Communication Between Allies*

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Abstract

I study the problems of diplomatic communication between states that have a common purpose, such as in a joint military effort. I analyze a formal model to characterize the conditions under which a state can credibly reveal private information to its ally. The results call into question the supposition that alliances play a general information-sharing role. No informative communication is possible when the states' private information concerns their cost of contributing to the project or, equivalently, their willingness to contribute. In this case, states have an incentive to pretend to be relatively unwilling to participate, so as to induce greater contributions by their partners. Similarly, when the source of private information is intelligence about how much effort will be required for the joint project to succeed, communication is possible only under highly restrictive conditions. On the other hand, communication is broadly possible if there are multiple avenues of contribution (e.g., ground and air power) and states have private information about their specialty.

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1 Introduction

When is diplomatic communication between allies effective? More precisely, under what conditions can states with a common purpose credibly reveal private information to each other through "cheap talk"? Many international political processes depend on whether states with common interests can work together in the face of uncertainty. Efforts to provide global public goods, such as carbon emissions reduction, require a division of labor among countries that are unsure of each other's ability and willingness to comply. Multilateral military cooperation poses a similar problem: states may have to make deployment decisions without knowing how much their allies will contribute or how long they will remain committed. Such coordination is not uncommon in international conflicts, especially the most serious ones: in about 15 percent of militarized international disputes between 1816 and 2001, including 50 percent of those that eventually resulted in war, at least one side consisted of multiple states.[1] Nevertheless, the extensive theoretical literature on the efficacy of diplomatic communication has focused on the credibility of threats that adversaries issue to each other in crises,[2] to the exclusion of the problem of communication between allies.

This paper uses a formal model to examine cheap talk communication between countries with a common purpose. Drawing from Olson and Zeckhauser (1966), the model treats international cooperation, including extended deterrence and joint military ventures, as a public goods problem. In the model, two states individually decide how much effort to expend on a project whose success benefits both. As is common in models of international politics, their decision is complicated by uncertainty: at least one state possesses private information that may affect how much it will contribute. Before making their contributions, the states may use cheap talk messages to communicate about their private information. The goal of the analysis is to characterize when this kind of diplomatic communication may be effective. Specifically, I look for equilibria of the model in which (1) one state partially or fully reveals its private information, and (2) this revelation affects its partner's contribution decision.

The crucial variable in the analysis is the source of private information. In the main portion of the paper, I assume that states have private information

[1] Calculated using version 4.01 of the Militarized International Dispute data from the Correlates of War project (Ghosn, Palmer and Bremer 2004). The statistics are similar if one instead uses International Crisis Behavior data: 22 percent of all crises between 1918 and 2007, and 45 percent of those resulting in war, involved three or more actors (Brecher and Wilkenfeld 2000).

[2] For example, see Fearon (1995, 1997), Sartori (2002), Guisinger and Smith (2002), Trager (2010, 2011), and Ramsay (2011).

about their own cost of contributing to the joint project—or, equivalently, the amount they are willing to contribute. In this setting, there is a pervasive incentive to misrepresent, precluding the existence of an equilibrium with meaningful communication, even though the chance of the joint effort succeeding would be greater if each state knew each other's true willingness to contribute. The impossibility of communication is closely related to the incentive to free ride in public goods problems. A state will contribute little to a joint cause if it knows its partner is willing to assume most of the burden. Therefore, by falsely claiming to be relatively unwilling to contribute (or to have a high cost of doing so), a state can prompt its partner to take up a greater share of the work. Regardless of its actual willingness to contribute, a state always benefits from greater contributions by its partner, meaning it is profitable to pretend to be incapable. This incentive to understate one's ability to contribute undermines the possibility of meaningful communication.

This result is closely related to the well-known incentive to misrepresent strength or resolve in crisis bargaining between adversaries. When a state is negotiating with an enemy, it has a strong incentive to overstate its willingness to fight, in hopes of eliciting a better offer at the bargaining table. The pervasive incentive to misrepresent prevents states from finding negotiated settlements that all involved would prefer to war (Fearon 1995). On the surface, crisis bargaining between adversaries and public good provision between allies seem like opposite situations: adversaries have contrary preferences, whereas allies have common interests. However, these are both actually mixed-motive problems with similar underlying structures. In crisis bargaining, the states involved have a mutual interest in avoiding war, but face distributional conflict over the division of spoils in a negotiated settlement. Similarly, in a joint venture between allies, the states have a mutual interest in success, but face distributional conflict over how to split the cost among partners. A state always wants an adversary to offer better terms, and always wants an ally to take on a greater share of the burden. In both cases, the barriers to honest communication are similar.

After ruling out communication about willingness to contribute, I examine two alternative sources of incomplete information. In the first of these extensions, I assume that one state gains additional information about how much effort the partners must put forth in order for their joint project to succeed. As in the main model, I look for equilibria in which the better-informed state fully or partially reveals its intelligence to its partner through cheap talk. I find that such communication is indeed possible, but only under restrictive conditions. If the informed country learns exactly how much effort is required for success, there is an equilibrium in which it always honestly reveals this intelligence to its partner. However, the possibility of influential communication breaks down if the informed state's knowledge is even slightly less than perfect. Cheap-talk intelligence sharing is possible only in the most fragile of circumstances.

The second extension concerns a scenario in which the success of the joint project depends on two distinct venues of contribution. For example, in a military venture, these might be ground forces and air power. I assume that one state has private information about which of the two methods it can more easily partake in. I find broad conditions under which the state may honestly reveal this information to its partner in equilibrium. The reason why communication is possible here is intuitive: a country that is better at air power may prefer for its partner to focus on the ground war, and vice versa. By contrast, when there is only one avenue of contribution, a country always wants its partner to do the same thing—namely, to contribute as much as possible.

This paper contributes to the literature on the informational role of military alliances. Previous work in this area takes information sharing among allies as given. Bearce, Flanagan and Floros (2006) argue that allied states learn each other's strength and are thus less likely to go to war with one another, as uncertainty is one of the main hindrances to reaching negotiated settlements in crises. Similarly, Konrad (2012) develops a theoretical model in which members of an alliance learn each other's military budget, finding that informational alliances may reduce overall conflict. The results of the present analysis cast some doubt on the assumption that alliances facilitate information transmission in general, particularly about states' capabilities or resolve. If they do, it must be through some channel other than ordinary communication among their members. Interest in a common goal is not a sufficient condition for a set of states to be honest with each other about their resources.

This paper also contributes to the political economy literature on communication in public goods dilemmas. The main finding in this literature is that the addition of a cheap-talk stage, in which the players may send costless messages about their willingness to contribute, can improve the efficiency of outcomes in a public goods game (Agastya, Menezes and Sengupta 2007). A crucial assumption of Agastya, Menezes and Sengupta (2007) is that all players know the cost of provision—the total amount that needs to be contributed in order for the project to succeed. This assumption is problematic in the context of international military cooperation, as states may be uncertain of exactly how much force is required to defeat a common opponent. To allow for this possibility, I use a model in which the states are uncertain of the threshold required for success (see Nitzan and Romano 1990). To my knowledge, this paper is the first to add cheap-talk messaging to a public goods game with threshold uncertainty. The closest other works are Barbieri and Malueg (2010) and Bag and Roy (2011), which introduce private information, but not communication, to continuous public goods models.

The remainder of the paper proceeds as follows. Section 2 sets up the main model, in which states have private information about their cost of contributing to the joint project. I demonstrate the impossibility of influential communication in this setting and analyze its consequences for social welfare in Sections 3 and 4, which cover one-sided and two-sided incomplete information, respectively. I move on to the intelligence-sharing extension in Section 5.1, followed by the analysis of multiple avenues of contribution in Section 5.2. Brief concluding remarks appear in Section 6. Appendix A contains formal results that do not appear in the body of the paper.

2 Setup

2.1 Contribution Game

The core of the model is a public goods game, in which two players individually choose how much costly effort to devote to the achievement of a common goal. In line with the application to military alliances, I refer to the players as country 1 and country 2, and to the public good as a joint military venture; however, the results apply to a broad class of continuous public goods problems. In the *contribution game*, each country chooses how much military effort, denoted x_i, to devote to the joint venture.[3] Let $X_i = \mathfrak{R}_+$ denote the set of feasible contributions for country i. Whether the venture succeeds depends on how much the states contribute. The probability of success is given by $p(x_1 + x_2)$, where the function $p : \mathfrak{R}_+ \to [0, 1]$ is strictly increasing, twice differentiable, and strictly concave.[4]

It is costly for a state to contribute to the joint effort. The preparation and mobilization of military resources have direct financial costs that the state must pay through tax collection or borrowing. When a country devotes fiscal resources to international aims, it thereby reduces the amount left over for domestic consumption or the pursuit of other political priorities (Powell 1993). To represent this trade-off in the model, I assume that each country pays a cost

[3]Throughout the paper, generic countries are labeled i and j, with the understanding that $i \neq j$.

[4] As in Bag and Roy (2011), these properties do not need to hold on the entire domain of p. It is sufficient that they hold on $[0, \bar{X}]$, where $\bar{X}$ is the maximal total contribution that can be realized when neither country employs a strictly dominated strategy. Specifically, this is $\bar{X} = x^*(\underline{c}_1) + x^*(\underline{c}_2)$, where the standalone contribution function x^* and the minimal types $\underline{c}_i$ are defined as below.

$c_i > 0$ for each unit of military resources it contributes. Although success is determined by the total contribution, each country pays only for its own effort.

Both of the allied countries want the joint venture to succeed. Without loss of generality, I assume that a country receives 1 in case of success and 0 in case of failure.[5] Given its marginal cost of effort c_i, a country's utility function is

$$u_i(x_i, x_j \mid c_i) = p(x_i + x_j) - c_i x_i. \tag{2.1}$$

A country's payoff may be non-monotonic in its own effort, as higher values of x_i increase both the chance of success and the total cost to country i. However, a country always benefits from higher contributions by its partner.

A country's marginal cost of effort determines its *standalone contribution*—the amount it would contribute if it were the only player in the game or, equivalently, if it expected its ally to contribute $x_j = 0$. Because p is strictly concave and continuously differentiable, the standalone contribution of a country with marginal cost c is given by the continuous, non-increasing function

$$x^*(c) = \begin{cases} (p')^{-1}(c) & c < p'(0), \\ 0 & c \geq p'(0). \end{cases} \tag{2.2}$$

It is immediate from this definition that $x^*(c) < 1/c$ for all $c > 0$ and that any contribution $x_i > x^*(c_i)$ is strictly dominated (Bag and Roy 2011).

Under complete information, the standalone contributions determine the outcome of the game. Each country, given its ally's contribution strategy, will contribute just enough that its own standalone contribution is met. If the two countries have different marginal costs of effort, this means the unique Nash equilibrium is for the lower-cost country to contribute its standalone amount and for the higher-cost country to contribute nothing. On the other hand, if the cost of effort is the same across countries, there are many equilibria, all of which entail the total contribution equaling the two countries' common standalone amount. These equilibrium results are stated formally in the following proposition.

[5] This assumption is made for expositional convenience; it does not imply that the two countries value success equally. To see that the assumption of unit valuation is without loss of generality, consider a model in which each country values success at $v_i > 0$ and has a marginal contribution cost of $\kappa_i > 0$. The utility function for this model is

$$u_i(x_i, x_j) = v_i p(x_i + x_j) - \kappa_i x_i.$$

Because Von Neumann–Morgenstern utility functions are preserved by affine transformations, this is equivalent to our utility function (2.1) with $c_i = \kappa_i / v_i$. In other words, higher valuations of success are equivalent to a lower marginal cost of military effort.

Proposition 2.1. *Suppose the marginal contribution costs c_1 and c_2 are common knowledge. Without loss of generality, let $c_1 \leq c_2$. If $c_1 < c_2$, the unique Nash equilibrium of the contribution game is $(x_1, x_2) = (x^*(c_1), 0)$. If $c_1 = c_2 = c$, then (x_1, x_2) is a Nash equilibrium of the contribution game if and only if $x_1 + x_2 = x^*(c)$.*

Proof. I begin by ruling out mixed-strategy equilibria. Fix country j's strategy as a mixed strategy given by the probability measure ξ. As p is strictly concave, country i's objective function,

$$Eu_i(x_i) = \int_{X_j} p(x_i + x_j)\, d\xi - c_i x_i,$$

is strictly concave as well (Hildreth 1974). This implies any country has a unique best response to any mixed strategy by its ally, so there can be no mixed-strategy equilibrium. A further consequence of strict concavity of the objective function is that the pure strategy profile (x_1, x_2) is an equilibrium if and only if the first-order condition

$$\frac{\partial u_i(x_i, x_j \mid c_i)}{\partial x_i} = p'(x_i + x_j) - c_i = 0 \tag{2.3}$$

is satisfied for $i = 1, 2$. The proposition follows immediately. □

2.2 Uncertainty and Communication

One of the fundamental problems of international relations is that states are uncertain of each other's capabilities and intentions (Schelling 1960; Jervis 1976; Fearon 1995). This observation applies to relations between allies as well as those between adversaries. The primary goal of this paper is to analyze whether ordinary communication, or cheap talk, can allow allied countries to resolve their mutual uncertainty and improve their odds of success. To this end, I introduce incomplete information and cheap talk communication to the public goods model described above.

Throughout the main portion of the paper, I assume that one or both of the countries has private information about its marginal cost of contribution to the joint venture.[6] To represent this asymmetric information formally, let each country's cost parameter c_i be a random variable whose cumulative distribution function is denoted F_i. These distributions are common knowledge. I assume

[6] This may also represent private information about how much a country values success; see footnote 5.

that the support of each c_i is an interval, denoted $T_i = [\underline{c}_i, \bar{c}_i]$, and that c_1 and c_2 are independent. In the case of *one-sided incomplete information about costs*, only country 1's cost of contribution is private information: $\underline{c}_1 < \bar{c}_1$ and $\underline{c}_2 = \bar{c}_2 = c_2$, meaning F_2 is a degenerate distribution on c_2. On the other hand, under *two-sided incomplete information about costs*, we have both $\underline{c}_1 < \bar{c}_1$ and $\underline{c}_2 < \bar{c}_2$. I consider sources of incomplete information other than the cost of contribution in Section 5.

To model the possibility of communication, I introduce a *messaging stage* that occurs after the countries learn their types but prior to the contribution game. In the messaging stage, each country selects a message m_i from its message space M_i. For ease of exposition, I assume that each country's message space is identical to its type space, so $M_i = T_i$.[7] A *messaging strategy* is a function $\mu_i : T_i \to M_i$ that specifies the message sent by each type of country i. Given μ_i, let $\mathscr{C}_i(m_i)$ be the set of types that send the message m_i; i.e., $\mathscr{C}_i(m_i) = \mu_i^{-1}(\{m_i\})$.

Messages are cheap talk, as in Crawford and Sobel (1982): each type of country i may send any message in M_i, and the choice of message has no direct effect on either country's payoff. A country's message is important insofar as it shapes its ally's beliefs, which in turn may affect the ally's contribution. Let $\lambda_j(m_i)$ denote country j's updated beliefs about c_i after receiving the message m_i, where each $\lambda_j(m_i)$ is a probability measure on T_i.

In parts of the analysis, it will be convenient to restrict attention to messaging strategies that partition the type space into intervals. Under this kind of strategy, there is never a "gap" in the set of types that send a particular message. In other words, every message can be interpreted as a statement like "my marginal cost of contribution is c" or "my marginal cost is between c^L and c^H". Formally, an *interval messaging strategy* is a messaging strategy μ_i such that every $\mathscr{C}_i(m_i)$ is convex. It is common in the analysis of cheap talk with a continuous type space to restrict attention to interval strategies (e.g., Crawford and Sobel 1982; Barbieri 2012). For any interval partition of the type space, including infinite partitions, there exists a corresponding interval messaging strategy in which any message sent by a set of types is contained within that set: if $\mathscr{C}_i(m_i)$ is nonempty, then $m_i \in \mathscr{C}_i(m_i)$ (Fey, Kim and Rothenberg 2007). Under such a strategy, each μ_i is weakly increasing, so higher messages correspond to higher-cost types. Throughout the remainder of the paper, I assume that any interval messaging strategy satisfies this property. This assumption is solely for notational convenience and does not substantively affect any of the

[7] In the case of one-sided incomplete information, this means country 2 only has one message available, $M_2 = T_2 = \{c_2\}$, so we may omit the analysis of its choice in the messaging stage.

results about interval messaging strategies.

Because the messaging stage occurs first, a country's choice in the contribution subgame may depend on which messages were sent. A country's *contribution strategy* is a function $s_i : T_i \times M_i \times M_j \to X_i$, where $s_i(c_i, m_i, m_j)$ denotes the contribution by country i of type c_i after sending the message m_i and receiving m_j. I write $\Gamma(m_1, m_2)$ to denote the contribution subgame that follows a history in which each country sent the given messages.

To summarize, given the messaging strategies μ_i, the beliefs λ_i, and the contribution strategies s_i for each $i = 1, 2$, the sequence of play is as follows:

1. Nature privately informs each country of its type, $c_i \in T_i$.
2. The two countries simultaneously send their messages, $\mu_i(c_i) \in M_i$.
3. Each country observes its ally's message, $\mu_j(c_j) \in M_j$, and updates its beliefs about its ally's type c_j to the probability measure $\lambda_i(\mu_j(c_j))$.
4. The two countries simultaneously choose how much to contribute to the joint effort, $s_i(c_i, \mu_i(c_i), \mu_j(c_j)) \geq 0$.
5. The game ends and payoffs are realized.

An *assessment* $\sigma = (\mu_1, \mu_2, s_1, s_2, \lambda_1, \lambda_2)$ is a tuple containing both countries' strategies and beliefs.

2.3 Solution Concept

As this is a multistage game of incomplete information with observed actions, the appropriate solution concept is perfect Bayesian equilibrium. An assessment σ is an *equilibrium* if it satisfies the following sequential rationality and consistency conditions for each country i:

- Each type's choice of message is optimal, given the contribution strategies:

$$\mu_i(c_i) \in \operatorname*{arg\,max}_{m_i \in M_i} \int_{T_j} u_i \left(s_i(c_i, m_i, \mu_j(c_j)), s_j(c_j, \mu_j(c_j), m_i) \,|\, c_i \right) \, dF_j. \quad (2.4)$$

- In every contribution subgame $\Gamma(m_1, m_2)$, each type's choice of contribution is optimal, given its own beliefs and its ally's strategy:

$$s_i(c_i, m_i, m_j) \in \operatorname*{arg\,max}_{x_i \in X_i} \int_{T_j} u_i(x_i, s_j(c_j, m_j, m_i) \,|\, c_i) \, d\lambda_i(m_j). \quad (2.5)$$

- The beliefs $\lambda_i(m_j)$ are updated in accordance with Bayes' rule whenever possible (i.e., for all m_j such that $\mathscr{C}_j(m_j) \neq \emptyset$).

Equilibria can be classified in terms of how much information is revealed in the messaging stage. At one end of the informational spectrum is a *babbling equilibrium*, in which no meaningful communication takes place, because neither country's messaging strategy reveals anything about its type. Formally, an equilibrium σ is babbling if both μ_1 and μ_2 are constant functions, meaning each country sends the same message regardless of its type. In a babbling equilibrium, each country's updated beliefs on the path of play are the same as its prior beliefs. At the other extreme is a *fully separating equilibrium*, in which each country's messaging strategy always reveals its type: $\mu_i(c_i) = c_i$ for all c_i.[8] Babbling and fully separating messaging strategies are both special cases of interval messaging strategies. Any equilibrium that is neither babbling nor fully separating is *partially separating*.

Proposition 2.2. *A babbling equilibrium exists.*

Proof. Let each μ_i be a babbling messaging strategy, and let every $\lambda_j(m_i)$ be the same as the prior distribution of c_i. Consider any contribution subgame $\Gamma(m_1, m_2)$. By Lemma A.1 (in the Appendix), there exists an equilibrium of this subgame. Let each country's strategy in every subgame be the same as in the chosen equilibrium. As both countries' beliefs are the same in every subgame, these strategies satisfy sequential rationality. Since the outcome of every subgame is the same, no type of either country has an incentive to deviate from its prescribed message. Finally, beliefs are updated in accordance with Bayes' rule wherever possible. Therefore, the assessment is an equilibrium. $\square$

One of the main tasks of the analysis is to investigate whether there are equilibria in which the outcome of the messaging stage influences the countries' decisions in the contribution stage. An *influential equilibrium* is one in which at least two types of a country send distinct messages and some types of its ally make different contributions in response to these messages. Formally, an equilibrium σ is influential if there exist types $c_i, c_i' \in T_i$ and a set of types $\tilde{T}_j \subseteq T_j$ such that $s_j(c_j, \mu_j(c_j), \mu_i(c_i)) \neq s_j(c_j, \mu_j(c_j), \mu_i(c_i'))$ and $\Pr(c_j \in \tilde{T}_j) > 0$. Any influential equilibrium is non-babbling by definition, but some non-babbling

[8] In general, any one-to-one function μ_i is fully separating. The given form is the only such function that satisfies the aforementioned requirement for interval messaging strategies that $m_i \in \mathscr{C}(m_i)$.

equilibria may not be influential. For example, an equilibrium in which country 1 fully reveals its type but country 2 always contributes $x_2 = 0$ on the path of play is non-babbling and non-influential.

A key features of the model is that all of a country's types have the same preferences over its ally's actions. The more country 2 contributes, the better off country 1 is, regardless of its marginal cost of effort. Commonality of preferences across types is typically, though not always, an obstacle to influential communication in cheap talk models (Baliga and Morris 2002). In equilibrium, each type must weakly prefer sending its prescribed message over any other. Since all types want their ally to contribute more, this means in equilibrium a country will never send a message that guarantees a lower contribution by its partner than it would receive from a different message. In other words, country i's messaging strategy cannot contain two messages m_i and m_i' such that no type of country j contributes less, and some types contribute more, after receiving m_i' than after m_i. This logic is formalized in the following proposition.

Proposition 2.3. *There is no equilibrium that satisfies all of the following criteria:*

1. *There is a pair of messages $m_i, m_i' \in M_i$ such that $m_i = \mu_i(c_i)$ for some $c_i \in T_i$ and $m_i' = \mu_i(c_i')$ for some $c_i' \in T_i$.*

2. *For almost all $c_j \in T_j$,*

$$s_j(c_j, \mu_j(c_j), m_i') \geq s_j(c_j, \mu_j(c_j), m_i).$$

3. *There is a set of types $\tilde{T}_j \subseteq T_j$ such that $\Pr(c_j \in \tilde{T}_j) > 0$ and*

$$s_j(c_j, \mu_j(c_j), m_i') > s_j(c_j, \mu_j(c_j), m_i),$$

   for all $c_j \in \tilde{T}_j$.

Proof. Take any assessment that meets the conditions of the proposition. Consider a deviation whereby type c_i sends the message m_i' and then, after receiving country j's message, makes the same contribution as if he had sent the prescribed message m_i. For ease of exposition, let $s_{i|c_i}(c_j) = s_i(c_i, \mu_i(c_i), \mu_j(c_j))$ denote the contribution of type c_i that is realized on the path of play when its ally is of type c_j, and define $s_{j|c_j}$ analogously. As p is strictly increasing, the difference in expected utility between the deviation and type c_i's proposed strategy is

$$\begin{aligned}
&\int_{T_j} \left[p\left(s_{i\,|\,c_i}(c_j) + s_{j\,|\,c_j}(c_i') \right) - p\left(s_{i\,|\,c_i}(c_j) + s_{j\,|\,c_j}(c_i) \right) \right] \, dF_j \\
&\qquad \geq \int_{\tilde{T}_j} \left[p\left(s_{i\,|\,c_i}(c_j) + s_{j\,|\,c_j}(c_i') \right) - p\left(s_{i\,|\,c_i}(c_j) + s_{j\,|\,c_j}(c_i) \right) \right] \, dF_j \\
&\qquad > 0.
\end{aligned}$$

Therefore, the assessment is not an equilibrium. □

This result places a stringent condition on influential equilibria. If some types of country j contribute more after receiving m_i' than after m_i, then there must be other types that contribute less. In order for there to be an influential equilibrium, different types of a country must have opposite reactions to the same information. Intuitively, it is difficult to see conditions under which this might be the case. It is easier to imagine that every type contributes less as it learns its ally is willing to contribute more. Subsequent proofs of the non-existence of influential equilibria, specifically in the case of two-sided incomplete information examined in Section 4, rely on exactly this logic.

3 One-Sided Uncertainty

In this section, I consider the case in which one country's type is private information and the other's is common knowledge. I find that, under broad conditions, influential communication is not possible in equilibrium. When a country knows its partner's type, it thereby knows exactly how its partner will respond to any potential message in equilibrium. Since every type of a country prefers that its partner contribute more, the informed country will always choose the message that induces the greatest contribution by its ally. As every type of the informed country prefers to send the same message, any potential influential equilibrium unravels. After ruling out influential equilibria, I examine whether the countries are worse off because of their failure to communicate. I show that the total contribution in equilibrium is never higher—and sometimes is less—than what it would be if the informed country's type were common knowledge.

It is straightforward to prove that there is no influential equilibrium in the case of one-sided uncertainty. According to the main result of the previous section, Proposition 2.3, there cannot be an equilibrium in which every type of a country contributes more after receiving one message than another. But under one-sided incomplete information, there is only one type of country 2.

Therefore, it is not possible for different types of country 2 to react differently to country 1's message, as required in an influential equilibrium. I restate this logic formally in the proof of the following proposition.

Proposition 3.1. *There is no influential equilibrium in the game with one-sided uncertainty about willingness to contribute.*

Proof. In an influential assessment, there exist types c_1, c_1' such that

$$s_2(c_2, \mu_2(c_2), \mu_1(c_1)) > s_2(c_2, \mu_2(c_2), \mu_1(c_1')).$$

Since country 2's type is c_2 with probability 1, by Proposition 2.3, such an assessment cannot be an equilibrium. □

This result holds under rather general conditions. It requires no assumption about the form of the messaging strategy—i.e., it is not restricted to interval messaging. In fact, the proposition would still hold if many of the background assumptions of the model were relaxed. Among these are the assumption that the probability of success function, p, is differentiable and concave, and that the distribution of country 1's type is continuous on an interval.[9]

I now turn to the question of whether the players in the model would be better off if communication were possible. I compare the equilibrium outcome that results for each possible type of country 1 to what the outcome would have been if types were common knowledge, as given by Proposition 2.1. I find that country 1's inability to communicate indeed hinders the allies' chances of success: the total contribution is never greater in the game with one-sided incomplete information than in the complete-information benchmark. Moreover, unless country 1's marginal cost of effort is almost surely greater than country 2's (or vice versa), the equilibrium outcome is sometimes strictly worse in the game with incomplete information.

Proposition 3.2. *Let $\{(\mu_i, s_i, \lambda_i)\}_{i=1,2}$ be any equilibrium of the game with one-sided uncertainty about costs. For all possible realizations of country 1's type $c_1 \in T_1$, the total contribution on the path of play is no greater than the equilibrium total contribution if c_1 were common knowledge:*

$$s_1(c_1, \mu_1(c_1), c_2) + s_2(c_2, c_2, \mu_1(c_1)) \leq \max\{x^*(c_1), x^*(c_2)\}. \tag{3.1}$$

If $\underline{c}_1 < c_2 < \bar{c}_1$ and $x^*(c_2) > 0$, then the above inequality holds strictly for all $c_1 \in (c_2, \bar{c}_1]$.

[9] Although these assumptions are not necessary for Proposition 3.1, they are required for the efficiency results that follow.

Proof. By Proposition 3.1, there exists $\tilde{x}_2 \geq 0$ such that $s_2(c_2, c_2, \mu_1(c_1)) = \tilde{x}_2$ for all $c_1 \in T_1$. The first-order conditions for an optimal choice by country 1 then give

$$s_1(c_1, \mu_1(c_1), c_2) = \max\{0, x^*(c_1) - \tilde{x}_2\} \tag{3.2}$$

Since any contribution above a country's standalone contribution is strictly dominated, $x_2 \leq x^*(c_2)$. The inequality (3.1) follows immediately.

Now suppose $\underline{c}_1 < c_2 < \bar{c}_1$ and $x^*(c_2) > 0$. To prove that the inequality (3.1) holds strictly for all $c_1 \in (c_2, \bar{c}_1]$, it suffices to show that $\tilde{x}_2 \neq x^*(c_2)$. As $\tilde{x}_2 \leq x^*(c_2)$, equation (3.2) gives $s_1(c_1, \mu_1(c_1), c_2) > 0$ for all $c_1 < c_2$. Therefore, the first-order condition for a best response by country 2 cannot be satisfied at $x_2 = x^*(c_2)$:

$$\int_{T_1} p'\left(s_1(c_1, \mu_1(c_1), c_2) + x^*(c_2)\right) dF_1 - c_2$$
$$< \int_{T_1} p'\left(x^*(c_2)\right) dF_1 - c_2 = 0. \qquad \square$$

Technically, this result does not address whether the countries would be better off, in terms of their utility, under complete information. In fact, because of the free-riding incentives inherent in a public goods dilemma, the result does not imply that the equilibrium outcomes are Pareto inferior to their full-information counterparts. For example, suppose $\underline{c}_1 < c_2$ and consider an equilibrium in which country 2 contributes a positive amount, $\tilde{x}_2 > 0$, on the path of play. When country 1's type is $\underline{c}_1$, the equilibrium outcome is that country 1 contributes $x^*(\underline{c}_1) - \tilde{x}_2$ and country 2 contributes $\tilde{x}_2$. If country 1's type were common knowledge, the equilibrium outcome would be that country 1 contributes $x^*(\underline{c}_1)$ and country 2 contributes nothing. As the total contribution is the same in either case, country 1 strictly prefers the former outcome, in which it is responsible for a smaller share of the burden, and country 2 prefers the converse. Therefore, neither outcome is Pareto superior to the other.

However, by the weaker criterion of social welfare maximization, equilibrium outcomes under one-sided incomplete information are indeed inefficient relative to full revelation. Define the *ex post* social welfare function as the sum of the countries' payoffs, given their types:

$$U(x_1, x_2 \,|\, c_1, c_2) = 2p(x_1 + x_2) - c_1 x_1 - c_2 x_2.$$

Let (x_1', x_2') be the outcome that results for type c_1 in the one-sided incomplete information game, and let (x_1'', x_2'') be the corresponding complete information outcome. *Ex post* inefficiency of the incomplete information outcome follows from two facts. First, under incomplete information, the higher-cost country may contribute; this never occurs in the complete information benchmark. Second, the total contribution under incomplete information may be less than under full revelation, which in turn is less than the social welfare maximizer, $x^*(\min\{c_1, c_2\}/2)$. Therefore, we have

$$\begin{aligned}
U(x_1', x_2' \mid c_1, c_2) &= 2p(x_1' + x_2') - c_1 x_1' - c_2 x_2' \\
&\leq 2p(x_1' + x_2') - \min\{c_1, c_2\}(x_1' + x_2') \\
&\leq 2p(x_1'' + x_2'') - \min\{c_1, c_2\}(x_1'' + x_2'') \\
&= U(x_1'', x_2'' \mid c_1, c_2).
\end{aligned}$$

as claimed. A sufficient condition for the final inequality to hold strictly is that equation (3.2) hold strictly. Unless it is common knowledge that the informed country's cost of contribution is higher than its partner's (or vice versa), social welfare in equilibrium is sometimes strictly less than if all costs were common knowledge.

4 Two-Sided Uncertainty

I now consider the case in which both countries have private information about their marginal cost of effort. As in the analysis of the one-sided case, I focus on whether influential communication is possible in equilibrium. The logic of Proposition 3.1, which rules out influential equilibria under one-sided incomplete information, does not carry over to this setting. Each country is uncertain of its partner's type in the messaging stage, meaning it cannot know exactly how its partner will react to any given message. However, the other barriers to communication are still present. An influential equilibrium requires that a country prefer to send different messages depending on its type, even though each country always wants the same thing—namely, for its ally to contribute as much as possible—regardless of its type.

As in the previous section, I obtain negative results about the possibility of influential communication. What hinders information revelation is the incentive for a country to overstate its cost of contributing to the joint effort. A country expects its ally to contribute more if its cost of effort is relatively low. It follows that a country contributes less if it believes its ally is a low-cost type than if it believes it is a high-cost type. Therefore, it is always in the interest of a country for its ally to believe its cost is high, i.e., that it is relatively unwilling to contribute to the joint effort. Even countries that are willing to contribute a lot on their own are better off if their ally takes on more of the burden. When this universal incentive to misrepresent is present, there cannot be influential communication in equilibrium.

I begin by ruling out the extreme case of a fully separating equilibrium, in which each type truthfully reports its exact willingness to contribute. The logic of the result is as follows. If both countries honestly reveal their types, then there is complete information in the contribution stage. The equilibrium outcome is thus the same as in the complete information game, as given by Proposition 2.1. Specifically, the equilibrium outcome is that the country with a lower cost of effort contributes its standalone value, and the other country contributes nothing. Therefore, by dishonestly claiming to be the highest-cost type, a country can ensure that its partner will contribute as much as possible.

Proposition 4.1. *There is no fully separating influential equilibrium in the game with two-sided uncertainty about willingness to contribute.*

Proof. Consider a fully separating equilibrium, in which $\mu_i(c_i) = c_i$ for all c_i. By Proposition 2.1, each type's contribution on the path of play is given by

$$s_i(c_i, c_i, c_j) = \begin{cases} x^*(c_i) & c_i < c_j, \\ x^*(c_i) - s_j(c_j, c_j, c_i) & c_i = c_j, \\ 0 & c_i > c_j. \end{cases}$$

This expression is weakly decreasing in c_j. Therefore, if the assessment is influential, it meets the conditions of Proposition 2.3, contradicting the assumption of equilibrium. $\square$

This result only rules out influential equilibria in which the countries fully reveal their types. What remains to be seen is whether the logic of the result extends to coarser forms of communication. In other cheap talk settings, including the canonical model of Crawford and Sobel (1982), there may be partially separating equilibria even if there is no equilibrium with full revelation. In such equilibria, a country's messaging strategy may reveal only that its type lies within a particular interval, rather than its exact value. In the remainder of this section, I investigate whether this coarser kind of communication may occur in equilibrium.

To make the analysis of more general messaging strategies tractable, I impose an additional assumption on the functional form of the model. In general, when a country is uncertain of its ally's type in the contribution stage, its objective is to select x_i so as to maximize its expected utility,

$$E_{\lambda_i(m_j)}\left[u_i(x_i,s_j(c_j,m_j,m_i))\right]=\int_{T_j}u_i(x_i,s_j(c_j,m_j,m_i))\,d\lambda_i(m_j).$$

The solution to this maximization problem depends on the whole distribution of the ally's contribution, $s_j(c_j,m_j,m_i)$, making it difficult to characterize a general solution. As in Barbieri and Malueg's (2010; 2013) analysis of similar public goods models with incomplete information, one way to facilitate characterization of the solution is to impose conditions such that the maximum depends only on the expected value of the ally's contribution:

$$\begin{aligned}\arg\max_{x_i} E_{\lambda_i(m_j)}&\left[u_i(x_i,s_j(c_j,m_j,m_i))\right]\\&=\arg\max_{x_i} u_i\left(x_i,E_{\lambda_i(m_j)}[s_j(c_j,m_j,m_i)]\right).\end{aligned}\tag{4.1}$$

When this condition holds, we may write a country's best response as a decreasing function of the expected value of its ally's contribution. A sufficient condition for equation (4.1) to hold is that p, the function for the probability of success, be quadratic on the set of undominated contribution profiles. The result is summarized in the following lemma.

Lemma 4.1. *If p is quadratic on $[0,\frac{1}{\underline{c}_1}+\frac{1}{\underline{c}_2}]$, then in any equilibrium*

$$s_i(c_i,m_i,m_j)=\max\left\{0,x^*(c_i)-E_{\lambda_i(m_j)}[s_j(c_j,m_j,m_i)]\right\}\tag{4.2}$$

for all $c_i\in T_i$, $m_i\in M_i$, $m_j\in M_j$.

Proof. The quadraticity assumption implies p' is linear on $[0,\frac{1}{\underline{c}_1}+\frac{1}{\underline{c}_2}]$. Moreover, in equilibrium, $s_j(c_j,m_j,m_i)\le x^*(c_j)<\frac{1}{\underline{c}_j}$ for all $c_j\in T_j$. The first-order condition for $x_i\in(0,x^*(c_i)]$ to be a best response for type c_i in the subgame $\Gamma(m_i,m_j)$ is thus

$$\begin{aligned}0&=E_{\lambda_i(m_j)}\left[p'\left(x_i+s_j(c_j,m_j,m_i)\right)-c_i\right]\\&=p'\left(x_i+E_{\lambda_i(m_j)}[s_j(c_j,m_j,m_i)]\right)-c_i.\end{aligned}$$

This implies $x_i+E_{\lambda_i(m_j)}[s_j(c_j,m_j,m_i)]=x^*(c_i)$, as claimed. $\square$

Under this additional assumption, it is possible to rule out influential communication in any equilibrium in which the countries use interval messaging strategies. The proof generalizes that of Proposition 4.1. In an influential equilibrium, if some types of a country contribute more after receiving m_j than after m_j', other types must do the opposite, per Proposition 2.3. This condition cannot be met under interval messaging. Whenever a country receives a "high-cost" message from its ally, it expects its partner to contribute less—and thus contributes more itself, per equation (4.2)—than if it had received a "low-cost" message. This gives each country an incentive to overstate its own cost of effort, so as to induce the greatest possible contribution by its ally. Therefore, there is no influential equilibrium in interval messaging.

Proposition 4.2. *If p is quadratic on $[0, \frac{1}{\underline{c}_1} + \frac{1}{\underline{c}_2}]$, there is no influential equilibrium in interval messaging strategies in the game with two-sided uncertainty about willingness to contribute.*

Proof. Suppose, for the purposes of a proof by contradiction, that there exists an influential equilibrium in interval messaging strategies. Without loss of generality, let country 1 be the influential one, so there exist messages m_1 and m_1' in the range of μ_1 such that $m_1' > m_1$ and $s_2(c_2, \mu_2(c_2), m_1) \neq s_2(c_2, \mu_2(c_2), m_1')$ on a positive-measure subset of T_2.[10] By Proposition 2.3 and the assumption of equilibrium, there is a type $\tilde{c}_2 \in T_2$ that contributes more after receiving the message m_1 than after receiving m_1'; i.e., $s_2(\tilde{c}_2, m_2, m_1) > s_2(\tilde{c}_2, m_2, m_1')$, where $m_2 = \mu_2(\tilde{c}_2)$.

I claim that the expected total contribution in the subgame $\Gamma(m_1, m_2)$ is no greater than that in $\Gamma(m_1', m_2)$. Let $\bar{x}_j(m_i, m_j)$ denote country i's expectation of what country j will contribute, given the outcome of the messaging stage:

$$\bar{x}_j(m_i, m_j) = E_{\lambda_i(m_j)}\left[s_j(c_j, m_j, m_i)\right].$$

By Lemma 4.1, $s_2(\tilde{c}_2, m_2, m_1) > s_2(\tilde{c}_2, m_2, m_1')$ implies $\bar{x}_1(m_1, m_2) < \bar{x}_1(m_1', m_2)$, as country 2's optimal contribution is inversely related to its expectation of what country 1 will contribute. Again applying Lemma 4.1, this in

[10]Under the form of the messaging strategy defined in Section 2.2, $m_1' > m_1$ implies that m_1' is sent by higher-cost types than m_1.

turn gives

$$
\begin{aligned}
&\left[\bar{x}_1(m_1', m_2) + \bar{x}_2(m_1', m_2)\right] - \left[\bar{x}_1(m_1, m_2) + \bar{x}_2(m_1, m_2)\right] \\
&\quad = \bar{x}_1(m_1', m_2) - \bar{x}_1(m_1, m_2) \\
&\qquad\quad + \int_{T_2} \max\left\{0, x^*(c_2) - \bar{x}_1(m_1', m_2)\right\} d\lambda_1(m_2) \\
&\qquad\quad - \int_{T_2} \max\left\{0, x^*(c_2) - \bar{x}_1(m_1, m_2)\right\} d\lambda_1(m_2) \\
&\quad \geq \bar{x}_1(m_1', m_2) - \bar{x}_1(m_1, m_2) + \int_{T_2} \left[\bar{x}_1(m_1, m_2) - \bar{x}_1(m_1', m_2)\right] d\lambda_1(m_2) \\
&\quad = 0.
\end{aligned}
$$

Therefore, we have

$$\bar{x}_1(m_1', m_2) + \bar{x}_2(m_1', m_2) \geq \bar{x}_1(m_1, m_2) + \bar{x}_2(m_1, m_2). \tag{4.3}$$

To conclude the proof, I show that equation (4.3) contradicts the assumption that $m_1' > m_1$. Take any $\tilde{c}_1 \in \mathscr{C}_1(m_1)$ such that $s_1(\tilde{c}_1, m_1, m_2) = \bar{x}_1(m_1, m_2)$. If $\mathscr{C}_1(m_1)$ is a singleton, then $\tilde{c}_1 = m_1$ by definition; otherwise, $\mathscr{C}_1(m_1)$ is an interval, and the Second Mean Value Theorem for Integrals guarantees the existence of an appropriate value of $\tilde{c}_1 \in \mathscr{C}_1(m_1)$. Let $\tilde{c}_1' \in \mathscr{C}_1(m_1')$ be chosen analogously. Lemma 4.1 gives

$$\bar{x}_1(m_1, m_2) = s_1(\tilde{c}_1, m_1, m_2) \geq x^*(\tilde{c}_1) - \bar{x}_2(m_1, m_2), \tag{4.4}$$

$$\bar{x}_1(m_1', m_2) = s_1(\tilde{c}_1', m_1', m_2) = x^*(\tilde{c}_1') - \bar{x}_2(m_1', m_2). \tag{4.5}$$

By definition of a monotone messaging strategy, $\tilde{c}_1' > \tilde{c}_1$. Moreover, by equation (4.5), $x^*(\tilde{c}_1) \geq \bar{x}_1(m_1', m_2) > 0$. As the standalone contribution function x^* is weakly decreasing everywhere and strictly decreasing wherever it is positive, $x^*(\tilde{c}_1') < x^*(\tilde{c}_1)$. However, combining equations (4.3)–(4.5) gives

$$x^*(\tilde{c}_1') = \bar{x}_1(m_1', m_2) + \bar{x}_2(m_1', m_2) \geq \bar{x}_1(m_1, m_2) + \bar{x}_2(m_1, m_2) \geq x^*(\tilde{c}_1),$$

a contradiction. $\square$

Although I only prove this result for quadratic p, it is reasonable to suspect that the basic finding—that there is no influential equilibrium in interval messaging—holds for some broader class of functional forms. The key to the proof is that no type of a country contributes more after receiving a low-cost message from its partner than a high-cost message. In other words, a country contributes less when it knows its ally can contribute more. It is the same property that drives the proof that there is no fully separating equilibrium, Proposition 4.1, just with noisier beliefs. The assumption of quadratic payoffs makes it feasible to show that this property holds, insofar as it simplifies the form of the best response function, but there is no immediately apparent reason to suspect that the property itself would not hold for other concave functional forms.

The final topic of concern in the case of two-sided incomplete information about costs is efficiency. Under one-sided incomplete information, Proposition 3.2 showed that a babbling equilibrium is *ex post* inefficient compared to the outcome with exogenous full revelation. This strong form of inefficiency, which requires that the outcome be weakly worse for all possible realizations of the countries' types, does not hold in the two-sided case. It is easy to contrive examples where, for some realizations of types, the outcome of a babbling equilibrium exceeds the full-revelation outcome. For example, suppose the distributions of both countries' types are left-skewed, so each country expects its partner to have a high cost of effort and thus contribute relatively little. The lowest-cost type of each country will then contribute close to its own standalone contribution in equilibrium. In the event that both countries are the lowest type, $c_1 = \underline{c}_1$ and $c_2 = \underline{c}_2$, their total contribution may exceed either of their standalone amounts.

However, it is possible to demonstrate *ex ante* inefficiency, which requires only that the expected value of the outcome, taken over the distribution of the countries' types, be worse than the full-revelation benchmark. The outcome of any babbling equilibrium is also a Bayesian Nash equilibrium of the contribution game with two-sided incomplete information and no communication. Bag and Roy (2011, Proposition 1) consider the efficiency of equilibria of such a game in which at least one country almost surely contributes a positive amount.[11] They prove that any such equilibrium is *ex ante* inefficient relative to the outcome of a game with sequential contributions. Specifically, the expected total contribution in the equilibrium of the simultaneous game is weakly less than in any equilibrium of any sequential game form. Any equilibrium outcome of a sequential contribution game can, in turn, be shown to be *ex post* inefficient (and hence also *ex ante* inefficient) compared to the outcome with exogenous full revelation, using logic similar to the proof of Proposition 3.2. Therefore, if Bag and Roy's conditions are met, a babbling equilibrium of the game with two-sided incomplete information about costs yields a lower ex-

[11]The authors impose some additional conditions on p to obtain their results, notably the requirement that p' be concave on $[0, \frac{1}{\underline{c}_1} + \frac{1}{\underline{c}_2}]$.

pected total contribution than would obtain under exogenous full revelation.

5 Extensions

In this section, I consider two sources of uncertainty other than a state's marginal cost of contribution. First, I look at an extension in which one state gains extra intelligence about how much effort is required for the joint venture to succeed. There is only slightly more room for influential communication in this setting than in the original model. If the informed state learns the precise amount that is necessary for success, there is an equilibrium in which it always honestly reveals its information to its partner. However, influential communication is impossible in equilibrium if the signal also contains a small amount of noise. In the second extension, there are two avenues of contribution to the joint project, and one state has private information about which method it can provide more easily. In contrast to the previous results, there are broad conditions under which an influential equilibrium exists in this case.

5.1 Intelligence Sharing

The original model assumed that both allies have the same beliefs about how much effort is required for the joint venture to succeed. I now relax that assumption by allowing one country to have additional intelligence about the likelihood of success. For example, a country may have proprietary access to satellite photos showing the location and capabilities of enemy military bases. With this information in hand, a country can more accurately estimate how much strength is required to overwhelm the enemy. I examine the conditions under which a country that receives this sort of intelligence can honestly share it with an ally. As in the original model, the main obstacle to communication is that a state always prefers that its ally take on more of the burden, regardless of what it may have learned about the total amount required for success.

For the purposes of this extension, it will be convenient to interpret the allies' joint project as a military conflict against a common enemy. The allies succeed if their total contribution meets or exceeds the enemy's strength, denoted $y \geq 0$. At the start of the game, the allies do not know the value of y; it is a random variable drawn according to the cumulative distribution function F_y. The *ex ante* probability of success, as a function of the total contribution, is

$$\Pr(x_1 + x_2 \geq y) = F_y(x_1 + x_2).$$

Therefore, if the allies receive no additional information about their enemy's strength, this extension is equivalent to the original contribution game, with $p = F_y$.

In the *intelligence-sharing game*, Nature sends country 1 a signal of the common enemy's strength before contributions are chosen, and country 1 has the opportunity to pass this intelligence along to its ally. The enemy's strength and the signal are drawn from the joint probability distribution $F_{y,z}$ on $Y \times Z$, where $Z \subseteq Y \subseteq \Re_+$ and F_y is the marginal distribution of y. This distribution is common knowledge, but only country 1 observes the realized value of z, and neither country observes the realized value of y. For each $z \in Z$, let $F_y(\cdot \,|\, z)$ denote the conditional CDF of the common enemy's strength given the signal. The signal is *perfectly informative* if $z = y$, meaning country 1 learns the exact contribution level required for success. In this case, the conditional CDF is a step function,

$$F_y(y' \,|\, z) = \begin{cases} 0 & y' < z, \\ 1 & y' \geq z. \end{cases}$$

The common enemy's strength and the signal received by country 1 are the only sources of uncertainty in the intelligence-sharing game. In particular, the marginal costs of contribution, c_1 and c_2, are now common knowledge.

The sequence of play in the intelligence-sharing game is essentially the same as in the original model. After observing the signal z, country 1 sends a message $m_1 \in M_1$ to its ally, where the set of potential messages is the same as the set of signals, $M_1 = Z$. A messaging strategy for country 1 is a function $\mu_1 : Z \to M_1$. After receiving country 1's message, country 2 updates its beliefs about the common enemy's strength to $\lambda_2(m_1)$, a probability measure on Y. Finally, each country simultaneously selects its contribution x_i from $X_i = \Re_+$. Country 1's contribution depends both on the signal it received and the message it sent, so its contribution strategy is a function $s_1 : Z \times M_1 \to X_1$. Country 2's contribution strategy is a function solely of the message it received, $s_2 : M_1 \to X_2$.

The solution concept, as in the original model, is perfect Bayesian equilibrium, which requires sequential rationality of strategies and consistency of beliefs. A key requirement is that no type of country 1 be able to strictly increase its payoff by sending a message other than what is prescribed. This requirement is summarized formally in the following incentive compatibility condition:

$$\begin{aligned} &F_y\left(s_1(z, \mu_1(z)) + s_2(\mu_1(z)) \,|\, z\right) - c_1 s_1(z, \mu_1(z)) \\ &\qquad \geq \max_{(m_1, x_1)} \left\{ F_y\left(x_1 + s_2(m_1) \,|\, z\right) - c_1 x_1 \right\} \quad \text{for all } z \in Z. \end{aligned} \tag{5.1}$$

In two important respects, the intelligence sharing game resembles the game with one-sided incomplete information about costs examined in Section 3. First, all types of the informed country weakly prefer greater contributions by its partner. Second, the informed country knows exactly how its partner, which has no private information, will respond to each potential message. In light of these similarities, it may seem that the logic of Proposition 3.1 would also apply here—that there can be no influential equilibrium. On the contrary, in the special case of perfectly informative signals, there is a fully separating, influential equilibrium. In this equilibrium, the informed country always honestly reports its signal, z, which equals the contribution necessary for success. If this cost is no greater than the maximum its uninformed partner is willing to contribute, that country supplies all of the necessary effort. If the cost is too much for the uninformed country on its own but does not exceed the total willingness of both countries, the uninformed country contributes as much as it is willing to and the informed one covers the remainder. Otherwise, if the cost is too high for completion to be in the countries' joint interest, neither contributes at all. The equilibrium strategies are illustrated in Figure 1 and stated formally in the following proposition.

Proposition 5.1. *If the signal is perfectly informative, then there is a fully separating, influential equilibrium of the intelligence-sharing game in which:*

$$
\begin{aligned}
\mu_1(z) &= z, \\
s_1(z,z) &= \begin{cases} 0 & z < \frac{1}{c_2}, \\ z - \frac{1}{c_2} & \frac{1}{c_2} \leq z \leq \frac{1}{c_1} + \frac{1}{c_2}, \\ 0 & z > \frac{1}{c_1} + \frac{1}{c_2}, \end{cases} \\
s_2(m_1) &= \begin{cases} m_1 & m_1 < \frac{1}{c_2}, \\ \frac{1}{c_2} & \frac{1}{c_2} \leq m_1 \leq \frac{1}{c_1} + \frac{1}{c_2}, \\ 0 & m_1 > \frac{1}{c_1} + \frac{1}{c_2}, \end{cases} \\
\lambda_2(m_1) &= \delta_{m_1}.
\end{aligned}
$$

Proof. To construct an equilibrium, we must complete the definition of country 1's strategy, s_1, for subgames that are off the path of play—i.e., after country 1 falsely reports the signal z. Consider the subgame that follows after country 1 sends the message m_1. Country 1's interim utility function is

$$u_1(x_1 \mid m_1, z) = \mathbf{1}\{x_1 + s_2(m_1) \geq z\} - c_1 x_1.$$

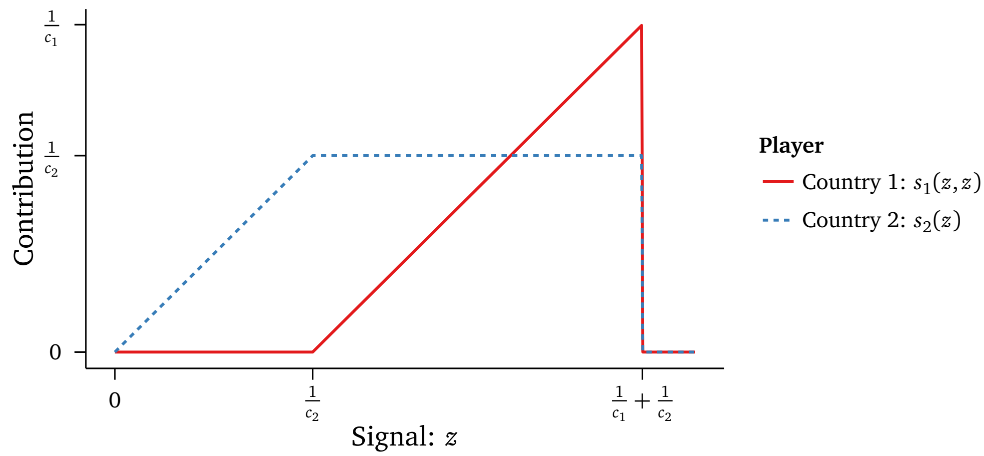

Figure 1. Contributions made by each country on the path of play in the equilibrium given in Proposition 5.1, as a function of the signal received by country 1.

This function is maximized at 0 or at $z - s_2(m_1)$, so a best response exists. For each case in which $m_1 \neq z$, let $s_1(z, m_1)$ be any such best response.

I proceed by showing that the proposed contribution strategies are sequentially rational. Consider country 2's strategy. After receiving the message m_1, country 2's belief about the enemy's strength Y is a point mass on m_1. Country 2's interim utility function is thus

$$u_2(x_2 \mid m_1) = \mathbf{1}\{s_1(m_1, m_1) + x_2 \geq m_1\} - c_2 x_2,$$

which is maximized at 0 or at $z - s_1(m_1, m_1)$. Specifically, 0 is a maximizer if country 1's contribution is sufficient for success ($s_1(m_1, m_1) \geq m_1$) or the remaining amount necessary to succeed exceeds country 2's willingness to contribute ($z - s_1(m_1, m_1) \geq \frac{1}{c_2}$). Conversely, if $0 \leq z - s_1(m_1, m_1) \leq \frac{1}{c_2}$, then $z - s_1(m_1, m_1)$ is a maximizer. It is then immediate from the definitions of $s_1(z, z)$ and s_2 that country 2's contribution strategy is optimal, given its beliefs. It is analogous to confirm that country 1's contribution strategy along the path of play ($m_1 = z$) is optimal, given country 2's strategy. In cases off the path of play ($m_1 \neq z$), country 1's contribution is optimal by construction.

The next step is to show that country 1's messaging strategy is incentive compatible—i.e., that country 1 never has an incentive to falsely report the signal it has received. If $z \leq \frac{1}{c_2}$, then country 1's proposed messaging strategy yields its first-best outcome (guaranteed success at no cost to itself), so no profitable deviation is available. If $\frac{1}{c_2} < z \leq \frac{1}{c_1} + \frac{1}{c_2}$, then deviating to any message other than z would cause country 2 to contribute weakly less, so no such deviation may be profitable. Lastly, if $z > \frac{1}{c_1} + \frac{1}{c_2}$, then the most country 1 could induce country 2 to contribute by deviating to a different message is $x_2 = \frac{1}{c_2}$. Even in that case, country 1's best response is to contribute 0, assuring failure of the joint venture and a payoff of 0, the same as under the proposed messaging strategy. Therefore, there is no profitable deviation available.

The final step is to show that country 2's beliefs are consistent with the application of Bayes' rule, which is immediate from the definition of λ_2. □

What keeps influential communication from unraveling in this equilibrium, as it did in the earlier case of one-sided incomplete information about costs? The key is that the informed country does not always strictly benefit from a higher contribution by its partner. Whether its ally contributes just enough to assure success or even more, the informed country's payoff is the same. By the same token, the informed country's payoff from failure is the same no matter what its partner contributes. This indifference results from assuming that the signal is perfectly informative. If every increase in the total contribution raised the chance of success even slightly, then country 1 would always strictly prefer the message that induced the highest contribution by its partner. A sufficient condition for country 1 never to be indifferent is that its belief about the enemy's strength have full support on the set of feasible contributions following every signal, as stated in the following proposition.

Proposition 5.2. *If* $[0, \frac{1}{c_1} + \frac{1}{c_2}] \subseteq \operatorname{supp} F_y(\cdot \,|\, z)$ *for all signals* z*, then there is no influential equilibrium of the intelligence-sharing game.*

Proof. In an influential equilibrium, there exist signals z' and z'' such that $s_2(\mu_1(z')) < s_2(\mu_1(z''))$. Contributions $x_j > \frac{1}{c_j}$ are strictly dominated for country j, so $s_1(\mu_1(z'), z') + s_2(\mu_1(z'')) \leq \frac{1}{c_1} + \frac{1}{c_2}$. Moreover, $F_y(\cdot \,|\, z)$ is strictly increasing on $[0, \frac{1}{c_1} + \frac{1}{c_2}]$ by construction. Therefore, country 1 can profit by sending the message $\mu(z'')$ when the true signal is z':

$$
\begin{aligned}
F_y\left(s_1(z', \mu_1(z')) + s_2(\mu_1(z')) \,|\, z'\right) - c_1 s_1(z', \mu_1(z')) \\
< F_y\left(s_1(z', \mu_1(z')) + s_2(\mu_1(z'')) \,|\, z'\right) - c_1 s_1(z', \mu_1(z')).
\end{aligned}
$$

This contradicts the incentive compatibility condition (5.1). □

This result shows that the influential communication that can occur with a perfectly informative signal is fragile. For example, consider the case of an "almost perfectly informative" signal, in which, after receiving the signal z, country 1's is a mixture with probability $1-\epsilon$ on $y = z$ and probability ϵ that y is uniformly distributed on $[0, \frac{1}{c_1} + \frac{1}{c_2}]$. For any $\epsilon > 0$, there is no influential equilibrium, according to Proposition 5.2. In this sense, the existence of an influential equilibrium under perfectly informative signaling is not robust to a small perturbation in the information environment. Although it is interesting that influential communication can occur at all in the intelligence-sharing game, particularly in contrast with the game with one-sided incomplete information about costs, the finding is knife-edge and should not be interpreted as a general feature of intelligence sharing.

5.2 Multiple Contribution Methods

I now consider an extension in which there are multiple ways a state can contribute to the joint effort. For example, the success of a foreign invasion may depend on both ground troops and air support. States may vary in their ability or willingness to contribute to each method—continuing the example, some may have better flight technology, while others have better-trained infantries. In this extension, I assume that one state has private information about its relative cost of two different methods of contribution. In contrast with the results of previous sections, I show that influential communication is possible under broad conditions in this extension. The finding is similar to Trager's (2011) results about multidimensional bargaining between adversaries.

In the contribution stage of the *two-method game*, each state decides whether to devote effort to either or both of two avenues of contribution. To keep the analysis simple, I assume the contribution to each method is a binary, all-or-nothing decision. Formally, each state's action in the contribution stage is a pair $x_i = (x_i^A, x_i^B)$ of binary decisions, denoting the country's decision to contribute to methods A and B respectively. Each country's action space is $X_i = \{0, 1\}^2$. The cost to country i of participating in method A is α_i, and its cost of participating in method B is β_i. The probability of success is now a function of the total contribution to each method, denoted $p(x^A, x^B)$, which is strictly increasing in both arguments. A country's utility function in the two-method contribution game is

$$u_i(x_i, x_j \mid \alpha_i, \beta_i) = p(x_i^A + x_j^A, x_i^B + x_j^B) - \alpha_i x_i^A - \beta_i x_i^B. \tag{5.2}$$

Figure 2 contains the payoff matrix for the contribution subgame.

I consider a form of the game with one-sided incomplete information, in which the source of uncertainty is country 1's cost of contributing to method B. Country 1's type β_1 equals β_1^L with probability $\pi \in (0, 1)$ and β^H with probabil-

		Country 2			
		$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
Country 1	$(0,0)$	$p(0,0)$ $p(0,0)$	$p(1,0)$ $p(1,0)-\alpha_2$	$p(0,1)$ $p(0,1)-\beta_2$	$p(1,1)$ $p(1,1)-\alpha_2-\beta_2$
	$(1,0)$	$p(1,0)-\alpha_1$ $p(1,0)$	$p(2,0)-\alpha_1$ $p(2,0)-\alpha_2$	$p(1,1)-\alpha_1$ $p(1,1)-\beta_2$	$p(2,1)-\alpha_1$ $p(2,1)-\alpha_2-\beta_2$
	$(0,1)$	$p(0,1)-\beta_1$ $p(0,1)$	$p(1,1)-\beta_1$ $p(1,1)-\alpha_2$	$p(0,2)-\beta_1$ $p(0,2)-\beta_2$	$p(1,2)-\beta_1$ $p(1,2)-\alpha_2-\beta_2$
	$(1,1)$	$p(1,1)-\alpha_1-\beta_1$ $p(1,1)$	$p(2,1)-\alpha_1-\beta_1$ $p(2,1)-\alpha_2$	$p(1,2)-\alpha_1-\beta_1$ $p(1,2)-\beta_2$	$p(2,2)-\alpha_1-\beta_1$ $p(2,2)-\alpha_2-\beta_2$

Figure 2. Payoff matrix for the contribution stage of the two-method game.

ity $1-\pi$, where $\beta_1^L < \alpha_1 < \beta_1^H$. I call country 1 "$B$-advantaged" if $\beta_1 = \beta_1^L$ and "A-advantaged" if $\beta_1 = \beta_1^H$. The prior distribution of β_1 is common knowledge, but only country 1 observes its realized value. The other cost parameters, α_1, α_2, and β_2, are common knowledge. I assume it is never in a state's interest to contribute to a method that its ally is also contributing to.[12] This condition is equivalent to

$$p(2, x^B) - p(1, x^B) \leq \min\{\alpha_1, \alpha_2\} \quad \text{for all } x^B = 0, 1, 2,$$
$$p(x^A, 2) - p(x^A, 1) \leq \min\{\beta_1^L, \beta_2\} \quad \text{for all } x^A = 0, 1, 2.$$

As in the original model, country 1 has the opportunity to send its ally a message about its type prior to the contribution stage. Let $T_1 = \{\beta_1^L, \beta_1^H\}$ denote country 1's type space, and, as before, suppose its message space is the same, $M_1 = T_1$. The definitions of strategies and beliefs are similar to those in the intelligence-sharing game. A messaging strategy for country 1 is a function $\mu_1 : T_1 \to M_1$. After receiving country 1's message m_1, country 2 updates its beliefs to $\lambda_1(m_1)$, which now is a scalar denoting the probability that $\beta_1 = \beta_1^L$. Country 1's contribution strategy is a function of both its type and the message it sent, $s_1 : T_1 \times M_1 \to X_1$. Country 2's contribution strategy is a function solely of the message it received, $s_2 : M_1 \to X_2$.

In the remainder of this section, I characterize the conditions under which there exists a *division of labor equilibrium*, which I define as an equilibrium with the following properties. First, country 1 honestly reveals its type in the messaging stage: $\mu_1(\beta_1) = \beta_1$ for all β_1. Second, both methods of contribution are always supplied on the equilibrium path: $s_1(\beta_1, \mu_1(\beta)) + s_2(\mu_1(\beta_1)) = (1, 1)$ for all β_1. Finally, there is a division of labor between the countries that depends on country 1's type: if country 1 is B-advantaged, then it contributes to method B and country 2 contributes to method A, and vice versa. The last requirement implies that a division of labor equilibrium is influential, though there may be influential equilibria of different forms as well.

In order to delineate the conditions under which there is a division of labor equilibrium, it is useful to think of country 1's messages as statements about what it will do in the contribution stage. Consider the B-advantaged type's message, which can be interpreted as the statement "I will contribute to method B." For this to be credible, the B-advantaged type of country 1 must indeed prefer to do as promised, given that its ally believes the promise and chooses its own action accordingly. Farrell and Rabin (1996) call such a statement *self-committing*. It must not be in country 1's interest to trick its partner

[12] Influential equilibria would still exist if this condition were relaxed, though the statement of the conditions for Proposition 5.4 would become more cumbersome.

into thinking it will contribute to method B and then choose a different course of action. In formal terms, country 1's announced action and country 2's best response must form a Nash equilibrium of the game where country 1's type is common knowledge. The following proposition establishes the conditions under which it is an equilibrium for country 1 to provide B and country 2 to provide A, or vice versa, given that country 1's type is known. Specifically, neither country's contribution may cost more than the amount it increases the probability of success, holding fixed its ally's contribution.

Proposition 5.3. *In the contribution stage of the two-method game, suppose β_1 is common knowledge. The strategy profile $\{(x_1^A, x_1^B), (x_2^A, x_2^B)\} = \{(1,0), (0,1)\}$ is a Nash equilibrium if and only if the following conditions are met:*

$$p(1,1) - p(0,1) \geq \alpha_1, \tag{5.3}$$

$$p(1,1) - p(1,0) \geq \beta_2. \tag{5.4}$$

The strategy profile $\{(x_1^A, x_1^B), (x_2^A, x_2^B)\} = \{(0,1), (1,0)\}$ is a Nash equilibrium if and only if:

$$p(1,1) - p(0,1) \geq \alpha_2, \tag{5.5}$$

$$p(1,1) - p(1,0) \geq \beta_1. \tag{5.6}$$

Proof. I will prove the first statement; the proof of the second is analogous. Under the given strategy profile, since $x_2^B = 1$, it cannot be profitable for country 1 to deviate to any strategy with $x_1^B = 1$. Therefore, the only deviation we must consider for country 1 is $(x_1^A, x_1^B) = (0,0)$. This deviation is unprofitable if and only if the condition (5.3) holds. Similarly, because $x_1^A = 1$ under the given strategy profile, the only deviation we must consider for country 2 is $(x_2^A, x_2^B) = (0,0)$. This is unprofitable if and only if the condition (5.4) holds. □

Self-commitment is necessary, but not sufficient, for a cheap-talk statement to be credible. To determine if a statement about what action a speaker will take is self-committing, we assume the statement's recipient believes the statement and examine whether the speaker indeed prefers to take the promised action. But we cannot just consider a speaker's preferences given the recipient's beliefs; we must also address its preferences about those beliefs. To this end, Farrell and Rabin (1996) say a statement is *self-signaling* if a speaker wants it to be believed when, and only when, it is true. In the context of a division of labor equilibrium, self-signaling requires that the B-advantaged type of country 1, which promises to contribute B, would not be better off if country 2 instead believed country 1 would contribute A and acted accordingly. Similarly, the A-advantaged type must not prefer to induce country 2 to believe it will contribute B instead of A. The following proposition summarizes the conditions under which both the self-commitment and self-signaling requirements are satisfied, and therefore a division of labor equilibrium exists.

Proposition 5.4. *In the two-method game, suppose the following conditions are satisfied:*

$$p(1,1) - p(0,1) \geq \max\left\{\alpha_1, \alpha_2, \beta_1^L\right\}, \tag{5.7}$$

$$p(1,1) - p(1,0) \geq \max\left\{\alpha_1, \beta_1^L, \beta_2\right\}. \tag{5.8}$$

There exists a division of labor equilibrium, in which:

$$\begin{aligned}
\mu_1(\beta_1) &= \beta_1, \\
s_1(\beta_1, \beta_1) &= \begin{cases} (0,1) & \beta_1 = \beta_1^L, \\ (1,0) & \beta_1 = \beta_1^H, \end{cases} \\
s_2(m_1) &= \begin{cases} (1,0) & m_1 = \beta_1^L, \\ (0,1) & m_1 = \beta_1^H, \end{cases} \\
\lambda_2(m_1) &= \mathbf{1}\{m_1 = \beta_1^L\}.
\end{aligned}$$

Proof. Let the assessment $(\mu_1, s_1, s_2, \lambda_2)$ be defined as in the proposition. I begin by completing the definition of country 1's contribution strategy, namely for subgames that are off the path of play. For the cases in which $m_1 \neq \beta_1$, let $s_1(\beta_1, m_1)$ be any best response for country 1 to $s_2(m_1)$; a best response is guaranteed to exist, as the game is finite. The goal of the remainder of the proof is to show that this assessment is an equilibrium.

I proceed by showing that the proposed contribution strategies are sequentially rational. The conditions (5.7) and (5.8) imply that equations (5.3)– (5.6) are satisfied. Therefore, by Proposition 5.3, country 2's proposed actions are best responses given its beliefs, as are country 1's proposed actions in the subgames on the equilibrium path. In the subgames off the equilibrium path, country 1's actions are best responses by construction.

Next, I show that country 1's messaging strategy is incentive compatible: neither type of country 1 can strictly improve its payoff by deviating from its messaging strategy. Consider the maximal payoff to the B-advantaged type after falsely reporting β_1^H. Since country 2 contributes $x_2^B = 1$ after receiving the A-advantaged message, we only need to consider actions with $x_1^B = 0$. The condition for the deviation to be unprofitable is thus

$$p(1,1) - \beta_1^L \geq \max\left\{p(0,1), p(1,1) - \alpha_1\right\},$$

which follows from equation (5.7) and $\beta_1^L < \alpha_1$. Similarly, the condition for it to be unprofitable for the A-advantaged type to deviate in the messaging stage is

$$p(1,1) - \alpha_1 \geq \max\left\{p(1,0), p(1,1) - \beta_1^H\right\},$$

which follows from equation (5.8) and $\beta_1^H > \alpha_1$.

The final step is to confirm that country 2's updated beliefs are consistent with Bayes' rule wherever possible, which is immediate from the definition of λ_2. □

The difference between this result and those for other forms of uncertainty is striking. I have characterized an influential equilibrium, unlike in the case of incomplete information about costs, and the conditions for it to exist are not knife-edge, unlike in the intelligence-sharing game. The crucial difference is that in the two-method game, different types of the informed country may have different preferences over its ally's actions. In particular, under the conditions of Proposition 5.4, the B-advantaged type of country 1 would rather that country 2 only contribute A than that it only contribute B, and conversely for the A-advantaged type. Therefore, neither type has an incentive to mimic the other—and, in fact, each may be strictly better off if it reports its type honestly than if it mimics the other.

6 Conclusion

I have examined the possibility of meaningful communication between allied countries about three separate sources of uncertainty. Although the players in the model have a common interest in the success of a joint project in all three cases, the results about influential communication in equilibrium vary. First, the results are uniformly negative when states have private information about how much it would cost to contribute to the joint venture. Each country has an incentive to pretend its cost of contribution is relatively high, so as to induce its partner to contribute as much as possible. The incentive to free-ride creates an incentive to misrepresent that precludes honest communication, even though the chance of success would be greater if the countries revealed their private information. Second, when one state receives intelligence about the strength of the allies' common enemy (or, more generally, the amount required for their joint project to succeed), influential communication is possible only under highly restrictive conditions. Unless the intelligence is precise to an unrealistic degree, no influential equilibrium exists. Finally, communication is possible when there are two avenues of contribution and one state has private information about its relative willingness to devote effort to each. In this case, unlike the others, different types of the informed country may have different preferences over its ally's actions, so communication is not hampered by incentives to misrepresent.

There are many fruitful directions for future research on international diplomatic communication. One interesting problem to examine would be cheap-talk communication in the presence of both allies and adversaries. Previous research has demonstrated that there is an incentive to overstate one's strength when communicating solely with an adversary (Fearon 1995), whereas this paper has shown that there is an incentive to understate capabilities to one's allies. In reality, states that send public signals are communicating with both audiences at once—a more complex problem that merits further examination. Another direction for future study on diplomacy between allies is how a state discovers which states it shares common interests with in the first place. The model in this paper presupposes that it is common knowledge that both players are on the same side (i.e., they both want the project to succeed rather than fail), but this may not always be clear *a priori*. It would be interesting to model the process by which a country searches for and identifies potential allies without revealing its weaknesses or specific interests to potential enemies.

A Appendix

Lemma A.1. *Consider any contribution subgame $\Gamma(m_1, m_2)$ of the game with incomplete information about costs. If each $\lambda_i(m_j)$ is either a degenerate distribution or a continuous distribution on an interval, then there exists a Bayesian Nash equilibrium of the subgame.*

Proof. First, suppose country 2's belief about country 1's type is a point mass on c_1'; i.e., $\lambda_2(m_1) = \delta_{c_1'}$. (The proof for the opposite case is identical.) Because country 2 believes with certainty that $c_1 = c_1'$, its strategy $s_2(\cdot, m_2, m_1)$ is a best response if and only if

$$s_2(c_2, m_2, m_1) = \max\left\{0, x^*(c_2) - s_1(c_1', m_1, m_2)\right\}.$$

To show that an equilibrium exists, we must find a value of $s_1(c_1', m_1, m_2)$ that is a best response for type c_1' of country 1 when country 2's strategy is defined as above. Let $\Psi(x_1, c_1)$ denote the best response for a given type of country 1 when it expects each type of country 2 to employ a best response to x_1:

$$\Psi(x_1, c_1) = \arg\max_{x_1' \in X_1} \left\{ \int_{T_2} p\left(x_1' + \max\left\{0, x^*(c_2) - x_1\right\}\right) d\lambda_1(m_2) - c_1 x_1' \right\}.$$

Because contributions above a type's standalone contribution are strictly dominated, we have $\Psi(x_1, c_1') \in [0, x^*(c_1')]$ for all x_1. Moreover, by the Berge Maximum Theorem (Aliprantis and Border 2006, Theorem 17.31), Ψ is continuous in x_1. Then, by the Brouwer fixed-point theorem, there exists $\hat{x} \in [0, x^*(c_1')]$ such that $\Psi(\hat{x}, c_1') = \hat{x}$. The following strategies therefore constitute an equilibrium of $\Gamma(m_1, m_2)$:

$$\begin{aligned} s_1(c_1, m_1, m_2) &= \Psi(\hat{x}, c_1), \\ s_2(c_2, m_2, m_1) &= \max\left\{0, x^*(c_2) - \hat{x}\right\}. \end{aligned}$$

On the other hand, suppose neither message is fully revealing, so $\lambda_2(m_1)$ and $\lambda_1(m_2)$ are both continuous distributions over an interval. Each country's expected utility in the subgame, holding fixed the strategy of the other country, is

$$Eu_i(x_i \mid c_i) = \int_{T_j} p(x_i + s_j(c_j, m_j, m_i))\, d\lambda_i(m_j) - c_i x_i.$$

Differentiating gives

$$\frac{\partial^2 Eu_i(x_i \mid c_i)}{\partial x_i \partial c_i} = -1,$$

so each country's best response is non-increasing in its type. In addition, because contributions above a type's standalone contribution are strictly dominated and x^* is decreasing, we may restrict each country's action space in the contribution stage to $[0, x^*(\underline{c}_i)]$ without loss of generality. Therefore, by Corollary 2.1 of Athey (2001), a pure-strategy equilibrium of the subgame exists. □

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