Laboratory Manual

PHSC 12700 Stars

The University of Chicago

Autumn 2020

Labs

LAB 1

Bending light to see into space[1]

Optical telescopes are one way that astronomers use to better observe the cosmos. In this lab, you will build up your understanding about how telescopes help us do this.

1.1 Learning Goals

- Learn how light behaves when traveling between mediums.
- Create an image with a lens.
- Configure a refracting telescope and explain how it helps for observing the sky.

1.2 Equipment needed

- Some sort of clear container with parallel sides that you can fill with water (e.g. Tupperware, fish tank)
- Two transparent hollow cylindrical containers (e.g. drinking glass, Nalgene bottle, Mason jar) of different diameters
- A small bright disperse light source (e.g. cell phone light, pen flashlight, NOT a laser pointer)
- A cardboard or similar box that you can cut a small slit into so that you only let a thin beam of light pass through it
- Something to measure distances with (e.g. meter stick, measuring tape, standard-sized objects you can look up the dimensions for like coins, credit cards, paper. You can even use an online ruler on your phone, e.g. https://www.piliapp.com/actual-size/cm-ruler/)

1.3 Team roles

Decide on roles for each group member. The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded

[1] Initial lab written by Brent Barker. Home measurement setup, photos, and text provided by Chris Varney.

- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

these roles can (will?) rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role.

1.4 How do we use materials to bend light?

We use lenses all the time to shape the path that light takes, either with eyeglasses or with optical telescopes. These activities are intended to help you understand how we use materials to bend light.

Observing light refraction at home

1. You need an object that will refract the light. Find some sort of square or rectangular clear container (rounded edges are fine) and fill it with water. Small Tupperware is perfect for this.
2. We need to create a beam of parallel light. Find something like a cardboard box (anything that will not let any light through and that nobody will mind if you cut chunks out of it). In the middle of one edge, cut a thin (~ 1 mm thick) slice 7–10 centimeters tall (we can cut it a little further later if we need). If you are clever about it, you can find a way to make this "wall" stand up on its own. Figure 1.1 shows the one Chris made out of a box from a microwave dinner. They stuck a single staple in the bottom corner that you can see to hold everything together so that it would hold itself upright.
3. Set your container on a sheet of paper and set it just beyond the slit. Set it up so that the light in incident to your container at an angle. A correct setup might look like Figure 1.2.

   Note that to get such a clear beam Chris had to tilt their light source slightly downward. They also needed to build their slit a little taller than they originally had made it. Try to find something that works for you. **Take a picture of your setup to include in your report.**
4. Play with the angle at which the beam strikes the prism and observe what happens to its direction as it enters the prism (ignore the reflections or transmission through the other side, for now). How does the path of the laser beam change when it enters the prism? **Record your observations. Be specific about any patterns you notice.**

Careful observation with a simulated prism

5. It's hard to see the light in the prism, so let's use a simulation. Go to `https://phet.colorado.edu/en/simulation/bending-light` and select the play button to launch the simulation. Select the leftmost "Intro" box. This is a side view of the interface between two materials, currently air on the top half of the screen and water on the bottom. A laser is positioned above the interface.
6. Play with the controls on this screen until you have an idea of how to move the laser, turn it on and off, and adjust the materials on the top and bottom. To reset the simulation, select the orange circular button on the lower right.
7. Observe what happens to the laser beam when you change the a) angle, b) type of material on top and bottom, and c) indices of refraction. In each case, does the beam get deflected from its original straight-line path more, less, or the same amount? **Record your observations for your lab report in a table format.**

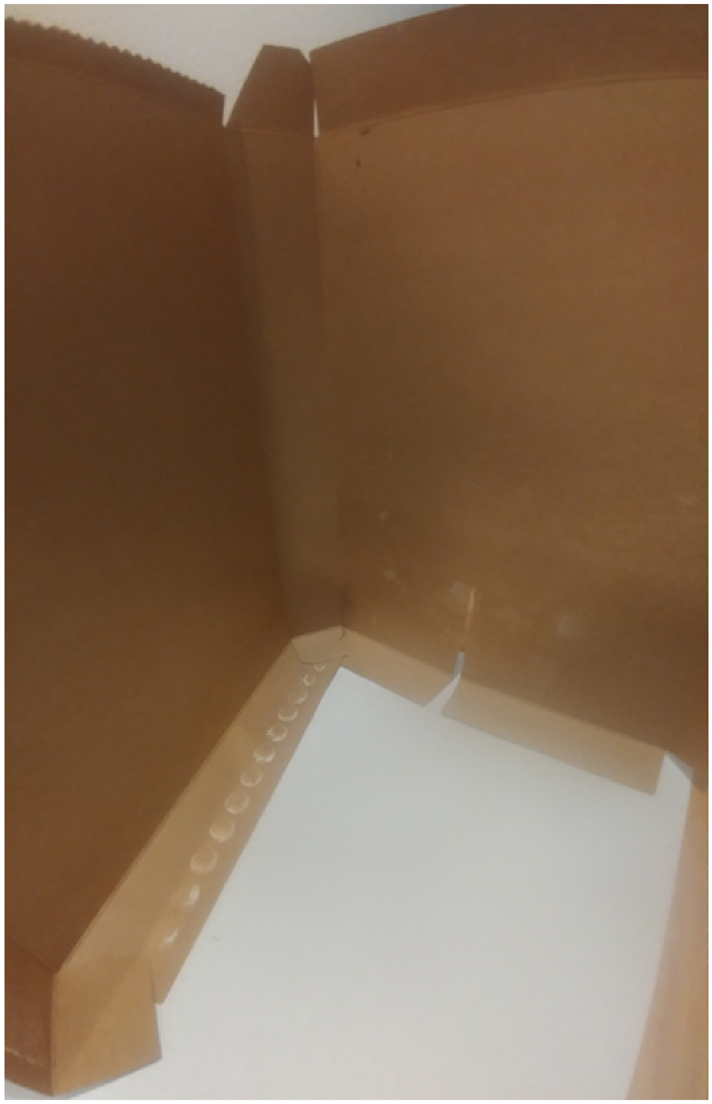


Figure 1.1: Thin slit cut into a cardboard box.

8. Open the second screen at the bottom of the sim. Lenses are wider in the middle and thinner at the top and bottom, and we can model this with a circular prism. Drag a circle up and shine a ray through it. Set the laser to output several parallel rays and aim it so they hit the middle of the circle. **Record your observations of what the rays do when the exit the other side of the prism.**

This setup of parallel rays is convenient for seeing precisely what the lens does. It also happens to be the situation when we observe things that are very far away compared to length scales of the lens. Consider two stakes driven into the ground next to each other, both perpendicular to the ground (and thus pointing directly at the center of the Earth). Since the Earth is a sphere, those stakes can't actually be both pointing directly toward the center of the Earth and also parallel to each other. For the former to be true, they must be angled slightly away from each other. But since the distance between them is so short compared to the distance away from the Earth's center, they are effectively parallel. *This is the same with light arriving from distant objects like stars.*

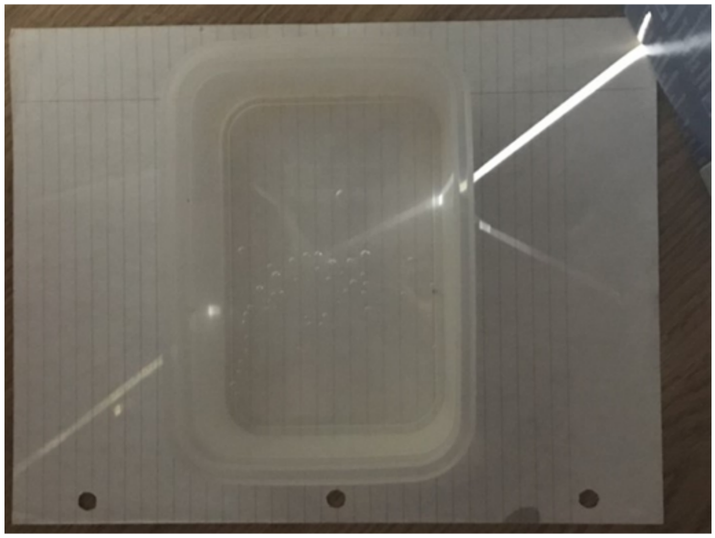


Figure 1.2: A ray of light enters one side of the water-filled container and exits the other side.

Imaging with a real lens

Goal: Use a real lens to create an image, and calculate the focal length of the lens.

9. Fill one of your cylindrical containers with water. Note that it needs to be as smooth and cylindrical as possible – tapered or textured edges will distort the image and affect your results. Place it about 30 cm away from a wall or some sort of screen that you can project an image onto. Move your small light source to be several container radii away from the container, on the other side from the wall/screen. Shine your small light source through the container "lens" so that the light should shine through it onto the wall/screen behind it. Move the container back and forth until you see a clear image on the screen. You should notice a thin vertical line of light that comes into focus if you hold the light at a specific distance from the lens. The point where this line is the thinnest is the point at which it is focused. **Draw a sketch of your setup, labeling the parts. Take a picture of your setup and of the image, if possible.**

If the object (light source) is a long distance away from the lens, compared to the distance from the lens to the image, then this intersection distance is called the focal length of the lens. The focal length is a property of the lens, based on its material and curvature. If an object is a long distance away compared to the focal length, then its image is formed at the focal length (if the object is closer, the image is formed further away than the focal length).

This principle works in reverse too — if an object is placed at the focal length of the lens, the rays come out parallel on the other side. The image is effectively formed an infinite distance away.

10. Design and conduct an experiment to measure the focal length of the lens you just used. Decide as a group how to measure, and how to estimate an uncertainty for your measurement. See Appendix A for detailed information about estimating uncertainty. **Record a sketch of your setup, a description of your procedure for gathering and analyzing your data, and the data itself, including your value of focal length with its uncertainty.**

This lens setup is great for producing images, for example to record onto photographic film or a digital camera's image sensor. It's less good for looking through the lens to magnify and gather more light the way we want to with a telescope. For that, we'll need at least two lenses.

1.5 Your first telescope

Telescopes come in many different configurations. Here you'll construct one that is simple by comparison, a refracting telescope, using just 2 lenses.

11. Here's the principle for building this telescope: the image created by the first lens, called the objective lens, is the object for the second lens, which is called the eyepiece lens. The thing we are wanting to look at (the object for the objective lens) is far away. We want the image created by the eyepiece lens to be an infinite distance away on the near side (just trust me on this). **Given these design goals and the information about lenses above, where should the two lenses be positioned with respect to each other? Sketch your proposal, labeling each lens and drawing the focal lengths of each lens.**
12. Acquire a second "lens" (cylindrical container filled with water), one with a different radius than the first one.
13. Find the focal length of this lens the same way you found the length of other one.
14. Construct your telescope by positioning the two lens on your surface according to your design.
15. Test your telescope. Look at a distant (across the room) object through it. If you don't get a sharp image of it by looking through the eyepiece lens, iterate on your design until you get it.
16. Find the magnification of your telescope. Hint: if you have two identical objects, how close does one need to be to look the same size/distance as the one you see through the telescope? Experimentally determine this magnification factor (1 is no magnification, 2 means it looks twice as close, 0.5 means that it looks twice as far away, etc), and estimate the uncertainty of your magnification. **Record this.**
17. Magnification should be related to the properties of the two lenses. Make up a formula that relates the focal lengths of your lenses to the magnification of your telescope. You may need to switch the lenses around or use different ones to test your formula. **Record this.**

1.6 Report checklist and grading

Each item below is worth 10 points. See Appendix B for guidance on writing the report and formatting tables and graphs.

1. Picture of setup and detailed observations from Steps 3–4.
2. Sketch and picture of your setup and the image formed by your lens in Step 9.
3. Detailed observations from Steps 7–8.
4. Sketch and picture of your setup and procedure for finding the focal length in Step 10, as well as value, with uncertainty, of the focal length.
5. Detailed sketch of your working telescope design, from Steps 11–15.
6. Experimental determination of the magnification of your telescope, with uncertainty.
7. Formula relating the focal lengths to the magnification.
8. Discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

9. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

10. Write a paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps?

LAB 2

Answering questions with astronomical images

Last week, you learned how we use lenses and telescopes to bend light. This week, you will analyze images that were taken by a research-grade robotic telescope, learn how we record these images, and learn one way to analyze those images to create knowledge about the distance between stars.

2.1 Learning goals

- Answer questions by analyzing data that you took yourself.
- Experience and analyze the light gathering and angular resolution benefits of telescopes.
- Use pixel scale and length measurements to determine the angular separation of objects in a digital image.

Other skills learned

- Calculate pixel scale, plate scale from an image of known objects
- Measure distances with DS9
- Convert between angular separation and length
- Use trig functions with triangles for the above conversion

2.2 Lab Team Roles

Decide which team members will hold each role this week: facilitator, scribe, technician, skeptic. If there are three members, consider distributing the first three roles, then adding the skeptic role to someone as a second role.

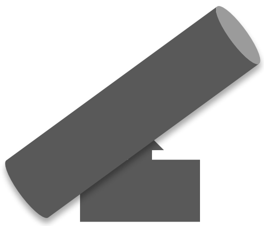
2.3 Telescopes and angular size

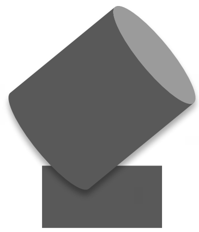
To gain some intuition for the functions of telescopes and for the concept of angular size, **complete the worksheets on the next 4 pages**. For ease, you can answer them in your lab report instead of drawing right on them, if you'd like.

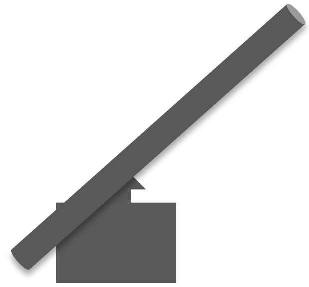
Telescopes 14

It is difficult to image doing astronomy without a telescope. The three main tasks of a telescope are to: (i) gather and focus light from distant objects; (ii) see fine details; and (iii) magnify nearby objects

LIGHT GATHERING POWER: The ability of a telescope to gather and focus light from distant objects is closely related to its diameter.


Telescope A


Telescope B

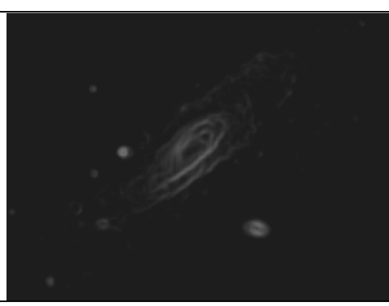
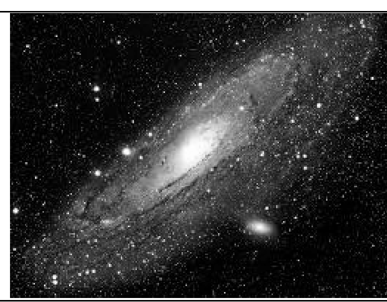

Telescope C

1. Rank order these telescopes (A, B, and C) from greatest to lowest light gathering power.

Greatest Light Gathering Power	_____ _____ _____	Lowest Light Gathering Power

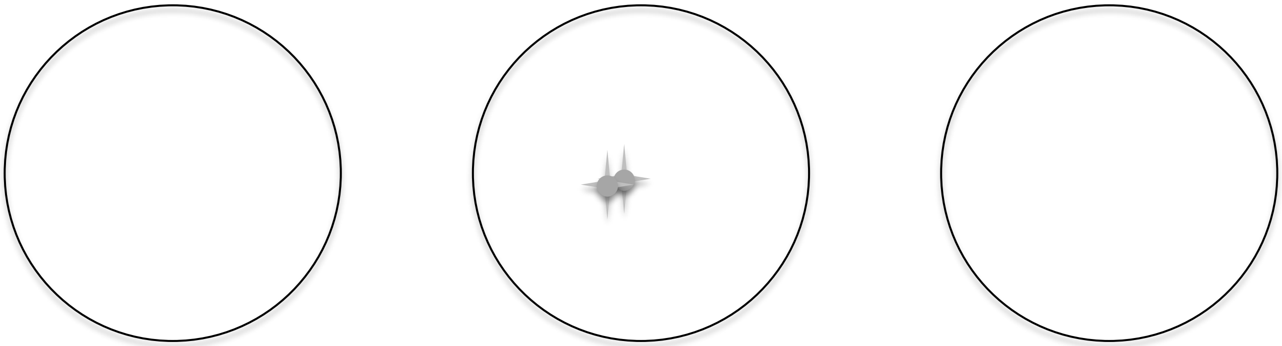
2. A telescope's light gathering power is largely based on its total collecting area, which can be calculated with the simple formula, πr^2, where "r" is the radius. How much more light gathering power does an 8-m telescope compared to a 2-m telescope?

SEEING FINE DETAIL: Better telescopes are able to resolve fine detail.

		
Picture from Tiger Telescope	Picture from Canary Telescope	Picture from Gomez Telescope

3. These three pictures are of the same galaxy of stars, taken by three different telescopes. Circle the one with the greatest ability to resolve fine detail.

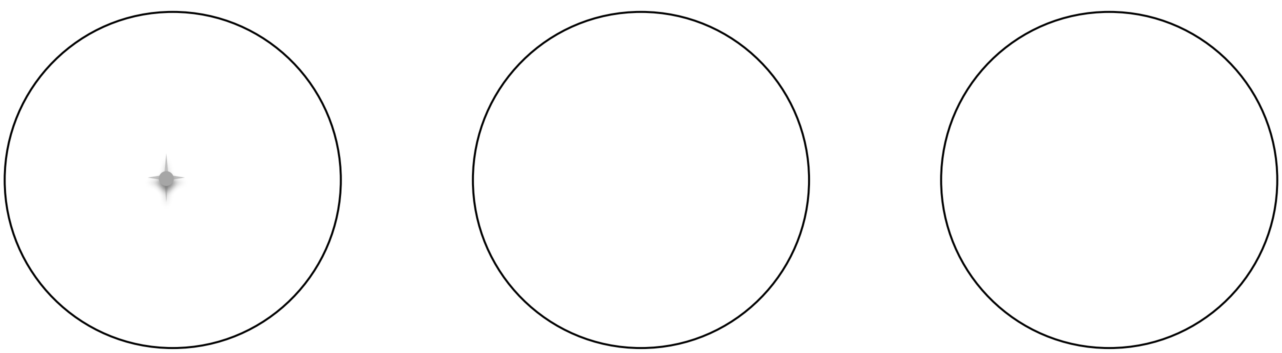
4. Below is a sketch of how a binary star system looks through a telescope—where two stars are found very close together. On the left, sketch what a lower resolution would look like, and on the right, a higher resolution.



Picture from Green Telescope (*low resolution*)	Picture from Red Telescope (*medium resolution*)	Picture from Blue Telescope (*high resolution*)

MAGNIFICATION: The least valuable part of a telescope is its ability to magnify. This is because even the largest stars are so far away, that most stars will still look like the same tiny pinpoints of light, regardless of the telescope.

5. Here is a sketch of a star as seen in a low magnification telescope. In the remaining two circles, first sketch what a medium magnification would look like, and then on the right, a higher magnification would look like.

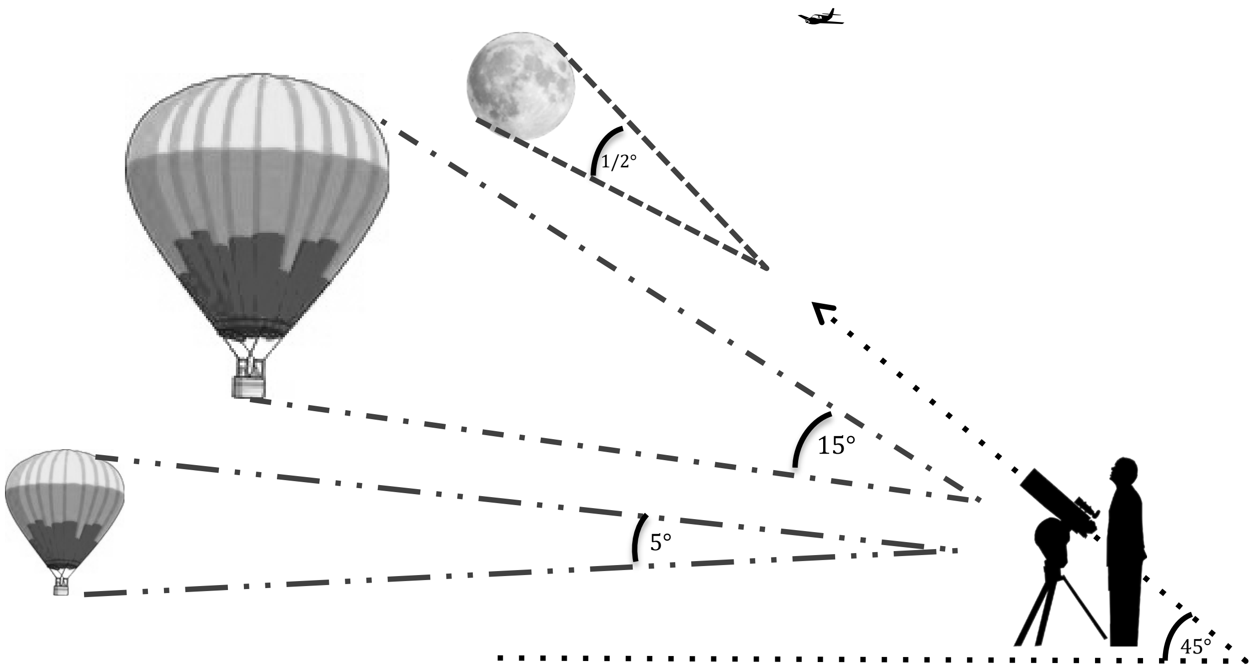


Sketch from a low magnification telescope	Sketch from a medium magnification telescope	Sketch from a high magnification telescope

Your brain normally wants to think that bigger—more magnification—must be better. In many cases that is true. However, in astronomy, we're usually looking at very distant objects whose size is too small to expand. Worse, by using magnification we can actually spread out the little light that is being captured, making the star harder to see.

Angular Size

When talking about how big objects appear in the sky, it doesn't make sense to say that the Moon is 2-in across or 6-m above the horizon. Instead, astronomers use angular sizes, measuring in degrees. Something that takes up the whole sky would be 180°, and something that takes up half the sky is 90°.



What is the apparent angular size or height for each of the items shown in the Figure above?

SMALL SIZES: In astronomy, many sizes and distances are smaller, and can be fractions of a degree.

In using angular measures, we often subdivide a degree of arc into 60 minutes, and subdivide a minute of arc into 60 seconds of arc.

1. Altitude of the Full Moon above horizon?

Circle one: ½° 5° 15° 45°

2. Apparent angular size of Full Moon?

Circle one: ½° 5° 15° 45°

3. Apparent angular size of nearby 5-story tall, hot air balloon?

Circle one: ½° 5° 15° 45°

4. Apparent angular size of 5-story tall, hot air balloon off in the distance?

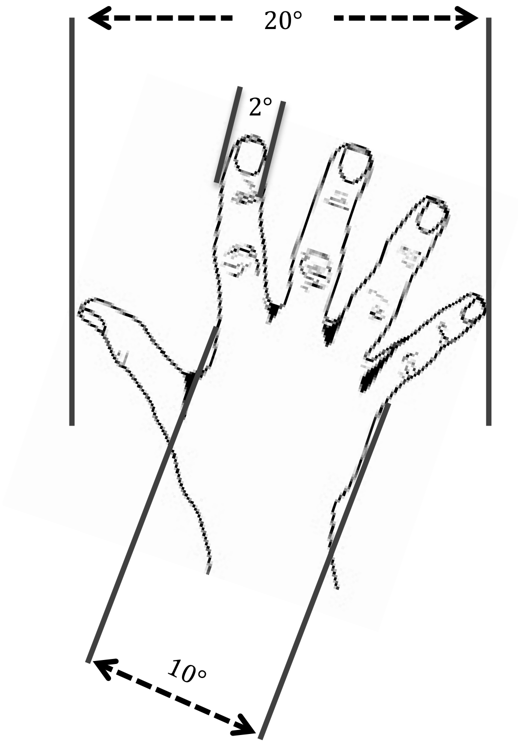
Circle one: ½° 5° 15° 45°

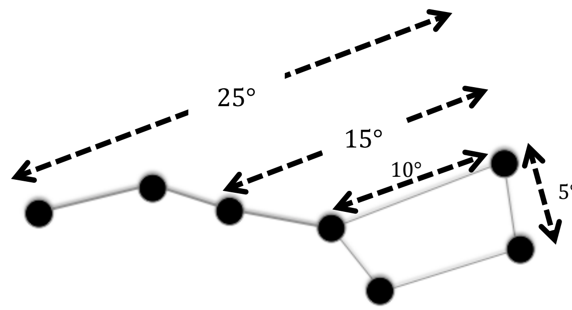
5. If the Full Moon extends about ½ of arc, how many minutes of arc is this?

Circle one: 30' 50' 3600"

6. The 20-m (60-ft) long airplane flying in the distance appears to be about ½ the size of the much larger 3,500 km (2,000 mile) diameter Moon. What is this airplane's apparent angular size?

Stretching out your arm as far as it will go, you can make angular size estimates using your hand as a sort of angular ruler. (*You can also do this with the Big Dipper, as shown at right.*)





7. What is the angular size of a cell phone at arm's length? (*measure the longest side*)

8. How big does that that same cell phone appear from about 5 feet away?

9. How big does the cell phone appear 10 feet away?

10. What is the angular size of the nearest window or door?

2.4 How big is your face?

This is a simple question, so that you can learn some astronomical concepts and tools while answering it and still keeping the subject matter intuitive.

Background: reflecting telescopes and CCDs

The primary utility of a telescope is its ability to gather light, thereby enabling visualization and analysis of the faint astronomical objects we are trying to observe. This requires focusing light incident on a large surface area. We will be using a **reflecting** telescope, which means that light rays from observation targets are focused into an eyepiece or onto a detector with reflecting mirrors. This is in contrast to refracting telescopes, which use refracting lenses to focus light rays. Figure 2.1 shows schematically how this kind of telescope works.

To take images during this lab with your smartphone or digital camera, and for the images taken by the Stone Edge Observatory telescope, we make use of a **Charge Coupled Device (CCD)**. CCDs are the standard detectors for taking astronomical images at wavelengths blueward of (lower wavelength than) approximately 1 micron. Think of the observatory's CCD a very advanced low-noise digital detector, not wholly dissimilar from the digital detector in your smartphone. Every time a photon within a certain energy range hits the detector, an electron is knocked off of the incident pixel, charging that pixel's capacitor. Thus, for each pixel, more photons $\implies$ more electrons $\implies$ more charge, and the charge can be read off into a digital signal that is then processed as an image.

Finding your pixel scale

To find out how big your face is the same way you might find out how big a planet is, you need to know the pixel scale of your camera, which is what angle a single pixel covers (this quantity is also your optical system's maximum angular resolution). A simple way is to take an image of something with known angular size, then divide that by the number of pixels it takes up.

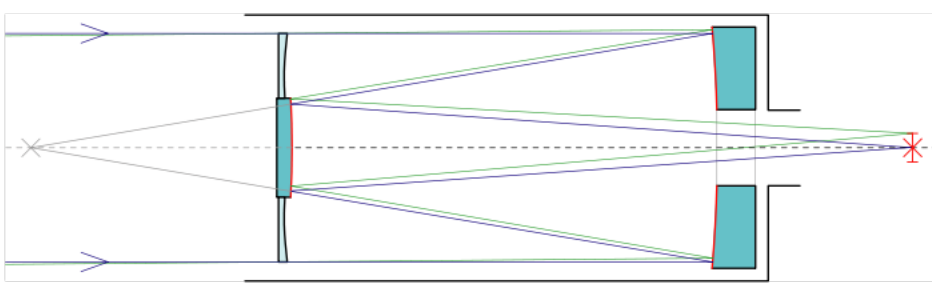


Figure 2.1: Schematic for a reflecting telescope of a Schmidt-Cassegrain design. Light rays from astronomical objects enter the telescope in parallel because their source is effectively at infinity. They are then reflected by a parabolic primary mirror onto a secondary mirror that again reflects the light to a focus. An eyepiece or a camera is placed at the focal plane of the resulting image. Image source: https://en.wikipedia.org/wiki/Cassegrain_reflector#/media/File:Schmidt-Cassegrain-Telescope.svg

Equipment: Smartphone or digital camera, object of known length, ruler or other method of measuring distance across your room, the software SAOImage DS9.

1. Download and install DS9 from `http://ds9.si.edu/site/Download.html`. SAOImage DS9, or DS9 for short, is an image viewer, analyzer, and processor written and used by astronomers for working with astronomical images.

   If you click the link to download, it might say "redirecting" while never actually redirecting. In this case, copy the link into the address bar directly.

> **For MacOS**, unless you know otherwise, choose from the top set of choices (to the right of the blue apple logo). To find your version, from the Apple menu in the corner of the screen, choose "About This Mac".
>
> If it displays a warning and prevents you from installed from an unidentified developer, follow the instructions at the following link to create an exception:
>
> `https://support.apple.com/guide/mac-help/open-a-mac-app-from-an-unidentified-developer-mh4061`
> `mac`

2. Set up an object of known length at a known distance from the camera. For example, US Letter sized paper has a standard size, as does currency. You can measure the distance from the camera with a tape measure or other standardized objects. **Record a length of the object and the distance to the camera, including uncertainty.** Discuss with your group how to estimate the measurement uncertainty.

3. Calculate the angular size of the object and propagate the uncertainty (see Appendix A.2) from the two measured quantities to report the angular size in the format $A \pm \delta A$.

   To find the angular size of the object, as seen by the camera, you can use the formula

   $$\tan\theta = \frac{\text{physical size}}{\text{distance}}\,. \tag{2.1}$$

   For situations where the object is far away compared to its size ($\gtrsim 5$ times more distant), you can apply the small-angle approximation, where $\tan\theta \approx \theta$, to simplify the formula to

   $$\theta = \frac{\text{physical size}}{\text{distance}}\,. \tag{2.2}$$

   Note that the resulting angle will be in radians, not degrees.

4. Take an image of the object with your camera and copy it to the computer that has DS9 installed on it. **Include this image in your lab report and label the known object.**

5. The standard image format for astronomical images is FITS. Convert your image to the FITS format at `https://www.online-utility.org/image/convert/to/FITS`

6. Open the FITS file in DS9.

Notice that the image displays in grayscale, and their is another window that opens titled "Cube", that has a slider in it. When you move the slider, the image will change slightly. What is happen is that a CCD pixel, by itself, does not know the wavelength of the photon (and thus the color) that it receives. To resolve color, a color filter is placed over the pixel. In consumer cameras, red, green, and blue color filters are attached permanently over different pixels, in a pattern so that all parts of the CCD have equal coverage of the different colors. DS9 reads this and separates the three filters so that each color filter band can be analyzed separately.

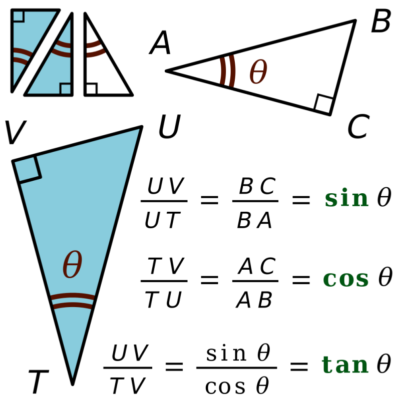


Figure 2.2: Trigonometric relations for triangles. "UV" is the distance between vertices U and V. (Image by https://commons.wikimedia.org/wiki/User:Baelde)

7. Measure the length of the known object in pixels using DS9. There are two ways to do this:
   a) One is to place the cursor at each end of the object, record the x and y pixel coordinates at each end, and find the straight-line distance between them (find the x difference and y difference, then use those in Pythagorean's theorem to find the hypotenuse).
   b) The other way is to use the ruler tool. To measure a length, select Region > Shape > Ruler from the drop-down menu. Then select Edit > Region. Now if you click and drag on the image, it will draw a line and display the distance in pixels, or in angular separation if the image file has been calibrated withe pixel scale already.

   Record the length in pixels and your estimated uncertainty. To get the uncertainty, you can try doing the above measurement several times and calculate the mean value and the uncertainty of the mean (see Appendix A.1).

8. Calculate the pixel scale, which is the angular range that each pixel sees. Express it in units of degrees (or arcminutes or arcseconds) per pixel. Find the uncertainty by propagating the uncertainties. **Record the pixel scale of your camera**.

9. Based on the pixel scale alone, how far apart in angle would two stars need to be in order to be distinguished as two objects instead of one?

Measuring your face

Now that you have the pixel scale, you can image another object and find its angular size. And if you know the distance to it, you can find its physical size.

10. Now take a selfie.

11. Use the pixel scale that you determined, along with that image, to find the angular separation between two points in the image — for example, what is the angle subtended by your eye? Use the distance from the camera to the person to also determine the distance between those two points, for example in millimeters.

12. Estimate the uncertainty, and **record the angular separation, physical size, and uncertainties for each**.

2.5 How far apart are these stars?

Now that you have experience measuring distances for objects you can directly measure, you'll try your hand at measuring distant objects that none of us can measure directly — stars that were imaged by our robotic telescope at Stone Edge Observatory. The goal here is to pick two stars in your image and find the angular separation between them. To check your work, you can compare to the known positions of these stars. To get that, you'll need to match up those stars in your image to known stars. You can use Stellarium for this, a free virtual observatory app. Then you can select those stars in the app and easily find information about them.

13. Download the FITS file for 51 Pegasi from the Labs module in Canvas. 51 Peg is the star system where humans detected our first exoplanet.
14. Load the image in DS9. You can adjust the contrast of the image by holding down the right-click and dragging left, right, up, and down. In the upper right window, you can drag the frame around to view different parts of the image.
15. Also open `stellarium-web.org` and search for 51 Pegasi.
16. The field of view (total angular size of image) for SEO is 27', so zoom in Stellarium to the same approximate field of view.
17. Use the Stellarium and the downloaded image to identify two stars, 51 Peg and one other. Note that the two pictures might be rotated or flipped relative to each other.
18. For those two stars, use DS9 to measure the angular separation (angular distance) between them. You may need to use the pixel scale of the Stone Edge Observatory telescope, which is 0.76"/pix (arcseconds per pixel). The image might have "binning" applied, which means that the photons from groups of neighboring pixels were put in the same "bin" (added up), resulting in a larger pixel scale. For example, if the image filename has "bin2" in it, then groups of 2x2 pixels were binned together, so the pixel scale of the image is double the original. **Record your procedure, value, and uncertainty for the angular separation.**
19. In Stellarium, select each of the two stars in turn and record their equatorial coordinates (RA/Dec). Calculate the angular separation using a web form, for example `http://hea.iki.rssi.ru/AZT22/ENG/cgi-bin/c_dist.htm`.
20. Use the t' statistic (see Appendix A.3) to compare these two values of angular separation.

2.6 Report checklist and grading

Each item below is worth 10 points. See Appendix B for guidance on writing the report and formatting tables and graphs.

1. Completed telescopes and angular size worksheets.
2. Image of the known object, with object labeled, along with measurements and calculations for the angular size of the known object, with uncertainties (Steps 3–4)
3. Length calculated with DS9, with the calculation of pixel scale and its uncertainty (Steps 7–8)
4. Image of selfie, calculation of angular separation, physical size, and uncertainties (Steps 10–12)
5. Image from SEO, with your two stars marked, along with the calculation of angular separation. (Step 18)
6. Calculation of angular separation from Stellarium's coordinates, along with the comparison using the t' statistic (Steps 19–20)

7. Discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

8. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

9. Write a paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps?

LAB 3

Inventing Color

[1]

Everything glows (gives off electromagnetic radiation) when it has a temperature — particles are wiggling around randomly (faster if it's hotter) and giving off energy as they wiggle. This **thermal radiation**, also called blackbody radiation, is the same for any object at the same temperature. This means we can take something's temperature by looking at the spectrum of light it gives off. This is great for learning about something's temperature at a distance, like stars.

3.1 Lab Team Roles

Decide which team members will hold each role this week: facilitator, scribe, technician, skeptic. If there are three members, consider having the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or thinking of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

3.2 Exploring thermal radiation

1. Load the interactive thermal radiation simulation at https://phet.colorado.edu/sims/html/blackbody-spectrum/latest/blackbody-spectrum_en.html
2. Play with the controls and see what happens. Discuss among your team.
3. Find a pattern — how does the shape change as the temperature goes up? For example, how does the peak wavelength change, and how about the total radiated power (area under curve)? **Record your observations.**
4. At what peak wavelength do you radiate? Use the sim to determine this and estimate your uncertainty. **Record your findings.**
5. Comparing the spectra of the light bulb, Sun, and Sirius A, how can you use the spectrum to determine what color each one appears as?

[1] Contributions from Brent Barker, Mike Gladders, Amanda Pagul, and others.

3.3 CCDs and filters

Cameras have come a long way since the days of developing film, but sensors are not as smart you might think. They register light, but they can't usually tell the color (i.e. wavelength) of the incoming photons. So then how do you get a color image? In astronomy (and in your cell phone!) color images are generated by measuring the amount of light at specific colors and then combining these measurements to create a colorful image. A filter is used to select bands of color to allow through. In consumer cameras and phone cameras, these filters are permanently attached to the front of individual pixels. In astronomical imaging, there is no permanent filter, and different filters are moved into place.

Filters

Light is composed of photons with energies that determine their wavelengths (shorter wavelength $\Longrightarrow$ higher energy). Thus every light source exhibits a **spectrum** of energies based on its energetic components, determined by the physics of the light emission process. Thus, observing the energetic constituents of light from astronomical objects - a.k.a. observing the spectrum of emitted radiation - is a fundamental tool in observational astrophysics. However, obtaining the specific intensity of radiation as a function of energy from an astronomical source is challenging. An easier way to asses the electromagnetic energies observed is to image them in different filters: materials that are transparent to a known range of wavelengths and opaque to all others. Thus, one can image the same object with multiple different filters to get a sense of the wavelength regimes that make the strongest contributions to the overall electromagnetic output.

A filter is characterized by its **transmission function**: a function that characterizes the amount of light that is transmitted by the filter at each wavelength. Figure 3.1 shows the transmission functions for standard astronomical filters that are used in the Sloan Digital Sky Survey and now used as standard filters elsewhere.

6. Open the Google Sheet found here: https://docs.google.com/spreadsheets/d/1qgTqiqWOFCzYuSmHeT-5hwy_lBYOOTvu24kM955nfq4/edit?usp=sharing
7. Make a copy of this sheet for your group to use.

This spreadsheet calculates the same theoretical thermal radiation curve (according to Planck's Law) as the PhET simulation above, seen in the first two columns. Then it applies the transmission functions of the SDSS filters and plots the remaining power spectrum that gets through each filter. Finally, it sums up each filter's power spectrum and gives a total pixel value and magnitude. The pixel value is the relative number of counts a pixel with that filter would read. This is proportional to the brightness it sees.

Since brightness as perceived by humans is logarithmic (10 times the number of photons yields 2 times the perceived brightness), astronomers express brightness in a logarithmic form called *magnitude*. The relationship between intensity (power per unit area) I and magnitude m is defined as

$$m \equiv -2.5 \log_{10}\left(\frac{I}{I_{\text{ref}}}\right) , \tag{3.1}$$

where I_{ref} is the intensity of some standard reference object. In the spreadsheet, these values marked as magnitudes are not technically magnitudes, since we are not using the dimensions of intensity, but it will help your analysis to treat them as such.

Color index

In Section 3.2, you learned that the color of a thermally radiating object is related to its temperature. Here, you will develop a quantitative color index and use it to determine the temperature of the Sun.

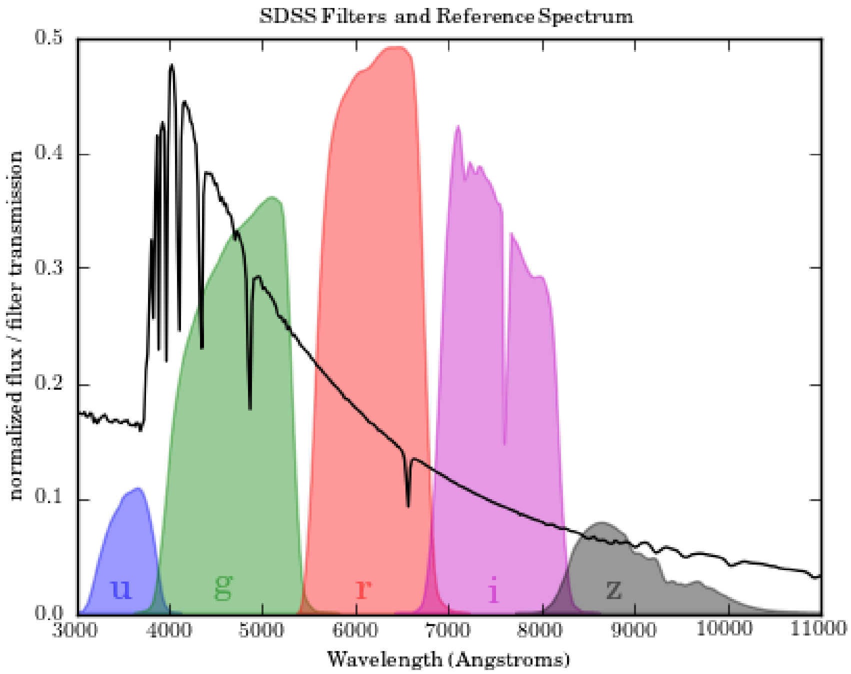


Figure 3.1: Filter Transmission Functions for the Sloan Digital Sky Survey, overlaid on a stellar spectrum (in this case the spectrum is probably an A-type star). The magnitude observed by each filter will be proportional to the integrated spectrum multiplied by the filter transmission. It's clear in this image that the underlying spectrum of the star will cause the different filters to have different magnitudes. How would these magnitudes change if the spectrum was, say, of an M-star instead? (Image source: http://www.astroml.org/_images/fig_sdss_filters_1.png)

8. In the spreadsheet, change the temperature of the blackbody (edit cell B1) and watch how the spectrum changes, and how the filter magnitudes change with respect to each other. **Record your observations.**

The magnitude depends not just on the temperature, but also on the size of the star and the distance away from us. So to characterize a star's color quantitatively, independent of the size and distance, astronomers use a ratio of brightnesses of different filters. This is equivalent to subtracting the magnitudes. The subtraction of two broadband filter magnitudes is called the *color index*.

Correlating color index and temperature

In order to use the color index of a star to find its temperature, you need to determine how the two related to each other. If you assume the EM radiation emitted by the star is entirely due to thermal radiation, then you can find the theoretical color index of a blackbody at various temperatures, and then find where the star's color index sits on that graph.

9. Choose the filters from which to make your color index. Do you want to choose filters that detect wavelengths right next to each other, or farther apart? If you're not sure, try both and see which is better for this application. You can call the resulting color index "$x - y$", for example $g' - r'$ if you are subtracting the r' filter value from the g' value.

10. Make a table and graph of various temperatures and color indices. **Record your table and graph.**

Taking the Sun's temperature

11. Switch to the "Sun" tab in the spreadsheet. This has experimentally determined values for the intensity of EM radiation impinging on the Earth from the Sun, alongside the same filtering system as in the theoretical tab.

12. Using the filter magnitudes here, calculate the color index of the Sun.

13. Use your theoretical data from the previous section to find the temperature of the Sun.

14. Look up the effective temperature of the Sun's photosphere on Wikipedia.

15. Calculate the percent difference between your value and the one that Wikipedia references:

$$\text{percent difference} = \frac{|a-b|}{\frac{a+b}{2}} \times 100\% \tag{3.2}$$

3.4 Creating a color image

To create the beautiful color astronomy images we like to see, one must manually combine images taken with different filters. There are always choices to be made that will change how the combined image looks.

16. Using the directions below, create a false color image of either the Orion nebula (M42), a stellar nursery, or the Crab nebula (M1), the remnant of a supernova. You can find the relevant images on Canvas in the Lab module.

Now you'll create a color image from three separate images of the same target. You can use the broadband filters that you used in the previous section, and you can also try using the narrowband filters h-alpha, oiii, and sii. These filters only allow specific wavelengths through, that correspond to particular electronic excitations of hydrogen, oxygen, and silicon, respectively. Different features can be seen with different filters, as seen in Figure 3.2.

Since there is not a 1-to-1 correspondence between the red-green-blue options in DS9 and the filters available, you will assign a different, arbitrary filter to each color in DS9. This makes the image a *false color image*, since the colors we will see in it do not correspond to the actual wavelength of the light captured.

Loading and manipulating images in ds9 consists of:

- loading an image (file > open)
- setting lower, upper limits (z1,z2) on an image (scale > various algorithms; use scale > scale parameters for full control). See Figures 3.3–3.4 for examples.
- controlling the intensity mapping within those bounds (mouse right click-and-hold and drag)

You can change the zoom and center location in an image by

- moving around in image (mouse middle click if edit>point is set[the default], or edit>pan and mouse left click)
- zooming in and out (mouse wheel, zoom> +,- etc.)

To build a color image

- Identify and download from SEO your three different filter image FITS files.
- Open a color (rather than monochrome) frame: Frame > new rgb

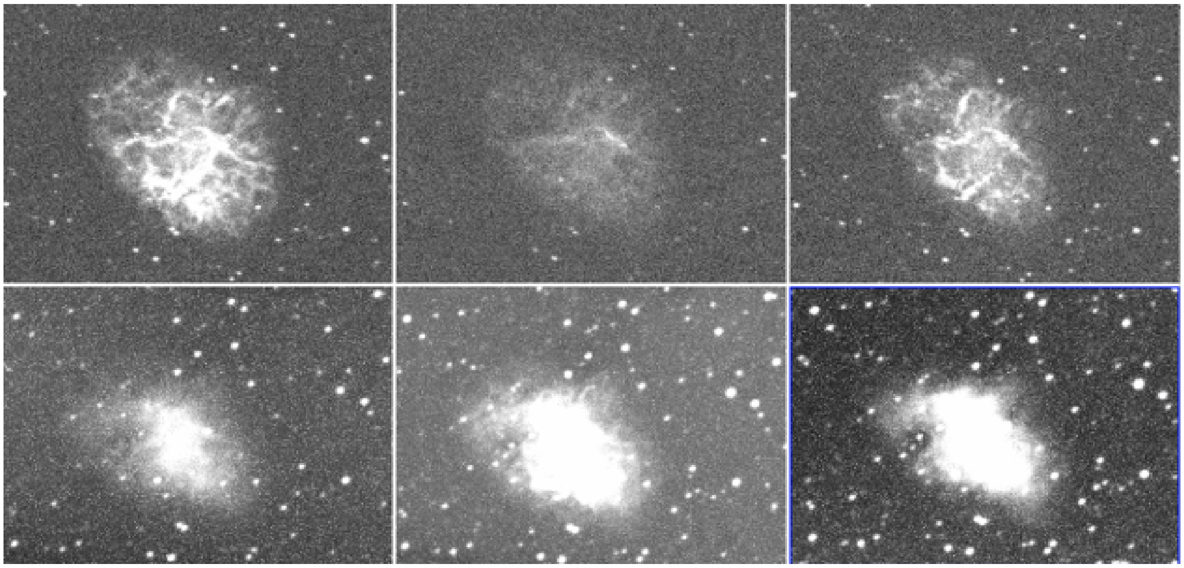


Figure 3.2: Images of M1, the Crab Nebula, in different wavelength filters, revealing different features. Top row from left to right are the narrowband h-alpha, oiii, and sii filters. Bottom row is the broadband g', r', and i'. The structure comes out nicely in the narrowband, while there are more stars in broadband.

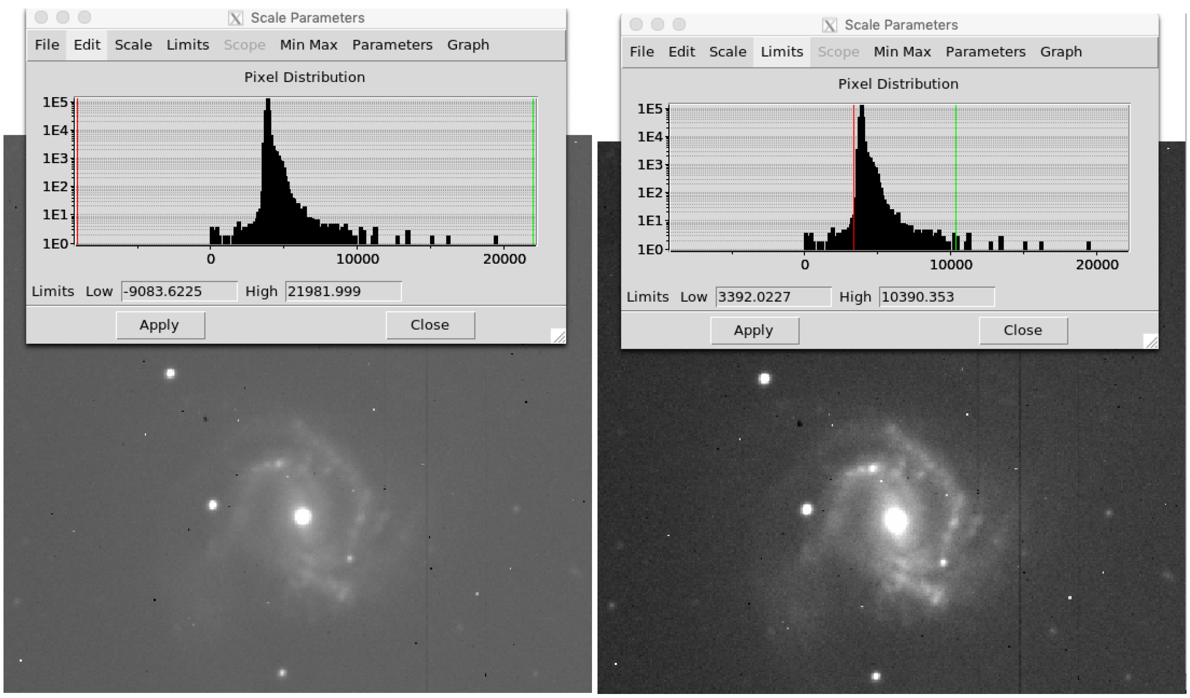


Figure 3.3: Proper choice of data ranges is important. The default for ds9 is often the min/max values in the image, which can be a poor choice if there are outlier pixels, as shown here on the left. The red line shows the lower limit z1, which is mapped to no color (black here), and the green line is the upper limit z2, mapped to full color (white here). Pixel values between these are shown in various brightnesses of the color. On the right, z1 and z2 are more tuned to the distribution of pixel values, which more effectively uses the dynamic range of the display for pixel values where there are significant amounts of data.

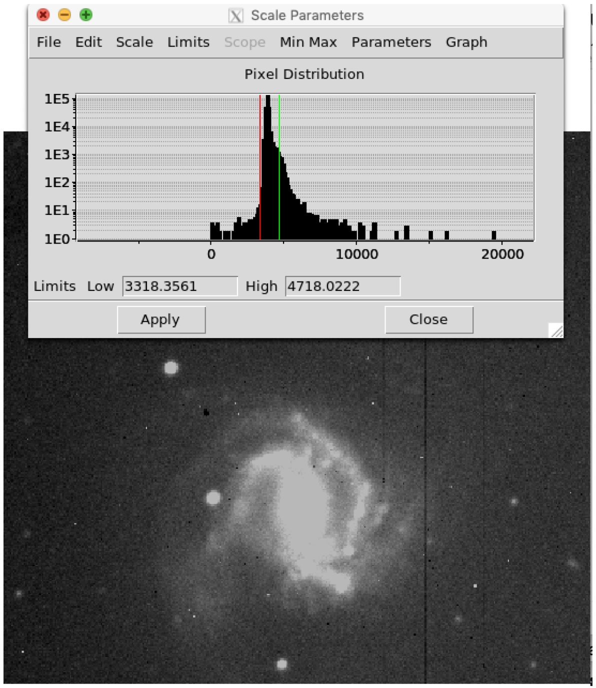


Figure 3.4: Smaller values of z2 will emphasize fainter values in the target. Compare the image to the left to the right-hand image above.

- Open the red, green, infrared files using the rgb subwindow to select which channel you are working in, and then scale and control intensities on each one.
- There are many possible ways to scale the images. Some testing suggests that choosing Scale > ASINH (or Linear or Square Root as other choices) and Scale > 99.5% (or maybe 99% or 98%) produces reasonable results. Experiment!
- One thing to note: the rgb subwindow allows you to control how the images are aligned spatially via the "align" menu at the top. There are three relevant choices: "WCS", "Image" or "Physical". The latter two should give the same result in this instance. "WCS" alignment uses information in the image header that has been added by the processing pipeline, that establishes a World Coordinate System (this tells ds9 and other programs how pixel x,y values are mapped into the sky coordinates - typically Right Ascension [east/west] and Declination [north/south]). The default is WCS and should work fine, if the images processed correctly. If that doesn't look good, you could try the others. If that still doesn't look good, note that you tried your best, and give an example of how it didn't work will in either mode. Examples of good and bad image alignment are shown in Figure 3.5.

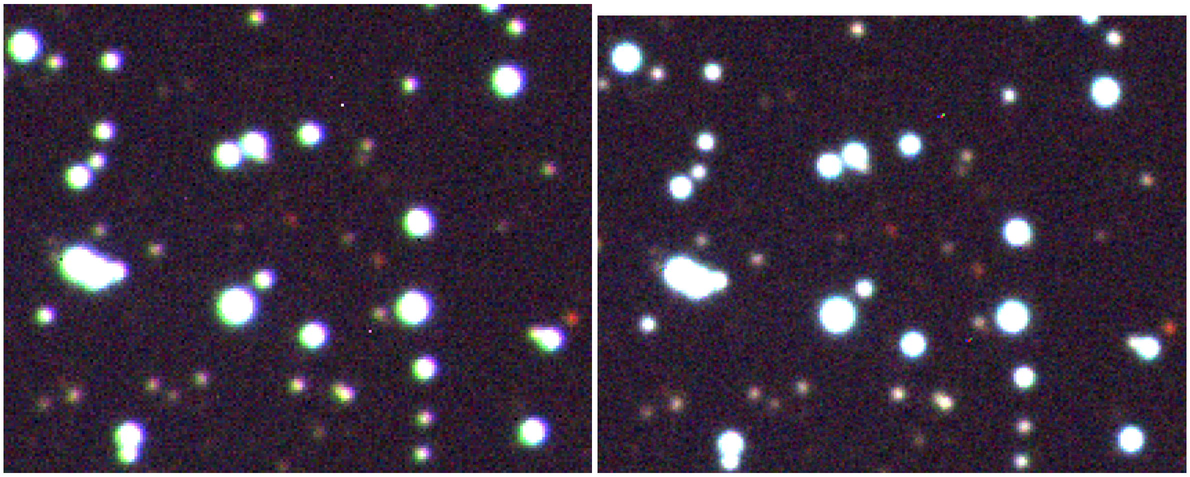


Figure 3.5: Zoom-in on a color image, showing poor (left) and good (right) image alignment across filters. Note how objects are shifted between different color channels in the poorly aligned image.

3.5 Report checklist and grading

Each item below is worth 10 points. See Appendix B for guidance on writing the report and formatting tables and graphs.

1. Pattern of how shape of spectrum changes with temperature, your peak wavelength, and how to use spectrum to predict color (Steps 1–5).
2. Observations of how spectrum and filter magnitudes change with temperature (Step 8).
3. Table and graph of color index versus temperature (Step 10).
4. Procedure, calculations, and determination of solar temperature with percent difference (Steps 12–15).
5. Your beautiful color image (Step 16).
6. Discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").
7. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?
8. Write a paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps?

LAB 4

How old are star clusters?

4.1 Introduction

In this lab you will use a Hertzsprung-Russell (H-R) Diagram to learn about a star cluster. This will give us insight into the stellar populations in the clusters we observe, and allow us to estimate the age of their constituent stars.

4.2 Learning goals

- Gain an understanding of astronomical observation, image analysis and photometry
- Gain practice performing basic calculations on datasets and making informative scientific figures from those data
- Retrieve data from astronomical databases.
- Learn where different stellar populations lie on the HR diagram, and understand the physical reasons behind these localizations
- Estimate stellar cluster ages using the predictions of stellar evolutionary models.

4.3 Lab Team Roles

Decide which team members will hold each role this week: facilitator, scribe, technician, skeptic. If there are three members, consider having the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or thinking of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

4.4 Scientific background

Stars evolve (i.e., change their properties) over millions and often billions of years – too slow for us to see the evolution over a human lifespan. Such impressive longevity is due to the fact that stars are powered by thermonuclear reactions, which are very efficient in generating abundant energy and have quite a bit of fuel to last for a long time. Stars like our Sun last about 10 billion years (so the Sun is in its middle age). The long timescale of evolution also means that we have to develop a different way to study stellar evolution.

Astronomers explore evolution of stars by observing large populations of stars where different stars are in different stages of evolution. Of course, in order to do this we need to be able to tell which star is in what stage. This is done by a combination of observations – which measure luminosity and surface temperatures – and theoretical models – which predict how luminosity and surface temperature change as stars evolve. The key is that luminosity and temperature at a certain age are determined by star's mass, chemical composition, and details of thermonuclear reactions (which elements are burning, over what fraction of star's volume, etc.).

Luminosity and temperature of stars are related because they are both determined by their internal structure, which, in turn, is determined by the basic physical properties (mass, chemical composition, age). Therefore, stars are not scattered randomly in the luminosity and temperature space but follow well-defined sequences, which reflect the ranges of the controlling parameters in a given stellar population.

The surface temperatures of stars can be deduced by fitting a blackbody radiation spectrum to their spectra. Even for stars that do not have spectra measured, their temperatures can be deduced from their colors (Recall: does bluer color correspond to cooler or hotter temperature?). Our eyes and brain perceive color by analyzing spectral composition of the incoming light. In astronomy, a star's color is defined as the difference between its magnitudes measured through two different filters that block out all light except light within a fairly narrow range of wavelengths.

In order to interpret evolutionary states, we look at physical groupings of stars called stellar clusters, which are located at the same distance from us and were born at the same time from the same cloud of dense gas. The spread in their properties will thus not be due to different ages or initial compositions, but mainly due to different masses. As you will see, stars occupy distinct regions of the observable equivalent of the luminosity-temperature space — the magnitude-color space called the Hertzsprung-Russell (H-R) diagram. We will make this diagram for a star cluster.

Filters

Light is composed of energy-packets termed **photons** with energies that determine their wavelengths (shorter wavelength $\implies$ higher energy). Thus every light source exhibits a **spectrum** of energies based on its components, which are determined by the physics of the light emission process. Observing the spectrum of radiation emitted from astronomical objects is a fundamental tool in observational astrophysics. However, obtaining the specific intensity of radiation as a function of energy for many dim sources is challenging. An easier way to assess the electromagnetic energies observed is to image them in different **filters**: materials placed at the opening of a telescope that are transparent to a known range of wavelengths and opaque to all others (thus "filtering" the light). Thus, one can image the same object with multiple different filters to get a sense of the wavelength regimes that dominate the light from a source.

A filter is characterized by its **transmission function**: a function that characterizes the amount of light that is transmitted by the filter at each wavelength. Figure 4.4 shows the transmission functions for some standard astronomical filters (similar to the one's you'll be using in this class).

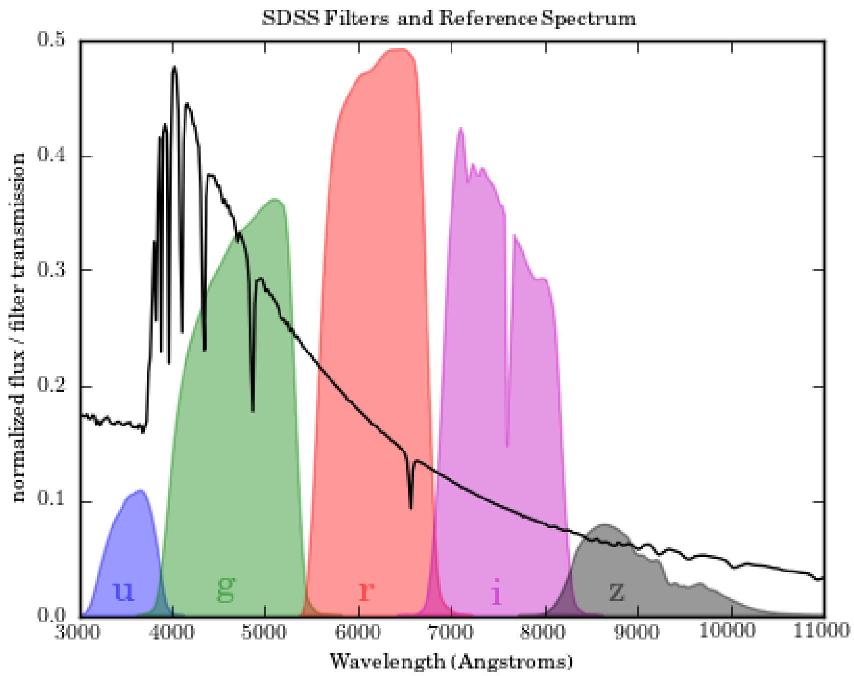


Figure 4.1: Filter Transmission Functions (taking into account atmospheric transmission and instrumental effects such as CCD efficiency) for the Sloan Digital Sky Survey, overlaid on a stellar spectrum. The magnitude observed by each filter will be proportional to the integrated spectrum multiplied by the filter transmission. It's clear in this image that the underlying spectrum of the star will cause the different filters to have different magnitudes. Image source: http://www.astroml.org/book_figures/appendix/fig_sdss_filters.html

4.5 Defining color

Since different bands measure brightness in different wavelengths, the ratio of flux in two bands is a measure of an object's color. Therefore, the difference between astronomical magnitudes of an object in different bands is a measure of its color, since the magnitude scale is logarithmic:

$$m_A - m_B = -2.5 \log \left(\frac{F_A}{F_B} \right). \tag{4.1}$$

An H-R diagram is a plot of stellar magnitude vs. color — aka a "color-magnitude diagram". A star's color is an observational indication of it's surface temperature. Since all of the stars in a star cluster are at approximately the same distance, their apparent magnitude gives a good relative indicator of luminosity. Therefore, a color-magnitude diagram of stars at roughly constant distance is effectively a temperature-luminosity diagram, and stars fall in characteristic regions of this parameter space based on their mass, age, and metallicity.

We will have data in two wavelength bands, so we can subtract those magnitudes to get a color index. With the software or coding language of your choice, you will plot g on the vertical axis and $g - r$ on the horizontal axis.

4.6 Making an HR Diagram

To learn about a star cluster, you'll make a color-magnitude diagram of the stars in that cluster. You'll be analyzing M15, a globular star cluster in the Pegasus constellation. To make the diagram, you need magnitudes in at least two different filters for many stars in the cluster. Since this would be incredibly time-consuming to do by hand, you will retrieve this information from an online database.

1. Open the SkyServer Search Form in a browser: http://cas.sdss.org/dr7/en/tools/search/form/Default.aspx.
2. Select "Show me [stars] in the region [around:]". Note that you will need to know the celestial coordinates of the part of the sky that you want retrieve data about, as well as the radius of a circle that captures all the objects you want.
3. Look up M15 in Stellarium (stellarium-web.org), change the date to 2000-01-01, and record its RA and Dec. It may be helpful to turn off the ground and atmosphere to get a better view. Then, determine a radius in arcminutes that includes all of the cluster. You can either use the Field Of View (FOV) listed on the screen, or you can turn on the Equatorial Grid by selecting that option in the bottom toolbar and use the contour lines.
4. The Search Form wants the RA and Dec in decimal degrees. So convert the RA and Dec that you recorded to decimal degrees and enter this (for example, an RA of 15h30m20s converts to 232.5833 degrees). You can use this website to assist: https://www.swift.psu.edu/secure/toop/convert.htm.
5. In the Form, tell it to output 1000 objects, and the magnitudes of the objects.
6. Select "Generate" or "Update Query" and it will convert all the options into a database query that you should then "Submit". This opens a new tab with the output from the query.
7. If the outcome from the query has much fewer than 1000 objects, then increase the radius (and/or check to ensure you have the correct RA and Dec entered).
8. Once you have lots of objects, change the form to output a CSV file.
9. Plot the data from the CSV file using your favorite spreadsheet or programming environment. Specifically plot 'g' vs. 'g-r', with 'g' on the vertical axis, as a scatter plot. Reverse the vertical axis, so that brighter stars with lower magnitudes are towards the top of the plot.
10. Label the different stages of stellar evolution on your plot. *Note that your H-R diagram may not have examples of stars in all stages of stellar evolution.*

4.7 Comparing to stellar evolution models

The last portion of this lab involves estimating the age of these stellar populations by comparing their color magnitude diagrams with predictions from a stellar evolution model. Stellar physics is sufficiently well-understood that accurate evolutionary models have been constructed to calculate the observable properties of stars across their lifetimes. Because stars evolve in their position on the HR diagram, we can estimate the age of our observed cluster by comparing these models to our observations.

Several files containing model predictions for the magnitudes of stars at different ages are contained in files on the course website. These have been calculated using the observed metallicity of the star cluster to predict the positions of stars at ages 10^7, 10^8, 10^9, and 10^{10} years. These are called isochrones - stellar properties as a function of stellar mass for a fixed age (and metallicity). Note that these magnitudes were calculated using prior knowledge of the star cluster metallicity, and have been corrected for cluster distance and extinction due to intervening galactic dust (taken from [1] for M15 and [2] for NGC 869).

11. Plot these isochrones on your H-R diagrams and compare them to your data to estimate the cluster age. Be sure to estimate an error on this value and explain your method in your lab report.

12. What changes in stellar properties are represented by the different locations of isochrones of different ages?

13. What physical processes occuring inside the star underlie those changes?

4.8 Identifying trends in color-magnitude diagrams

Now that you have seen and used theoretical models for H-R diagrams of of a particular star cluster of different ages, you can use your understanding to learn about other star clusters.

Four unlabeled color-magnitude diagrams are shown in Figure 4.2. They have been modified so that the vertical axis is the absolute magnitude rather than apparent magnitude so that they can be compared. These four clusters were not all formed at the same time. In fact, they have distinct ages that are quite different from one another.

14. What trends do you see among these four diagrams?

15. **Determine the sequence from youngest to oldest, and provide an explanation.**

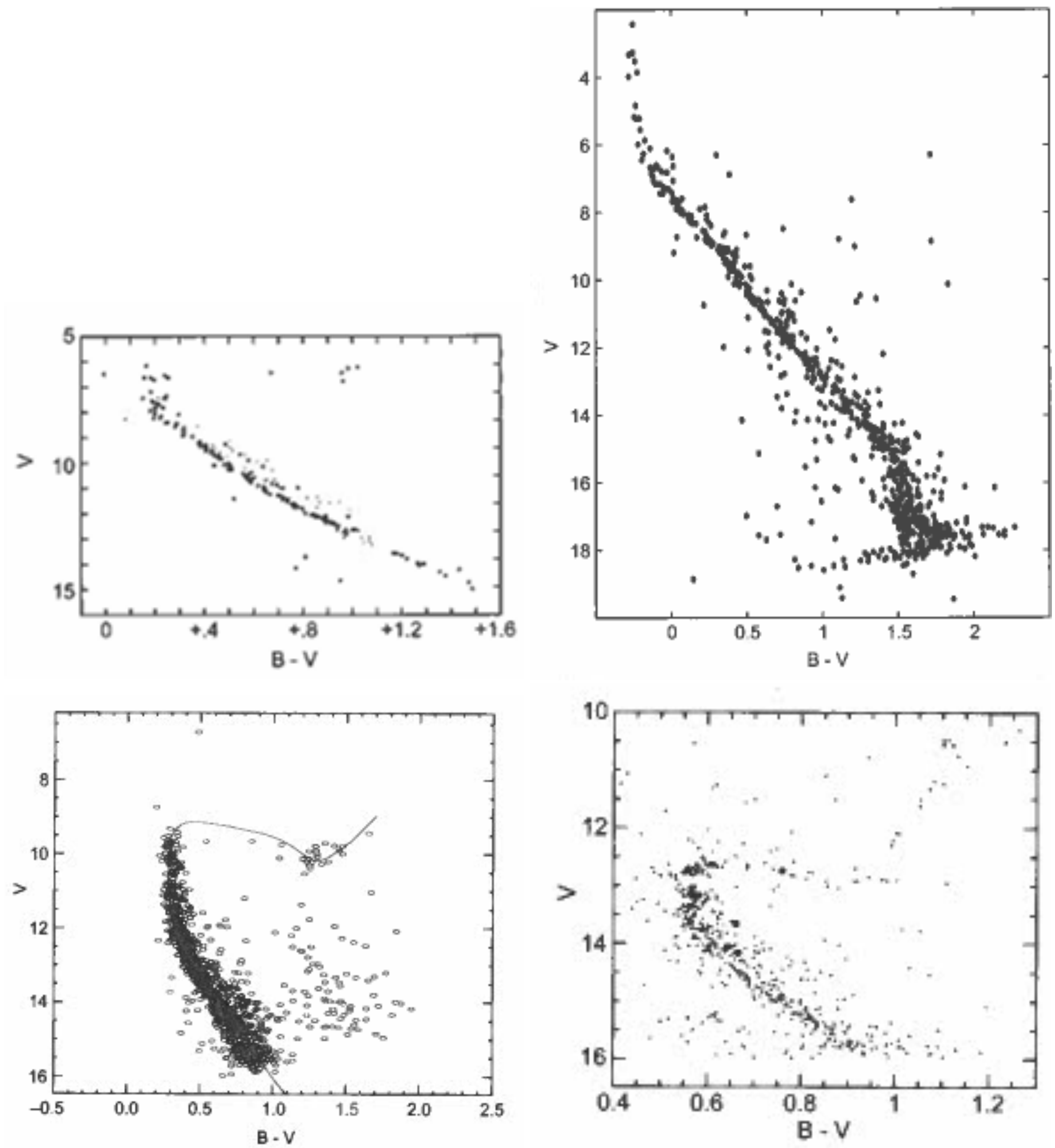


Figure 4.2: H-R diagrams for several clusters of stars. The vertical axes have been changed to absolute magnitude for comparison purposes.

4.9 Report checklist and grading

Each item below is worth 10 points. See Appendix B for guidance on writing the report and formatting tables and graphs.

1. The coordinates of M15 and the radius you choose, along with how you found these (Step 3).
2. The plot of g vs. $g - r$ with labels of different stages of stellar evolution (Steps 9–10).
3. Plot of H-R diagram with isochrones (Step 11).
4. Analysis and determination of cluster age, with uncertainty (Step 11).
5. Descriptions of stellar property changes and physical processes leading to change in isochrone shapes (Steps 12–13).
6. Trends and ordering of the H-R diagrams of different star clusters (Steps 14–15).
7. Discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").
8. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?
9. Write a paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps?

LAB 5

Origin of the Crab Nebula

The Crab Nebula was discovered in 1731 by John Bevis, an English doctor, electrical researcher, and astronomer. The French astronomer Charles Messier found the object while searching for comets. He decided to start cataloging these fuzzy objects, thus the Crab Nebula being marked as "M1". In 1921, Carl Otto Lampland, a first-generation Norwegian American immigrant astronomer, discovered that the nebula was changing its structure over time.

In this lab, you will investigate how the Crab Nebula is changing and what it means for its history.

5.1 Learning goals

- Identify and use analysis skills and tools to answer an astronomical question.
- Become familiar with the Crab Nebula.

5.2 Lab Team Roles

Decide which team members will hold each role this week: facilitator, scribe, technician, skeptic. If there are three members, consider having the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or thinking of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

5.3 How is it changing?

Goal

Describe qualitatively and quantitatively how the Crab Nebula is changing over time.

Available equipment

- Historical images of the Crab Nebula from the STScI Sky Survey, found at `https://archive.stsci.edu/cgi-bin/dss_form`.
- Any software or other tools used in previous labs.

Steps

1. Navigate to the URL above.
2. To get the object's coordinates, type in "M1" in the Object Name field and select "GET COORDINATES". The Target name "MESSIER 001" should appear under the "Retrieve an Image" section.
3. Each of the selections in the "Retrieve from" menu is from a different sky survey and filter. Different surveys were done at different times.
4. Download a FITS file from two different surveys for the same filter.
5. Open these files in DS9 and see if you notice any change in shape or structure between these (you might not see any difference). **Record your observations.**

In fact, over time, it was found that the nebula was expanding, increasing in size over time. If this is true, then at some time in the past, the nebula would have been its smallest size. In fact, we know that this nebula is actually the remnant of a supernova — an exploding star!

6. Design an experiment to determine how fast the nebula is expanding (in arcsec/year), and how long ago the star went supernova. Discuss your experiment with a TA. Once you have decided on your experiment, **record your experimental procedure and analysis plan**.
   - Detailed information about the image, including the plate scale, time of observation, and pixel size, can be found in the FITS header, viewable in DS9 with File > Display Header...
   - When comparing images, it can be useful to identify structures that have the same shape in each image.
7. Conduct your experiment, collecting the data, taking relevant screenshots to show what you are doing.
8. Analyze your data to determine the speed of expansion and year in which the star exploded. **Record this analysis and your results with uncertainties**.

Once astronomers calculated when the star had exploded, they looked back into historical records and found that indeed, astronomers from around the world (China, Japan, Iraq, North America) had observed and recorded this new or "guest" star. You can read about even more historical observations at the Wikipedia page for SN 1054.

5.4 Report checklist and grading

Each item below is worth 10 points. See Appendix B for guidance on writing the report and formatting tables and graphs.

1. Initial observation notes (Step 5)
2. Detailed description of experimental design, including data collection and analysis (Step 6)
3. Data collection results (Step 7)
4. Analysis of data with final speed of expansion (in arcsec/year) and year of supernova, both with uncertainties (Step 8)
5. Discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

6. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

7. Write a paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps?

APPENDIX A

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, "$5 \pm 1\,\mathrm{m}$" means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters[1]. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

A.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument's scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment's manual often gives the instrument's resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

[1] The phrase "most likely" can mean different things depending on who is writing. If a physicist gives the value and does not given a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet, or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}}\,, \tag{A.1}$$

where $\{x_1, x_2, \ldots, x_N\}$ are the measured values, $\bar{x}$ is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called "standard deviation of the mean", which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}\,. \tag{A.2}$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as 52 ± 3 m.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn't change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

A.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction), } \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{A.3}$$

$$\text{For } c = ab \text{ (or for division), } \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \tag{A.4}$$

$$\text{For } c = a^n,\ \frac{\delta c}{c} = n\frac{\delta a}{a} \tag{A.5}$$

For other calculations, there is a more general formula not discussed here.

Expression	Implied uncertainty
12	0.5
12.0	0.05
120	5
120.	0.5

Table A.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π, is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table A.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value "`0.0322857142857143`". Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as "`9.65683578099600e-04`", where the '`e`' stands for "times ten to the". I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as "0.032 ± 0.001 km/s". You may also see the equivalent, more succinct notation "0.032(1) km/s".

A.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced "tee prime"). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as[2]

$$t' = \frac{|a-b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \tag{A.6}$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

[2] Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.

APPENDIX B

Lab Report Format

B.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.

B.2 Organizing the report

The report should follow the sequence of the report checklist. Answers to questions and inclusion of tables and figures should appear in the order they are referenced in the manual. In general, include the following:

- For any calculations that you perform using your data, and the final results of your calculation, you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92\,\mathrm{m/s^2})t + 0.2\,\mathrm{m/s}$."
- Answers to any questions that appear in the lab handout. Each answer requires providing justification for your answer.

B.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like "distance (pc)". This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like "distance (pc)".
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called "error bars", along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.

Bibliography

[1] P. R. Durrell and W. E. Harris, "A color-magnitude study of the globular cluster M15", Astronomical Journal **105**, 1420 (1993).

[2] T. Currie, J. Hernandez, J. Irwin, S. J. Kenyon, S. Tokarz, Z. Balog, A. Bragg, P. Berlind, and M. Calkins, "The Stellar Population of h and χ Persei: Cluster Properties, Membership, and the Intrinsic Colors and Temperatures of Stars", The Astrophysical Journal Supplement **186**, 191 (2010).