Social Conflict and the Predatory State*

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Abstract

Empirical scholarship identifies social cleavages as a predictor of slower economic growth, greater civil conflict, and other socioeconomic ills. Meanwhile, accounts of divide-and-rule politics claim that predatory rulers may benefit politically from exploiting internal schisms. Altogether, then, does internal fractionalization work to the benefit or detriment of a rent-seeking ruler? To answer this question, I model the political economy of predatory governance in a fractionalized society. For rentier states, whose revenues derive from control over natural resources or a similar exogenous source, internal divisions work to the ruler's benefit by depressing collective resistance. The opposite is true for a state financed by endogenous labor output, as the negative effect of social fractionalization on economic production outweighs the decrease in collective action against predatory rule. The analysis thus highlights a new channel by which rentier states are distinctive: they are the sole beneficiaries of divide-and-rule politics.

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Scholars have identified social cleavages—ethnic, religious, linguistic, and other divisions among a country's population—as the source of various political and economic maladies. Across a variety of measures, ethnic cleavages are correlated with slower rates of economic growth (Easterly and Levine 1997; Alesina et al. 2003; Posner 2004; Montalvo and Reynal-Querol 2005*a*). Most civil conflicts are organized on ethnic lines (Denny and Walter 2014) and there is some evidence that ethnic and religious diversity are correlated with civil conflict onset (Montalvo and Reynal-Querol 2005*b*; Esteban, Mayoral and Ray 2012).[1] Greater social divisions are also associated with lower public goods provision (Habyarimana et al. 2007) and worse property rights protection (Keefer and Knack 2002). The negative effects of social cleavages are most serious in environments with high levels of natural resource wealth (Hodler 2006; Baggio and Papyrakis 2010; Wegenast and Basedau 2014; Berman et al. 2017).

Despite the socioeconomic ills associated with a divided population, a strand of research in political science and political economy argues that autocratic rulers benefit from the presence of social cleavages. Theoretical models of divide-and-rule politics show kleptocratic rulers benefiting from internal divisions, as mutual jealousies among the populace increase the ruler's bargaining power (Acemoglu, Verdier and Robinson 2004; Padró i Miquel 2007). De Luca, Sekeris and Vargas (2018) claim that the relationship between resource wealth and civil conflict in weak states can be traced to weak states' financial incentives to stoke communal violence. In an analysis of Middle Eastern rentier states, Bellin (2004) argues that ethnic divide-and-rule tactics allow rulers to maintain patrimonial control over militaries, deterring insider threats (see also Paine 2021). Though these studies differ as to the precise mechanisms by which social cleavages are supposed to benefit the state, they are united by a focus on predatory rulers, whose primary objective is to profit from power (Levi 1989).

The divide-and-rule literature shows how a ruler facing a divided population may benefit from political tactics that dissuade collective action against government predation. Does this imply that predatory rulers are better off—i.e., that they can extract more revenue—

[1]Fearon and Laitin (2003) and Collier and Hoeffler (2004), using fractionalization measures with arguably weaker theoretical foundations (Esteban and Ray 2011), find no correlation.

when the extent of social cleavages in the underlying population is greater? Insofar as social divisions are a cause of economic sluggishness and communal violence, there is good reason for skepticism. Even predatory rulers benefit from development, as the long-run prospects for revenue extraction are greater when the state's economy is more productive (North 1981). In principle, then, social cleavages may work to the benefit or the detriment of the predatory state. When does one effect dominate the other?

To bring theoretical clarity to the relationship between social divisions and state revenues, this paper develops a game-theoretic model of the predatory governance of a divided society. In the model, a revenue-maximizing ruler sets a tax rate, and then competing factions within the society choose how to allocate their labor. The society is characterized by weak property rights protections, as each faction's holdings are subject to both "horizontal" appropriation by other factions and "vertical" appropriation by the state. The model incorporates a stark form of a tradeoff driving divide-and-rule political incentives: the more labor a faction devotes to competition or conflict with the other social factions, the less it has left over to resist government expropriation. However, internal competition may also detract from economically productive activity.

I find that state revenues increase with the extent of social cleavages *only* in rentier states, whose government is financed by a fixed source of wealth rather than the variable product of the population's labor (see Bellin 2004; Morrison 2009; Ross 2012; Colgan 2015). As social cleavages increase (operationalized in the model as the number of politically organized factions), so does the amount of labor devoted to internal competition among groups. In line with the traditional logic of collective action (Olson 1965), the increase in internal competition comes at the expense of collective resistance against predatory taxation, ultimately working to the benefit of the ruler. All else equal, a rentier ruler whose revenues derive from oil reserves, mineral wealth, or foreign aid benefits from the same underlying conditions that promote social conflict.

Some of the same logic applies for non-rentier states, which are funded by taxing the endogenous output of the population's labor rather than an exogenous source of wealth. The incentive for collective action still decreases with the extent of social cleavages, allowing the ruler to impose a greater tax rate without engendering popular resistance. However, this effect is now offset by a decrease in the state's revenue base. In an environment with weak property rights protection, productive activity is effectively a collective good (Skaperdas 1992; Grossman and Kim 1995), so the incentive to produce decreases with the extent of social cleavages just as the incentive to resist the state does. Altogether, as social cleavages increase, the ruler extracts a larger share of a smaller pie. The net effect on predatory revenues is negative, meaning non-rentier rulers are overall better off the *less* divided the population is.

Two key features of the strategic environment drive the finding that social cleavages cause a net reduction in non-rentier rulers' revenues. First, the incentive to engage in collective resistance depends on the state's total revenue, not simply the tax rate. Second, while internal divisions reduce the incentive to divert labor from internal competition to collective resistance, the same is not true for diverting labor from productive activity to resistance. Therefore, the increased equilibrium tax rate is not high enough to make up for the productive losses due to higher social cleavages.

Broadly speaking, the analysis highlights that the distinctiveness of rentier state governance (see Bellin 2004; Morrison 2009) extends to their management of divided societies. States whose revenues come from exogenous resource stocks benefit from heightening the salience of internal social distinctions, whereas predatory states funded by labor taxation do not. While earlier theories have focused on the role of social divisions in military organization and coup-proofing (Bellin 2004), my model demonstrates that cleavages also benefit rentier states by deterring mass action against the regime. The analysis also yields novel implications about predatory states' incentives to protect property rights, a key concern for theories of state predation (North 1981; Tornell and Lane 1999; Bates, Greif and Singh 2002). In divided societies, these incentives are critically dependent on the nature of the state's revenue base: labor taxation or resource wealth. I discuss these empirical implications further after presenting the main model analysis.

Following the discussion of empirical implications, I extend the baseline model to address additional substantive questions and confirm the robustness of the main results. Two key extensions are worth noting here. The first concerns whether a non-rentier government has an incentive to stoke conflict by creating inequality between groups through unequal taxation, as in Acemoglu, Verdier and Robinson (2004). Inequality increases the incentives for unproductive rent-seeking behavior (Hirshleifer 1991), which ultimately comes at the expense of the revenue base and is thus unprofitable for the ruler. The second extension models the process of initially gaining control over the society, prior to taxation and predatory governance. Recovering the divide-and-conquer logic commonly associated with Cortés's conquest of the Aztec empire, I find that social cleavages increase the likelihood of successful outside conquest even when they would reduce state revenues afterward.

1 Related Literature

My theory draws from the literatures on social conflict, predatory governance, and natural resource rents. Here I review the existing points of intersection between these literatures and outline how my work innovates on them.

Predatory governance and social cleavages. Theories of predatory governance in divided societies emphasize how rulers may politically benefit from exploiting rivalries among the population. Acemoglu, Verdier and Robinson (2004) identify divide-and-rule politics as an explanation for the persistence of kleptocratic rule. In their model, a kleptocrat facing a fractured opposition can credibly threaten to divert resources to the rival group of any potential challenger, thereby deterring a challenge in the first place. Padró i Miquel (2007) similarly finds that rulers gain bargaining leverage from the fear of political succession turning over to another group. Introducing an informational mechanism, Debs (2007) finds that it is in a ruler's interest to manipulate the media so as to polarize social preferences over policy.

Weakly institutionalized states are associated with poor protection of claims to property (Besley and Persson 2009), and property rights protection is a central concern in general theories of state predation (North 1981; Olson 1993; Moselle and Polak 2001; North, Wallis and Weingast 2009). Despite this, the aforementioned models of divide-and-rule politics neglect the possibility of predation and rent-seeking behavior *among* the factions within a divided society. In these models, the ruler may appropriate from the groups within society, but the groups cannot attempt to appropriate from their rivals. But it is important to account for this possibility, as this kind of horizontal rent-seeking behavior may undercut state revenues by reducing productive activity. I bridge the gap in the literature by allowing for this type of horizontal conflict in my model, drawing from the formal literature on social conflict in environments with weak property rights protection (e.g., Skaperdas 1992; Grossman and Kim 1995). This allows me to examine the net effect of social cleavages on predatory state revenues, balancing the political advantages against the productivity drawbacks.

The structure of my model also parallels recent work by Tyson (2020). He describes two security problems that the state must solve: the horizontal problem, whereby citizens might appropriate from each other, and the vertical problem, whereby the state might appropriate from the citizenry. These security problems also form the core of the political economy that I study. But whereas Tyson focuses on the conditions that allow for a social contract to form between the ruler and the ruled, I take the predatory nature of the government as given and analyze how its choices and payoffs are affected by underlying political and economic variables.

Predatory governance and resource rents. A substantial literature argues for the political distinctiveness of rentier states—those whose revenues derive primarily from resource rents and similar fixed streams, rather than taxing their population's productive output—

even when compared to other autocratic regimes. Focusing on the Middle East, Bellin (2004) argues that rentier states can sustain robust coercive mechanisms due to their easy access to revenues (see also Ross 2012). Morrison (2009) argues that rentier states are less subject to conflict between elites and the masses, as their governments can pay for social services without raising taxes on the wealthy. Rentier income has also been used to explain variation in civil war incidence and outcomes. Colgan (2015) finds that civil conflict is relatively unlikely to result in democratization in petrostates, while Paine (2016) shows that states with high oil revenues have an advantage in deterring center-seeking rebellions.

I identify a novel way in which rentier states are politically distinctive—their ability to benefit from social cleavages and the application of divide-and-rule politics. In the model, the relationship between predatory extraction and the degree of internal cleavage turns out to critically depend on whether state revenues come from labor output or from fixed resources. States dependent on labor taxation suffer from the productivity losses associated with greater tensions between social groups, while rentier states do not. The analysis highlights how social cleavages may reduce mass collective threats to rentier rulers, a mechanism that differs from (but may complement) the manipulation of cleavages to guard against insider threats identified by Bellin (2004). The model also yields empirical implications about the intensity of rebellion in rentier versus non-rentier states (Colgan 2015) and on the effects of resource lootability (Snyder and Bhavnani 2005; Thies 2010), which I discuss further after presenting the results.

Social cleavages and resource rents. A strand of political economy scholarship examines how resource wealth affects the likelihood of social conflict in divided societies. Using a model similar to the one presented in this paper, Hodler (2006) identifies resource wealth as a cause of social conflict in divided societies with weak property rights protection. Resource wealth reduces the incentive to engage in productive activity, which in turn frees up labor for unproductive rent-seeking or appropriation directed at other social groups. To analyze how these dynamics interact with predatory state revenues, I extend Hodler's modeling framework to include a revenue-maximizing ruler and the possibility of collective resistance against state predation. I show that the "curse" of fractionalization afflicts state and citizens alike when state revenue is drawn from labor output, but that rentier states benefit from this dynamic. The results of the model also comport with Baggio and Papyrakis's (2010) finding of less effective property rights protection in resource-dependent states with high social cleavages.

In a recent analysis whose theoretical concerns mirror this paper's, De Luca, Sekeris and Vargas (2018) argue that rulers in resource-rich societies may benefit from internal conflict. Their model also extends the Hodler framework to include a predatory ruler, but not collective resistance against extractive taxation. By including this possibility, my analysis provides a stronger basis for the distinctiveness of resource wealth: even though social cleavages reduce collective action against the ruler in all cases, only rentier rulers end up benefiting from this dynamic in terms of net revenue.

2 The Model

I model the political economy of predatory governance in a divided society. The players are the government, denoted G, and a set of $N \geq 2$ identical factions within the society, denoted $\mathcal{N} = \{1, \ldots, N\}$. Let $i \in \mathcal{N}$ denote a generic faction.

The interaction proceeds in two stages. First, the government chooses a tax rate, $t \in [0, 1]$.[2] Second, after observing the tax rate, each faction simultaneously allocates its labor among activities that affect the level and distribution of economic output. These are production, denoted p_i; resistance against the government, r_i; and internal competition (or conflict), c_i. Each faction has finite labor available, facing the budget constraint

$$\frac{p_i}{\pi^p} + \frac{r_i}{\pi^r} + \frac{c_i}{\pi^c} = \frac{L}{N}, \tag{1}$$

[2]The tax rate is the same for all factions. I relax this assumption in an extension.

where $L > 0$ denotes the total size of the population and each $\pi^p, \pi^r, \pi^c > 0$ denotes the society's productivity in the respective activity.[3] For example, the greater π^r is, the less labor is required to produce the same amount of resistance; total resistance cannot exceed $\pi^r L$. After these choices are made, the game ends and each player receives her payoffs. Let $p = (p_1, \ldots, p_N)$ denote the vector of production choices, and define the vectors r and c similarly.

The goal of each player, including the government, is to maximize their own consumption. The nature of this consumption depends on the state's revenue base. If the state is *capital-financed*, there is a fixed resource of value $X > 0$ that the players compete over.[4] To simplify the presentation of this case, I assume production by the factions is payoff-irrelevant when the state is capital-financed, which implies $p = 0$ in equilibrium. In formal terms, the production function is $f(p) = X$, a constant. On the other hand, a *labor-financed* state depends solely on the productive output of the population. In this case, I assume a linear production function,

$$f(p) = \sum_{i=1}^{N} p_i, \tag{2}$$

where $p = (p_1, \ldots, p_N)$ denotes the vector of each faction's production choice.[5] Whether the state is capital- or labor-financed, each player's utility is ultimately a fraction of $f(p)$. The other choice variables—the tax rate, resistance, and internal conflict—determine what these fractions are.

Resistance, the second way the factions can expend their labor, determines how much the government can actually collect in taxes. Given the nominal tax rate t and the vector

[3]I write each faction's constraint as a fraction of L to take comparative statics on the number of factions while holding fixed the total size of the society. In the Appendix, I derive equilibrium existence and uniqueness results for the more general case in which factions may differ in their size and productivities.

[4]Different actors might value the same resources differently. For example, control of oil fields might be more valuable in an absolute sense to the government than to rebel groups (Le Billon 2013, 29–30). The results of the analysis would not change if each actor valued the pie at a potentially different level $X_i > 0$, as this would simply entail the multiplication of each actor's utility function by a positive constant.

[5]The linear production technology rules out complementarity between different factions' economic activities (see Silve 2018). This is a reasonable assumption for historically common forms of labor extraction by predatory states, such as staple crop production and mining of raw materials or precious metals, but would be less applicable to more developed economies.

of resistance allocations r, let $\tau(t, r)$ denote the proportion of economic output that goes to the government, or the effective tax rate, and let $\bar{\tau} = 1 - \tau$ denote the share that remains to be divided among the factions. Resistance determines what proportion of the nominal tax rate the government can collect:

$$\tau(t, r) = t \times g\left(\sum_{i=1}^{N} r_i\right), \tag{3}$$

where $g : [0, \pi^r L] \to [0, 1]$ is a strictly decreasing function. Given total resistance $R = \sum_{i=1}^{N} r_i$, the function $g(R)$ may represent either the proportion of t that the government can collect or the probability that it collects t as opposed to nothing. To ensure the existence of an equilibrium and ease its characterization, I assume g is twice continuously differentiable, convex, and log-concave.[6] Convexity implies that resistance has diminishing returns, so the factions face a collective action problem: the more each faction expects the others to contribute to resistance, the less it prefers to contribute itself. To simplify the exposition, I assume the government fully collects the announced tax rate if there is no resistance: $g(0) = 1$. The linear function $g(R) = 1 - \frac{R}{\pi^r L}$ is one example of the many functional forms that meet these conditions.

Internal competition, the third and final outlet for the factions' labor, determines the share each group receives of what is left over after the government takes its cut. I model the internal distribution of consumption as a contest, in which each faction expends costly effort to increase its share of the pie. Given the allocations $c = (c_1, \ldots, c_N)$ to the internal competition, a faction's share of post-tax output is given by the contest success function

$$\omega_i(c) = \frac{\phi(c_i)}{\sum_{j=1}^{N} \phi(c_j)}, \tag{4}$$

where $\phi : [0, \pi^c L/N] \to \mathbb{R}_+$ is strictly increasing. Factions that devote more effort to internal competition end up with larger shares of the output. If every faction spends the same

[6]Given the first condition, the latter two are equivalent to $0 \leq g''(R) \leq g'(R)^2/g(R)$ for all $R \in [0, \pi^r L]$.

amount, or they all spend nothing, they all end up with equal shares, $\omega_i(c) = 1/N$.[7] Again, as regularity conditions to ensure equilibrium existence, I assume ϕ is twice continuously differentiable and log-concave. Both of the most popular contest success functions satisfy these criteria: the ratio form, with $\phi(c_i) = c_i$, and the difference form, with $\phi(c_i) = e^{c_i}$ (Hirshleifer 1989).

Each faction's utility is simply the amount of economic output it receives. This is a function of how much is produced, how much the government extracts through taxation, and the faction's standing in the internal contest. Together, these yield faction i's utility function,

$$u_i(t, p, r, c) = \omega_i(c) \times \bar{\tau}(t, r) \times f(p). \tag{5}$$

The multiplicative payoff structure is similar to that of Hirshleifer (1991) and Skaperdas (1992). It is a strategic environment with weak protection of property rights, in which possession is determined through appropriation (Skaperdas 1992).[8]

The government in the model is predatory insofar as its motivation is to increase revenues for its own consumption (Levi 1989). Its utility is how much of the economic output it receives, accounting for reductions in the effective tax rate due to resistance:

$$u_G(t, p, r, c) = \tau(t, r) \times f(p). \tag{6}$$

The government does not use tax revenues to provide public goods or redistribute wealth within society. In addition, the government does not have preferences over the distribution of post-tax consumption among the factions. Internal competition does not directly enter the state's utility, though it matters indirectly insofar as it reduces production or resistance.

[7] If $\phi(0) = 0$, in which case (4) is not well-defined at $c = 0$, let each $\omega_i(0) = 1/N$.

[8] For a model of conflict with (endogenously) partial property rights, see Grossman and Kim (1995).

2.1 Notes on the Model Setup

The model sets up stark strategic tradeoffs for both the factions and the government. Each faction must choose between increasing the total size of the post-tax pie, namely through production or resistance, and securing its own share of that pie through internal competition. They face a collective action problem, as production and resistance are collectively beneficial while one's share of the internal contest only has private benefits. For the government, the key tradeoff comes in how it calibrates its extractive demand. Holding fixed the behavior of the factions, the government would always prefer a higher tax rate. But a high rate is counterproductive if it diverts social effort away from production and into resistance.

By separating resistance and internal competition into separate choices, the model assumes that effort spent resisting government predation does not help a faction in resource competition with other groups, and vice versa. In reality, some activities, such as building fortifications, may serve both purposes. The current setup, with a stark separation between competition among factions and resistance against the government, is the environment in which the ruler is least likely to benefit from social fractionalization. To see why, imagine that labor devoted to internal competition had a positive spillover for resistance, causing a reduction in the effective tax rate. Then any exogenous shock that increased incentives for internal strife—such as an increase in fractionalization—would directly reduce the amount the ruler can extract. Allowing for this type of spillover would strengthen my results for labor-financed governments by introducing an additional channel through which fractionalization reduces predatory revenues. By the same token, however, it would weaken the results for capital-financed governments. With strong enough spillovers, such a state might find additional social divisions to be a detriment rather than a benefit.

It may seem natural to interpret the internal competition in the model as the eruption of violent conflict between factions. However, that interpretation raises the question of why the factions do not simply determine the distribution of post-tax surplus through peaceful negotiation (Fearon 1995). For the model, what matters is not whether violence occurs, but the extent to which competition between factions results in them spending labor that neither increases the level of economic output nor contributes to collective resistance against predatory rule. The mere threat of violence might result in this kind of labor allocation, even when that threat remains latent, as groups would need to spend time and resources acquiring the coercive capability to maintain their leverage in intergroup negotiations (Coe 2011; Fearon 2018).

3 Rentier State Revenues

I begin the model analysis by focusing on capital-financed governments, including contemporary rentier states, in which the object of both predatory taxation and internal competition is to control a resource of fixed value. In the absence of concerns over labor productivity, the government's equilibrium revenue increases as the factions have greater incentives to engage in internal competition. I use this baseline case to identify important comparative statics on the structural determinants of internal strife. Social fractionalization, here conceived of as the number of distinct factions, raises the level of internal competition and thereby works to the benefit of a rentier state. Efficiency improvements in the technology of conflict between factions have countervailing effects, however.

The clearest example of a capital-financed government is a rentier state funded by oil wealth. Oil exploitation is capital-intensive, often takes place offshore, and can be conducted by workers imported from abroad (Le Billon 2013, 28–30). In the absence of mass resistance, a government can extract value from oil deposits with minimal regard for local labor contributions. Similarly, an empire that seeks land for settlement—rather than for agricultural or mineral exploitation by a local labor force—is capital-financed in this sense. For example, the English colonial empire in North America, which sought sparsely populated territories and fought to expel American Indians where they settled, was capital-financed by this definition (Elliott 2007, 36–38). By contrast, the Spanish colonial empire, with its notorious *encomienda* system of native labor exploitation, was labor-financed.

3.1 Equilibrium

I solve the model via backward induction, first identifying the factions' equilibrium labor allocations given the government's choice of tax rate. When the state is capital-financed, the factions essentially face a tradeoff between two dimensions of competition, both over the fixed prize X. There is a vertical dimension, consisting of competition between the government and the factions collectively over how much the government is to expropriate. Resistance determines whether, or to what extent, this competition is resolved in favor of the governed society rather than the government. The other dimension of competition is horizontal, consisting of the competition among factions over their division of whatever the state fails to expropriate. By diverting labor from collective resistance to competition against other factions, an individual faction reduces overall post-tax output (by allowing the government to collect a greater fraction of t), but increases its own share of what is left.

Regardless of the government's choice of tax rate, all factions spend the same amount on internal competition in equilibrium. This is a consequence of the symmetry in the factions' productivity and budget constraints.[9] To see formally why this holds, imagine a strategy profile in which $c_i > c_j$ for some pair of factions. The log-concavity of the contest function then implies $\frac{\partial u_i}{\partial c_i} \leq \frac{\partial u_j}{\partial c_j}$. Meanwhile, because i's equilibrium share of consumption is greater than j's, we have $\frac{\partial u_i}{\partial r_i} > \frac{\partial u_j}{\partial r_j}$. But then if i's allocation satisfies the first-order condition for optimality, j's does not:

$$\pi^c \frac{\partial u_j}{\partial c_j} \geq \pi^c \frac{\partial u_i}{\partial c_i} \geq \pi^r \frac{\partial u_i}{\partial r_i} > \pi^r \frac{\partial u_j}{\partial r_j},$$

which implies that it would be profitable for j to divert some of its resistance labor into internal competition instead. Therefore, no strategy profile with unequal internal competition can be an equilibrium.

As the government's tax demand increases, the vertical dimension of competition weighs

[9] See Lemma A.7 in the Appendix.

more heavily in each individual faction's decision calculus. At low levels of taxation, the return to resistance is low, and the factions devote all labor to internal competition. Formally, this is an equilibrium as long as

$$\pi^r \frac{\partial u_i}{\partial r_i} \leq \pi^c \frac{\partial u_i}{\partial c_i} \quad \Leftrightarrow \quad \overbrace{\frac{t}{1-t}}^{\text{increases with } t} \times \underbrace{(-\pi^r g'(0))}_{>0} \leq \pi^c \times \frac{N-1}{N} \times \frac{\phi'(\frac{\pi^c L}{N})}{\phi(\frac{\pi^c L}{N})}.$$

This expression shows that there is a cutpoint $\hat{t}_0^X < 1$ such that resistance occurs if and only if $t > \hat{t}_0^X$. Just above this threshold, the equilibrium among the factions involves a mixture of internal competition and resistance. The level of each is determined by the condition

$$\pi^r \frac{\partial u_i}{\partial r_i} = \pi^c \frac{\partial u_i}{\partial c_i} \quad \Leftrightarrow \quad \pi^r \times \underbrace{\frac{t(-g'(R))}{1-tg(R)}}_{\text{decreases with } R} = \pi^c \times \frac{N-1}{N} \times \underbrace{\frac{\phi'(C)}{\phi(C)}}_{\text{decreases with } C}, \tag{7}$$

where $C = \frac{\pi^c}{N}(L - \frac{R}{\pi^r})$ per the budget constraint. Because the marginal benefit of resistance increases with t, higher tax rates are associated with greater resistance and lower internal competition within this range. Depending on the specific parameters, there may come a point at which even low levels of internal competition are strategically unsustainable. In particular, this is true if the marginal benefit of resistance exceeds that of internal competition when the tax demand is $t = 1$ and the factions devote all labor to resistance:

$$\pi^r \frac{\partial u_i}{\partial r_i} \geq \pi^c \frac{\partial u_i}{\partial c_i} \quad \Leftrightarrow \quad \pi^r \times \frac{-g'(\pi^r L)}{1-g(\pi^r L)} \geq \pi^c \times \frac{N-1}{N} \times \frac{\phi'(0)}{\phi(0)}.$$

In this case, there is an additional cutpoint $\hat{t}_1^X \leq 1$ such that internal competition occurs if and only if $t < \hat{t}_1^X$. The following proposition summarizes the equilibrium, and Figure 1 illustrates.[10]

Proposition 1. *If the government is capital-financed, every labor allocation subgame has a*

[10] All proofs are in the Appendix.

unique equilibrium. There exists a tax rate $\hat{t}_0^X \in (0,1)$ *such that each* $r_i = 0$ *in equilibrium if and only if* $t \leq \hat{t}_0^X$*. There exists* $\hat{t}_1^X > \hat{t}_0^X$ *such that each* $c_i = 0$ *in equilibrium if and only if* $t \geq \hat{t}_1^X$*. For* $t \in (\hat{t}_0^X, \hat{t}_1^X)$*, in equilibrium each* $r_i = \tilde{R}_X(t)/N > 0$ *(strictly increasing in t) and each* $c_i = \tilde{c}_X(t) > 0$ *(strictly decreasing).*

The model thus recovers a well-known dynamic in extraction by autocratic governments: a high tax rate galvanizes the population, giving the factions an incentive to act in concert to reduce government expropriation. In other words, predatory state behavior endogenously reduces the structural impediments to collective action within a divided society. As an example of this dynamic, excessive extraction by colonial empires led to unified resistance both among the American colonies during the Stamp Act crisis of 1765 and between creoles and Indians during a contemporaneous tax revolt in Quito (Elliott 2007, 310–314).

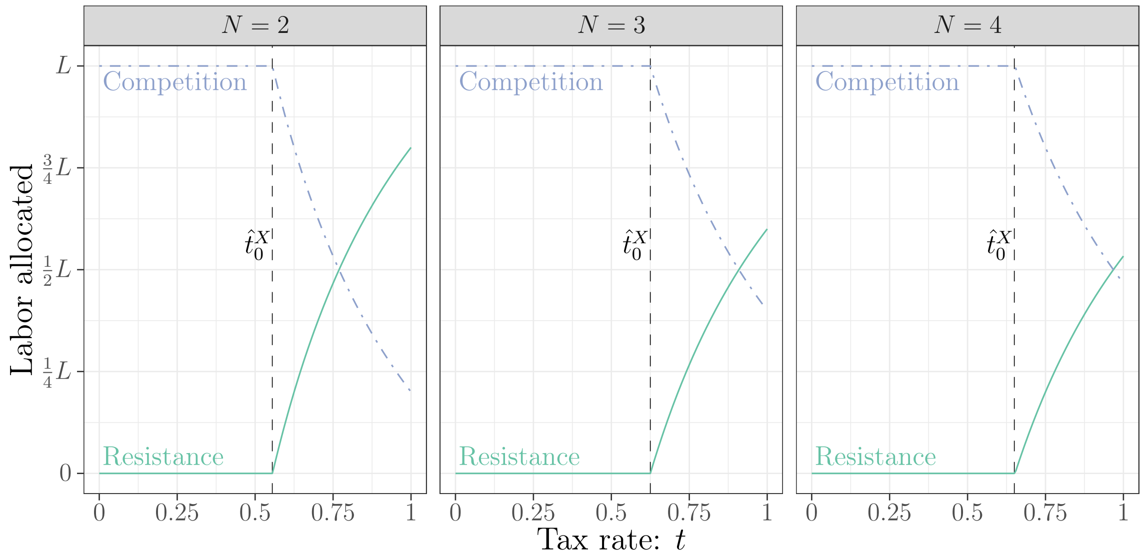
Moving up the game tree, the government selects a tax rate in anticipation of the factions' responses. Given the factions' equilibrium response to each potential tax rate, the government's induced utility function is

$$Eu_G(t) = \overbrace{t}^{\text{increasing}} \times \underbrace{g(R^*(t))}_{\text{decreasing}} \times \overbrace{X}^{\text{constant}}.$$

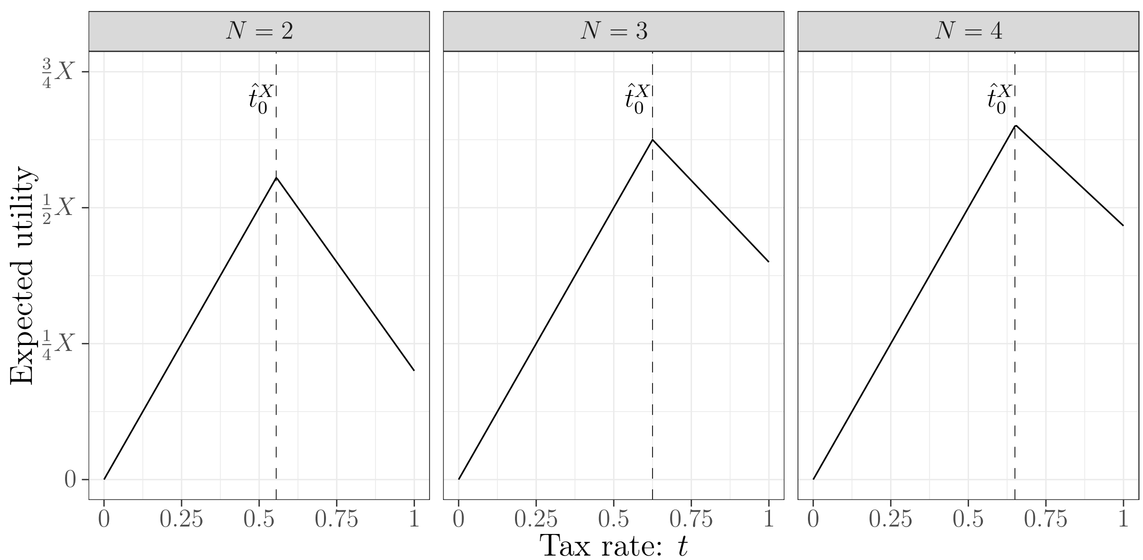
where R^* is total resistance in the corresponding equilibrium. Here we see countervailing effects of the tax rate on the government's revenues. All else equal, greater t means greater revenues. But all else is not equal: equilibrium resistance increases with the tax rate, partially or even fully offsetting whatever gains the government might reap. While there is no general expression for a capital-financed government's optimal choice of tax rate, some results are evident.[11] First, because total resistance is continuous as a function of t, an optimal rate exists.[12] Second, no $t < \hat{t}_0^X$ is an equilibrium, as the government could raise the rate to the cutpoint without provoking resistance. Similarly, no $t \in (\hat{t}_1^X, 1)$ may be an equilibrium

[11]In Figure 1 the equilibrium tax rate is always $\hat{t}_0^X$, but this is an artifact of the particular functional forms used.

[12]See Lemma A.6 in the Appendix.



(a) Equilibrium responses by the factions.



(b) Government utility given equilibrium faction responses.

Figure 1. Labor allocation with a capital-financed government as a function of the tax rate and the number of factions (Proposition 1). For each case plotted here, $\hat{t}_X^1 > 1$. This and subsequent figures use the parameters $\pi^p = \pi^r = \pi^c = 1$ and $L = X = 2.5$ and the functional forms $g(R) = 1 - \frac{R}{\pi^r L}$ and $\phi(c_i) = \exp(c_i)$.

either, as resistance is at its maximum and thus cannot further increase with taxes in this range.

3.2 Comparative Statics

I now consider the structural determinants of social conflict and their effects on a capital-financed government's equilibrium revenue. The first is the number of factions, N, which I take to represent the extent of social fractionalization. I find that greater fractionalization results in a greater portion of the factions' labor spent on internal competition. In the capital-financed case, this must come at the expense of collective resistance, thereby benefiting the government. To see why resistance decreases with fractionalization, remember that in equilibrium, each faction receives $1/N$ of post-tax wealth, and thus only internalizes $1/N$ of its efforts toward collective resistance. As this fraction grows smaller, so does the individual incentive to engage in collective resistance. Therefore, by the classic logic of group size and collective action (Olson 1965), high fractionalization discourages social mobilization against predatory rule. Because a capital-financed predatory state is indifferent to internal competition except insofar as it affects resistance, this in turn means that such states gain from governing fractionalized populations. Heightened internal tension works to the benefit of the rentier state.

Proposition 2. *A capital-financed government's equilibrium payoff is increasing in the number of factions, N.*

To see formally why this result holds, observe that the level of resistance in an equilibrium with nonzero internal competition is determined by the first-order condition in Equation 7 above. An exogenous increase in N essentially increases the relative marginal utility of internal competition (the right-hand side of the condition) and thus must result in lower total resistance. To maintain the equality, C must increase and R must decrease.

The effects of improvements in the technology of internal competition, π^c, on the equilibrium level of resistance—and thus on government revenues—are more complicated. An increase in π^c has two countervailing effects. The first is a *labor-saving effect*: greater π^c means that a faction can achieve the same result in the internal competition with less labor devoted to it. This promotes resistance by allowing each faction to maintain its current position in the conflict at a strictly lower labor cost. The second is an *incentive effect*: effectively, the marginal utility of internal competition increases with π^c. This increases the incentives for fierce competition in the contest among factions, discouraging resistance.

Neither the labor-saving effect nor the incentive effect is guaranteed to dominate. Their relative influence depends on the equilibrium level of internal competition and the functional form of the contest success function. To see why, remember that $r_i = \pi^r(\frac{L}{N} - \frac{c_i}{\pi^c})$, so the effect of a marginal change in competitive effectiveness on equilibrium resistance is

$$\frac{dr_i^*}{d\pi^c} = \frac{\pi^r}{\pi^c}\left[\overbrace{\frac{c_i^*}{\pi^c}}^{\text{labor-saving effect}} - \underbrace{\frac{dc_i^*}{d\pi^c}}_{\text{incentive effect}}\right].$$

If there is minimal competition in equilibrium ($c_i^* \approx 0$), then the incentive effect dominates, and an increase in π^c results in less resistance. More generally, the incentive effect dominates if

$$\frac{d \log \phi(c_i^*)}{dc_i^*} + c_i^* \frac{d^2 \log \phi(c_i^*)}{d(c_i^*)^2} > 0, \tag{8}$$

which may hold at some tax rates but not others.[13]

Proposition 3. *If there is a unique equilibrium tax rate t^*, the government's equilibrium payoff is locally increasing in competitive effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect (i.e., Equation 8 holds) at the corresponding equilibrium level of internal competition.*

[13] For the derivation of this expression, see Lemma A.9 in the Appendix. Additionally, in Lemma A.10 in the Appendix, I verify that the incentive effect always outweighs the labor-saving effect for the "difference" contest success function ($\phi(C) = \theta e^{\lambda C}$) and that the effects are exactly offsetting for the "ratio" contest success function ($\phi(C) = \theta C^{\lambda}$).

This result implies that a rentier state might choose not to protect factions' property rights claims against each other, even if it were costless to do so. In fact, if the initial level of social conflict were low enough, the state might try to increase it—e.g., by dismantling extant justice-providing institutions, or by arming the population—if the cost were low enough.

These results for capital-financed governments rely on the assumption that internal competition has no *direct* externality on the state's utility. Conflict among factions only matters to the government insofar as it affects resistance. If instead there were negative externalities from internal conflict, such as if it leads to the destructiveness of open violence, then fractionalization and conflict effectiveness would have countervailing effects on the state's equilibrium utility. For example, if civil conflict makes it harder for an occupying power to maintain control over a territory in a resource-rich state, then the occupier might benefit from unity rather than fractionalization.

4 Revenues from Labor Extraction

I now examine labor-financed states, in which the payoffs of all players depend on endogenous production choices by the factions. The dependence on endogenous production introduces new strategic considerations. As with a rentier state, greater internal conflict may come at the cost of collective resistance, thereby increasing the state's ability to expropriate. But competition among factions may now also detract from production, having the opposite effect on state revenues. In fact, in equilibrium both effects are evident. A greater baseline level of internal competition—meaning the amount that would occur in the absence of state predation—is associated with lower production, but allows the government to impose a greater tax rate without provoking resistance. Despite these countervailing effects, I find a sharp converse of the comparative statics from the previous case: structural conditions favoring social order work to the *benefit* of a labor-financed predatory state.

In order to focus on the most substantively relevant applications of the model, I impose the following assumption throughout this section:

Assumption 1. $\frac{N-1}{N} > \frac{\phi(0)}{\phi'(0)} \cdot \frac{1}{\pi^c L}$.

If Assumption 1 does not hold, then for every tax rate $t \in [0,1]$, the equilibrium of the subsequent labor allocation subgame entails each faction spending nothing on internal competition. This case is of relatively little substantive interest for the analysis of the interplay between government predation and social order.

4.1 Equilibrium

The factions' equilibrium responses to each possible tax rate are structurally similar to the capital-financed case (Proposition 1). For the same reasons as before, every faction spends an equal amount on internal competition, resulting in each $\omega_i(c) = \frac{1}{N}$ regardless of the tax rate. The incentive to resist again increases with t, and the additional resistance comes at the expense of internal competition. The key difference is that now the factions always spend nonzero effort on economic production as well.

Equilibrium production is greatest when the tax demand is at its lowest. In the absence of state predation, with $t = 0$, the factions divide their labor between production and internal competition. Total production $\bar{P}_0$ is the unique solution to the first-order condition

$$\pi^p \frac{\partial u_i}{\partial p_i} = \pi^c \frac{\partial u_i}{\partial c_i} \quad \Leftrightarrow \quad \bar{P}_0 = \pi^p \times \frac{N}{N-1} \times \underbrace{\frac{\phi(\frac{\pi^c}{N}(L - \frac{\bar{P}_0}{\pi^p}))}{\phi'(\frac{\pi^c}{N}(L - \frac{\bar{P}_0}{\pi^p}))}}_{\text{decreases with } \bar{P}_0}.$$

This allocation remains sustainable for small tax rates, as long as the marginal benefit of resistance continues to be lower than that of production. The formal condition for this to hold is

$$\pi^r \frac{\partial u_i}{\partial r_i} \leq \pi^p \frac{\partial u_i}{\partial p_i} \quad \Leftrightarrow \quad \overbrace{\frac{t}{1-t}}^{\text{increases with } t} \times \underbrace{(-\pi^r g'(0))}_{>0} \leq \frac{\pi^p}{\bar{P}_0}.$$

Similar to the capital-financed case, this expression implies the existence of a cutpoint $\hat{t}_0$ such that resistance occurs if and only if $t \leq \hat{t}_0$. Just above this cutpoint, the equilibrium labor allocation involves positive levels of production, resistance, and internal competition. The level of each is uniquely pinned down by the requirement that $\pi^p \frac{\partial u_i}{\partial p_i} = \pi^r \frac{\partial u_i}{\partial r_i} = \pi^c \frac{\partial u_i}{\partial c_i}$.[14]

As t increases beyond the cutpoint, equilibrium resistance increases and internal competition declines for the same reasons as in the capital-financed case. Production also goes down (or at least does not increase), as the marginal benefit of production decreases with t. Finally, if the tax demand is high enough, the factions may cease all internal competition. To characterize whether this occurs, let $\bar{P}_1$ and $\bar{R}_1$ represent equilibrium resistance and production at $t = 1$ if the factions were restricted to $c_i = 0$.[15] There is then zero internal competition at the highest tax rate if

$$\pi^r \frac{\partial u_i}{\partial r_i} \geq \pi^c \frac{\partial u_i}{\partial c_i} \quad \Leftrightarrow \quad \pi^r \times \frac{-g'(\bar{R}_1)}{1 - g(\bar{R}_1)} \geq \pi^c \times \frac{N-1}{N} \times \frac{\phi'(0)}{\phi(0)}.$$

If this condition holds, then there is a cutpoint $\hat{t}_1 \leq 1$ such that internal competition occurs if and only if $t < \hat{t}_1$. Proposition 4 summarizes the form of the labor allocation equilibrium, and Figure 2 illustrates.

Proposition 4. *Assume the government is labor-financed. There exist tax rates $\hat{t}_0 \in (0, 1)$ and $\hat{t}_1 > \hat{t}_0$ such that in every equilibrium of the labor allocation subgame with tax rate t:*

- *If $t \leq \hat{t}_0$, then each $p_i = \bar{P}_0/N > 0$, each $r_i = 0$, and each $c_i = \bar{c}_0 > 0$.*
- *If $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = \tilde{P}_1(t) > 0$ (weakly decreasing in t), $\sum_i r_i = \tilde{R}_1(t) > 0$ (strictly increasing), and each $c_i = \tilde{c}_1(t) > 0$ (strictly decreasing).*
- *If $t \geq \hat{t}_1$, then $\sum_i p_i = \tilde{P}_2(t) > 0$ (strictly decreasing in t), $\sum_i r_i = \tilde{R}_2(t) > 0$ (strictly*

[14] Although *total* production and resistance are uniquely pinned down, the individual allocations to them are not. However, this multiplicity does not affect the analysis of the government's choice of tax rate, as total production and resistance are the only payoff-relevant quantities.

[15] Specifically, these will be the unique solutions to $\pi^p \frac{\partial u_i}{\partial p_i} = \pi^r \frac{\partial u_i}{\partial r_i}$ subject to the constraint $R = \pi^r (L - \frac{P}{\pi^p})$.

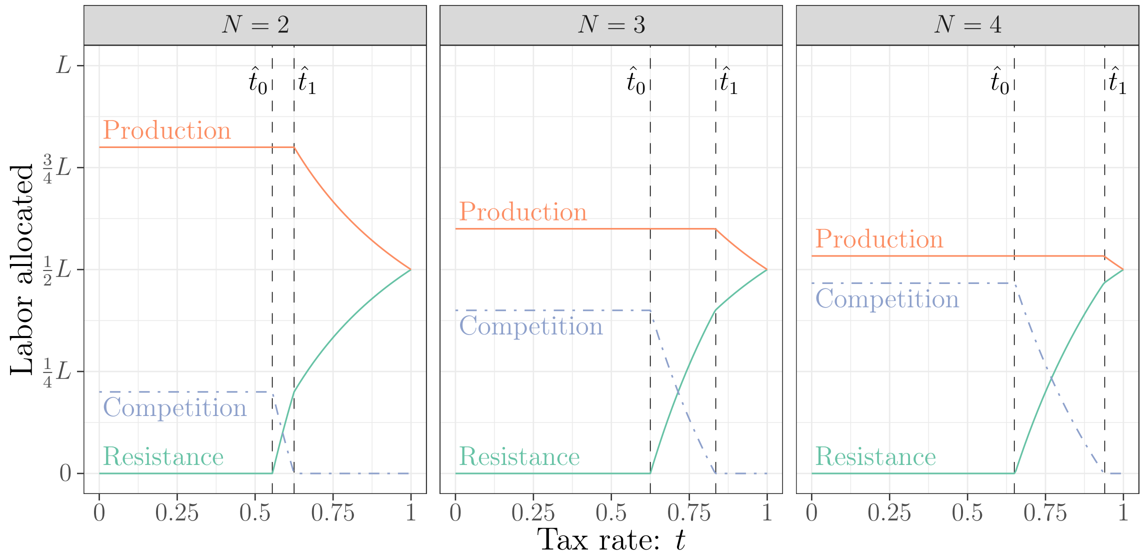
increasing), and each $c_i = 0$.

Even when $t = 0$, the outcome is social welfare inefficient *and* Pareto inefficient from the perspective of the factions. Every faction would receive the same share, $1/N$, of a larger pie if they devoted all their labor to production. A kind of prisoner's dilemma logic explains why this Pareto efficient allocation of labor is not sustainable: if no faction planned to spend on the internal conflict, then any single faction could obtain a large share by spending relatively little. Under Assumption 1, the temptation is large enough that every faction has an incentive to deviate from a strategy profile without competition.

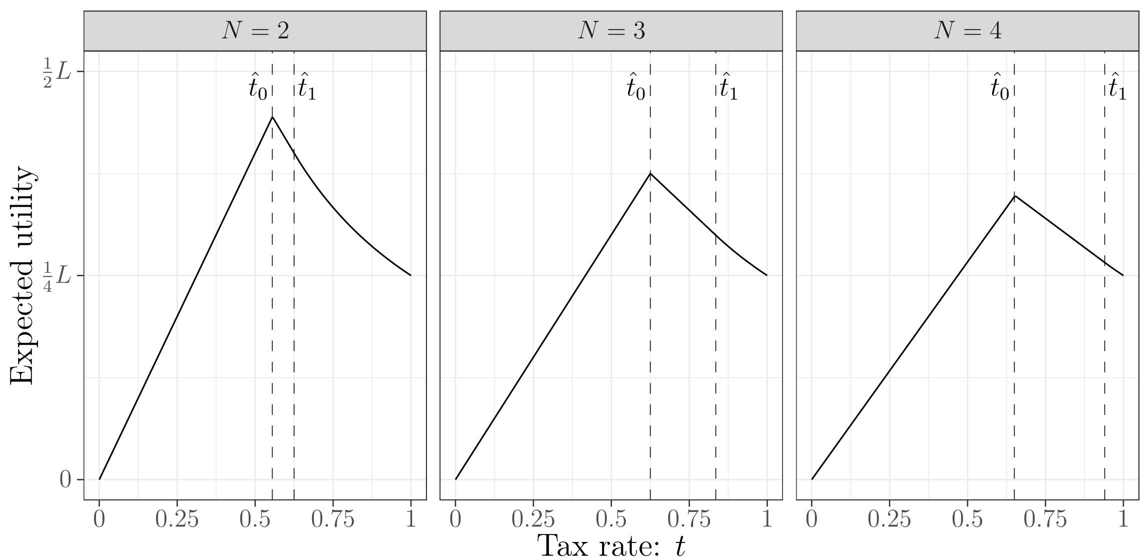
Having solved for labor allocations as a function of the tax rate, I now solve for the government's optimal choice of t. One basic tradeoff from the capital-financed case carries over here: more taxes (above the cutpoint $\hat{t}_0$) lead to more resistance, at least partially offsetting the intended increase in the government's share of output. But now there is an additional downside to predatory taxation. Collective resistance no longer merely siphons away from internal competition—it also reduces economic production, thereby shrinking the pool of output from which government revenues are drawn. The combination of these effects allows me to pin down the equilibrium tax rate for a labor-financed government more clearly than in the capital-financed case. A labor-financed government prefers $t = \hat{t}_0$, the greatest rate that engenders no resistance. Figure 2 illustrates equilibrium labor allocations as a function of the chosen tax rate, showing how government revenues are maximized at $\hat{t}_0$.

Proposition 5. *If the government is labor-financed, there is an equilibrium in which the government chooses the greatest tax rate that engenders no resistance,* $t = \hat{t}_0$*. If* g *or* ϕ *is strictly log-concave, this is the unique equilibrium tax rate.*

To see why it is optimal for the government to avoid engendering resistance, consider a tax rate $t' > \hat{t}_0$ that does lead to resistance. This will result in an effective tax rate of $\tau(t', r) < t$, with some of the increase in resistance coming at the expense of production. It would be better for the government to make the announced tax rate equal the effective one,



(a) Equilibrium responses by the factions.



(b) Government utility given equilibrium faction responses.

Figure 2. Labor allocation with a labor-financed government as a function of the tax rate and the number of factions (Proposition 4).

$t'' = \tau(t', r)$, and reap the gains of the additional production.[16]

At a glance, this result might appear to imply that a labor-financed government benefits from internal schisms. It is evident from Figure 2 that the equilibrium tax rate corresponds to a high point for competition among factions. But this correlation is not causal. Where internal competition is at its maximum, production is also at its maximum, and resistance is at its minimum. To analyze whether a labor-financed government benefits from social order or disorder, it is more apt to examine how its equilibrium utility varies with an exogenous shock to the structural conditions favoring social conflict.

4.2 Comparative Statics

Per Proposition 5, a labor-financed government will choose the highest tax rate at which there is no resistance, $\hat{t}_0$. This implies that equilibrium production will equal its baseline value, $\bar{P}_0$, defined in Proposition 4. The government's equilibrium payoff is thus $\hat{t}_0 \bar{P}_0$. I now examine the effects of fractionalization and internal conflict technology on taxation, production, and overall state revenue in equilibrium. In a sharp contrast with the results above for rentier states, I find that non-rentier states benefit from conditions that reduce the baseline level of internal competition.

On the equilibrium path, all the factions' labor is divided between internal competition and economic production. Anything that raises the labor devoted to internal conflict therefore reduces production. Following the same logic as in the capital-financed case, I find that greater fractionalization (higher N) increases internal competition in equilibrium. Figure 3 illustrates this result. Meanwhile, the effect of improvements in conflict technology again depend on the relative strengths of the incentive effect and the labor-saving effect. These in turn depend on the specific contest success function and the equilibrium level of internal conflict, as described by Equation 8 above.

[16] Of course, if $t'' > \hat{t}_0$, there will still be positive resistance at t'' and the government will not recoup the full share. Nonetheless, as I show in the proof of this proposition in the Appendix, the increase in production by moving to a lower rate is great enough to be profitable for the government.

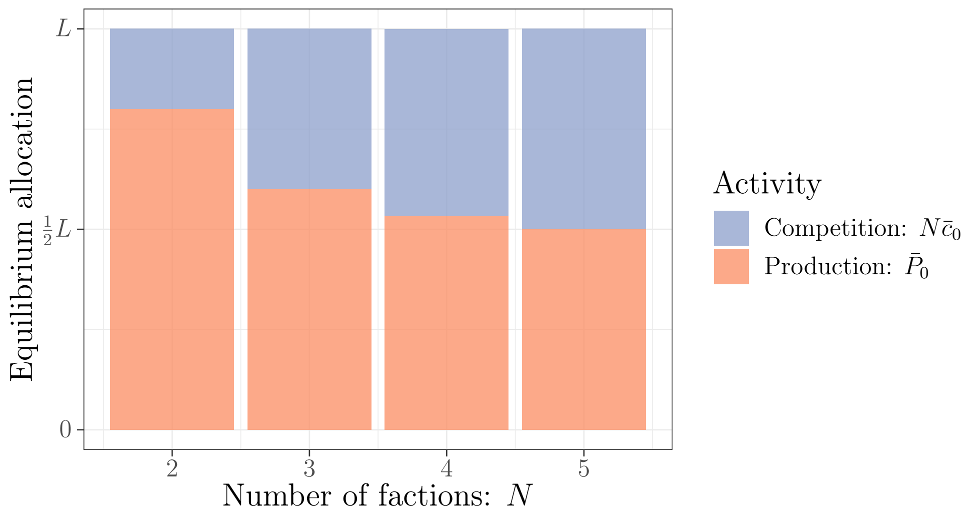


Figure 3. Division of labor on the equilibrium path under a labor-financed government (Proposition 4) as a function of the number of factions.

Proposition 6. *Equilibrium production with a labor-financed government, $\bar{P}_0$, is strictly decreasing in the number of factions, N. It is locally decreasing in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

This result comports with the broad empirical literature showing that social cleavages reduce a society's economic potential (Easterly and Levine 1997; Alesina et al. 2003; Posner 2004; Montalvo and Reynal-Querol 2005*a*). If the tax rate were exogenously fixed, this result would directly imply that a labor-financed government benefits from conditions that discourage internal competition.

The picture looks different when we look at the equilibrium tax rate. Even though there is zero resistance on the equilibrium path, the threat of resistance remains important here, as it determines how highly the government can set taxes without engendering a backlash. And just as in the case of a capital-financed government, the threat to resist grows weaker as the marginal utility of internal competition increases. The equilibrium tax rate, $\hat{t}_0$, is the point at which the marginal benefit of resistance equals that of internal competition. Consequently, as structural conditions increasingly favor internal conflict, the state is able to extract a greater share of output in equilibrium.

Proposition 7. *The equilibrium tax rate of a labor-financed government,*

$$\hat{t}_0 = \frac{\pi^p}{\pi^p - \pi^r \bar{P}_0 g'(0)},$$

is strictly increasing in fractionalization, N. It strictly increases with a marginal increase in competitive effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.

From this perspective, labor-financed governments begin to look more like rentier states. By discouraging collective resistance, greater fractionalization allows the government to extract a greater proportion of economic production. If the size of the pie were exogenous, as with a rentier state, this would imply the government benefits from internal division.

Proposition 6 and Proposition 7 identify countervailing forces on the predatory state's revenues. Structural conditions that promote internal competition, such as fractionalization, result in the state extracting a larger share of a smaller pie. What is the net effect on government revenues? I find that on balance, a revenue-maximizing ruler would benefit from structural conditions that favor internal peace rather than social conflict. Even for an essentially kleptocratic state, it is worse to rule a divided society than a unified one, as long as the state's revenue is based on the economic output of its subjects. This is evident from inspecting the government's revenue curve in Figure 2: even though the equilibrium tax rate increases with N, overall revenue decreases.

Proposition 8. *A labor-financed government's equilibrium payoff is strictly decreasing in the number of factions, N. It strictly decreases with a marginal increase in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

This result arises from two key features of the factions' incentives in the labor allocation subgame. The first is that the marginal benefit of production decreases with the tax rate. Letting (p^*, r^*, c^*) be an equilibrium of the subgame with $t = \hat{t}_0$, the marginal utility of production is

$$\pi^p \frac{\partial u_i(\hat{t}_0, p^*, r^*, c^*)}{\partial p_i} = \frac{\pi^p}{N}(1 - \hat{t}_0).$$

The second key feature of the factions' incentives is that the marginal benefit of resistance increases with the amount the government can extract through taxation. Specifically, the marginal benefit of resistance in equilibrium is

$$\pi^r \frac{\partial u_i(\hat{t}_0, p^*, r^*, c^*)}{\partial r_i} = \frac{-\pi^r g'(0)}{N}\hat{t}_0 \bar{P}_0,$$

where $g'(0) < 0$ represents how quickly the effective tax rate shrinks with a marginal increase in resistance.

Because the equilibrium tax rate pushes the factions just to the point where resistance would become profitable, in equilibrium the marginal benefits of production and resistance must be equal even though no resistance takes place. Importantly, while N and π^c reduce the incentive to divert labor from internal conflict into collective resistance, they do not directly affect the incentive to divert labor from economic production. The formal condition for production and resistance to have equal marginal benefits in equilibrium is

$$\pi^p \times \overbrace{(1 - \hat{t}_0)}^{\text{decreases with } N} = \underbrace{(-\pi^r g'(0))}_{>0} \times \overbrace{\hat{t}_0 \bar{P}_0}^{\text{govt.'s equilibrium payoff}}.$$

Proposition 7 shows that fractionalization increases $\hat{t}_0$ and thereby decreases the marginal return to production (the left-hand side of the above expression). Therefore, in order to maintain the equality of marginal benefits, production $\bar{P}_0$ must decrease enough with fractionalization that the government's payoff, $\hat{t}_0 \bar{P}_0$, also decreases. Fractionalization directly reduces the benefits of resistance, but it also reduces the benefits of production. Therefore, even as fractionalization increases, the factions' collective ability to threaten resistance remains strong enough to prevent the government from profiting.

The upshot of Proposition 8 is that social order increases the profitability of extraction from the population's labor. A labor-financed government is better off governing a society where structural conditions are favorable to low internal competition. If we think of these conditions as at least partially endogenous, this implies that it is in the government's interest to promote social order if the costs of doing so are low enough. For example, the government might seek to divert labor away from looting and into productive economic activity by enforcing subjects' claims to property against appropriation by other factions.

The 17th-century experience of the Dutch East India Company in the Sulawesi region of present-day Indonesia is a clear example of a labor-financed predatory government benefiting from social order rather than division. When the Company arrived in Sulawesi in the 17th century, the region was beset with raiding and other violence, largely between neighboring rival villages (Schouten 1998). Instead of encouraging these conflicts, the Dutch sought to reduce looting and protect property rights. Warring parties regularly called on the Dutch to arbitrate their disputes, making the Company a kind of "stranger king" in Sulawesi society. And the Dutch found it in their interest to do so, as "any conflict quickly tended to interfere with the production and supply of the Minahasan rice which ... formed the Company's main economic interest in the area" (Henley 2004, 105). In other words, with its revenues in Sulawesi funded by agricultural output, the ruling class did not profit from social strife, and in fact sought to reduce it.

Similarly, on the frontiers of Latin Christendom prior to Frankish conquest in the late Middle Ages, "direct predation ... was not the occasional excess of the lawless but the prime activity of the free adult male population" (Bartlett 1993, 303). By establishing free villages and improving the protection of property rights, the immigrant Frankish nobility was able to profit by directing labor into productive activity rather than banditry. Later in Europe, the Ottoman empire maintained an advantage in trade and revenue in part by maintaining peace among the diverse religious and cultural groups that constituted its subjects (Burbank and Cooper 2010, 132–133). In yet another example, the Mughal empire that preceded British rule on the Indian subcontinent "defined their task as to keep an ordered balance between the different forces which constituted Indian society" (Wilson 2016, 17).

5 Empirical Implications

The clearest empirical implication of the formal analysis is that the relationship between social cleavages and government revenues in predatory states is conditional on the nature of the tax base. All else equal, greater fractionalization should increase the net revenues of rentier states whose wealth derives from natural resources or other exogenous sources, but should have the opposite effect in states reliant on domestic labor for their tax base. The mechanism driving these predictions is how internal cleavages and the structure of the domestic economy affect the returns to different forms of labor. As the society becomes more divided, the incentive to divert resources from intergroup competition toward collective resistance decreases—but the incentive to divert resources away from economic production is unchanged.

The equilibrium results of my model are consistent with previous empirical findings on the resilience of rentier states against internal challenges. Colgan (2015) observes that autocratic regimes in petrostates tend to be relatively long-lived, despite also facing comparatively high levels of armed rebellion. The greater frequency of rebellion in petrostates mirrors the equilibrium analysis here, where labor-financed states prefer to set tax rates that engender no resistance (Proposition 5), while capital-financed states may choose higher taxes in equilibrium despite incurring resistance. The extent of domestic challenge is greater for rentier regimes in the model precisely because they can better afford it: domestic resistance, while obviously posing a first-order threat to the regime, at least does not have the second-order effect of shrinking the tax base. By contrast, in the largely labor-financed non-settler colonies of the British empire, the threat of resistance placed hard constraints on the state's extractive ability (Frankema 2010). Colgan (2015) also finds in an auxiliary analysis that autocracies in petrostates are most secure against democratization when religious fractionalization is relatively high, consistent with my comparative statics on social divisions in capital-financed regimes (Proposition 2).

The model also predicts variation in the incentives of predatory regimes to enact policies that dampen competition among social factions, such as increased property rights protection. North (1981) famously observed that even predatory states may benefit from protecting subjects' property rights, as doing so enlarges the long-run revenue stream through greater economic growth. A similar logic emerges for labor-financed regimes in my model (Proposition 5 and Proposition 8), though with two important distinctions. First, restraint in predatory taxation is not driven exclusively by productivity concerns, but also by the possibility of direct resistance. Second, the labor-financed state benefits not just from limiting the extent of government expropriation, but also from protecting factions against communal threats to property from rival social groups.

Rentier states have distinctly less incentive to provide for property rights protection, according to the formal analysis here. While even a rentier regime may have some incentive to limit its own predations due to the threat of collective resistance, there is no such incentive to limit economic predation among competing social factions (Proposition 3). This finding is consistent with empirical evidence from Baggio and Papyrakis (2010), who find a negative interaction effect of resource wealth and ethnic polarization on the quality of property rights protection. In other words, resource-rich states with deep communal divisions have particularly weak incentives to guarantee the security of property claims, just as predicted in the model here. Wegenast and Basedau (2014) find a similar interaction between oil wealth and fractionalization in a study of the determinants of armed ethnic conflict.

An important determinant of communal economic conflict not directly incorporated into my model is the lootability of resources and commodities (Snyder and Bhavnani 2005; Thies 2010). In terms of the model, lootability could be thought of as a factor that increases the labor productivity of both internal conflict and collective resistance, insofar as it takes relatively little effort for a social faction to gain possession of a lootable resource initially controlled by another group or the government. If the labor-saving effect of this greater productivity outweighs the incentive effect, then lootability in this sense would unambiguously lower rentier state revenues by increasing effective resistance (Proposition 3), consistent with the findings by Snyder and Bhavnani (2005) on the risk of state collapse. If instead the incentive effect were dominant (which depends on the technology of internal competition and the labor allocated toward it in equilibrium; see Equation 8), then lootability would have countervailing effects, making resistance more effective while making internal competition more attractive.

A final question to address is whether the analysis here implies that rentier states will actively seek to stoke communal violence, so as to reduce the extent of resistance against predation by the state. Working with a formal model similar to mine, De Luca, Sekeris and Vargas (2018) make this argument, and present some empirical corroboration: resource wealth is correlated with ethnic conflict in states governed by weak rulers. But while rentier states do extract greater revenues under conditions of high fractionalization in my model, this does not imply that they benefit from the outbreak of actual violence. As noted above in the discussion of the model setup, even if violence does not break out, deterring other factions from predation or maintaining one's own bargaining power may take labor away from collective resistance. This diversion of labor is what a rentier state benefits from. Insofar as violence has spillover effects that reduce the value of the resources at stake, then the outbreak of armed conflict among groups would effect a direct reduction in state revenues, holding fixed the allocation of labor toward internal competition. If fractionalization increased the likelihood of violence, then this would mean it has countervailing effects, rather than benefiting rentier states. However, whether internal competition among social factions actually turns violent should depend primarily on the extent of informational or commitment problems among them (Fearon 1995), neither of which are clearly functions of the extent of fractionalization.

6 Extensions

To address additional substantive questions and to confirm the robustness of the main results when key assumptions are relaxed, I extend the model in various directions. I summarize the key details here, leaving all formal details to the Appendix.

Endogenous Inequality. The baseline model assumes that all factions have identical incentives and that taxation affects each one equally. I extend the model to examine whether a labor-financed government might profit by creating inequality between groups, namely by taxing them at different rates. The extension is similar to the model of divide-and-rule politics in Acemoglu, Verdier and Robinson (2004), the crucial difference being that here the factions' economic output is subject to appropriation by other factions as well as the government.

In contrast with Acemoglu, Verdier and Robinson (2004), I find no incentive for the government to create inequality through unequal taxation. The logic of the result follows from the paradox of power that arises in contests over endogenously valued prizes (Hirshleifer 1991). If the government taxes one faction more than another, the more-taxed faction responds by shifting labor away from production, toward both resistance and internal competition. The less-taxed faction then shifts its own labor away from resistance and production in order to keep up with the internal competition. Because the marginal benefit of resistance remains a function of the government's payoff, the decrease in the less-taxed faction's resistance is not enough to make up for the decrease in production by both factions and the increase in resistance by the more-taxed faction. See Appendix A.5.1 for details.

Conquest. The baseline analysis takes the identity of the ruler as fixed. Yet classic examples of divide-and-rule politics, such as Cortés with the Aztecs (Elliott 2007; Burkholder and Johnson 2015) or the Dutch East India Company in present-day Indonesia (Scammell 1989, 20), concern the likelihood of taking power in the first place. I extend the model to analyze the effects of internal fractionalization on the likelihood of an outsider taking control. In the extended model, the baseline game is preceded by a conquest stage, in which social factions divide their efforts between resisting outside takeover and competing for influence in case the takeover fails.

Whereas internal fractionalization is only conditionally beneficial for predatory governance, namely when the government is capital-financed, it unconditionally increases the prospects of an outsider seeking to gain control in the first place. Resistance against an outsider's attempt to take control is effectively a public good. As the number of factions increases, the incentive to provide this public good rather than to fend for oneself decreases (Olson 1965). Therefore, an outsider can more easily take control of a divided society than a unified one. That said, fractionalization may still be a net negative for *ex ante* expected revenues if the main source of revenues is labor output—a more fractionalized society is easier to take over, but harder to extract from.

Informal Sector Production. In the baseline model with a capital-financed government, the only outlets for the factions' labor are internal competition or resistance against taxation. I extend the capital-financed model to allow for a more realistic environment in which the factions may also engage in productive activity, but where government finance still depends entirely on a fixed resource endowment. In the extended model, in addition to the original choices of resistance and internal competition, the factions may allocate labor to informal production. Unlike economic output in the labor-financed model, informal production is not subject to appropriation by the government or competing factions. In effect, the factions' utility becomes

$$\underbrace{\omega_i(c) \times \bar{\tau}(t, r) \times X}_{\text{original utility}} + \overbrace{\kappa o_i}^{\text{informal production}},$$

where o_i is the proportion of a faction's labor devoted to informal production and $\kappa > 0$ is the marginal product of that labor. The key comparative static from the baseline model holds up in this environment: government revenue increases with the extent of social fractionalization.

The full extension appears in Appendix A.5.3.

Incomplete Information. No resistance occurs on the path of play in the baseline model with a labor-financed government, as the government chooses the highest tax rate that engenders zero resistance (Proposition 5). This finding depends on the assumption that the government is completely informed about the factions' incentives, such that it can perfectly calibrate its tax demand to avoid costly resistance. I rely on this property of the equilibrium when showing that a labor-financed government's revenues decrease with fractionalization (Proposition 8).

In an extension to the model, I consider a more realistic environment in which the government has incomplete information about the productivity of the factions' resistance technology, and thus about the exact outcome that will follow from any given tax demand. The government may now overshoot in equilibrium, selecting a tax rate that engenders popular resistance on the path of play. Even so, the government's expected revenues continue to decrease with the extent of social fractionalization, as in the baseline model. See Appendix A.5.4 for full details.

Combined Capital and Labor Financing. In the baseline analysis, I consider the extreme cases of a government financed entirely by resource rents and one financed entirely by labor output. I extend the model to allow revenues to derive from both sources. Specifically, in the extended model, the production function becomes $f(p) = \sum_{i=1}^{N} p_i + X$, where $X > 0$ is fixed. Details are in Appendix A.5.5.

As in the baseline model with a purely labor-financed government, endogenous production strictly decreases with the extent of social fractionalization. At low levels of fractionalization, the government chooses the lowest tax rate that engenders no resistance, just as in the baseline labor-financed model. When fractionalization is higher, endogenous production vanishes, and the government's incentives become the same as in the baseline capital-financed model. Overall, then, fractionalization has a nonmonotonic relationship with revenues; the government's equilibrium payoff is U-shaped as a function of N. Additionally, the location of the low point decreases with the magnitude of exogenous resources, X. In other words, the greater the value of the exogenous resource stock, the more likely it is that a marginal increase in fractionalization benefits increases the amount the government can extract.

7 Conclusion

I have studied the political economy of predatory rule in a divided society. The key finding is that internal fractionalization increases the extractive capacity of rentier states, but has the opposite effect on regimes funded by endogenous labor output. The results here highlight an additional channel by which rentier states are distinctive among autocratic regimes: they are able to profit from divide-and-rule politics. I demonstrate that the political upside of internal schisms for a rentier government is not limited to the prevention of insider threats from the military apparatus (see Bellin 2004). The model shows how rentier states benefit from a broad reduction in the society's capacity for collective action—a benefit that does not accrue to labor-financed states, due to the corresponding reduction in economic output.

In this paper, I have assumed the shape of the society being governed is exogenously fixed. A natural direction for future research would be to examine how internal competition of the type modeled here interacts with territorial conflict and the drawing of political boundaries. The analysis here highlights how the *economic* value of territory cannot be disentangled from its *social* structure. According to the model, natural resource wealth becomes all the more valuable—and thus more likely to be the source of costly conflict—when the local society is politically fractionalized. The opposite would be true, however, for territory that is primarily valued for its agricultural productivity, as was the case in various colonial conflicts. In addition to their implications for the study of territorial conflict, these are important issues for political economy models of state size and endogenous border formation (e.g., Alesina and Spolaore 2005; Acharya and Lee 2018).

To focus on how fractionalization and the technology of internal conflict affect a predatory state's payoffs, I have assumed away any *ex ante* inequality in size or productivity between social groups. Of course, inequality itself might increase social conflict (Esteban and Ray 2011). In light of the analysis here, we thus might expect capital-financed predatory states to prefer inequality and labor-financed ones to prefer equality. However, if productivity inequalities between factions lead to specialization in particular activities, then different strategic tradeoffs might emerge than in the symmetric case. For example, in a labor-financed state, if one group specializes in resistance while the other specializes in production, increased conflict between them may benefit a predatory government on net. An important question for future research is how these inequalities might interact with the political-economic incentives studied here, and to what extent they alter the baseline relationship between social conflict and the profits of predatory rule.

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A Appendix to “Social Conflict and the Predatory State”

Contents

A.1 Additional Notation

Throughout the appendix, let $\Phi(c) = \sum_i \phi(c_i)$. We have $\log \omega_i(c) = \log \phi(c_i) - \log \Phi(c)$ and thus

$$\begin{aligned}
\frac{\partial \log \omega_i(c)}{\partial c_i} &= \frac{\phi'(c_i)}{\phi(c_i)} - \frac{\phi'(c_i)}{\Phi(c)} \\
&= \frac{\phi'(c_i)}{\phi(c_i)} \left(1 - \frac{\phi(c_i)}{\Phi(c)}\right) \\
&= \hat{\phi}'(c_i)(1 - \omega_i(c)),
\end{aligned}$$

where $\hat{\phi} = \log \phi$. Because ϕ is strictly increasing and log-concave, $\hat{\phi}' > 0$ and $\hat{\phi}'' \leq 0$.

A.2 Equilibrium Existence and Uniqueness

For the existence and uniqueness results, I consider a more general version of the model presented in the text. I allow groups to be asymmetric in their size and productivities, which entails generalizing each faction i's budget constraint (1) to

$$\frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} = L_i, \tag{A.1}$$

where $\pi_i^p, \pi_i^r, \pi_i^c, L_i > 0$. I assume a labor-financed government throughout the existence and uniqueness results, as this is the more difficult case; all claims here also apply to a capital-financed state in which $f(p) = X$. In addition, the results here do not depend on Assumption 1.

Let $\Gamma(t)$ denote the subgame that follows the government's selection of t, in which the factions simultaneously decide how to allocate their labor. Let $\sigma_i = (p_i, r_i, c_i)$ be a strategy for faction i in the subgame, and let

$$\Sigma_i = \left\{ (p_i, r_i, c_i) \;\middle|\; \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} = L_i \right\}$$

denote the strategy space. Let $\sigma = (\sigma_1, \ldots, \sigma_N)$ and $\Sigma = \bigtimes_{i=1}^N \Sigma_i$.

I begin by proving that a Nash equilibrium exists in each subgame. The task is complicated by the potential discontinuity of the factions' payoffs, namely at $c = 0$ when $\phi(0) = 0$. I rely on Reny's (1999) conditions for the existence of pure strategy equilibria in a discontinuous game. The key condition is *better-reply security*—informally, that at least one player can assure a strict benefit by deviating from any non-equilibrium strategy profile, even if the other players make slight deviations.

Lemma A.1. $\Gamma(t)$ *is better-reply secure.*

Proof. Let $U^t : \Sigma \to \mathbb{R}_+^N$ be the vector payoff function for the factions in $\Gamma(t)$, so that $U^t(\sigma) = (u_1(t, \sigma), \ldots, u_N(t, \sigma))$. Take any convergent sequence in the graph of U^t, call it $(\sigma^k, U^t(\sigma^k)) \to (\sigma^*, U^*)$, such that σ^* is not an equilibrium of $\Gamma(t)$. Because production and the effective tax rate are continuous in (p, r), we have

$$U_i^* = w_i^* \times \bar{\tau}(t, r^*) \times f(p^*)$$

for each i, where $w_i^* \geq 0$ and $\sum_{i=1}^N w_i^* = 1$. I must show there is a player i who can secure a payoff $\bar{U}_i > U_i^*$ at σ^*; i.e., there exists $\bar{\sigma}_i \in \Sigma_i$ such that $u_i(t, \bar{\sigma}_i, \sigma'_{-i}) \geq \bar{U}_i$ for all σ'_{-i} in a neighborhood of σ^*_{-i} (Reny 1999, 1032).

If $N = 1$ or $\Phi(c^*) > 0$, then U^t is continuous in a neighborhood of σ^*, so the conclusion is immediate. If $\bar{\tau}(t, r^*) \times f(p^*) = 0$, then each $U_i^* = 0$ and each faction can assure a strictly greater payoff by deviating to a strategy with positive production, resistance, and conflict.

For the remaining cases, suppose $N > 1$, $\bar{\tau}(t, r^*) \times f(p^*) > 0$, and $\Phi(c^*) = 0$, the latter of which implies $c^* = 0$ and $\phi(0) = 0$. Since $N > 1$, there is a faction i such that $w_i^* < 1$. Take any $\epsilon \in (0, (1 - w_i^*)/2)$ and any $\delta_1 > 0$ such that

$$\bar{\tau}(t, r') \times f(p') \geq (w_i^* + 2\epsilon) \times \bar{\tau}(t, r^*) \times f(p^*)$$

for all σ' in a δ_1-neighborhood of σ^*. Since $w_i^* + 2\epsilon < 1$ and $\bar{\tau}(t, r) \times f(p)$ is continuous in (p, r), such a δ_1 exists. Then let $\bar{\sigma}_i = (\bar{p}_i, \bar{r}_i, \bar{c}_i)$ be any strategy in a δ_1-neighborhood of σ_i^* such that $\bar{c}_i > 0$. Because $c_{-i}^* = 0$ and ϕ is continuous, there exists $\delta_2 > 0$ such that

$$\omega_i(\bar{c}_i, c'_{-i}) = \frac{\phi(\bar{c}_i)}{\phi(\bar{c}_i) + \sum_{j \in \mathcal{N} \setminus \{i\}} \phi(c'_j)} \geq \frac{w_i^* + \epsilon}{w_i^* + 2\epsilon}$$

for all σ'_{-i} in a δ_2-neighborhood of σ_{-i}^*. Therefore, for all σ'_{-i} in a $\min\{\delta_1, \delta_2\}$-neighborhood of σ_{-i}^*, we have

$$u_i(t, \bar{\sigma}_i, \sigma'_{-i}) \geq (w_i^* + \epsilon) \times \bar{\tau}(t, r^*) \times f(p^*) > U_i^*,$$

establishing the claim. $\square$

The other main condition for equilibrium existence is that each faction's utility function be quasiconcave in its own actions. I prove this by showing that the logarithm of a faction's utility function is concave in its actions.

Lemma A.2. $\Gamma(t)$ *is log-concave.*

Proof. Take any (p, r, c) such that $u_i(t, p, r, c) > 0$, and let $P = \sum_j p_j$ and $R = \sum_j r_j$. First, assume $\sum_{j \neq i} \phi(c_j) > 0$, so that u_i is continuously differentiable in (p_i, r_i, c_i). We have

$$\begin{aligned}
\frac{\partial \log u_i(t, p, r, c)}{\partial p_i} &= \frac{1}{P}, \\
\frac{\partial \log u_i(t, p, r, c)}{\partial r_i} &= \frac{-t g'(R)}{1 - t g(R)}, \\
\frac{\partial \log u_i(t, p, r, c)}{\partial c_i} &= \hat{\phi}'(c_i)(1 - \omega_i(c)),
\end{aligned}$$

and therefore

$$\begin{aligned}
\frac{\partial^2 \log u_i(t, p, r, c)}{\partial p_i^2} &= \frac{-1}{P^2} < 0, \\
\frac{\partial^2 \log u_i(t, p, r, c)}{\partial r_i^2} &= \frac{-t g''(R)(1 - t g(R)) - (t g'(R))^2}{(1 - t g(R))^2} \leq 0,
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial^2 \log u_i(t,p,r,c)}{\partial c_i^2} &= \hat{\phi}''(c_i)(1-\omega_i(c)) - \hat{\phi}'(c_i)\frac{\partial \omega_i(c)}{\partial c_i} \leq 0,\\
\frac{\partial^2 \log u_i(t,p,r,c)}{\partial p_i \partial r_i} &= \frac{\partial^2 \log u_i(t,p,r,c)}{\partial p_i \partial c_i} = \frac{\partial^2 \log u_i(t,p,r,c)}{\partial r_i \partial c_i} = 0,
\end{aligned}
$$

so $\log u_i$ is concave in (p_i, r_i, c_i). By the same token, $\bar{\tau}(t,r) \times f(p)$ is log-concave in (p,r).

Now assume $\sum_{j \neq i} \phi(c_j) = 0$. Take any (p_i', r_i', c_i') such that $u_i(t,p',r',c') > 0$, where $(p',r',c') = ((p_i', p_{-i}), (r_i', r_{-i}), (c_i', c_{-i}))$. Take any $\alpha \in [0,1]$, and let $(p^\alpha, r^\alpha, c^\alpha) = \alpha(p,r,c) + (1-\alpha)(p',r',c')$. If $c_i = c_i' = 0$, then $\omega_i(c^\alpha) = \omega_i(c) = \omega_i(c') = 1/N$ and thus

$$
\begin{aligned}
\log u_i(t,p^\alpha,r^\alpha,c^\alpha) &= \log \frac{1}{N} + \log \bar{\tau}(t,r^\alpha) + \log f(p^\alpha)\\
&\geq \log \frac{1}{N} + \alpha\left(\log \bar{\tau}(t,r) + \log f(p)\right) + (1-\alpha)\left(\log \bar{\tau}(t,r') + \log f(p')\right)\\
&= \alpha \log u_i(t,p,r,c) + (1-\alpha)\log u_i(t,p',r',c'),
\end{aligned}
$$

where the inequality follows from the log-concavity of $\bar{\tau}(t,r) \times f(p)$ in (p,r). If $c_i > 0$ and $c_i' = 0$, then $\omega_i(c^\alpha) = \omega_i(c) = 1$, $\omega_i(c') = 1/N$, and thus

$$
\begin{aligned}
\log u_i(t,p^\alpha,r^\alpha,c^\alpha) &= \log \bar{\tau}(t,r^\alpha) + \log f(p^\alpha)\\
&\geq \alpha\left(\log \bar{\tau}(t,r) + \log f(p)\right) + (1-\alpha)\left(\log \frac{1}{N} + \log \bar{\tau}(t,r') + \log f(p')\right)\\
&= \alpha \log u_i(t,p,r,c) + (1-\alpha)\log u_i(t,p',r',c').
\end{aligned}
$$

The same argument holds in case $c_i = 0$ and $c_i' > 0$. It is easy to see that the same conclusion holds if $c_i > 0$ and $c_i' > 0$, in which case $\omega_i(c^\alpha) = \omega_i(c) = \omega_i(c') = 1$. Therefore, $\log u_i$ is concave in (p_i, r_i, c_i). □

Equilibrium existence follows immediately from the two preceding lemmas.

Proposition A.1. $\Gamma(t)$ *has a pure strategy equilibrium.*

Proof. The strategy space Σ is compact, each payoff function u_i is bounded on Σ, and $\Gamma(t)$ is better-reply secure (Lemma A.1) and quasiconcave (Lemma A.2). Therefore, a pure strategy equilibrium exists (Reny 1999, Theorem 3.1). □

I now turn to the question of uniqueness. I show that although $\Gamma(t)$ may have multiple equilibria, these equilibria are identical in terms of three essential characteristics: total production, $\sum_i p_i$; total resistance, $\sum_i r_i$; and the vector of individual expenditures on internal conflict, c.

To prove essential uniqueness, I must characterize the equilibrium more fully than I have up to this point. The following result rules out equilibria in which (1) a faction's share in the internal competition is zero or (2) a faction could raise its share to one by an infinitesimal change in strategy.

Lemma A.3. *If $N > 1$, then each $\phi(c_i) > 0$ in any equilibrium of $\Gamma(t)$.*

Proof. Assume $N > 1$, and let (p, r, c) be a strategy profile of $\Gamma(t)$ in which $c_i = 0$ for some $i \in \mathcal{N}$. The claim holds trivially if $\phi(0) > 0$, so assume $\phi(0) = 0$. If $\Phi(c) > 0$ or $\bar{\tau}(t, r) \times f(p) = 0$, then $u_i(t, p, r, c) = 0$. But i could ensure a strictly positive payoff with any strategy that allocated nonzero labor to production, resistance, and conflict, so (p, r, c) is not an equilibrium. Conversely, suppose $\Phi(c) = 0$, which implies $c_j = 0$ for all $j \in \mathcal{N}$, and $\bar{\tau}(t, r) \times f(p) > 0$. Then $u_i(t, p, r, c) = (\bar{\tau}(t, r) \times f(p))/N$. But i could obtain a payoff arbitrarily close to $\bar{\tau}(t, r) \times f(p)$ by diverting an infinitesimal amount of labor away from production or resistance and into internal conflict, so (p, r, c) is not an equilibrium. □

This result is important because it implies utility functions are continuously differentiable in the neighborhood of any equilibrium. Equilibria can therefore be characterized in terms of first-order conditions.

Lemma A.4. *(p', r', c') is an equilibrium of $\Gamma(t)$ if and only if, for each $i \in \mathcal{N}$,*

$$p_i' \left(\pi_i^p \frac{\partial \log f(p')}{\partial p_i} - \mu_i \right) = 0, \tag{A.2}$$

$$r_i' \left(\pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} - \mu_i \right) = 0, \tag{A.3}$$

$$c_i' \left(\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} - \mu_i \right) = 0, \tag{A.4}$$

$$\frac{p_i'}{\pi_i^p} + \frac{r_i'}{\pi_i^r} + \frac{c_i'}{\pi_i^c} - L_i = 0, \tag{A.5}$$

where

$$\mu_i = \max \left\{ \pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i}, \pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} \right\}.$$

Proof. In equilibrium, each faction's strategy must solve the constrained maximization problem

$$\begin{aligned} \max_{p_i, r_i, c_i} \quad & \log u_i(t, p, r, c) \\ \text{s.t.} \quad & \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} + \frac{c_i}{\pi_i^c} - L_i = 0, \end{aligned}$$

$$p_i \geq 0, r_i \geq 0, c_i \geq 0.$$

It follows from Lemma A.3 that each u_i is C^1 in (p_i, r_i, c_i) in a neighborhood of any equilibrium. This allows use of the Karush–Kuhn–Tucker conditions to characterize solutions of the above problem. The "only if" direction holds because (A.2)–(A.5) are the first-order conditions for the problem and the linearity constraint qualification holds. The "if" direction holds because $\log u_i$ is concave in (p_i, r_i, c_i), per Lemma A.2. □

A weak welfare optimality result follows almost immediately from this equilibrium characterization. If (p', r', c') is an equilibrium of $\Gamma(t)$, then there is no other equilibrium (p'', r'', c'') such that $c'' = c'$ and $\bar{\tau}(t, r'') \times f(p'') > \bar{\tau}(t, r') \times f(p')$. In other words, taking as fixed the factions' allocations toward internal conflict, there is no inefficient misallocation of labor between production and resistance.

Corollary A.1. *If (p', r', c') is an equilibrium of $\Gamma(t)$, then (p', r') solves*

$$\begin{aligned} \max_{p,r} \quad & \log \bar{\tau}(t, r) + \log f(p) & \\ s.t. \quad & \frac{p_i}{\pi_i^p} + \frac{r_i}{\pi_i^r} = L_i - \frac{c_i'}{\pi_i^c}, & i = 1, \ldots, N, \\ & p_i \geq 0, r_i \geq 0, & i = 1, \ldots, N. \end{aligned}$$

Proof. This is a C^1 concave maximization problem with linear constraints, so the Karush–Kuhn–Tucker first-order conditions are necessary and sufficient for a solution. The result then follows from Lemma A.4. □

I next prove that if post-tax output is weakly greater in one equilibrium of $\Gamma(t)$ than another, then each of the two individual components (production and the factions' total share) is weakly greater. The proof relies on the fact that if $c_i' \leq c_i''$ and $\omega_i(c') \leq \omega_i(c'')$, then

$$\frac{\partial \log \omega_i(c')}{\partial c_i} = \hat{\phi}'(c_i')(1 - \omega_i(c')) \geq \hat{\phi}'(c_i'')(1 - \omega_i(c'')) = \frac{\partial \log \omega_i(c'')}{\partial c_i}.$$

If in addition $\omega_i(c') < \omega_i(c'')$, the inequality is strict.

Lemma A.5. *If (p', r', c') and (p'', r'', c'') are equilibria of $\Gamma(t)$ such that $\bar{\tau}(t, r') \times f(p') \geq \bar{\tau}(t, r'') \times f(p'')$, then $\bar{\tau}(t, r') \geq \bar{\tau}(t, r'')$ and $f(p') \geq f(p'')$.*

Proof. Suppose the claim of the lemma does not hold, so there exist equilibria such that $\bar{\tau}(t, r') \times f(p') \geq \bar{\tau}(t, r'') \times f(p'')$ but $\bar{\tau}(t, r') < \bar{\tau}(t, r'')$. Together, these inequalities imply

$f(p') > f(p'')$. (The proof in case $\bar{\tau}(t, r') > \bar{\tau}(t, r'')$ and $f(p') < f(p'')$ is analogous.)

I will first establish that $p'_i > 0$ implies $r''_i = 0$. Per Lemma A.4 and the log-concavity of f and $\bar{\tau}$, $p'_i > 0$ implies

$$\pi_i^p \frac{\partial \log f(p'')}{\partial p_i} > \pi_i^p \frac{\partial \log f(p')}{\partial p_i} \geq \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} > \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i}.$$

Therefore, again by Lemma A.4, $r''_i = 0$.

Next, I establish that $\Phi(c'') > \Phi(c')$. Since $f(p') > f(p'')$, there is a faction $i \in \mathcal{N}$ such that $p'_i > p''_i$. As this implies $r''_i = 0$, the budget constraint gives $c''_i > c'_i$. If $\Phi(c'') \leq \Phi(c')$, then $\omega_i(c'') > \omega_i(c')$ and thus by Lemma A.4

$$\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} > \pi_i^c \frac{\partial \log \omega_i(c'')}{\partial c_i} \geq \pi_i^p \frac{\partial \log f(p'')}{\partial p_i} > \pi_i^p \frac{\partial \log f(p')}{\partial p_i}.$$

But this implies $p'_i = 0$, a contradiction. Therefore, $\Phi(c'') > \Phi(c')$.

Using these intermediate results, I can now establish the main claim by contradiction. Since $\bar{\tau}(t, r'') > \bar{\tau}(t, r')$, there is a faction $j \in \mathcal{N}$ such that $r''_j > r'_j$. This implies $p'_j = 0$, so the budget constraint gives $c''_j < c'_j$. Since $\Phi(c'') > \Phi(c')$, this in turn gives $\omega_j(c'') < \omega_j(c')$ and thus

$$\pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j} > \pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} \geq \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j} > \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}.$$

But this implies $r''_j = 0$, a contradiction. □

I can now state and prove the essential uniqueness of the equilibrium of each labor allocation subgame.

Proposition A.2. *If (p', r', c') and (p'', r'', c'') are equilibria of $\Gamma(t)$, then $f(p') = f(p'')$, $\bar{\tau}(t, r') = \bar{\tau}(t, r'')$, and $c' = c''$.*

Proof. First I prove that $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$. Suppose not, so that, without loss of generality, $\bar{\tau}(t, r') \times f(p') > \bar{\tau}(t, r'') \times f(p'')$. Then Lemma A.5 implies $\bar{\tau}(t, r') \geq \bar{\tau}(t, r'')$ and $f(p') \geq f(p'')$, at least one strictly so, and thus

$$\max \left\{ \pi_i^p \frac{\partial \log f(p'')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i} \right\} \geq \max \left\{ \pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i} \right\}$$

for all $i \in \mathcal{N}$, strictly so for some $j \in \mathcal{N}$. Since $\bar{\tau}(t, r'') \times f(p'') < \bar{\tau}(t, r') \times f(p')$, it follows from Corollary A.1 that the set $\mathcal{N}^+ = \{i \in \mathcal{N} \mid c''_i > c'_i\}$ is nonempty. For any $i \in \mathcal{N}^+$ such that $\omega_i(c'') > \omega_i(c')$,

$$\begin{aligned}
\pi_i^c \frac{\partial \log \omega_i(c')}{\partial c_i} &> \pi_i^c \frac{\partial \log \omega_i(c'')}{\partial c_i} \\
&\geq \max\left\{\pi_i^p \frac{\partial \log f(p'')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_i}\right\} \\
&\geq \max\left\{\pi_i^p \frac{\partial \log f(p')}{\partial p_i}, \pi_i^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_i}\right\}.
\end{aligned}$$

But this implies $p'_i = r'_i = 0$, contradicting $c''_i > c'_i$. So $\omega_i(c'') \leq \omega_i(c')$ for all $i \in \mathcal{N}^+$. Since $\mathcal{N}^+$ is nonempty and the conflict shares are increasing in c_i and sum to one, this can hold only if $\mathcal{N}^+ = \mathcal{N}$ and $\omega_i(c'') = \omega_i(c')$ for all $i \in \mathcal{N}$. This implies

$$\begin{aligned}
\pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} &\geq \pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j} \\
&\geq \max\left\{\pi_j^p \frac{\partial \log f(p'')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}\right\} \\
&> \max\left\{\pi_j^p \frac{\partial \log f(p')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j}\right\},
\end{aligned}$$

which in turn implies $p'_j = r'_j = 0$, contradicting $c''_j > c'_j$. I conclude that $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$ and thus, by Lemma A.5, $\bar{\tau}(t, r') = \bar{\tau}(t, r'')$ and $f(p') = f(p'')$.

Next, I prove that $c' = c''$. Suppose not, so $c' \neq c''$. Without loss of generality, suppose $\Phi(c') \geq \Phi(c'')$. Since $\bar{\tau}(t, r') \times f(p') = \bar{\tau}(t, r'') \times f(p'')$ yet $c' \neq c''$, by Corollary A.1 there exists $i \in \mathcal{N}$ such that $c'_i > c''_i$ and $j \in \mathcal{N}$ such that $c'_j < c''_j$. It follows from $\Phi(c') \geq \Phi(c'')$ that $\omega_j(c') < \omega_j(c'')$ and therefore

$$\begin{aligned}
\pi_j^c \frac{\partial \log \omega_j(c')}{\partial c_j} &> \pi_j^c \frac{\partial \log \omega_j(c'')}{\partial c_j} \\
&\geq \max\left\{\pi_j^p \frac{\partial \log f(p'')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r'')}{\partial r_j}\right\} \\
&= \max\left\{\pi_j^p \frac{\partial \log f(p')}{\partial p_j}, \pi_j^r \frac{\partial \log \bar{\tau}(t, r')}{\partial r_j}\right\}.
\end{aligned}$$

But this implies $p'_j = r'_j = 0$, contradicting $c''_j > c'_j$. □

Proposition A.2 allows me to write the equilibrium values of total production, total resistance, and individual conflict allocations as functions of the tax rate. For each $t \in [0, 1]$, let $P^*(t) = P$ if and only if there is an equilibrium (p, r, c) of $\Gamma(t)$ such that $\sum_i p_i = P$. Let the functions $R^*(t)$ and $c^*(t)$, the latter of which is vector-valued, be defined analogously.

The only remaining step to prove the existence of an equilibrium in the full game is to show that an optimal tax rate exists. An important consequence of Proposition A.2 is that the optimal tax rate (if one exists) does not depend on the equilibrium that is selected in each labor allocation subgame, since the government's payoff depends only on total production and resistance. The main step toward proving the existence of an optimal tax rate is to show that total production and resistance are continuous in t.

Lemma A.6. *P^*, R^*, and c^* are continuous.*

Proof. Define the equilibrium correspondence $E : [0,1] \rightrightarrows \Sigma$ by

$$E(t) = \{(p, r, c) \mid (p, r, c) \text{ is an equilibrium of } \Gamma(t)\}.$$

Standard arguments (e.g., Fudenberg and Tirole 1991, 30–32) imply that E has a closed graph.[17] This in turn implies E is upper hemicontinuous, as its codomain, Σ, is compact. Let $F : \Sigma \to \mathbb{R}_+^{N+2}$ be the function defined by $F(p, r, c) = (\sum_i p_i, \sum_i r_i, c)$. Since F is continuous as a function, it is upper hemicontinuous as a correspondence. Then we can write the functions in the lemma as the composition of F and E:

$$(P^*(t), R^*(t), c^*(t)) = \{F(p, r, c) \mid (p, r, c) \in E(t)\} = (F \circ E)(t).$$

As the composition of upper hemicontinuous correspondences, (P^*, R^*, c^*) is upper hemicontinuous (Aliprantis and Border 2006, Theorem 17.23). Then, as an upper hemicontinuous correspondence that is single-valued (per Proposition A.2), (P^*, R^*, c^*) is continuous as a function. □

Continuity of total production and resistance in the tax rate imply that the government's payoff is continuous in the tax rate, so an equilibrium exists.

Proposition A.3. *There is a pure strategy equilibrium.*

Proof. For each labor allocation subgame $\Gamma(t)$, let $\sigma^*(t)$ be a pure strategy equilibrium of $\Gamma(t)$. Proposition A.1 guarantees the existence of these equilibria. By Proposition A.2, the government's payoff from any $t \in [0,1]$ is

$$u_G(t, \sigma^*(t)) = t \times g(R^*(t)) \times P^*(t).$$

[17] The only complication in applying the usual argument is that the model is discontinuous at $c = 0$ in case $\phi(0) = 0$. However, by the same arguments as in the proof of Lemma A.1, if $\phi(0) = 0$ there cannot be a sequence $(t^k, (p^k, r^k, c^k))$ in the graph of E such that $c^k \to 0$.

This expression is continuous in t, per Lemma A.6, and therefore attains its maximum on the compact interval $[0,1]$. A maximizer t^* exists, and the pure strategy profile $(t^*, (\sigma^*(t))_{t\in[0,1]})$ is an equilibrium. $\square$

A.3 Symmetric Game Properties

In the remainder of the appendix, I consider the special symmetric case of the model discussed in the text, in which each $\pi_i^p = \pi^p$, $\pi_i^r = \pi^r$, $\pi_i^c = \pi^c$, and $L_i = L/N$. Let $\Gamma(t)$ denote the labor allocation subgame with tax rate t in the model with a labor-financed government, and let $\Gamma_X(t)$ denote the same with a capital-financed government.

An important initial result for the symmetric case is that in every equilibrium of every labor allocation subgame, every faction spends the same amount on internal conflict.

Lemma A.7. *If the game is symmetric and (p, r, c) is an equilibrium of $\Gamma(t)$ or $\Gamma_X(t)$, then $c_i = c_j$ for all $i, j \in \mathcal{N}$.*

Proof. Because the game is symmetric, there exists an equilibrium (p', r', c') in which $(p'_i, r'_i, c'_i) = (p_j, r_j, c_j)$, $(p'_j, r'_j, c'_j) = (p_i, r_i, c_i)$, and $(p'_k, r'_k, c'_k) = (p_k, r_k, c_k)$ for all $k \in \mathcal{N} \setminus \{i, j\}$. Proposition A.2 then implies $c_i = c_j$. $\square$

This means we can characterize an equilibrium of the labor allocation subgame in terms of just three variables: total production, total resistance, and the (common across factions) individual allocation to internal conflict. Using the characterization result of Lemma A.4, we can identify an equilibrium as a solution to some subset of the following system of equations derived from the first-order conditions. Each Q^{xy} represents marginal equality of the returns to x and y, while Q^b is the budget constraint. I write these as functions of t as well as the exogenous parameters $\pi = (\pi^p, \pi^r, \pi^c, L, N)$ to allow for comparative statics via implicit differentiation:

$$Q^{pr}(P, R, C; t, \pi) = \pi^p(1 - tg(R)) + \pi^r tPg'(R) = 0, \tag{A.6}$$

$$Q^{pc}(P, R, C; t, \pi) = \frac{\pi^p}{P} - \frac{N-1}{N}\pi^c\hat{\phi}'(C) = 0, \tag{A.7}$$

$$Q^{rc}(P, R, C; t, \pi) = \frac{\pi^r tg'(R)}{1 - tg(R)} + \frac{N-1}{N}\pi^c\hat{\phi}'(C) = 0, \tag{A.8}$$

$$Q^b(P, R, C; t, \pi) = L - \frac{P}{\pi^p} - \frac{R}{\pi^r} - \frac{NC}{\pi^c} = 0. \tag{A.9}$$

The condition (A.8) is redundant when the government is labor-financed and (A.6) and (A.7) both hold, but it is useful for characterizing equilibrium with a capital-financed government.

A.4 Proofs of Named Results

A.4.1 Proof of Proposition 1

The quantities defined in Proposition 1 are as follows. $(\tilde{R}_X(t), \tilde{c}_X(t))$ is the solution to the system

$$Q^{rc}(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi) = \frac{\pi^r t g'(\tilde{R}_X(t))}{1 - tg(\tilde{R}_X(t))} + \frac{N-1}{N}\pi^c \hat{\phi}'(\tilde{c}_X(t)) = 0, \tag{A.10}$$

$$Q^b(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi) = L - \frac{\tilde{R}_X(t)}{\pi^r} - \frac{N\tilde{c}_X(t)}{\pi^c} = 0. \tag{A.11}$$

The cutpoint tax rates are

$$\hat{t}_0^X = \frac{\eta \pi^c \hat{\phi}'(\pi^c L/N)}{\eta \pi^c \hat{\phi}'(\pi^c L/N) - \pi^r g'(0)}, \tag{A.12}$$

$$\hat{t}_1^X = \frac{\eta \pi^c \hat{\phi}'(0)}{\eta \pi^c g(\pi^r L)\hat{\phi}'(0) - \pi^r g'(\pi^r L)}, \tag{A.13}$$

where $\eta = (N-1)/N$. Observe that

$$\begin{aligned} \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} &= \eta \pi^c \hat{\phi}'(\pi^c L/N) = \pi^c \frac{\partial \log \omega_i((\pi^c L/N)\mathbf{1}_N)}{\partial c_i}, \\ \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1^X, (\pi^r L/N)\mathbf{1}_N)}{\partial r_i} &= \eta \pi^c \hat{\phi}'(0) \qquad = \pi^c \frac{\partial \log \omega_i(0)}{\partial c_i} \end{aligned}$$

for each $i \in \mathcal{N}$, where $\mathbf{1}_N$ is an N-vector of 1s.

Proposition 1. *If the government is capital-financed, every labor allocation subgame has a unique equilibrium. There exists a tax rate $\hat{t}_0^X \in (0,1)$ such that each $r_i = 0$ in equilibrium if and only if $t \leq \hat{t}_0^X$. There exists $\hat{t}_1^X > \hat{t}_0^X$ such that each $c_i = 0$ in equilibrium if and only if $t \geq \hat{t}_1^X$. For $t \in (\hat{t}_0^X, \hat{t}_1^X)$, in equilibrium each $r_i = \tilde{R}_X(t)/N > 0$ (strictly increasing in t) and each $c_i = \tilde{c}_X(t) > 0$ (strictly decreasing).*

Proof. $\Gamma_X(t)$ has an equilibrium (Proposition A.1), and there exists $c_X^*(t)$ such that each $c_i = c_X^*(t)$ in all of its equilibria (Proposition A.2 and Lemma A.7). The budget constraint then implies each $r_i = \pi^r(L/N - c_X^*(t)/\pi^c)$ in every equilibrium of $\Gamma_X(t)$, so the equilibrium is unique.

Let (r, c) be the equilibrium of $\Gamma_X(t)$. To simplify expressions in what follows, let $C = c_i = c_X^*(t)$ and $R = \sum_i r_i = \pi^r(L - NC/\pi^c)$. If $t \leq \hat{t}_0^X$ and $R > 0$, then the first-order conditions give

$$\pi^r \frac{\partial \log \bar{\tau}(t,r)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} = \pi^c \frac{\partial \log \omega_i((\pi^c L/N)\mathbf{1}_N)}{\partial c_i} \leq \pi^c \frac{\partial \log \omega_i(c)}{\partial c_i}$$

for each $i \in \mathcal{N}$. But this implies each $r_i = 0$, contradicting $R > 0$. Therefore, if $t \leq \hat{t}_0^X$, then $R = 0$. Similarly, if $t > \hat{t}_0^X$ and $R = 0$, then each $c_i = \pi^c L/N$ and thus

$$\pi^c \frac{\partial \log \omega_i(c)}{\partial c_i} = \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0^X, 0)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(t,r)}{\partial r_i}.$$

But this implies each $c_i = 0$, a contradiction. Therefore, if $t > \hat{t}_0^X$, then $R > 0$. The proof that $C > 0$ if and only if $t < \hat{t}_1^X$ is analogous.

For $t \in (\hat{t}_0^X, \hat{t}_1^X)$, the first-order conditions imply that R and C solve $Q^{rc}(0, R, C; t, \pi) = Q^b(0, R, C; t, \pi) = 0$; therefore, $R = \tilde{R}_X(t)$ and $C = \tilde{c}_X(t)$. To reduce clutter in what follows, I omit the evaluation point $(0, \tilde{R}_X(t), \tilde{c}_X(t); t, \pi)$ from all partial derivative expressions. The Jacobian of the system defining $(\tilde{R}_X(t), \tilde{c}_X(t))$ is

$$\begin{aligned} \mathbf{J}_X &= \begin{bmatrix} \partial Q^{rc}/\partial R & \partial Q^{rc}/\partial C \\ \partial Q^b/\partial R & \partial Q^b/\partial C \end{bmatrix} \\ &= \begin{bmatrix} \pi^r t \dfrac{g''(\tilde{R}_X(t)) - tg(\tilde{R}_X(t))^2 \hat{g}''(\tilde{R}_X(t))}{(1 - tg(\tilde{R}_X(t)))^2} & \eta \pi^c \hat{\phi}''(\tilde{c}_X(t)) \\ -1/\pi^r & -N/\pi^c \end{bmatrix} \end{aligned}$$

where $\eta = (N-1)/N$ and $\hat{g} = \log g$. Its determinant is

$$|\mathbf{J}_X| = \frac{\pi^c}{\pi^r} \left(\eta \hat{\phi}''(\tilde{c}_X(t)) - N \pi^r t \frac{g''(\tilde{R}_X(t)) - tg(\tilde{R}_X(t))^2 \hat{g}''(\tilde{R}_X(t))}{(1 - tg(\tilde{R}_X(t)))^2} \right) < 0.$$

By the implicit function theorem and Cramer's rule,

$$\begin{aligned} \frac{d\tilde{R}_X(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{rc}/\partial t & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial t & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X|} \\ &= \frac{\begin{vmatrix} -\pi^r g'(\tilde{R}_X(t))/(1 - tg(\tilde{R}_X(t)))^2 & \eta \pi^c \hat{\phi}''(\tilde{c}_X(t)) \\ 0 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X|} \\ &= \frac{N \pi^r g'(\tilde{R}_X(t))}{\pi^c (1 - tg(\tilde{R}_X(t)))^2 |\mathbf{J}_X|} \end{aligned}$$

$$> 0,$$

as claimed. The budget constraint then implies $d\tilde{c}_X(t)/dt < 0$, as claimed. □

A.4.2 Proof of Propositions 2 and 3

Before proving the results, I separately derive the comparative statics of $\hat{t}_0^X$ and $\tilde{R}_X(t)$ in N and π^c.

Lemma A.8. *The lower cutpoint $\hat{t}_0^X$ is strictly increasing in the number of factions, N. It is locally decreasing in the effectiveness of conflict, π^c, if and only if*

$$\hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N}\hat{\phi}''\left(\frac{\pi^c L}{N}\right) \geq 0.$$

Proof. Recall that

$$\hat{t}_0^X = \frac{((N-1)/N)\pi^c\hat{\phi}'(\pi^c L/N)}{((N-1)/N)\pi^c\hat{\phi}'(\pi^c L/N) - \pi^r g'(0)}.$$

Observe that $g'(0) < 0$, $(N-1)/N$ is strictly increasing in N, and $\hat{\phi}'(\pi^c L/N)$ is weakly increasing in N. Therefore, $\hat{t}_0^X$ is strictly increasing in N. Notice that

$$\frac{\partial}{\partial \pi^c}\left[\pi^c\hat{\phi}'\left(\frac{\pi^c L}{N}\right)\right] = \hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N}\hat{\phi}''\left(\frac{\pi^c L}{N}\right),$$

so $\hat{t}_0^X$ is locally increasing in π^c if and only if the above expression is positive. □

Lemma A.9. *For fixed $t \in (\hat{t}_0^X, \hat{t}_1^X)$, total resistance, $\tilde{R}_X(t)$, is strictly decreasing in the number of factions, N. It is locally decreasing in the effectiveness of conflict, π^c, if and only if*

$$\hat{\phi}'(\tilde{c}_X(t)) + \tilde{c}_X(t)\hat{\phi}''(\tilde{c}_X(t)) \geq 0.$$

Proof. I will treat N as if it were continuous in order to obtain comparative statics by implicit differentiation. Throughout the proof I write $\tilde{R}_X(t)$ and $\tilde{c}_X(t)$ as functions of (N, π^c).

I first consider comparative statics in N. To reduce clutter in what follows, I omit the evaluation point $(0, \tilde{R}_X(t; N, \pi^c), \tilde{c}_X(t; N, \pi^c); t, \pi)$ from all partial derivative expressions. By the implicit function theorem and Cramer's rule,

$$\frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial N} = \frac{\begin{vmatrix} -\partial Q^{rc}/\partial N & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial N & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X(t; N, \pi^c)|}$$

$$
\begin{aligned}
&= \frac{\begin{vmatrix} -\pi^c \hat{\phi}'(\tilde{c}_X(t;N,\pi^c))/N^2 & ((N-1)/N)\pi^c \hat{\phi}''(\tilde{c}_X(t;N,\pi^c)) \\ \tilde{c}_X(t;N,\pi^c)/\pi^c & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X(t;N,\pi^c)|} \\
&= \frac{\hat{\phi}'(\tilde{c}_X(t;N,\pi^c)) - (N-1)\tilde{c}_X(t;N,\pi^c)\hat{\phi}''(\tilde{c}_X(t;N,\pi^c))}{N|\mathbf{J}_X(t;N,\pi^c)|} \\
&< 0,
\end{aligned}
$$

as claimed, where $|\mathbf{J}_X(t;N,\pi^c)| < 0$ is defined as in the proof of Proposition 1.

I now consider comparative statics in π^c. Again by the implicit function theorem and Cramer's rule,

$$
\begin{aligned}
\frac{\partial \tilde{R}_X(t;N,\pi^c)}{\partial \pi^c} &= \frac{\begin{vmatrix} -\partial Q^{rc}/\partial \pi^c & \partial Q^{rc}/\partial C \\ -\partial Q^b/\partial \pi^c & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_X(t;N,\pi^c)|} \\
&= \frac{\begin{vmatrix} -((N-1)/N)\hat{\phi}'(\tilde{c}_X(t;N,\pi^c)) & ((N-1)/N)\pi^c \hat{\phi}''(\tilde{c}_X(t;N,\pi^c)) \\ -N\tilde{c}_X(t;N,\pi^c)/(\pi^c)^2 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_X(t;N,\pi^c)|} \\
&= \frac{(N-1)\left(\hat{\phi}'(\tilde{c}_X(t;N,\pi^c)) + \tilde{c}_X(t;N,\pi^c)\hat{\phi}''(\tilde{c}_X(t;N,\pi^c))\right)}{\pi^c|\mathbf{J}_X(t;N,\pi^c)|}.
\end{aligned}
$$

Therefore, $\partial \tilde{R}_X(t;N,\pi^c)/\partial \pi^c \leq 0$ if and only if

$$
\hat{\phi}'(\tilde{c}_X(t;N,\pi^c)) + \tilde{c}_X(t;N,\pi^c)\hat{\phi}''(\tilde{c}_X(t;N,\pi^c)) \geq 0,
$$

as claimed. □

The propositions, which I prove together, follow mainly from these lemmas.

Proposition 2. *A capital-financed government's equilibrium payoff is increasing in the number of factions, N.*

Proposition 3. *If there is a unique equilibrium tax rate t^*, the government's equilibrium payoff is locally increasing in competitive effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect (i.e., Equation 8 holds) at the corresponding equilibrium level of internal competition.*

Proof. Throughout the proof I write various equilibrium quantities, including the cutpoints $\hat{t}_0^X$ and $\hat{t}_1^X$, as functions of (N,π^c). Let the government's equilibrium payoff as a function of these parameters be

$$u_G^*(N, \pi^c) = \max_{t \in [0,1]} t \times g(R^*(t; N, \pi^c)) \times X.$$

I begin with the comparative statics on N. First, suppose $t = \hat{t}_0^X(N', \pi^c)$ is an equilibrium for all N' in a neighborhood of N.[18] Then $u_G^*(N', \pi^c) = \hat{t}_0^X(N', \pi^c) \times X$ in a neighborhood of N, which by Lemma A.8 is strictly increasing in N'. Next, suppose there is an equilibrium with $t \in (\hat{t}_0^X(N', \pi^c), \hat{t}_1^X(N', \pi^c))$ for all N' in a neighborhood of N. Then, by the envelope theorem,

$$\frac{\partial u_G^*(N, \pi^c)}{\partial N} = g'(\tilde{R}_X(t; N, \pi^c)) \frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial N} \times X > 0,$$

where the inequality follows from Lemma A.9. Finally, suppose $\hat{t}_1^X(N', \pi^c) < 1$ and $t = 1$ is an equilibrium for all N' in a neighborhood of N. Then $u_G^*(N, \pi^c) = g(\pi^r L) \times X$ is locally constant in N, and thus weakly increasing.

I now consider the comparative statics on π^c. First, suppose $t = \hat{t}_0^X(N, \pi^{c\prime})$ is an equilibrium for all $\pi^{c\prime}$ in a neighborhood of π^c. Then $u_G^*(N, \pi^{c\prime}) = \hat{t}_0^X(N, \pi^{c\prime}) \times X$ in a neighborhood of $\pi^{c\prime}$, which by Lemma A.8 is locally increasing at π^c if and only if

$$\hat{\phi}'\left(\frac{\pi^c L}{N}\right) + \frac{\pi^c L}{N} \hat{\phi}''\left(\frac{\pi^c L}{N}\right) \geq 0.$$

Next, suppose there is a unique equilibrium with $t \in (\hat{t}_0^X(N, \pi^{c\prime}), \hat{t}_1^X(N, \pi^{c\prime}))$ for all $\pi^{c\prime}$ in a neighborhood of π^c. Then, by the envelope theorem,

$$\frac{\partial u_G^*(N, \pi^c)}{\partial \pi^c} = g'(\tilde{R}_X(t; N, \pi^c)) \frac{\partial \tilde{R}_X(t; N, \pi^c)}{\partial \pi^c} \times X.$$

This is positive if and only if $\hat{\phi}'(\tilde{c}_X(t)) + \tilde{c}_X(t)\hat{\phi}''(\tilde{c}_X(t)) \geq 0$, per Lemma A.9. Finally, suppose $\hat{t}_1^X(N, \pi^{c\prime}) < 1$ and $t = 1$ is an equilibrium for all $\pi^{c\prime}$ in a neighborhood of π^c. Then $u_G^*(N, \pi^c) = g(\pi^r L) \times X$ is locally constant in π^c, and thus weakly increasing. □

I also prove the claim in footnote 13.

Lemma A.10. *Let $\theta, \lambda > 0$. If $\phi(C) = \theta \exp(\lambda C)$, then the incentive effect outweighs the labor-saving effect for all $C \geq 0$. If $\phi(C) = \theta C^\lambda$, then the incentive and labor-saving effects are exactly offsetting for all $C > 0$.*

Proof. First consider the difference contest success function, $\phi(C) = \theta \exp(\lambda C)$. Then

[18] There cannot be an equilibrium tax rate $t < \hat{t}_0^X$ or (if $\hat{t}_1^X < 1$) $t \in [\hat{t}_1^X, 1)$, as R^* is constant on $[0, \hat{t}_0^X]$ and on $[\hat{t}_1^X, 1]$.

$\hat{\phi}(C) = \log\theta + \lambda C$, $\hat{\phi}'(C) = \lambda$, and $\hat{\phi}''(C) = 0$ for all $C \geq 0$. Therefore,

$$\hat{\phi}'(C) + C\hat{\phi}''(C) = \lambda > 0.$$

Now consider the ratio contest success function, $\phi(C) = \theta C^{\lambda}$. Then $\hat{\phi}(C) = \log\theta + \lambda \log C$, $\hat{\phi}'(C) = \lambda/C$, and $\hat{\phi}''(C) = -\lambda/C^2$. Therefore,

$$\hat{\phi}'(C) + C\hat{\phi}''(C) = \frac{\lambda}{C} + C\left(\frac{-\lambda}{C^2}\right) = 0. \qquad \square$$

A.4.3 Proof of Proposition 4

The quantities defined in Proposition 4 are as follows. $(\bar{P}_0, \bar{c}_0)$ is the solution to the system

$$Q^{pc}(\bar{P}_0, 0, \bar{c}_0; t, \pi) = \frac{\pi^p}{\bar{P}_0} - \frac{N-1}{N}\pi^c\hat{\phi}'(\bar{c}_0) = 0, \tag{A.14}$$

$$Q^{b}(\bar{P}_0, 0, \bar{c}_0; t, \pi) = L - \frac{\bar{P}_0}{\pi^p} - \frac{N\bar{c}_0}{\pi^c} = 0. \tag{A.15}$$

$(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t))$ is the solution to the system

$$Q^{pr}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = \pi^p(1 - tg(\tilde{R}_1(t)) + \pi^r t\tilde{P}_1(t)g'(\tilde{R}_1(t)) = 0, \tag{A.16}$$

$$Q^{pc}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = \frac{\pi^p}{\tilde{P}_1(t)} - \frac{N-1}{N}\pi^c\hat{\phi}'(\tilde{c}_1(t)) = 0, \tag{A.17}$$

$$Q^{b}(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi) = L - \frac{\tilde{P}_1(t)}{\pi^p} - \frac{\tilde{R}_1(t)}{\pi^r} - \frac{N\tilde{c}_1(t)}{\pi^c} = 0. \tag{A.18}$$

$(\tilde{P}_2(t), \tilde{R}_2(t))$ is the solution to the system

$$Q^{pr}(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi) = \pi^p(1 - tg(\tilde{R}_2(t)) + \pi^r t\tilde{P}_2(t)g'(\tilde{R}_2(t)) = 0, \tag{A.19}$$

$$Q^{b}(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi) = L - \frac{\tilde{P}_2(t)}{\pi^p} - \frac{\tilde{R}_2(t)}{\pi^r} = 0. \tag{A.20}$$

The first cutpoint tax rate is

$$\hat{t}_0 = \frac{\pi^p}{\pi^p - \pi^r\bar{P}_0 g'(0)}. \tag{A.21}$$

Lemma A.11 below shows that $\bar{P}_0 > 0$ and therefore, since $g'(0) < 0$, that $\hat{t}_0 < 1$. The second cutpoint tax rate is

$$\hat{t}_1 = \frac{\pi^p}{\pi^p g(\bar{R}_1) - \pi^r\bar{P}_1 g'(\bar{R}_1)}, \tag{A.22}$$

where

$$\bar{P}_1 = \frac{N}{N-1}\frac{\pi^p}{\pi^c}\frac{1}{\hat{\phi}'(0)}, \tag{A.23}$$

$$\bar{R}_1 = \pi^r\left(L - \frac{\bar{P}_1}{\pi^p}\right). \tag{A.24}$$

The next three lemmas give conditions on the tax rate under which there is positive production, resistance, and internal conflict in the equilibrium of the labor allocation subgame. Jointly, these lemmas constitute the bulk of the proof of Proposition 4. The proofs rely on the following equalities:

$$\begin{aligned}
\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, 0)}{\partial r_i} &= -\pi^r \frac{\hat{t}_0 g'(0)}{1-\hat{t}_0} &&= \frac{\pi^p}{\bar{P}_0}, \\
\pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, (\bar{R}_1/N)\mathbf{1}_N)}{\partial r_i} &= -\pi^r \frac{\hat{t}_1 g'(\bar{R}_1)}{1-\hat{t}_1 g(\bar{R}_1)} &&= \frac{\pi^p}{\bar{P}_1}
\end{aligned}$$

for all $i \in \mathcal{N}$.

Lemma A.11. *If the game is symmetric, Assumption 1 holds, and (p, r, c) is an equilibrium of $\Gamma(t)$, then $0 < \sum_i p_i \leq \bar{P}_0 < \pi^p L$.*

Proof. Assumption 1 implies

$$Q^{pc}(\pi^p L, 0, 0; 0, \pi) = \frac{1}{L} - \frac{N-1}{N}\pi^c \hat{\phi}'(0) < 0.$$

Since Q^{pc} is decreasing in P and weakly increasing in C, this gives $\bar{P}_0 < \pi^p L$.

Let $P = \sum_i p_i$, and suppose $P > \bar{P}_0$. The budget constraint and Lemma A.7 then give $c_i = C < \bar{c}_0$ for each $i \in \mathcal{N}$. But then we have

$$\pi^c \frac{\partial \log \omega_i(c)}{\partial c_i} \geq \frac{N-1}{N}\pi^c \hat{\phi}'(\bar{c}_0) = \frac{\pi^p}{\bar{P}_0} > \pi^p \frac{\partial \log f(p)}{\partial p_i}$$

for each $i \in \mathcal{N}$. By Lemma A.4, this implies each $p_i = 0$, a contradiction. Therefore, $P \leq \bar{P}_0$.

Finally, since $P = 0$ implies each $u_i(t, p, r, c) = 0$, but any faction can assure itself a positive payoff with any $(p_i, r_i, c_i) \gg 0$, in equilibrium $P > 0$. □

Lemma A.12. *If the game is symmetric, Assumption 1 holds, and (p, r, c) is an equilibrium of $\Gamma(t)$, then $\sum_i r_i > 0$ if and only if $t > \hat{t}_0$.*

Proof. Let $P = \sum_i p_i$ and $R = \sum_i r_i$. To prove the "if" direction, suppose $t > \hat{t}_0$ and $R = 0$. Since $P \leq \bar{P}_0$, this implies each $c_i = C > 0$; the first-order conditions of Lemma A.4 then give $P = \bar{P}_0$ and $C = \bar{c}_0$. It follows that

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, r)}{\partial r_i} = \frac{\pi^p}{\bar{P}_0} = \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

This implies each $p_i = 0$, a contradiction.

To prove the "only if" direction, suppose $t \leq \hat{t}_0$ and $R > 0$. For each $i \in \mathcal{N}$,

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} < \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_0, 0)}{\partial r_i} = \frac{\pi^p}{\bar{P}_0} \leq \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

This implies each $r_i = 0$, a contradiction. □

Lemma A.13. *If the game is symmetric and Assumption 1 holds, then $\hat{t}_1 > \hat{t}_0$. If, in addition, (p, r, c) is an equilibrium of $\Gamma(t)$, then each $c_i > 0$ if and only if $t < \hat{t}_1$.*

Proof. To prove that $\hat{t}_1 > \hat{t}_0$, note that

$$\bar{P}_1 = \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(0)} \leq \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(\bar{c}_0)} = \bar{P}_0$$

by log-concavity of ϕ. This implies $\bar{R}_1 > 0$, so $g(\bar{R}_1) < g(0) = 1$ and $g'(0) \leq g'(\bar{R}_1) < 0$. Therefore,

$$\pi^p - \pi^r \bar{P}_0 g'(0) > \pi^p g(\bar{R}_1) - \pi^r \bar{P}_1 g'(\bar{R}_1) > 0,$$

which implies $\hat{t}_1 > \hat{t}_0$.

Let $P = \sum_i p_i$ and $R = \sum_i r_i$. To prove the "if" direction of the second statement, suppose $t \geq \hat{t}_1$ and some $c_i > 0$. By Lemma A.7, $c_j = c_i = C > 0$ for each $j \in \mathcal{N}$. Since $P > 0$ by Lemma A.11, the first-order conditions give

$$P = \frac{N}{N-1} \frac{\pi^p}{\pi^c} \frac{1}{\hat{\phi}'(C)} \geq \bar{P}_1.$$

The budget constraint then gives $R < \bar{R}_1$ and thus

$$\pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, (\bar{R}_1/N)\mathbf{1}_N)}{\partial r_i} = \frac{\pi^p}{\bar{P}_1} \geq \pi^p \frac{\partial \log f(p)}{\partial p_i}.$$

But this implies each $p_i = 0$, a contradiction.

To prove the "only if" direction, suppose $t < \hat{t}_1$ and each $c_i = 0$. The first-order conditions then give $P \leq \bar{P}_1$, so $R \geq \bar{R}_1 > 0$ by the budget constraint. This in turn gives

$$\pi^p \frac{\partial \log f(p)}{\partial p_i} = \frac{\pi^p}{P} \geq \frac{\pi^p}{\bar{P}_1} \geq \pi^r \frac{\partial \log \bar{\tau}(\hat{t}_1, r)}{\partial r_i} > \pi^r \frac{\partial \log \bar{\tau}(t, r)}{\partial r_i}$$

for each $i \in \mathcal{N}$. But this implies each $r_i = 0$, a contradiction. $\square$

The last thing we need to prove the propositions is how the labor allocations change with the tax rate when $t > \hat{t}_0$.

Lemma A.14. *Let the game be symmetric and Assumption 1 hold. For all $t \in (\hat{t}_0, \hat{t}_1)$,*

$$\frac{d\tilde{P}_1(t)}{dt} = \frac{-(N-1)\pi^p \pi^c \hat{\phi}''(\tilde{c}_1(t))}{N\pi^r t \Delta_1(t)} \leq 0, \tag{A.25}$$

$$\frac{d\tilde{R}_1(t)}{dt} = \frac{-\pi^p \left(N\pi^p/\pi^c \tilde{P}_1(t)^2 - (N-1)\pi^c \hat{\phi}''(\tilde{c}_1(t))/N\pi^p\right)}{t\Delta_1(t)} > 0, \tag{A.26}$$

$$\frac{d\tilde{c}_1(t)}{dt} = \frac{(\pi^p)^2}{\pi^r t \tilde{P}_1(t)^2 \Delta_1(t)} < 0, \tag{A.27}$$

where

$$\begin{aligned}\Delta_1(t) = {} & \left(\pi^p t g'(\tilde{R}_1(t)) - \pi^r t \tilde{P}_1(t) g''(\tilde{R}_1(t))\right)\left(\frac{N\pi^p}{\pi^c \tilde{P}_1(t)^2} - \frac{N-1}{N}\frac{\pi^c}{\pi^p}\hat{\phi}''(\tilde{c}_1(t))\right) \\ & - \frac{N-1}{N}\pi^c t g'(\tilde{R}_1(t))\hat{\phi}''(\tilde{c}_1(t)) \\ < {} & 0.\end{aligned} \tag{A.28}$$

For all $t > \hat{t}_1$,

$$\frac{d\tilde{P}_2(t)}{dt} = \frac{-\pi^p}{\pi^r t \Delta_2(t)} < 0, \tag{A.29}$$

$$\frac{d\tilde{R}_2(t)}{dt} = \frac{1}{t\Delta_2(t)} > 0, \tag{A.30}$$

where

$$\Delta_2(t) = \frac{\pi^r}{\pi^p} t \tilde{P}_2(t) g''(\tilde{R}_2(t)) - 2tg'(\tilde{R}_2(t)) > 0. \tag{A.31}$$

Proof. Throughout the proof, let $\eta = (N-1)/N$.

First consider $t \in (\hat{t}_0, \hat{t}_1)$. To reduce clutter in what follows, I omit the evaluation point $(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t); t, \pi)$ from all partial derivative expressions. The Jacobian of the system of equations that defines $(\tilde{P}_1(t), \tilde{R}_1(t), \tilde{c}_1(t))$ is

$$\begin{aligned}
\mathbf{J}_1(t) &= \begin{bmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R & \partial Q^{pr}/\partial C \\ \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial R & \partial Q^{pc}/\partial C \\ \partial Q^{b}/\partial P & \partial Q^{b}/\partial R & \partial Q^{b}/\partial C \end{bmatrix} \\
&= \begin{bmatrix} \pi^r t g'(\tilde{R}_1(t)) & \pi^r t \tilde{P}_1(t) g''(\tilde{R}_1(t)) - \pi^p t g'(\tilde{R}_1(t)) & 0 \\ -\pi^p/\tilde{P}_1(t)^2 & 0 & -\eta\pi^c\hat{\phi}''(\tilde{c}_1(t)) \\ -1/\pi^p & -1/\pi^r & -N/\pi^c \end{bmatrix}.
\end{aligned}$$

It is easy to verify that $|\mathbf{J}_1(t)| = \Delta_1(t) < 0$. Notice that

$$\begin{aligned}
\frac{\partial Q^{pr}}{\partial t} &= \pi^r \tilde{P}_1(t) g'(\tilde{R}_1(t)) - \pi^p g(\tilde{R}_1(t)) \\
&= \pi^r \left(-\frac{\pi^p(1 - t g(\tilde{R}_1(t)))}{\pi^r t g'(\tilde{R}_1(t))} \right) g'(\tilde{R}_1(t)) - \pi^p g(\tilde{R}_1(t)) \\
&= -\frac{\pi^p}{t}.
\end{aligned}$$

Then, by the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{d\tilde{P}_1(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial R & \partial Q^{pr}/\partial C \\ -\partial Q^{pc}/\partial t & \partial Q^{pc}/\partial R & \partial Q^{pc}/\partial C \\ -\partial Q^{b}/\partial t & \partial Q^{b}/\partial R & \partial Q^{b}/\partial C \end{vmatrix}}{|\mathbf{J}_1(t)|} \\
&= \frac{-\eta\pi^p\pi^c\hat{\phi}''(\tilde{c}_1(t))}{\pi^r t \Delta_1(t)} \\
&\leq 0, \\
\frac{d\tilde{R}_1(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial C \\ \partial Q^{pc}/\partial P & -\partial Q^{pc}/\partial t & \partial Q^{pc}/\partial C \\ \partial Q^{b}/\partial P & -\partial Q^{b}/\partial t & \partial Q^{b}/\partial C \end{vmatrix}}{|\mathbf{J}_1(t)|} \\
&= \frac{-\pi^p \left(N\pi^p/\pi^c\tilde{P}_1(t)^2 - \eta\pi^c\hat{\phi}''(\tilde{c}_1(t))/\pi^p \right)}{t\Delta_1(t)} \\
&> 0,
\end{aligned}$$

$$
\begin{aligned}
\frac{d\tilde{c}_1(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R & -\partial Q^{pr}/\partial t \\ \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial R & -\partial Q^{pc}/\partial t \\ \partial Q^{b}/\partial P & \partial Q^{b}/\partial R & -\partial Q^{b}/\partial t \end{vmatrix}}{|\mathbf{J}_1(t)|} \\
&= \frac{(\pi^p)^2}{\pi^r t \tilde{P}_1(t)^2 \Delta_1(t)} \\
&< 0,
\end{aligned}
$$

as claimed.

Now consider $t > \hat{t}_1$. Again to reduce clutter in what follows, I omit the evaluation point $(\tilde{P}_2(t), \tilde{R}_2(t), 0; t, \pi)$ from all partial derivative expressions. The Jacobian of the system of equations that defines $(\tilde{P}_2(t), \tilde{R}_2(t))$ is

$$
\begin{aligned}
\mathbf{J}_2(t) &= \begin{bmatrix} \partial Q^{pr}/\partial P & \partial Q^{pr}/\partial R \\ \partial Q^{b}/\partial P & \partial Q^{b}/\partial R \end{bmatrix} \\
&= \begin{bmatrix} \pi^r t g'(\tilde{R}_2(t)) & \pi^r t \tilde{P}_2(t) g''(\tilde{R}_2(t)) - \pi^p t g'(\tilde{R}_2(t)) \\ -1/\pi^p & -1/\pi^r \end{bmatrix}.
\end{aligned}
$$

It is easy to verify that $|\mathbf{J}_2(t)| = \Delta_2(t) > 0$. As before, $\partial Q^{pr}/\partial t = -\pi^p/t$. So by the implicit function theorem and Cramer's rule,

$$
\begin{aligned}
\frac{d\tilde{P}_2(t)}{dt} &= \frac{\begin{vmatrix} -\partial Q^{pr}/\partial t & \partial Q^{pr}/\partial R \\ -\partial Q^{b}/\partial t & \partial Q^{b}/\partial R \end{vmatrix}}{|\mathbf{J}_2(t)|} \\
&= \frac{-\pi^p}{\pi^r t \Delta_2(t)} \\
&< 0, \\
\frac{d\tilde{R}_2(t)}{dt} &= \frac{\begin{vmatrix} \partial Q^{pr}/\partial P & -\partial Q^{pr}/\partial t \\ \partial Q^{b}/\partial P & -\partial Q^{b}/\partial t \end{vmatrix}}{|\mathbf{J}_2(t)|} \\
&= \frac{1}{t \Delta_2(t)} \\
&> 0,
\end{aligned}
$$

as claimed. □

The proof of Proposition 4 follows almost immediately from these lemmas.

Proposition 4. *Assume the government is labor-financed. There exist tax rates $\hat{t}_0 \in (0,1)$ and $\hat{t}_1 > \hat{t}_0$ such that in every equilibrium of the labor allocation subgame with tax rate t:*

- *If $t \leq \hat{t}_0$, then each $p_i = \bar{P}_0/N > 0$, each $r_i = 0$, and each $c_i = \bar{c}_0 > 0$.*
- *If $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = \tilde{P}_1(t) > 0$ (weakly decreasing in t), $\sum_i r_i = \tilde{R}_1(t) > 0$ (strictly increasing), and each $c_i = \tilde{c}_1(t) > 0$ (strictly decreasing).*
- *If $t \geq \hat{t}_1$, then $\sum_i p_i = \tilde{P}_2(t) > 0$ (strictly decreasing in t), $\sum_i r_i = \tilde{R}_2(t) > 0$ (strictly increasing), and each $c_i = 0$.*

Proof. For fixed t, every equilibrium of $\Gamma(t)$ has the same total production, total resistance, and individual conflict allocations, per Proposition A.2. Consider any $t \in [0,1]$ and let (p, r, c) be an equilibrium of $\Gamma(t)$.

If $t \leq \hat{t}_0$, then $\sum_i p_i = P > 0$, $\sum_i r_i = 0$, and each $c_i = C > 0$ by Lemmas A.11–A.13. The first-order conditions (Lemma A.4) imply that P and C solve $Q^{pc}(P, 0, C; t, \pi) = Q^b(P, 0, C; t, \pi) = 0$; therefore, $P = \bar{P}_0$ and $C = \bar{c}_0$. Since each $r_i = 0$, each $p_i = \pi^p(L/N - \bar{c}_0/\pi^c) = \bar{P}_0/N$, so the equilibrium is unique.

Similarly, if $t \in (\hat{t}_0, \hat{t}_1)$, then $\sum_i p_i = P > 0$, $\sum_i r_i = R > 0$, and each $c_i = C > 0$ by Lemmas A.11–A.13. The first-order conditions then imply that these solve the system (A.6)–(A.9); therefore, $P = \tilde{P}_1(t)$, $R = \tilde{R}_1(t)$, and $C = \tilde{c}_1(t)$. The comparative statics on $\tilde{P}_1$, $\tilde{R}_1$, and $\tilde{c}_1$ follow from Lemma A.14.

Finally, if $t \geq \hat{t}_1$, then $\sum_i p_i = P > 0$, $\sum_i r_i = R > 0$, and each $c_i = 0$ by Lemmas A.11–A.13. The first-order conditions then imply that P and R solve $Q^{pr}(P, R, 0; t, \pi) = Q^b(P, R, 0; t, \pi) = 0$; therefore, $P = \tilde{P}_2(t)$ and $R = \tilde{R}_2(t)$. The comparative statics on $\tilde{P}_2$ and $\tilde{R}_2$ follow from Lemma A.14. □

A.4.4 Proof of Proposition 5

Proposition 5. *If the government is labor-financed, there is an equilibrium in which the government chooses the greatest tax rate that engenders no resistance, $t = \hat{t}_0$. If g or ϕ is strictly log-concave, this is the unique equilibrium tax rate.*

Proof. As in the proof of Lemma A.14, let $\eta = (N-1)/N$.

For each $t \in [0,1]$, fix an equilibrium $(p(t), r(t), c(t))$ of $\Gamma(t)$. By Propositions 4 and A.2, the government's induced utility function is

$$u_G^*(t) = u_G(t, p(t), r(t), c(t)) = \begin{cases} t\bar{P}_0 & t \le \hat{t}_0, \\ tg(\tilde{R}_1(t))\tilde{P}_1(t) & \hat{t}_0 < t < \hat{t}_1, \\ tg(\tilde{R}_2(t))\tilde{P}_2(t) & t \ge \hat{t}_1. \end{cases}$$

It is immediate from the above expression that $u_G^*(t) < u_G^*(\hat{t}_0)$ for all $t < \hat{t}_0$.

Now consider $t \in (\hat{t}_0, \hat{t}_1)$. By Lemma A.14,

$$\begin{aligned}
\frac{du_G^*(t)}{dt} &= g(\tilde{R}_1(t))\tilde{P}_1(t) + tg'(\tilde{R}_1(t))\frac{d\tilde{R}_1(t)}{dt}\tilde{P}_1(t) + tg(\tilde{R}_1(t))\frac{d\tilde{P}_1(t)}{dt} \\
&= g(\tilde{R}_1(t))\tilde{P}_1(t) - \frac{\pi^p g'(\tilde{R}_1(t))\tilde{P}_1(t)\left(N\pi^p/\pi^c\tilde{P}_1(t)^2 - \eta\pi^c\hat{\phi}''(\tilde{c}_1(t))/\pi^p\right)}{\Delta_1(t)} \\
&\quad - \frac{\eta\pi^p\pi^c g(\tilde{R}_1(t))\hat{\phi}''(\tilde{c}_1(t))}{\pi^r\Delta_1(t)},
\end{aligned}$$

where $\Delta_1(t)$ is defined by (A.28). To reduce clutter in what follows, let $\tilde{P} = \tilde{P}_1(t)$, $\tilde{R} = \tilde{R}_1(t)$, and $\tilde{c} = \tilde{c}_1(t)$. Since $\Delta_1(t) < 0$, the sign of the above expression is the same as that of

$$\begin{aligned}
& g'(\tilde{R})\tilde{P}\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) + \frac{\eta\pi^p\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})}{\pi^r} - g(\tilde{R})\tilde{P}\Delta_1(t) \\
&= \tilde{P}g'(\tilde{R})\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) + \frac{\eta\pi^p\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})}{\pi^r} \\
&\quad - \tilde{P}g(\tilde{R})\left(\pi^p tg'(\tilde{R}) - \pi^r t\tilde{P}g''(\tilde{R})\right)\left(\frac{N\pi^p}{\pi^c\tilde{P}^2} - \eta\frac{\pi^c}{\pi^p}\hat{\phi}''(\tilde{c})\right) \\
&\quad + \eta\pi^c t\tilde{P}g(\tilde{R})g'(\tilde{R})\hat{\phi}''(\tilde{c}) \\
&= \tilde{P}\left(g'(\tilde{R}) - \frac{g(\tilde{R})\left(\pi^p tg'(\tilde{R}) - \pi^r t\tilde{P}g''(\tilde{R})\right)}{\pi^p}\right)\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) \\
&\quad + \eta\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})\left(\frac{\pi^p}{\pi^r} + t\tilde{P}g'(\tilde{R})\right) \\
&= \frac{\tilde{P}}{g'(\tilde{R})}(1 - tg(\tilde{R}))\left(g'(\tilde{R})^2 - g(\tilde{R})g''(\tilde{R})\right)\left(\frac{N(\pi^p)^2}{\pi^c\tilde{P}^2} - \eta\pi^c\hat{\phi}''(\tilde{c})\right) \\
&\quad + \eta\pi^c g(\tilde{R})\hat{\phi}''(\tilde{c})\left(\frac{\pi^p}{\pi^r}tg(\tilde{R})\right).
\end{aligned}$$

The first term is weakly negative, strictly so if g is strictly log-concave. The second term is weakly negative, strictly so if ϕ is strictly log-concave. Therefore, $du_G^*(t)/dt \le 0$ for all

$t \in (\hat{t}_0, \hat{t}_1)$, strictly so if g or ϕ is strictly log-concave. This implies $u_G^*(\hat{t}_0) \geq u_G^*(t)$ for all $t \in (\hat{t}_0, \hat{t}_1]$, strictly so if g or ϕ is strictly log-concave.

Finally, consider $t > \hat{t}_1$. Again by Lemma A.14,

$$\begin{aligned}
\frac{du_G^*(t)}{dt} &= g(\tilde{R}_2(t))\tilde{P}_2(t) + tg'(\tilde{R}_2(t))\frac{d\tilde{R}_2(t)}{dt}\tilde{P}_2(t) + tg(\tilde{R}_2(t))\frac{d\tilde{P}_2(t)}{dt} \\
&= g(\tilde{R}_2(t))\tilde{P}_2(t) + \frac{g'(\tilde{R}_2(t))\tilde{P}_2(t)}{\Delta_2(t)} - \frac{\pi^p g(\tilde{R}_2(t))}{\pi^r \Delta_2(t)},
\end{aligned}$$

where $\Delta_2(t)$ is defined by (A.31). To reduce clutter in what follows, let $\tilde{P} = \tilde{P}_2(t)$ and $\tilde{R} = \tilde{R}_2(t)$. Since $\Delta_2(t) > 0$, the sign of the above expression is the same as that of

$$\begin{aligned}
&\tilde{P}g(\tilde{R})\Delta_2(t) + \tilde{P}g'(\tilde{R}) - \frac{\pi^p g(\tilde{R})}{\pi^r} \\
&\quad = \tilde{P}g(\tilde{R})\left(\frac{\pi^r t\tilde{P}g''(\tilde{R})}{\pi^p} - 2tg'(\tilde{R})\right) + \tilde{P}g'(\tilde{R}) - \frac{\pi^p g(\tilde{R})}{\pi^r} \\
&\quad = \frac{\pi^p}{\pi^r tg'(\tilde{R})^2}\left((1 - tg(\tilde{R}))^2\left(g(\tilde{R})g''(\tilde{R}) - g'(\tilde{R})^2\right) - \left(tg(\tilde{R}g'(\tilde{R}))\right)^2\right) \\
&\quad < \frac{\pi^p(1 - tg(\tilde{R}))^2\left(g(\tilde{R})g''(\tilde{R}) - g'(\tilde{R})^2\right)}{\pi^r tg'(\tilde{R})^2} \\
&\quad \leq 0.
\end{aligned}$$

Therefore, $u_G^*(\hat{t}_0) \geq u_G^*(\hat{t}_1) > u_G^*(t)$ for all $t > \hat{t}_1$.

Combining these findings, $u_G^*(\hat{t}_0) \geq u_G^*(t)$ for all $t \in [0, 1] \setminus \{\hat{t}_0\}$, strictly so if g or ϕ is strictly log-concave. Therefore, there is an equilibrium in which $t = \hat{t}_0$, and every equilibrium has this tax rate if g or ϕ is strictly log-concave. □

A.4.5 Proof of Proposition 6

Proposition 6. *Equilibrium production with a labor-financed government, $\bar{P}_0$, is strictly decreasing in the number of factions, N. It is locally decreasing in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

Proof. I will treat N as if it were continuous in order to obtain comparative statics by implicit differentiation. Throughout the proof I write $\bar{P}_0$ and $\bar{c}_0$ as functions of (N, π^c).

Recall that $(\bar{P}_0(N, \pi^c), \bar{c}_0(N, \pi^c))$ is defined as the solution to (A.14) and (A.15). To reduce clutter in what follows, I omit the evaluation point $(\bar{P}_0(N, \pi^c), 0, \bar{c}_0(N, \pi^c); t, \pi)$ from all partial derivative expressions. The Jacobian of the system is

$$\mathbf{J}_0 = \begin{bmatrix} \partial Q^{pc}/\partial P & \partial Q^{pc}/\partial C \\ \partial Q^b/\partial P & \partial Q^b/\partial C \end{bmatrix} = \begin{bmatrix} -\pi^p/\bar{P}_0(N,\pi^c)^2 & -(N-1)\pi^c\hat{\phi}''(\bar{c}_0(N,\pi^c))/N \\ -1/\pi^p & -N/\pi^c \end{bmatrix},$$

with determinant

$$|\mathbf{J}_0| = \frac{N\pi^p}{\pi^c\bar{P}_0(N,\pi^c)^2} - \frac{N-1}{N}\frac{\pi^c}{\pi^p}\hat{\phi}''(\bar{c}_0(N,\pi^c)) > 0.$$

By the implicit function theorem and Cramer's rule,

$$\begin{aligned}
\frac{\partial \bar{P}_0(N,\pi^c)}{\partial N} &= \frac{\begin{vmatrix} -\partial Q^{pc}/\partial N & \partial Q^{pc}/\partial C \\ -\partial Q^b/\partial N & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_0|} \\
&= \frac{\begin{vmatrix} \pi^c\hat{\phi}'(\bar{c}_0(N,\pi^c))/N^2 & -(N-1)\pi^c\hat{\phi}''(\bar{c}_0(N,\pi^c))/N \\ \bar{c}_0(N,\pi^c)/\pi^c & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_0|} \\
&= \frac{1}{|\mathbf{J}_0|}\left(\frac{N-1}{N}\bar{c}_0(N,\pi^c)\hat{\phi}''(\bar{c}_0(N,\pi^c)) - \frac{\hat{\phi}'(\bar{c}_0(N,\pi^c))}{N}\right) \\
&< 0,
\end{aligned}$$

as claimed. Similarly,

$$\begin{aligned}
\frac{\partial \bar{P}_0(N,\pi^c)}{\partial \pi^c} &= \frac{\begin{vmatrix} -\partial Q^{pc}/\partial \pi^c & \partial Q^{pc}/\partial C \\ -\partial Q^b/\partial \pi^c & \partial Q^b/\partial C \end{vmatrix}}{|\mathbf{J}_0|} \\
&= \frac{\begin{vmatrix} (N-1)\hat{\phi}'(\bar{c}_0(N,\pi^c))/N & -(N-1)\pi^c\hat{\phi}''(\bar{c}_0(N,\pi^c))/N \\ -N\bar{c}_0(N,\pi^c)/(\pi^c)^2 & -N/\pi^c \end{vmatrix}}{|\mathbf{J}_0|} \\
&= -\frac{N-1}{\pi^c|\mathbf{J}_0|}\left(\hat{\phi}'(\bar{c}_0(N,\pi^c)) + \bar{c}_0(N,\pi^c)\hat{\phi}''(\bar{c}_0(N,\pi^c))\right),
\end{aligned}$$

which is negative if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0(N,\pi^c)$. $\square$

A.4.6 Proof of Proposition 7

Proposition 7. *The equilibrium tax rate of a labor-financed government,*

$$\hat{t}_0 = \frac{\pi^p}{\pi^p - \pi^r \bar{P}_0 g'(0)},$$

is strictly increasing in fractionalization, N. It strictly increases with a marginal increase in competitive effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.

Proof. Immediate from Proposition 6, as $\hat{t}_0$ is strictly decreasing in $\bar{P}_0$, and N and π^c enter the expression for $\hat{t}_0$ only via $\bar{P}_0$. □

A.4.7 Proof of Proposition 8

Proposition 8. *A labor-financed government's equilibrium payoff is strictly decreasing in the number of factions, N. It strictly decreases with a marginal increase in conflict effectiveness, π^c, if and only if the incentive effect outweighs the labor-saving effect at $\bar{c}_0$.*

Proof. By Proposition 5, the government's equilibrium payoff is

$$\hat{t}_0 \bar{P}_0 = \frac{\pi^p \bar{P}_0}{\pi^p - \pi^r \bar{P}_0 g'(0)} = \frac{\pi^p}{(\pi^p / \bar{P}_0) - \pi^r g'(0)}.$$

This expression is strictly increasing in $\bar{P}_0$. Since N and π^c only enter through the equilibrium value of $\bar{P}_0$, the claim follows from Proposition 6. □

A.5 Extensions

A.5.1 Endogenous Inequality

In the model with *asymmetric taxation*, the government is labor-financed and taxes each faction's production separately. To keep the analysis simple, I assume throughout the extension that $N = 2$. The government chooses a pair of tax rates, t_1 and t_2, where each $t_i \in [0, 1]$ as before. The factions then respond as before, by allocating their labor among production, resistance, and internal conflict, (p_i, r_i, c_i), subject to the budget constraint, Equation 1. I consider tax schemes such that $t_1 \geq t_2$; as the factions remain identical *ex ante*, this restriction is without loss of generality. The utility functions for the government and the factions are now

$$u_G(t,p,r,c) = \tau(t_1,r)p_1 + \tau(t_2,r)p_2,$$
$$u_i(t,p,r,c) = \omega_i(c)\left[\bar{\tau}(t_1,r)p_1 + \bar{\tau}(t_2,r)p_2\right].$$

If the government chooses the same tax rate for both groups, $t_1 = t_2$, then each player's utility is the same as in the original model with that rate. Throughout the analysis of this extension, I impose an additional technical condition on the function that translates c_i into effective strength in the internal conflict: I assume ϕ'/ϕ is convex.[19]

Similar to above, let $\Gamma(t_1, t_2)$ denote the labor allocation subgame following the government's choice of the given tax rates. Notice that in the model with asymmetric taxation we have

$$\begin{aligned}
\frac{\partial u_i(t,p,r,c)}{\partial p_i} &= \frac{\phi(c_i)}{\phi(c_i)+\phi(c_j)}(1 - t_i g(r_i + r_j)), \\
\frac{\partial u_i(t,p,r,c)}{\partial r_i} &= \frac{\phi(c_i)}{\phi(c_i)+\phi(c_j)}(-g'(r_i + r_j))(t_i p_i + t_j p_j), \\
\frac{\partial u_i(t,p,r,c)}{\partial c_i} &= \frac{\phi'(c_i)\phi(c_j)}{(\phi(c_i)+\phi(c_j))^2}\left[(1 - t_i g(r_i + r_j))p_i + (1 - t_j g(r_i + r_j))p_j\right].
\end{aligned}$$

To analyze the extension, I first consider how the factions would respond to the choice of unequal tax rates. Naturally, as taxation reduces the marginal benefit of production, the faction that is taxed more produces less in equilibrium. The more highly taxed faction then shifts some of the labor it would have spent on economic production into resistance and internal conflict. This has the counterintuitive implication that the equilibrium payoff for the more-taxed faction is no less than that of the less-taxed faction. By reducing a group's incentive to produce, the government increases its incentive to appropriate from the other group, resulting in it taking home a disproportionate share of the total post-tax output. This result is reminiscent of the "paradox of power" characterized by Hirshleifer (1991), wherein seemingly weaker groups expend disproportionate effort on appropriation. The following proposition summarizes the equilibrium responses to unequal taxation.

Proposition A.4. *In the game with asymmetric taxation, if the government chooses $t_1 > t_2$, then $p_1 \le p_2$, $r_1 \ge r_2$, and $c_1 \ge c_2$ in any equilibrium of the subsequent labor allocation subgame.*

Proof. First I will prove $c_1 \ge c_2$. To this end, suppose $c_i > c_j$; I will show this implies $t_i > t_j$

[19] The baseline assumptions imply that ϕ'/ϕ is positive and decreasing, so convexity is a natural restriction. The difference and ratio functional forms described above in footnote 13 both satisfy this condition.

and thus $i = 1$. Log-concavity of ϕ implies $\phi'(c_i)\phi(c_j) \leq \phi'(c_j)\phi(c_i)$, so we have

$$\pi^c \frac{\partial u_i(t,p,r,c)}{\partial c_i} \leq \pi^c \frac{\partial u_j(t,p,r,c)}{\partial c_j}.$$

The first-order conditions of equilibrium then imply

$$\max\left\{\pi^p \frac{\partial u_i(t,p,r,c)}{\partial p_i}, \pi^r \frac{\partial u_i(t,p,r,c)}{\partial r_i}\right\} \leq \max\left\{\pi^p \frac{\partial u_j(t,p,r,c)}{\partial p_j}, \pi^r \frac{\partial u_j(t,p,r,c)}{\partial r_j}\right\}.$$

As $\phi(c_i) > \phi(c_j)$, this can hold only if $1 - t_i g(r_i + r_j) < 1 - t_j g(r_i + r_j)$; i.e., $t_i > t_j$.

I now prove $r_1 \geq r_2$. First suppose $c_1 > 0$. The first-order conditions of equilibrium, combined with the fact that $c_1 \geq c_2$, imply

$$\pi^c \frac{\partial u_2(t,p,r,c)}{\partial c_2} \geq \pi^c \frac{\partial u_1(t,p,r,c)}{\partial c_1} \geq \pi^r \frac{\partial u_1(t,p,r,c)}{\partial r_1} > \pi^r \frac{\partial u_2(t,p,r,c)}{\partial r_2}.$$

It then follows from the first-order conditions that $r_2 = 0$, which implies $r_1 \geq r_2$. On the other hand, suppose $c_1 = 0$, in which case $c_2 = 0$ per above. It is then immediate from the budget constraint that $r_1 \geq r_2$ if $p_1 = 0$, so suppose $p_1 > 0$. The first-order conditions and $t_1 > t_2$ imply

$$\pi^p \frac{\partial u_2(t,p,r,c)}{\partial p_2} > \pi^p \frac{\partial u_1(t,p,r,c)}{\partial p_1} \geq \pi^r \frac{\partial u_1(t,p,r,c)}{\partial r_1} = \pi^r \frac{\partial u_2(t,p,r,c)}{\partial r_2}.$$

It then follows from the first-order conditions that $r_2 = 0$, which implies $r_1 \geq r_2$.

Finally, under the budget constraint, $c_1 \geq c_2$ and $r_1 \geq r_2$ imply $p_1 \leq p_2$. □

The factions' responses show why asymmetric taxation is ultimately unprofitable for a labor-financed government. There is obviously no profit to be made from the more highly taxed faction, as it reduces its production in response to the greater taxation. But as the more-taxed faction increases its appropriative efforts, the less-taxed faction also loses some of its incentive to engage in productive activity. The decrease in the less-taxed faction's incentive to resist does not make up the difference, as the marginal benefit of resistance remains a function of the government's overall payoff, just as in the baseline model. Ultimately, then, the government is no better off having the ability to set unequal tax rates across groups.

Proposition A.5. *Asymmetric taxation does not raise the equilibrium payoff of a labor-financed government.*

Proof. Assume $t_1 > t_2$, and let (p, r, c) be an equilibrium of $\Gamma(t_1, t_2)$. Let $\hat{t}_0$, $\bar{P}_0$, and $\bar{c}_0$ be defined as in Proposition 4. In addition, let $P = p_1 + p_2$, $R = r_1 + r_2$, and $C = c_1 + c_2$.

My first task is to prove $P \leq \bar{P}_0$. As any equilibrium entails $P > 0$, it follows from Proposition A.4 that $p_2 > 0$. If $p_1 = 0$, in which case $P = p_2$, the first-order conditions for equilibrium imply

$$\pi^p \phi(c_2) \geq \frac{\pi^c \phi'(c_2)\phi(c_1)}{\phi(c_1) + \phi(c_2)} P.$$

Rearranging terms and applying the fact that $\phi(c_1) \geq \phi(c_2)$ (per Proposition A.4) gives

$$P \leq \frac{\pi^p(\phi(c_1) + \phi(c_2))\phi(c_2)}{\pi^c \phi(c_1)\phi'(c_2)} \leq \frac{2\pi^p \phi(c_2)}{\pi^c \phi'(c_2)}.$$

Under this inequality, $P > \bar{P}_0$ would imply $c_2 > \bar{c}_0$, violating the budget constraint. Therefore, $P \leq \bar{P}_0$. Next, suppose $p_1 > 0$, so the first-order conditions for equilibrium imply

$$\begin{aligned} \pi^p(1 - t_1 g(R)) &\geq \frac{\pi^c \phi'(c_1)\phi(c_2)}{\phi(c_1)(\phi(c_1) + \pi(c_2))} \left[P - (t \cdot p)g(R)\right], \\ \pi^p(1 - t_2 g(R)) &\geq \frac{\pi^c \phi'(c_2)\phi(c_1)}{\phi(c_2)(\phi(c_1) + \pi(c_2))} \left[P - (t \cdot p)g(R)\right]. \end{aligned} \tag{A.32}$$

As $\log \phi$ is concave and its derivative is convex, $c_1 \geq c_2$ (per Proposition A.4) implies $\phi'(c_1)/\phi(c_1) \leq \phi'(c_2)/\phi(c_2)$ and

$$\frac{1}{2}\left(\frac{\phi'(c_1)}{\phi(c_1)} + \frac{\phi'(c_2)}{\phi(c_2)}\right) \geq \frac{\phi'(C/2)}{\phi(C/2)}.$$

In addition, $p_1 \leq p_2$ and $t_1 > t_2$ imply

$$\left(\frac{t_1 + t_2}{2}\right) P \geq t_1 p_1 + t_2 p_2.$$

Summing the conditions in (A.32) and applying these inequalities gives

$$\begin{aligned} &2\pi^p \left(1 - \frac{t_1 + t_2}{2} g(R)\right) \\ &\quad \geq \pi^c \left[P - (t \cdot p)g(R)\right] \left[\left(\frac{\phi(c_2)}{\phi(c_1) + \phi(c_2)}\right) \frac{\phi'(c_1)}{\phi(c_1)} + \left(\frac{\phi(c_1)}{\phi(c_1) + \phi(c_2)}\right) \frac{\phi'(c_2)}{\phi(c_2)}\right] \\ &\quad \geq \pi^c \left[P - (t \cdot p)g(R)\right] \left[\frac{1}{2}\left(\frac{\phi'(c_1)}{\phi(c_1)} + \frac{\phi'(c_2)}{\phi(c_2)}\right)\right] \\ &\quad \geq \pi^c \left[P - (t \cdot p)g(R)\right] \left(\frac{\phi'(C/2)}{\phi(C/2)}\right) \\ &\quad \geq \pi^c P \left(1 - \frac{t_1 + t_2}{2} g(R)\right) \left(\frac{\phi'(C/2)}{\phi(C/2)}\right) \end{aligned}$$

Simplifying and rearranging terms gives

$$P \leq \frac{2\pi^p \phi'(C/2)}{\pi^c \phi(C/2)}.$$

As in the previous case, the budget constraint then implies $P \leq \bar{P}_0$.

If $t_i g(R) \leq \hat{t}_0$ for each $i = 1, 2$ such that $p_i > 0$, then $P \leq \bar{P}_0$ implies $u_G(t, c, p, r) \leq \hat{t}_0 \bar{P}_0$, as claimed. So suppose there is a group i such that $p_i > 0$ and $t_i g(R) > \hat{t}_0$. The first-order conditions for equilibrium imply

$$\pi^p(1 - t_i g(R)) \geq \pi^r(-g'(R))(t_i p_i + t_j p_j).$$

This inequality, combined with log-concavity of g and the assumption that $t_i g(R) > \hat{t}_0$, gives

$$\begin{aligned}
u_G(t, p, c, r) &= (t_i p_i + t_j p_j) g(R) \\
&\leq -\left(\frac{g(R)}{g'(R)}\right) \frac{\pi^p(1 - t_i g(R))}{\pi^r} \\
&< -\left(\frac{1}{g'(0)}\right) \frac{\pi^p(1 - \hat{t}_0)}{\pi^r} \\
&= \left(\frac{\pi^p}{\pi^p - \pi^r \bar{P}_0 g'(0)}\right) \bar{P}_0 \\
&= \hat{t}_0 \bar{P}_0,
\end{aligned}$$

as claimed. □

This brief extension demonstrates that the earlier results for labor-financed extraction do not depend on the assumption of equal tax rates across factions. All else equal, a labor-financed government benefits from social order and has no incentive to create inequality where none exists before. What this extension does not answer is how asymmetric tax rates would interact with *ex ante* asymmetries in productivity or size among factions, a topic that is beyond the scope of the present analysis.

A.5.2 Conquest

In the *conquest model*, a set of N factions compete with each other and with an outsider, denoted O, for the chance to be the government in the future. The incremental value of being the government is $v(N) > 0$, which may increase with N (when capital is the main source of revenue) or decrease (when labor is the main source of revenue). Each faction has L/N units of labor, which it may divide between two activities: $s_i \geq 0$, to prevent the outsider from taking over; and $d_i \geq 0$, to influence its own chance of becoming the government if the outsider fails. Each faction's budget constraint is[20]

$$s_i + d_i = \frac{L}{N}. \tag{A.33}$$

The success of the attempted takeover depends on how much the factions spend to combat the outsider. I assume the outsider's military strength is a fixed value, $\bar{s}_O > 0$, so the outsider is not a strategic player here. The assumption that the outsider's strength is exogenous is of course a simplification, but it is plausible in situations where the outsider marshals its forces before fully understanding the internal political situation—such as in Cortés's incursion into the Mexican mainland, and other early maritime colonial ventures. The probability that the outsider becomes the government is

$$\frac{\bar{s}_O}{\bar{s}_O + \chi(\sum_{i=1}^N s_i)}, \tag{A.34}$$

where $\chi : [0, L] \to \mathbb{R}_+$ represents the translation of society's labor into its strength against the outsider. In case the outsider fails, the probability that faction i becomes the government is

$$\frac{\psi(d_i)}{\sum_{j=1}^N \psi(d_j)}, \tag{A.35}$$

where $\psi : [0, L/N] \to \mathbb{R}_+$ represents the translation of an individual faction's labor into its proportional chance of success against other factions. As with the function ϕ in the original model, I assume χ and ψ are strictly increasing and log-concave.

The factions simultaneously choose how to allocate their labor, subject to the budget constraint (A.33). A faction's utility function is

$$u_i(s, d) = \frac{\psi(d_i)}{\sum_{j=1}^N \psi(d_j)} \times \frac{\chi(\sum_{j=1}^N s_j)}{\bar{s}_O + \chi(\sum_{j=1}^N s_j)} \times v(N), \tag{A.36}$$

where $s = (s_1, \ldots, s_N)$ and $d = (d_1, \ldots, d_N)$.

I begin by characterizing the equilibrium of the conquest game. Throughout the proofs, let $\hat{\chi} = \log \chi$ and $\hat{\psi} = \log \psi$. I will characterize equilibria in terms of the criterion function

$$Q^{ds}(S; N) = \frac{N-1}{N}(\psi(S) + \bar{s}_O)\hat{\chi}'\left(\frac{L-S}{N}\right) - \hat{\psi}'(S)\bar{s}_O, \tag{A.37}$$

[20] The assumption of unit productivity for each activity is without loss of generality. The model here with functional forms $\chi(S) = \tilde{\chi}(\pi^s S)$ and $\psi(D) = \tilde{\psi}(\pi^d D)$ is isomorphic to a model with the common budget constraint $s_i/\pi^s + d_i/\pi^d = L/N$ and functional forms $\tilde{\chi}$ and $\tilde{\psi}$.

which is strictly increasing in both S and N.

Lemma A.15. *The conquest game has a unique equilibrium in which each*

$$s_i = \begin{cases} 0 & Q^{ds}(0; N) \geq 0, \\ \tilde{S}(N)/N & Q^{ds}(0; N) < 0, Q^{ds}(L; N) > 0, \\ L/N & Q^{ds}(L; N) \leq 0, \end{cases}$$

and each $d_i = L/N - s_i$, where $\tilde{S}(N)$ is the unique solution to $Q^{ds}(\tilde{S}(N); N) = 0$.

Proof. Like the original game, the conquest game is log-concave, so a pure-strategy equilibrium exists can be characterized by first-order conditions. In addition, the proof of Lemma A.7 carries over to the conquest game, so in equilibrium each $d_i = d_j$ for $i, j \in \mathcal{N}$. The claim then follows from the first-order conditions for maximization of each faction's utility. $\square$

The strategic tradeoff for the factions here is analogous to the tradeoff between resistance and internal conflict in the baseline model. Critically, the relative marginal benefit of fighting the outsider declines as the number of factions increases. When the number of factions is large, any individual faction's chance of becoming the government if the outsider loses is small, which in turn reduces its incentive to contribute to the collective effort against the outsider. Consequently, as the following result states, the outsider is more likely to win the more divided the society is.

Proposition A.6. *In the conquest model, the probability that the outsider wins is increasing in the number of factions, N.*

Proof. I will prove that the equilibrium value of $\sum_i s_i$ decreases with the number of factions. Let (d, s) and (d', s') be the equilibria at N and N' respectively, where $N' > N$, and let $S = \sum_i s_i$ and $S' = \sum_i s_i'$. If $S = 0$, then $Q^{ds}(0; N) \geq 0$ and thus $Q^{ds}(0; N') \geq 0$, so $S' = 0$ as well. If $S \in (0, L)$, then $S = \tilde{S}(N)$, which implies $Q^{ds}(L; N) > 0$ and thus $Q^{ds}(L; N') > 0$. This in turn implies either $S' = \tilde{S}(N') < \tilde{S}(N)$ or $S' = 0 < \tilde{S}(N)$. Finally, if $S = L$, then it is trivial that $S' \leq S$. $\square$

A.5.3 Informal Sector Production

In the baseline model with a capital-financed government, the factions' only choices are resistance and internal competition. Here I extend the model to allow factions to allocate labor to the informal sector, which has a fixed marginal value and cannot be expropriated by the government or by other factions. Let o_i denote informal sector production, and let $\kappa > 0$ denote the per-unit consumption value. Each faction's utility function in the extended model is

$$u_i(t, o, r, c) = \omega_i(c) \times \bar{\tau}(t, r) \times X + \kappa o_i, \tag{A.38}$$

and they are subject to the budget constraint

$$o_i + \frac{r_i}{\pi^r} + \frac{c_i}{\pi^c} = \frac{L}{N}. \tag{A.39}$$

(There is no loss of generality in normalizing the labor cost of informal production to 1.) Because the government cannot expropriate informal production, its utility function remains as in Equation 6.

The existence and uniqueness arguments from the baseline model do not immediately carry over to the extended model, as quasiconcavity is not necessarily preserved under addition. If u_i is strictly quasiconcave in a faction's own choices,[21] then the arguments from Proposition A.1 and Proposition A.2 can be adapted to prove that the labor allocation subgame in the extended model has a unique equilibrium characterized by first-order conditions, in which $c_i = C$ and $o_i = O$ for all factions i. From there, the argument of Lemma A.6 can be adapted to show that total equilibrium resistance is continuous in t, which in turn implies existence of an optimal tax rate per the argument of Proposition A.3. For the remainder of the analysis of this extension, I will proceed under the assumption that u_i is strictly quasiconcave in a faction's own choices, so that the existence and continuity of equilibria in the labor allocation subgame can be presumed. Additionally, I will assume $\phi(0) > 0$ to ensure the utility function is everywhere continuously differentiable.

My goal is to show that the main substantive result of the baseline analysis with a capital-financed government—namely, that the government's equilibrium payoff increases with the extent of social fractionalization (Proposition 2)—holds up in the environment with informal production. I first consider a global result, showing that the government extracts the full resource endowment in equilibrium if fractionalization is great enough.

Proposition A.7. *In the model with a capital-financed government and informal production, if $N \geq \frac{-\pi^r g'(0) X}{\kappa}$, then there is an equilibrium where $t^* = 1$ and each $(o_i^*(1), r_i^*(1), c_i^*(1)) = (\frac{L}{N}, 0, 0)$.*

Proof. I begin by proving that each $o_i^*(1) = \frac{L}{N}$ (and thus $r_i^*(1) = c_i^*(1) = 0$ by the budget

[21] A sufficient condition is that $\omega_i(c) \cdot \bar{\tau}(t, r)$ is strictly concave in (c_i, r_i).

constraint) under the hypothesis of the proposition. It will suffice to show that the first-order conditions for a best response are satisfied for each faction at the proposed allocation. The relevant partial derivatives are:

$$\frac{\partial u_i(t,o,r,c)}{\partial o_i} = \kappa,$$
$$\frac{\partial u_i(t,o,r,c)}{\partial r_i} = \frac{-g'(0)}{N} \cdot X,$$
$$\frac{\partial u_i(t,o,r,c)}{\partial c_i} = 0.$$

If $N \geq \frac{-\pi^r g'(0) X}{\kappa}$, then we have

$$\frac{\partial u_i(t,o,r,c)}{\partial o_i} \geq \max\left\{\pi^r \frac{\partial u_i(t,o,r,c)}{\partial r_i}, \pi^c \frac{\partial u_i(t,o,r,c)}{\partial c_i}\right\},$$

confirming that the proposed allocation is an equilibrium.

To conclude, I must prove that $t^* = 1$ is optimal for 1. Because $\sum_i r_i^*(1) = 0$, the government's expected utility from $t^* = 1$ is $u_G(1, o^*(1), r^*(1), c^*(1)) = X$. Regardless of the factions' responses, no other tax rate can yield a strictly greater payoff for the government, so the proposed tax rate is optimal. □

I next prove a local result, showing that the government's utility decreases with fractionalization when there is positive informal production in equilibrium. As in the proof of Proposition 2 above, I treat N as though it were continuous so as to obtain comparative statics via implicit differentiation.

Proposition A.8. *In the model with a capital-financed government and informal production, if there is an equilibrium where $t = t^*$ and each $o_i^*(t^*) > 0$, then the government's equilibrium utility is locally non-decreasing in N.*

Proof. As in the proof of Proposition 2 above, it will suffice to show that $R^*(t^*)$ locally decreases with N. The claim holds trivially if $R^*(t^*) = 0$, so consider the case where $R^*(t^*) > 0$. In this case, equilibrium resistance is defined by the condition

$$\pi^r \frac{\partial u_i(t^*,o,r,c)}{\partial r_i} = \frac{\pi^r(-t^* g'(R^*(t^*)))}{N} = \kappa = \frac{\partial u_i(t^*,o,r,c)}{\partial o_i}.$$

Convexity of g implies $-g'$ decreases with $R^*(t^*)$. Therefore, as N increases, $R^*(t^*)$ must decrease in order to maintain the condition. □

A.5.4 Incomplete Information

In the baseline model with a labor-financed government, the equilibrium tax rate is $\hat{t}_0$, the highest level at which there is no resistance (Proposition 5). The proof that such a government's equilibrium payoff decreases with fractionalization (Proposition 8) depends on this property. In this section, to probe the robustness of the result relating fractionalization to government revenues, I consider a simple extension of the baseline labor-financed environment in which the optimal tax rate may engender positive resistance. The main result still holds true in this environment: the government's equilibrium payoff decreases with the extent of fractionalization.

To introduce the possibility of positive resistance in equilibrium, I assume there is incomplete information about the labor cost of resistance at the time that the government chooses the tax rate. After the government chooses t, Nature draws π^r from a distribution over $\{\pi^r_\ell, \pi^r_h\}$, where $\pi^r_\ell < \pi^r_h$, and reveals its value to all players. The factions then play the labor subgame as usual. The prior distribution of π^r and the values of all other parameters are common knowledge from the outset of the game. Let $\xi \equiv \Pr(\pi^r = \pi^r_h)$, and to avoid trivialities assume $\xi \in (0, 1)$.

The incomplete-information setup creates the possibility of nonzero resistance occurring with positive probability along the equilibrium path. Let $\hat{t}_{0\ell}$ denote the greatest tax rate that engenders no resistance if $\pi^r = \pi^r_\ell$ (Equation A.21), and let $\hat{t}_{0h}$ be defined analogously. We naturally have $\hat{t}_{0h} \leq \hat{t}_{0\ell}$, so the government can guarantee no resistance by choosing $t = \hat{t}_{0h}$. A choice of $t \in (\hat{t}_{0h}, \hat{t}_{0\ell}]$ will raise the government's payoff in case $\pi^r = \pi^r_\ell$ but will lower it in case $\pi^r = \pi^r_h$. Following the same risk-reward tradeoff logic as in bargaining models with incomplete information (Fearon 1995), it is optimal to choose $t > \hat{t}_{0h}$ if ξ is sufficiently small.

For tractability, I impose specific functional forms to analyze the extended model. Let $\phi(c_i) = c_i$, so that ω_i is a ratio-form contest success function. Let $g(R) = 1 - R$, so that the relationship between total resistance and the effective tax rate is linear. Finally, to reduce clutter in the analysis, I normalize $L = 1$. Consequently, I also assume $\pi^r_h \leq 1$, so that $g(R) \geq 0$ for all feasible $R \in [0, \pi^r L]$.

With these functional forms, the labor allocation subgame has a closed-form solution for each value of π^r. Lemma A.3 implies each $c_i > 0$ in equilibrium, so each $\hat{t}_1 > 1$; the equilibrium following any $t > \hat{t}_0$ will thus be characterized by (A.16)–(A.18). It is straightforward to verify that the equilibrium conditions imply the following:

$$\bar{P}_{0\ell} = \bar{P}_{0h} = \frac{\pi^p}{N},$$
$$\hat{t}_{0\ell} = \frac{N}{N + \pi^r_\ell},$$

$$\hat{t}_{0h} = \frac{N}{N + \pi_h^r},$$
$$\tilde{P}_{1h}(t) = \frac{\pi^p \left(\frac{1}{t} - 1 + \pi_h^r\right)}{\pi_h^r (N+1)},$$
$$\tilde{R}_{1h}(t) = \frac{N\left(1 - \frac{1}{t}\right) + \pi_h^r}{N+1}.$$

For any $t \geq \hat{t}_{0h}$, the effective tax rate in case $\pi^r = \pi_h^r$ is

$$t\left(1 - \tilde{R}_{1h}(t)\right) = t\left[\frac{[N+1] - [N(1-\frac{1}{t}) + \pi_h^r]}{N+1}\right] = \frac{N + (1-\pi_h^r)t}{N+1}.$$

The government's utility in this case is therefore

$$\begin{aligned} t\left(1 - \tilde{R}_{1h}(t)\right)\tilde{P}_{1h}(t) &= \frac{N + (1-\pi_h^r)t}{N+1} \times \frac{\pi^p \left(\frac{1}{t} - 1 + \pi_r^h\right)}{\pi_h^r (N+1)} \\ &= \frac{\pi^p N}{\pi_h^r (N+1)^2} \underbrace{\left[\frac{1}{t} - 1 + \pi_h^r\right]}_{>0} + \frac{\pi^p (1-\pi_h^r)[1 - (1-\pi_h^r)t]}{\pi_h^r (N+1)^2}. \end{aligned} \tag{A.40}$$

I now prove that the government's expected utility strictly decreases with N in the incomplete information game, even when the equilibrium entails positive probability of resistance along the equilibrium path. Arguments from the baseline analysis imply that no $t < \hat{t}_{0h}$ or $t > \hat{t}_{0\ell}$ may be optimal, so we may restrict attention to $t \in [\hat{t}_{0h}, \hat{t}_{0\ell}]$. In this range, the government's expected utility from any tax rate is

$$\begin{aligned} &\mathbb{E}[u_G(t, p^*(t), r^*(t), c^*(t))] \\ &\quad = \xi\left[t\left(1 - \tilde{R}_{1h}(t)\right)\tilde{P}_{1h}(t)\right] + (1-\xi)\left[t\bar{P}_{0\ell}\right] \\ &\quad = \frac{\xi \pi^p N}{\pi_h^r (N+1)^2}\left[\frac{1}{t} - 1 + \pi_h^r\right] + \frac{\xi \pi^p (1-\pi_h^r)[1-(1-\pi_h^r)t]}{\pi_h^r (N+1)^2} + \frac{(1-\xi)\pi^p t}{N}. \end{aligned}$$

This expression is strictly convex in t, so it is maximized at one of the boundary points, $t \in \{\hat{t}_{0h}, \hat{t}_{0\ell}\}$. At the lower boundary, $t = \hat{t}_{0h}$, the tax demand engenders no resistance regardless of the factions' type. This yields a government payoff of

$$\xi \hat{t}_{0h} \bar{P}_{0h} + (1-\xi)\hat{t}_{0h}\bar{P}_{0\ell} = \frac{\pi^p}{N + \pi_h^r},$$

which is strictly decreasing in N. At the upper boundary, $t = \hat{t}_{0\ell}$, the government's payoff is

$$\xi \hat{t}_{0\ell} \left(1 - \tilde{R}_{1h}(\hat{t}_{0\ell})\right) \tilde{P}_{1h}(\hat{t}_{0\ell}) + (1-\xi)\hat{t}_{0\ell}\bar{P}_{0\ell}.$$

It is evident from Equation A.40 that $t(1 - \tilde{R}_{1h}(t))\tilde{P}_{1h}(t)$ is strictly decreasing in both N and t. Since $\hat{t}_{0\ell}$ is increasing in N, this means the first term of the expression here is strictly decreasing in N. Meanwhile, the baseline model result Proposition 8 implies that the second term is strictly decreasing in N, as it is simply a scalar multiple of the government's utility from the complete-information game where $\pi^r = \pi^r_\ell$. Altogether, we have that the government's equilibrium expected utility,

$$\begin{aligned}
&\max_{t \in [0,1]} \mathbb{E}[u_G(t, p^*(t), r^*(t), c^*(t))] \\
&\qquad = \max \left\{ \mathbb{E}[u_G(\hat{t}_{0h}, p^*(\hat{t}_{0h}), r^*(\hat{t}_{0h}), c^*(\hat{t}_{0h}))], \mathbb{E}[u_G(\hat{t}_{0\ell}, p^*(\hat{t}_{0\ell}), r^*(\hat{t}_{0\ell}), c^*(\hat{t}_{0\ell}))] \right\},
\end{aligned}$$

is strictly decreasing in N, just as in the baseline model.

A.5.5 Combined Capital and Labor Financing

In the baseline model, each player's utility is a fraction of either exogenous resources X or endogenous production $\sum_i p_i$. I now extend the model to allow these to be combined, altering the production function to $f(p) = X + \sum_i p_i$ (where $X > 0$). In the extended model, the government's utility is U-shaped as a function of social fractionalization. Marginal increases in fractionalization reduce government revenues at low levels (when equilibrium behavior resembles the baseline model with a labor-financed government), but increase revenue once N is high enough (when behavior is more like in the capital-financed baseline). Additionally, as the value of the exogenous resource X increases, the portion of the parameter space where fractionalization increases revenues grows.

As in the extension with incomplete information (subsubsection A.5.4), I impose particular functional forms to allow for a closed-form solution. Specifically, I again let $\phi(c_i) = c_i$ and $g(R) = 1 - R$, and I assume $L = 1$ and $\pi^r \leq 1$ to ensure that $g(R) \geq 0$.

Equilibrium with low fractionalization. If $N < 1 + \frac{\pi^p}{X}$, then the equilibrium closely resembles that of the baseline model with a labor-financed government. (All of the following statements can be verified by checking them against the first-order conditions for optimal labor allocation.) Behavior in the labor allocation subgame is determined by two cutpoints on the tax rate. If $t \leq \hat{t}_0 \equiv \frac{N\pi^p}{N\pi^p + \pi^r(\pi^p + X)}$, then we have

$$
\begin{aligned}
P^*(t) &= \frac{\pi^p - (N-1)X}{N}; \\
R^*(t) &= 0.
\end{aligned}
$$

If $\hat{t}_0 < t < \hat{t}_1 \equiv \frac{\pi^p}{\pi^p(1-\pi^r) + N\pi^r X}$, then we have

$$
\begin{aligned}
P^*(t) &= \frac{\pi^p(\pi^r + \frac{1}{t} - 1) - N\pi^r X}{(N+1)\pi^r}; \\
R^*(t) &= \frac{\pi^r(1 + \frac{X}{\pi^p}) - N(\frac{1}{t} - 1)}{N+1}.
\end{aligned}
$$

Finally, if $t \geq \hat{t}_1$, then we have

$$
\begin{aligned}
P^*(t) &= 0; \\
R^*(t) &= \frac{\pi^r - (N-1)\left(\frac{1}{t} - 1\right)}{N}.
\end{aligned}
$$

Note that $\hat{t}_1 \geq 1$ if and only if $N \leq \frac{\pi^p}{X}$.

I now solve for the government's optimal tax rate. Clearly no $t < \hat{t}_0$ may be optimal. For $t \in [\hat{t}_0, \hat{t}_1]$, government revenues given equilibrium responses by the factions are

$$
\begin{aligned}
&u_G(t, p^*(t), r^*(t), c^*(t)) \\
&\quad = t \times (1 - R^*(t)) \times (X + P^*(t)) \\
&\quad = \frac{\pi^p}{(N+1)^2}\left[t\left(1 - \pi^r - \frac{\pi^r X}{\pi^p}\right) + N\right]\left[\frac{1}{t} + \pi^r\left(1 + \frac{X}{\pi^p}\right) - 1\right] \\
&\quad = \frac{\pi^p}{(N+1)^2}\left[\left(1 - \pi^r - \frac{\pi^r X}{\pi^p}\right) + \frac{N}{t} - t\left(1 - \pi^r - \frac{\pi^r X}{\pi^p}\right) - N\right]
\end{aligned}
$$

Because $\pi^r \leq 1$ and $\frac{X}{\pi^p} < \frac{1}{N-1}$, we have

$$
\begin{aligned}
\frac{du_G(t, p^*(t), r^*(t), c^*(t))}{dt} &= \frac{\pi^p}{(N+1)^2}\left[-\frac{N}{t^2} - 1 + \pi^r + \frac{\pi^r X}{\pi^p}\right] \\
&\leq \frac{\pi^p}{(N+1)^2}\left[\frac{X}{\pi^p} - \frac{N}{t^2}\right] \\
&< \frac{\pi^p}{(N+1)^2}\left[\frac{1}{N-1} - N\right] \\
&< 0.
\end{aligned}
$$

Therefore, no $t \in (\hat{t}_0, \hat{t}_1]$ may be optimal for the government. Meanwhile, for $t > \hat{t}_1$ the government's payoff is

$$\begin{aligned} u_G(t, p^*(t), r^*(t), c^*(t)) &= t \times (1 - R^*(t)) \times X \\ &= \frac{(1-\pi^r)t + N - 1}{N} \times X, \end{aligned}$$

which is weakly increasing in t (strictly if $\pi^r < 1$). Therefore, no $t \in [\hat{t}_1, 1)$ may be optimal for the government.

If $N \leq \frac{\pi^p}{X}$, so that $\hat{t}_1 \geq 1$, then the argument above implies that $t^* = \hat{t}_0$. Otherwise, if $\frac{\pi^p}{X} < N < 1 + \frac{\pi^p}{X}$, then the optimal tax rate depends on the government's preference between $t = \hat{t}_0$ and $t = 1$. We have

$$\begin{aligned} u_G(\hat{t}_0, p^*(\hat{t}_0), r^*(\hat{t}_0), c^*(\hat{t}_0)) = \hat{t}_0 \times (X + P^*(\hat{t}_0)) &= \frac{\pi^p(\pi^p + X)}{N\pi^p + \pi^r(\pi^p + X)}; \\ u_G(1, p^*(1), r^*(1), c^*(1)) = (1 - R^*(1)) \cdot X &= \frac{(N - \pi^r)X}{N}. \end{aligned}$$

It is evident from these expressions that the government's utility from $t = \hat{t}_0$ is decreasing in N, while its utility from $t = 1$ is increasing in N. This implies that there exists a cutpoint $N^* \in [\frac{\pi^p}{X}, 1 + \frac{\pi^p}{X}]$ such that $t^* = \hat{t}_0$ if $N \in [0, N^*]$ and $t^* = 1$ if $N \in (N^*, 1 + \frac{\pi^p}{X})$. Additionally, the government's equilibrium utility as a function of N is decreasing on $[2, N^*)$ and increasing on $(N^*, 1 + \frac{\pi^p}{X})$.

Equilibrium with high fractionalization. If $N \geq 1 + \frac{\pi^p}{X}$, then the equilibrium resembles that of the baseline model with a capital-financed government. If $t \leq \hat{t}_X \equiv \frac{N-1}{N-1+\pi^r}$, then $P^*(t) = R^*(t) = 0$. Otherwise, if $t > \hat{t}_X$, then equilibrium labor allocation is the same as in the case above with $t > \hat{t}_1$:

$$\begin{aligned} P^*(t) &= 0; \\ R^*(t) &= \frac{\pi^r - (N-1)\left(\frac{1}{t} - 1\right)}{N}. \end{aligned}$$

It then follows from the same arguments as above that the government's optimal tax rate is $t^* = 1$. Combined with the results from above, we now have that the government's equilbrium utility as a function of N is decreasing on $[2, N^*)$ and increasing on (N^*, ∞).

Exogenous resources and the cutpoint. The last step of the argument is to show that the government benefits from fractionalization under a wider set of parameters as the resource

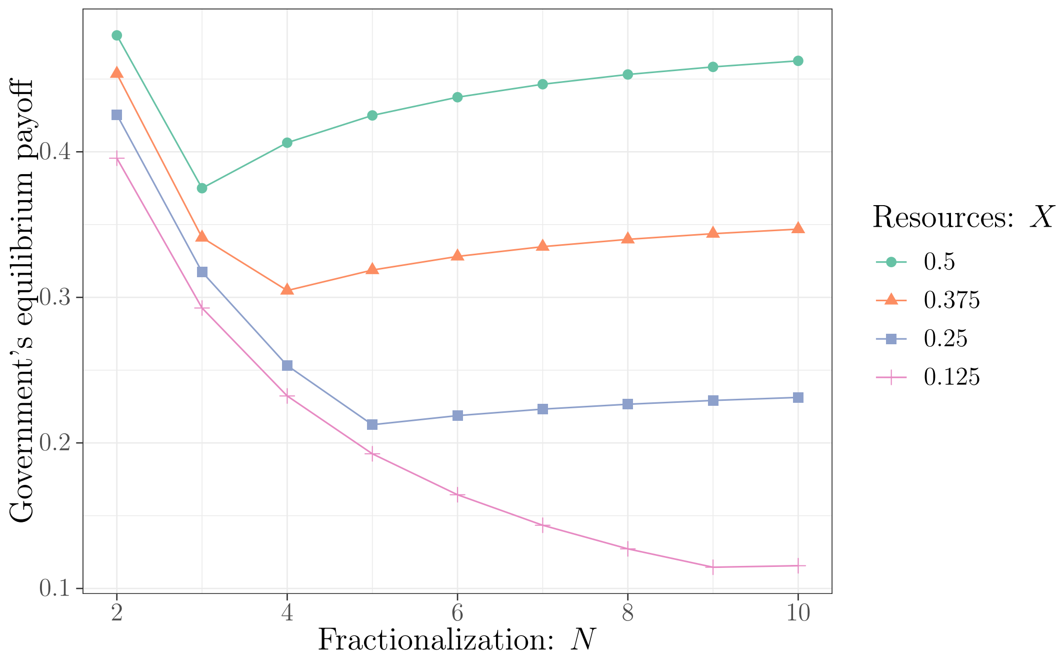

Figure A.1. Government's equilibrium payoff as a function of N and X in the extension with combined resources and production.

endowment increases; i.e., that the cutpoint N^* is decreasing in X. This is immediate in case $N^* = \frac{\pi^p}{X}$ or $N^* = 1 + \frac{\pi^p}{X}$. The last case to consider is when $N^* \in (\frac{\pi^p}{X}, 1 + \frac{\pi^p}{X})$, in which case N^* is defined as the root of

$$u_G(1, p^*(1), r^*(1), c^*(1)) - u_G(\hat{t}_0, p^*(\hat{t}_0), r^*(\hat{t}_0), c^*(\hat{t}_0)) = \frac{(N - \pi^r)X}{N} - \frac{\pi^p(\pi^p + X)}{N\pi^p + \pi^r(\pi^p + X)}.$$

Since this expression is increasing in N, to prove that N^* decreases with X it will suffice to prove that this expression is also increasing in X. We have

$$\begin{aligned}\frac{d}{dX}\left[\frac{(N - \pi^r)X}{N} - \frac{\pi^p(\pi^p + X)}{N\pi^p + \pi^r(\pi^p + X)}\right] &= 1 - \frac{\pi^r}{N} - \frac{N(\pi^p)^2}{[N\pi^p + \pi^r(\pi^p + X)]^2} \\ &> \frac{N - \pi^r - 1}{N} \\ &\geq 0,\end{aligned}$$

where the final inequality holds because $\pi^r \leq 1$ and $N \geq 2$. Therefore, N^* decreases with X. Figure A.1 illustrates how the government's utility is U-shaped in N, as well as how the range where N increases revenues expands as X increases.