Laboratory Manual

PHSC 12600 Matter, Energy, Space, & Time

The University of Chicago

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Labs

LAB 1

Observing Falling Filters

The ability to observe without evaluating is the highest form of intelligence.
Jiddu Krishnamurti

While Mr. Krishnamuri may be making a stretch with his superlative, it remains true that observing without evaluating is essential for the creation of knowledge. In our lives, we have bias (conceptions, self-constructed mental models) that we use as our lens to view the world. These models are based on how each of us were socialized and on our subsequent experience. To learn and create new knowledge, we must develop skill in observation. In this lab, we will direct you to make detailed, careful quantitative observations, describe the patterns you find with mathematics, and finally make some wild guesses ("hypotheses") about a more universal principle that explains this pattern that one could use to make predictions. Due to time and brain constraints, we will not, in this lab, test those hypotheses.

Learning Goals

- Become familiar with measurement, uncertainty, and writing lab reports.
- Learn how to conduct an observational experiment, including collecting data and analyzing the data to find and describe a pattern quantitatively.
- Use measurement uncertainty to describe physical quantities meaningfully.
- Format a lab report in a helpful way.

The Scientific Cycle[1]

One way of describing science is the process of incrementally improving a shared model of how our universe works. In different fields of science, different methods and cycles are used, so there is no "One True Scientific Method." One can still create a model for the process of science, and we describe here one such cycle, summarized in Figure 1.1.

In this cycle, there are three types of experiments, each one representing a different stage of the scientific effort. One stage, often started when encountering a novel phenomenon, is the **observational experiment**. This is an experiment that consists of deciding what to observe and how to observe it, collecting data, finding a pattern, and brainstorming possible explanations for what is observed (also called "hypotheses").

[1] adapted from [1]

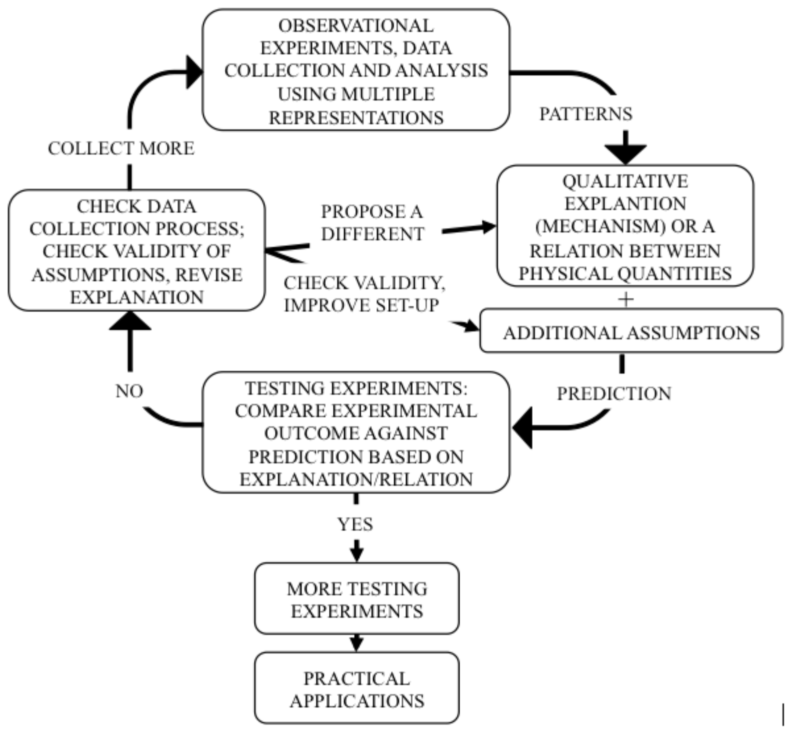


Figure 1.1: A model of the process some scientists go through to create knowledge.[2]

Once one has some trial explanations, one can test one or more of those with a **testing experiment**. Here, one designs a new experimental procedure and uses each hypothesis to predict what will happen. Then the prediction is compared to the procedure's outcome. If they are different, then the hypothesis is judged to be not a helpful explanation for that phenomenon. If they are the same, then it is still helpful. Throughout this stage, one may make various assumptions that would need to be validated, as they can effect the prediction or outcome.

Once a hypothesis has been tested enough for people to find it useful, then it can be applied to solve practical problems, or to determine properties of particular situations, in an "application experiment."

Observation experiment: Observing falling filters

In today's lab, you will investigate the relationship between the size of coffee filters and how long it takes them to fall. In the first section, you will determine the size of the coffee filters. In the second, you will determine how long each take to fall, controlling for other variables, and then find a mathematical pattern that describes the relationship. Note that this lab does not include any hypothesis testing.

Self-assessment: To help you improve your scientific abilities, we provide you with self-assessment rubrics. A rubric is a scoring system. Self-assessment is determining how well you performed a particular task. So, these self-assessment rubrics are designed to help you evaluate your performance while you are designing and performing your experiment.

The complete set of rubrics is available in Appendix B. In each lab, your report will be assessed using Rubric F, found in Table B.5, as well as 5 additional rubric rows listed in that lab. Each week, read through these and use them to evaluate your work as you design and perform the experiment. Your instructor will use the same rubrics to determine part of your grade for the lab. In particular, each row will be worth 3 possible points (from "Missing" being 0 points to "Adequate" being 3 points).

Rubrics to focus on during this experiment: B5, B7, B8, F1, F2, G1, G2. See Appendix B for details.

Available equipment: several differently-sized coffee filters, meter stick, balance or scale, stop-watch, scratch paper

You may want to **decide on roles** for each group member. Example roles include Facilitator (ensures time and group focus are efficiently used), Scribe (ensures work is recorded), Technician (oversees apparatus assembly, usage), Skeptic (ensures group is questioning itself). Note that each role is responsible for ensuring that the thing happens, rather than necessarily doing it themselves.

How big are the filters?

Goal: Find the cross-sectional area of each coffee filter and make a determination of that area, including uncertainty in that area, for use in the next section.

1. Review Rubric G (Table B.6) and discuss any unclear expectations with your group and the instructor.

2. Brainstorm different methods you could use to determine the cross-sectional area. Feel free to play with the equipment as desired. Here are some things to consider:

   - Will you measure the area directly, or will you measure something else and use that to calculate the area?
   - With any method, you will probably make one or more assumptions about the shape of the filter. How valid are those assumptions?
   - For each method you consider, there may be different sources of uncertainty — the resolution of the measuring devices themselves, how you use them to measure, etc. If there is a source of random uncertainty, then you will need to take several measurements and use Appendix A.1 to determine the uncertainty. The decision of how many measurements to take is a trade-off between increasing precision (decreasing the uncertainty of the mean) and decreasing the time the measurement process takes.
   - If you make a measurement and use that measurement in an equation to find the area, you will need to propagate uncertainty as described in Appendix A.2.

3. Decide on your method and discuss it with an instructor before you begin. They will help increase the chances that your method will lead to successful results, or at least that the unhelpful path that you choose will take a short enough amount of time for you to change it when you discover it does not work. We want you to have productive failure that you have time to learn from.

4. Write down an outline of your intended procedure. You might end up changing this as you go, but it is helpful to start with a plan and then change it, rather than having no plan at all.

5. For your procedure, list the sources of uncertainty involved with each measurement. For each source, identify whether it is a random or instrumental uncertainty.

6. Execute your procedure, including setup, data collection, calculation of area, uncertainty estimation and propagation.

At the end of this step, you should have a table of coffee filter cross-sectional areas, with uncertainties.

7. Once you are done collecting this data, review your written procedure and correct it to match what you actually did, and ensure you have sketched any measurement setups, so you can include it in the lab report. In particular, ensure that you have enough written so you can demonstrate Rubric Rows F1, G1 and G2 in your report (see Tables B.5 and B.6).

How fast do the filters fall?

Goal: Determine how long it takes each coffee filter to fall.

1. Review Rubric B (Table B.2) and discuss any unclear expectations with your group and the instructor.
2. Identify any variables (things that could change between measurements — either between measurements of the same filter, or among different filters) that could affect the fall time other than the coffee filter's cross-sectional area. If there is controversy in the group, feel free to test what variables might affect that fall time.
3. Since you are testing how the fall time is related to the filter's area, you should hold the other variables constant, so that they affect all the filters in the same way. For each variable identified in the previous step, decide how to keep that constant.
4. Brainstorm different methods you could use to determine the time it takes for the filter to fall. Feel free to play with the equipment as desired. Here are some things to consider:
   - Will you measure the fall time directly, or will you measure something else and use that to calculate the time?
   - For each method you consider, there may be different sources of uncertainty — the resolution of the measuring devices themselves, how you use them to measure, etc. If there is a source of random uncertainty, then you will need to take several measurements and use Appendix A.1 to determine the uncertainty.
   - If you make a measurement and use that measurement in an equation to find the time, you will need to propagate uncertainty as described in Appendix A.2.
5. Decide on your method and discuss it with an instructor before you begin. They will help increase the chances that your method will lead to successful results, or at least that the unhelpful path that you choose will take a short enough amount of time for you to change it when you discover it does not work. We want you to have productive failure that you have time to learn from.
6. Write down an outline of your intended procedure. You might end up changing this as you go, but it is helpful to start with a plan and then change it, rather than having no plan at all.
7. For your procedure, list the sources of uncertainty involved with each measurement. For each source, identify whether it is a random or instrumental uncertainty.
8. Execute your procedure, including setup, data collection, calculation of area, uncertainty estimation and propagation.

> At the end of this step, you should have a table of coffee filter cross-sectional areas, with uncertainties, with another column for fall time, with uncertainty in the fall time.

9. Once you are done collecting this data, review your written procedure and correct it to match what you actually did, and ensure you have sketched any measurement setups, so you can include it in the lab report. In particular, ensure that you have enough written so you can demonstrate Rubric Rows B5, F1, G1 and G2 in your report (see Tables B.2, B.5, and B.6).

Now that you have these measurements, it is time to find a pattern.

Finding a pattern

The penultimate step in an observational experiment is to find a pattern. Note that we are not explaining why this pattern is happening yet — we are focusing on describing it first.

Goal: Find a pattern in the data and describe it mathematically. **Available equipment:** Computer with spreadsheet software

1. Use a plotting program, for example LibreOffice Calc or Microsoft Excel, to plot a graph of fall time vs. filter area. The independent variable should be on the horizontal axis. The axes should each be labeled with the quantity name and the unit in parentheses. For example, if you measured fall time in seconds, then the axis label should be something like "fall time (s)".
2. In that graph, include also the uncertainty in each value. This usually involves right-clicking on a data point and selecting "error bars". Then you can highlight the column of cells that include the uncertainties.
3. Visually, discuss what shape the data points make. Speculate what kind of relationship you see. Is it proportional? Linear? Parabolic? Exponential? Logarithmic?
4. Create a line of best fit (or "trend line") in the graph using the software. Choose the equation type to match what your group guessed in the previous step. If the line obviously does not match the data, try again with a different equation type. Quantitatively, the goodness of fit of a line (how close the line is to your data points) can be represented by the correlation coefficient, given as r^2 in the software. If $r^2 \gtrsim 0.8$, then the equation that you found describes the data fairly well. Record that equation and the r^2 (or RMSE if given).
5. Make a final determination for describing in words the pattern found. If it applies, you should use one of the terms given in Step 3 in order to precisely describe the pattern.
6. Review Rubric Rows B7 and B8 in Table B.2 and ensure that you are demonstrating them here or have enough information to do so in your lab report.

Finishing up

Before you leave lab, be sure that you have reviewed the 7 rubric rows that are being used to assess your report and that you are equipped to do as well on them as you would like to. The visuals that you should definitely have in your report are sketches of your measurement setups and a graph of fall time vs. filter area. You may want to have others as well, but those you will definitely need.

Optional Fun Time Extra

To extend the lab just a little bit and break out of the "observational experiment" frame, and to check to see if your mathematical pattern (line of best fit) is consistent with other physics principles, you can extrapolate down to zero cross-sectional area. As an optional part of the lab with no added benefit to your grade (but with perhaps glee for your inner scientist), you can do this.

Using your equation for the relationship between the two variables, predict the freefall time of an object with zero cross-sectional area. You can compare this with the theoretical value for this time t, given for an object that is falling from height h, with the strength of the local gravitational field $g = 9.8\,\mathrm{m/s^2}$, with the following formula:

$$t = \sqrt{\frac{2h}{g}} \tag{1.1}$$

How close did you get?

1.1 Post-lab survey

To assist the instructors with understanding your experience in this lab, please also respond to the following questions on a scale from 1 to 7 (with 7 being the most)

1. To what extent was the instructor's assistance needed?
2. To what extent did you know what to do (goal of the task)?
3. To what extent did you know how to do it?
4. To what extent did you know how well you are doing?
5. to what extent was the assignment challenging?
6. To what extent did you feel knowledgeable and skillful during the lab?
7. To what extent was the lab fun and interesting?

1.2 If in Stars class, also do this:

Remote Observing with the Stone Edge Observatory

For two labs in this course, we will be taking observations remotely with the Stone Edge Observatory in Sonoma, California. We will use a queuing system to submit observations that are automatically scheduled and taken by the telescope. The data are then processed and typically available for analysis several days after they were obtained. To ensure that our data are taken and reduced in time for our in-lab analysis (which is subject to possible delays due to, e.g., the weather at the observing site), we will be submitting observations to the queue several weeks prior to lab in which they will be analyzed.

First, you will need to register for an account that will allow you to access the queue website. Make groups of two to three students so that there are no more than 5 groups in a section. Each group will use one member's email to sign up for an account in the queuing system. A TA will be present to manually add each group. Each group will receive an email that will allow them to create an account. Since you will be sharing an account, be sure to share the account password (and obviously don't re-use one from another personal account).

Once you have an account, you will be able to log onto the queue and submit observations. To do so, go to the website `https://queue.stoneedgeobservatory.com/` and log-in with your group's credentials. Then navigate to `OBSERVATIONS` ▸ `SUBMIT AN OBSERVATION`. This will take you to a form that allows you to input the specifics of your observation. These will be given to you for each lab.

HR Diagram: Taking observations

For this lab, each section will be taking observations of one of two star clusters — NGC 869 and M15. You will then use this data to make color-magnitude diagrams of these clusters, which can then be compared with stellar evolution models to determine when these clusters formed. Table 1.1 lists the parameters for the observations to be taken in this lab - each section will be assigned a cluster, and each group in the section should submit one observation. Each group will then analyze their data in-lab, and will combine datasets. If there are fewer groups than observations, omit the longest-exposure observations.

Table 1.1: HR Diagram Lab Observations

Program	**Target**	**Exp Time (s)**	**Exp Count**	**Bin**	**Filters**
General	M 15	1	1	2	Dark, g', r'
"	"	5	"	"	"
"	"	10	"	"	"
"	"	20	"	"	"
"	"	40	"	"	"
"	NGC 869	0.5	"	"	"
"	"	1	"	"	"
"	"	2	"	"	"
"	"	5	"	"	"
"	"	10	"	"	"

LAB 2

Intro to Radioactivity

2.1 Introduction

Over the next two weeks, we will explore a few basic properties of radioactive decay: (1) counting statistics, (2) types of radioactive decay, and (3) half-life. In the first week, we'll use samples of radioactive materials — isotopes of cobalt, strontium, cesium, and polonium — to generate energetic particles. We'll then make measurements to explore the properties of these particles and the counting statistics related to their decays. In the second week, we'll produce our own short-lived isotope of silver and watch the new atoms decay by counting the number of particles they emit. From that, we will measure the so-called half-life of the silver isotope we produce.

Learning Goals

- Become familiar with the statistics of counting events.
- Describe the four types of radioactive decay products
- Identify sources of random and systematic error.
- Make careful measurements.
- Use a spreadsheet application to calculate and plot data.
- Relate small scale physics (studying atoms in the lab) to large scale astrophysics (dating the big bang, supernovae, and the solar system)

Scientific Background

Nuclear reactions play an important role in astronomy, geophysical sciences, archaeology, and physical anthropology. They explain energy generation in stars, the relative abundance of chemical elements, and provide a method for determining the age of things — from a piece of wood, to a meteorite, to the universe itself.

Most elements exist in a number of different forms, called isotopes, some of which are unstable and can change from one type of element to another. When this occurs, a high-energy particle is usually emitted from the nucleus of the element as it changes. By measuring the ratios of isotopes with differing decay rates, one can infer the age of an object.

2.2 Testing experiment: Do radioactive isotopes decay randomly?

Goal: Answer the question, "Given what we understand about the standard deviation of measurements of random processes, do radioactive isotopes decay randomly?" Or, in the frame of a testing experiment, test the following hypothesis: "Radioactive sources decay randomly."

Rubrics to focus on during this experiment: C1, C4, C7, C8, F1, F2, G2. See Appendix B for details.

Available equipment: Stopwatch, SpecTech Geiger counter, computer with spreadsheet software, radioactive source (^{137}Cs)

Warning: Radioactive Material! The radiation levels are very low and they present no hazard for the short time that you are in the lab. We estimate that you will receive an additional 2–10 μSv dose of ionizing radiation for the time that you are in lab today, about the same as you receive every day normally. You can compare this to the example doses in Fig. 2.1. Here are tips to keep your exposure low:

- **Do not have any food, drink, food containers, or make-up on the lab bench, and do not consume any food or drink, and do not apply cosmetics, in the lab.** If the sealed button source were to leak, your food, drink, food container, or cosmetics could become contaminated with low levels of radioactive material that could be ingested.
- **Decrease time with and increase distance from sources.** Handle the sources only when you need to be for the lab, and return them to their container when not using them.

Caution: Fragile Equipment! The Geiger tube (the upright cylinder sitting in the plastic stand and connected to a coaxial cable at the top) hold a gas under vacuum, with a thin, fragile window at the bottom of the tube. Do not touch it, as it breaks extremely easily.

Theory of counting statistics

If you've ever worked in a not-too-busy retail environment, you've probably had the experience of realizing (perhaps) that random and uniform are two quite different things. You no doubt have sat waiting for customers, with nobody around for long stretches of time, and then, as though they'd coordinated in advance, a half dozen people all show up within a few minutes of each other. Each customer is indeed independent (no, they didn't have a Twitter call to organize a flashmob![1]) yet their arrival times clearly appear clustered. They are, in fact. But randomly. Uniform – one customer each minute – is quite dramatically different than a random procedure that yields one customer per minute on average. Retail customers (to a great extent), and elementary particles, act randomly, not uniformly.

Each atom in a radioactive sample has, per unit time, some probability of undergoing radioactive decay. That probability is independent of all the other atoms in the sample. Each atom decays, or not, based on its own probability and no other. The ensemble of particle decay counts that one would measure in the sample (using a Geiger counter as we will do, for example) is described by the something called the Poisson distribution, which gives, in this instance, the probability of a integer number of events occurring in a fixed interval of time, given an average rate. For large count rates, like we have here, the Poisson distribution is indistinguishable from the normal distribution (that is, a simple Gaussian function).

[1] Yes, popular interest in flash mobs peaked in 2011. Bear with us here.

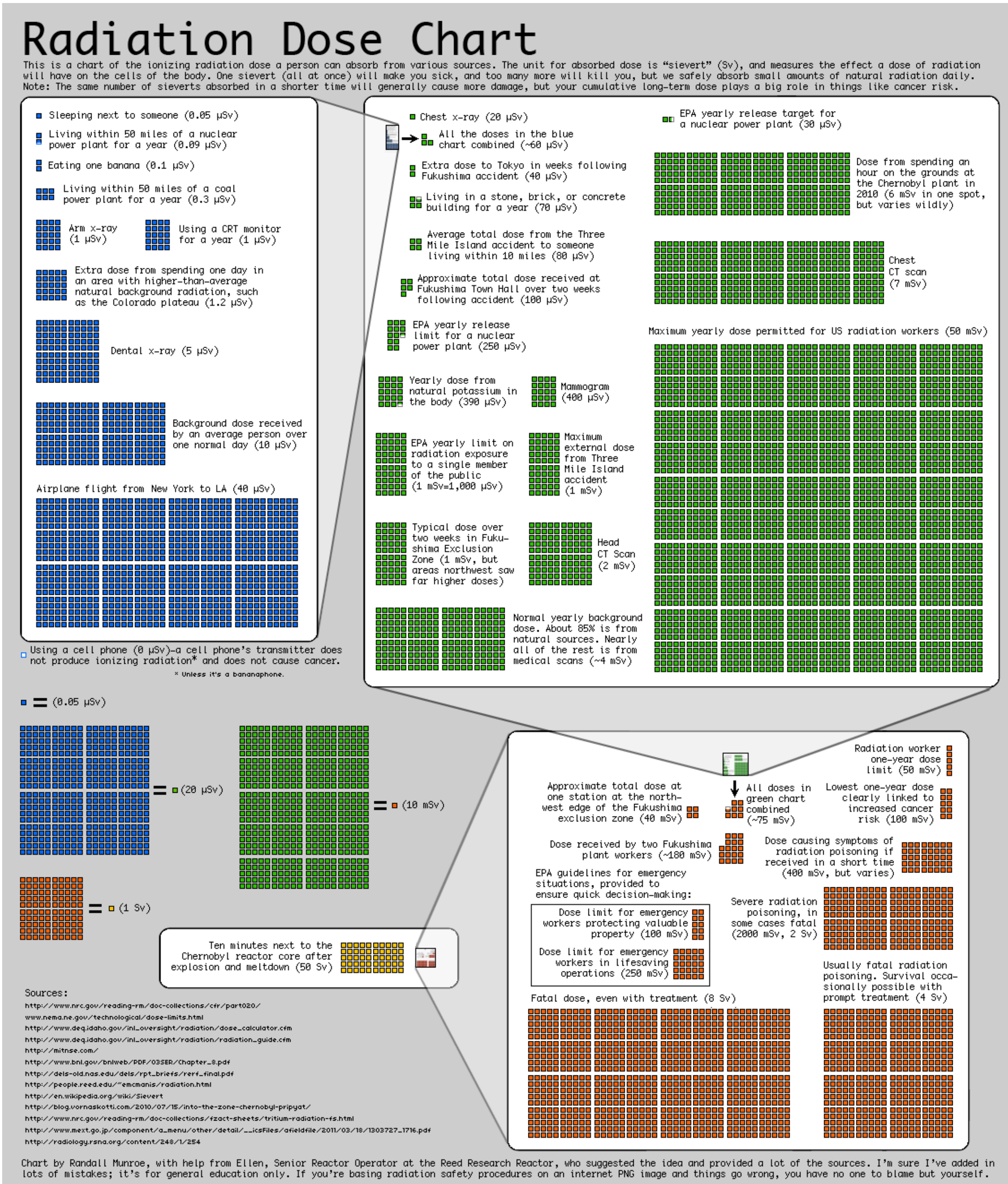


Figure 2.1: A chart of ionizing radiation dose from various sources. Source: https://xkcd.com/radiation/

Specifically, for a process that produces an average $\bar{N}$ counts in a certain time duration (in the case of large N), the expected measurement of that count for that time duration is a random draw from a normal distribution with mean $\bar{N}$ and a standard deviation of $\sqrt{N}$.

Procedure and analysis

1. To find the steps of a testing experiment, review Rubric C, found in Table B.3.
2. A prediction is what the hypothesis says will happen in the event of a particular experimental procedure. Given the hypothesis stated in the goal, and the theory described above, if you collect a series of counts of radioactive decay, what does the hypothesis predict the standard deviation of those counts should be equal to? Record this prediction in your lab notebook.
3. **Collecting data.** In order to find the standard deviation of a number of measurements, you will need to take those measurements. In the following steps, you will do 10 trials of measuring the count rate (counts per unit time) during periods of about 30 seconds, then take the standard deviation of those 10 trials. Since you will not be able to stop the counter after exactly 30 seconds, you will divide the displayed count by the displayed time to get a count rate (in units of "counts per second"), and then multiply this by 30 to get the count-rate per 30 seconds.

The Apparatus.

For this lab and the following one, you will be provided with a number of different radioactive materials packaged in small plastic disks. Note that each disk has the name of the isotope (e.g. ^{60}Co), the type of radiation, and the half-life printed on one side of the sample. Half lives can range from fractions of a second to billions of years, depending on the isotope.

The energetic particles produced in radioactive decay reactions can be detected by a device called a Geiger counter. This consists of a tube filled with inert gas with a wire running through it. A high voltage is applied between the wire and the tube. When a high- energy particle enters the tube, it can ionize the gas. The freed electrons produce a brief pulse of current at the output of the device. The pulses can then be counted.

The counter itself is controlled using buttons on the front panel. `COUNT` begins the count and starts the timer, `STOP` pauses the count and timer, and `RESET` sets the count and timer to zero. Use the dial on the counter to change the display between the count and the elapsed time. When you first turn on the Geiger counter you need to set the voltage to 1000 V — the TA will demonstrate how. **Caution:** Do not turn off the voltage during the experiment; use the `STOP` button to stop the count. **Do not touch the window on the bottom of the tube, as it breaks extremely easily.**

   a) Take one of the ^{137}Cs disks and place it printed side down in the plastic shelf shaped to hold it, and place that shelf in the slot second from the top in the stand under the Geiger counter. Note that if you have the Geiger counter on, that it is measuring a large count rate.
   b) Use the controls `COUNT`, `STOP`, and `RESET` to measure the number of counts in 10 approximately 30-second intervals. Use the dial on the display to access both the counts and the elapsed time. The provided stopwatch may also help you organize your effort — *but use the time from the Geiger counter apparatus directly* for your calculations.
   c) Record the counts and elapsed time in each interval in a spreadsheet and calculate the count rate in counts per half-minute.

4. **Analysis.** Note that the count rate is not identical in each trial, even though you accounted for the different measurement durations. Let's see how far away the standard deviation here is from

the prediction in Step 2. Calculate the standard deviation of these measurements. How close is it? Answer this quantitatively by calculating the *percent difference*,

$$\text{percent difference} = 200\% \times \frac{|A-B|}{A+B}\,, \tag{2.1}$$

where A and B are the two quantities that you are comparing. Without taking the time to find the uncertainty in the standard deviation, this is the best you can do to say how close it is.

5. Use this quantitative comparison to determine if the prediction and the outcome agree or not.

6. Based on that agreement, and taking into account the experimental conditions, make a judgment about the hypothesis. Is the radioactive decay happening randomly?

2.3 Observational experiment: types of radiation

Goal: Observe the effects of shielding on different types of radiation and generate some ideas about the patterns you see.

Rubrics to focus on: F1, F2, G2. See Appendix B for details. *Note that the C rubric rows from the previous experiment do not apply to an observational experiment.*

Available equipment: Stopwatch, SpecTech Geiger counter, computer with spreadsheet software, radioactive sources (^{60}Co, ^{137}Cs, ^{90}Sr), shielding of various thicknesses

Background: types of radiation

There are four basic types of radiation: alpha particles, which are the nuclei of helium atoms consisting of two protons and two neutrons; beta particles, which are energetic electrons; gamma rays, which are very energetic photons, *i.e.* particles of light; and finally neutrons. (The terms "alpha," "beta," and "gamma" were coined before anyone knew much about the actual particles involved.) We'll be working with gamma and beta radiation in what follows. Part 2 of this lab next week includes the use of a neutron radiation source.

Different types of radiation of the same energy interact with matter in different ways. Alpha is stopped by a piece of paper, beta is stopped by a book, while gamma rays are stopped by a shelf of books, or many inches of steel or lead.

Procedure

1. We will begin by attempting to measure how much material it takes to stop the beta particles from ^{90}Sr. Take a sample of ^{90}Sr and put it under the Geiger counter in the second slot down from the top, printed side down, removing the ^{137}Cs source you used previously if you still have it in place.

2. Count the number of particles detected in 60 seconds from this source, and record this number.

3. Qualitative observe the effect of different shields by placing them between the sample and the Geiger counter and recording the count rate (for example, in counts per minute) for each shield. You should try a few different shields with different thicknesses (measured in mg/cm^2).

4. One way to quantity the penetrating ability of radiation is to find it's "stopping length", which is the thickness of shielding that reduces the detected count to half the unshielded amount. Find the stopping length of the radiation emitted by ^{90}Sr and include uncertainties in your reported count rates.

5. Now find the stopping length for ^{60}Co. This time, place the sample holder in the third slot from the top instead of the second. If you need to use a thicker shield than the thickest one, you can use two of them and add the thicknesses together.

6. How do the two stopping lengths compare, qualitatively, to each other. What does this tell you about the relative penetrating ability of beta and gamma radiation?

7. As before, note your measured count rates, with uncertainties, in your lab report, as well as recording the stopping length.

8. Now, finally, we will examine the radioactive properties of the ^{137}Cs source. Place it in the sample holder as before, and record the unshielded count rate in a 60 second interval. The sample is noted as having both gamma and beta radiation. Use the shields to attempt to establish this experimentally, by comparison to the results you got for the ^{60}Co and ^{90}Sr samples. *Note that this last step is not a standard part of an observational experiment, but rather an application experiment.*

Questions for discussion, to include in your lab report

1. You probably didn't find a shield that produced an exact 50% reduction in counts. However, we can still estimate what the properties an exact 50% reduction shield would be. Describe a procedure to estimate the properties of such a shield and then perform that calculation.

2. What fraction of the total counts from the ^{137}Cs source are due to gamma rays?

3. Is the stopping length of the ^{137}Cs gamma rays similar to the stopping length of the ^{60}Co gamma rays? Would you expect it to be?

4. Is the stopping length of the beta particles from the ^{137}Cs source more, less, or similar to that for the Sr source? What might you have expected, if you'd known that the beta particles for the ^{137}Cs in most cases have energies of 0.512 MeV and those from the decay of ^{90}Sr have an energy of 0.546 MeV?

5. If you put the ^{90}Sr source in once more (this time in the third slot) and measured the counts, and then placed two of the shields previously determined to cut the source counts by a factor of 2 between the source and Geiger counter, what count rate would you expect, relative to the unshielded rate? Why? Would you expect the same behavior from the ^{137}Cs source?

6. How do the stopping lengths for beta particles compare to the stopping lengths for gamma rays? Can you come up with a physical explanation for the patterns you observe? Does this explanation extend to what you know about alpha particles?

LAB 3

Radioactive Half-Life

Building on our work from last week, we will continue to study the radioactive decay. The physical laws of radioactivity predict that the rate of decay (number of atoms decayed / time interval) is proportional to the number of radioactive nuclei present. This is due, as we saw in counting statistics previously, to the independence of the decay of each atom in the sample. The proportionality constant that describes the decay rate depends on the specific radioactive nucleus. A concise and suggestive way to characterize the nucleus is by its half-life, the time it takes for the number of radioactive nuclei to decrease to half of the initial value. You will obtain data to check the form of the law and to determine the half-life of one or more isotopes of silver.

We will use a device known as a neutron "Howitzer". It consists of a source of alpha particles and a material that absorbs alpha particles and immediately decays by emitting neutrons. (The neutrons shoot out from the barrel of the shielded volume, vaguely like shells from WWI artillery, hence the name.)

3.1 Background

We will let the neutrons bombard a small sample of the stable isotope of silver, $^{107}\mathrm{Ag}$, to produce $^{108}\mathrm{Ag}$, via the reaction

$$^{107}\mathrm{Ag} + \mathrm{n} \rightarrow {}^{108}\mathrm{Ag} + \gamma \,, \tag{3.1}$$

where n represents a neutron and γ a gamma particle — that is, a high energy photon.

The radioactive isotope of silver, $^{108}\mathrm{Ag}$, spontaneously decays to an isotope of cadmium with the same mass number, $^{108}\mathrm{Cd}$, by the reaction

$$^{108}\mathrm{Ag} \rightarrow {}^{108}\mathrm{Cd} + \mathrm{e}^- + \gamma \,, \tag{3.2}$$

where e^- is an electron (that is, a β particle, as we saw and measured last week).

Your TA will bombard silver foils with neutrons using the neutron howitzer. Some of the nuclei in the foil will have captured a neutron and transformed into a different isotope which is unstable and can be detected via their decay products. Each group will be given one of these silver foils.

3.2 Procedure

Rubric rows to be assessed: D1, D4, F1, F2, G2, G3 (ignore actually doing it), G4

Your task is to measure the half-life of $^{108}\mathrm{Ag}$. We will use the Geiger tube to count decays. That is, we will count the β particles — the gamma rays make only a small contribution to the counts in this instance. You should attempt to carry out the counting fairly quickly after the silver foil is removed from the howitzer as the decay time is quite short.

Before the neutron irradiation begins you will want to record the background rate. Press `STOP` and `RESET` on the counter to set the display to zero. Next, press `COUNT` with no sample below the Geiger tube and collect the total number of background counts, N_{bkg} , that accumulate in approximately 5 minutes. Once 5 minutes has elapsed press `STOP` to end the count. Turn the dial to `TIME` and record a precise measurement of the elapsed time, t, in seconds. The background rate R_{bkg} is found with

$$R_{\mathrm{bkg}} = N_{\mathrm{bkg}}/t \tag{3.3}$$

with an uncertainty given by Poisson statistics. Report both the background rate and uncertainty in your lab report.

While the samples are being irradiated, set up your measurement apparatus. Once the samples are ready, quickly place a silver foil sample in the tray below the Geiger tube. Using a stopwatch and the counter, record the number of counts and the time at 30 second intervals for about 10 minutes, continuously. Unlike last week's lab with the same apparatus, you will be recording data continuously, and so will need to use the watch to record times rather than timer built into the Geiger counter. Record your data.

3.3 Calculations

The experimental data will be used to determine a half-life (or half-lives). We know that the decay rate ($R = \Delta N/\Delta t$) of a radioactive nuclide is proportional to the number of nuclei present. The proportionality constant is called the decay constant λ, and the equation that describes what was just discussed is

$$R = \lambda N \,, \tag{3.4}$$

where N is the background-subtracted counts. Using integral calculus and the above equation, we find

$$\frac{N}{N_0} = e^{-\lambda t} \,, \tag{3.5}$$

where N_0 is the number of nuclei at the initial time $t = 0$. The half life $T_{1/2}$ is defined by the time it takes for $N = N_0/2$ and is related to the decay constant by $T_{1/2} = \ln(2)/\lambda$, where ln() is the natural logarithm function, and so $\ln(2) \approx 0.693$.

We can now write the radioactive decay equation as

$$R = \lambda N_0 e^{-t \ln(2)/T_{1/2}} \,. \tag{3.6}$$

Taking the logarithm of both sides and substituting for N gives

$$\ln(R) = -\left(\frac{\ln(2)}{T_{1/2}}\right) t + \mathrm{const}\,. \tag{3.7}$$

Your TA will help you to understand the details of this derivation.

Make a plot showing $\ln(R)$ on the vertical axis and elapsed time, t, along the horizontal axis. Don't forget to subtract the background where appropriate. Calculate the slope and use this value to solve for the half life using the above equations. Report this value along with a table of your decay rate data and the plot described above.

3.4 Questions (these should be included in your lab report)

1. Look up the half-lives of the various nuclides of silver. What is the published half-life of the nuclide you're observing? How does this compare with your calculated result? Calculate the percent difference in your result.

2. For the silver foil, how long would it take before you would expect to detect only one count per second, background corrected?

3. The detector only measures particles that travel up into the detector. The majority of particles traveling in the other directions escape detection. Will this short- coming affect the measured half-life? If so, how? If not, why not?

4. Describe one thing you could change in this experiment that could lead to a more accurate measurement of the half-life of the silver isotope.

5. The particular irradiated silver sample you used contained some unknown percentage of the unstable silver isotope, and was irradiated at some unmeasured time before you began your experiment. Does this matter to your results? Explain.

LAB 4

Local Gravitational Field

One of Newton's revelations was that physical laws that governed the movement of objects near Earth also predicted the movements of objects in the sky. The apocryphal story of an apple falling on Newton's head brings to mind the mechanism of gravity — the phenomenon of massive objects attracting each other. In this lab, you will measure the strength that gravity has where we are, near the Earth's surface. This measurement might also enable us to learn more about the mass of the Earth itself in a future lab.

4.1 Learning goals

- Understand Newton's law of universal gravitation and its linear approximation
- Identify sources of statistical and systematic error
- Demonstrate an ability to make careful measurements
- Demonstrate proficiency in basic calculations and plotting
- Explain the importance of repeated measurements and sufficiently large datasets
- Use experimentally derived quantities to obtain a mass for the Earth.

4.2 Scientific Background

The gravitational field strength and Newton's second law

The force of gravity, F, between two objects with mass m_1 and m_2 and whose centers are separated by a distance R is given by Newton's law,

$$F_{\text{gravity}} = G\frac{m_1\, m_2}{r^2}\,, \tag{4.1}$$

where the Newtonian constant of gravitation $G = 6.67408(31) \times 10^{-11}$ m^3 kg^{-1} s^{-2}. Astronomers apply Newton's law to infer fundamental information about astrophysical objects, for example the mass of binary stars. Indeed, this is one of the most common methods by which astronomers "weigh" astrophysical objects, including the Earth itself. For measuring the force acting on an object of mass m that is affected predominantly by the Earth's gravity, the force acting on it would be

$$F_{\text{Earth}} = \frac{GM_{\oplus}}{(R_{\oplus} + h)^2}m \tag{4.2}$$

where $M_\oplus$ and $R_\oplus$ are the mass and radius of the Earth, respectively, and h is the height above the Earth.

For objects near the Earth's surface, where h is much less than $R_\oplus$, h can be treated as zero, resulting in a constant gravitational force, with Equation 4.2 reducing to

$$F_{\text{Earth}} = \frac{GM_\oplus}{(R_\oplus)^2} m \,. \tag{4.3}$$

Notice that on the right-hand-side of this equation, the only variable is the mass. The others, together, constitute the *strength of the local gravitational field*, g (sometimes pronounced "little g"). So our simplified equation is

$$F_{\text{Earth}} = gm \,, \tag{4.4}$$

where we have made the substitution

$$g = \frac{GM_\oplus}{(R_\oplus)^2} \,. \tag{4.5}$$

Notice that we have taken a complicated inverse square equation (Equation 4.2) and converted it to a much simpler one (Equation 4.4). This process is called *linearization* and is a trick astronomers often use to make calculations more manageable. You will encounter this technique throughout this and other PHSC courses.

We see from Equation 4.5 that if we can make accurate measurements of g, G, and $R_\oplus$, we can calculate the mass of the Earth. We'll look up $R_\oplus$ online, and next week we will measure G. To find g, we note that Newton's second law of motion states that the acceleration a of an object is directly proportional to the net force F_{net} acting on it and inversely proportional to its mass, m, or, more succinctly and slightly rearranged,

$$F_{\text{net}} = ma \,. \tag{4.6}$$

If the Earth's gravity is the only force acting on our object, then $F_{\text{net}} = F_{\text{Earth}}$, and substituting Equation 4.4, we find that

$$gm = ma \,, \tag{4.7}$$

and thus, simplifying,

$$a = g \,. \tag{4.8}$$

So, the acceleration of an object that is subject only to the Earth's gravity is equal to the local gravitational field strength. If we can measure the acceleration, then we can find g, and get one step closer to determining the mass of the Earth.

Constantly accelerated motion

If an object is subject to a constant force, then according to Newton's second law, it undergoes constant acceleration. If an object undergoes constant acceleration a, and we know the object's initial position x_0 and velocity v_0, then after a time duration t, we can derive using calculus that the object's position x and velocity v are given by

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \tag{4.9}$$

and

$$v = v_0 + at \,. \tag{4.10}$$

4.3 Application experiment: determine g on the Earth's surface

Goal: Determine g near the Earth's surface by finding the acceleration of an object undergoing freefall (no substantial forces other than gravity) using two different methods:

Rubrics to be assessed: D4, D5, F1, F2, G1, G2, G4

Available equipment: stopwatch, dense object to drop, meter stick, camera (including the one on your phone), computer with Tracker[1] installed.

[1] Open Source Physics Tracker can be downloaded from `https://physlets.org/tracker` and is also installed on the lab computers.

Method 1: freefall time

1. Drop the object from a known height and measure the time to fall with a stopwatch. Do this as many times as makes sense to you.
2. List the sources of uncertainty and determine whether each is a random uncertainty or an instrumental uncertainty.
3. Calculate the average fall time.
4. Calculate the standard deviation of the average fall time (using Equation A.2), and report the latter as the uncertainty in the average fall time.
5. Use the average fall time and the initial position and velocity of the object to calculate the acceleration.
6. Propagate the uncertainty in the time and position to find the uncertainty of your measured acceleration (see Section A.2)
7. Report the acceleration found by this method as "value ± uncertainty [units]". For example, $9.73 \pm 0.04\,\mathrm{m/s^2}$.

Method 2: Video tracking

It is helpful to use two methods to find the same quantity, so that mistakes or incorrect assumptions made in one method do not carry over to the other, and are thus more likely to be detected. In this method, you will record a video of an object falling, make a position vs. time plot, and fit the constant acceleration equation (Equation 4.9). You will use a computer program to make this analysis easier.

Record the video

1. Find a good object to drop. It should be dense enough to not be slowed down significantly by air resistance.
2. Using the camera on one of your group member's phones, record a video of the object falling.

   Here are some tips to get a quality video:

   - Include an object of known length in the shot, at the same distance from the camera as the falling object. This gives a reference length, so that you can find how each camera pixel scales to the physical situation.
   - Avoid parallax error by having the object be at about the same distance from the camera throughout the fall. Having the camera be farther away can help. Also, you can ensure that the top and the bottom of the fall are the same distance from the camera.
   - Hold the camera steady.
3. Record that video and transfer the video to a computer that has Tracker installed.

Importing the data into Tracker

In this part, you'll use Tracker to record the position of the object at each timestep. To do this, you'll need to tell it what direction "down" is in, what the scale of the image is, and when time $t = 0$ is. Then you'll record the positions, find out what parameters best fit the curve that is produced, and use those to find the acceleration.

1. Open Tracker on a computer. You can install it on your own computer by visiting https://physlets.org/tracker.

2. In Tracker, open your video.

3. **Find frame when zero time is.** Move the slider below the video to the right to advance the frames until you find the first one in which the object is falling. Record that start frame number, which is found to the left of the slider bar in red.

4. **Find the last relevant frame.** Keep moving the slider to the right until you find the last frame before the object hits the floor. Record that end frame number.

5. To **tell Tracker about these frames**, click the 5th icon from the left on the toolbar above the video ("Clip settings") and enter the start frame and end frame.

6. **Tell Tracker how long things are.** In astronomy applications, this is known as the "pixel scale". Here we can just draw a line on the frame and tell Tracker how long that line is in real life. Click the 6th icon from the left (blue, with a "10") and select `New → Calibration Stick`. Shift-click to mark each end of your known length, and type in your known length, with units in the box that appears along the stick. Use "m" for meters.

7. **Align the coordinate system.** In the toolbar, click the 7th icon from the left (magenta crossed lines). Click and drag the coordinate system's origin (the intersection of long lines) to the location of the object in the start frame.

8. **Check to see if the camera was tilted.** Advance the video to see if the object moves along an axis. If it goes off at an angle, the camera was tilted compared to the direction of motion. In this case, rotate the coordinate system to align with the motion by clicking and dragging the small line that crosses one of the axes.

9. **Tell Tracker where the object is in every frame.**

   a) In the toolbar, click `Create → Point Mass`.

   b) Ensure the slider is at the start frame.

   c) Shift-click on the object. Notice that the frame advances to the next one automatically.

   d) Continue to shift-click to mark the object's position throughout the duration.

Analysis

1. **Ensure the correct axis is selected for analysis.** Look at the plot to the right of the video. If there is not a smooth-ish curved line, click on the axis label "x (m)" and choose instead "y (m)".

2. In the drop-down menu, select `View → Data Tool (Analyze...)`.

3. In the window that appears, above the plot, click `Analyze → Curve Fits`.

4. Notice that Eq. 4.9, which describes freefall, is a quadratic equation, which means the shape is a parabola. For "Fit Name", choose "Parabola" from the drop-down menu.

5. Use the Fit Equation and Parameter Values, comparing with Equation 4.9, to find the acceleration a, and thus the gravitational field strength g.

6. To get an uncertainty for this value, use the "rms dev" value, which describes the average deviation of the fit equation from the points, divide that by the average (mean) position, and multiply that by your value for the acceleration. You can find the mean position by selecting `Analyze → Statistics` and reading above the data table column.

Comparing the methods, final determination of $\boldsymbol{g}$

1. Compare the values of g from the two methods using the t' statistic as described in Appendix A.3.
2. Use that comparison and your assessment of which method had fewer questionable assumptions to decide on your final answer for g (including an uncertainty). How close is it to the average g described, for example, on Wikipedia?

LAB 5

Measuring the Gravitational Constant

Gravity is the least understood of the fundamental forces, and it is the one that is most easily noticed by humans. Part of the reason for this second part is that gravity is weak enough that we can easily push and move against it (for example, walking or launching a rocket). Compared with the electromagnetic bonds that are holding molecules together or the nuclear forces that hold atomic nuclei, gravity is extremely weak.

If it is universal, though, we should be able to measure the gravitational attraction between any two massive objects, if we are very careful and have sensitive equipment. In this lab, we will measure the gravitational force between lead spheres and use that to find the Newtonian constant of gravitation, G, which is a measure of how strong, in general, the force of gravity is.

5.1 Learning goals

- Follow the procedure of an important physical measurement.
- Identify sources of statistical and systematic error.
- Demonstrate an ability to make careful measurements.

Rubrics rows to be assessed: D4, D8, F1, F2, G1, G2, G4.

5.2 Scientific Background

In 1798, Henry Cavendish completed his measurements of “the density of Earth” using a simple yet extremely sensitive apparatus: a torsion balance. He placed a small lead sphere of mass $m = 0.73\,\text{kg}$ at each end of a wooden rod, which was suspended horizontally. Two large lead spheres, each of mass $M = 159\,\text{kg}$, were fixed in place, at a distance $R = 23\,\text{cm}$ from each of the smaller spheres. When the torsion balance was released and allowed to move freely, the lead spheres would be attracted by the gravitational force according to Newton’s law of universal gravitation,

$$F = G\frac{Mm}{R^2}\,. \tag{5.1}$$

To protect the balance from air currents and temperature changes, Cavendish placed it in a closed room (Fig. 5.1), appropriately equipped for remote operation and reading of the torsion angle.

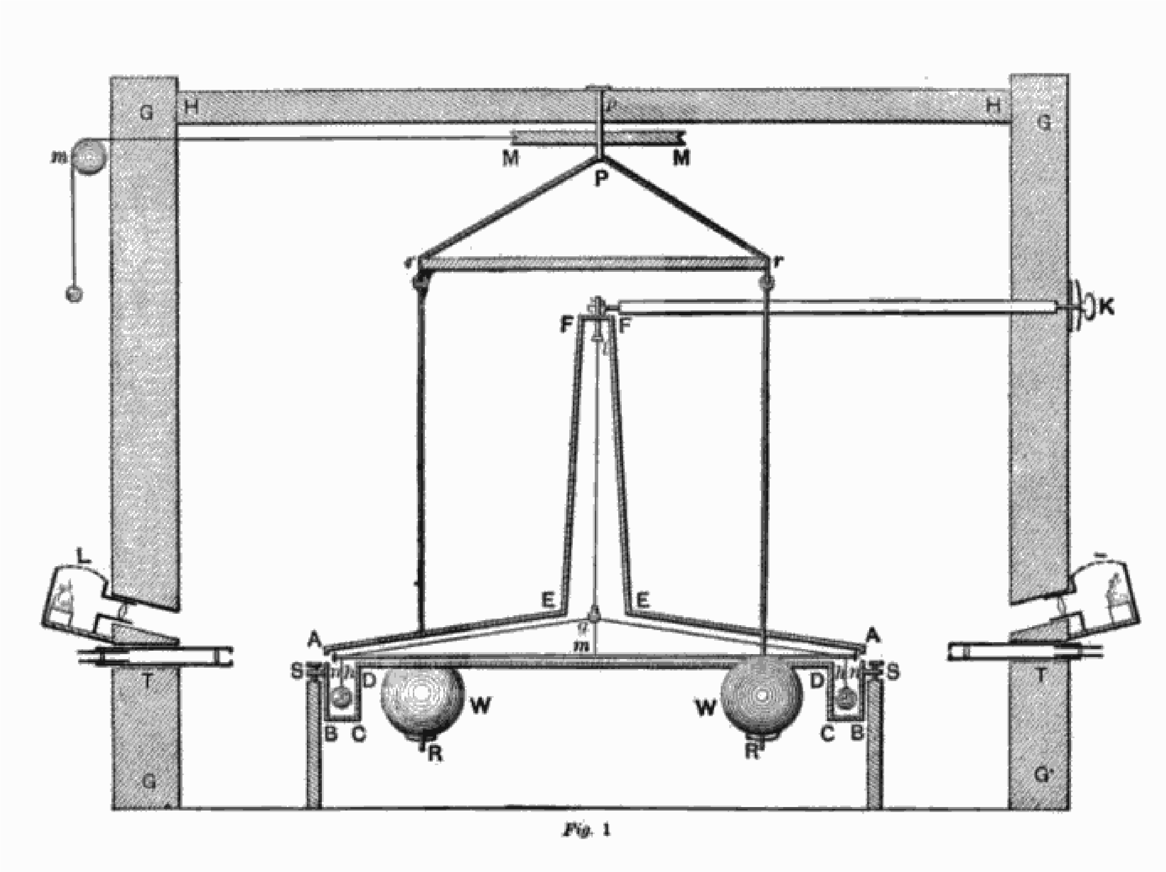


Figure 5.1: Henry Cavendish's sketch of his experimental setup.

While Cavendish's main objective was the measurement of the density of Earth, in fact he had performed a measurement of the Gravitational constant G to a remarkable accuracy ($\approx 1\%$).

High precision torsion balances are still used nowadays to test fundamental principles of general relativity (the equivalence of the inertial and gravitational mass) and possible deviations of the Newton's law of gravitation. In this lab, you will measure the Gravitational constant with a modern version of the Cavendish torsion balance.

5.3 Apparatus and Theory

The setup and measurement of the Gravitational constant is conceptually straightforward (Fig. 5.2). A boom with two small, identical lead spheres (labeled 'A' in the figure) of mass m is suspended by a thin tungsten wire. Since the masses are identical, the system is in equilibrium under the Earth gravitational field. At a certain moment, two large lead spheres (B) of mass M are placed close to the A spheres.

The gravitational force between spheres A and B is given by Eq. 5.1. This force induces a torque τ_G on the boom which starts to rotate. From the laws of physics, the torque is given by

$$\tau_G = 2F_G d \tag{5.2}$$

where d is the distance to the center of A from the center of the boom (the factor 2 comes from the fact that there are two B spheres each exerting a force F_G). The suspending wire, which is fixed at its

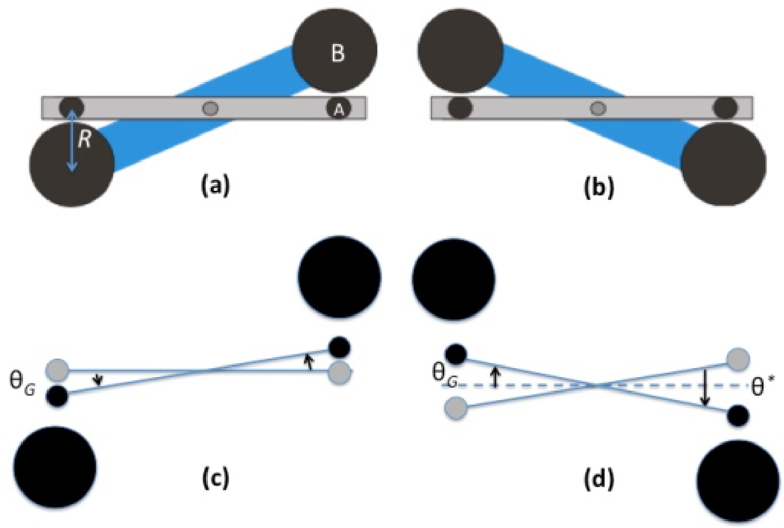


Figure 5.2: A torsion balance. In (a) and (b), top view of the Cavendish balance with two symmetric configurations of the lead spheres. In (c) and (d), the corresponding equilibrium positions and rotation angles.

top end, is twisted by the rotation of the boom (Fig. 5.3), and "tries" to recover its initial position by a counter-torque τ_w. For a small rotation angle θ, τ_w is proportional to θ,

$$\tau_w = k\theta \,, \tag{5.3}$$

where the torsion constant k depends on the wire's material and dimensions.

A new equilibrium position is achieved when the boom has rotated by an angle θ_G (Fig. 5.2(c)) such that the two torques equilibrate themselves, so

$$\tau_G = \tau_w \,. \tag{5.4}$$

From Eqs. (5.1–5.4), the gravitational constant is obtained:

$$G = \frac{k\theta_G R^2}{2Mmd} \,. \tag{5.5}$$

The symmetry of the torsion balance can be exploited to perform a more precise measurement of rotation angle θ_G (which is very small). After the boom has reached the equilibrium in the Fig. 5.2(c) configuration, the B spheres are placed in the symmetric position of Fig. 5.2(b), which results in a new equilibrium position as in Fig. 5.2(d). Thanks to the symmetry of the apparatus, the rotation angle θ_{rotation} between the two configurations is simply twice the angle θ_G. With the Cavendish balance described in Section 5.4, you will measure θ_{rotation} and then use the following equation to find θ_G:

$$\theta_G = \theta_{\text{rotation}}/2 \,. \tag{5.6}$$

When performing the experiment, you will notice that the torsion balance undergoes harmonic oscillations around its equilibrium position. The dampening of the oscillations (that is, the decrease of their amplitude with time) is due primarily to air dragging against the boom. The period T of the oscillations depends on the torsion constant k according to

$$k = 2md^2 \left(\frac{2\pi}{T}\right)^2 \,. \tag{5.7}$$

Note that the quantity $2md^2$ is the *moment of inertia* of the A spheres.

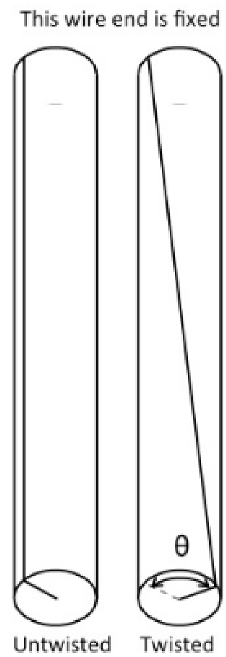


Figure 5.3: Twisted wire of a torsion balance.

5.4 Procedure: Measurement of G with the Cavendish Balance

You will perform a measurement of the Gravitational constant using data taken with a computerized Cavendish balance (Fig. 5.4). The apparatus is placed on a wooden box filled with sand which helps in dampening vibrations (of the building, when you walk close to the apparatus, etc.). The rotation angle is measured by a "differential capacitive sensor" which provides a more stable and convenient way of recording when compared to other methods (e.g. using a laser). The apparatus is extremely sensitive and requires several hours of preparation. For this reason, it is not possible to set it up and take good data during a standard two-hour lab session. Thus, you will find the apparatus ready for the measurements, which will be performed by the TA by moving the B spheres in the two configurations of Fig. 5.2. You will notice that after the TA has moved the spheres, the boom will start to oscillate and it will take about one hour for the oscillations to dampen around a new equilibrium position. The TA will then provide you with the data after the session. Sometimes the data collected during your session will not be of good enough quality for the measurement (this is indeed a very difficult experiment and we are trying to do it while you walk around the room causing vibrations), and in such case you will be provided a set of data from another session. An example of data is given in Fig. 5.5 (your data will be significantly noisier).

By analyzing the data, you can determine the Gravitational constant with the following procedure:

1. Measure the period T of oscillation (e.g. by measuring the time between two consecutive maxima of the damping oscillations), and determine k using Eq. (5.7). For a more accurate measurement of T, measure the time corresponding to several oscillations and divide it by the number of oscillations. Use Fig. 5.6 for the values of the masses and the geometrical parameters.

2. Measure the rotation angles θ_1 and θ_2 corresponding to the equilibrium positions of the first and second oscillation, respectively (see Fig. 5.5). Try various ways to measure the rotation angles (e.g. fitting by eye an horizontal line; sum the maxima and minima of consecutive oscillations).

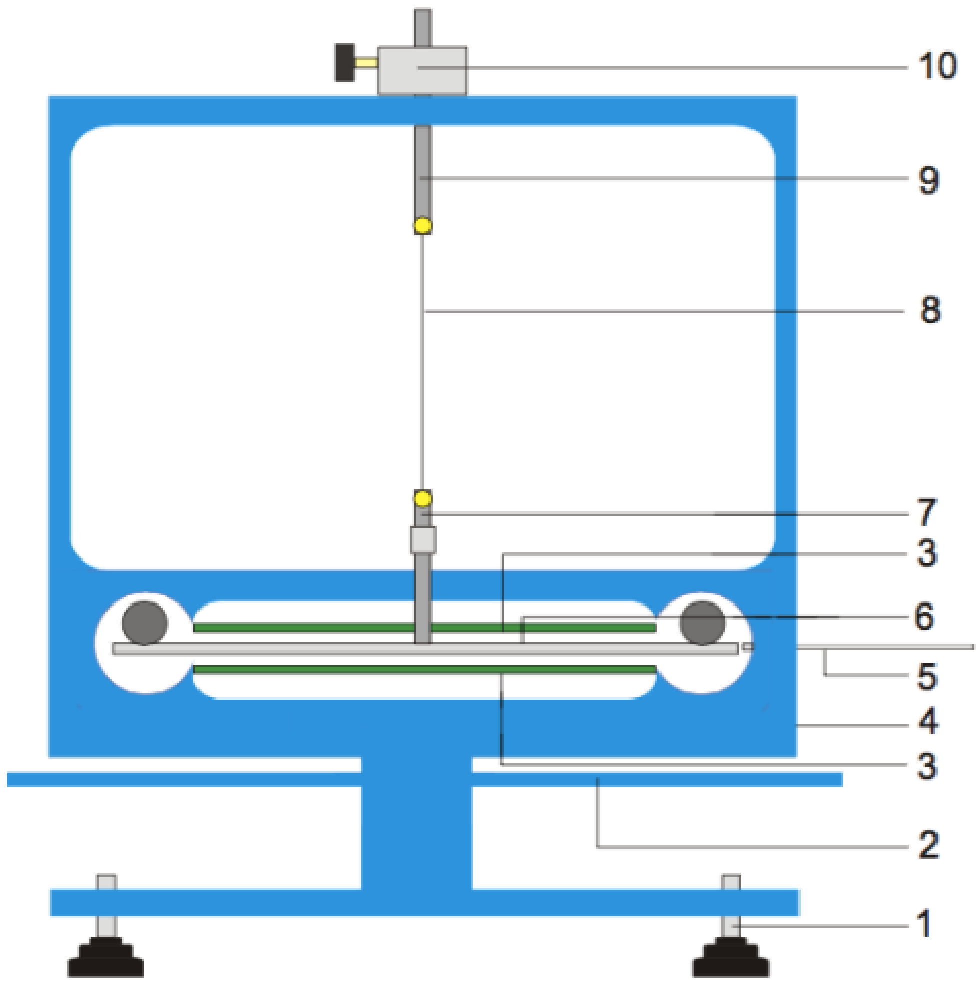


Fig. 1 Components of a Cavendish torsion balance

1 Adjustable feet, 2 Outer supporting beam for large lead spheres, 3 Capacitive differential sensor, 4 USB port, 5 Centring rod, 6 Inner supporting beam for small lead spheres, 7 Lower suspension rod with mirror, 8 Tungsten wire, 9 Upper suspension rod, 10 Rotatable mounting with protractor scale

Figure 5.4: The computerized Cavendish balance.

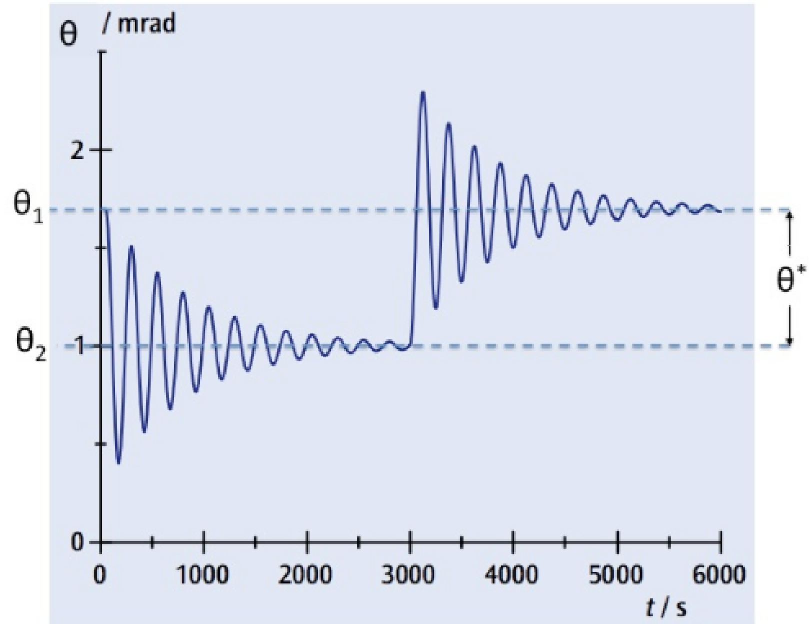


Figure 5.5: Example of data from the Cavendish balance.

You can then calculate

$$\theta_{\text{rotation}} = |\theta_1 - \theta_2| \tag{5.8}$$

and use Eq. (5.6) to derive θ_G.

3. Determine the gravitational constant G using Eq. (5.5) and compare your result with its nominal value.

4. Note that the mathematical procedure given here makes some assumptions about what is important to include. Discuss with your group to identify those assumptions and report these in your report.

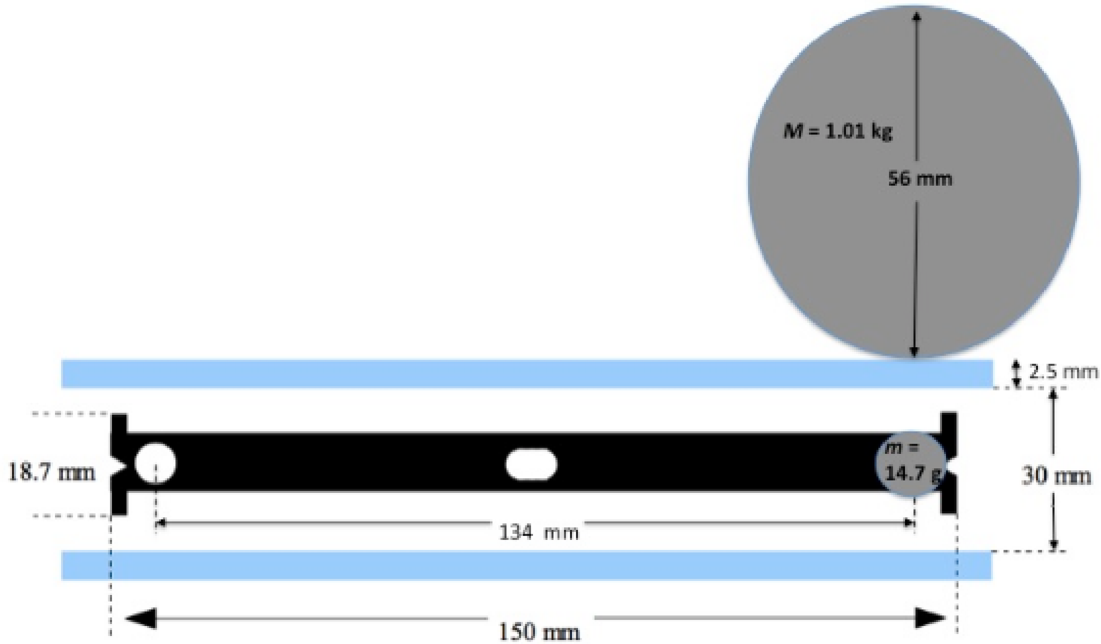


Figure 5.6: Geometry of the Cavendish balance and lead spheres masses.

5.5 Procedure: Build the Cavendish Balance

While the computerized Cavendish balance is acquiring data, you will build and calibrate a similar apparatus. This part of the lab will give you a deeper understanding of the Cavendish balance, and a hands-on experience on the extreme sensitivity of the apparatus. A description of the PASCO balance and instructions are given in the manual in Appendix D. Read pages 1–3 of the manual and familiarize yourself with the different components of the balance.

1. First, mount the thin tungsten wire which will be used to suspend the pendulum bob. The mounting mechanism is shown in Figs. 5.7–5.8. A good eye, cooperation within you group, and patience is required for this step; be careful in handling the wire since it can break (and probably will until you get enough practice).

2. To install the wire, screw the small aluminum block onto the tab as in Fig. 5.7 (you need to assemble two of these pieces).

3. Mount one of them on the brass rod (torsion wire head) as in Fig. 5.8.

4. Cut $\approx 50\,\mathrm{cm}$ of wire from the spool; be careful in keeping the wire straight avoiding kinks.

5. Take one end of the wire, slide it between the aluminum block and the plate (slightly loosen the screw to let the wire in) and make a loop around the screw; tighten the screw while keeping the wire (and the aluminum block) straight.

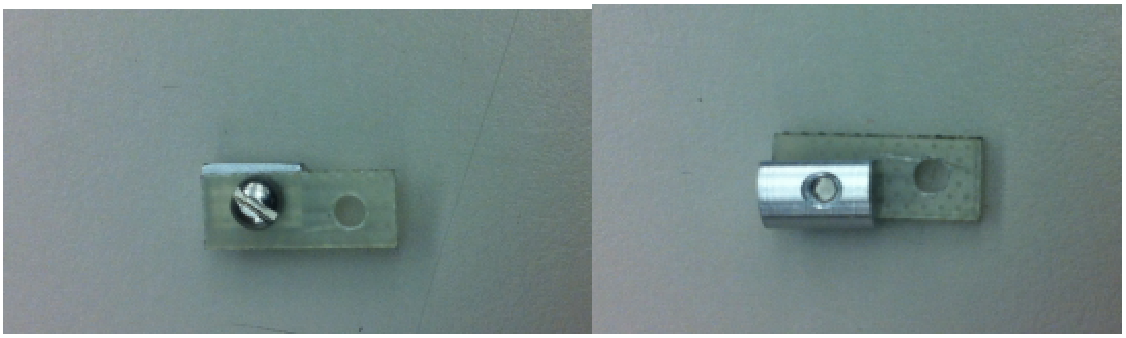


Figure 5.7: Wire mounting pieces, aluminum block and tab.

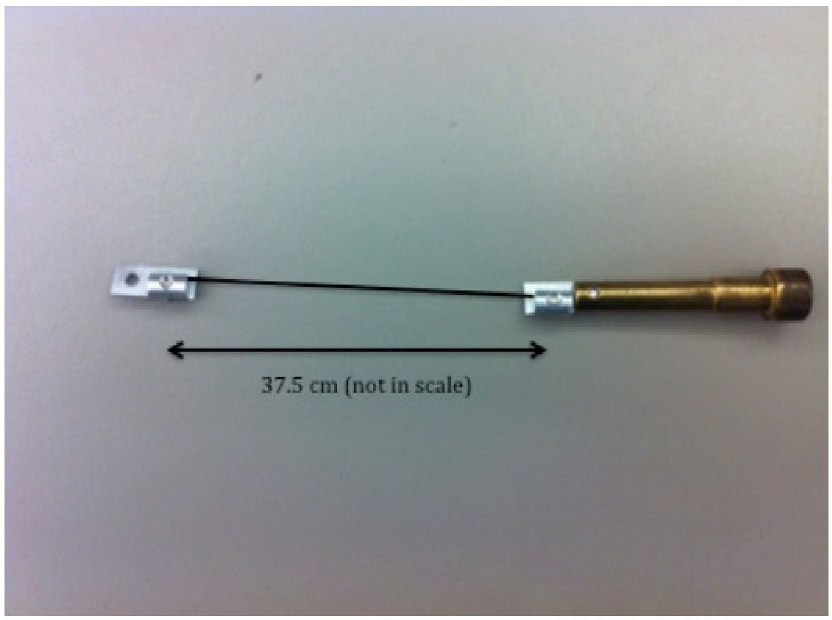


Figure 5.8: Assembly of torsion wire.

6. Take the other end of the wire and do the same with the other mounting piece; the length of the wire between the two mounting screws must be 37.5 cm (Fig. 5.8).
7. Thread the wire through the shaft of the Cavendish apparatus (you may need the help of a thin screwdriver to slide the mounting piece in).
8. Using the zero adjust knob (see PASCO manual), align the bottom tab with the face of the pendulum bob.
9. Tighten the screw on the top of the balance to secure the torsion wire head.
10. Attach the bottom tab to the pendulum bob using the Philips screw.

Also, check the Section "Maintenance" of the PASCO manual which provides a description of a similar procedure using a torsion ribbon.

Once you have successfully completed this step, you can proceed to leveling the Gravitational torsion balance, to perform the vertical adjustment of the pendulum and the rotational alignment of the pendulum bob arms. Follow the instructions on pages 3–6 of the PASCO manual. Ask the TA for advice and help if necessary. At the end of the session, show to the TA which steps you have been able to complete.

5.6 Procedure: Measure *G* with the PASCO Cavendish balance

Once you have your own Cavendish balance built and leveled (see the previous section), you are ready to take your own data.

Since the expected deflection angle θ_G is on the order of milliradians, measuring it with a protractor directly is not a practical solution (the instrumental uncertainty of a typical protractor is $0.5°\approx 9$ milliradians). Instead, you will direct a laser beam toward a mirror mounted on the pendulum bob and measure the linear displacement of reflected ray on a screen on the other side of the room (the larger the distance to the screen, L, the larger the linear displacement d for the same bob deflection angle θ_d, which leads to a smaller uncertainty). Then, you will use that displacement length to calculate that angle, according to

$$\theta_d = \frac{d}{2L}\,. \tag{5.9}$$

Here is a suggested step-by-step procedure:

1. Ensure that the laser, pendulum, and screen are arranged such that the laser can strike the pendulum mirror and reflect onto the screen, which is set up across the room (the larger the distance, the less uncertainty in the angle determination).

2. Measure and record the distance L from the pendulum to the screen.

3. Rotate the large-mass support arm so that it makes an approximately 90°angle with the pendulum arm.

4. Setup and level the balance according to pages 3–6 of the equipment manual in Appendix D, sections Initial Setup, Leveling the Gravitational Torsion Balance, Vertical Adjustment of the Pendulum, and Rotational Alignment of the Pendulum Bob Arms. In this last section, you do not need to be extremely precise about alignment — aim for "roughly aligned". We will correct for any misalignment by using differences in angles, rather than the angles themselves.

5. Decide how to collect the data. Your aim is to measure the linear displacement of the beam from an arbitrarily marked zero point on the screen, as a function of time. The result should be a table of (time,displacement) values.

   - To take data manually, use a stopwatch, meter stick, and teamwork to take data at regular time intervals that are close enough together to capture the periodic motion.
   - To take data with video tracking, record the movement of the laser beam on the screen with a camera whose line of sight is at roughly a right angle to the surface of the screen. Load the video in Open Source Physics Tracker and record the position of the laser similar to the local gravitational field ("little gee") lab.

6. Quiet the pendulum by following Steps 5a and 5b from page 5 of the equipment manual.

7. Slowly and gently rotate the large-mass support arms counter-clockwise until one of the large spheres is touching the case plate.

8. Collect the data according to your chosen method. Ensure that you record several complete cycles of oscillation.

9. Rotate the large-mass support arms clockwise until a sphere is touching a case plate.

10. Continue collecting data, ensuring that you record several complete cycles of oscillation.

11. Using Tracker or the manual method, create a spreadsheet with columns Time (s) and Displacement (m).

12. Create another column "Deflection Angle (rad)" and calculate this angle for each row using Eq. 5.9.

13. Follow the numbered steps in Section 5.4 to calculate G and its uncertainty.

5.7 Calculations & Analysis

The data acquired by the computerized Cavendish balance are provided as an Excel file, which also includes a plot of this data. The plot shows rotation angle (in milliradians, where 1 milliradian = 0.001 radians, on the vertical axis) as a function of time (in seconds, on the horizontal axis). Follow Section 5.3 to do the necessary data analysis and measure the gravitational constant G. You will need to measure (1) the period of oscillations T and (2) the rotation angles corresponding to the two equilibrium positions. You are interested in the difference between the angles.

APPENDIX A

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, "$5 \pm 1\,\mathrm{m}$" means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters[1]. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

A.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument's scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment's manual often gives the instrument's resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

[1] The phrase "most likely" can mean different things depending on who is writing. If a physicist gives the value and does not given a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet, or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}}\,, \tag{A.1}$$

where $\{x_1, x_2, \ldots, x_N\}$ are the measured values, $\bar{x}$ is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called "standard deviation of the mean", which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}\,. \tag{A.2}$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as 52 ± 3 m.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn't change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

A.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction), } \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{A.3}$$

$$\text{For } c = ab \text{ (or for division), } \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \tag{A.4}$$

$$\text{For } c = a^n,\ \frac{\delta c}{c} = n\frac{\delta a}{a} \tag{A.5}$$

For other calculations, there is a more general formula not discussed here.

Expression	**Implied uncertainty**
12	0.5
12.0	0.05
120	5
120.	0.5

Table A.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π, is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table A.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value "`0.0322857142857143`". Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as "`9.65683578099600e-04`", where the '`e`' stands for "times ten to the". I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as "0.032 ± 0.001 km/s". You may also see the equivalent, more succinct notation "0.032(1) km/s".

A.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced "tee prime"). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as[2]

$$t' = \frac{|a-b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \tag{A.6}$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

[2]Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.

APPENDIX B

Rubrics

Each lab is graded 50% on attendance and participation during the lab, and providing evidence in the lab report of completing all steps of the lab, including answering every question. The other 50% is based on a selection of scientific abilities. Each scientific ability rubric row assessed is worth a possible 3 points, with “Missing” being 0 points, “Inadequate” 1 point, “Needs Improvement” 2 points, and “Adequate” 3 points.

The scientific abilities rubrics are found on the following pages.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
A11	**Graph**	No graph is present.	A graph is present but the axes are not labeled. There is no scale on the axes.	The graph is present and axes are correctly labeled, but the axes do not correspond to the independent and dependent variables, or the scale is not accurate.	The graph has correctly labeled axes, independent variable is along the horizontal axis and the scale is accurate.

Table B.1: Rubric A: Ability to represent information in multiple ways

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B1	**Is able to identify the phenomenon to be investigated**	No phenomenon is mentioned	The description of the phenomenon to be investigated is confusing, or it is not the phenomenon of interest.	The description of the phenomenon is vague or incomplete.	The phenomenon to be investigated is clearly stated.
B2	**Is able to design a reliable experiment that investigates the phenomenon**	The experiment does not investigate the phenomenon.	The experiment may not yield any interesting patterns.	Some important aspects of the phenomenon will not be observable.	The experiment might yield interesting patterns relevant to the investigation of the phenomenon.
B3	**Is able to decide what physical quantities are to be measured and identify independent and dependent variables**	The physical quantities are irrelevant.	Only some of physical quantities are relevant.	The physical quantities are relevant. However, independent and dependent variables are not identified.	The physical quantities are relevant and independent and dependent variables are identified.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B4	**Is able to describe how to use available equipment to make measurements**	At least one of the chosen measurements cannot be made with the available equipment.	All chosen measurements can be made, but no details are given about how it is done.	All chosen measurements can be made, but the details of how it is done are vague or incomplete.	All chosen measurements can be made and all details of how it is done are clearly provided.
B5	**Is able to describe what is observed without trying to explain, both in words and by means of a picture of the experimental setup.**	No description is mentioned.	A description is incomplete. No labeled sketch is present. Or, observations are adjusted to fit expectations.	A description is complete, but mixed up with explanations or pattern. Or the sketch is present but is difficult to understand.	Clearly describes what happens in the experiments both verbally and with a sketch. Provides other representations when necessary (tables and graphs).
B6	**Is able to identify the shortcomings in an experiment and suggest improvements**	No attempt is made to identify any shortcomings of the experiment.	The shortcomings are described vaguely and no suggestions for improvement are made.	Not all aspects of the design are considered in terms of shortcomings or improvements.	All major shortcomings of the experiment are identified and reasonable suggestions for improvement are made.
B7	**Is able to identify a pattern in the data**	No attempt is made to search for a pattern.	The pattern described is irrelevant or inconsistent with the data.	The pattern has minor errors or omissions. Terms like “proportional” used without clarity, e.g. is the proportionality linear, quadratic, etc.	The pattern represents the relevant trend in the data. When possible, the trend is described in words.
B8	**Is able to represent a pattern mathematically (if applicable)**	No attempt is made to represent a pattern mathematically.	The mathematical expression does not represent the trend.	No analysis of how well the expression agrees with the data is included, or some features of the pattern are missing.	The expression represents the trend completely and an analysis of how well it agrees with the data is included.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B9	**Is able to devise an explanation for an observed pattern**	No attempt is made to explain the observed pattern.	An explanation is vague, not testable, or contradicts the pattern.	An explanation contradicts previous knowledge or the reasoning is flawed.	A reasonable explanation is made. It is testable and it explains the observed pattern.

Table B.2: Rubric B: Ability to design and conduct an observational experiment [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C1	**Is able to identify the hypothesis to be tested**	No mention is made of a hypothesis.	An attempt is made to identify the hypothesis to be tested but it is described in a confusing manner.	The hypothesis to be tested is described but there are minor omissions or vague details.	The hypothesis is clearly, specifically, and thoroughly stated.
C2	**Is able to design a reliable experiment that tests the hypothesis**	The experiment does not test the hypothesis.	The experiment tests the hypothesis, but due to the nature of the design it is likely the data will lead to an incorrect judgment.	The experiment tests the hypothesis, but due to the nature of the design there is a moderate chance the data will lead to an inconclusive judgment.	The experiment tests the hypothesis and has a high likelihood of producing data that will lead to a conclusive judgment.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C4	**Is able to make a reasonable prediction based on a hypothesis**	No prediction is made. The experiment is not treated as a testing experiment.	A prediction is made, but it is identical to the hypothesis, OR prediction is made based on a source unrelated to the hypothesis being tested, or is completely inconsistent with hypothesis being tested, OR prediction is unrelated to the context of the designed experiment.	Prediction follows from hypothesis but is flawed because relevant assumptions are not considered, OR prediction is incomplete or somewhat inconsistent with hypothesis, OR prediction is somewhat inconsistent with the experiment.	A prediction is made that follows from hypothesis, is distinct from the hypothesis, accurately describes the expected outcome of the experiment, and incorporates relevant assumptions if needed.
C5	**Is able to identify the assumptions made in making the prediction**	No attempt is made to identify assumptions.	An attempt is made to identify assumptions, but the assumptions are irrelevant or are confused with the hypothesis.	Relevant assumptions are identified but are not significant for making the prediction.	Sufficient assumptions are correctly identified, and are significant for the prediction that is made.
C6	**Is able to determine specifically the way in which assumptions might affect the prediction**	No attempt is made to determine the effects of assumptions.	The effects of assumptions are mentioned but are described vaguely.	The effects of assumptions are determined, but no attempt is made to validate them.	The effects of assumptions are determined and the assumptions are validated.
C7	**Is able to decide whether the prediction and the outcome agree/disagree**	No mention of whether the prediction and outcome agree/disagree.	A decision about the agreement/disagreement is made but is not consistent with the results of the experiment.	A reasonable decision about the agreement/disagreement is made but experimental uncertainty is not taken into account.	A reasonable decision about the agreement/disagreement is made and experimental uncertainty is taken into account.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C8	**Is able to make a reasonable judgment about the hypothesis**	No judgment is made about the hypothesis.	A judgment is made but is not consistent with the outcome of the experiment.	A judgment is made, is consistent with the outcome of the experiment, but assumptions are not taken into account.	A judgment is made, is consistent with the outcome of the experiment, and assumptions are taken into account.

Table B.3: Rubric C: Ability to design and conduct a testing experiment [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D1	**Is able to identify the problem to be solved**	No mention is made of the problem to be solved.	An attempt is made to identify the problem to be solved but it is described in a confusing manner.	The problem to be solved is described but there are minor omissions or vague details.	The problem to be solved is clearly stated.
D2	**Is able to design a reliable experiment that solves the problem.**	The experiment does not solve the problem.	The experiment attempts to solve the problem but due to the nature of the design the data will not lead to a reliable solution.	The experiment attempts to solve the problem but due to the nature of the design there is a moderate chance the data will not lead to a reliable solution.	The experiment solves the problem and has a high likelihood of producing data that will lead to a reliable solution.
D3	**Is able to use available equipment to make measurements**	At least one of the chosen measurements cannot be made with the available equipment.	All of the chosen measurements can be made, but no details are given about how it is done.	All of the chosen measurements can be made, but the details about how they are done are vague or incomplete.	All of the chosen measurements can be made and all details about how they are done are provided and clear.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D4	**Is able to make a judgment about the results of the experiment**	No discussion is presented about the results of the experiment.	A judgment is made about the results, but it is not reasonable or coherent.	An acceptable judgment is made about the result, but the reasoning is incomplete, OR uncertainties are not taken into account, OR assumptions are not discussed, OR the result is written as a single number.	An acceptable judgment is made about the result, with clear reasoning. The effects of assumptions and experimental uncertainties are considered. The result is written as an interval.
D5	**Is able to evaluate the results by means of an independent method**	No attempt is made to evaluate the consistency of the result using an independent method.	A second independent method is used to evaluate the results. However there is little or no discussion about the differences in the results due to the two methods.	A second independent method is used to evaluate the results. The results of the two methods are compared correctly using experimental uncertainties. But there is little or no discussion of the possible reasons for the differences when the results are different.	A second independent method is used to evaluate the results and the evaluation is correctly done with the experimental uncertainties. The discrepancy between the results of the two methods, and possible reasons are discussed.
D7	**Is able to choose a productive mathematical procedure for solving the experimental problem**	Mathematical procedure is either missing, or the equations written down are irrelevant to the design.	A mathematical procedure is described, but is incorrect or incomplete, due to which the final answer cannot be calculated. Or units are inconsistent.	Correct and complete mathematical procedure is described but an error is made in the calculations. All units are consistent.	Mathematical procedure is fully consistent with the design. All quantities are calculated correctly with proper units. Final answer is meaningful.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D8	**Is able to identify the assumptions made in using the mathematical procedure**	No attempt is made to identify any assumptions.	An attempt is made to identify assumptions, but the assumptions are irrelevent or incorrect for the situation.	Relevant assumptions are identified but are not significant for solving the problem.	All relevant assumptions are correctly identified.

Table B.4: Rubric D: Ability to design and conduct an application experiment [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
F1	**Is able to communicate the details of an experimental procedure clearly and completely**	Diagrams are missing and/or experimental procedure is missing or extremely vague.	Diagrams are present but unclear and/or experimental procedure is present but important details are missing. It takes a lot of effort to comprehend.	Diagrams and/or experimental procedure are present and clearly labeled but with minor omissions or vague details. The procedure takes some effort to comprehend.	Diagrams and/or experimental procedure are clear and complete. It takes no effort to comprehend.
F2	**Is able to communicate the point of the experiment clearly and completely**	No discussion of the point of the experiment is present.	The experiment and findings are discussed but vaguely. There is no reflection on the quality and importance of the findings.	The experiment and findings are communicated but the reflection on their importance and quality is not present.	The experiment and findings are discussed clearly. There is deep reflection on the quality and importance of the findings.

Table B.5: Rubric F: Ability to communicate scientific ideas [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
G1	**Is able to identify sources of experimental uncertainty**	No attempt is made to identify experimental uncertainties.	An attempt is made to identify experimental uncertainties, but most are missing, described vaguely, or incorrect.	Most experimental uncertainties are correctly identified. But there is no distinction between random and instrumental uncertainty.	All experimental uncertainties are correctly identified. There is a distinction between instrumental and random uncertainty.
G2	**Is able to evaluate specifically how identified experimental uncertainties affect the data**	No attempt is made to evaluate experimental uncertainties.	An attempt is made to evaluate uncertainties, but most are missing, described vaguely, or incorrect. Or the final result does not take uncertainty into account.	The final result does take the identified uncertainties into account but is not correctly evaluated. Uncertainty propagation is not used or is used incorrectly.	The experimental uncertainty of the final result is correctly evaluated. Uncertainty propagation is used appropriately.
G3	**Is able to describe how to minimize experimental uncertainty and actually do it**	No attempt is made to describe how to minimize experimental uncertainty and no attempt to minimize is present.	A description of how to minimize experimental uncertainty is present, but there is no attempt to actually minimize it.	An attempt is made to minimize the uncertainty in the final result is made but the method is not very effective.	The uncertainty is minimized in an effective way.
G4	**Is able to record and represent data in a meaningful way**	Data are either absent or incomprehensible.	Some important data are absent or incomprehensible. They are not organized in tables or the tables are not labeled properly.	All important data are present, but recorded in a way that requires some effort to comprehend. The tables are labeled but labels are confusing.	All important data are present, organized, and recorded clearly. The tables are labeled and placed in a logical order.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
G5	**Is able to analyze data appropriately**	No attempt is made to analyze the data.	An attempt is made to analyze the data, but it is either seriously flawed or inappropriate.	The analysis is appropriate but it contains errors or omissions.	The analysis is appropriate, complete, and correct.

Table B.6: Rubric G: Ability to collect and analyze experimental data [3].

APPENDIX C

Lab Report Format

Following the last session of a particular lab, each person will be responsible for turning in a lab report. In a general sense, the labs should demonstrate Rubric Rows F1 and F2 (see Table B.5), in addition to the other rubric rows listed in the lab write-up.

C.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the other members of your lab team.
- If the rubric row refers to a particular part of your lab report, clearly label that part of the report with that rubric row. For example, you should label the section where you demonstrate uncertainty propagation with "G2" if that rubric row is being assessed in that lab.

C.2 Organizing the report

If the lab is clearly framed as an observational, testing, or application experiment, you can follow the corresponding rubric for the elements to include in the report (see, respectively, Rubrics B, C, and D in Appendix B).

In general, the report **must** include the following sections:

1. **Introduction.** A written description of what the lab is designed to investigate and a brief summary of the procedure used. This section should be at least a full paragraph long, and not more than 3 double-space pages.

   You don't need to include too much detail here, but it should be a complete and concise description of the purpose and general method used in the lab. Imagine a classmate who hasn't seen the lab writeup asked you, "what is this lab about? What do you do?" You should be able to hand them your introduction, and they'd be able to understand the purpose and general structure of the lab. But you need not mention every step and calculation here.

2. **Analysis and discussion.** For most labs that have more than one part, this should be broken up into parts and labeled in order. This section must include all of the following, in the same order in which these elements appear in the lab instructions.

- Any data that you've collected: tables, figures, measured values, sketches. Whenever possible, include an estimate of the uncertainty of measured values.
- Any calculations that you perform using your data, and the final results of your calculation. Note that you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92\,\mathrm{m/s^2})t + 0.2\,\mathrm{m/s}$.
- Answers to any questions that appear in the lab handout.

3. **Conclusion.** This can be very short, and will generally only require one or two paragraphs. In your conclusion, you should summarize the point of the lab and what you learned, both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

C.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like "distance (pc)". This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like "distance (pc)".
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called "error bars", along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.

APPENDIX D

Manual: PASCO Cavendish Balance

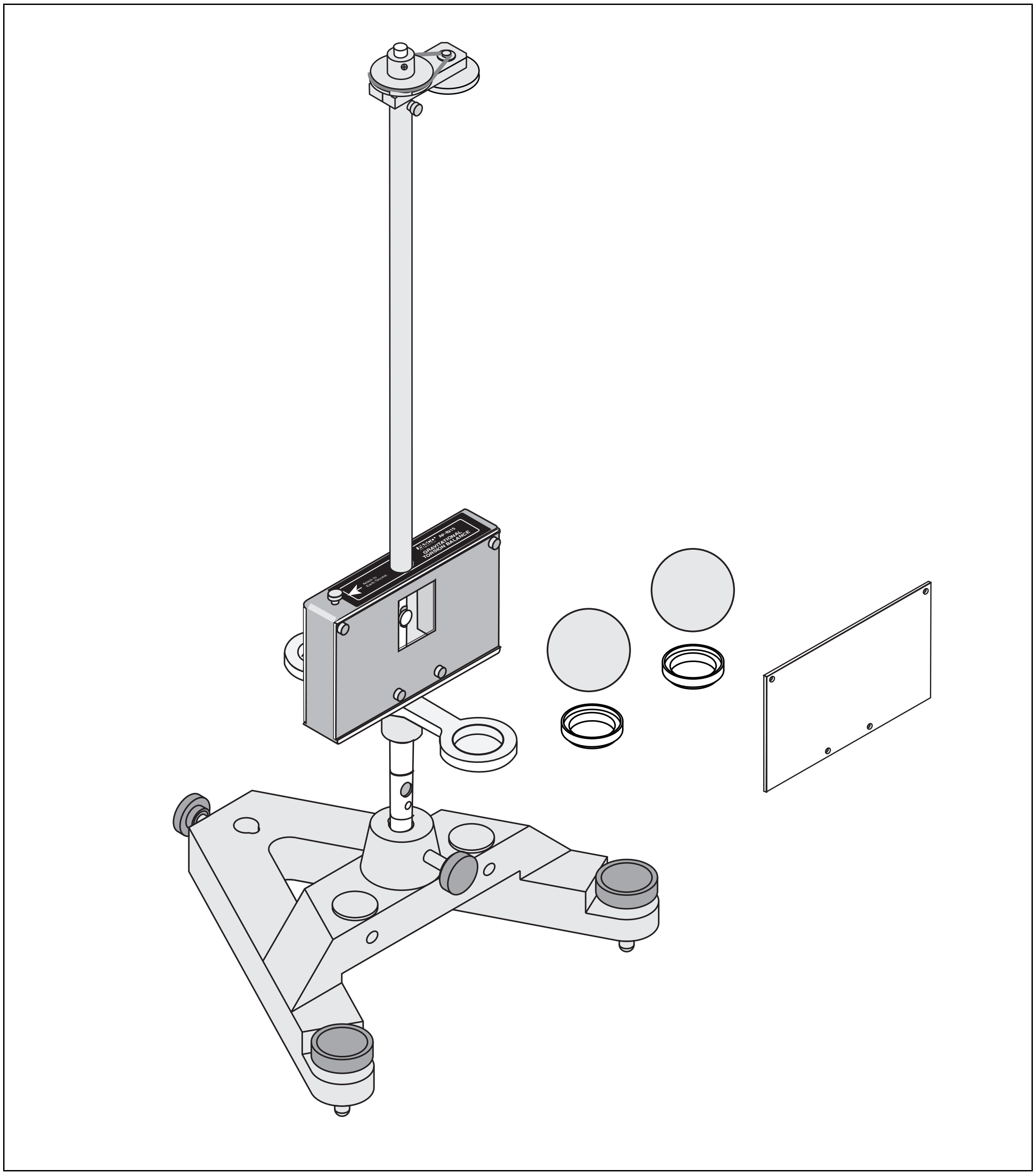
On the following pages is the manual for the PASCO Cavendish Balance.

PASCO®

Instruction Manual with
Experiment Guide and
Teachers' Notes
012-11032D

Gravitational Torsion Balance

AP-8215A



Introduction

The PASCO scientific AP-8215A Gravitational Torsion Balance reprises one of the great experiments in the history of physics—the measurement of the gravitational constant, as performed by Henry Cavendish in 1798.

The Gravitational Torsion Balance consists of two 38.3 gram masses suspended from a highly sensitive torsion ribbon and two 1.5 kilogram masses that can be positioned as required. The Gravitational Torsion Balance is oriented so the force of gravity between the small balls and the earth is negated (the pendulum is nearly perfectly aligned vertically and horizontally). The large masses are brought near the smaller masses, and the gravitational force between the large and small masses is measured by observing the twist of the torsion ribbon.

An optical lever, produced by a laser light source and a mirror affixed to the torsion pendulum, is used to accurately measure the small twist of the ribbon. Three methods of measurement are possible: the final deflection method, the equilibrium method, and the acceleration method.

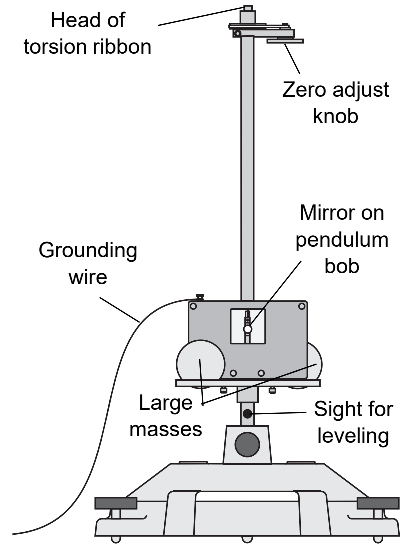


Figure 1: Assembled Gravitational Torsion Balance, ready to begin Henry Cavendish's classic experiment to determine the gravitational constant.

A Little Background

The gravitational attraction of all objects toward the Earth is obvious. The gravitational attraction of every object to every other object, however, is anything but obvious. Despite the lack of direct evidence for any such attraction between everyday objects, Isaac Newton was able to deduce his law of universal gravitation.

However, in Newton's time, every measurable example of this gravitational force included the Earth as one of the masses. It was therefore impossible to measure the constant, *G*, without first knowing the mass of the Earth (or vice versa).

The answer to this problem came from Henry Cavendish in 1798, when he performed experiments with a torsion balance, measuring the gravitational attraction between relatively small objects in the laboratory. The value he determined for *G* allowed the mass and density of the Earth to be determined. Cavendish's experiment was so well constructed that it was a hundred years before more accurate measurements were made.

Newton's Law of Universal Gravitation:

$$F = G\frac{m_1 m_2}{r^2}$$

where m_1 and m_2 are the masses of the objects, r is the distance between their centers, and G is the universal gravitational constant, 6.67×10^{-11} Nm^2/kg^2.

Equipment

Included:

Gravitational Torsion Balance	Plastic Plate
Support Base with Leveling Feet	Replacement Torsion Ribbon*
1.5 kg Tungsten Balls (2)	2-56 x 1/8 Phillips head screws (4)
Adapter Rings (2)	Phillips screwdriver (not shown)

(*Model No. AP-8218)

Additional Required:

Laser light source (such as the PASCO OS-9171 Helium-Neon Laser)

Meter stick

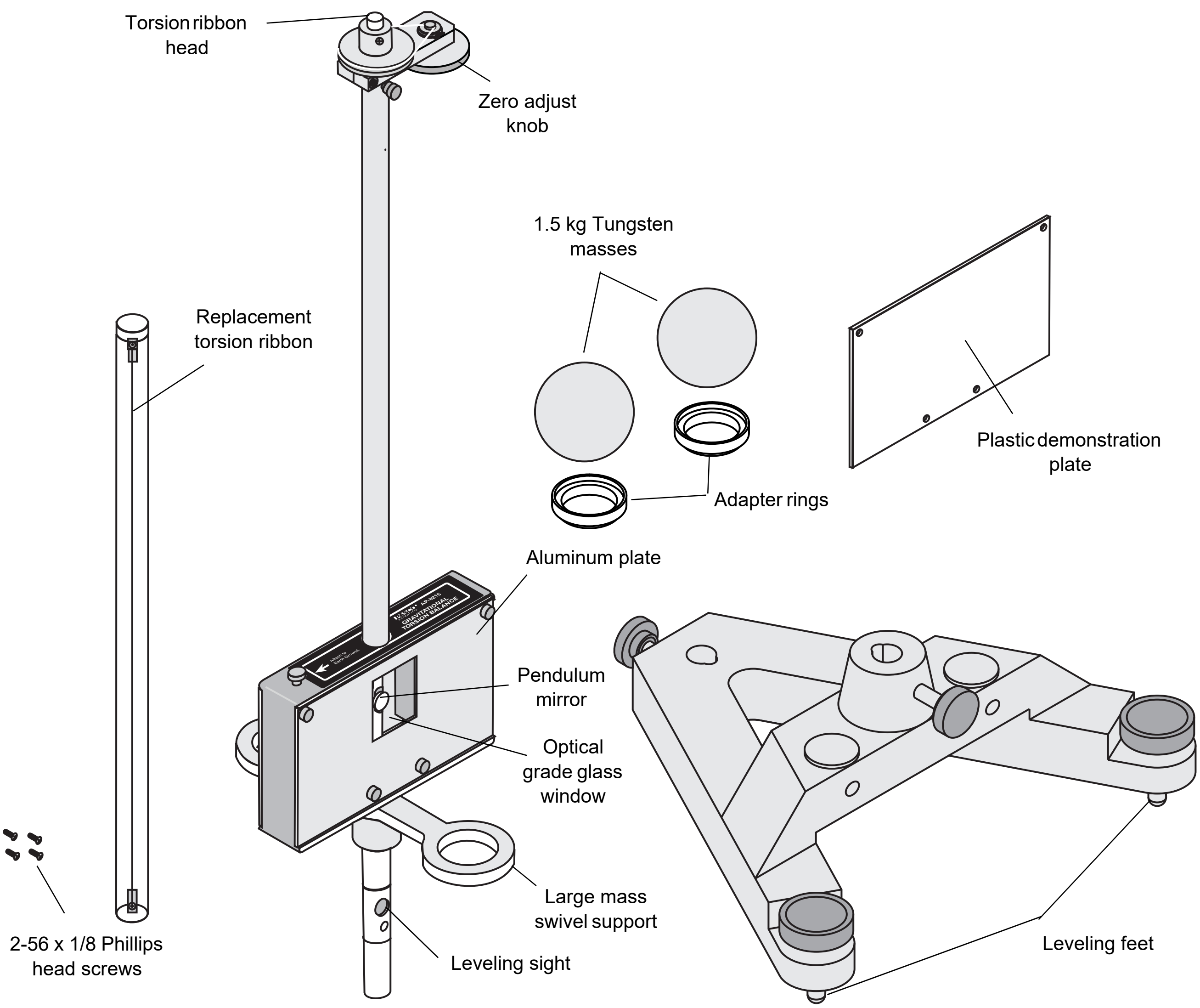


Figure 2: Equipment included

Equipment Parameters

IMPORTANT NOTES

1. The Gravitational Torsion Balance is a delicate instrument. We recommend that you set it up in a relatively secure area where it is safe from accidents and from those who don't fully appreciate delicate instruments.

2. The first time you set up the torsion balance, do so in a place where you can leave it for at least one day before attempting measurements, allowing time for the slight elongation of the torsion ribbon that will occur initially.

3. Keep the pendulum bob secured in the locking mechanisms at all times, except while setting up and conducting experiences.

- Small tungsten balls

  Mass: 38.3 g ±0.2 g (m_2)
  Radius: 8.19 mm
  Distance from ball center to torsion axis: d = 50.0 mm

- Large tungsten balls

  Mass: 1500 ±10 g (m_1)
  Radius: 27.6 mm

- Distance from the center of the large ball to the center of mass of the small ball when the large ball is against the aluminum plate and the small ball is in the center position within the case: b = 42.2 mm. (Note: Tolerances may vary depending on the accuracy of the horizontal alignment of the pendulum.)
- Distance from the surface of the mirror to the outer surface of the glass window: 11.4 mm
- Torsion Ribbon

  Material: Beryllium Copper
  Length: approximately 260 mm
  Cross-section: 0.017 by 0.150 mm

Equipment Setup

Initial Setup

1. Place the support base on a flat, stable table that is located such that the Gravitational Torsion Balance will be at least 5 meters away from a wall or screen.

***Note**:* For best results, use a very sturdy table, such as an optics table.

2. Carefully remove the Gravitational Torsion Balance from the box, and secure it in the base.

3. Remove the front aluminum plate by removing the thumbscrews (Figure 3), and carefully remove the packing foam from the pendulum chamber.

Note: Save the packing foam, and reinstall it each time the Gravitational Torsion Balance is transported.

4. Fasten the clear plastic plate to the pendulum chamber with the thumbscrews.

Note: Do not touch the mirror on the pendulum.

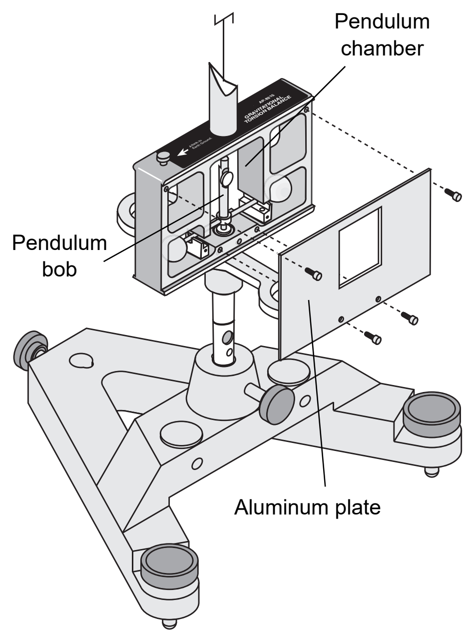


Figure 3: Removing a plate from the chamber box

Leveling the Gravitational Torsion Balance

1. Release the pendulum from the locking mechanism by unscrewing the locking screws on the case, lowering the locking mechanisms to their lowest positions (Figure 4).

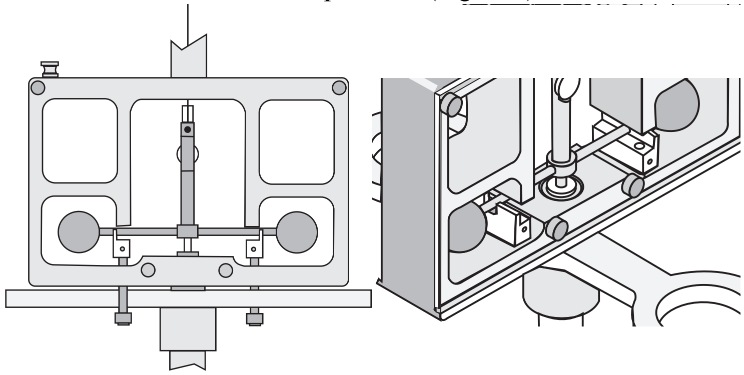


Figure 4: Lowering the locking mechanism to release the pendulum bob arms

2. Adjust the feet of the base until the pendulum is centered in the leveling sight (Figure 5). (The base of the pendulum will appear as a dark circle surrounded by a ring of light).

3. Orient the Gravitational Torsion Balance so the mirror on the pendulum bob faces a screen or wall that is at least 5 meters away.

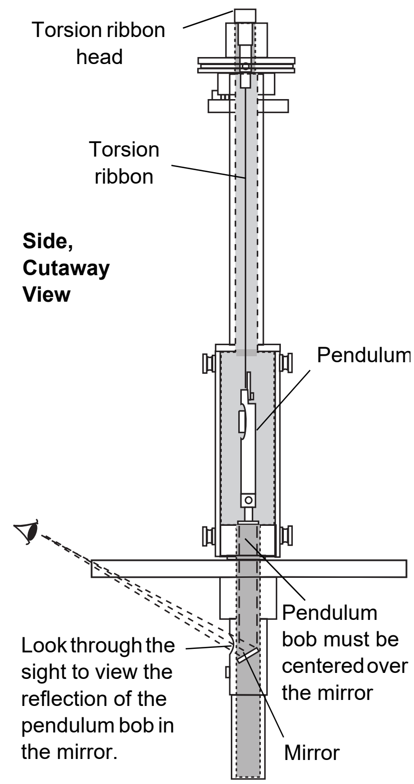


Figure 5: Using the leveling sight to level the Gravitational Torsion Balance.

Vertical Adjustment of the Pendulum

The base of the pendulum should be flush with the floor of the pendulum chamber. If it is not, adjust the height of the pendulum:

1. Grasp the torsion ribbon head and loosen the Phillips retaining screw (Figure 6a).

2. Adjust the height of the pendulum by moving the torsion ribbon head up or down so the base of the pendulum is flush with the floor of the pendulum chamber (Figure 6b).

3. Tighten the retaining (Phillips head) screw.

***Note**:* Vertical adjustment is only necessary at initial setup and when you change the torsion ribbon or if someone has loosened the retaining screw by mistake; it is not normally done during each experimental setup.

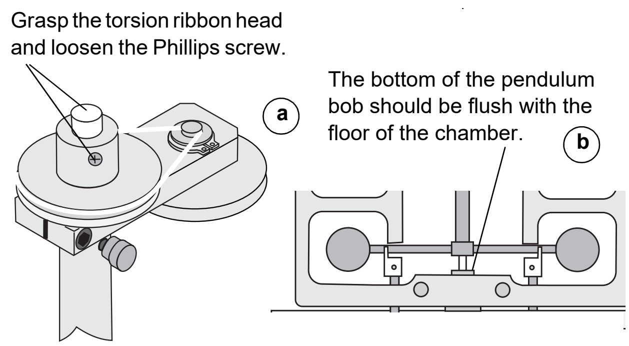


Figure 6: Adjusting the height of the pendulum bob

Rotational Alignment of the Pendulum Bob Arms (Zeroing)

The pendulum bob arms must be centered rotationally in the case — that is, equidistant from each side of the case (Figure 7). To adjust them:

1. Mount a metric scale on the wall or other projection surface that is at least 5 meters away from the mirror of the pendulum.

2. Replace the plastic cover with the aluminum cover.

3. Set up the laser so it will reflect from the mirror to the projection surface where you will take your measurements (approximately 5 meters from the mirror). You will need to point the laser so that it is tilted upward toward the mirror and so the reflected beam projects onto the projection surface (Figure 8). There will also be a fainter beam projected off the surface of the glass window.

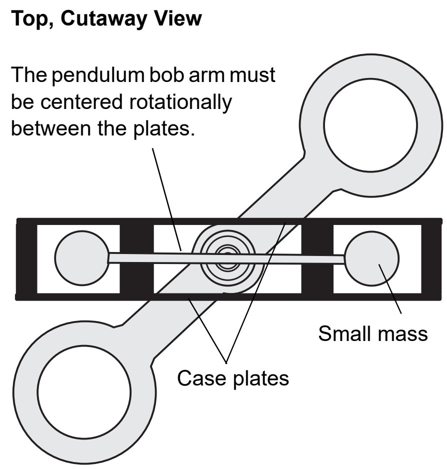


Figure 7: Aligning the pendulum bob rotationally

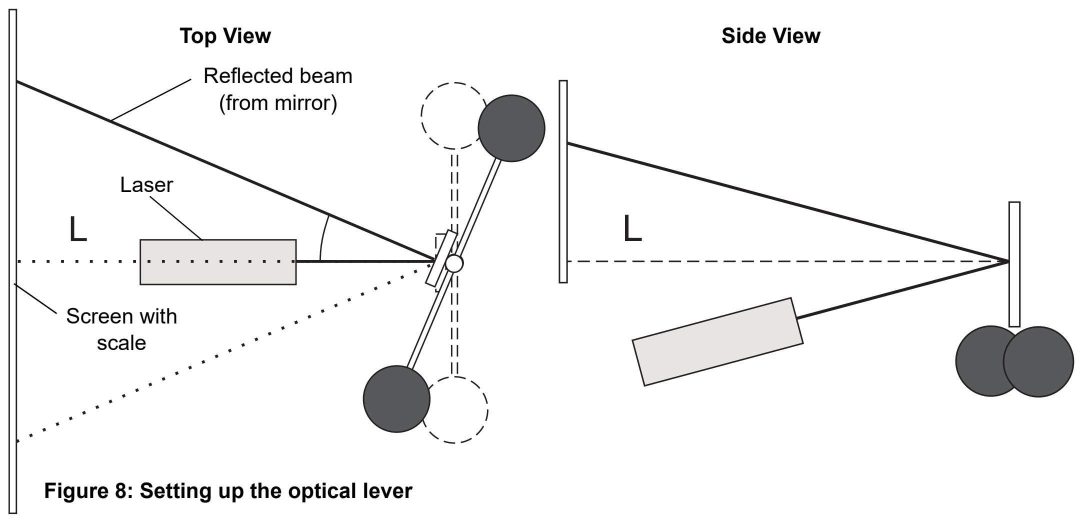


Figure 8: Setting up the optical lever

4. Rotationally align the case by rotating it until the laser beam projected from the glass window is centered on the metric scale (Figure 9).

5. Rotationally align the pendulum arm:

   a. Raise the locking mechanisms by turning the locking screws until both of the locking mechanisms barely touch the pendulum arm. Maintain this position for a few moments until the oscillating energy of the pendulum is dampened.

   b. Carefully lower the locking mechanisms slightly so the pendulum can swing freely. If necessary, repeat the dampening exercise to calm any wild oscillations of the pendulum bob.

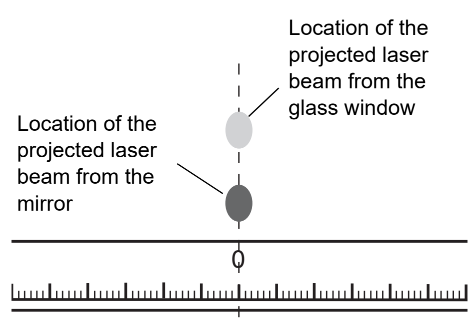


Figure 9: Ideal rotational alignment (zeroing) of the pendulum

   c. Observe the laser beam reflected from the mirror. In the optimally aligned system, the equilibrium point of the oscillations of the beam reflected from the mirror will be vertically aligned below the beam reflected from the glass surface of the case (Figure 9).

d. If the spots on the projection surface (the laser beam reflections) are not aligned vertically, loosen the zero adjust thumbscrew, turn the zero adjust knob slightly to refine the rotational alignment of the pendulum bob arms (Figure 10), and wait until the movement of the pendulum stops or nearly stops.

e. Repeat steps 4a – 4c as necessary until the spots are aligned vertically on the projection surface.

6. When the rotational alignment is complete, carefully tighten the zero adjust thumbscrew, being careful to avoid jarring the system.

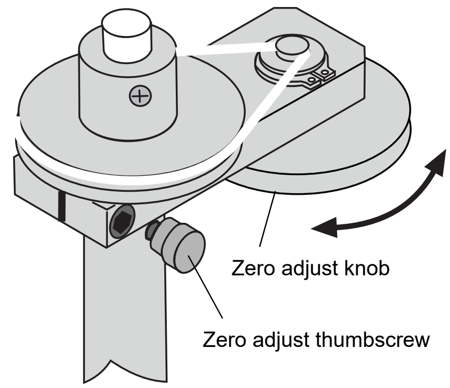


Figure 10: Refining the rotational alignment of the pendulum bob

Hints for speedier rotational alignments:

- Dampen any wild oscillations of the pendulum bob with the locking mechanisms, as described;
- Adjust the rotational alignment of the pendulum bob using small, smooth adjustments of the zero adjust knob;
- Exercise patience and finesse in your movements.

Setting up for the Experiment

1. Take an accurate measurement of the distance from the mirror to the zero point on the scale on the projection surface (L) (Figure 8). (The distance from the mirror surface to the outside of the glass window is 11.4 mm.)

Note: Avoid jarring the apparatus during this setup procedure.

2. Attach copper wire to the grounding screw (Figure 11), and ground it to the earth.

3. Place the adapter rings on the support arm and place the large tungsten masses on the adapter rings, and rotate the arm to Position I (Figure 12), taking care to avoid bumping the case with the masses.

4. Allow the pendulum to come to resting equilibrium.

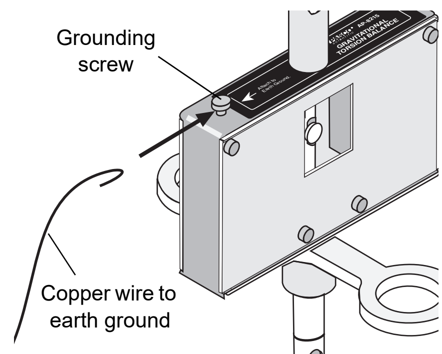
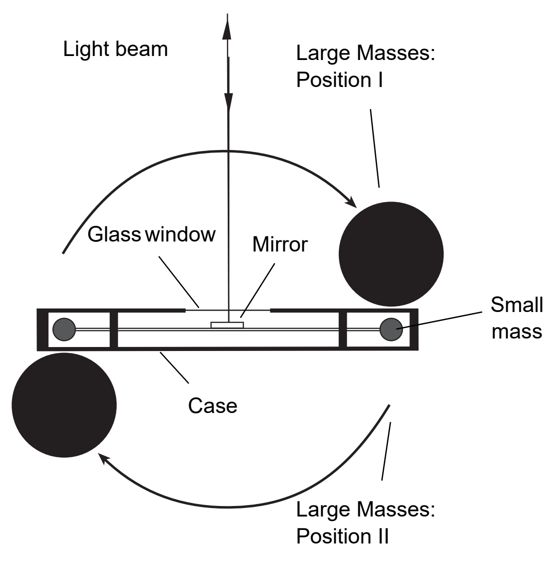


Figure 11: Attaching the ground strap to the grounding screw

You are now ready to make a measurement using one of three methods: the final deflection method, the equilibrium method, or the acceleration method.

Note: The pendulum may require several hours to reach resting equilibrium. To shorten the time required, dampen the oscillation of the pendulum by smoothly raising the locking mechanisms up (by turning the locking screws) until they just touch the crossbar, holding for several seconds until the oscillations are dampened, and then carefully lowering the locking mechanisms slightly.



Maintenance

Replacing the Torsion Ribbon Assembly

If the torsion ribbon breaks, replace it as follows:

1. Remove the plates, and raise the locking mechanism using the locking screws until the pendulum arms are securely anchored (Figure 21a).

2. Grasp the pendulum bob near the bottom ribbon tab to stabilize it.

3. Loosen the Phillips screw on the bottom tab of the torsion ribbon assembly (Figure 21a), and remove the bottom half of the broken ribbon assembly.

4. Loosen the Phillips screw at the top of the balance assembly (Figure 21b).

5. Grasp the torsion ribbon head and remove the top portion of the broken torsion ribbon assembly.

6. Attach the top tab of the new torsion ribbon to the torsion ribbon head using the Phillips screw, being sure the copper disc on the tab is in contact with the torsion ribbon head (Figure 22). Align the tab with the face of the torsion ribbon head.

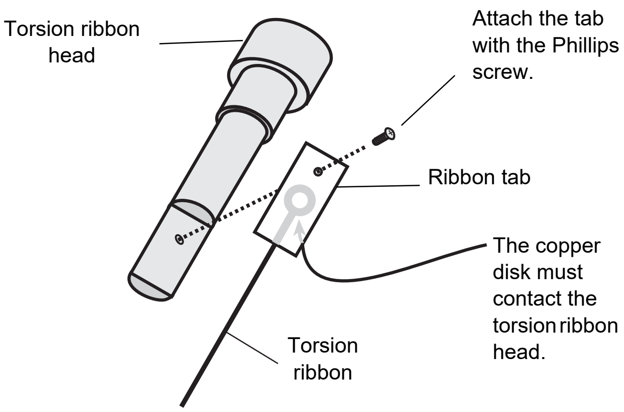


Figure 22: Attaching the top tab of the torsion ribbon assembly to the torsion ribbon head.

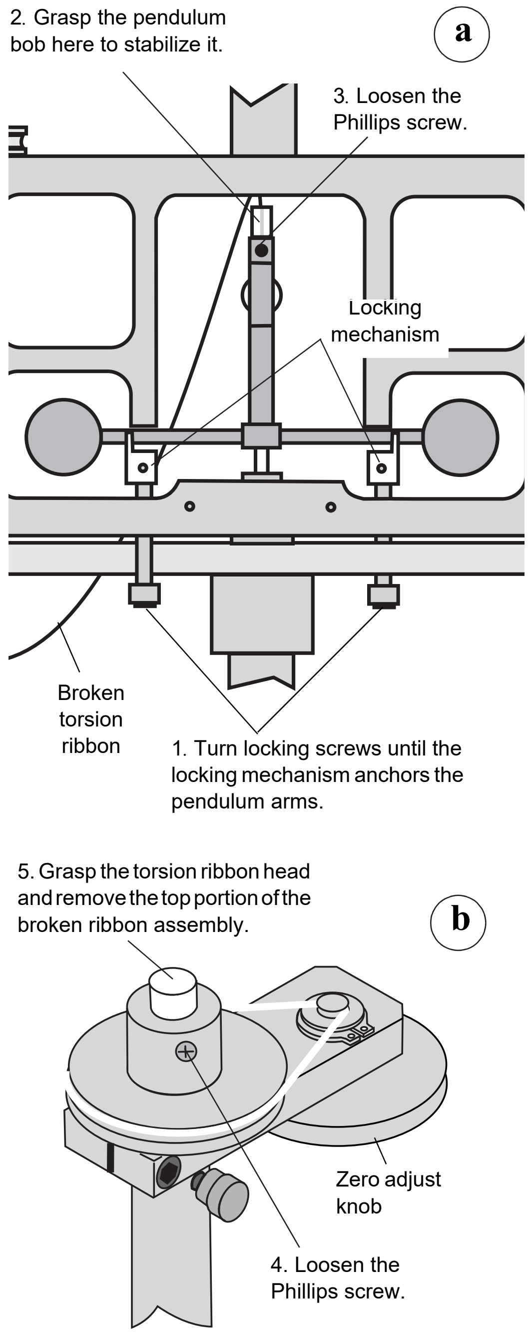


Figure 21: Securing the pendulum bob before removing a broken torsion ribbon, and loosening the torsion ribbon head.

7. Thread the ribbon through the shaft.

8. Using the zero adjust knob, align the bottom tab with the face of the pendulum bob.

⚠ ***Note:* Be sure the ribbon is not twisted.**

9. Tighten the Phillips screw on the top of the balance to secure the torsion ribbon head.

10. Attach the bottom tab of the ribbon to the pendulum bob using the Phillips screw.

11. Replace the back plate.

12. Level and align the pendulum according to the instructions in the *Equipment Setup* section of this manual.

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[2] E. Etkina, "Millikan award lecture: students of physics—listeners, observers, or collaborative participants in physics scientific practices?", American Journal of Physics **83**, 669 (2015).

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