Regression with interaction terms

PSCI 2301: Quantitative Political Science II

Prof. Brenton Kenkel

brenton.kenkel@gmail.com

Vanderbilt University

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Recap

Last time — regression for causal inference

1. The linear model with controls
    - Reduce standard errors by isolating treatment effect
    - Control for selection bias
2. Interpreting regression output
    - “All else equal” comparison
    - Don’t try to interpret control variable coefficients

Today's agenda

Regression with interaction terms

1. Heterogeneous effects and why we might look for them
2. Interaction with a categorical variable
3. Interaction with a continuous variable
    - Linear interaction approach
    - Discretization approach

Heterogeneous effects and the interactive model

Heterogeneous effects: What they are, why we care

We've mostly focused on overall average treatment effects

Sometimes we suspect different average effects across subpopulations

→ e.g., does social pressure affect turnout differently for men and women?

Reasons to look for these **heterogeneous effects**

- Theoretical
  - → Think about mechanisms — why you expect treatment to be effective
  - → Would the mechanism be stronger/weaker for some groups?
- Practical
  - → How can the treatment be targeted most effectively?
  - → e.g., can we get most of the votes for a fraction of the cost?

Heterogeneous effects in randomized experiments

Potential outcomes Y_{1i}, Y_{0i}

Treatment indicator D_i

Binary group indicator X_i

→ e.g., $X_i = 1$ for men, $X_i = 0$ for women

Overall average treatment effect $\mathbb{E}[Y_{1i} - Y_{0i}]$

Average treatment effect among men $\mathbb{E}[Y_{1i} - Y_{0i} \mid X_i = 1]$

Average treatment effect among women $\mathbb{E}[Y_{1i} - Y_{0i} \mid X_i = 0]$

→ same subgroup effects we calculate for subclassification, now just treating them as interesting on their own

Capturing heterogeneous effects with a linear model

Regression with an interaction term:

$$\mathbb{E}[Y_i \mid D_i, Z_i] = \alpha + \beta_1 D_i + \beta_2 X_i + \underbrace{\beta_3 (D_i \times X_i)}_{\text{interaction term}}$$

- α: average outcome in control condition among $X_i = 0$ subgroup
  - → voter turnout among women who don't receive the GOTV mailer
- β_1: average treatment effect in $X_i = 0$ subgroup
  - → average effect of mailer on turnout among women
- β_2: difference in avg outcome in control cond. b/w $X_i = 1$ and $X_i = 0$
  - → among those w/ no mailer, how much more/less is men's turnout?
- β_3 difference in ATE b/w $X_i = 1$ and $X_i = 0$
  - → how much more/less does mailer affect turnout among men?

Recap: The GGL turnout data

```r
print(df_ggl)
```

```
# A tibble: 344,084 × 16
  sex      yob g2000 g2002 g2004 p2000 p2002 p2004 treatment cluster voted hh_id
  <chr>  <dbl> <chr> <chr> <chr> <chr> <chr> <chr> <chr>       <dbl> <dbl> <dbl>
1 male    1941 yes   yes   yes   no    yes   No    Civic Du…       1     0     1
2 female  1947 yes   yes   yes   no    yes   No    Civic Du…       1     0     1
3 male    1951 yes   yes   yes   no    yes   No    Hawthorne       1     1     2
4 female  1950 yes   yes   yes   no    yes   No    Hawthorne       1     1     2
5 female  1982 yes   yes   yes   no    yes   No    Hawthorne       1     1     2
# ℹ 344,079 more rows
# ℹ 4 more variables: hh_size <dbl>, numberofnames <dbl>, p2004_mean <dbl>,
#   g2004_mean <dbl>
```

To begin — just look at Neighbors treatment vs no mailer

```r
df_ggl_subset <- df_ggl |>
  filter(treatment %in% c("Neighbors", "Control")) |>
  mutate(treat = if_else(treatment == "Neighbors", 1, 0))
```

Estimating an interactive model in R

```
fit_interact <- lm(voted ~ treat * sex, data = df_ggl_subset)
tidy(fit_interact)
```

```
# A tibble: 4 × 5
  term          estimate std.error statistic   p.value
  <chr>            <dbl>     <dbl>     <dbl>     <dbl>
1 (Intercept)   0.290      0.00149   194.     0
2 treat         0.0809     0.00366    22.1    3.02e-108
3 sexmale       0.0123     0.00211     5.85   5.05e-  9
4 treat:sexmale 0.000849   0.00517     0.164  8.70e-  1
```

Term	Value	Interpretation
(Intercept)	0.290	29% turnout among women in Control group
treat	0.081	8.1% ATE among women
sexmale	0.012	1.2% higher turnout among men in Control group (i.e., 30.2%)
treat:sexmale	0.0008	0.08% higher ATE among men (i.e., 8.18%)

Another discrete interaction example

Divide into three categories based on household size:

```r
df_ggl_subset <- df_ggl_subset |>
  mutate(hh_type = case_when(
          hh_size == 1 ~ "Single",
          hh_size == 2 ~ "Couple",
          TRUE ~ "More than two"))

fit_hh_type <- lm(voted ~ treat * hh_type, data = df_ggl_subset)
tidy(fit_hh_type)
```

```
# A tibble: 6 × 5
  term                       estimate std.error statistic   p.value
  <chr>                         <dbl>     <dbl>     <dbl>     <dbl>
1 (Intercept)                 0.303     0.00134   227.    0        
2 treat                       0.0878    0.00328    26.8   4.29e-158
3 hh_typeMore than two       -0.0423    0.00254   -16.7   1.64e- 62
4 hh_typeSingle               0.0277    0.00313     8.84  9.54e- 19
5 treat:hh_typeMore than two -0.0303    0.00622    -4.87  1.15e-  6
6 treat:hh_typeSingle         0.00418   0.00764     0.547 5.84e-  1
```

Heterogeneous effects across a continuous variable

What if we wanted to see whether the treatment effect varies by age?

Linear interactive effect model

$$\mathbb{E}[Y_i \mid D_i, X_i] = \alpha + \beta_1 D_i + \beta_2 X_i + \beta_3 (D_i \times X_i)$$

Same equation as before, but new interpretation

Average treatment effect when $X_i = x$:

$$\mathbb{E}[Y_i \mid D_i = 1, X_i = x] - \mathbb{E}[Y_i \mid D_i = 0, X_i = x] = \beta_1 + \beta_3 x$$

β_3 = how much does treatment effect increase/decrease with each unit of X_i?

Heterogeneous effects across a continuous variable

```
fit_interact_age <- lm(voted ~ treat * yob, data = df_ggl_subset)
tidy(fit_interact_age)
```

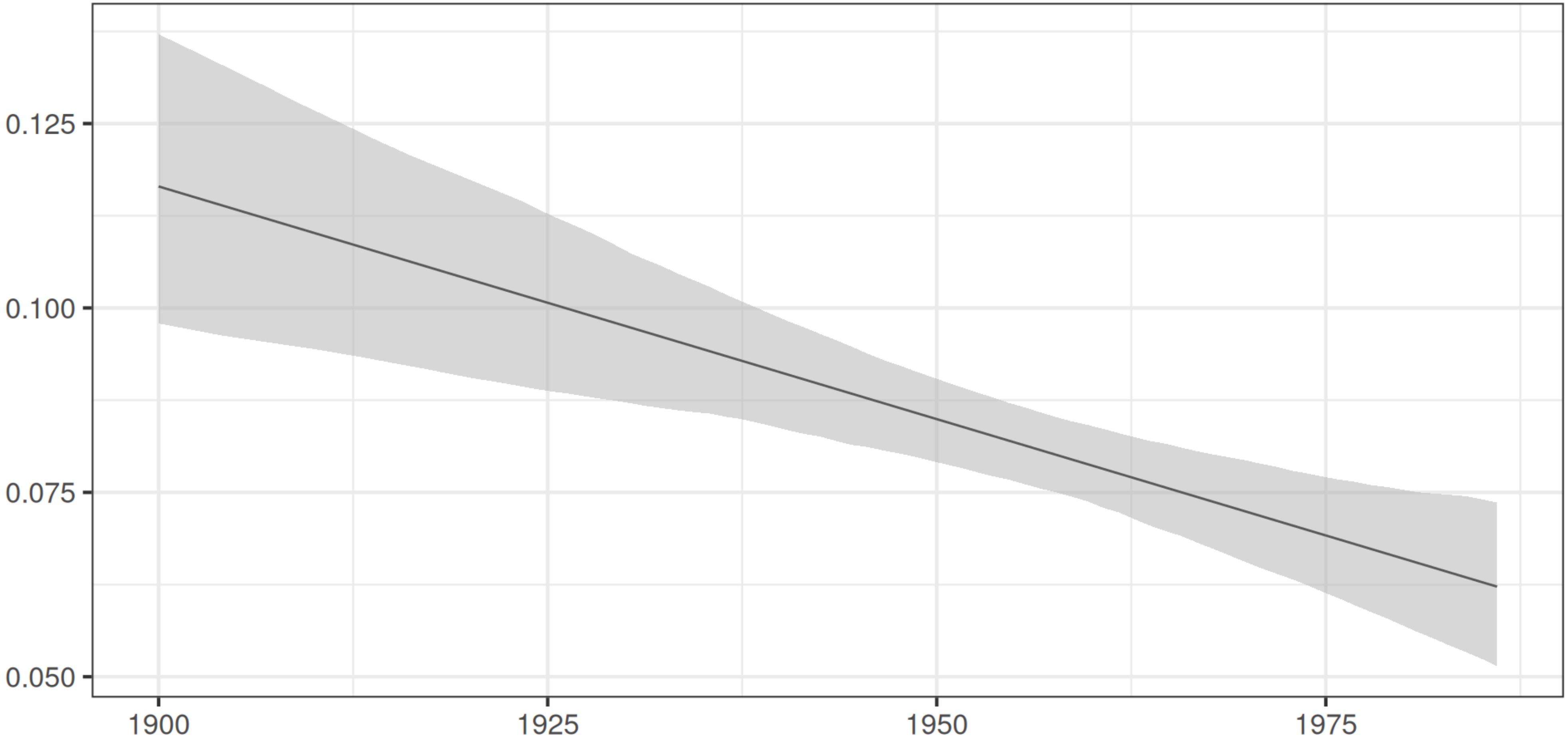
```
# A tibble: 4 × 5
  term         estimate std.error statistic  p.value
  <chr>           <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept)  8.12     0.142          57.2  0
2 treat        1.31     0.345           3.80 0.000145
3 yob         -0.00400  0.0000725     -55.1  0
4 treat:yob   -0.000628 0.000176       -3.57 0.000364
```

ATE = 1.31 - 0.000628 $\times$ birth year

- born 1925: 10.11% turnout increase
- born 1950: 8.54% turnout increase
- born 1975: 6.97% turnout increase

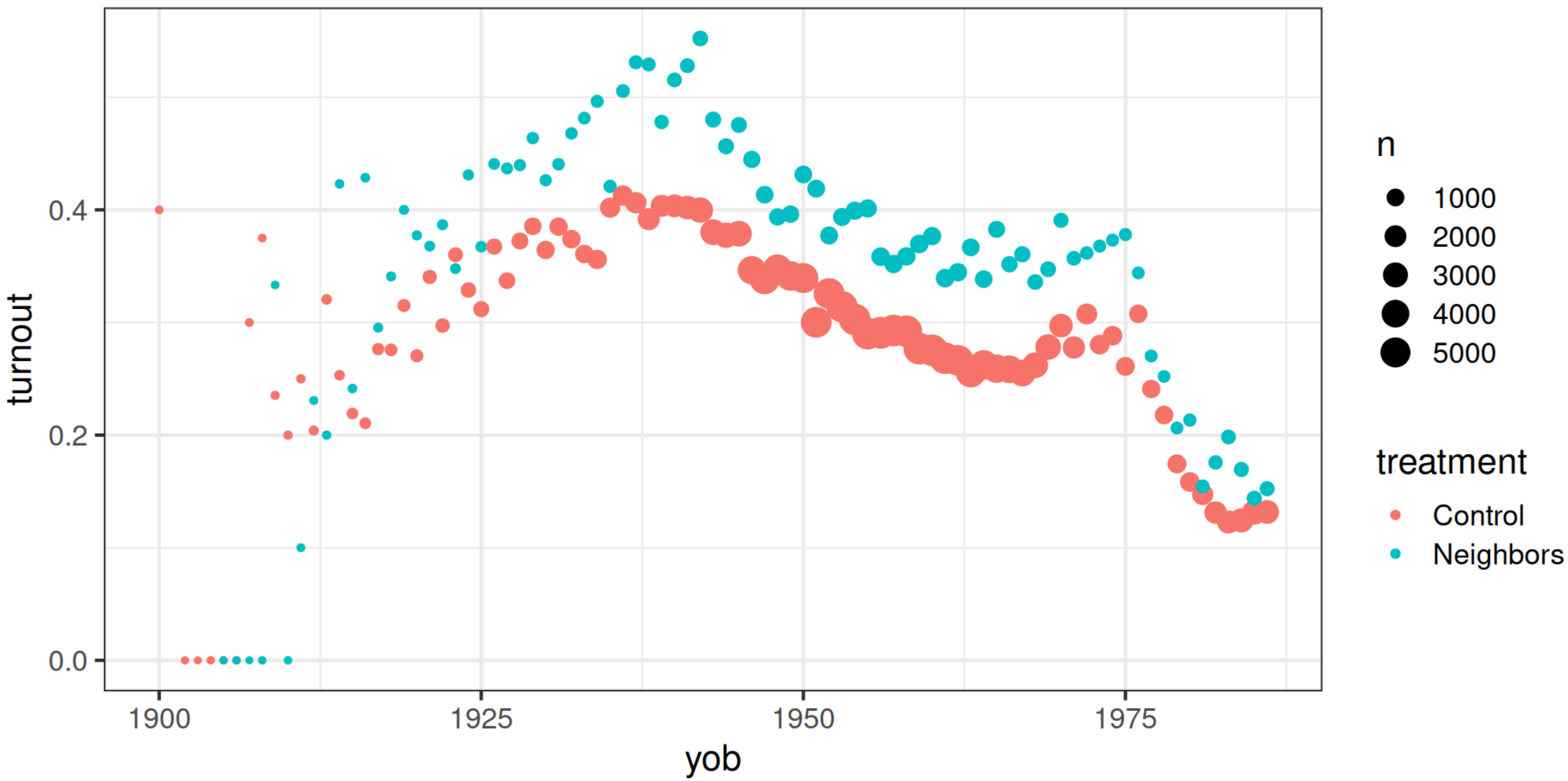
Heterogeneous effects across a continuous variable

```r
library("interplot")
interplot(m = fit_interact_age, var1 = "treat", var2 = "yob")
```



Heterogeneous effects across a continuous variable

Problem with the standard approach — is ATE linear in age?



Heterogenous effects across a continuous variable

Another approach — discretize the continuous variable

```r
df_ggl_subset <- df_ggl_subset |>
  mutate(yob_cat = cut(yob - 1900, breaks = seq(0, 90, by = 10)))

df_ggl_subset |> relocate(yob_cat, yob)
```

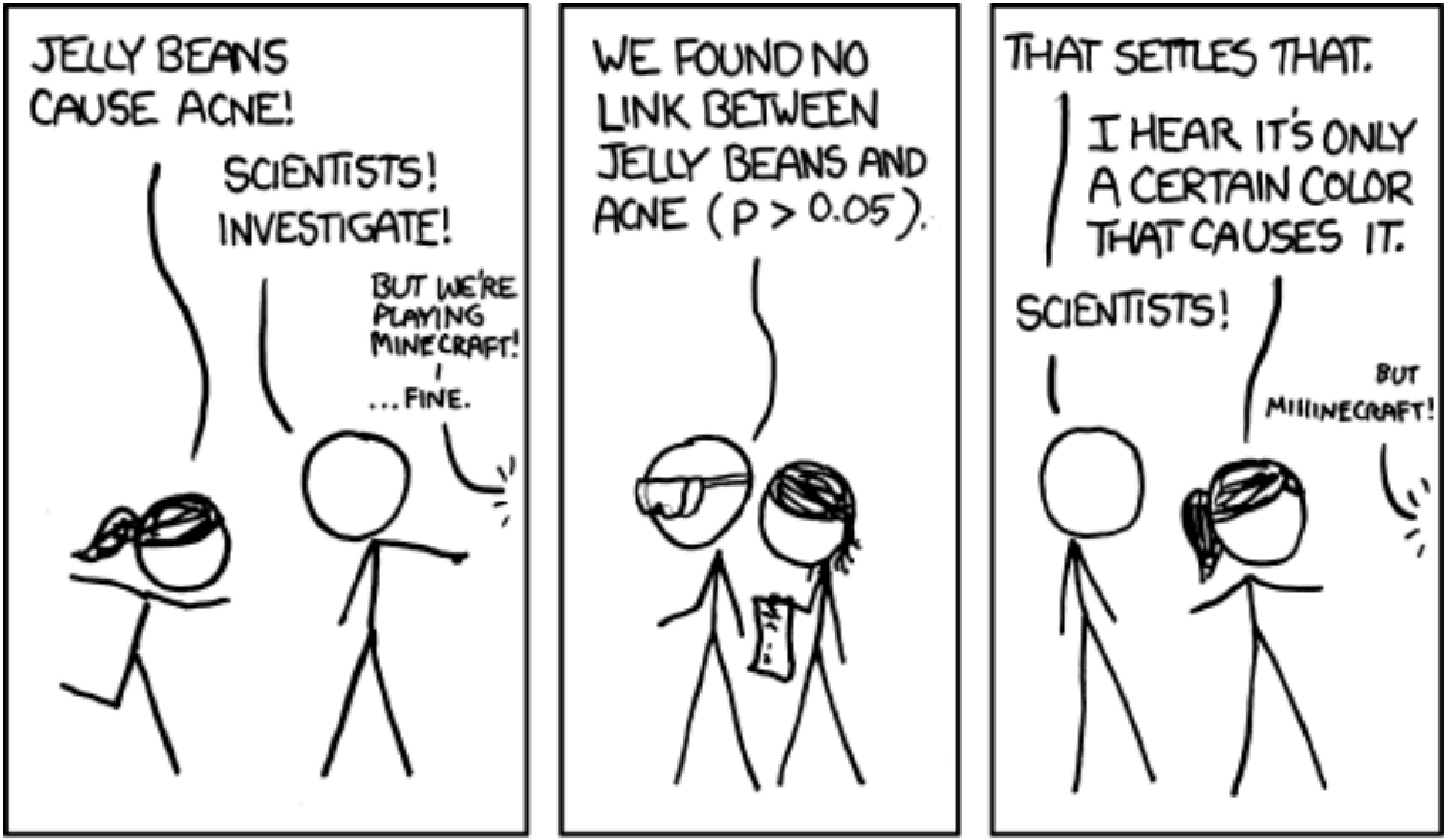
```
# A tibble: 229,444 × 19
  yob_cat   yob sex    g2000 g2002 g2004 p2000 p2002 p2004 treatment cluster
  <fct>   <dbl> <chr>  <chr> <chr> <chr> <chr> <chr> <chr> <chr>       <dbl>
1 (80,90]  1981 male   no    no    yes   no    no    No    Control         1
2 (50,60]  1959 female yes   yes   yes   no    yes   No    Control         1
3 (50,60]  1956 male   yes   yes   yes   no    yes   No    Control         1
4 (60,70]  1968 female no    no    yes   no    yes   No    Control         1
5 (60,70]  1967 male   yes   yes   yes   no    yes   No    Control         1
# i 229,439 more rows
# i 8 more variables: voted <dbl>, hh_id <dbl>, hh_size <dbl>,
#   numberofnames <dbl>, p2004_mean <dbl>, g2004_mean <dbl>, treat <dbl>,
#   hh_type <chr>
```

Heterogeneous effects across a continuous variable

```r
fit_yob_cat <- lm(voted ~ treat * yob_cat, data = df_ggl_subset)
tidy(fit_yob_cat)
```

```
# A tibble: 18 × 5
   term                 estimate std.error statistic  p.value
   <chr>                   <dbl>     <dbl>     <dbl>    <dbl>
 1 (Intercept)            0.197     0.0542     3.64  0.000275
 2 treat                 -0.0795    0.123     -0.645 0.519
 3 yob_cat(10,20]         0.0712    0.0554     1.28  0.199
 4 yob_cat(20,30]         0.154     0.0545     2.83  0.00464
 5 yob_cat(30,40]         0.196     0.0543     3.60  0.000319
 6 yob_cat(40,50]         0.163     0.0543     3.00  0.00268
 7 yob_cat(50,60]         0.0986    0.0542     1.82  0.0690
 8 yob_cat(60,70]         0.0677    0.0542     1.25  0.212
 9 yob_cat(70,80]         0.0574    0.0543     1.06  0.291
10 yob_cat(80,90]        -0.0664    0.0543    -1.22  0.221
11 treat:yob_cat(10,20]   0.147     0.126      1.16  0.246
12 treat:yob_cat(20,30]   0.147     0.124      1.19  0.235
13 treat:yob_cat(30,40]   0.179     0.124      1.45  0.148
```

Be careful when looking at lots of interactions!



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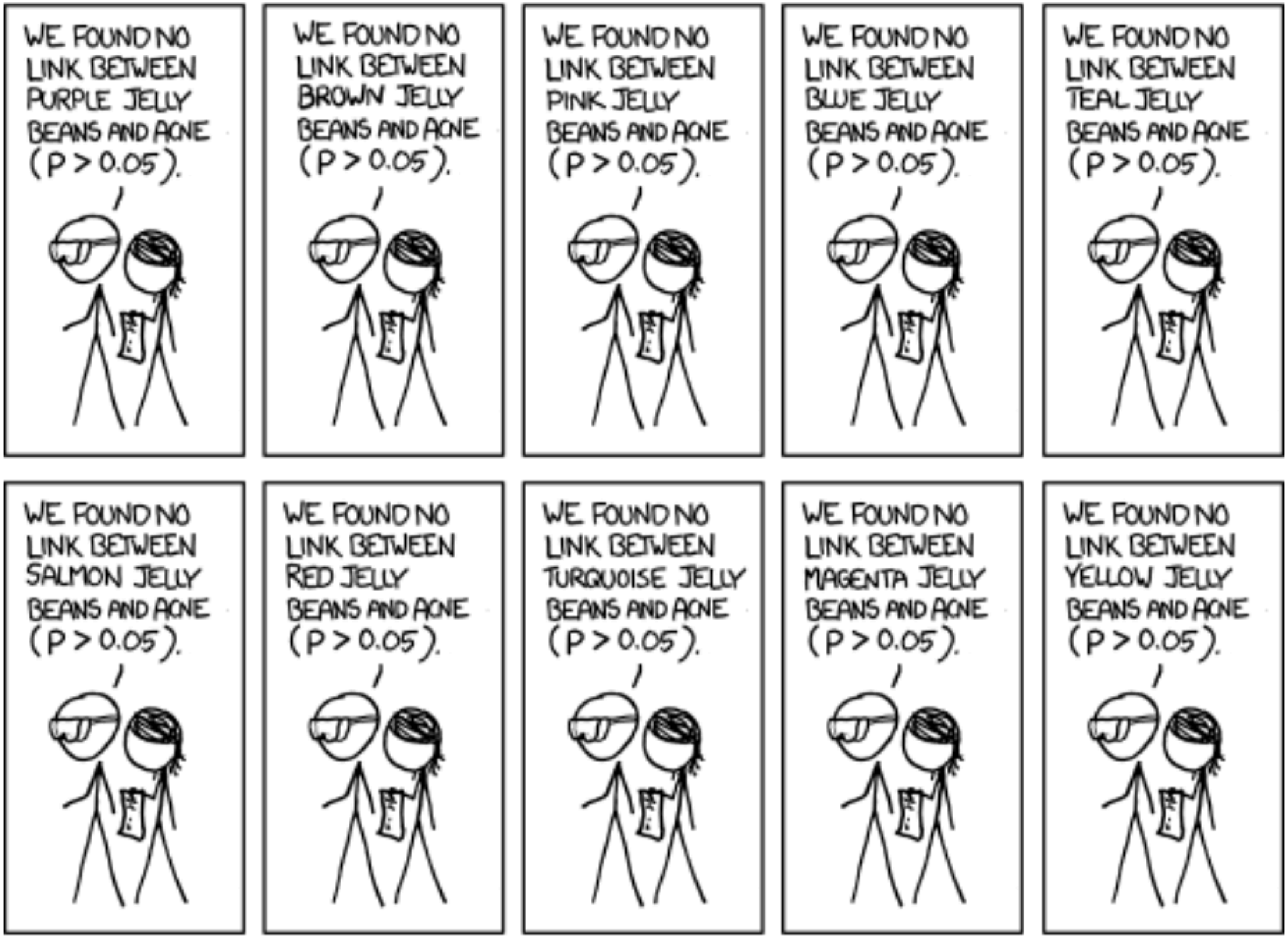

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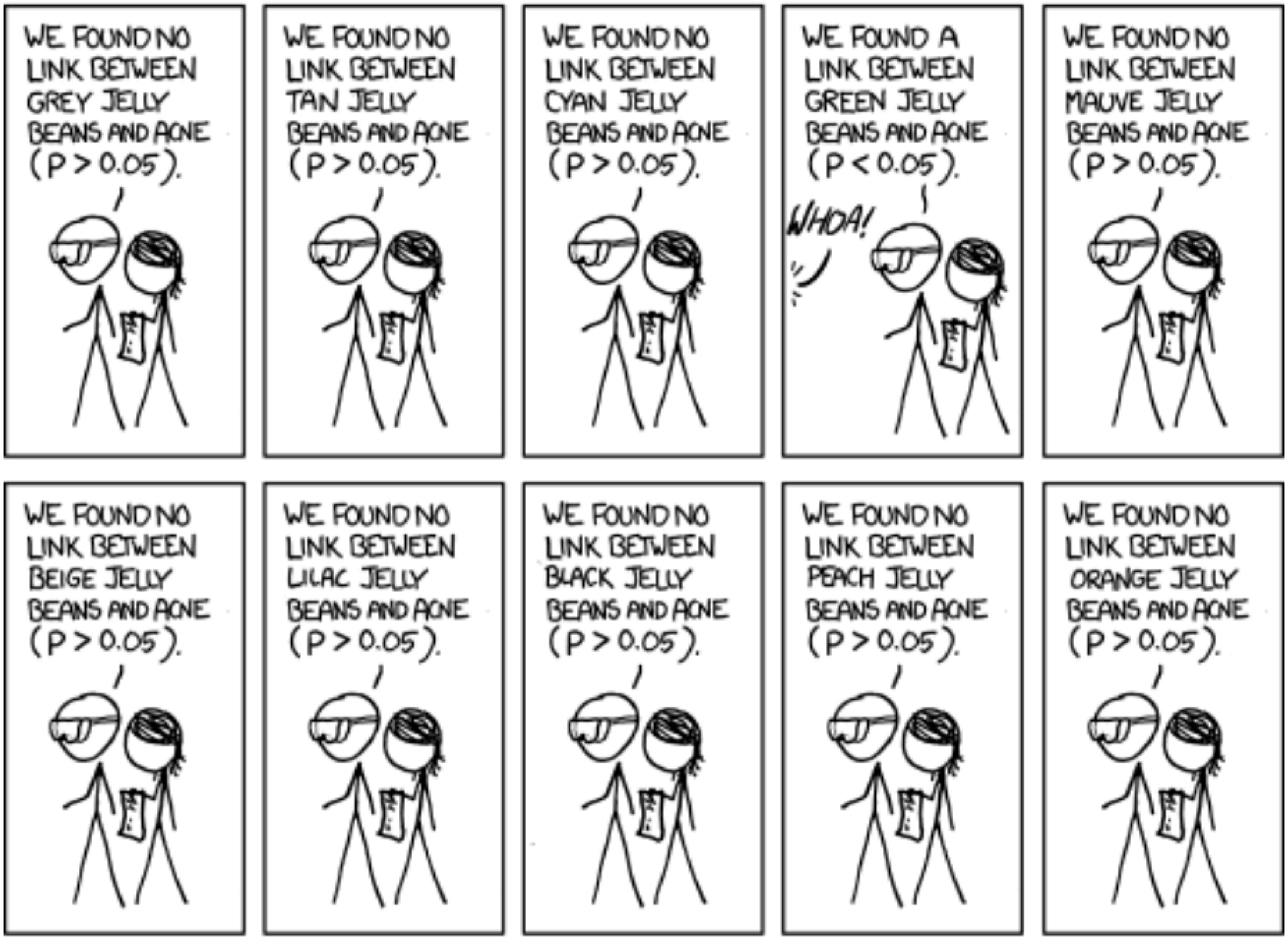
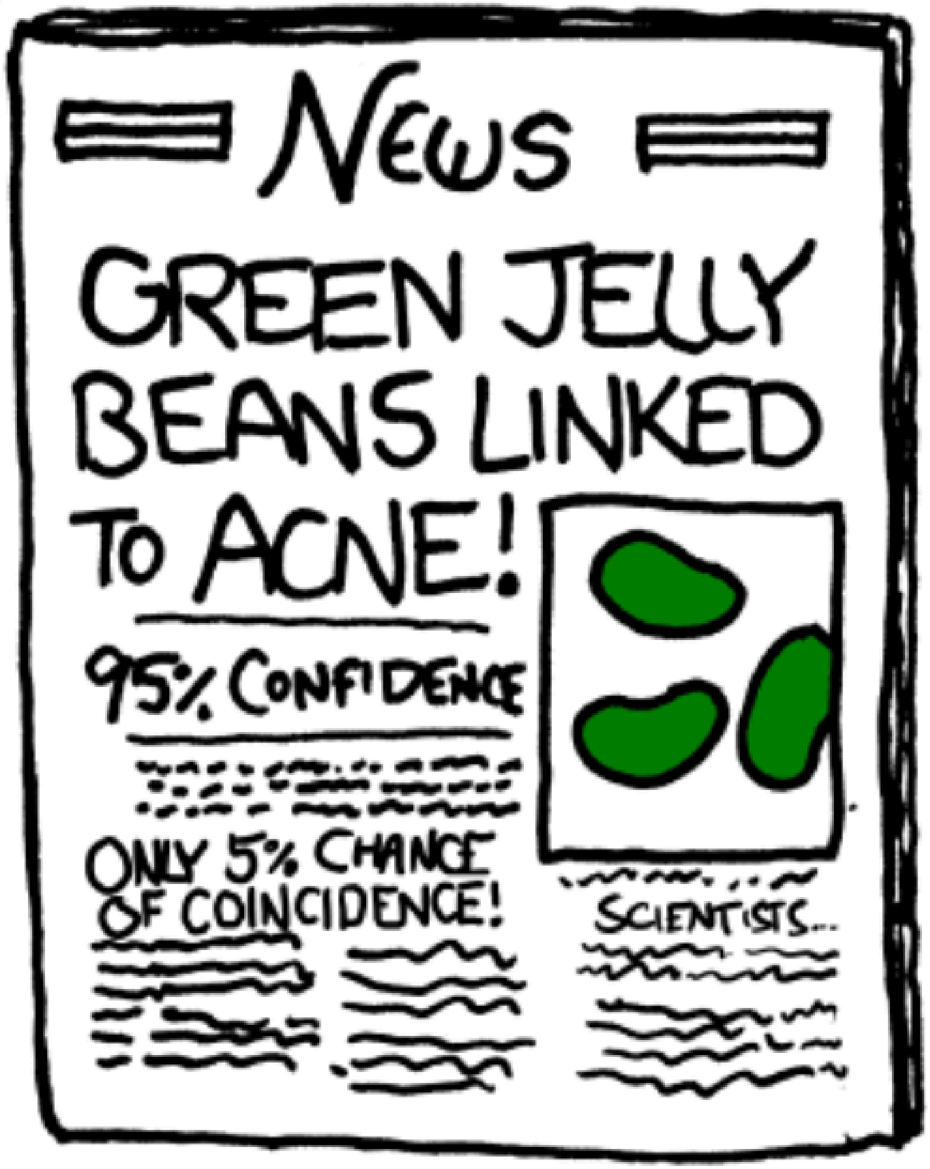


Be careful when looking at lots of interactions!



Heterogeneous effects in observational data

What if you're looking for heterogeneous effects when treatment isn't randomly assigned?

Methods are essentially the same — just include controls alongside the interaction

You *can* include multiple interactions but interpretation gets tricky

Wrapping up

What we did today

1. Reasons to look for heterogeneous effects
    - Theoretical: where we expect mechanisms to be stronger/weaker
    - Practical: targeting implementation to get max effect
2. Interpreting interactive regressions
    - Coefficient on interaction term = *difference* in treatment effects
    - Special considerations for interaction w/ continuous variable

Next week

What if we can't measure and control for every plausible confounder?

One way to deal with unobserved confounding — instrumental variables

→ Essentially, random *influences* on non-random treatment assignment

1. Turn in project proposal this Friday
2. Read Acemoglu, Johnson, Robinson, "The Colonial Origins of Economic Development"
3. Read chapter 3 of *Mastering 'Metrics*