	0.11	0.11	[−0.13, 0.35]
Urban Population	−0.09	0.08	[−0.21, 0.11]
Force Multiplier m_i/c_i: Economic			
Energy Consumption	0.02	0.06	[−0.13, 0.14]
GDP	0.03	0.13	[−0.29, 0.23]
Import Percentage	−0.15	0.18	[−0.53, 0.15]
Iron and Steel Production	0.02	0.05	[−0.08, 0.11]
Force Multiplier m_i/c_i: Political			
Democracy	0.01	0.01	[−0.01, 0.05]
Force Multiplier m_i/c_i: Geopolitical			
Distance to Dispute	−0.01	0.03	[−0.05, 0.04]
Nuclear Weapons	−0.39	0.36	[−1.37, 0.19]
Audience Cost Mean μ_K			
Intercept	−68.16	27.93	[−136.41, −42.09]
Contiguity	−5.67	4.63	[−14.99, 3.95]
Democracy	−3.20	1.40	[−6.39, −1.52]
Interest Similarity	3.55	5.83	[−9.07, 16.63]
Major Power	−2.51	7.34	[−15.93, 15.22]
Peace Years	−6.23	3.69	[−12.50, −0.32]
Audience Cost Scale σ_K			
Intercept	2.54	1.79	[2.10, 8.61]
Coalition Size	−0.37	0.82	[−2.29, 0.58]
Disputes	2101		
Participants	4962		
Log-likelihood	−919.13		

Table A.4. Structural estimation results for the originators model.

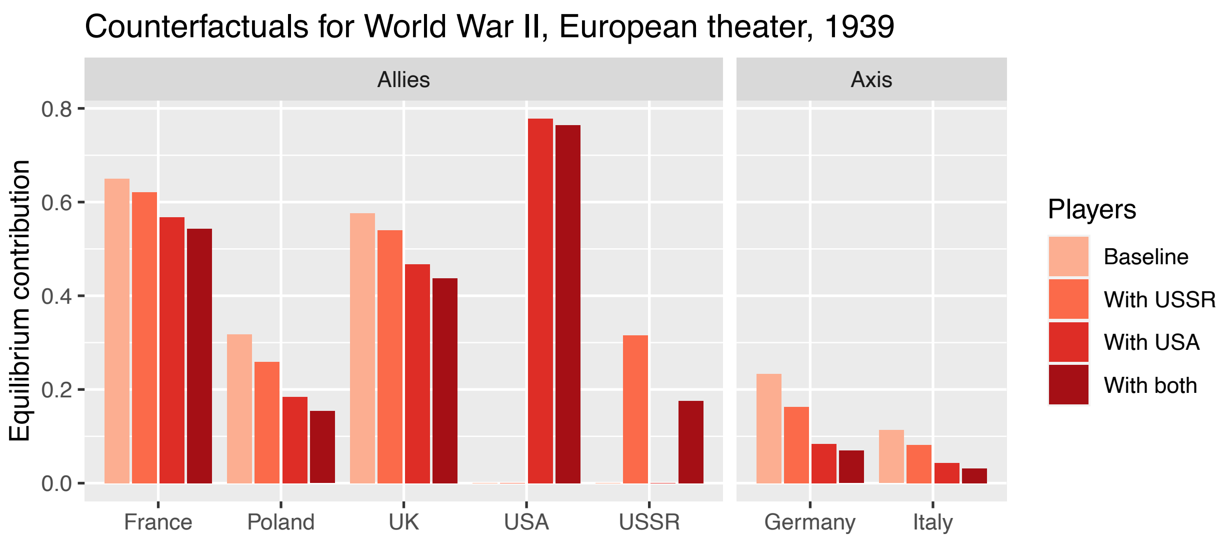


Figure A.2. Counterfactual choices of war effort in the European theater of World War II, 1939.

H Additional Counterfactual Analyses

H.1 World War II

Counterfactual	Pr(Allies Win)
Baseline	0.78
With USSR	0.84
With USA	0.91
With both	0.93

Table A.5. Counterfactual equilibrium quantities in the European theater of World War II, 1939.

We analyze counterfactuals for World War II similar to the analysis for World War I in the main text. First, in the European theater, we analyze equilibrium war efforts and the probability of victory if the Soviet Union, the United States, or both had contributed to the Allies from the war's outset in 1939. Figure A.2 plots the predicted contributions, and Table A.5 reports the dispute-level equilibrium quantities for each scenario.

Soviet contribution to the Allies in 1939 would have had a moderate effect, raising the Allied probability of victory by 6 percentage points. Meanwhile, the United States would have made a substantial difference, a 13 percentage point increase, by joining the war effort two years earlier. Conditional on the United States joining, the participation of the Soviet Union would have been even less pivotal, making only a 2 percentage point difference in the Allies' chances of winning. In this way, American participation would have significantly reduced the Soviet Union's incentive to contribute. The same is true for the other powers, as seen in Figure A.2. As before, the United States has a larger effective force multiplier than its prospective allies. It compares favorably to the UK and France in terms of total population and import dependence, while its advantage over the Soviet Union is largely due to its democratic political system.

Counterfactual	Pr(Allies Win)
Baseline	0.73
With USSR	0.75

Table A.6. Counterfactual equilibrium quantities in the Pacific theater of World War II, 1941.

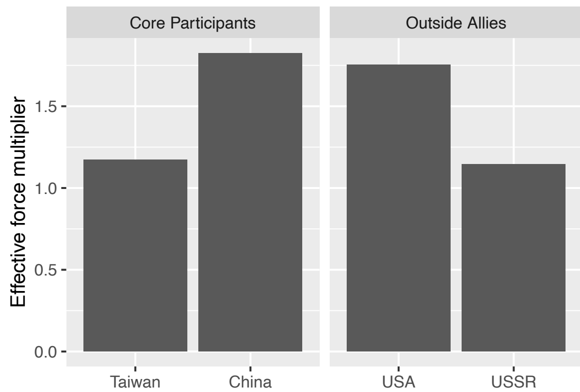
Turning to the Pacific theater, we find that the participation of the Soviet Union would not have made much difference as of 1941. As shown in Table A.6, Soviet involvement would have increased the probability of victory in the Pacific by just 2 percentage points. We predict that the Soviet Union would have contributed relatively little effort and that even this small contribution would have been largely offset by decreases in British and Chinese contributions.

H.2 Taiwan Strait

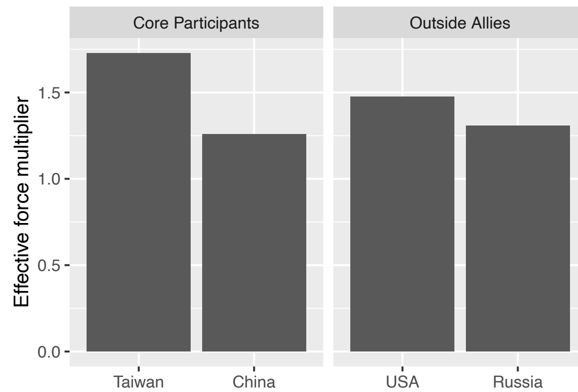
The 1954 dispute between China and Taiwan over control of islands in the Taiwan Strait has been widely studied as an example of moral hazard in alliances (Benson 2012). According to the theory of moral hazard, allied support raises a state's likelihood of winning a conflict and thus gives it an incentive to behave more aggressively in disputes with adversaries. Here, we use our structural model to quantify key variables in the moral hazard explanation when applied to a crisis between Taiwan and China in the Taiwan Strait. First, in light of the potential incentive to free-ride, how much would allied support—from the United States in Taiwan's case, or the Soviet Union/Russia in China's—actually affect the military balance in case of outright war? Second, at the bargaining table, how would additional allies change the incentives to take risks and the probability that negotiations end in war? We calculate these counterfactual quantities both for the 1954 dispute and for a hypothetical present-day dispute (using covariate values from 2012, the most recent year in our data).

Figure A.3 plots the force multipliers for the potentially involved countries, and Table A.7 reports our estimates of the key equilibrium quantities in prewar bargaining. We treat Taiwan's coalition as side A, so higher estimated offer values reflect more aggressive bargaining behavior by Taiwan.[37]

[37] China is coded as side A in the 1954 dispute in the MID data. We treat Taiwan as side A here so that we can estimate its optimal bargaining offer, so as to directly speak to accounts of American involvement creating moral hazard for Taiwan.



(a) Actual crisis (1954).



(b) Hypothetical recent crisis (2012).

Figure A.3. Estimated force multipliers for a dispute in the Taiwan Strait.

Participants	p_A	x^*	μ_A	σ_A	μ_B	σ_B	Pr(War)
Baseline	0.38	1.4	−1.5	0.5	−1.5	0.5	0.20
With US	0.55	1.5	−2.2	1.2	−1.4	0.5	0.22
With USSR	0.24	1.9	−2.2	0.5	−2.2	1.2	0.31
With both	0.49	1.8	−2.6	1.2	−2.0	1.2	0.33

(a) Actual crisis (1954).

Participants	p_A	x^*	μ_A	σ_A	μ_B	σ_B	Pr(War)
Baseline	0.57	2.2	−3.8	0.5	−2.5	0.5	0.09
With US	0.77	2.2	−3.7	1.2	−2.3	0.5	0.12
With Russia	0.45	2.8	−4.9	0.5	−4.0	1.2	0.16
With both	0.66	2.8	−4.6	1.2	−3.7	1.2	0.20

(b) Hypothetical recent crisis (2012).

Table A.7. Bargaining quantities and outcomes for a dispute in the Taiwan Strait. A = Taiwan's side, B = China's side.

In both scenarios, we estimate that American assistance would materially increase Taiwan's military fortunes. In the 1954 dispute, with the US having a substantially larger force multiplier than Taiwan, US participation increases Taiwan's chance of victory by 17 percentage points, even though Taiwan's equilibrium effort would decrease by half. For a hypothetical present-day dispute, the effect of US participation increases to 20 percentage points in favor of Taiwanese victory. This is in part because of less free-riding: the force multipliers of Taiwan and the US are closer to each other in 2012, and Taiwan's equilibrium effort barely decreases as a result of US involvement. The effects of Soviet/Russian involvement on China's military fortunes are similar: a 14 percentage point increase in 1954, 12 percentage points in 2012.

Despite the substantial effect of US involvement on the military balance, we see fairly muted indications of an overall moral hazard effect in a Taiwan Strait crisis. In both scenarios, US participation barely increases the Taiwanese side's optimal offer,[38] and only increases the probability of war by 2–3 percentage points. The small effect here reflects a few important facets of the strategic interaction. Because the increase in the Taiwanese side's power is common knowledge, the main effect is to move the bargaining range in favor of Taiwan rather than to increase the risk of war. Additionally, the prospect of splitting the spoils of victory with the US decreases the expected value of military competition for Taiwan, partly countering the effect of the greater chance of winning. Interestingly, we estimate that Soviet/Russian involvement on China's side would have a stronger effect on the chances of war. This operates through the uncertainty mechanism. The inclusion of a second actor vastly increases the uncertainty around the Chinese side's audience cost, increasing the Taiwanese side's incentive to make an offer that carries a risk of war rather than making large concessions to guarantee peace.

I Model Comparison

Before describing the models used in our comparison analysis, we note the connection between our structural model and a logistic regression on capability differences. Consider a bilateral dispute, with $I_A = I_B = 1$. Proposition 1 implies that both players exert positive effort in equilibrium and that $e_B^* = \frac{c_A}{c_B} e_A^*$. This in turn implies

$$p_A(e^*) = \frac{\frac{m_A}{c_A}}{\frac{m_A}{c_A} + \frac{m_B}{c_B}}.$$

[38]One notable potential point of departure between our model and theories of moral hazard is that we assume each side's objective in bargaining is to maximize the total payoff of the coalition. A possible direction for future work is to compare our structural model to one in which the originator of a dispute makes bargaining offers solely to maximize their own individual payoff, while accounting for the likely contributions of their coalition partners in case conflict occurs.

	Bilateral Only	Summed Capabilities	Max Capabilities	Equilibrium Model
GDP	0.166	0.140	0.202	0.019
	(0.211)	(0.164)	(0.162)	(0.134)
Iron and Steel	0.033	−0.004	0.007	0.016
	(0.071)	(0.057)	(0.058)	(0.058)
Energy Consumption	0.064	0.081	0.075	0.018
	(0.100)	(0.083)	(0.082)	(0.081)
Total Population	−0.014	0.116	0.058	0.102
	(0.195)	(0.152)	(0.152)	(0.143)
Urban Population	−0.098	−0.169	−0.171	−0.077
	(0.126)	(0.111)	(0.106)	(0.099)
Distance to Dispute	−0.032	0.007	0.007	−0.008
	(0.043)	(0.035)	(0.033)	(0.034)
Nuclear Weapons	0.443	0.149	−0.207	−0.378
	(0.560)	(0.302)	(0.411)	(0.372)
Import Percentage	0.037	0.032	−0.096	−0.095
	(0.253)	(0.214)	(0.236)	(0.194)
Democracy	−0.029	−0.009	−0.007	0.015
	(0.021)	(0.018)	(0.016)	(0.015)
No. Wars	160	250	250	250
LL: Bilateral Wars	−0.656	−0.663	−0.668	−0.694
LL: Coalition Wars	–	−0.651	−0.640	−0.611

Table A.8. Full results for the model comparison analysis.

Now, as in the structural model, let each $\frac{m_i}{c_i} = \exp(X_i^\top \beta)$. Then the probability of victory by A is given by a logistic regression model, where the covariates are the differences in the individual components of X_i:

$$p_A(e^*) = \frac{1}{1 + \exp\left(-\left[(X_A - X_B)^\top \beta\right]\right)}. \tag{10}$$

Therefore, with a sample of exclusively bilateral wars, we can estimate the determinants of the force multiplier with a simple logistic regression. The first column of Table A.8 reports the results of such a model, using the 160 bilateral wars in our dataset. The standard errors for the first three columns of Table A.8 are derived from the nominal logistic regression standard errors, using the multiple imputation formula from King et al. (2001).

In the second column of Table A.8, we report the model where each coalition's capabilities are treated as the sum of its member's capabilities. To estimate this model, we again use the logistic regression specification of Equation 10. However, we now sum each side's capabilities. Specifically, we define the k'th entry of X_A as $X_A k = \sum_{i \in A} X_i k$, and we do the same for X_B. Finally, in the third column of Table A.8, we define X_A using the capabilities of the A member with the highest CINC score, and we do the same for X_B.

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Table A.9 reports results from a model run using only the 1946–2010 subsample of disputes. The point estimate for nuclear weapons in the force multiplier is virtually identical to the estimate in the model from the full sample. Consequently, the negative estimate in the baseline analysis is not an artifact of including disputes from years when nuclear weapons did not exist.

Term	Estimate	Std. Err.	Conf. Int.
Force Multiplier m_i/c_i: Demographic			
Total Population	0.13	0.19	[−0.27, 0.48]
Urban Population	−0.07	0.14	[−0.33, 0.21]
Force Multiplier m_i/c_i: Economic			
Energy Consumption	0.04	0.13	[−0.25, 0.29]
GDP	−0.06	0.19	[−0.42, 0.32]
Import Percentage	−0.01	0.25	[−0.52, 0.40]
Iron and Steel Production	0.04	0.08	[−0.12, 0.19]
Force Multiplier m_i/c_i: Political			
Democracy	0.02	0.05	[−0.04, 0.06]
Force Multiplier m_i/c_i: Geopolitical			
Distance to Dispute	−0.01	0.06	[−0.07, 0.13]
Nuclear Weapons	−0.35	0.48	[−1.38, 0.46]
Audience Cost Mean μ_K			
Intercept	−2.08	8.98	[−3.98, 26.87]
Contiguity	−1.99	2.73	[−9.61, −3.05]
Democracy	−0.06	0.30	[−0.20, 0.79]
Interest Similarity	−2.72	1.81	[−6.25, −4.37]
Major Power	−3.87	3.57	[−8.32, −2.85]
Peace Years	−1.95	3.04	[−3.83, 6.24]
Audience Cost Scale σ_K			
Intercept	−0.19	0.79	[−1.98, 2.05]
Coalition Size	1.23	0.28	[0.81, 2.36]
Disputes	1498		
Participants	3487		
Log-likelihood	−596.18		

Table A.9. Structural estimation results for the model fit only to disputes from 1946 onward.

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