Laboratory Manual

PHSC 12620 The Big Bang

The University of Chicago

Spring 2020

Labs

LAB 1

Making groups, installing the needed software

In this lab, you will ensure that you have groups, and that you know how you will communicate with each other. You will also ensure that you have the software installed that you need to complete the labs this quarter. This software includes

1. Zoom, for video collaboration and joining meetings
2. DS9, for analyzing astronomical images
3. Spreadsheet software (e.g. LibreOffice Calc or Microsoft Excel), for analyzing data and making plots
4. Flash, for using a web-based simulation

1.1 Forming Groups

1. Fill out the group introductions spreadsheet found here: https://docs.google.com/spreadsheets/d/1h3V1AqgsETIhE1cOZ7OZxpSnK9WcJQotNosq4jZkSds/edit?usp=sharing, including taking the DOPE Bird Personality Quiz, linked to in the spreadsheet.
2. Contact others in your lab section to form groups. Consider what kind of Bird Types you think would add to your group's effectiveness, and also what time zone people are in and when they are generally available to work.

If you are attending the lab session live and do not yet have a group, one way the TA could assist is to arrange "speed networking" among those who still need a group. This would involve the TA organizing Zoom Breakout Rooms, where each room is 2-3 students, and each group talks about how they work and what they are looking for in a group member. Then after 5 minutes or so, the Rooms are changed so people are with different people. This could help people get to know each other enough to form lab groups.

3. Once you have a group, meet with each other and decide a) what tools you will use to communicate and collaborate, b) when you will meet, c) what you will do when you need to change an agreement, and d) what you will do when you a person has an issue with how the group is functioning. **Write this in your lab report. This part counts as data collection and analysis, so it can be identical in each member's report.**

1.2 Zoom

Zoom is a tool for video conferencing. You probably already it installed.

4. Download and install the application from your app store, or from here: https://zoom.us/download#client_4meeting.

5. Some teachers might require students to log in with their uchicago.edu email to access their Zoom meetings, so ensure that you can log in with that email address. You can also change your settings by logging in through the browser: uchicago.zoom.us

1.3 Spreadsheet software

Unless you already have data analysis experience with other software, spreadsheet software will be useful for you to collect and analyze data, including plotting and curve fitting. As a UChicago student, you have free access to Microsoft Office 365 Excel (through portal.office.com). This can work, but is sometimes less intuitive for doing curve fitting than a free open source office software, LibreOffice Calc.

6. Ensure you have access to Microsoft Excel through portal.office.com

7. (optional, but recommended) Install LibreOffice. You can download it for free from https://www.libreoffice.org/download/download/ (get version 6.3.5, and if you're not sure whether you need the 32-bit or 64-bit version, you almost certainly want the 64-bit version). You can also get it from the Microsoft Store or Mac Store, for a small fee.

1.4 SAOImage DS9

SAOImage DS9, or DS9 for short, is an image viewer, analyzer, and processor written and used by astronomers for working with astronomical images.

8. Download and install DS9 from http://ds9.si.edu/site/Download.html.

   - For MacOS, unless you know otherwise, choose from the top set of choices (to the right of the blue apple logo). To find your version, from the Apple menu in the corner of the screen, choose "About This Mac".

1.5 Flash

Adobe Flash is software that is currently being phased out, but is still used to enable interactive animations in web browsers.

9. Test if you already have Flash installed by going to http://www.zombo.com/. If you see some faintly flashing circles and hear an inspirational message welcoming you to zombocom, then Flash is installed and working.

   If it does not automatically run, then depending on your browser, try the following:

   - Safari: https://helpx.adobe.com/flash-player/kb/enabling-flash-player-safari.html
   - Microsoft Edge: https://helpx.adobe.com/flash-player/kb/flash-player-issues-windows-10-edge.html

1.6 Report checklist

Include the following in your lab report. See Appendix A for formatting details. Each item below is worth 10 points.

1. List of your lab group members and decisions you've made about collaborating (Step 3).
2. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

LAB 2

Measuring distant objects with parallax

2.1 Introduction

Since it takes time for light to travel to us from objects in the universe, the further out an object is, the further back in time we see it. So for us to have an accurate picture of how the universe was in the past, we need to know how far away things are. For things that are nearby on Earth, we can travel there and see how far we went, or how long we took to get there. For things further away like the moon, we can use Kepler's laws, or we can bounce a beam of light off of it and see how long it takes to get back. For objects outside of our solar system, it would take too long, and the light would disperse too much, for us to use this last technique. For those objects that are still relatively nearby, we can use the parallax technique as the first rung on our distance ladder.

2.2 Procedure

First, complete the worksheet "The Parsec" on the following pages. You can draw the diagrams needed and include a picture of your diagrams, or use a drawing program to draw on them. Each member of the group will turn in their own worksheet as part of their lab report, and you can work in groups to complete them.

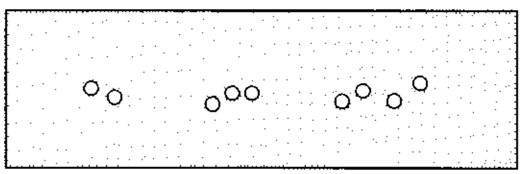
Part I: Stars in the Sky

Consider the diagram to the right.

1) Imagine that you are looking at the stars from Earth in January. Use a straightedge or a ruler to draw a straight line from Earth in January, through the Nearby Star (Star A), out to the Distant Stars. Which of the distant stars would appear closest to Star A in your night sky in January? Circle this distant star and label it "Jan."

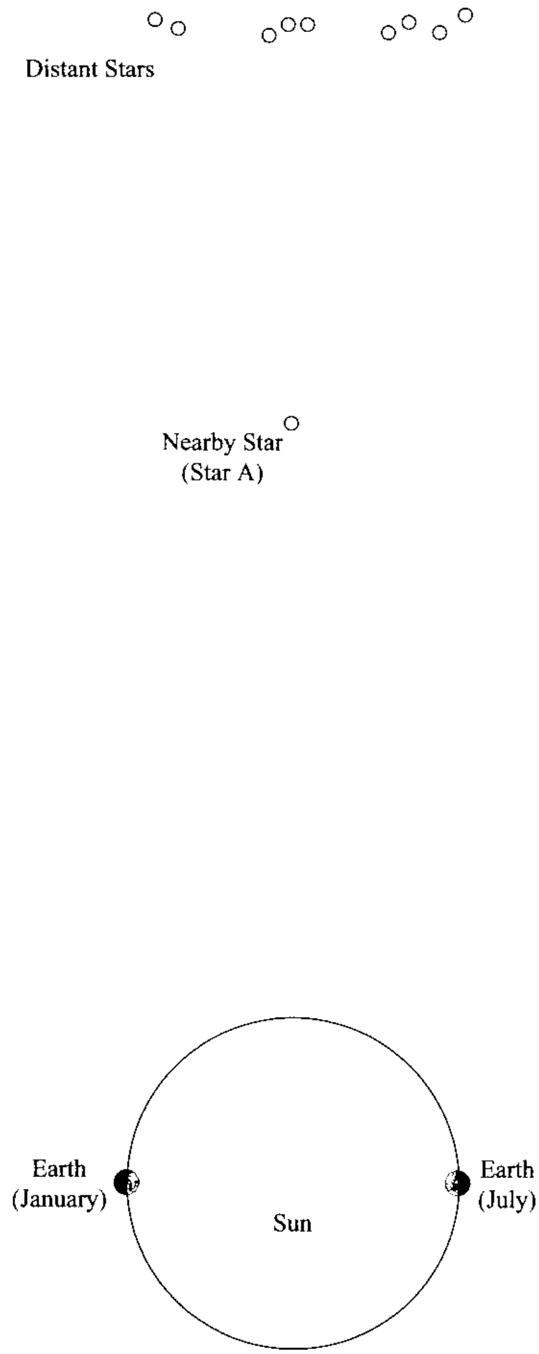
2) Repeat Question 1 for July and label the distant star "July."

3) In the box below, the same distant stars are shown as you would see them in the night sky. Draw a small × to indicate the position of Star A as seen in January and label it "Star A Jan."



4) In the same box, draw another × to indicate the position of Star A as seen in July and label it "Star A July."

5) Describe how Star A would appear to move among the distant stars as Earth orbits the Sun counterclockwise from January of one year, through July, to January of the following year.



The apparent motion of nearby objects relative to distant objects, which you just described, is called **parallax.**

6) Consider two stars (C and D) that both exhibit parallax. If Star C appears to move back and forth by a greater amount than Star D, which star do you think is actually closer to you? If you're not sure, just take a guess. We'll return to this question later in this activity.

Part II: What's a Parsec?

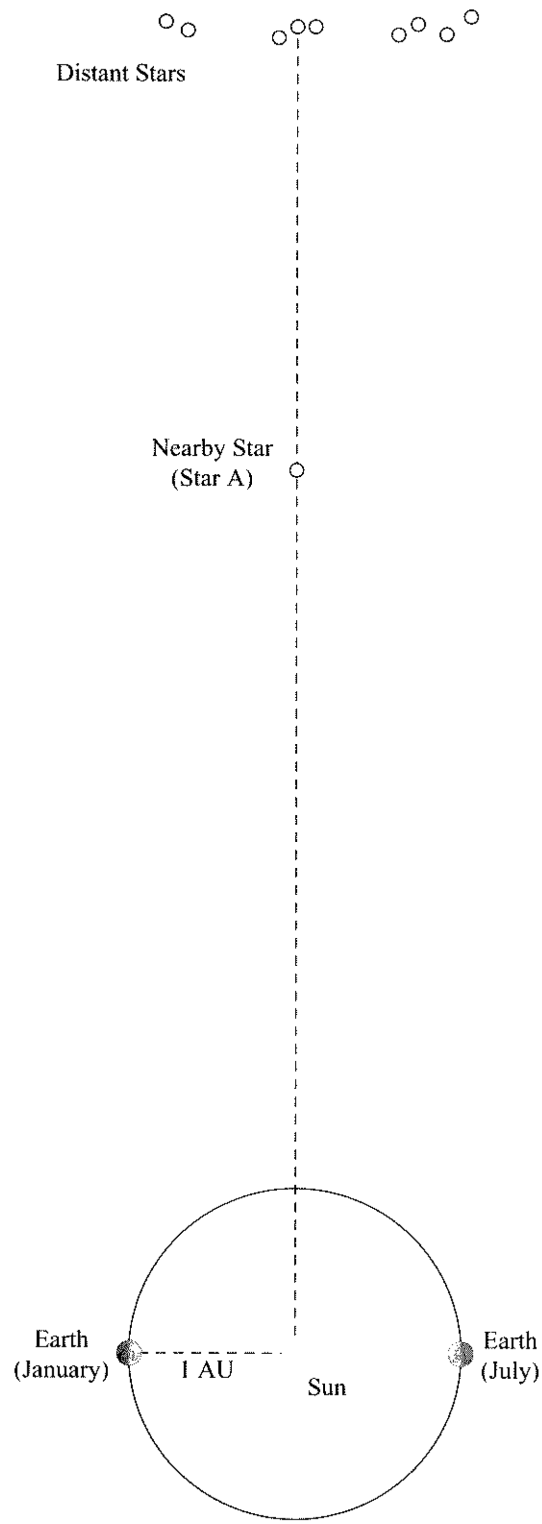
Consider the diagram to the right.

7) Starting from Earth in January, draw a line through Star A to the top of the page.

8) There is now a narrow triangle, created by the line you drew, the dotted line provided in the diagram, and the line connecting Earth and the Sun. The small angle, just below Star A, formed by the two longest sides of this triangle is called the **parallax angle** for Star A. Label this angle **"p_A."**

Knowing a star's parallax angle allows us to calculate the distance to the star. Since even the nearest stars are still very far away, parallax angles are extremely small. These parallax angles are measured in "arc-seconds" where an arcsecond is 1/3600 of 1 degree.

To describe the distances to stars, astronomers use a unit of length called the **parsec.** One parsec is defined as the distance to a star that has a **par**allax angle of exactly 1 arc**sec**ond. The distance from the Sun to a star 1 parsec away is 206,265 times the Earth–Sun distance or 206,265 AU. (Note that the diagram to the right is not drawn to scale.)



9) If the parallax angle for Star A ($\mathbf{p}_A$) is 1 arcsecond, what is the distance from the Sun to Star A? (Hint: Use parsec as your unit of distance.) Label this distance on the diagram.

10) Is a parsec a unit of length or a unit of angle? It can't be both.

Note: Since the distance from the Sun to even the closest star is so much greater than 1 AU, we can consider the distance from Earth to a star and the distance from the Sun to that star to be approximately equal.

Part III: Distances

11) Consider the following debate between two students regarding the relationship between parallax angle and the distance we measure to a star.

Student 1: *If the distance to the star is more than 1 parsec, then the parallax angle must be more than 1 arcsecond. So a star that is many parsecs away will have a large parallax angle.*

Student 2: *If we drew a diagram for a star that was much more than 1 parsec away from us, the triangle in the diagram would be pointier than the one we just drew in Part II. That should make the parallax angle smaller for a star farther away.*

Do you agree or disagree with either or both of the students? Explain your reasoning.

12) On your diagram from Part II, draw a second star along the dotted line farther from the Sun than Star A and label this faraway star "Star B." Repeat steps 7 and 8 from Part II, except label the parallax angle for this Star B with $\mathbf{p}_B$.

13) Which star, the closer one (Star A) or the farther one (Star B), has the larger parallax angle?

14) Check your answers to Questions 6 and 11 and resolve any discrepancies.

2.3 A quick measurement with hand tools

You will now use the parallax technique to measure the distance to an object in your environment without needed to travel to it.

1. Identify an object to be your "nearby star" and another as your "distant star", by analogy to the figure in the worksheet. Also choose the positions to view from as "Earth (January)" and "Earth (July)". When picking the stars and your Earth positions, check to see that from each Earth position, the distant star does not appear to move much, compared to the nearby star, and your guessed distance to the nearby star is at least 10 times greater than the distance between your two Earth positions.

You'll first use crude measuring tools to compute the nearby star's distance with the parallax technique, and then use a more precise method. First, you'll use the little finger on your outstretched arm as a measurement of angular size — the width of the index finger covers ("subtends") about 1 degree.

2. Identify an object to be your "nearby star" and another as your "distant star", by analogy to the figure in the worksheet. Find a place where you can stand and move a meter or two side to side and still see both "stars". The movement will simulate the Earth moving from its January to its July position. When picking the stars and your Earth positions, check to see that from each Earth position, the distant star does not appear to move much, compared to the nearby star, and your guessed distance to the nearby star is at least 10 times greater than the distance between your two Earth positions.

3. Looking from just one eye, move so that the two stars appear to be just touching each other. Mark your current position as Earth (January). Hold up your smallest finger at arm's length and move to your left or right until your finger fits just in between the two stars. This means that they are 1 degree away from each other in angular separation. Mark this position as Earth (July).

4. Draw a diagram, similar to the second figure in the worksheet, and find your own Earth-Sun distance (half the distance between your Earth positions). Calculate the parallax angle in radians, which is half the 1 degree you measured with your finger. For the distance measurement, you can use a ruler, measuring tape, or objects that have standard lengths like coins, paper money, or sheets of paper.

Now the distance to the nearby star can be found using the triangle formed by the line segments Sun-Earth, Earth-Star, and Star-Sun (see Figure 2.1). Trigonometry relates these lengths to each other according to

$$\tan p = \frac{a}{d}\,, \tag{2.1}$$

where p is the parallax angle in radians, d is the distance to the star, and a is half of the distance between the two measurement positions. Since the length d is much greater than a, the angle p is very small, and so we can use the small angle approximation $\tan u \approx u$, and therefore

$$p = \frac{a}{d}\,. \tag{2.2}$$

5. Use the above equation to calculate the distance to the nearby star.

6. Conduct the data collection twice more to get two more Earth-Star distances. You can use this to calculate an uncertainty for your measurement — the uncertainty is half the range of values, and the distance is the average of the 3 distances you found.

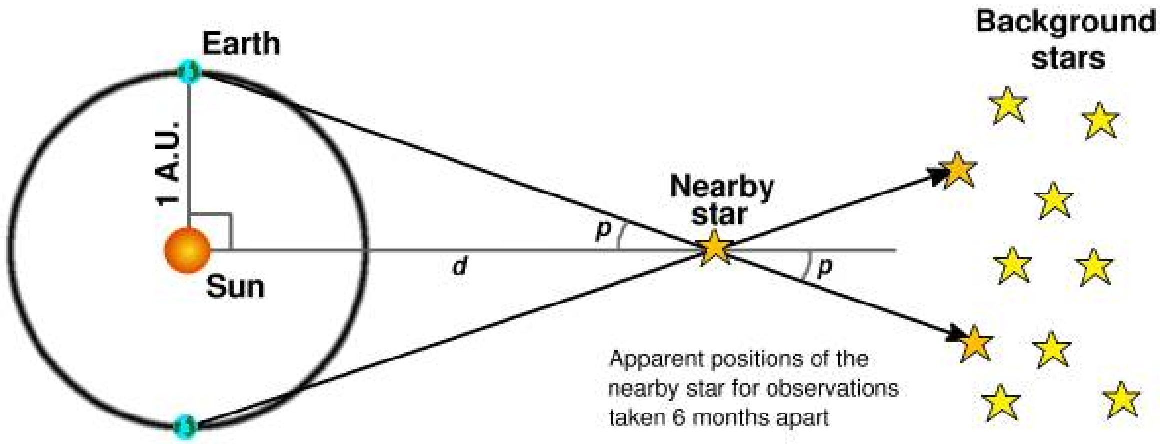


Figure 2.1: Illustration of the geometry involved in a parallax measurement to determine d, the distance to a nearby star.

7. Find the distance to the nearby star in a more direct way — measure the distance with a measuring tape or pieces of paper, or for more distant objects, find them on Google Maps.

One way of describing how different two values are, without considering the uncertainty of those values, is to calculate the percent difference:

$$\text{percent difference} = \frac{d_1 - d_2}{\left(\frac{d_1+d_2}{2}\right)} \times 100\% \tag{2.3}$$

8. How close is your parallax distance measurement to the direct measurement? Report the percent difference.

2.4 Measuring with more precise equipment

Using a finger for measuring angular separation is not very precise. Here you'll use the parallax technique to determine the same distance to the nearby star, but using a camera and analysis software instead.

9. Take a picture from a digital camera (likely your phone camera) from the vantage point of each Earth position used above.
10. Upload your photographs to a computer. The simplest way to do this may be to email the images to yourself from your phone. Give each file a descriptive name (e.g. "`parallax_left_telescope`").

Finding the pixel scale

Notice that with the finger, I told you that your little finger, outstretched, is about 1 degree wide. This was a conversion between the linear size of your finger, for example 1 cm, to an angular size, 1 degree. With the camera, we need to find out the similar conversion — how many pixels in an image corresponds to what angular size, also known as the *pixel scale* of the image. To do this, you can take a known angular size and measure its length in pixels.

11. Find an object of known length and place it a known distance from the camera (distance to camera should be 10 times or more the length of the object). Take a picture of that object.

You will now convert the images from their default format (likely .png or .jpeg) into .fits files, a format commonly used by astronomers. This format will be readable by SAO Image DS9, an astronomical image analysis tool.

12. Convert the image files to a .fits format using your favorite image processing software, or the software "GIMP" (Gnu Image Manipulation Program), or an image conversion website like `https://www.files-conversion.com/image/fits`. For using GIMP, open the file. From the FILE menu, select EXPORT AS, change the file extension to ".fits," and then click EXPORT. Repeat this procedure for each of your images.

13. Open a saved .fits image of the pixel scale image in DS9. Your first task is to measure the pixel scale. From the menu at the top of the screen, select REGION, SHAPE, LINE. On the first row of buttons in the DS9 window, click EDIT then on the second row click REGION. Draw a line along the known length of the object. On the first row of buttons, click REGION then on the second row click INFORMATION. A window should pop up that will give you the length of the line in physical units, that is, in pixels. **Record this value in your lab notebook.**

14. Find the angular size of the known object. Since the object is far away, we can again use the small angle approximation for the triangle involved and find that the angular size of the object is equal to its length divided by the distance to the object. This angle is in radians.

15. Find the pixel scale by dividing the number of pixels in the length by the angular size of the known object. This gives the pixel scale in pixels per radian.

Finding the parallax angle

Now that you have the pixel scale of the image, you can use that to measure the parallax angle of the your nearby star and thus find the distance to that object like you did with the little finger method.

16. Open the first parallax image. Measure the displacement from the nearby star to the distant star. Convert that displacement to radians using the pixel scale. Make sure to record both the X- and Y- offsets. Repeat these measurements for the second parallax image using the same distant star.

17. Find the total angular distance the star moved between images. To do this, subtract the X-offsets from each other, and subtract the Y- offsets from each other. Then use the Pythagorean theorem to find the total distance moved ($d = \sqrt{x^2 + y^2}$). Divide this by two to get the parallax angle.

18. Use Equation 2.2 to find your new calculation for the distance to the nearby star.

19. Calculate the percent difference between this and the direct measurement like you did in the previous section.

2.5 Questions

These should be included in your lab report.

20. Your parallax measurements depend on an incorrect implicit assumption. What is this assumption, and how will it bias your results? How would you change the procedure in order to minimize this bias?

21. What were the primary sources of uncertainty? How would you improve the procedure for future measurements?

2.6 Report checklist

Include the following in your lab report. See Appendix A for formatting details. Each item below is worth 10 points.

1. The completed worksheet "The Parsec".
2. Work and final answer for your distance measurements using your finger, with uncertainty.
3. Work and final answer for direct distance measurement, along with percent difference.
4. A figure with your three images (pixel scale image and two parallax images).
5. The displacement vectors from distant star to nearby star.
6. Final determined value of the distance and comparison with the direct distance. Show your work (see Appendix A).
7. Answers to the questions in Section 2.5, with justification.
8. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

LAB 3

Measuring the distance to a galaxy using globular clusters

In this lab, we will use a globular cluster in the Milky Way called M5 (see Figure 3.1) as the first step of our distance ladder to other galaxies. By comparing this globular cluster to another in a nearby galaxy called M87 in the Virgo cluster, we will estimate its distance, adding another rung to our distance ladder. Ideally, we should compare many globular clusters in the Milky Way to M87, here we will use just one cluster, which is enough to demonstrate the principle.

If two identical lightbulbs are placed with one close to you and another farther away, the more distant one will appear dimmer. This is because the light from a spherical emitting source spreads out over a spherical shell that gets larger as the light gets more distant from the source. So if sources 1 and 2 have the same luminosity, then their distances d and apparent brightness b are related by

$$\frac{b_1}{b_2} = \left(\frac{d_2}{d_1}\right)^2 . \tag{3.1}$$

Since the numbers we will extract from the images are either in brightness or magnitudes, it is convenient to re-cast this relation in terms of magnitudes. Magnitudes m_1 and m_2 are related to brightness b_1 and b_2 by

$$m_2 - m_1 = 2.5 \log \left[\left(\frac{b_1}{b_2}\right)^2\right] . \tag{3.2}$$

Combining the two equations, we get

$$\log(d_1/d_2) = 0.2(m_1 - m_2)\,. \tag{3.3}$$

This says that once we have measured magnitudes m_1 and m_2 for two sources, then we can derive the ratio of their distances from us, *as long as they have the same luminosity.*

Figure 3.1: Messier 5 (M5) is a globular cluster (a gravitationally bound collection of stars) of more than 100,000 stars in the Milky Way Galaxy. Located at Right Ascension (RA) = 229.640°, and Declination (Dec) = 2.075°. The above image is 2.85 arcmin on a side, or about 1/20th of a degree. Image source: ESA/Hubble & NASA, http://www.spacetelescope.org/images/potw1118a/

3.1 Road Map

To keep track of the steps in this lab, we will fill in Table 3.1. In this table, the entry for the magnitude of M5 refers to the sum of all of its stars. In principle, we could measure this ourselves with the rooftop telescope, but for this lab we take a value from a catalog of such data. The SDSS data cannot be used because the stars are too crowded together for an accurate measurement.

The first step is to make a *color-magnitude diagram* for the stars in M5 to find a star that has similar to the Sun; we assume that such a star has the same luminosity as the Sun. The magnitude of the star (specifically its r-band magnitude) gets entered into the above table, and you derive the distance to M5.

Object	**Magnitude**	**Distance (AU)**
Sun	-26.89	1
Sun-like stars in M5		
M5 itself	5.65	
M87 globular clusters		

Table 3.1: Table of magnitude and distance.

The second step is to identify faint things surrounding the galaxy M87 that are likely to be globular clusters associated with it, and get their magnitudes (again the r-band magnitude) from the database. Some value that properly represents the ensemble gets entered into the above table and you derive the distance to M87 by comparing the magnitudes of M5 and M87.

To summarize: the distance to the Virgo cluster depends on two assumptions: 1) stars with Sun-like colors in the globular cluster M5 have the same luminosity of the Sun. 2) Globular clusters like M5 in the Milky Way have luminosities that are comparable to the globular clusters in M87. Neither of these assumptions is necessarily well justified based on information available to you, but there are checks that reassure us that the assumptions are good enough for at least a first estimate of distance.

3.2 Analyzing the M5 globular cluster

First you'll retrieve from an online database the magnitude of stars in the region of sky where M5 is. In the window at http://skyserver.sdss.org/dr13/en/tools/search/sql.aspx, enter the following query:

```
SELECT TOP 200
   objid,ra,dec,u,g,r,i,z
FROM Star
WHERE
   r BETWEEN 10 AND 23
   AND ra between 229.50 and 229.78
   AND dec between 2.2 and 2.3
```

Questions and results for your report

1. From the data above, create a .csv file, rename it M5.csv. Read it into a spreadsheet, and make columns for the colors $g-r$, $r-i$, and $g-i$. The Sun has colors $g-r=0.44$, $r-i=0.11$, and $g-i=0.55$. Plot the r magnitude vs. one of the colors (e.g. $g-r$), and reverse the r magnitude axis, since lower magnitudes represent brighter objects. On this color-magnitude diagram, identify the *main sequence* of stars. This plot is called a color-magnitude diagram, which is similar to an H-R diagram as seen in Figure 3.2.

2. Begin to fill out Table 3.1 with the magnitude and distance to a Sun-like star in M5. When you finish this lab and turn in this lab report, this table will be completely filled out.

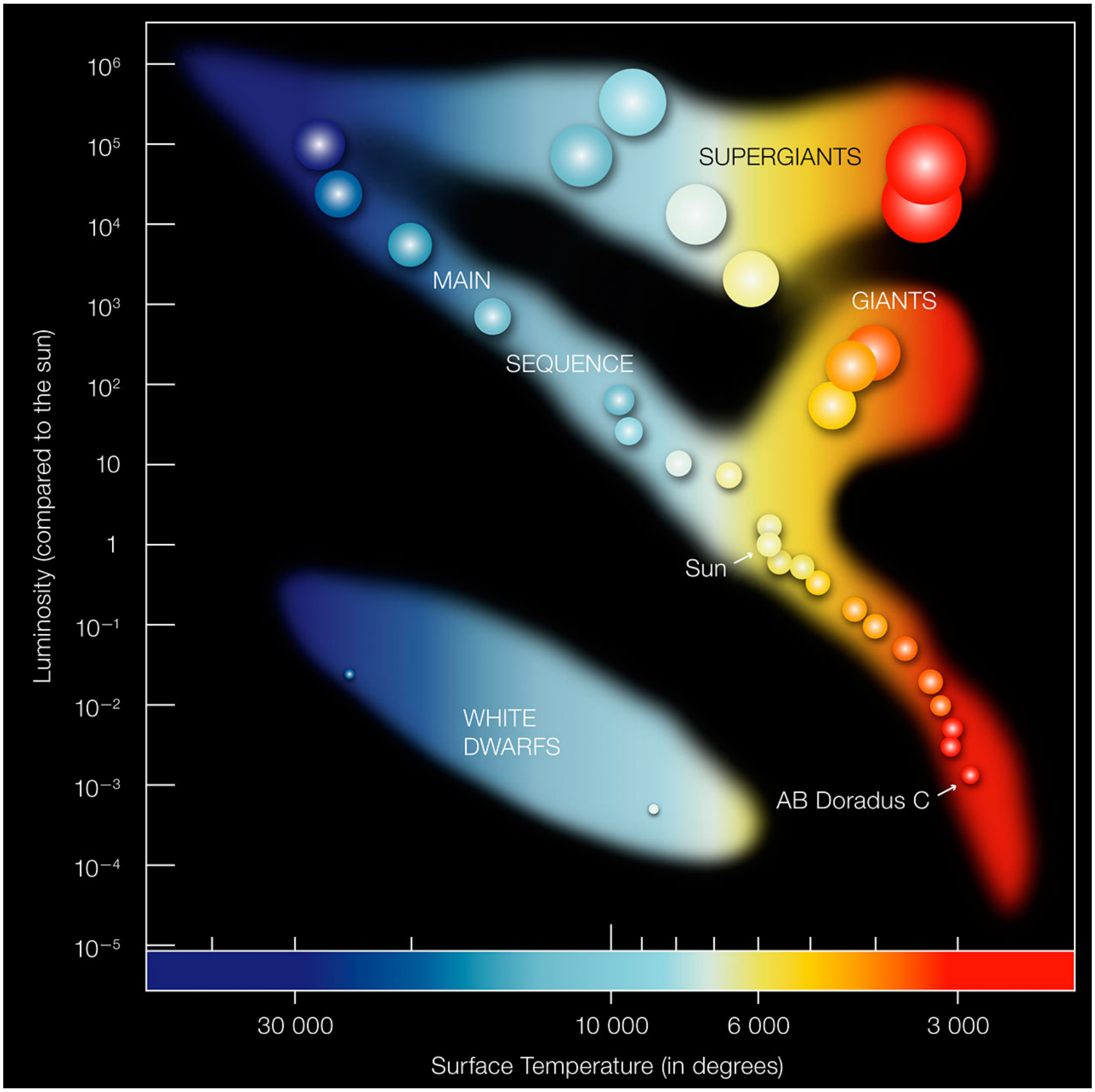


Figure 3.2: Herzsprung-Russell (H-R) diagram, plotting stars according to their luminosity and surface temperature. Luminosity is related to magnitude, and surface temperature is related to color. Image Source: ESO (https://www.eso.org/public/images/eso0728c/

3.3 Analyzing the globular clusters near the M87 galaxy

Figure 3.3 shows the field surrounding the giant Virgo galaxy M87, also known as NGC 4486.

The task is to find the magnitudes for the faint speckles surrounding M87 that are barely visible in Figure 3.3, namely its globular clusters. We set up a similar query to that used for M5, except of course the coordinates (RA, Dec) are different. Enter the following query:

```
SELECT TOP 200
   objid,ra,dec,u,g,r, i,z
FROM Star
WHERE
   r BETWEEN 10 AND 23
   AND ra between 187.591 and 187.821
   AND dec between 12.278 and 12.504
```

The globular clusters are so far away that each cluster of stars appears as, and is categorized as, stars in the SDSS database. As a cross-check, also run the above query in a random piece of sky at least 5°away.

Questions and results for your report

3. Make color-magnitude diagrams for both samples (M87 and random-sky) and compare them. Does either either color-magnitude diagram show any evidence for a correlation between brightness (magnitude) and color for the plotted points?

4. Once you have identified which of the sources on your M87 color-magnitude diagram can be identified with a population of globular clusters surrounding M87, argue which apparent magnitude should be selected to enter into the table, and do so. Why is there a range of magnitudes? How would you make this process more precise? What other sources of uncertainty do you think there are?

5. Calculate the distance to M87 using Equation 3.3, comparing its magnitude to that of M5. If you have not already converted AU to parsecs, do so now to get the distance in mega-parsecs ($1\,\mathrm{Mpc} = 2.06 \times 10^{11}$ AU). The accepted value for the distance to the Virgo cluster is 16.4 Mpc. From your uncertainties above, how well do these two values agree within your expected level of uncertainty? See Appendix B.3 for details of how to determine this.

6. Based on the distance you found, when did the light from the Virgo galaxy arrive at the sky survey's telescope? What was happening on Earth at that time?

Figure 3.3: Messier 87 (M87) is a nearby elliptical galaxy in the constellation Virgo. It is known for having a large population ($\sim 10,000$) of globular clusters, about 100 times more than the Milky Way Galaxy. Centered at RA=187.706°and Dec=12.391°, the above image is 97 arcminutes across. Image source: Chris Mihos (Case Western Reserve University)/ESO, http://www.eso.org/public/images/eso1525a/.

3.4 Include in your report

Each of the following items will be graded out of 10 points.

- Completed table
- Three color magnitude graphs
- Questions 1–2 from the first section
- Questions 3–6 from the second section
- A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

LAB 4

Galactic distances and the Hubble diagram

4.1 Introduction

In 1929, Edwin Hubble measured that distant galaxies were systematically redshifted relative to galaxies that were closer. From this data, Hubble inferred that the universe was expanding, an idea initially worked out by Georges Lemaitre using Einstein's theory of gravity.

In this lab, you will conduct a measurement similar to Hubble's and will produce your own version of his famous Hubble diagram shown below.

4.2 Building intuition

The graph in Figure 4.1 illustrates the impact of an expanding universe of photons emitted from distant objects. Because the speed of light is constant, photons that we measure today were emitted in the past, with photons originating from objects that are further away being emitted earlier in time. This means that photons from objects that are further away are older, and thus, those photons have experienced more expansion by the universe.

1. Sketch the plot from Figure 4.1, then sketch on the plot two more lines corresponding to the relationship for 1) a contracting universe, and 2) a static universe.

From this relationship, we can determine whether the universe is expanding, contracting, or static by looking at a number of galaxies and measuring their distance (corresponding to the horizontal axis) and the expansion experienced by their photons (corresponding to the vertical axis).

For this lab, we will use galaxy images and spectra listed in an online table.

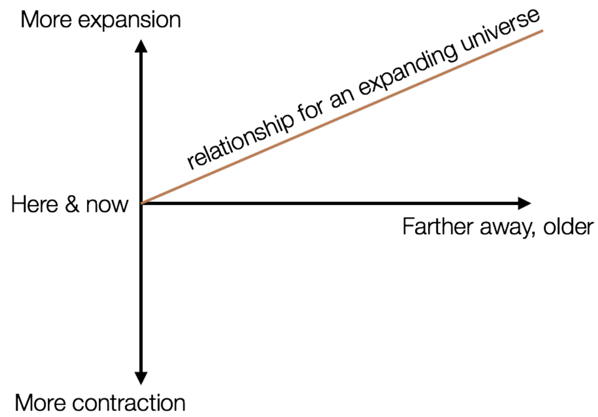


Figure 4.1: Schematic of a Hubble diagram plot. It illustrates the relationship between the expansion experienced by a photon and the distance of its emitter.

4.3 Measuring distance

We will use geometry to measure the distance of our galaxies. Galaxies that are closer will look bigger and will subtend a larger angle whereas galaxies that are further will looks smaller and will subtend a smaller angle. This relationship between the angular size of the galaxy and its distance is illustrated in Figure 4.2.

Our galaxies will all be nearly the same size (22 kpc). Using the geometry shown in the illustration, we can arrive at the following relationship:

$$\text{angular size} = \frac{22\ \text{kpc}}{\text{distance}}\,. \tag{4.1}$$

So, by measuring the angular size of our galaxy images, we can use the above equation to determine the distance to the galaxy.

From the Modules > Lab section of the Canvas site, download and extract to a folder `HubbleDataWebpage.zip`. In that folder, open `HubbleDataPage.html`. You will find a list of galaxy names. Click on the "Image" link for the first galaxy, NGC 1357. In the new tab, you will see an image of galaxy NGC 1357 similar to Figure 4.3.

2. What kind of galaxy is it (spiral, elliptical, unclear)? Note this in your spreadsheet.

3. Are there any noteworthy features in the image? Use your spreadsheet to record your answers.

We want to measure the angular size of NGC 1357, which you can do by measuring the angular separation between two appropriate points spanning the entire galaxy. In the lower left hand corner

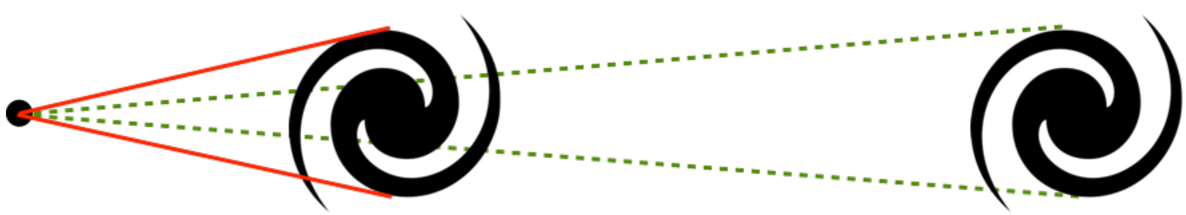


Figure 4.2: Looking from the dot on the left, there are two galaxies that are the same size, one further away than the other. The more distant galaxy subtends a smaller angle (dashed green lines) than the closer galaxy (solid red lines).

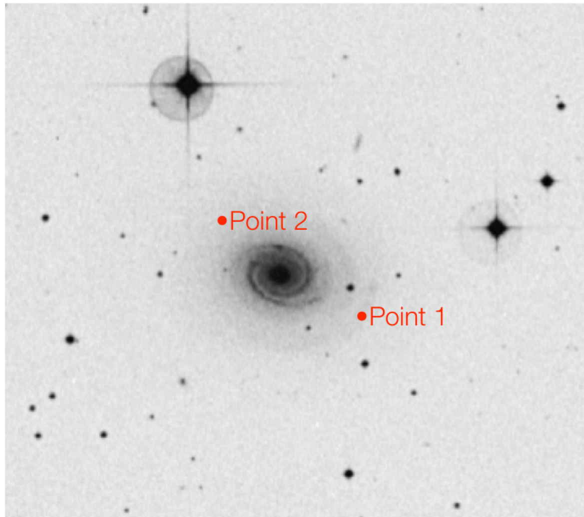


Figure 4.3: Example of galaxy image. Colors are inverted here, and Points 1 and 2 mark the furthest extent of the galaxy

of google skymaps, google shows you the coordinates of your cursor. Record the coordinates (RA and DEC) for two points spanning the galaxy. Then use an online calculator (e.g. `http://cads.iiap.res.in/tools/angularSeparation`) to calculate the angular separation between the points. Be careful and make sure that you don't choose points that are too far outside or inside the galaxy image.

4. Enter the value for the angular size (in radians, not degrees) in a spreadsheet.
5. Divide up the remaining galaxy images between your groupmates and repeat Steps 2–4 for each of the galaxies recording your notes and measurements in the spreadsheet.
6. Once you have measured the angular size of all the galaxies, use Equation 4.1 to estimate the distance for each galaxy and record the values in a "distance" column.

4.4 Measuring expansion

The wavelength of light changes as the universe expands, an effect known as cosmological redshifting. If the universe expands, the wavelength is stretched, becoming longer and redder. For a contracting universe, the wavelength will be compressed becoming bluer. We define the redshift, z, as

$$z = \frac{\lambda_{\text{measured}} - \lambda_{\text{original}}}{\lambda_{\text{original}}}, \tag{4.2}$$

where λ represents the wavelength. The redshift is a measure of how much the wavelength has been stretched or compressed.

We can measure the redshift by examining spectra (the energy emitted in different wavelengths) of the same galaxies we just measured. For NGC 1357, click on the link "Ca Spectra." The link will show spectra associated with Ca absorption which produces dips at wavelengths of 3933.7 angstroms and 3968.5 angstroms. You should see two prominent dips in the data. See Figure 4.4 for example spectra.

Go back to the galaxy list and click on the link "H-alpha spectrum" to bring up the spectra associated with Hydrogen alpha emission of light with wavelength 6562.8 Angstroms. You should see a clear peak in the data. The H-alpha peak is the leftmost of the distribution. Since we know the original wavelengths for these processes, we can compare the measured wavelength of these features with their original wavelength to determine how much the light has stretched.

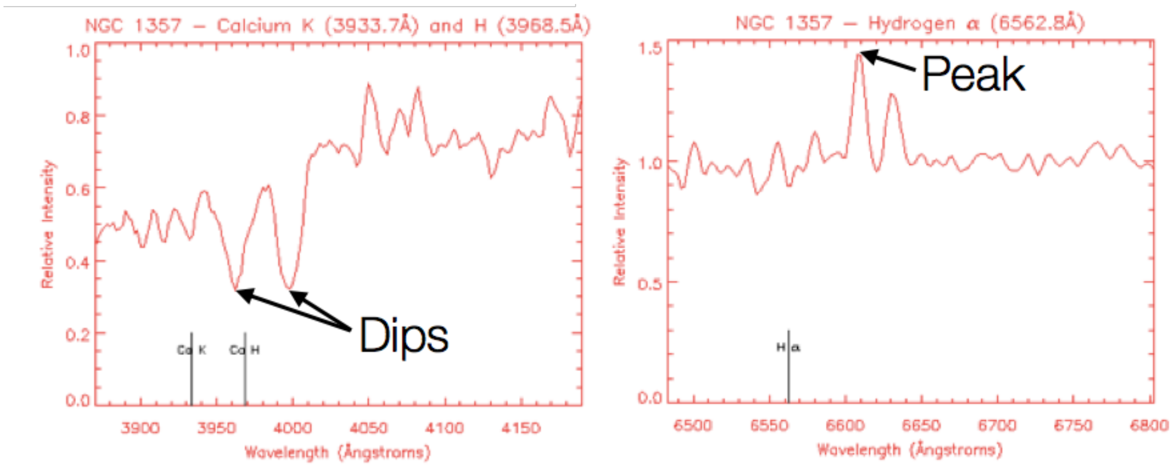


Figure 4.4: Spectrum of light detected from NGC 1357. The dips and peak that you will use to identify redshift are identified.

7. For each of the spectra estimate the value for the two Ca dips and the H-alpha peak. Record the values for the two Ca absorption lines and the H-alpha emission line in the excel worksheet.
8. Use Equation 4.2 to calculate the redshift for each of the lines and take the average to estimate the redshift for the galaxy. Record the redshift in an "average redshift" column in the spreadsheet.
9. Repeat this process for each of your galaxies.

4.5 The Hubble diagram

10. Using the measurements in your worksheet, make a plot of redshift versus distance for your galaxies.
11. Is there a trend in your data? Is the trend clear?
12. Compare your plot with the sketches from Step 1. Does your data indicate that the universe is expanding? Contracting? Static? Why?

Hubble's constant H_0 gives a relation between the recessional speed v of an object and its distance D, according to the equation

$$v = H_0 D\,. \tag{4.3}$$

You will determine Hubble's constant from your data. You have the distances of the galaxies already. To find the velocities, multiply the redshift by the speed of light c,

$$v = zc\,. \tag{4.4}$$

This equation is valid for low values of redshift.

13. Make a Hubble diagram by plotting velocity (in km/s) vs. distance (in Mpc).
14. Use this plot and Equation 4.3 to fit a line to the data and find Hubble's constant, which should be the slope of that line. If you are doing a linear fit, you need to force the y-intercept of the line to be zero. You may also need to specify that the fit equation be displayed on the plot.
15. Do some research on the history and background of Hubble's measurement. Write a one paragraph summary of this lab and discuss the following:

a) What was the historical context of Hubble's measurement?

b) Why was it important?

c) What did you do in this lab and how do your measurements and conclusions compare with Hubble's?

4.6 Report checklist and grading

Each item below is worth 10 points.

1. Data table
2. Sketch of predicted relations for expanding, contracting, and static universes
3. Your Hubble diagram (velocity vs. distance)
4. Your Hubble constant
5. Answers to Questions 11 and 15.
6. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

LAB 5

Modeling the Cosmic Microwave Background

In this lab, you will analyze the power spectrum of the cosmic microwave background (CMB). It is assumed that you have already heard of it. If you don't know what it is yet, search online and learn a little about it. Once you're back, continue with this lab.

1. Go to the CMB Analyzer simulation at `https://map.gsfc.nasa.gov/resources/camb_tool/index.html`.

This interactive simulation allows you to model the CMB angular power spectrum. You can change the different parameters that go into the model and see how it affects the spectrum. You can even change the parameters until your model spectrum matches the experimental spectrum. This would give you confidence that your chosen parameters are correct.

2. Play with the sliders and read through the text that appears when you hover on a slider. Move a few sliders around and observe how they affect the power spectrum.
3. To understand the CMB and the power spectrum, read through the "Parameters of Cosmology" here: `https://map.gsfc.nasa.gov/mission/sgoals_parameters.html`. There are 7 pages to this document, and you can navigate using the "next page" link in the lower right of the text. While you learn about each parameter, go to the CMB Analyzer and vary that parameter to become more familiar with it.
4. For each parameter that can be varied in the model, describe what the parameter means for the universe and how changing that variable changes the power spectrum. **Record this for your report.**

For complicated models like this, scientists learn about the universe, for example the proportion of matter, dark matter, and dark energy, by changing the free parameters in their model to see what best fits the observed data. If they trust that their model contains the right physics, then the parameters that fit best are likely to be correct.

5. Move the sliders to find the parameter values that make the model fit the experimental data best. **Take a screenshot of the best fit and record the values.**

5.1 Testing the Big Bang Theory

Here is a hypothetical observation that is *not* predicted by the Big Bang theory:[1]

> "evidence for an increase in the cosmic microwave background temperature with time"

6. Imagine what would happen if it were actually observed — whether it could be explained with the existing Big Bang theory, could be explained with a revision to the Big Bang theory, or would force us to abandon the Big Bang theory. **Write down your team's reasoning.**

   Note: you may want to assume these four roles for your discussion: *Scribe* — takes notes on the group's activities; *Proposer* — suggests tentative explanations to the group; *Skeptic* — points out weaknesses in proposed explanations; *Moderator* — leads group discussion and makes sure everyone contributes and no one is dominating the discussion.

5.2 Report checklist and grading

Each item below is worth 10 points. Every item except the last one can be identical between lab group members.

1. Parameter descriptions and effect on power spectrum (Step 4)
2. Best fit parameters and screenshot of best fit (Step 5)
3. Conclusion and reasoning for effects of hypothetical observation on Big Bang theory (Step 6)
4. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

[1] This discussion is from Bennett, Donahue, Schneider, Voit, The Cosmic Perspective, 9th ed. (2020)

LAB 6

The accelerating universe

6.1 Introduction

In 1929, Edwin Hubble discovered that the universe was expanding. At the end of the 20 th century, astronomers made another stunning discovery associated with the expansion of the universe. In 1998 two independent projects obtained results showing that the expansion of the universe was accelerating, a result that eventually led to Nobel prizes for Perlmutter, Riess, and Schmidt in 2011. The two teams used the same technique: measuring the distance and redshift of Type Ia supernova. In this lab, you will work through some supernova data looking for evidence of the accelerated expansion of the universe.

A supernova is essentially an exploding star. This “explosion” produces a tremendous amount of light which then slowly fades over a period of weeks to months. Measuring the light associated with the supernova over time produces what is known as the supernova light curve. You can use the following link to build some intuition about supernovae and their light curves: https://youtu.be/TY6Y5_7xQ8o. An example of a supernova light curve is shown in Figure 6.1. The vertical axis is in “magnitudes,” the standard astronomical measure of brightness where a larger magnitude corresponds to a fainter object.

Supernova are used to measure distance in a manner similar to how we measured distance in the Hubble lab. In the Hubble lab, we used the fact that objects that are father away look smaller, that is, they subtend a smaller angle on the sky. So, if we know the physical size of the object, we can measure the object’s angular size and from that determine its distance. For supernova, astronomers utilize the fact that objects that are closer, look brighter, and objects that are farther away, look dimmer. So, if we know the intrinsic brightness of an object (called its absolute luminosity), we can then compare that to our measured brightness (called its apparent brightness) to determine the object’s distance. This concept for measuring distance is illustrated in Figure 6.2 showing how an object of a known brightness (or size) looks fainter (smaller) when it is farther away.

The 1998 supernova teams studied Type Ia supernova because it is possible to use the shape of the supernova light curve to deduce the intrinsic brightness of the supernova. By comparing our inferred intrinsic brightness with our measured apparent brightness, we can then use the supernova to measure the distance.

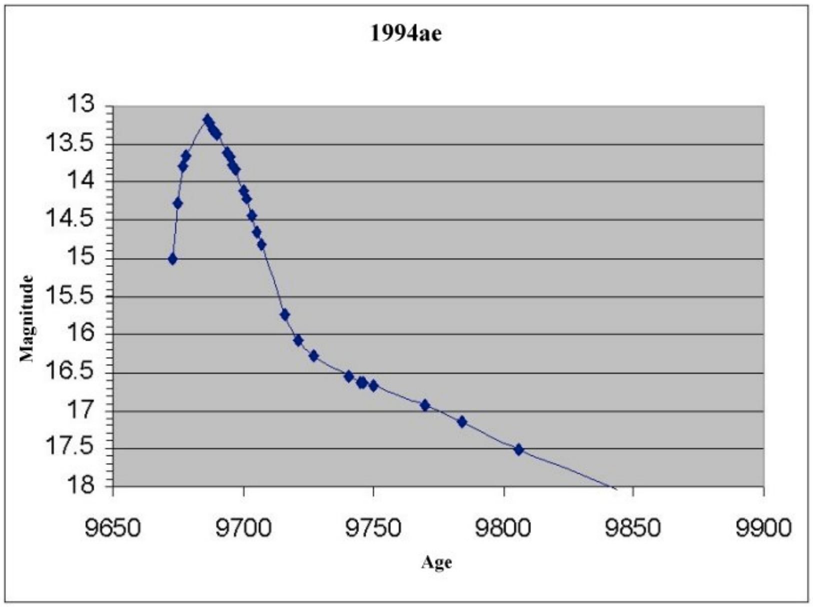


Figure 6.1: Light curve for supernova 1994ae. The age is given in days.

Figure 6.2: A method to determine the distance of an object. For objects with the same absolute luminosity, those are are further away will appear dimmer.

6.2 Making a supernova Hubble diagram

The first activity for this lab is to make a Hubble diagram, like in the Hubble lab, using Type Ia supernovae. Then you will analyze your Hubble diagram to qualitatively measure cosmic acceleration. Your data is in the Excel spreadsheet (available on Canvas in the Lab module in the compressed file DarkEnergyLab.zip) named SNe_Reiss_2004, and consists of distance moduli and redshifts for 186 supernova from Reiss, et al., 2004.

As discussed above, the apparent magnitude of the supernova can be used as a measurement of distance if we know the intrinsic brightness (also called absolute brightness) of the supernova. In astronomy, we call the difference between apparent and absolute brightness the "distance modulus," μ, which is defined as

$$\mu = m - M\,, \tag{6.1}$$

where m is the apparent brightness (in magnitudes), and M is the absolute brightness (in magnitudes). The distance to the object is related to the distance modulus by

$$\text{distance} = 10^{(\mu-25)/5}\,\text{Mpc}\,. \tag{6.2}$$

1. In the spreadsheet, in a new column, calculate the distance for each supernova from the supernova's distance modulus.

After determining distances for objects, the next part of the Hubble diagram involves making a measurement of the expansion of space. As with the Hubble lab, we will estimate the expansion using the redshift distortion of an object's spectrum. Recall that we measured the redshift by looking for a specific spectral feature (either a dip or a peak) which occurs at a known wavelength. A larger redshift indicates a faster recession velocity.

A constant expansion rate for the universe means that the amount of expansion is proportional to the amount of time. Mathematically, we can write this as

$$\text{expansion} \propto \text{time}\,. \tag{6.3}$$

As expected, light emitted by more distant (and therefore older) objects will undergo more expansion. This expansion leads to the redshifting we discussed above. So, we can write that for a constant expansion rate, the relationship between redshift and distance is

$$\text{redshift} = \frac{H}{c} \times \text{distance}\,, \tag{6.4}$$

where c is the speed of light (300,000 km/s) and H is called the Hubble constant. This relation is known as "Hubble's Law." It describes a linear relationship between redshift and distance, the two axes of the Hubble diagram.

We can measure the expansion rate of the universe by using our Hubble diagrams and fitting the data to Hubble's law. The slope of the line that fits the data will give a measurement of the Hubble constant, H, which parameterizes the expansion rate of the universe.

2. In the spreadsheet, sort data from closest to furthest. Then separately fit linear relationships to the 40 nearest and 40 furthest supernovae to obtain two Hubble constant measurements (Note that in Equation 6.4, the y-intercept is set to zero, so be sure to do this in your fit as well). Plot both linear relationships, as well as the data being fit, on a single graph. In order to see both sets of data more easily, set both axes to log scale to make a log-log plot.

3. What value of the Hubble constant best fits the data for the 40 closest supernova?

4. What value of the Hubble constant best fits the data for the 40 farthest supernova?

5. How do these two results show that the expansion rate is accelerating?

6. If the expansion rate is accelerating, what do you predict you should see if you fit data for 40 supernovae at intermediate distances?

7. What value of the Hubble constant best fits the data for 40 intermediate distance supernovae? Add this to your plot.

6.3 Finding and measuring supernovae

Supernova are random events. In order to use supernova for cosmological studies, astronomers must find them first. In this final section of the lab, you will look at real data from the Dark Energy Survey. You will search for a supernova and measure its light curve. Dr. Daniel Scolnic, a former KICP fellow at the University of Chicago, has graciously provided the images for this section.

8. In the file you downloaded earlier, find the folder "SNe_search" and open the two image files SNe1_search.jpeg and SNe2_search.jpeg. Compare these two images. These images correspond to pictures taken of the same patch of sky on two different nights. A supernova is in one of the images. Can you see it?

In general, it is difficult to find supernova in a raw image of the sky because of all the other objects in the image. Take a moment to look at the different objects. Almost all of them correspond to stars and galaxies.

Since the majority of objects in an image of the sky are not supernovae, astronomers can try to remove them by generating a template image for that patch of sky and then subtracting it from the image. Once these non-supernova objects are removed (or mostly removed), it becomes easier to search for supernovae.

9. Open the file SNe1_template.jpeg and compare it to SNe1_search.jpeg.

SNe1_template.jpeg is the template file and it should look very similar (though not exactly the same) to SNe1_search.jpeg. Subtracting the two yields a "difference" image which is file SNe1_diff.jpeg.

10. Open SNe1_diff.jpeg.

You should notice two things, 1) most of the features are now gone, and 2) the subtraction is imperfect. The imperfect subtraction introduces some artifacts in the differenced image such as the spiderweb-like patterns and perfect geometric shapes (like uniformly dark or bright squares and circles). You will note that most of the artifacts from the imperfect subtraction are in locations where there were very bright objects in the original image.

A supernova in the difference images will look like a round solid blob that is very bright. Because a supernova is transient, the brightness of the blob will not be the same in all images. In fact, there should be a few images where there is no blob at all.

11. Open the two difference images SNe1_diff.jpeg and SNe2_diff.jpeg. Compare these two difference images and identify the supernova. Remember, the supernova will look like a bright solid round blob in one of the images.

Once you have found the supernova, the next step is to measure the supernova light curve. We will do this using the DS9 software on your computer.

12. Open DS9 and press the "File" button to bring up the file menu. Press "open" and open the file 1-25-2014.diff.fits. This is the original file for the image with the supernova above.

In DS9, you will need to press the "scale" button followed by the "zscale" button so that the image will appear properly. You can drag the box to look more closely at different parts of the image.

13. Play around with the settings associated with the zoom, scale, and color buttons and examine different parts of the image.

14. Find the supernova in this file (it is in the same location as in the earlier jpeg images). As you move the mouse around on top of the supernova, the "value" field will show you the value of the pixel underneath the mouse. Find the location on the supernova where this value is maximaland record the (X,Y) coordinates (the numbers for Image X and Y) below. We will use these (X, Y) coordinates as the coordinates for the supernova. Also record the pixel value.

15. To measure the supernova's light curve, load up each of the other *.fits files. Move the mouse to the (X, Y) coordinates you determined for the supernova and record the date and pixel value in your spreadsheet. Remember, the supernova is a transient object and it may not be present in all images. The filename tells you the date the image was taken.

16. In the spreadsheet software, make a plot of the pixel value versus time. Make sure you use the date of the image to put your data in chronological order and to determine the time between each of the images.

6.4 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation.

1. Your Hubble diagram with all three sets of 40 supernovae and the three best-fit lines with their equations.

2. Your three determinations of the Hubble constant (near, far, and mid), with work showing how you found them.

3. Answers to questions 5 and 6.

4. The table and plot of your experimental supernova light curve.

5. Write one paragraph summarizing this lab. What is the significance and historical context of the discovery of cosmic acceleration (also called Dark Energy)? Why was it important? How does your analysis of the data in this lab demonstrate that the expansion rate of the universe is accelerating?

6. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

APPENDIX A

Lab Report Format

A.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.

A.2 Organizing the report

The report should follow the sequence of the report checklist. Answers to questions and inclusion of tables and figures should appear in the order they are referenced in the manual. In general, include the following:

- For any calculations that you perform using your data, and the final results of your calculation, you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92\,\mathrm{m/s^2})t + 0.2\,\mathrm{m/s}$."
- Answers to any questions that appear in the lab handout. Each answer requires providing justification for your answer.

A.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like "distance (pc)". This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like "distance (pc)".
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called "error bars", along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.

APPENDIX B

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, "$5 \pm 1\,\mathrm{m}$" means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters[1]. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

B.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument's scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment's manual often gives the instrument's resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

[1] The phrase "most likely" can mean different things depending on who is writing. If a physicist gives the value and does not given a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet, or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}}\,, \tag{B.1}$$

where $\{x_1, x_2, \ldots, x_N\}$ are the measured values, $\bar{x}$ is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called "standard deviation of the mean", which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}\,. \tag{B.2}$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as $52 \pm 3\,\text{m}$.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn't change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

B.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction)}, \ \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{B.3}$$

$$\text{For } c = ab \text{ (or for division)}, \ \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \tag{B.4}$$

$$\text{For } c = a^n, \ \frac{\delta c}{c} = n\frac{\delta a}{a} \tag{B.5}$$

For other calculations, there is a more general formula not discussed here.

Expression	Implied uncertainty
12	0.5
12.0	0.05
120	5
120.	0.5

Table B.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π, is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table B.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value "`0.0322857142857143`". Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as "`9.65683578099600e-04`", where the '`e`' stands for "times ten to the". I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as "0.032 ± 0.001 km/s". You may also see the equivalent, more succinct notation "0.032(1) km/s".

B.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced "tee prime"). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as[2]

$$t' = \frac{|a-b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \tag{B.6}$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

[2] Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.