Instrumental variables in theory

PSCI 2301: Quantitative Political Science II

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Recap

Approaches to causal inference so far

1. Randomized treatment assignment
    - Selection bias eliminated by design
    - Diff of means $\approx$ average treatment effect
2. Control for all the confounders (conditional independence)
    - Matching
    - Regression with controls

But what if we can't randomize treatment assignment, and some confounders are unmeasured or unobserved?

Today's agenda

We'll use instrumental variables to estimate treatment effects

1. Motivating question — effect of institutions on growth
    - Correlational evidence
    - Difficulties for causal inference
2. The instrumental variables approach
    - Key assumptions
    - Local average treatment effects (LATE)

The puzzle: Institutions and growth

The Great Divergence

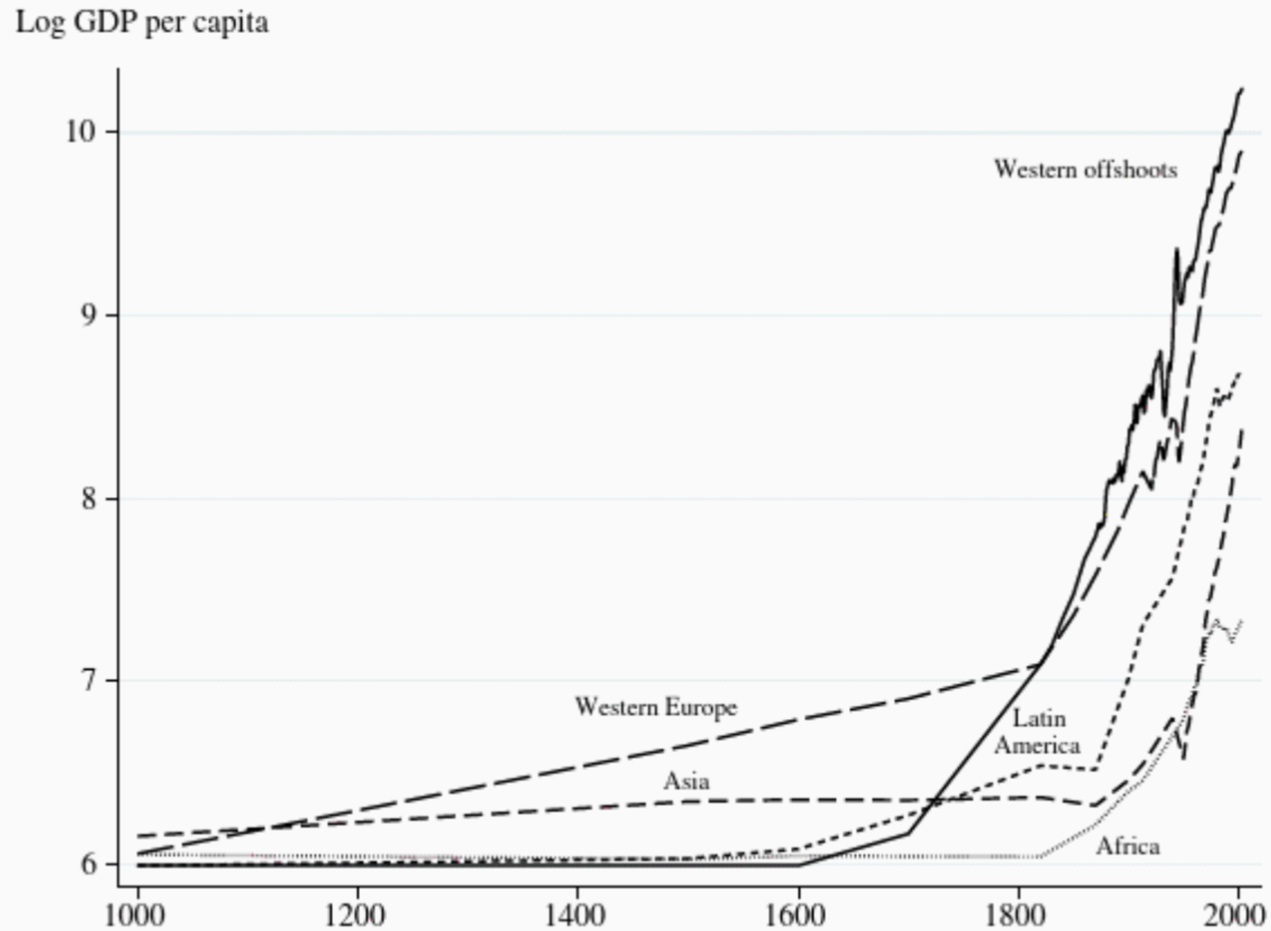

FIGURE 1.11 The evolution of average GDP per capita in Western offshoots, Western Europe, Latin America, Asia, and Africa, 1000–2000.

Do good political institutions cause faster growth?

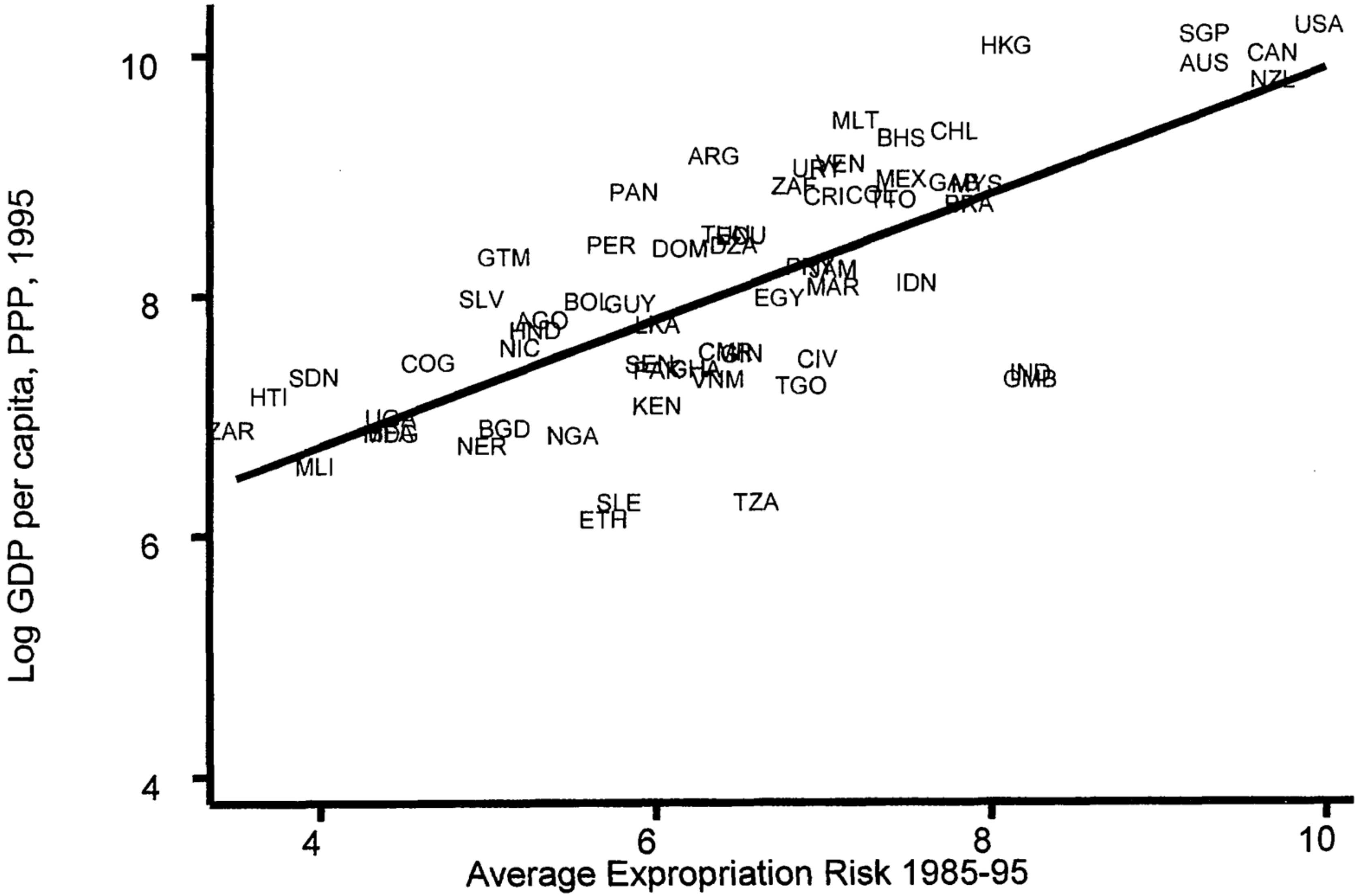

FIGURE 2. OLS RELATIONSHIP BETWEEN EXPROPRIATION RISK AND INCOME

Do good political institutions cause faster growth?

Obvious problem for causal inference: institutions aren't randomly assigned

More subtle problem: Many confounders are hard to measure

> **In-class exercise**
>
> Think of as many country-level factors as you can that might influence both *political institutions* and *economic growth rates*. Would any of these be difficult to measure systematically?

Unobserved confounding $\rightsquigarrow$ matching and regression are biased

- Conditional independence condition fails
- Differences b/w treated and untreated with same observables may not be due to treatment

AJR’s approach

Institutional qualities aren’t randomly assigned — but we can we find any as-if random <u>influences</u> on institutions?

AJR say yes: **European settler mortality** in the colonial period

- Disease environment varied across colonies
  - → Some diseases very fatal to newcomers, others less so
  - → Much less variation in effects on indigenous population
- The proposed causal chain
  1. Low settler mortality →
  2. More settlement by colonizers →
  3. Less extractive colonial institutions →
  4. Less extractive contemporary institutions

Why does an instrument help with causal inference?

For now, assume AJR are right that settler mortality is as-if random

Why does this one thing let us get around all the unobserved confounding?

1. Can isolate effect of settler mortality on economic development
2. Can isolate effect of settler mortality on political institutions

And, if settler mortality only affects development *through* institutions:

3. Can work backward to infer effect of institutions on development

Today's goal: Figure out why this works

Intro to instruments

Ingredients of the analysis

Potential outcomes Y_{1i}, Y_{0i}

- Y_{1i}: GDP per capita if institutions are good
- Y_{0i}: GDP per capita if institutions are bad

Instrumental variable Z_i

- $Z_i = 1$ for low settler mortality, $Z_i = 0$ for high settler mortality
- Need not be binary in general, but this is easiest case to think through

Potential treatment assignments D_{1i}, D_{0i}, given instrument assignment

- D_{1i}: Good institutions if low settler mortality?
- D_{0i}: Good institutions if high settler mortality?

What makes an instrumental variable?

An instrumental variable must satisfy three conditions

1. Independence
    - Instrument is as good as randomly assigned
    - Satisfies independence w.r.t. treatment assignment and outcome
2. First stage
    - Instrument has nonzero effect on treatment assignment
    - Bigger effect is better for inference
3. Exclusion restriction
    - Instrument has no *direct* effect on outcome
    - Nor any effect through any other channel besides D_i
    - This can be very difficult to justify in practice!

The IV estimator

Decomposing the “reduced form” effect of instrument on outcome:

$$\begin{aligned}\text{effect of } Z \text{ on } Y &= \underbrace{(\text{effect of } Z \text{ on } D) \cdot (\text{effect of } D \text{ on } Y)}_{\text{indirect effect}} \\ &\quad + \text{direct effect of } Z \text{ on } Y\end{aligned}$$

Exclusion restriction $\rightsquigarrow$ Direct effect = 0

First stage $\rightsquigarrow$ Effect of Z on D ≠ 0

$$\begin{aligned}\text{effect of } D \text{ on } Y &= \frac{\text{effect of } Z \text{ on } Y}{\text{effect of } Z \text{ on } D} \\ &\approx \frac{\text{avg}[Y_i \mid Z_i = 1] - \text{avg}[Y_i \mid Z_i = 0]}{\text{avg}[D_i \mid Z_i = 1] - \text{avg}[D_i \mid Z_i = 0]}\end{aligned}$$

Thinking through the first stage

Always takers: $D_{1i} = D_{0i} = 1$

→ Countries that would have good institutions regardless of settler mortality

Never takers: $D_{1i} = D_{0i} = 0$

→ Countries that would have bad institutions regardless of settler mortality

Compliers: $D_{1i} = 1$, $D_{0i} = 0$

→ Countries that would have good institutions with low settler mortality, but bad institutions with high settler mortality

Deniers: $D_{1i} = 0$, $D_{0i} = 1$

→ Countries that would have good institutions with high settler mortality, but bad institutions with low settler mortality

→ We typically assume there are no deniers

What IV is estimating

We typically don't want to assume the treatment effect is uniform

IV estimation relies heavily on the compliers

IV estimates a local average treatment effect (LATE)

- Average treatment effect among the set of compliers
- $\mathbb{E}[Y_{1i} - Y_{0i} \mid D_{1i} = 1, D_{0i} = 0]$

Fundamental prob of causal inference $\rightsquigarrow$ Can't discern which obs are compliers, always takers, or never takers

Why IV estimates the LATE

Imagine the data looks like this:

Subpopulation	Proportion	Baseline average Y_{0i}	Avg treatment effect
Always takers	1/2	10	2
Compliers	1/4	4	4
Never takers	1/4	2	0

Also imagine same fraction of each group gets Z = 1 (independence)

$$\mathbf{avg}[Y_i \mid Z_i = 1] = \frac{1}{2} \cdot 12 + \frac{1}{4} \cdot 8 + \frac{1}{4} \cdot 2 = 8.5$$

$$\mathbf{avg}[Y_i \mid Z_i = 0] = \frac{1}{2} \cdot 12 + \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 2 = 7.5$$

Reduced form: $\mathbf{avg}[Y_i \mid Z_i = 1] - \mathbf{avg}[Y_i \mid Z_i = 0] = 8.5 - 7.5 = 1$

Why IV estimates the LATE

Imagine the data looks like this:

Subpopulation	Proportion	Baseline average Y_{0i}	Avg treatment effect
Always takers	1/2	10	2
Compliers	1/4	4	4
Never takers	1/4	2	0

Also imagine same fraction of each group gets Z = 1 (independence)

First stage: $\mathbf{avg}[D_i \mid Z_i = 1] - \mathbf{avg}[D_i \mid Z_i = 0] = \mathbf{0.25}$ (% compliers)

IV estimate equals ATE among compliers:

$$\frac{\text{reduced form}}{\text{first stage}} = \frac{1}{0.25} = 4$$

Bonus slide: Why IV estimates the LATE, in general

Subpopulation	**Proportion**	**Baseline average** Y_{0i}	**Avg treatment effect**
Always takers	q_a	y_a	τ_a
Compliers	q_c	y_c	τ_c
Never takers	q_n	y_n	τ_n

$$
\begin{aligned}
\operatorname{avg}[Y_i \mid Z_i = 1] &= q_a(y_a + \tau_a) + q_c(y_c + \tau_c) + q_n y_n \\
\operatorname{avg}[Y_i \mid Z_i = 0] &= q_a(y_a + \tau_a) + q_c y_c + q_n y_n \\
\operatorname{avg}[D_i \mid Z_i = 1] &= q_a + q_c \\
\operatorname{avg}[D_i \mid Z_i = 0] &= q_a \\
\text{IV est} &= \frac{\operatorname{avg}[Y_i \mid Z_i = 1] - \operatorname{avg}[Y_i \mid Z_i = 0]}{\operatorname{avg}[D_i \mid Z_i = 1] - \operatorname{avg}[D_i \mid Z_i = 0]} \\
&= \frac{q_c \tau_c}{q_c} = \tau_c.
\end{aligned}
$$

Limitations of the IV technique

1. LATE ≠ ATE
    - Must either care about complier effect specifically
    - ...or have reason to think overall effect not much different
2. Big standard errors
    - Essentially throwing away data to isolate randomized part of the effect
    - Particularly acute if first stage is small (**weak instrument**)
3. Only works under stringent assumptions
    - Exclusion restriction can be very hard to justify
    - e.g., what if disease affected development of port infrastructure?

Wrapping up

What we did today

1. AJR's puzzle: Do institutions increase growth?
    - Present-day development strongly correlated w/ institutional quality
    - But tons of confounders, including many hard to observe
    - Settler mortality as potential instrument
2. Instrumental variables assumptions
    - Independence: IV is as-if random
    - First stage: IV affects treatment assignment
    - Exclusion restriction: IV only affects outcome through treatment
3. Local average treatment effect
    - IV only yields average among compliers

Next time: IV estimation in practice, including with control variables