$$\sigma_r = \frac{1}{r}\sqrt{\Delta\delta^2\,\sigma_{\Delta\delta}^2 + \Delta\alpha^2\,\sigma_{\Delta\alpha}^2\,\mathrm{Cos}[\delta avg]^4 + \Delta\alpha^4\,\sigma_{\delta avg}^2\,\mathrm{Cos}[\delta avg]^2\,\mathrm{Sin}[\delta avg]^2}$$

$$\sigma_\theta = \frac{1}{r^2}\sqrt{(\Delta\delta^2\,\sigma_{\Delta\alpha}^2 + \Delta\alpha^2\,\sigma_{\Delta\delta}^2)\,\mathrm{Cos}[\delta avg]^2 + \Delta\alpha^2\,\Delta\delta^2\,\sigma_{\delta avg}^2\,\mathrm{Sin}[\delta avg]^2}$$

$$\sigma_x = \sqrt{\frac{x^2}{r^2}\,\sigma_r^2 + y^2\,\sigma_\theta^2}$$

$$\sigma_y = \sqrt{x^2\,\sigma_\theta^2 + \frac{y^2}{r^2}\,\sigma_r^2}$$

$$\sigma_{M_A+M_B} = (M_A+M_B)\sqrt{\frac{9\,\sigma_a^2}{a^2} + \frac{4\,\sigma_P^2}{P^2}}$$

In[1]:= $d\delta avg = D\left[\sqrt{\mathrm{Cos}[\delta avg]^2\,\Delta\alpha^2 + \Delta\delta^2},\ \delta avg\right]$

Out[1]= $-\dfrac{\Delta\alpha^2\,\mathrm{Cos}[\delta avg]\,\mathrm{Sin}[\delta avg]}{\sqrt{\Delta\delta^2 + \Delta\alpha^2\,\mathrm{Cos}[\delta avg]^2}}$

In[2]:= $d\Delta\alpha = D\left[\sqrt{\mathrm{Cos}[\delta avg]^2\,\Delta\alpha^2 + \Delta\delta^2},\ \Delta\alpha\right]$

Out[2]= $\dfrac{\Delta\alpha\,\mathrm{Cos}[\delta avg]^2}{\sqrt{\Delta\delta^2 + \Delta\alpha^2\,\mathrm{Cos}[\delta avg]^2}}$

In[3]:= $d\Delta\delta = D\left[\sqrt{\mathrm{Cos}[\delta avg]^2\,\Delta\alpha^2 + \Delta\delta^2},\ \Delta\delta\right]$

Out[3]= $\dfrac{\Delta\delta}{\sqrt{\Delta\delta^2 + \Delta\alpha^2\,\mathrm{Cos}[\delta avg]^2}}$

In[4]:= $\sqrt{d\delta avg^2\,\sigma\delta avg^2 + d\Delta\alpha^2\,\sigma\Delta\alpha^2 + d\Delta\delta^2\,\sigma\Delta\delta^2}$ // FullSimplify

Out[4]= $\sqrt{\dfrac{\Delta\delta^2\,\sigma\Delta\delta^2 + \Delta\alpha^2\,\sigma\Delta\alpha^2\,\mathrm{Cos}[\delta avg]^4 + \Delta\alpha^4\,\sigma\delta avg^2\,\mathrm{Cos}[\delta avg]^2\,\mathrm{Sin}[\delta avg]^2}{\Delta\delta^2 + \Delta\alpha^2\,\mathrm{Cos}[\delta avg]^2}}$

$$\sigma_r = \frac{1}{r}\sqrt{(\Delta\delta^2\,\sigma_{\Delta\delta}^2 + \Delta\alpha^2\,\sigma_{\Delta\alpha}^2\,\mathrm{Cos}[\delta avg]^4 + \Delta\alpha^4\,\sigma_{\delta avg}^2\,\mathrm{Cos}[\delta avg]^2\,\mathrm{Sin}[\delta avg]^2)}$$

In[5]:= $d\delta avg = D\left[\mathrm{ArcTan}\left[\dfrac{\mathrm{Cos}[\delta avg]\,\Delta\alpha}{\Delta\delta}\right],\ \delta avg\right]$

Out[5]= $-\dfrac{\Delta\alpha\,\mathrm{Sin}[\delta avg]}{\Delta\delta\left(1 + \frac{\Delta\alpha^2\,\mathrm{Cos}[\delta avg]^2}{\Delta\delta^2}\right)}$

In[6]:=
```
dΔα = D[ArcTan[Cos[δavg] Δα/Δδ], Δα]
```

Out[6]= $$\frac{\text{Cos}[\delta\text{avg}]}{\Delta\delta\left(1+\frac{\Delta\alpha^2\,\text{Cos}[\delta\text{avg}]^2}{\Delta\delta^2}\right)}$$

In[7]:=
```
dΔδ = D[ArcTan[Cos[δavg] Δα/Δδ], Δδ]
```

Out[7]= $$-\frac{\Delta\alpha\,\text{Cos}[\delta\text{avg}]}{\Delta\delta^2\left(1+\frac{\Delta\alpha^2\,\text{Cos}[\delta\text{avg}]^2}{\Delta\delta^2}\right)}$$

In[8]:=
```
Sqrt[dδavg^2 σδavg^2 + dΔα^2 σΔα^2 + dΔδ^2 σΔδ^2] // FullSimplify
```

Out[8]= $$\sqrt{\left(\left(\left(\Delta\delta^2\,\sigma\Delta\alpha^2+\Delta\alpha^2\,\sigma\Delta\delta^2\right)\text{Cos}[\delta\text{avg}]^2+\Delta\alpha^2\,\Delta\delta^2\,\sigma\delta\text{avg}^2\,\text{Sin}[\delta\text{avg}]^2\right)\Big/\left(\Delta\delta^2+\Delta\alpha^2\,\text{Cos}[\delta\text{avg}]^2\right)^2\right)}$$

$$\sigma_\theta = \frac{1}{r^2}\sqrt{\left(\left(\Delta\delta^2\,\sigma_{\Delta\alpha}^2+\Delta\alpha^2\,\sigma_{\Delta\delta}^2\right)\text{Cos}[\delta\text{avg}]^2+\Delta\alpha^2\,\Delta\delta^2\,\sigma_{\delta\text{avg}}^2\,\text{Sin}[\delta\text{avg}]^2\right)}$$

In[9]:=
```
dr = D[r Cos[θ], r]
```

Out[9]= Cos[θ]

In[10]:=
```
dθ = D[r Cos[θ], θ]
```

Out[10]= -r Sin[θ]

In[11]:=
```
Sqrt[dr^2 σr^2 + dθ^2 σθ^2] // FullSimplify
```

Out[11]= $$\sqrt{\sigma r^2\,\text{Cos}[\theta]^2+r^2\,\sigma\theta^2\,\text{Sin}[\theta]^2}$$

$$\sigma_x = \sqrt{\frac{x^2}{r^2}\,\sigma_r^2+y^2\,\sigma_\theta^2}$$

In[12]:=
```
dr = D[r Sin[θ], r]
```

Out[12]= Sin[θ]

In[13]:=
```
dθ = D[r Sin[θ], θ]
```

Out[13]= r Cos[θ]

In[14]:=
```
Sqrt[dr^2 σr^2 + dθ^2 σθ^2] // FullSimplify
```

Out[14]= $$\sqrt{r^2\,\sigma\theta^2\,\text{Cos}[\theta]^2+\sigma r^2\,\text{Sin}[\theta]^2}$$

$$\sigma_y = \sqrt{x^2\,\sigma_\theta^2 + \frac{y^2}{r^2}\,\sigma_r^2}$$

In[15]:= **da = D[$\frac{4\,\pi^2\,a^3}{G\,P^2}$, a]**

Out[15]= $\frac{12\,a^2\,\pi^2}{G\,P^2}$

In[16]:= **dP = D[$\frac{4\,\pi^2\,a^3}{G\,P^2}$, P]**

Out[16]= $-\frac{8\,a^3\,\pi^2}{G\,P^3}$

In[17]:= **$\sqrt{da^2\,\sigma a^2 + dP^2\,\sigma P^2}$ // FullSimplify**

Out[17]= $4\,\pi^2\,\sqrt{\frac{a^4\,\left(9\,P^2\,\sigma a^2 + 4\,a^2\,\sigma P^2\right)}{G^2\,P^6}}$

$$\sigma_{M_A+M_B} = (M_A + M_B)\,\sqrt{\frac{9\,\sigma_a^2}{a^2} + \frac{4\,\sigma_P^2}{P^2}}$$