Regression discontinuity designs

PSCI 2301: Quantitative Political Science II

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Recap

Week before spring break — instrumental variables

1. Scope of application
   - When treatment assignment is nonrandom + some confounders unobserved
   - … but there's an observed random *influence* on treatment assignment
2. Conditions for instrument
   - Independence: instrument value not confounded
   - First stage: instrument affects treatment assignment (hopefully strongly)
   - Exclusion restriction: only affects outcome via treatment
3. IV estimation via two-stage least squares (2SLS)

Today's agenda

Another method for observational data with unmeasured confounders: regression discontinuity design (RDD)

- Used when treatment is assigned by a hard threshold on a continuous value
- Near the threshold, assignment is as-if random
- Can estimate LATE by comparing observations just above the threshold to those just below

Motivating question: Ideology and election success

Does moderation win elections?

Perpetual debate: Do moderates have an electoral advantage?

Some reasons to think so:

- Spatial model — people vote for ideologically closest candidate
- Plurality of Americans describe selves as moderate
- Moderates could raise more money from business interests

Some reasons to think not:

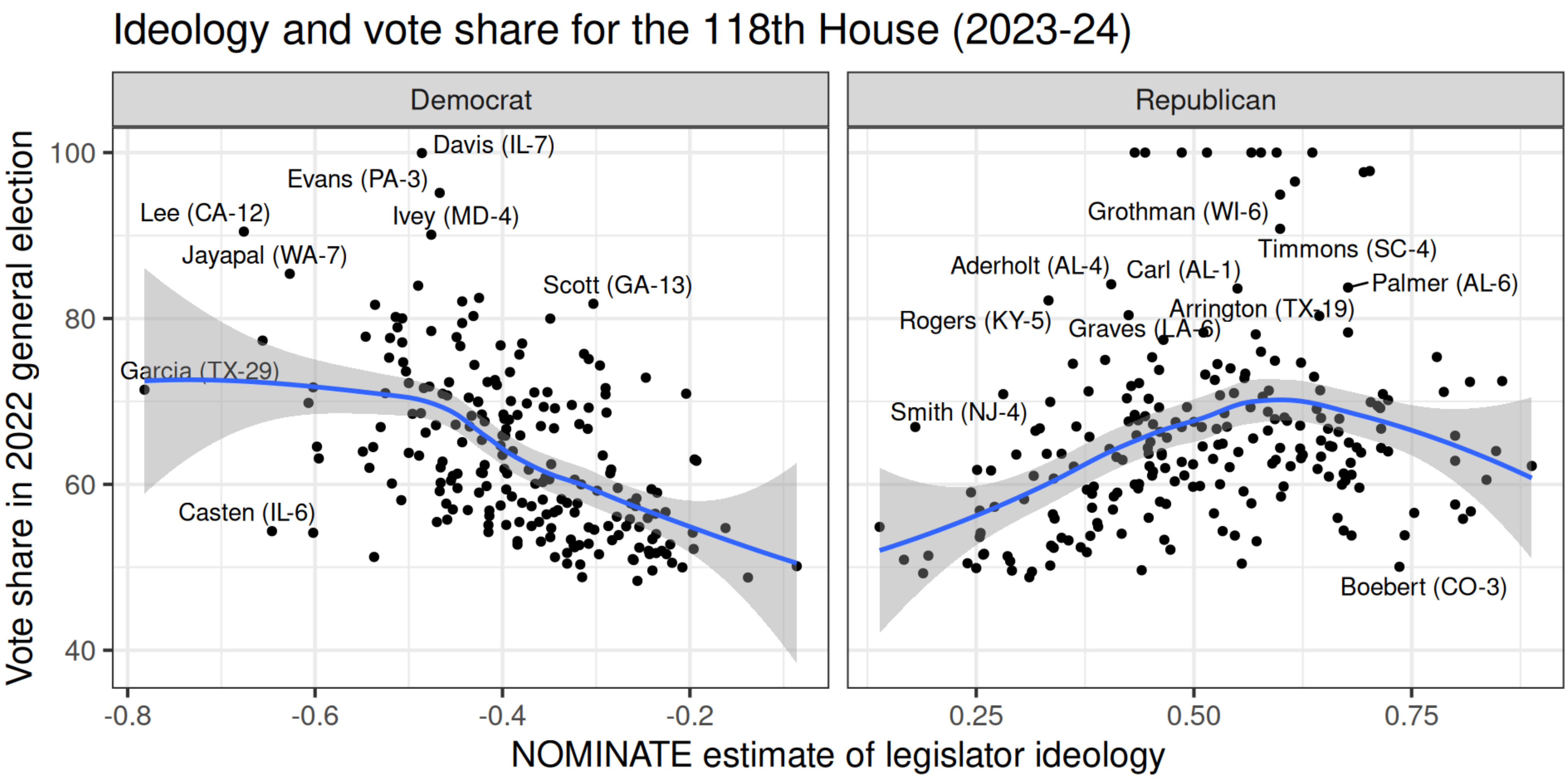
- Independents less informed, less likely to vote, less coherent in opinions
- Ideologues could raise more money from enthusiastic base
- Potential advantage of appearing principled, not calculating
- Moderation is correlated with **lower** vote shares among House members

A tale of two Democratic representatives

- Alexandria Ocasio-Cortez, NY-14
- Self-identified socialist
- Won 70.6% of vote in 2022

- Marie Gluesenkamp Perez, WA-3
- Centrist (e.g., didn’t endorse Harris)
- Won 50.1% of vote in 2022

Congressional moderates have lower vote shares



Does moderation *cause* lower vote shares?

The correlational question: Do moderates get fewer votes? (yes)

The causal question: If we replaced any given moderate with an extremist, would their party get fewer votes?

Potential confounders:

- District ideology
    - → Deep red/blue district ⇝ safe seat, electorate wants ideologue
    - → Purple district ⇝ close election, electorate wants moderate
- Candidate quality
    - → Charismatic candidate maybe can get away with more extreme views
- Opponent quality and ideology

Studying the effect of moderation on vote shares

Experimental manipulation not plausible

Difficulty for regression/matching approach — **unobserved confounders**

- District competitiveness fairly easy to observe
- District ideology kinda-sorta observable (presidential results, polls)
- Candidate and opponent quality pretty hard to measure

No obvious instruments

- Would need an as-if random influence on candidate ideology
- … that also doesn't affect election results through any other channel!

The regression discontinuity approach

General election candidates in contemporary US determined by primary elections

In general, districts where extremist wins primary are different from those where moderate wins

But districts where extremist <u>barely wins</u> are not so different from those where moderate <u>barely wins</u>

Hall's approach: only compare these barely-winners

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What Happens When Extremists Win Primaries?

ANDREW B. HALL *Harvard University*

This article studies the interplay of U.S. primary and general elections. I examine how the nomination of an extremist changes general-election outcomes and legislative behavior in the U.S. House, 1980–2010, using a regression discontinuity design in primary elections. When an extremist—as measured by primary-election campaign receipt patterns—wins a "coin-flip" election over a more moderate candidate, the party's general-election vote share decreases on average by approximately 9–13 percentage points, and the probability that the party wins the seat decreases by 35–54 percentage points. This electoral penalty is so large that nominating the more extreme primary candidate causes the district's subsequent roll-call representation to reverse, on average, becoming more liberal when an extreme Republican is nominated and more conservative when an extreme Democrat is nominated. Overall, the findings show how general-election voters act as a moderating filter in response to primary nominations.

...getting a general-election candidate who can win is the only thing we care about.[1]
—Rob Collins, National Republican Senatorial Committee

The road to hell is paved with electable candidates.[2]
—Joseph Ashby, conservative blogger

INTRODUCTION

With the rise of the Tea Party and the phenomenon of moderate incumbents "getting primaried," political scientists and journalists alike have placed added scrutiny on the role of primary elections in our system of representation.[3] As the first stage of candidate selection, primaries play an important role in choosing the people who will go on to represent voters in Congress. Primary voters exhibit a marked preference for more ideologically extreme candidates (Brady, Han, and Pope 2007; Hall and Snyder 2013), but general-election voters appear to prefer moderates (Ansolabehere, Snyder, and Stewart 2001; Burden 2004; Canes-Wrone, Brady, and Cogan 2002; Erikson and Wright 2000). As a stylized fact, primary voters who prefer extreme candidates are thus thought to face a tradeoff between voting for a candidate closer to their views, but less likely to win office, and a candidate farther from their views but perhaps more "electable." In this article, I study this tradeoff and its consequences for elections and representation in Congress.

How much does the party's electoral outlook suffer in a district where its primary voters nominate a more extreme candidate, relative to the counterfactual in which the same district nominates a more moderate candidate? If a district nominates a more extreme candidate, how much does the district's roll-call voting in the next Congress change relative to this same counterfactual—taking into account both the manner in which the extremist would vote and the probability that the extremist wins office? To answer these questions, I combine a scaling technique for estimating candidate positions based on campaign contributions with a regression discontinuity design in U.S. House primary elections, 1980–2010. This strategy allows me to obtain direct counterfactual comparisons between districts with an extreme or more moderate nominee without using assumptions to place districts and candidates on a single ideological scale and without asserting the exogeneity of differences in candidate positions.

I find that the "as-if" random nomination of the extremist candidate causes a substantial decrease in the party's vote share and probability of victory in the general election. These decreases are large enough to offset the more extreme roll-call voting that extremist candidates offer to primary voters, on average. They are also large enough to offset any other roll-call effects nominating extremists might have, e.g., inducing incumbents to strategically adopt positions like those of extremists. Indeed, the nomination of the more extreme candidate to the general election produces a *reversal* in observed roll-call voting for the district in the

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[1] http://firstread.nbcnews.com/_news/2013/11/05/21321519-in-shift-gop-vows-to-fight-for-more-electable-candidates-in-senate-primaries?lite

[2] http://www.americanthinker.com/2011/09/the_road_to_hell_is_paved_with_electable_candidates.html

[3] For example, a growing literature studies the link (or nonlink) between primary-election type (open vs. closed, etc.) and polarization (Gerber and Morton 1998; Hirano et al. 2010; McGhee et al. 2014; Rogowski 2013). For an overview of "getting primaried" see Boatright (2013).

Regression discontinuity design

Regression discontinuity: The basic idea

Some discrete treatments determined by continuous running variable

- Income threshold for government program eligibility
- Test score threshold for admission to an academic program
- Vote margin threshold for winning an election

Key assumption: No big differences in confounders around the threshold

- People making $31k not much different than those making $29k
- Students scoring 1490 on SAT not much different than those scoring 1510
- Candidates who win primary by 1% not much different than those who lose by 1%

$\Rightarrow$ Estimate treatment effect by comparing obs near threshold

Regression discontinuity: The model

Continuous running variable R_i

Treatment $D_i \in \{0, 1\}$, determined entirely by running variable

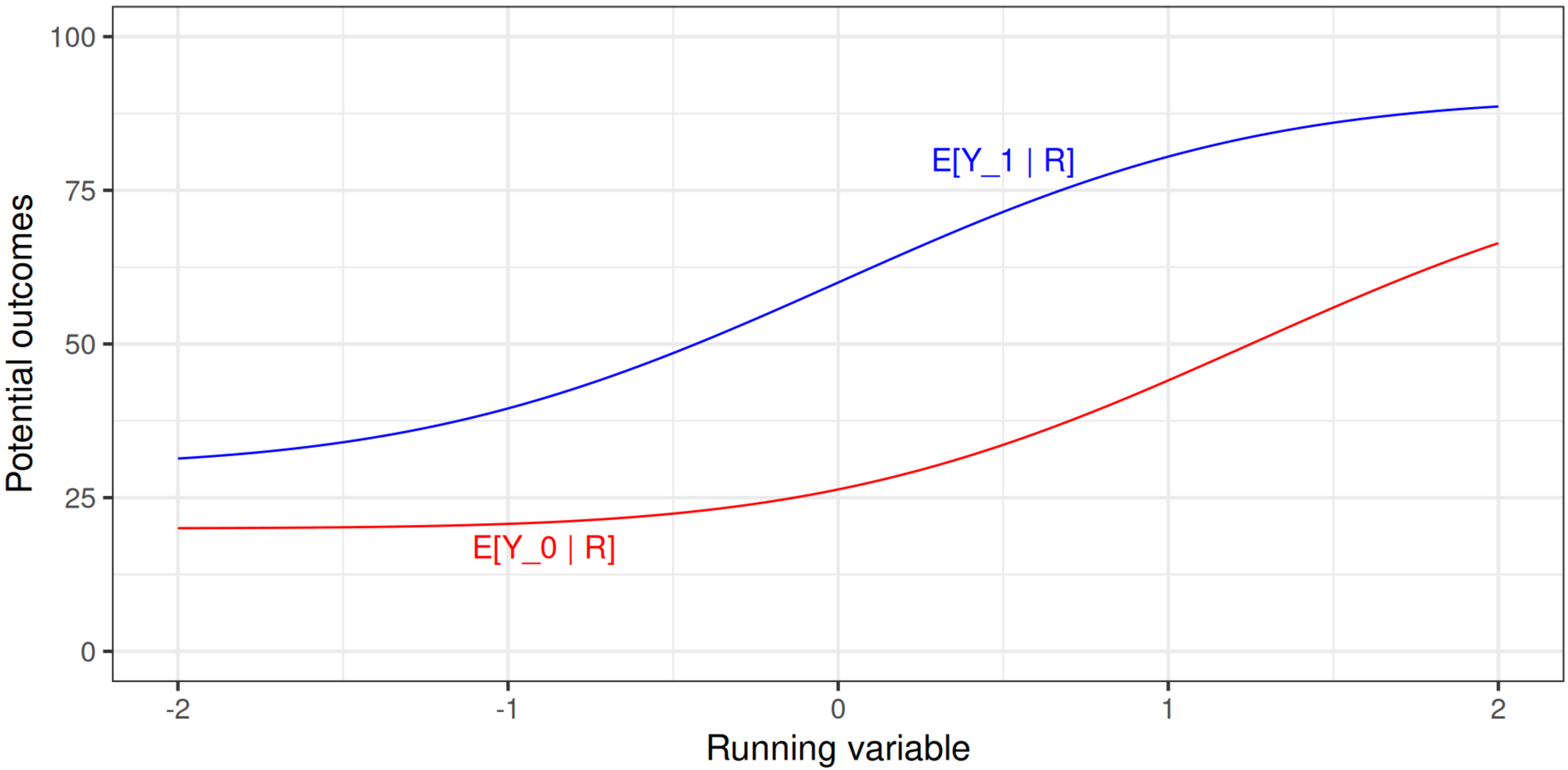
$$D_i = \begin{cases} 0 & \text{if } R_i < 0, \\ 1 & \text{if } R_i \geq 0. \end{cases}$$

Potential outcomes Y_{1i}, Y_{0i}

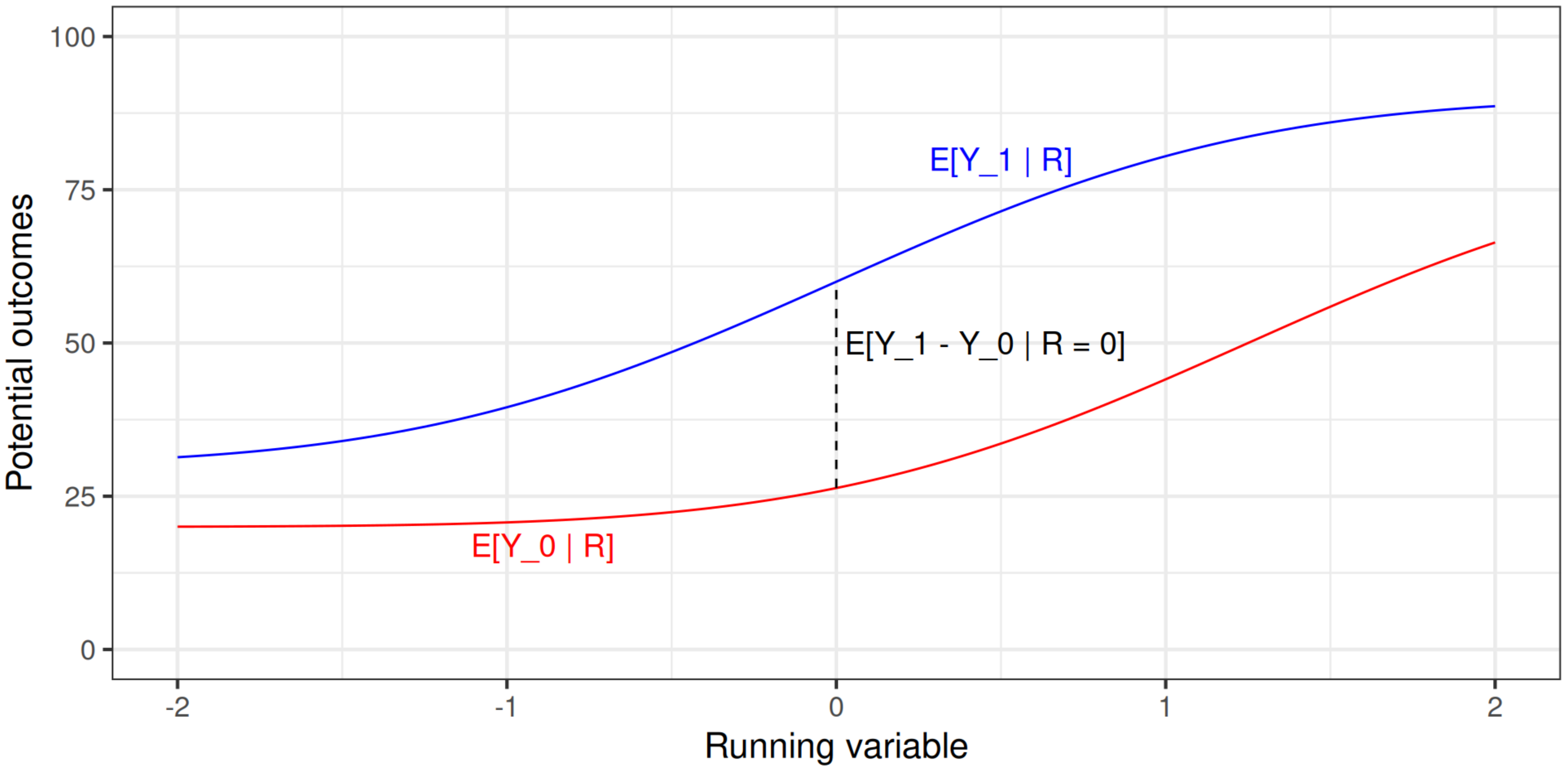
- Key assumption: $\mathbb{E}[Y_{1i} \mid R_i]$ and $\mathbb{E}[Y_{0i} \mid R_i]$ continuous functions of R_i
- Small change in running variable $\rightsquigarrow$ small change in potential outcome

Can estimate $\mathbb{E}[Y_{1i} - Y_{0i} \mid R_i \approx 0]$ by comparing outcomes near threshold

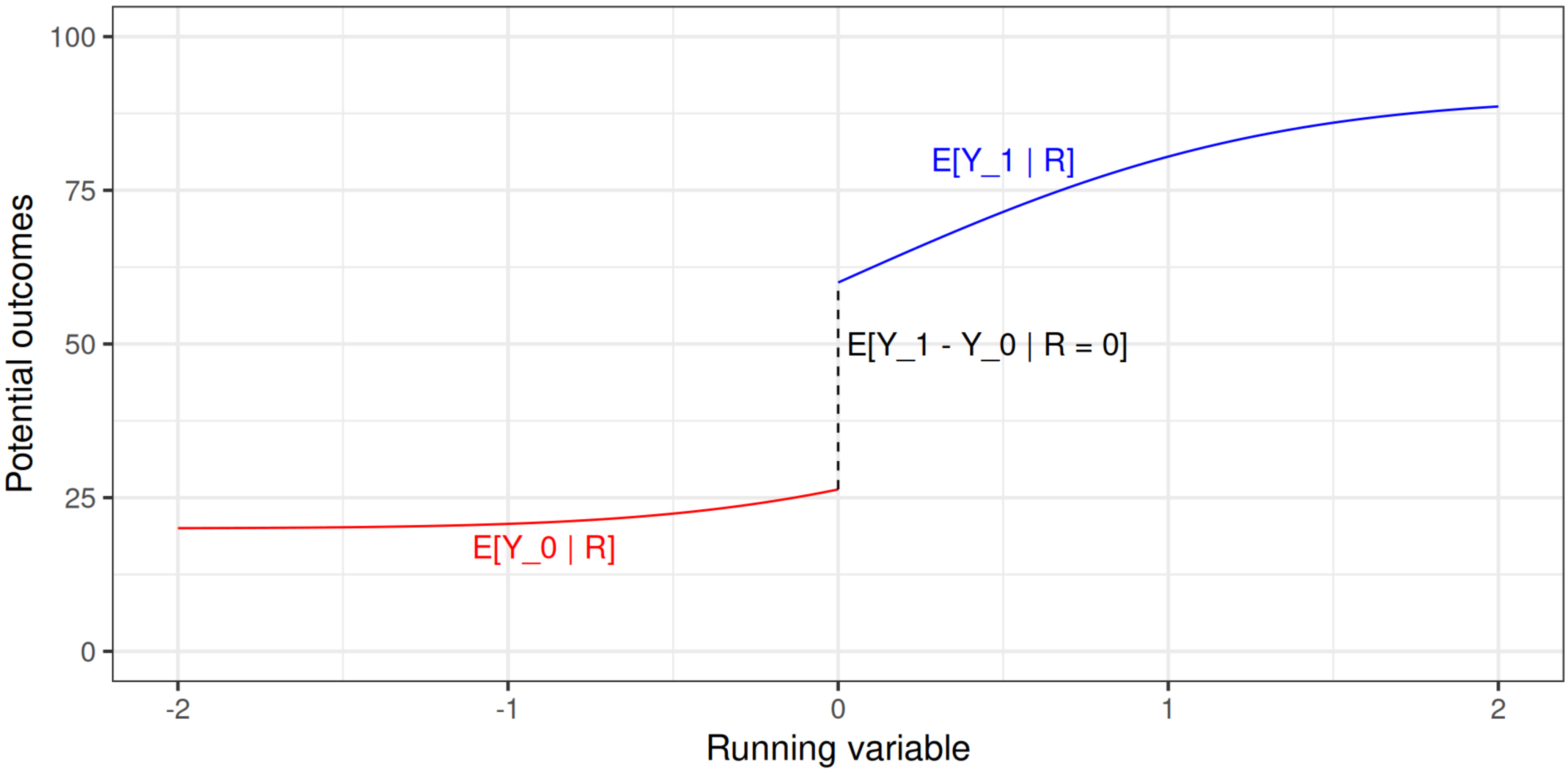
Regression discontinuity model, illustrated



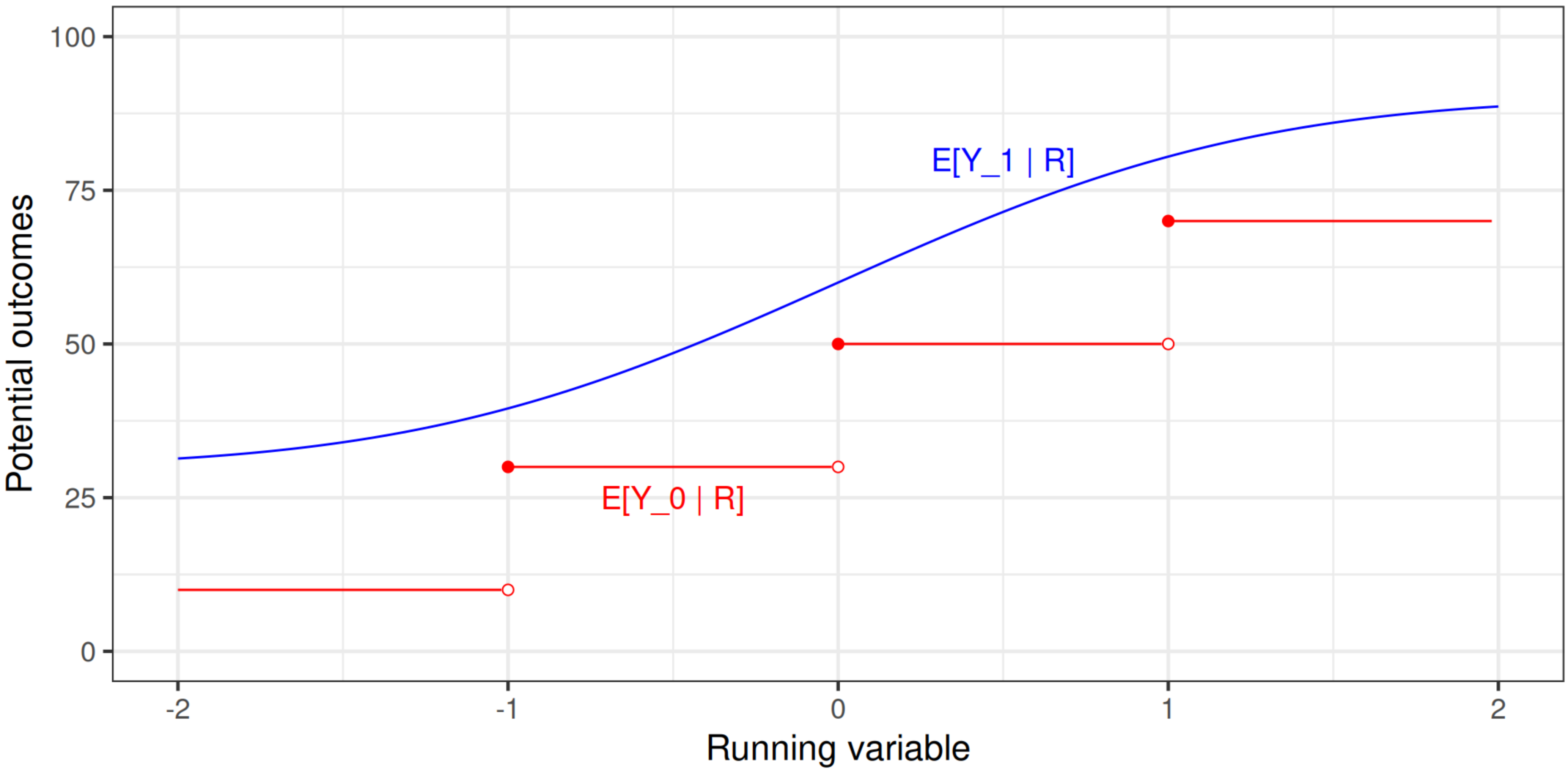
Regression discontinuity model, illustrated



Regression discontinuity model, illustrated



An invalid regression discontinuity model



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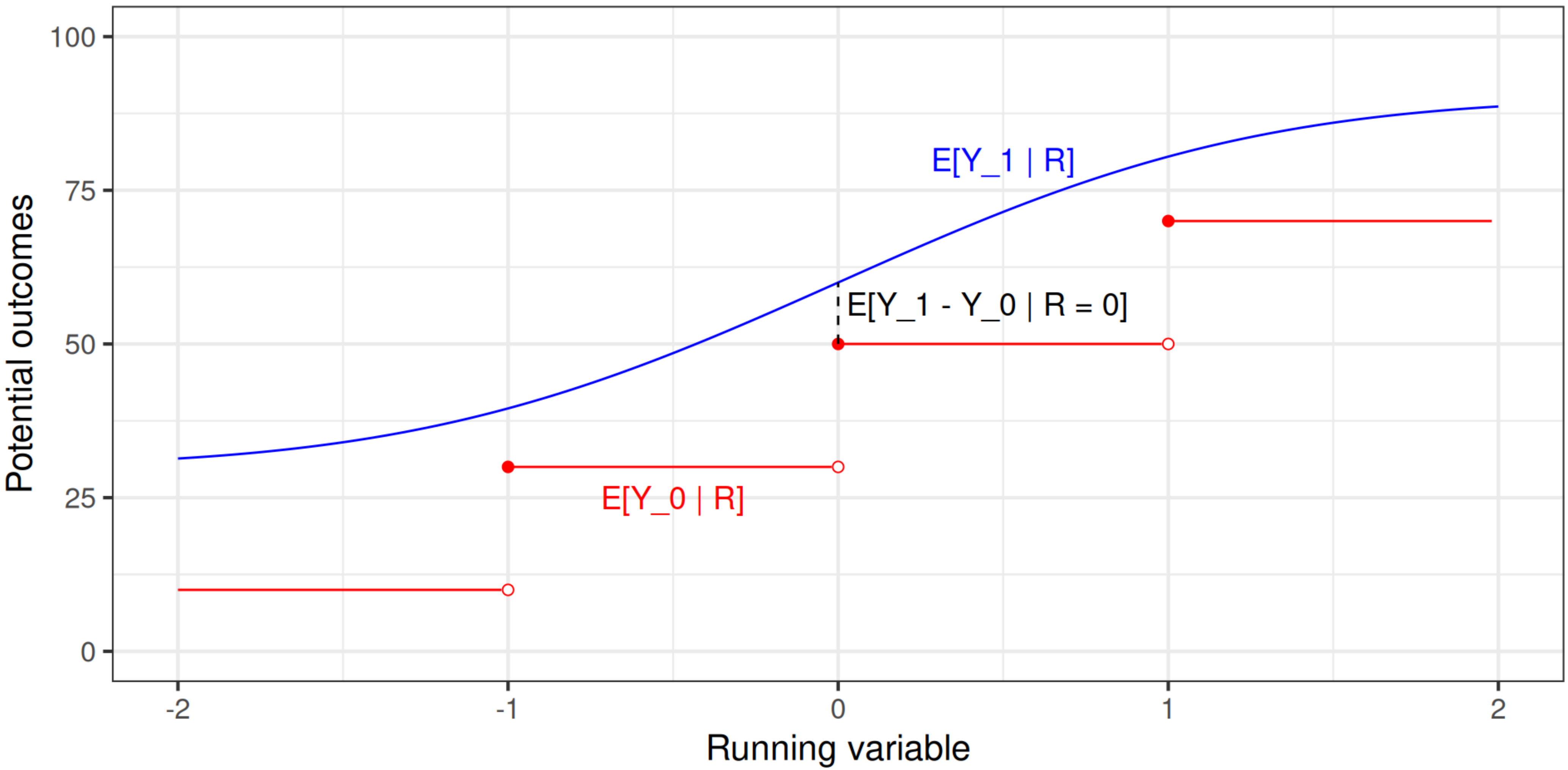

An invalid regression discontinuity model

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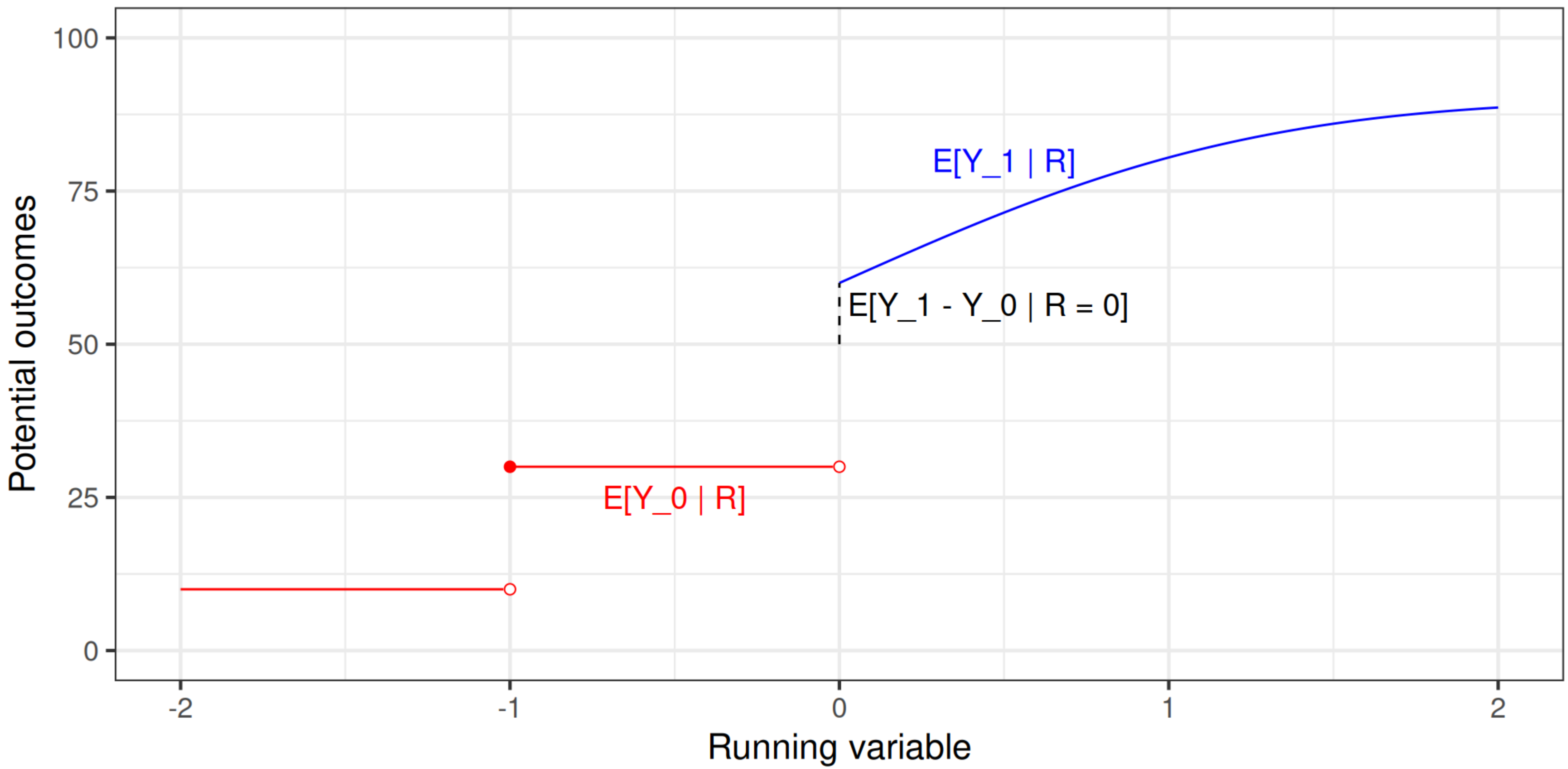

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Estimating the regression discontinuity model

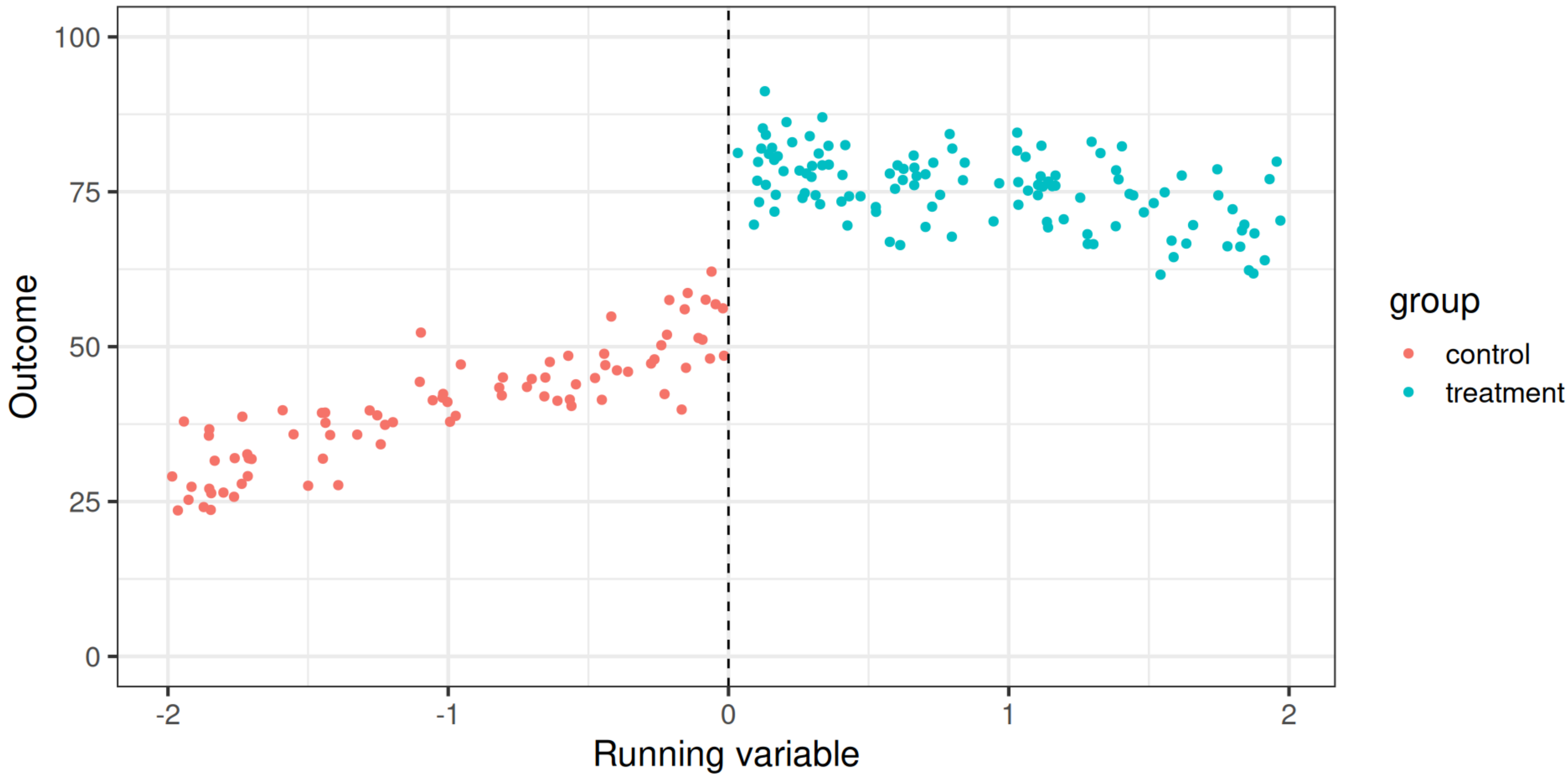
The general approach to estimation:

1. Model relationship b/w running variable and outcome below threshold
2. Model relationship b/w running variable and outcome above threshold
3. LATE estimate = difference in predictions at threshold

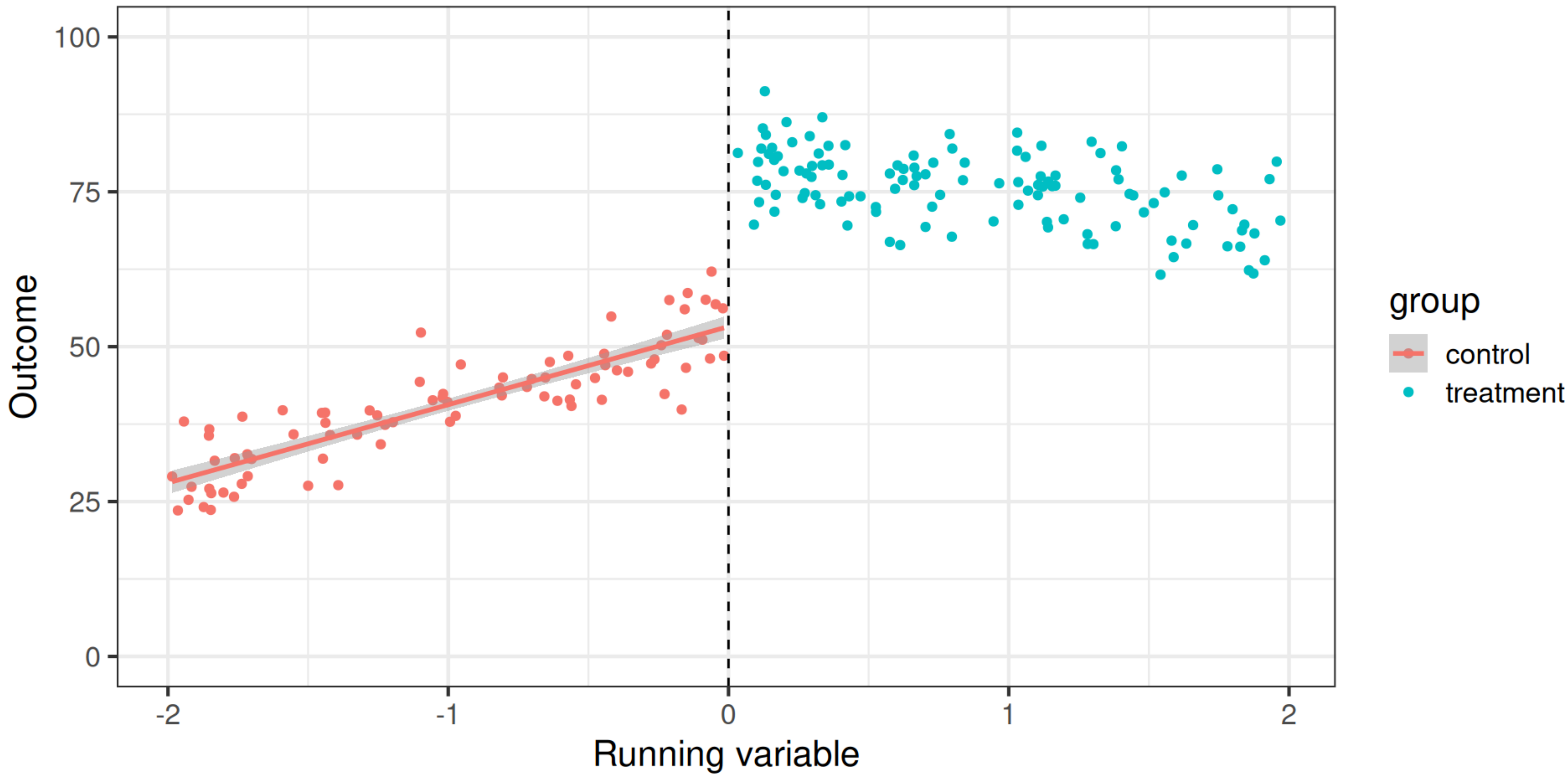
Specific implementation depends on shape of these relationships

1. If average outcome is a linear function of running variable
    - Linear regression with running/treatment interaction
2. If average outcome is a nonlinear function of running variable
    - Polynomial regressions
    - Linear regression within “bandwidth” of threshold

The linear case



The linear case



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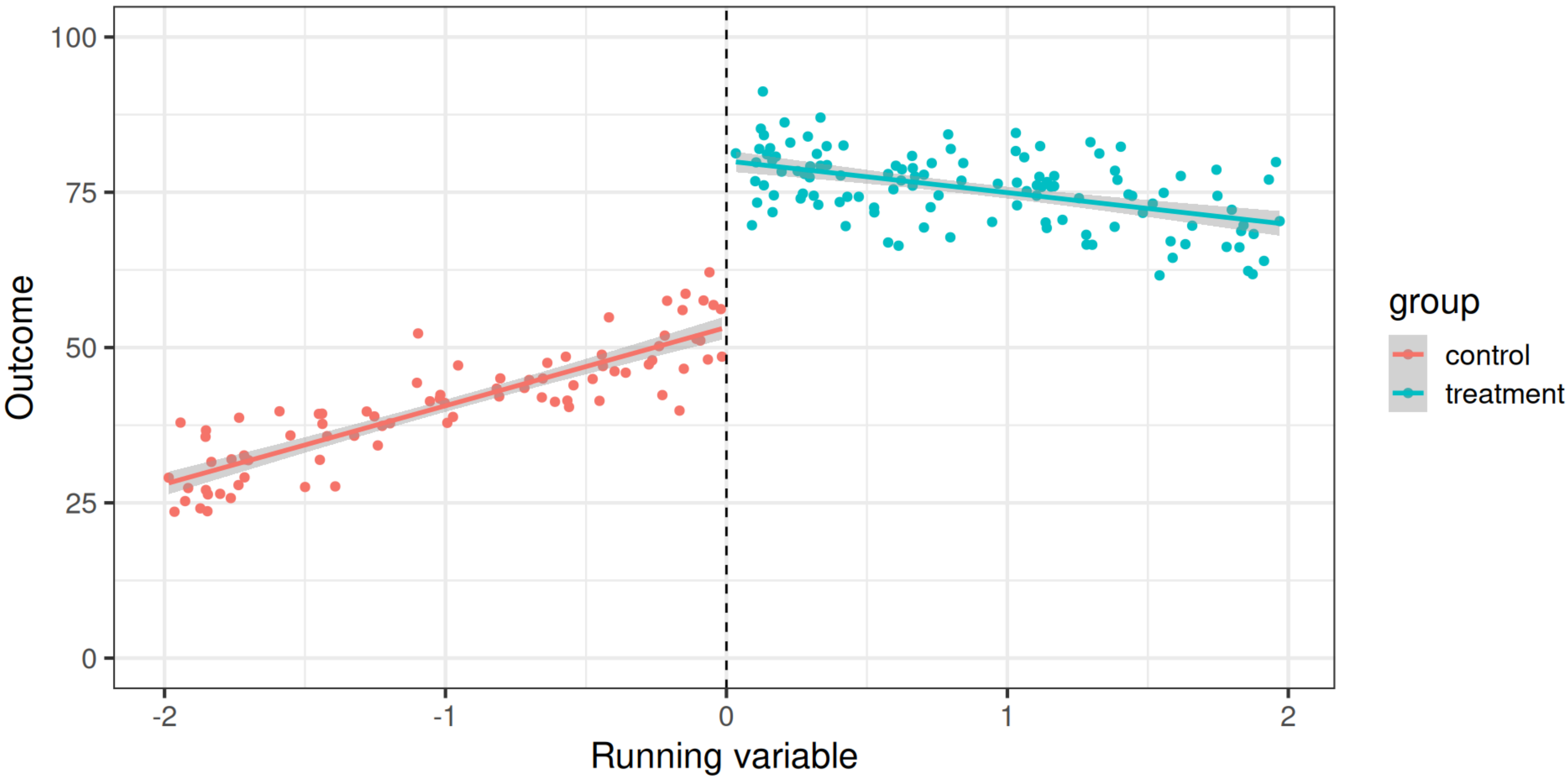

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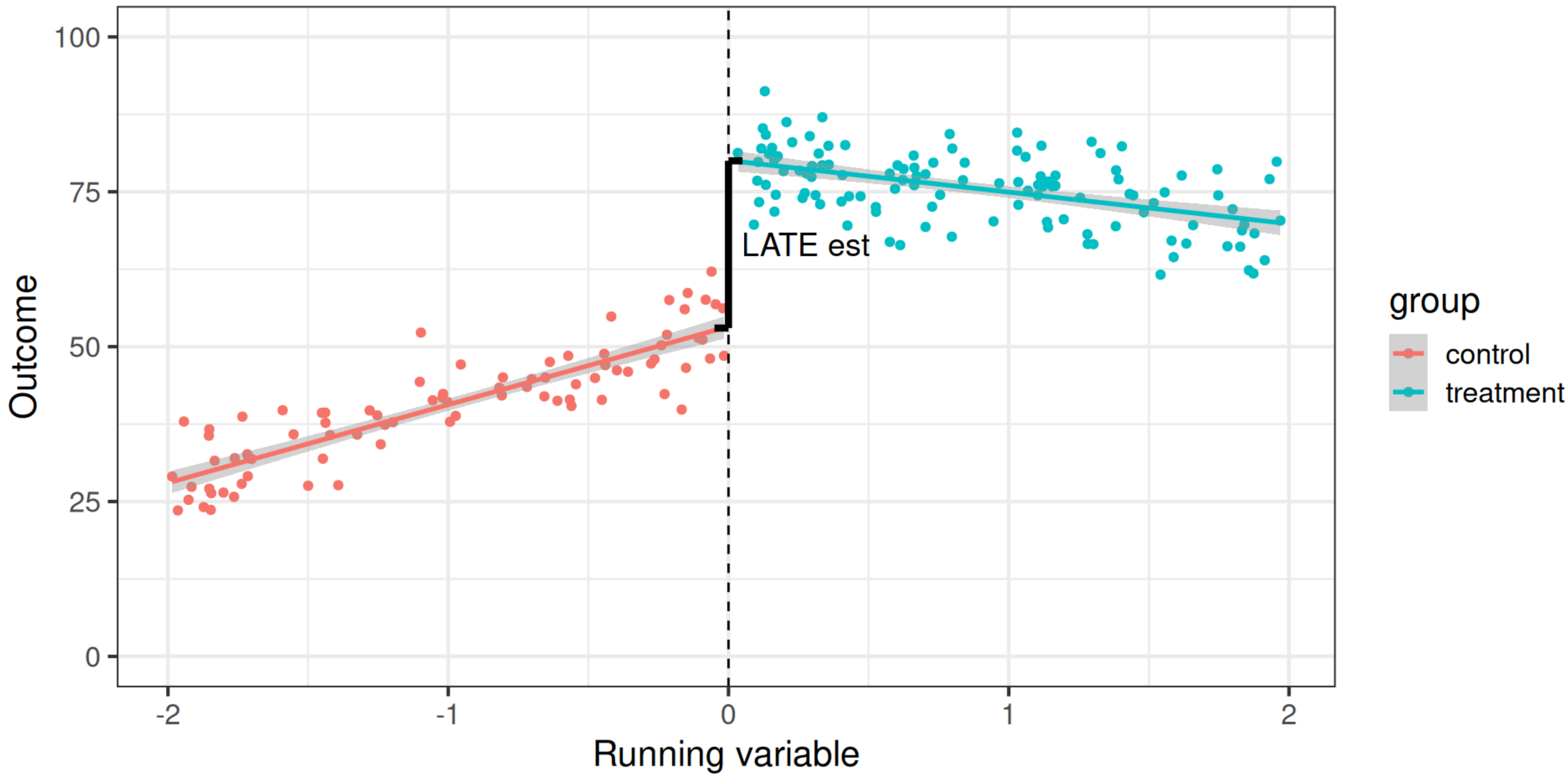


Regression model for the linear case

Regression equation to use:

$$\mathbb{E}[Y_i] = \alpha + \beta_1 R_i + \beta_2 D_i + \beta_3 (R_i \times D_i)$$

α: average control outcome at threshold

β_1: slope in control group

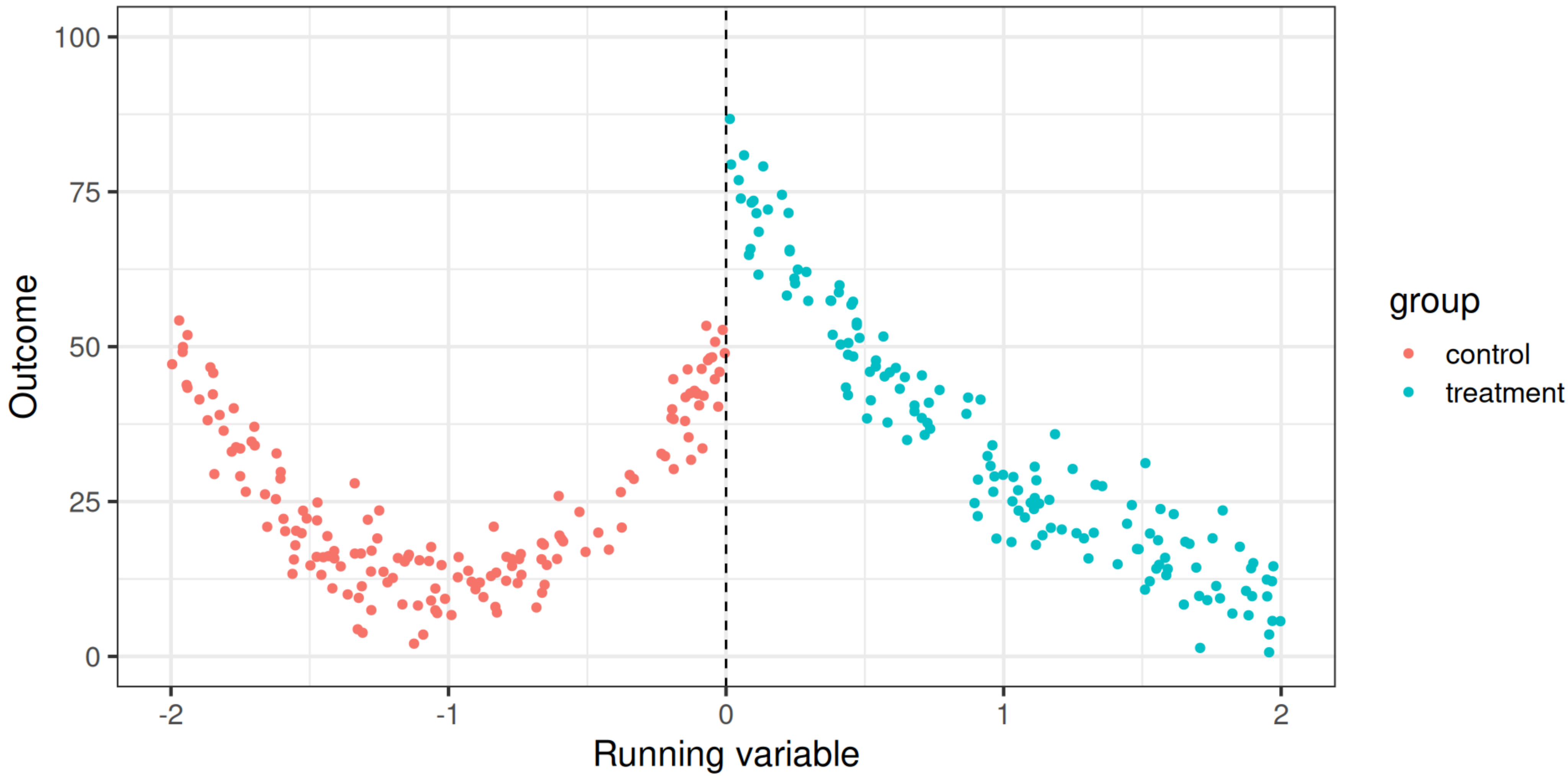
β_2: LATE

$\beta_1 + \beta_3$: slope in treatment group

```r
fit_linear <- lm(y ~ r * treat, data = df_linear)
tidy(fit_linear)
```

```
# A tibble: 4 × 5
  term        estimate std.error statistic
  <chr>          <dbl>     <dbl>     <dbl>
1 (Interce…       53.2     0.981      54.3
2 r               12.6     0.830      15.2
3 treat           26.8     1.28       21.0
4 r:treat        -17.7     1.14      -15.6
# i 1 more variable: p.value <dbl>
```

The nonlinear case: Polynomial regression approach



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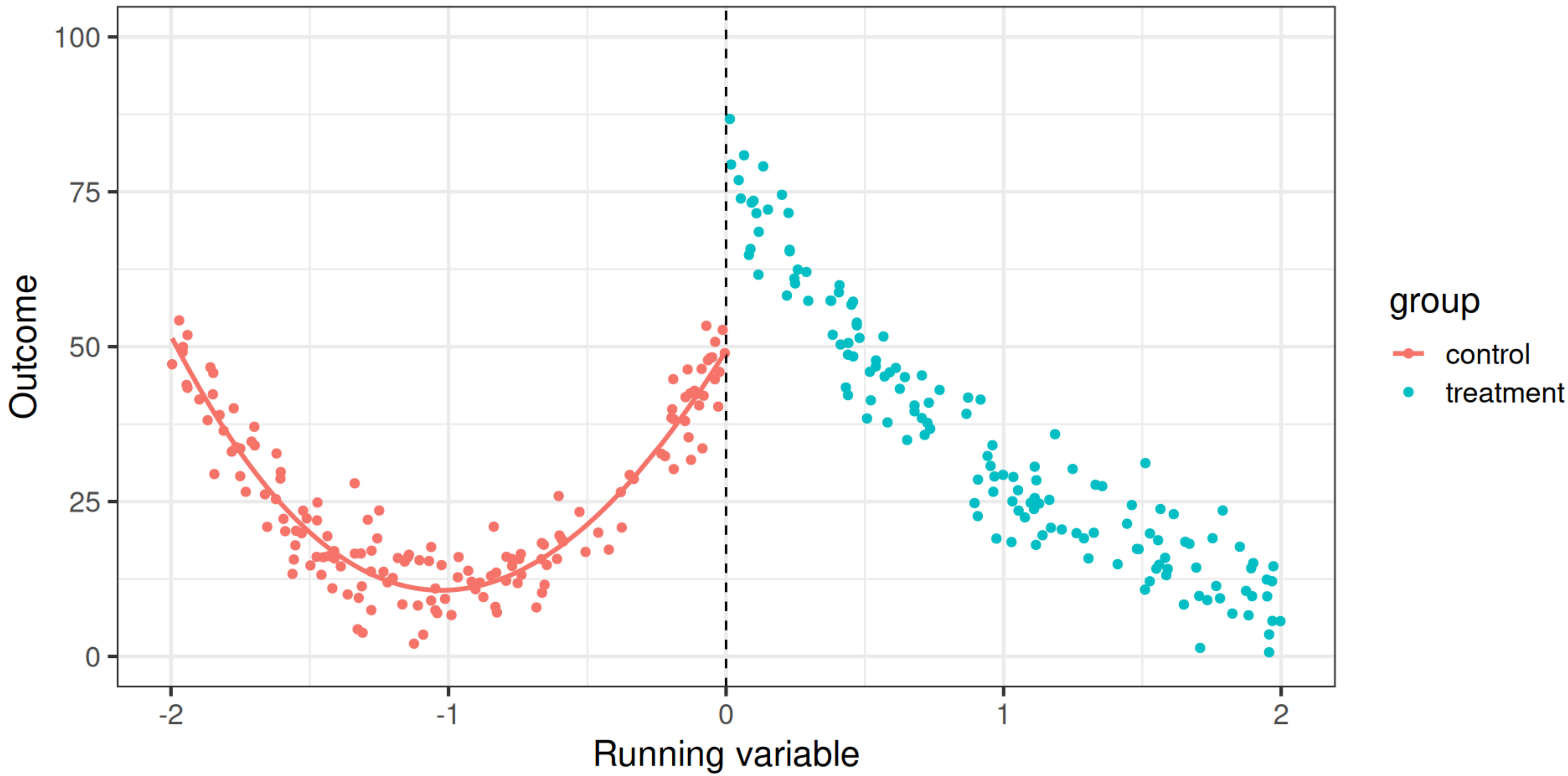

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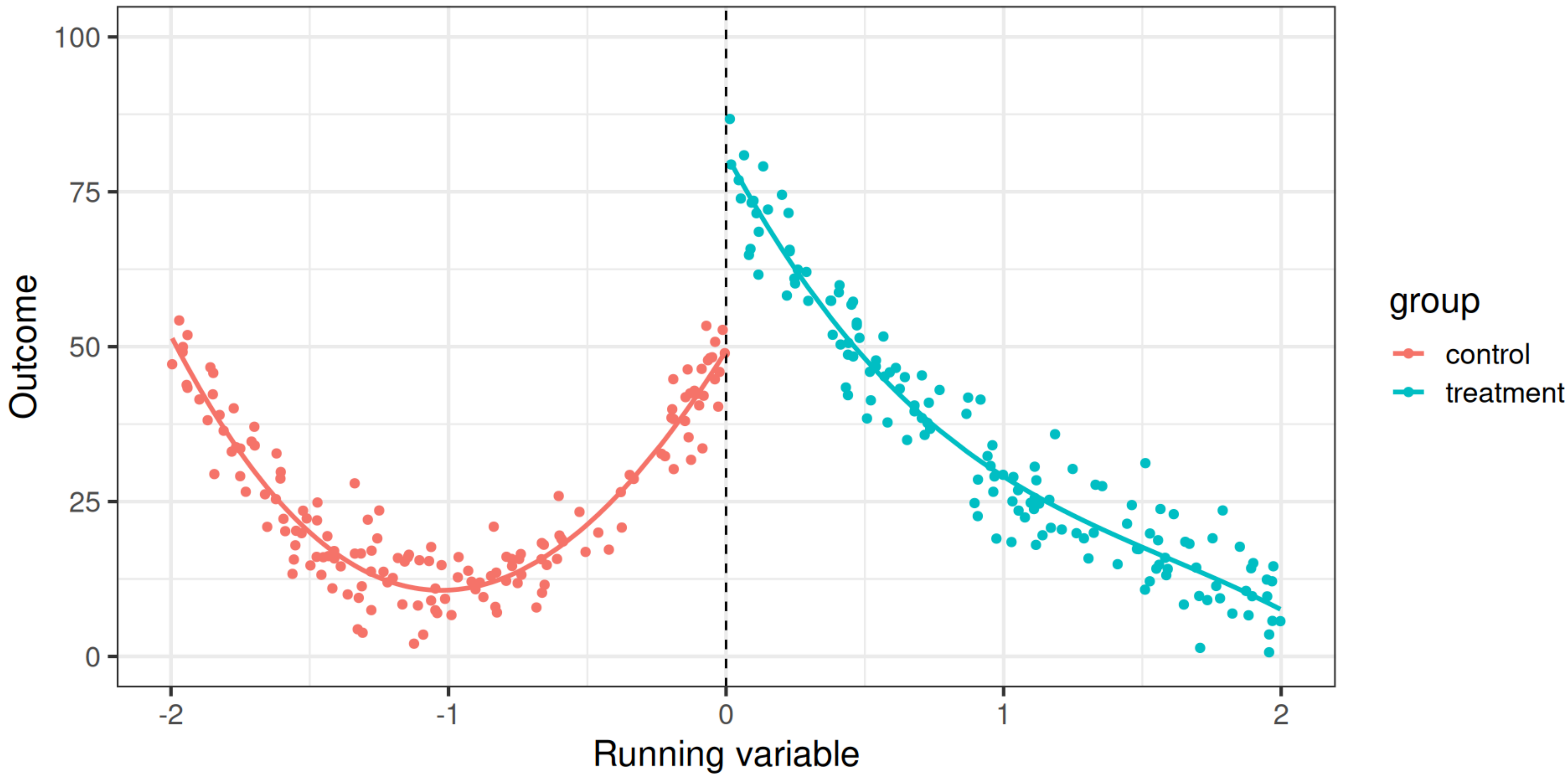
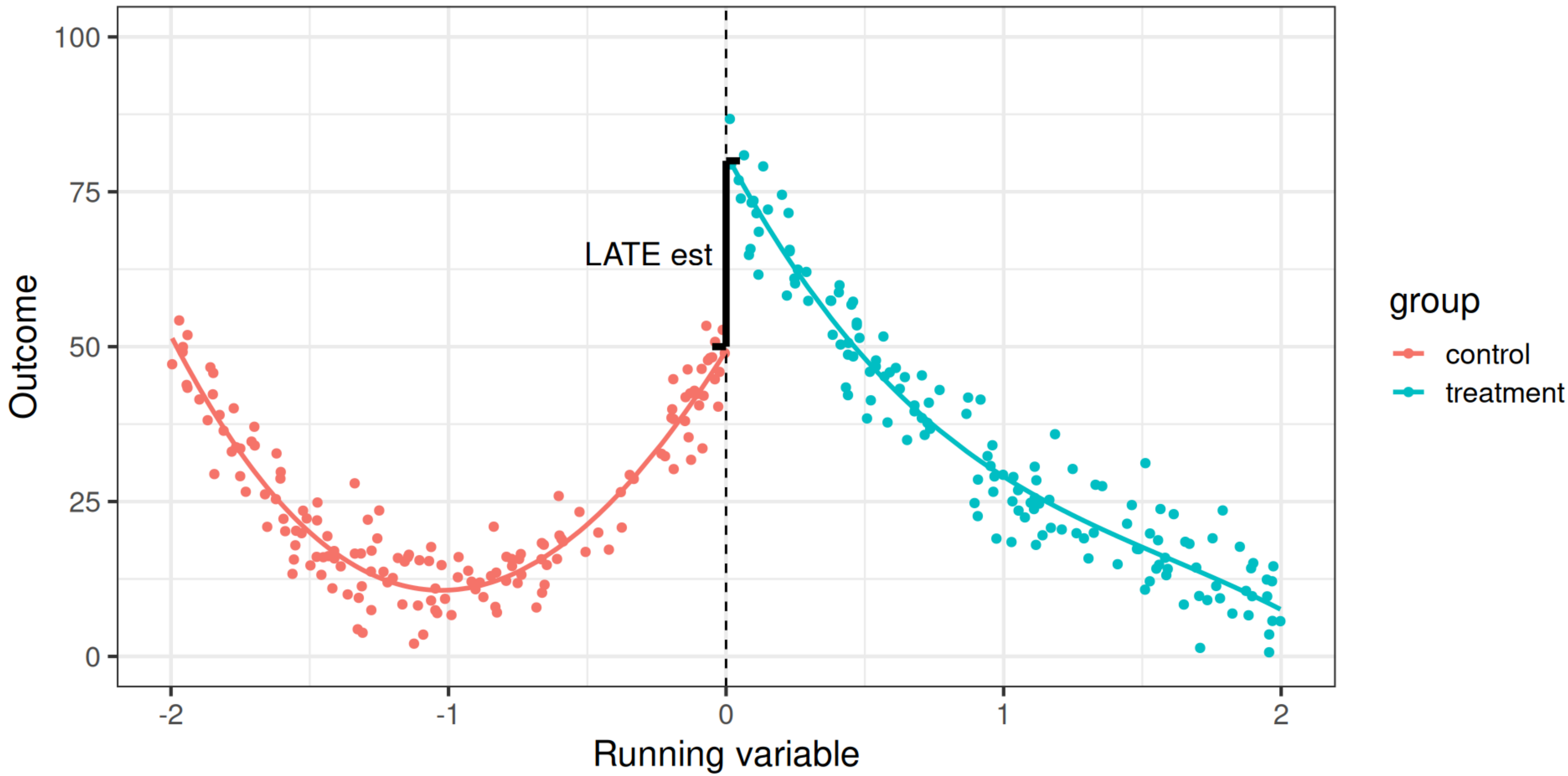


The nonlinear case: Polynomial regression approach



The problem with polynomials

Polynomial regression predictions *very* sensitive to small changes in data

- Example of the bias-variance tradeoff in statistics
- More flexible model $\rightsquigarrow$ higher standard errors, more overfitting

Predictions at boundary points are especially sensitive to small changes

- Exactly what we care about for RDD!

My heuristic: Would the discontinuity be obvious if you didn't plot the polynomial curves?

A questionable polynomial RDD

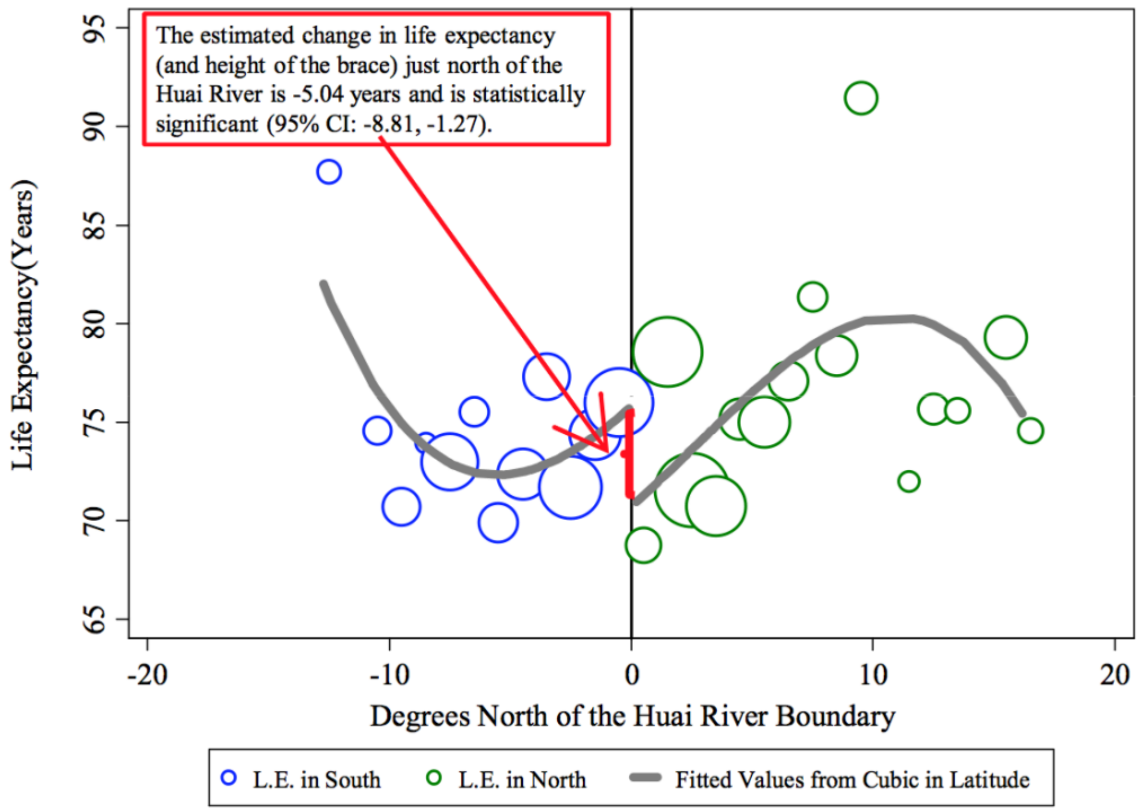


Fig. 3. The plotted line reports the fitted values from a regression of life expectancy on a cubic in latitude using the sample of DSP locations, weighted by the population at each location.

Example via the statistician Andrew Gelman's blog:

Alternative approach: Linear RDD within a bandwidth

When not assuming linearity, it doesn't make a lot of sense to use data far from boundary to make predictions close to boundary

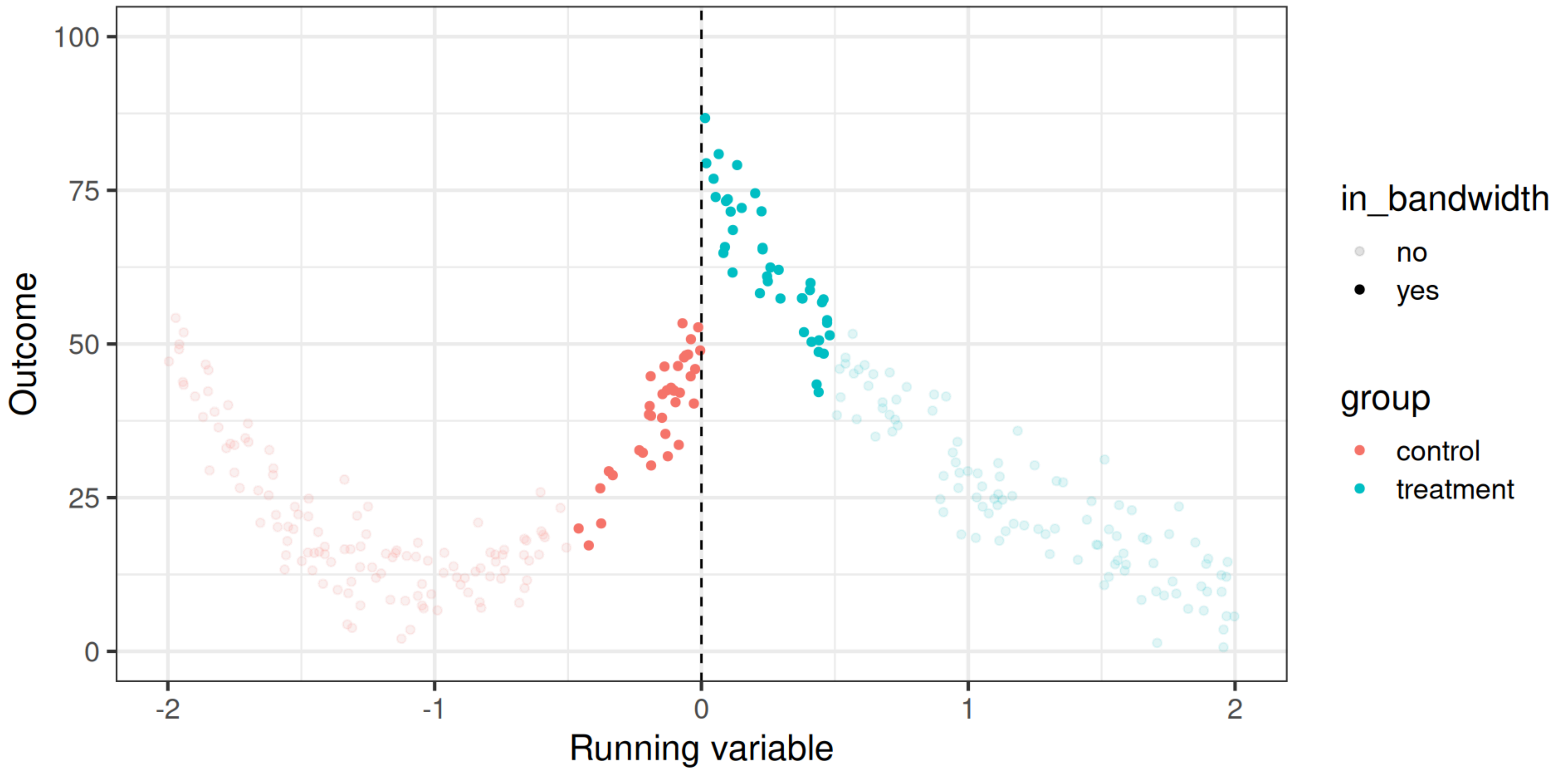
Even nonlinear functions are approximately linear on small intervals

Alternative to polynomials for RDD with nonlinear relationships:

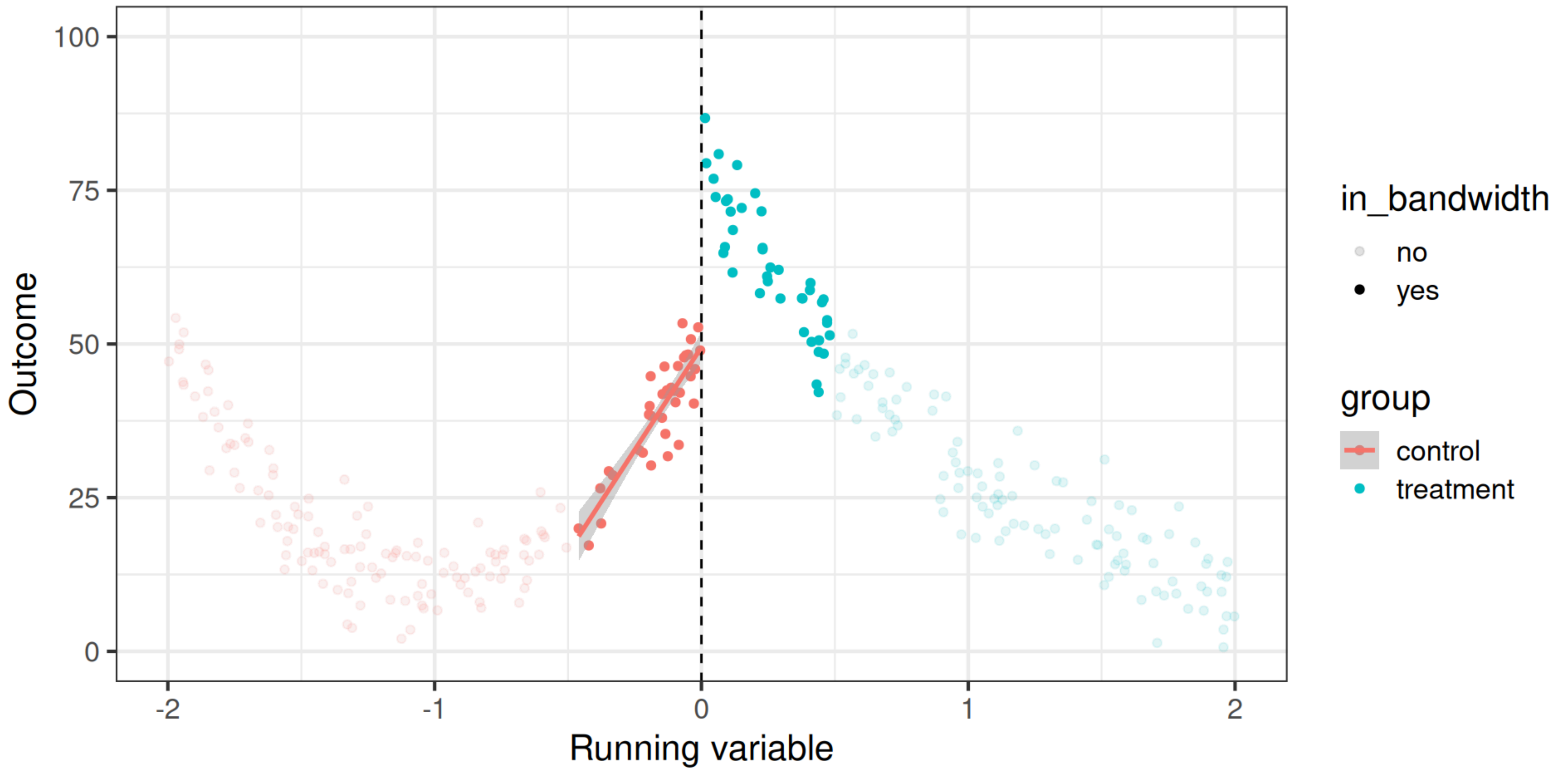
1. Restrict sample to a "bandwidth" h around threshold, $|R_i| < h$
2. Use linear RDD on restricted sample

Throwing away some data for sake of statistical precision!

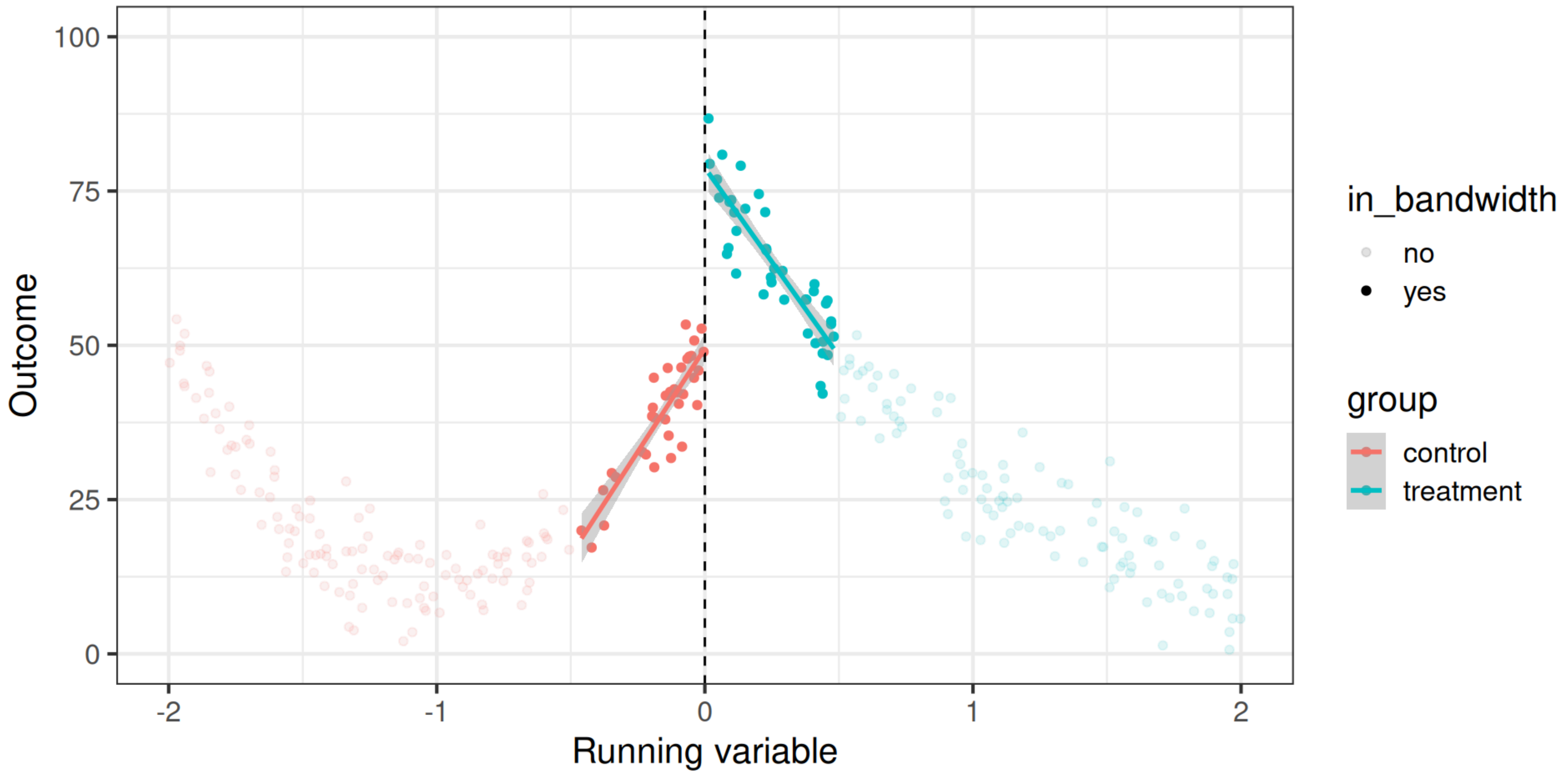
RDD within a bandwidth



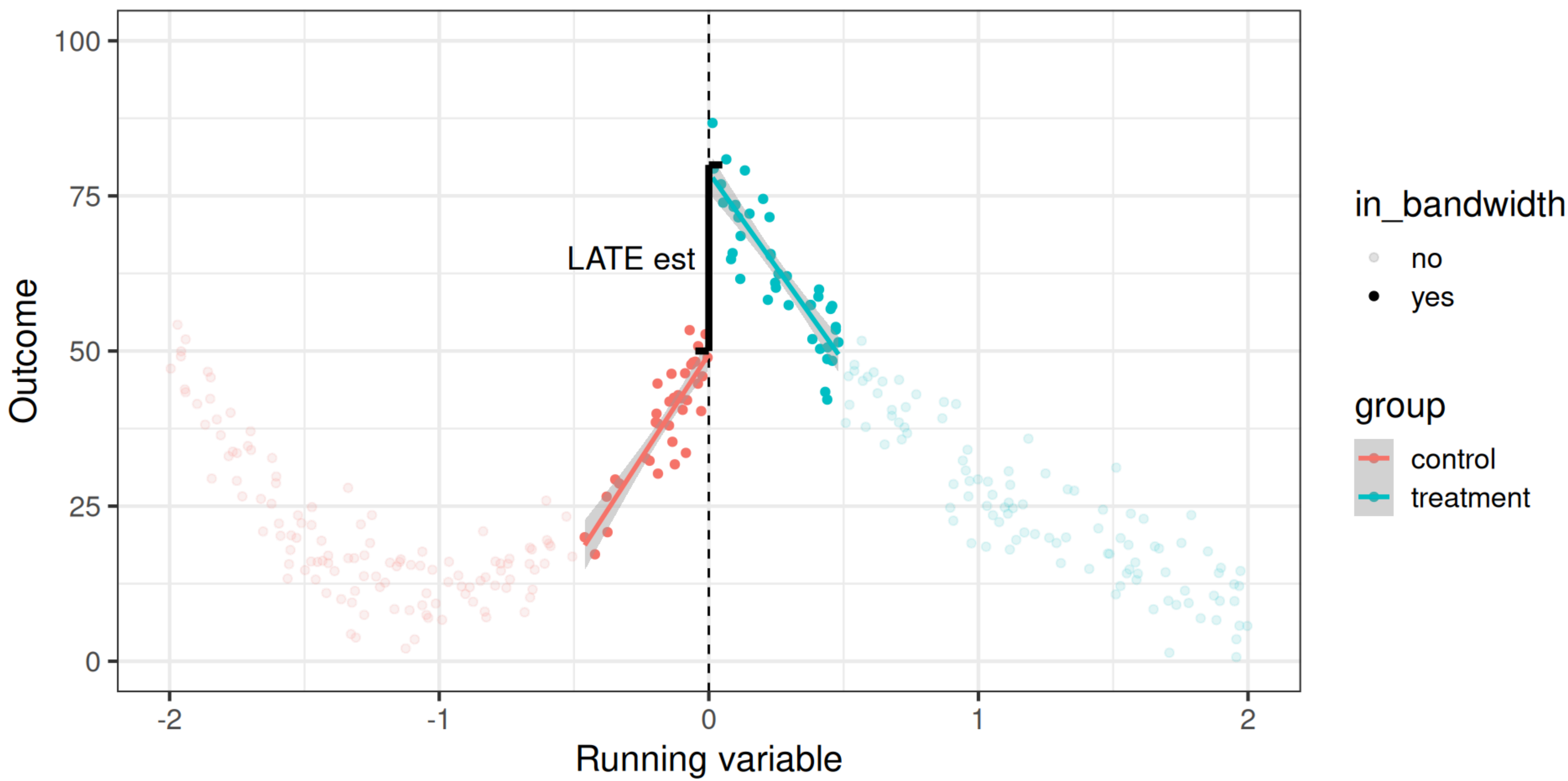
RDD within a bandwidth



RDD within a bandwidth



RDD within a bandwidth



Wrapping up

What we did today

Key requirements for RDD

- Treatment assignment determined by hard threshold on running variable
- Running variable is observed
- Potential outcomes vary continuously with running variable

RDD in practice

- Linear relationship between R and Y $\rightsquigarrow$ linear RDD
- Nonlinear relationship $\rightsquigarrow$ (usually) RDD within bandwidth

Next time: Hall's data, bandwidth selection, (maybe) fuzzy RDD