

Laboratory Manual

PHSC 12710 Galaxies

The University of Chicago

Winter 2021

Labs

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Measuring distant objects with parallax

1.1 Introduction

Since it takes time for light to travel to us from objects in the universe, the further out an object is, the further back in time we see it. So for us to have an accurate picture of how the universe was in the past, we need to know how far away things are. For things that are nearby on Earth, we can travel there and see how far we went, or how long we took to get there. For things further away like the moon, we can use Kepler's laws, or we can bounce a beam of light off of it and see how long it takes to get back. For objects outside of our solar system, it would take too long, and the light would disperse too much, for us to use this last technique. For those objects that are still relatively nearby, we can use the parallax technique as the first rung on our distance ladder.

1.2 Forming Groups

If you are attending the lab session live and do not yet have a group, one way the TA could assist is to arrange "speed networking" among those who still need a group. This would involve the TA organizing Zoom Breakout Rooms, where each room is 2-3 students, and each group talks about how they work and what they are looking for in a group member. Then after 5 minutes or so, the Rooms are changed so people are with different people. This could help people get to know each other enough to form lab groups.

1. Once you have a group, meet with each other and decide a) what tools you will use to communicate and collaborate, b) when you will meet, c) what you will do when you need to change an agreement, and d) what you will do when you a person has an issue with how the group is functioning.

1.3 Team roles

Decide on roles for each group member. The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded

- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

1.4 Installing SAOImage DS9

SAOImage DS9, or DS9 for short, is an image viewer, analyzer, and processor written and used by astronomers for working with astronomical images.

2. Download and install DS9 from <http://ds9.si.edu/site/Download.html>. SAOImage DS9, or DS9 for short, is an image viewer, analyzer, and processor written and used by astronomers for working with astronomical images.

If you click the link to download, it might say "redirecting" while never actually redirecting. In this case, copy the link into the address bar directly.

For MacOS, unless you know otherwise, choose from the top set of choices (to the right of the blue apple logo). To find your version, from the Apple menu in the corner of the screen, choose "About This Mac".

If it displays a warning and prevents you from installing from an unidentified developer, follow the instructions at the following link to create an exception:

<https://support.apple.com/guide/mac-help/open-a-mac-app-from-an-unidentified-developer-mh4061/mac>

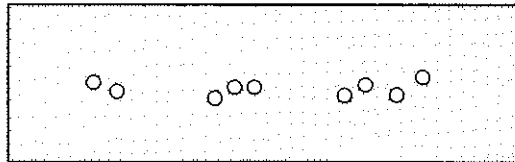
1.5 Worksheet

Complete the worksheet "The Parsec" on the following pages. You can draw the diagrams needed and include a picture of your diagrams, or use a drawing program to draw on them.

Part I: Stars in the Sky

Consider the diagram to the right.

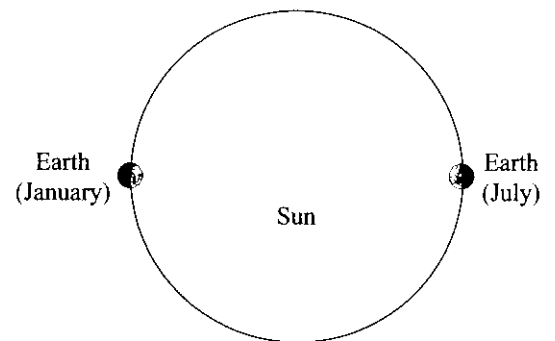
- 1) Imagine that you are looking at the stars from Earth in January. Use a straightedge or a ruler to draw a straight line from Earth in January, through the Nearby Star (Star A), out to the Distant Stars. Which of the distant stars would appear closest to Star A in your night sky in January? Circle this distant star and label it "Jan."
- 2) Repeat Question 1 for July and label the distant star "July."
- 3) In the box below, the same distant stars are shown as you would see them in the night sky. Draw a small \times to indicate the position of Star A as seen in January and label it "Star A Jan."



- 4) In the same box, draw another \times to indicate the position of Star A as seen in July and label it "Star A July."
- 5) Describe how Star A would appear to move among the distant stars as Earth orbits the Sun counterclockwise from January of one year, through July, to January of the following year.

Distant Stars

Nearby Star
(Star A)



The apparent motion of nearby objects relative to distant objects, which you just described, is called **parallax**.

- 6) Consider two stars (C and D) that both exhibit parallax. If Star C appears to move back and forth by a greater amount than Star D, which star do you think is actually closer to you? If you're not sure, just take a guess. We'll return to this question later in this activity.

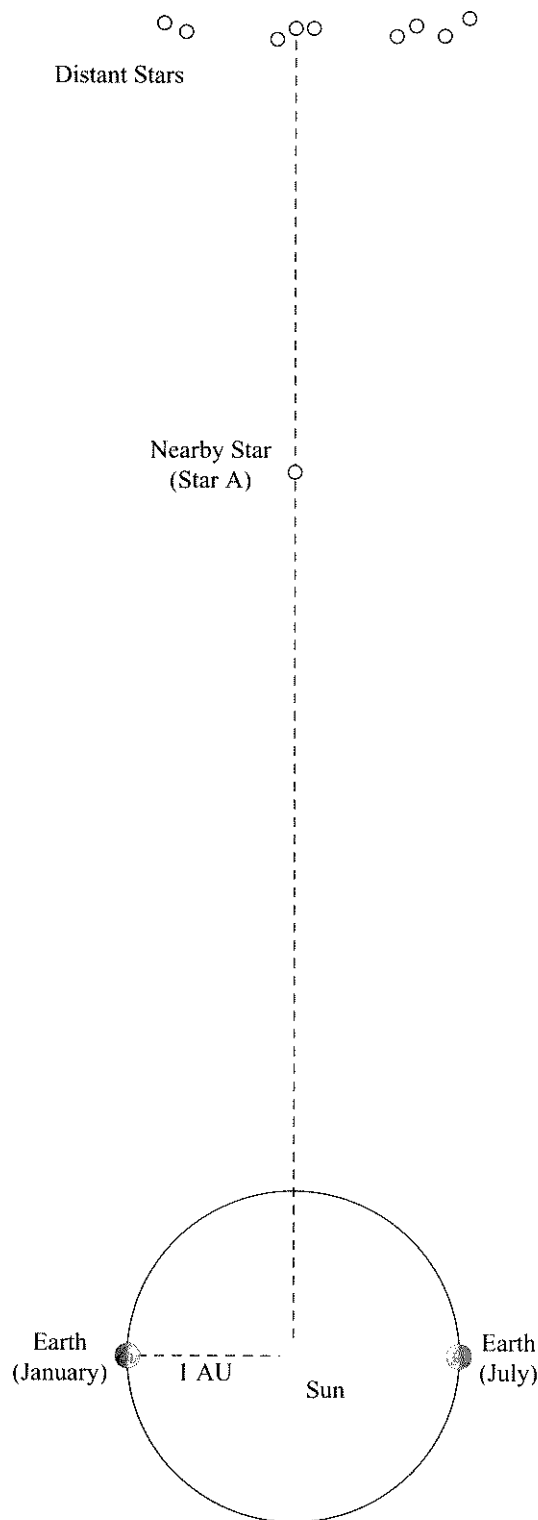
Part II: What's a Parsec?

Consider the diagram to the right.

- 7) Starting from Earth in January, draw a line through Star A to the top of the page.
- 8) There is now a narrow triangle, created by the line you drew, the dotted line provided in the diagram, and the line connecting Earth and the Sun. The small angle, just below Star A, formed by the two longest sides of this triangle is called the **parallax angle** for Star A. Label this angle " p_A ."

Knowing a star's parallax angle allows us to calculate the distance to the star. Since even the nearest stars are still very far away, parallax angles are extremely small. These parallax angles are measured in "arcseconds" where an arcsecond is $1/3600$ of 1 degree.

To describe the distances to stars, astronomers use a unit of length called the **parsec**. One parsec is defined as the distance to a star that has a **parallax angle** of exactly 1 arcsecond. The distance from the Sun to a star 1 parsec away is 206,265 times the Earth–Sun distance or 206,265 AU. (Note that the diagram to the right is not drawn to scale.)



1.6 A quick measurement with hand tools

You will now use the parallax technique to measure the distance to an object in your environment without needing to travel to it.

3. Identify an object to be your “nearby star” and another as your “distant star”, by analogy to the figure in the worksheet. Also choose the positions to view from as “Earth (January)” and “Earth (July)”. When picking the stars and your Earth positions, check to see that from each Earth position, the distant star does not appear to move much, compared to the nearby star, and your guessed distance to the nearby star is at least 10 times greater than the distance between your two Earth positions.

You’ll first use crude measuring tools to compute the nearby star’s distance with the parallax technique, and then use a more precise method. First, you’ll use the little finger on your outstretched arm as a measurement of angular size — the width of the index finger covers (“subtends”) about 1 degree.

4. Identify an object to be your “nearby star” and another as your “distant star”, by analogy to the figure in the worksheet. Find a place where you can stand and move a meter or two side to side and still see both “stars”. The movement will simulate the Earth moving from its January to its July position. When picking the stars and your Earth positions, check to see that from each Earth position, the distant star does not appear to move much, compared to the nearby star, and your guessed distance to the nearby star is at least 10 times greater than the distance between your two Earth positions.
5. Looking from just one eye, move so that the two stars appear to be directly overlapping. Mark your current position as Earth (January). Hold up your smallest finger at arm’s length and move to your left or right until your finger fits just in between the two stars. This means that they are 1 degree away from each other in angular separation. Mark this position as Earth (July).
6. Draw a diagram, similar to the second figure in the worksheet, and find your own Earth-Sun distance (half the distance between your Earth positions). Calculate the parallax angle in radians, which is half the 1 degree you measured with your finger. For the distance measurement, you can use a ruler, measuring tape, or objects that have standard lengths like coins, paper money, or sheets of paper (or Google Maps if the distance is very long, like for mountains or tall buildings in the distance).

Now the distance to the nearby star can be found using the triangle formed by the line segments Sun-Earth, Earth-Star, and Star-Sun (see Figure 1.1). Trigonometry relates these lengths to each other according to

$$\tan p = \frac{a}{d}, \quad (1.1)$$

where p is the parallax angle in radians, d is the distance to the star, and a is half of the distance between the two measurement positions. Since the length d is much greater than a , the angle p is very small, and so we can use the small angle approximation $\tan u \approx u$, and therefore

$$p = \frac{a}{d}. \quad (1.2)$$

7. Use Equation 1.2 to calculate the distance to the nearby star.
8. Conduct the data collection twice more to get two more Earth-Star distances. You can use this to calculate the *measurement uncertainty* — the uncertainty is half the range of values, and the distance is the average of the 3 distances you found.

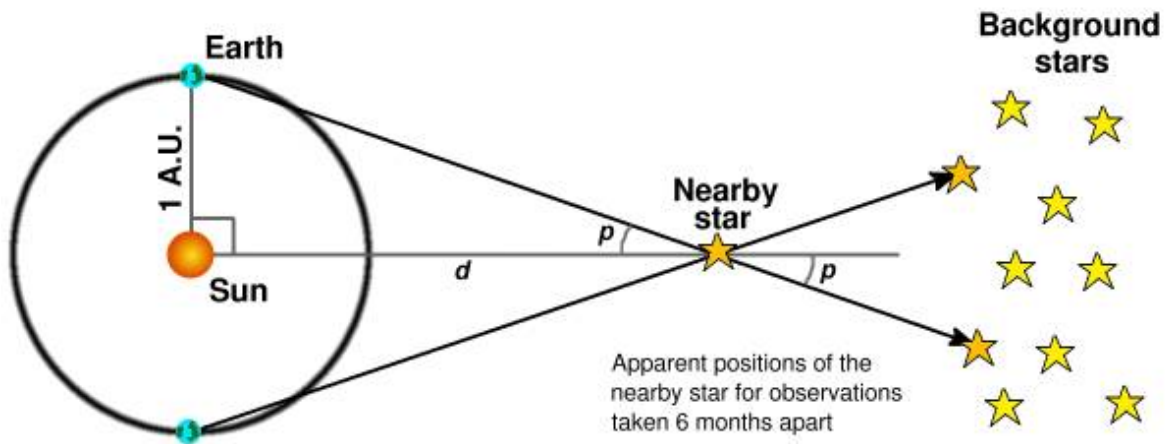


Figure 1.1: Illustration of the geometry involved in a parallax measurement to determine d , the distance to a nearby star.

9. Find the distance to the nearby star in a more direct way — measure the distance with a measuring tape or pieces of paper, or for more distant objects, find them on Google Maps. For this value, also measure this distance 3 times and find the value and uncertainty as in the previous step.

Notation for uncertain values

To be precise about how imprecise our measurements are, scientists often express the quantities as “ $\langle \text{value} \rangle \pm \langle \text{uncertainty} \rangle$ ”, followed by the units. For example, if you measured your average distance to be 3.3 meters, and your uncertainty of that distance to be 0.4 meters, then to express in a succinct way, you can write the distance as “ 3.3 ± 0.4 meters”.

One way of describing how different two values are, without considering the uncertainty of those values, is to calculate the percent difference:

$$\text{percent difference} = \frac{d_1 - d_2}{\left(\frac{d_1 + d_2}{2}\right)} \times 100\% \quad (1.3)$$

10. How close is your parallax distance measurement to the direct measurement? Report the percent difference.

In the next section, you will compare these values with each other using their uncertainties as well.

1.7 Measuring with more precise equipment

Using a finger for measuring angular separation is not very precise. Here you’ll use the parallax technique to determine the same distance to the nearby star, but using a camera and analysis software instead.

11. Take a picture from a digital camera (likely your phone camera) from the vantage point of each Earth position used above.
12. Upload your photographs to a computer. The simplest way to do this may be to email the images to yourself from your phone. Give each file a descriptive name (e.g. “parallax_left_telescope”).

Finding the pixel scale

Notice that with the finger, I told you that your little finger, outstretched, is about 1 degree wide. This was a conversion between the linear size of your finger, for example 1 cm, to an angular size, 1 degree. With the camera, we need to find out the similar conversion — how many pixels in an image corresponds to what angular size, also known as the *pixel scale* of the image. To do this, you can take a known angular size and measure its length in pixels.

13. Find an object of known length and place it a known distance from the camera (distance to camera should be 10 times or more the length of the object). Take a picture of that object.

You will now convert the images from their default format (likely .png or .jpeg) into .fits files, a format commonly used by astronomers. This format will be readable by SAO Image DS9, an astronomical image analysis tool.

14. Convert the image files to a .fits format using your favorite image processing software, or the software “GIMP” (Gnu Image Manipulation Program), or an image conversion website like <https://www.files-conversion.com/image/fits>. For using GIMP, open the file. From the FILE menu, select EXPORT AS, change the file extension to “.fits,” and then click EXPORT. Repeat this procedure for each of your images.
15. Open a saved .fits image of the pixel scale image in DS9. Your first task is to measure the pixel scale. From the menu at the top of the screen, select REGION, SHAPE, LINE. On the first row of buttons in the DS9 window, click EDIT then on the second row click REGION. Draw a line along the known length of the object. On the first row of buttons, click REGION then on the second row click INFORMATION. A window should pop up that will give you the length of the line in physical units, that is, in pixels. **Record this value in your lab notebook.**
16. Find the angular size of the known object. Since the object is far away, we can again use the small angle approximation for the triangle involved and find that the angular size of the object is equal to its length divided by the distance to the object. This angle is in radians.
17. Find the pixel scale by dividing the number of pixels in the length by the angular size of the known object. This gives the pixel scale in pixels per radian.

Finding the parallax angle

Now that you have the pixel scale of the image, you can use that to measure the parallax angle of the your nearby star and thus find the distance to that object like you did with the little finger method.

18. Open the first parallax image. Measure the displacement from the nearby star to the distant star. Convert that displacement to radians using the pixel scale. Make sure to record both the X- and Y- offsets. Repeat these measurements for the second parallax image using the same distant star.
19. Find the total angular distance the star moved between images. To do this, subtract the X- offsets from each other, and subtract the Y- offsets from each other. Then use the Pythagorean theorem to find the total distance moved ($d = \sqrt{x^2 + y^2}$). Divide this by two to get the parallax angle.
20. Use Equation 1.2 to find your new calculation for the distance to the nearby star.
21. Calculate the percent difference between this and the direct measurement like you did in the previous section.
22. Use the t' statistic described in Appendix A.3 to compare the two values and interpret the result — do these two ways of distance measurement really measure the same thing?

1.8 Report checklist

Include the following in your lab report. See Appendix B for formatting details. Each item below is worth 10 points.

1. Your group's agreements about communication.
2. The completed worksheet "The Parsec".
3. Work and final answer for your distance measurements using your finger, with uncertainty.
4. Work and final answer for direct distance measurement, along with percent difference.
5. A figure with your three images (pixel scale image and two parallax images).
6. The displacement vectors from distant star to nearby star.
7. Final determined value of the distance and comparison with the direct distance using percent difference and t' statistic. Show your work (see Appendix B).
8. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

Peculiarities of light: doppler shift, spectral lines, and limits of resolution

2.1 Looking at Things Far Away

On the surface, astronomy might seem like a relatively straightforward science. You simply point your telescope at an object and take the measurements you need. However, as is often the case with all areas of study, the reality is often far more complex than it seems. Observations which are simple to make on terrestrial objects suddenly become incredibly difficult to make for an object in space. For instance, how do you measure the velocity of something in space? On the Earth you simply measure how much distance it travels relative to the ground in a set time. When measuring the velocity for astronomical objects, however, the problem becomes a bit more complicated. Sometimes we don't have proper reference frames for the object's motion, other times the object is moving directly away from or towards us so we can't properly tell how much distance its traveled, and so on. Luckily, a lot more information is carried by the light which reaches us than simply how the object looks. In fact, using our knowledge of how light is transmitted, we can get very precise values for the velocity of objects in space. By using "light" beyond the visible spectrum detected through radio telescopes, we are able to gather a wealth of information we would not have access to otherwise.

This lab is the first in a sequence of three, building your knowledge and abilities so that you can measure the rotation of our galaxy and find evidence for the existence of dark matter.

2.2 Learning Goals

- Understand how the wavelength and frequency of a wave can be used to calculate the velocity of the source
- Use physical phenomena to make measurements of quantities which cannot be directly observed
- Relate the principles of optical telescope to radio telescopes

2.3 Observation Experiment: Doppler Shift

Goal

Understand the physics behind the Doppler Effect and be able to come up with a qualitative description of the relationship between the velocity of a source and the perceived frequency of the wave it emits

Available Equipment

- Doppler Effect Practical Example: Stationary Car Horn https://www.youtube.com/watch?v=Whj1FSaJhhI&ab_channel=SFXCloud Moving car horn https://www.youtube.com/watch?v=p-hBCcmCUPg&ab_channel=sm1thie
- Tone Generator: <https://www.szynalski.com/tone-generator/>
- Ripple Tank: <https://www.falstad.com/ripple/>

Steps

1. Open the two links under "Doppler Effect Practical Example" **These videos feature loud sounds, be sure to change your volume to an acceptable level before opening.** They should lead to youtube video examples of a stationary car horn and a moving car horn.
2. Once you have heard both examples, provide a qualitative description of the differences between both sounds. What does the car horn sound like and how does the sound change as it approaches the camera. How does it sound and how does the sound change as it moves away? **Write down your answers in the lab report.**
3. Now, go to the link labeled "Tone Generator". This should lead you to a website where you can generate pure sine wave tones.
4. Using the bar located under the "play" button, change the frequency of the tone generated. What relationship do you observe between the frequency of the tone and its pitch? How might this relate to the scenario of the car horns you saw in the example videos? Describe the car horn again, this time incorporating a description of the frequency. What is the frequency of the sound wave as the car is heading towards the camera? What is the frequency as it is heading away? **Record your answers.**

Up until now, you might be wondering why are we focusing so much on the noise a moving car makes. After all, this is a fairly common sound and you have probably heard it many times before. In fact, this seemingly mundane effect is actually the key to measuring the velocity of objects billions of miles away. What you are listening to is an example of a phenomenon called Doppler shift, which we commonly experience as the pitch of a sound coming from a car changing as it passes us by. As you might have guessed by now, this shift in the pitch of the sound is actually caused by a change in the frequency which is somehow related to the motion of the source. In the following steps, you will gain a better intuition for the physics surrounding this effect and learn why it is an invaluable tool for measuring the velocity of objects.

5. Open the link labeled "Ripple Tank" This is a simulation of a tank of water, where objects are placed to bob in and out of the water, produced ripples. On the top-center of the tank, notice a small clear square acting as the source of the wave. Right-click on the source and select "edit" in the menu that appears. In this menu you can change the frequency of the wave output by the source (how fast the bob dips into and out of the water). On the top-left of the screen, there is a series of menus. Select "add" and in the bottom of the menu select "add probe". In this case, imagine that the source is the car from the previous example and you are the probe.

6. Move the probe around the tank. How does the displayed waveform change based on different locations in the tank relative to the source? Now change the frequency of the source. What happens to the waveform now?
7. Now go to the drop-down menu in the upper-right corner of the window which says "Example: Single Source" and change it to "Doppler Effect 1". Right click the end node in order to open the edit menu for the source. Now you can change the frequency and the velocity, labeled as "move duration". **Note: The higher the move duration, the slower the source, and the lower the value, the faster the source.**
8. First, without changing its speed, describe how the waveform detected by the sensor changes as the source moves. What does the wave look like as the source is heading towards the probe? How does it look like moving away? Describe the wave in terms of its wavelength or frequency. **Write this down in your lab report**
9. Now, go to the source edit menu. Change the movement duration of the source to 1500 and select apply. This will make the source move faster between the nodes. After you decrease the speed, **record** what the average frequency is as the source is **a.** moving towards the probe and **b.** moving away from it.
10. Change the move duration several more times and record the frequency of the wave produced by the approaching and departing source. **Record your observations in a table.**
11. From your observations, find a relationship between the velocity of a sound source and the frequency detected by a stationary observer. See how precise you can describe the relationship. **Record your findings.**

2.4 Application Experiment: Redshift and Hydrogen Clouds

So far, you have learned how the Doppler effect works for sound waves. Now, you will learn about how the same effect can be applied to light for use in astronomical observations.

Goal

Using what you found in the Doppler Shift experiment, devise a method to measure the velocities of astronomical objects.

Background reading

- Openstax Astronomy: Section 5.1 <https://openstax.org/books/astronomy/pages/5-1-the-behavior-of-light>
- Openstax Astronomy: Section 20.2 <https://openstax.org/books/astronomy/pages/20-2-interstellar-gas>

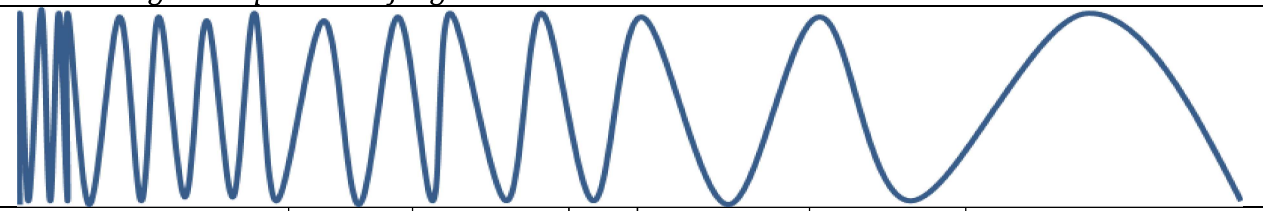
Steps

12. Open the first link in the equipment section. It should take you to the section 5.1 of the Openstax Astronomy textbook. Optionally read the subsection titled "The Wave-Like Characteristics of light".
13. Now, do the worksheet included in the page below.¹ **Write down the answers in your report.**

¹This is adapted from the CAPER Center for Astronomy and Physics Education Research's "Active Learning Tutorials for Astronomy & Planetary Sciences

All objects in the Universe give off energy in some form of light. Let's first consider the spectrum of different forms of light, by circling the correct relative values in the table below.

Electromagnetic Spectrum of Light Chart

		Gamma rays	X-rays	Ultra Violet	Visible	Infrared	Micro-waves	Radio Waves
1.	Circle one: Shorter or Longer	⇐ Wavelength ⇒						Circle one: Shorter or Longer
2.	Higher or Lower	⇐ Frequency ⇒						Higher or Lower
3.	Higher or Lower	⇐ Energy ⇒						Higher or Lower
4.	Bigger or Smaller	⇐ Changes in Electron State ⇒						Bigger or Smaller

Your brain might hiccup when talking about light because it seems like there are two different kinds of energy being emitted: rays of photon particles and waves of light. They are both the same thing, even though astronomers use different terms to talk about different parts of the spectrum. They are all forms of electromagnetic radiation. Don't let the words cause you confusion.

One mental cartoon about how light is emitted from an atom has to do with electrons releasing energy as light when they naturally move from higher-than normal outer positions around the center to closer, lower energy positions.

5. Compared to a small change in an electron's state, a large change results in emission at:

(Circle one) higher energy lower energy no difference

6. Compared to a small change in an electron's state, a large change results in emission at:

(Circle one) longer wavelength shorter wavelength no difference

7. Compared to a small change in an electron's state, a large change results in emission at:

(Circle one) higher frequency lower frequency no difference

8. Compared to a small change in an electron's state, a large change results in emission at:

(Circle one) higher speeds lower speeds no difference

14. Now, open the second link which takes you to section 20.2 of the Openstax Astronomy book. Optionally read the subsection "Neutral Hydrogen Clouds"
15. Given what you have read and learned so far, **answer the following questions:**
 - a) You spot a star which you know should be emitting a signal at 800nm. However, you instead detect a signal at 900nm. What does this tell you about the stars motion?
 - b) You spot two gas clouds which should both be emitting at frequencies of around 1400hz. However, for cloud A you detect a signal of 1500hz and for cloud B you detect 1350hz. Which cloud is moving towards you? Away from you? Which one is moving faster
 - c) If an astronomical object is moving away from you, will its light become more red or more blue? What if its moving towards you?
16. In section 20.2 of Openstax Astronomy, you read and learned about hydrogen clouds and the 21cm line. In your group, use your creativity and imagination to design an experiment to find the velocity of these hydrogen clouds.

2.5 Finding the resolution of a light-gathering device

The precision of our observation often depends on how finely we can resolve detail of objects that we are looking at. The *resolution* of an observing device is the the smallest angular separation between two objects where that device can see them as two instead of one. For example, the resolution of the human eye is larger than the angular separation between the pixels on a phone when held normally, so that we see a larger image, rather than individual pixels.

2.6 Testing Experiment: Resolution of the Eye

Goal

Find the resolution of your eye and compare it to a theoretical best resolution possible.

Equipment

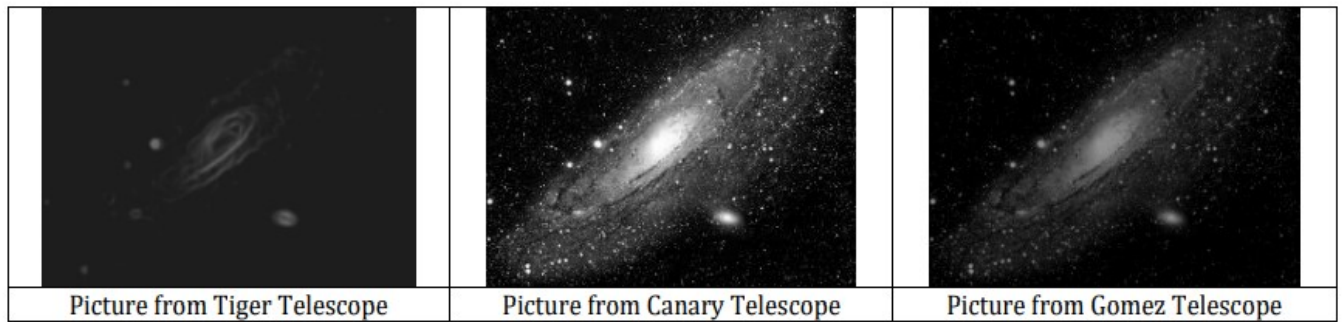
These include materials which you need to collect

- Openstax Astronomy 6.4 <https://openstax.org/books/astronomy/pages/6-4-radio-telescopes>
- Paper, preferably graphing paper
- Ruler or other measurement device.
- Measuring tape or meter stick.

Steps

17. Open the Openstax link and read up until the section titled "Major Radio Observatories Around the World"
18. Answer the questions below in your lab report.

SEEING FINE DETAIL: Better telescopes are able to resolve fine detail.



3. These three pictures are of the same galaxy of stars, taken by three different telescopes. Circle the one with the greatest ability to resolve fine detail.

19. Now, in the center of a sheet of paper, draw two points about .5cm (.25in). Make sure they are clearly visible
 - If you don't have access to a ruler but have graphing paper, then draw the points at either end of one square. If you have ruled paper, then this is roughly the distance between two lines. To measure things, you can also use standard sized objects, like coins, credit cards, and sheets of paper.
20. Find an area where you will be able to put a lot of distance between you and the sheet of paper. Attach the paper to a wall or prop it up on a surface such that you can see the points.
21. Start moving away from the sheet of paper looking directly at the points until you can no longer distinguish between the two points with your eyes. Measure your distance from the paper.
22. Repeat this several times with points at different distances from each other. Record your observations on a table which includes the distance between the points and the distance at which you could no longer distinguish between the points.
23. For each measurement of distance away, calculate the angular separation between the points, from the perspective of your eye, using the following equation:

$$\theta = \frac{d}{L}, \quad (2.1)$$

where d is the distance between the points on the paper and L is the distance between you and the paper. Note that θ is measured in radians here.

24. Average these angular separations to get the measured angular resolution of your eye.
25. Now, use Rayleigh's criterion $\theta = 1.22 \frac{\lambda}{D}$ to calculate the theoretical maximum angular resolution of your eye's pupil where λ is the wavelength of light, D is the diameter of the aperture (lens). To calculate the resolution use $\lambda = 550\text{nm}$ which is the wavelength for green light and $D = 5\text{mm}$ which is the average diameter of the pupil (if you can measure your own pupil, then use that).
26. Compare the theoretical resolution with the angular resolution you calculated. How different are they? What factors could account for the difference?

2.7 Report checklist

Include the following in your lab report. See Appendix B for formatting details. Each item below is worth 10 points.

1. Car horn observations (Steps 2–4)
2. Qualitative observations of the ripple tank (Steps 6, 8)
3. Table of wave frequencies in ripple tank (Steps 9–10)
4. Relationship between velocity of source and frequency detected (Step 11)
5. Nature of Light worksheet (Step 13)
6. Answers to doppler questions (Step 15)
7. Experimental design of hydrogen cloud velocity determination (Step 16)
8. Table of point separation distances, distance of distinguishability, and angular separations, and resulting average pupil resolution (Steps 21–23)

2. PECULIARITIES OF LIGHT: DOPPLER SHIFT, SPECTRAL LINES, AND LIMITS OF RESOLUTION

9. Calculation of theoretical maximum angular resolution and comparison with your measured value (Steps 25–26)
10. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

Rotating solar systems and radio astronomy

3.1 Introduction

You might think that being limited to radio signals, the kinds of observations you could make and the information you could gather would be relatively small. In fact, there are a wide range of observations and interesting phenomena which can be detected using a radio telescope. From the elemental make up of a distant planet's atmosphere, to the detection of extreme astronomical objects. In fact, we can use observations of signals from hydrogen clouds in the galactic plane and, using the knowledge we gained of the Doppler shift, obtain an estimate for the distribution of matter in the Milky Way. By now you know how to use radio signals to find velocity, but how do you use velocity to find mass? In this lab, not only will you learn how we can estimate the mass distribution of an orbital system, but you will also learn the basics of using the Small Radio Telescope, which lives on the roof of Kersten Physics Teaching Center.

3.2 Learning Goals

- Make predictions about large, gravitationally bound systems based on observations of smaller scale systems.
- Make observations using the Small Radio Telescope (SRT) and interpret the data
- Calibrate the SRT and understand the uncertainty by measuring background noise in the Chicago sky

3.3 Team roles

Decide on roles for each group member. The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded
- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

3.4 Masses and Orbital Velocity

One of the main labs you will carry out this quarter is a measurement of the motion of the Milky Way using the SRT. It is important that when you make these measurements that you understand the results you obtain. In this experiment, you will learn how the specific observations you will make can be used to make some predictions about the Milky Way.

Goal

Learn how new phenomena can be predicted or inferred from basic physical principles and how to interpret rotation curves.

Equipment

- Desmos Graphing Calculator: <https://www.desmos.com/calculator>

Steps

1. First, we know from Newton's first law that the force on an object is equal to its mass times its acceleration, $F = ma$. We also know that an object moving in a circle will experience an acceleration towards the center of its path which is given by $a = \frac{v^2}{r}$, where v is the velocity of the object and r is the radius the circle it travels along. Combine these two equations to find an equation for the force experienced by an object undergoing circular motion. **Record your derivation and final equation.**
2. Objects in orbit also move in an approximately circular path and the force of gravity they experience is given by $F = G \frac{Mm}{r^2}$, where M and m are the masses of the two objects and G is the gravitational constant. Using the equation you found in the previous step, derive an equation for the velocity of the orbiting object as a function of its radius. *Hint: plug equations into each other to remove variables that are common in both equations. Your final equation should be in terms of v , M , r , and G .* **Record your derivation and final equation.**
3. Create a plot of your equation for the velocity as a function of radius, for example using the online graphing calculator listed in the equipment. To plot it, type the equation in the left side of the screen. It will graph y on the vertical, x on the horizontal axis, so use y in place of your v and x in place of your r . For the variables M and G , click the "add slider" button so you can plot the shape of the curve. You can leave them set to equal 1, since we are using this to see the shape of the curve, not the absolute value. **Include this graph in your report.**
4. Adjust the mass slider to larger values and observe what happens to the curve. How does it change? **Record your answer.**

The graph you created is called a *rotation curve*, which plots the orbital velocities of objects in an orbital system against their distance from the center. Using these curves, it is possible to learn a lot about the system it describes.

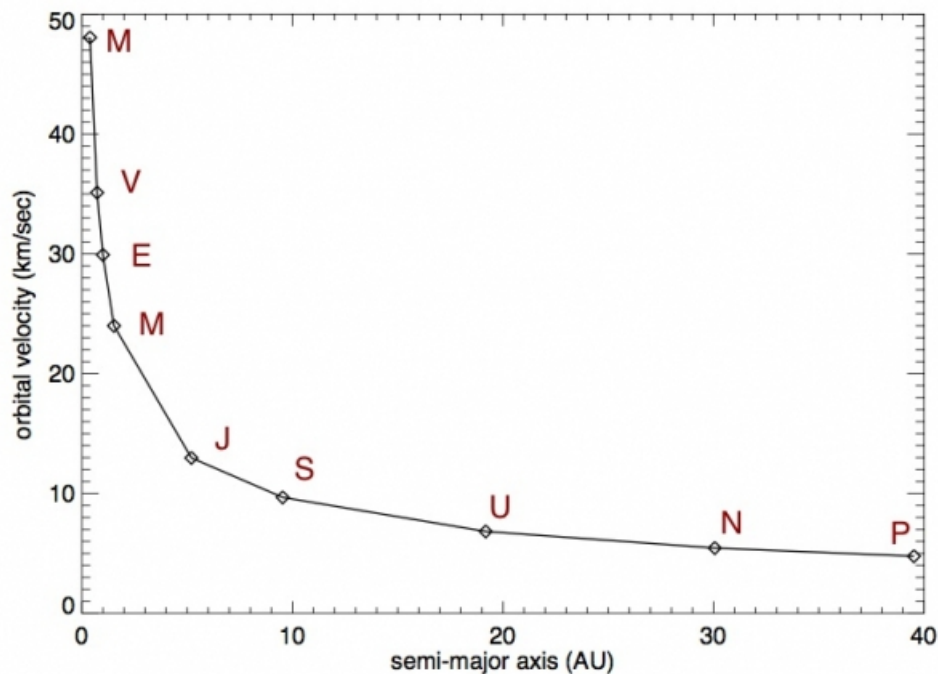


Figure 3.1: Rotation curve of an example orbital system, where each of the objects marked on the graph are orbiting the center of the system.

When you first derived the equation for rotational velocity, you assumed that M represented the mass for a single object at the center. Now we will assume that this equation holds true in the case that M represents the total mass contained within the orbital radius. This assumption is valid for a spherically symmetric system, which is a good enough approximation for us. Let's explore some cases for different mass distributions.

- For the case where the mass is uniformly distributed in a disk, the total enclosed mass increases as r^2 . To graph this, delete the M slider and create a new expression $M = cx^2$. How is this curve different from the previous one? **Include your answer and plot in your report.**
- Play with different mass distributions by changing the equation for M and observing how the curve changes.
- The rotation curve for a sample orbital system is shown in Figure 3.1. Change the mass formula in your plot to visually match the shape of this curve. How is the mass distributed in this orbital system? **Record your plot, mass equation, and answer.**
- Another rotation curve is shown in Figure 3.2 (line B). Change the mass formula in your plot to visually match the shape of this curve, especially the curve at medium and long distances. How is the mass distributed in this orbital system? **Record your plot, mass equation, and answer.**
- Look up information on the masses of the planets in the solar system as well as the Sun. Which of the rotation curves would you expect the solar system to have? *It might help to add up all the masses and see how much each planet contributes to the total mass.* **Record your answer.**

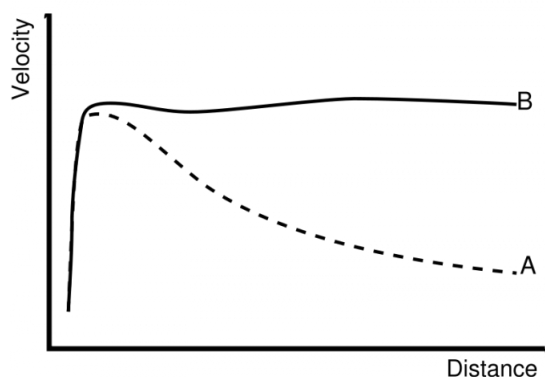


Figure 3.2: Rotation curves for two example orbital systems A and B.

3.5 How uniform is the sky in Hyde Park?

While learning about the theory behind making astronomical observations can be useful, the best way to gain an intuition for these concepts is through hands on experience. For this experiment, you will learn how to operate the small radio telescope located on top of the Kersten Physics Teaching Center.

Given that there are background sources that can interfere with our measurements of real sky signal, before we proceed to make any measurements, it is instructive to explore just how uniform the sky is in Hyde Park.

Background noise

The telescope is sensitive to all sources on the sky, as well as buildings, etc. Although the telescope is most sensitive over a small area of sky it does have some low sensitivity to sources at much larger angles from its pointing position. Radio power generated by cell phone towers and wireless devices within buildings can cause significant addition signals even though the 1420 MHz band that we are using is protected for radio astronomy by law against such interference. Communication devices operate near our frequency and do not always suppress power in our frequency bands; although the fraction of the radio power in these sidebands is tiny, our telescope is very sensitive and we can detect this interference. Human-caused interference can be distinguished from astronomical sources because it is usually fixed in position while astronomical sources move across the sky.

Telescope Control

Unfortunately, you will not be able to directly control the telescope and it will instead be controlled by a designated telescope technician. However, you should still learn the basics of controlling the telescope so you can give the technician proper observation instructions. In this case, the technician will act as your hands when operating the telescope, inputting the commands you give them.

Control Buttons

The control panel display, shown in Figure 3.3, has a line of control buttons at the top that set up a command for the telescope. For instance, if you want to change the receiver frequency, you click on the “freq” button. Instructions and help information are then displayed in the help panel below the map while the cursor is pointed to the button (This field is blank when you move the cursor away). If you want to change the frequency, then type data into the data line (e.g. “1420.4 4”), click “enter” or “return”. Be sure you leave a space between 1420.4 (which is the frequency) and the number 4 (which sets the bandwidth of the system). The current observing parameters will be updated in the appropriate box.

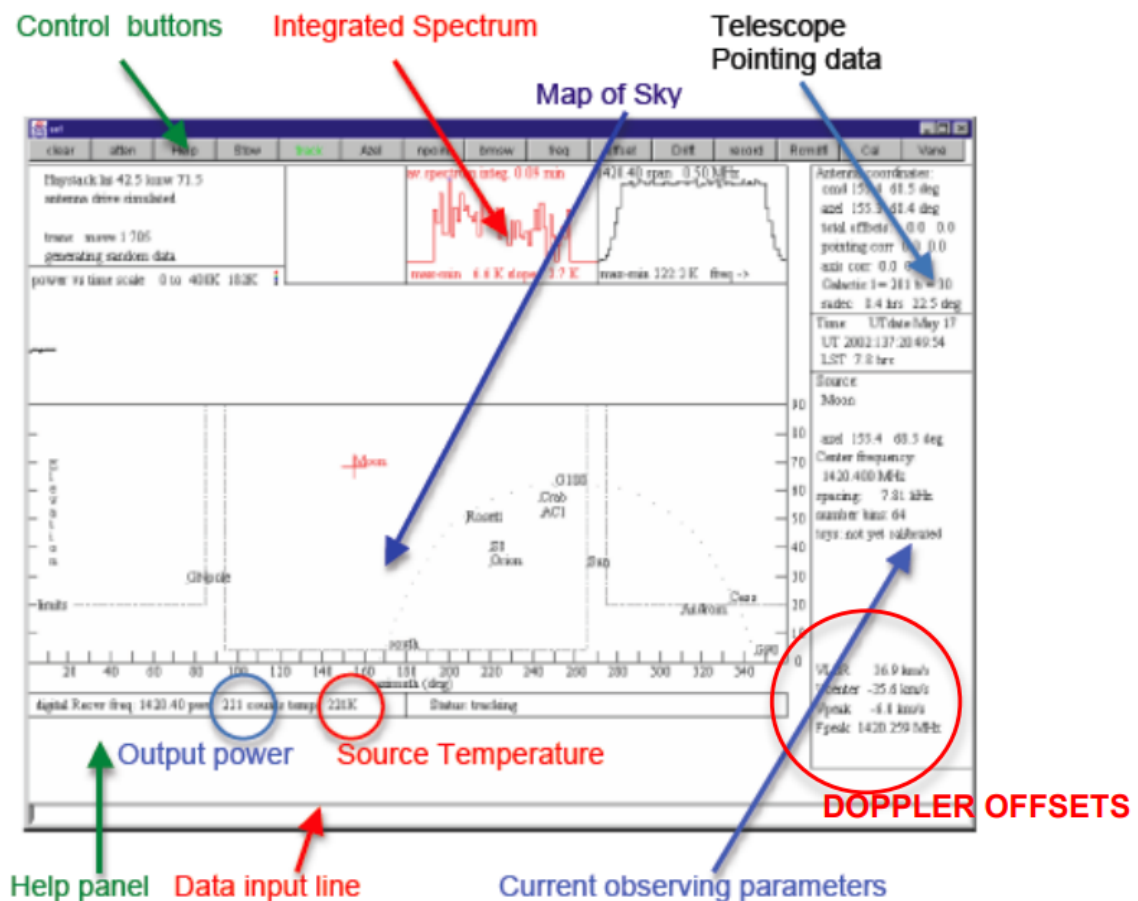


Figure 3.3: Screenshot of SRT control interface with various information labeled.

Control Display & Sky Map

The control panel shows a map of the current sky in Azimuth and Elevation units. 0 degrees azimuth points North, 90 degrees points East, 180 degrees points South and 270 degrees points West. The horizon is at 0 degrees elevation and Zenith is at 90 degrees. The telescope can point to about 85 degrees in elevation. The map displays objects visible at the current time. The software tracks an object or a given azimuth and altitude position, including corrections for the rotation of the earth. Also displayed on the map are various individual astronomical sources and a track of dots that show the plane of the Milky Way. The Longitude of points along the equator of the Milky Way are shown as Gxxx, where xxx is the Galactic longitude. We have an unobstructed view of all objects above about 20 degrees.

Pointing the Telescope

You can point to any given position in the sky by clicking on the “AzEl” button and then typing the desired Azimuth and Elevation in the command line at the bottom of the display. (e.g., 30 45 “enter” will send the telescope to a position of 30 degrees azimuth and 45 degrees elevation). When you enter a position you will see a yellow cross and the CMD numbers will change to these numbers.

When “track” shows up in green at the top of the display, the telescope has acquired the position and is tracking it, this holds for sources. If the telescope does not move, then you probably have to click on “track”. Occasionally the telescope motor will get stuck and stall. Normally it fixes itself automatically by going back to the “Stow” position (where it is stowed after every observation), but if it remains stalled for several minutes click “Stow” to do so manually. This can be a nuisance and time-consuming but you should still be able to obtain the necessary data if this happens.

Temperature and Calibration

In radio astronomy, it is usual to interpret the measured antenna signal in terms of temperature, as if the measured source is a blackbody, and fills the full field of view of the telescope. This isn’t necessarily the case, but it’s often a good approximation and gives a physical parameterization of the measured power. This is why the telescope outputs a temperature. This is not that the SRT is actually measuring the object’s temperature, rather, it is simply interpreting the power it receives as a temperature. Since the telescope itself has a temperature, we must perform a calibration step to have it account for it. Similarly, when calibrating, the telescope will display a system temperature T_{sys} which interferes with our measurements. When calibrating, we account for T_{sys} and it is subtracted from our measurements.

As a class, you will work together to collect all the necessary data. Each group will collect 2–3 points of data, and then you will share the data among the class.

Calibration Steps

10. Set the receiver frequency to 1416MHz by clicking on the Freq button, typing “1416 1” in the command line, and pressing Enter. At this frequency, you will detect the continuum radiation. That is, radiation emitted relatively uniformly over a broad band of frequencies.
11. Click on the AzEl button and type an Az and El for a blank part of the sky (e.g., azimuth and elevation: 10 35; check on the display to make sure you select coordinates away from the Galactic plane and from the Sun) into the data input line at the bottom of the control panel and then hit “enter”. You should see the telescope start to move in elevation. The telescope position is printed out in the top right panel of the control display. The top line gives the command position (which should reflect the position you typed in for AzEl) and the next line gives the actual position.
12. When the telescope position is settled (i.e., the cross on the sky display is red and the “Track” button at the top of the display is green), click “Clear” and write down the raw output power and temperature. Click on Cal button and watch these numbers. The system will record the output power itself and then switch on the noise source. The computer will then calculate the temperature of the sky at that point, which includes the system noise temperature. *Note that you should “Clear” before each new observation, for both calibration and data, since the telescope is continually integrating and thus keeps all the previous data since it was last cleared.*
13. Repeat the calibration a few times until you read a stable system temperature T_{sys} . Wait around 1 minute between calibrations.

Measuring the noise

14. Make sure the frequency is set to 1416 MHz by clicking on freq and typing in 1416 1
15. From the telescope pointing that you used in the previous exercise (say [az,el]=[10,35]) point the telescope 15 degrees lower in elevation (at the same azimuth) and 20, and 40 degrees higher in

elevation. Repeat the calibration at each of these positions 2–3 times and record azimuth and elevation of the pointings and values of the sky temperature and system temperature displayed by the SRT console. Avoid labeled astronomical objects.

16. After you are done with measurements at different elevation, return to your original elevation and change azimuth with step of 50 or 75 degrees (at constant elevation) and repeat calibrations 2–3 times at each pointing. Record azimuth and elevation of the pointings and values of the sky temperature and system temperature displayed by the SRT console.
17. How uniform is the radio sky around Hyde Park? In your lab report, discuss the uniformity of the sky signal and system temperature based on your measurements. Include a table of your measurements and your interpretation and conclusions.

3.6 Report checklist

Include the following in your lab report. See Appendix B for formatting details. Each item below is worth 10 points.

1. Derivation of $v(r)$ equation with plot and description of how curve changes (Steps 1–4)
2. Answer and graph of rotation curve with uniform disk (Step 5)
3. Graph, mass equation, and answers for two rotation curves (Steps 7–8)
4. Predicted rotation curve for the Solar System (Step 9)
5. Calibration raw output power, temperature, and final system temperature (Steps 12–13)
6. Table of azimuth, elevation, sky temperature, and system temperature (with time and date of each observation), with your group's observations identified (Steps 15–17)
7. Discussion of sky uniformity with conclusions (Step 17)
8. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

How fast is the galaxy rotating and what does it mean?

4.1 Introduction

Throughout the past few weeks, you have been performing labs which at times might have seemed disparate or only somewhat connected to each other. Now, you will bring all these concepts you have been working with in order to perform observations and make conclusions regarding the motion of the Milky Way. In this lab, you will be trying to measure how the rotational velocity of the Milky Way varies changes as you move away from the center. However, given the scale of the Milky Way, there is no way for us to directly observe its motion. However, we can use the motion of objects within the galaxy to calculate its rotational velocity. One useful object to do this are hydrogen clouds. In a previous lab you learned how shifts in frequency can be used to calculate the velocity of an object. In the case of hydrogen clouds, given that they emit at a very definite, known frequency we can calculate their velocity to a very high degree of accuracy. Moreover, given how common they are throughout the Milky Way, they can be used to make these measurements at many different points along the galactic plane. Using this data, you will calculate the rotational velocity of the galaxy at that point and create a rotation curve for the Milky Way. You will then use the curve to make inferences about the mass distribution of the Milky Way and see why dark matter is believed to play a role in its structure.

4.2 Learning Goals

- Use observation of hydrogen cloud velocities to determine motion of galactic plane
- Organize and present data in a logical and consistent manner and draw conclusions from it
- Interpret rotation curve data in order to make inferences about the mass distribution in the Milky Way

4.3 Team roles

Decide on roles for each group member. The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded

- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

4.4 Rotation of the Milky Way

Our star system, the Solar System, resides within the Milky Way Galaxy. When we observe it directly, it looks like the following:

https://en.wikipedia.org/wiki/Milky_Way#/media/File:ESO-VLT-Laser-phot-33a-07.jpg.

The galaxy has a spiral disc shape, and this image is looking towards the center of the galaxy. While we can't move outside our galaxy to take a picture of it, based on what we know, it probably looks like the artist's rendition here:

[https://en.wikipedia.org/wiki/Galactic_coordinate_system#/media/File:Artist's_impression_of_the_Milky_Way_\(updated_-_annotated\).jpg](https://en.wikipedia.org/wiki/Galactic_coordinate_system#/media/File:Artist's_impression_of_the_Milky_Way_(updated_-_annotated).jpg)

Locate the Sun in that image and notice the coordinate system that extends from it. This is the system of galactic longitude, shown as a schematic here:

https://en.wikipedia.org/wiki/Galactic_coordinate_system#/media/File:Galactic_coordinates.JPG

Throughout the galaxy, there are neutral hydrogen gas clouds in the spiral arms. The neutral hydrogen clouds emit light with a spectral line at a wavelength of 21 cm (frequency of 1420.4 MHz). Since they are all moving at different velocities, when we observe 21 cm line in the galactic disc (galactic latitude $b = 0$), we see different distinct peaks at wavelengths close to 21 cm, caused by their differing Doppler shifts. So we can use Doppler shift to find the velocity of those clouds. A sample observation is found in Figure 4.1.

Calculating Rotational Velocity

When measuring the velocity of these objects, we have to keep in mind that we are moving relative to them. As such, we are not measuring their rotational velocity, but their velocity relative to us. We therefore have to do several calculations to get their orbital velocity. To do this calculation, we will need several components. First, we will need the velocity of the sun in the line of sight for the galactic longitude l . This is because the Sun is also moving along the galactic plane, and thus we need to be able to account for it in our measurements. We already know that the velocity of the Local Standard

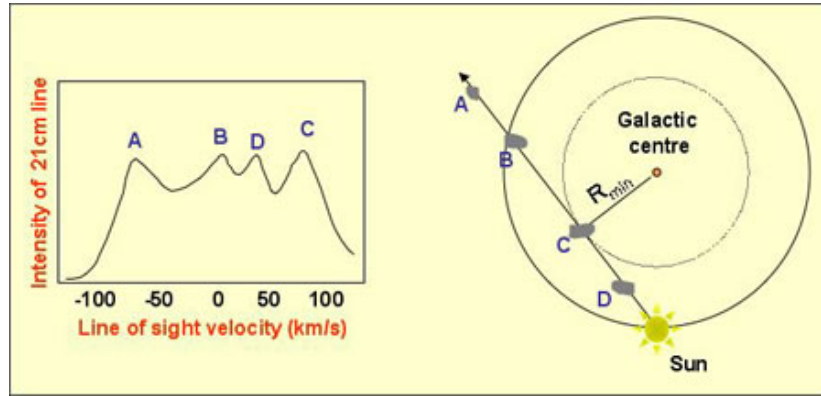


Figure 4.1: On the left, a graph of intensity vs line of sight velocity for 4 different gas clouds observed from Earth. The maximum line of sight velocity is from cloud C, since it's velocity orbiting the galactic center aligns with the line of sight. (Image from Swinburne University of Technology, <https://astronomy.swin.edu.au/cosmos/H/HI+cloud>)

of Rest (the local stellar environment around the Sun) is 220km/s, so using some trigonometry, we can see that the line of sight velocity is given by

$$V_{\text{sun}}(l) = (220 \text{ km/s}) \sin(l). \quad (4.1)$$

We also need to account for Earth's rotation around the sun as well as relative motion of the solar system compared to the LSR. These are given to you by the SRT altogether as the VLSR or Velocity relative to the Local Standard of Rest. From the graph generated by the telescope, we simply need the maximum VLSR, as it corresponds to the hydrogen cloud directly in our line of sight. The circular velocity of the cloud can thus be obtained by

$$V_c(r) = v_{\text{max}}(l) + v_{\text{sun}}(l). \quad (4.2)$$

Finally, for the final data processing, you will need the distance of that cloud from the center of the galaxy. Once again, from the geometry of the graph above, we can see that this can be found from the distance of the Sun to the center $r_0 = 8.5\text{kpc}$ (kiloparsec) and the galactic longitude l . The distance is then given by

$$r = r_0 \sin(l) \quad (4.3)$$

Goal

Measure the rotational velocity of the galactic plane along different orbital radii, create a rotation curve for the milky way and infer the mass distribution from it.

Equipment

- LAB sky survey: https://www.astro.uni-bonn.de/hisurvey/AllSky_profiles/
- Small Radio Telescope

Calibration

Before you begin your observations of the hydrogen clouds, you have to calibrate the telescope similarly to how you did in the previous lab. However, this time you have two options for calibrating: The offset frequency method and the off-source method. The two methods are described below.

Offset Frequency Method (recommended)

1. Move the telescope to one of the locations you want to measure on the galactic plane (marked by the Gxxx)
2. Change the telescope frequency to 1423MHz in mode 4 and clear the screen. This frequency is far enough from the hydrogen line that you will only be measuring the receiver noise and thermal background.
3. Once you have cleared the output, select "Cal" to begin the calibration procedure. Note the T_{sys} and T_{rec}
4. Clear the output again and repeat the calibration procedure 2-3 times or until you note a stable T_{sys} and T_{rec} .

Off-Source Method

1. Point the telescope to some location far away from the galactic plane (away from the labeled sources).
2. Change the receiver frequency to 1420.4MHz in mode 4 and clear the output
3. Once you have cleared the output, select "Cal" to begin the calibration procedure. Note the T_{sys} and T_{rec} .
4. Clear the output again and repeat the calibration procedure 2-3 times or until you not a stable T_{sys} and T_{rec} . **If there is a major hydrogen source in your pointing, the calibration will not work. Look out for strong spectral features when calibrating to make sure you are not pointed at a hydrogen source. If this were the case, try moving the telescope. You likely wont find a spot without any hydrogen emissions, but you can try to avoid significant sources. If not try the offset frequency method instead.**

Steps

1. In the control panel, the positions of different Galactic longitudes along the equator are indicated by Gxxx (where xxx is in degrees). If you see longitude of 90 degrees and smaller, start at the longitude of 90 degrees and work your way down to the smallest longitude you can observe down to zero of the center of our Galaxy (coincident with the source named Sgr A). If you cannot observe longitudes smaller than 90 degrees observe one or two longitudes that are up in the sky. Note that galactic coordinates labeled in the SRT display can be pointed to by clicking on them – for the rest you have to estimate (or look up) the Az El of the desired longitude on the galactic plane.
2. Move to your first pointing and press Clear button to clear the spectrum accumulated by the SRT.
3. Set the frequency to 1420.4MHz with mode 4 by typing in "1420.4 4" and obtain the spectrum for the galactic longitudes visible in the sky by integrating for 10–20 seconds (or longer) along the same pointing.
4. After integration click on the spectrum window to get a detailed plot of the spectrum in a separate window. You will see emission flux in units of kelvins (K) as a function of frequency and VSLR. Based on your understanding of how frequencies are affected by Doppler shift, estimate that maximum velocity of the clouds visible in the spectrum.
5. Record the spectrum by taking a screenshot of the spectrum plot. If necessary, crop the image in your preferred image editor such that only the spectrum viewer is visible. Your screenshot should look something like Figure 4.2.

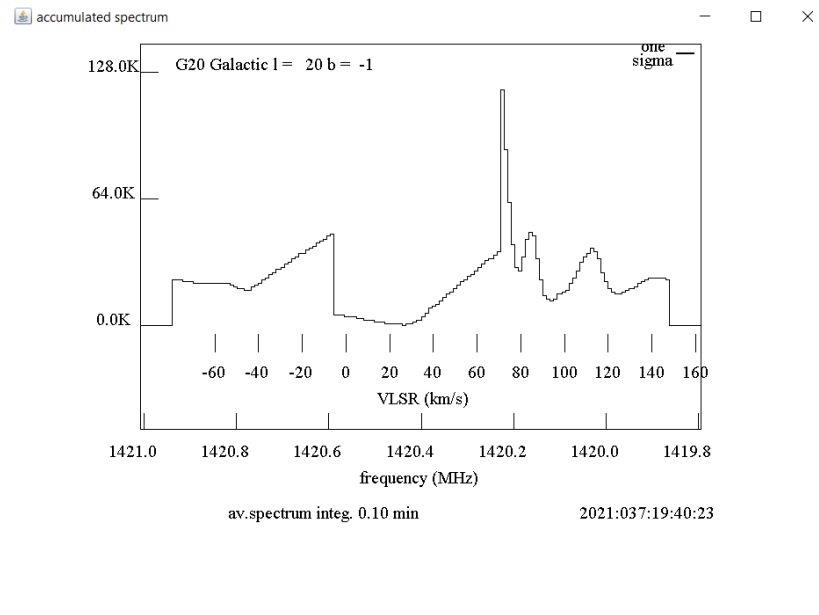


Figure 4.2: An example of a good capture of the spectrum plot. Note the different clear peaks that are seen.

Search Position	
Coordinate system	Galactic coordinates (l, b)
Center	RA [h m s]/ l [°] 50
	Dec [± ° ' "]/ b [°] 0
Effective beamsize FWHM [°] (must be < 1°) 0.2	
Surveys	EBHIS (δ > -4°) <input type="checkbox"/>
	GASS III (δ < 1°) <input type="checkbox"/>
	LAB <input checked="" type="checkbox"/>
Search	

Figure 4.3: The input form for accessing the LAB spectra. Dec is always set to 0, only LAB is selected, FWHM is set to 0.2 and galactic coordinate system is used.

- Proceed to the next Galactic longitude available for observation. Press Clear button to clear the spectrum before you make observation at each new longitude. Record longitude and spectrum. Given the limited time, each group should do 2-3 observations and supplement the rest of the data either by sharing with the other groups or using the LAB survey described below.
- After you have preformed all the possible observations, open the link to the LAB sky survey provided in the equipment section.
- In the search box, make sure that only the LAB survey box is checked and that Dec is always at 0. Figure 4.3 demonstrates how the display should look like.
- Obtain spectra from the LAB survey along the galactic equator for the longitudes that you could not observe with SRT. Measure the maximum radial velocity, V_{\max} (note these will be max positive for longitudes 10 to 90 deg. and max negative for longitudes -10 to -90 deg.). You

4. HOW FAST IS THE GALAXY ROTATING AND WHAT DOES IT MEAN?

will note that the spectrum does not provide a clean maximum velocity. Try to think about the best way to measure maximum velocity, given that you may be observing several hydrogen clouds all in a line, moving at different velocities.

10. Assuming a distance from the Sun to the galactic center of 8.5 kpc and a circular velocity of 220 km/s at this radius, make a spreadsheet that follows the template in Table 4.1 with the results of your calculations using your measurements of v_{max} and equations above.
11. Plot the orbital velocity versus the distance from the center in kpc using your plotting program of choice. **Include this graph in your report.**
12. From the rotation curve you obtain, how do you expect the matter to be distributed in the galaxy? **Record your answer.**
13. In our own solar system, the sun makes up most of the mass. As such, a safe assumption to make is that the mass in a given region of our galaxy is roughly proportional to its brightness (the brighter a region, the more stars and therefore the more mass there is). The brightness of our galaxy as a function of the radius can be roughly modeled using the equation $I(r) = e^{-r/2.1}$. In Desmos online graphing calculator, plot the equation.
14. Take a screenshot of the graph you obtain. Describe the behavior of the graph. How does brightness change as you move farther away from the center of the milky way?
15. Using the assumption above that mass is roughly proportional to brightness, how would you expect mass to be distributed in the Milky way? How does it compare to what you found from the rotation curve.
16. Think of possible explanations for the discrepancy between the distribution of mass suggested by the brightness curve and the distribution suggested by the rotation curve. Think about the assumptions that went into the equations and how you came to your original conclusions for each curve. **Record your discussion.**

Galactic Longitude (degrees)	Tangential Dis- tance r (kpc)	Maximum VLSR v_{\max} (km/sec)	Line of Sight So- lar Velocity V_{sun} (km/sec)	Circular Velocity v_c (km/sec)
10				
20				
30				
40				
50				
60				
70				
80				
90				
-10				
-20				
-30				
-40				
-50				
-60				
-70				
-80				
-90				

Table 4.1: Data Table for measurement of the rotation velocity of the Galaxy

4.5 Report Checklist

Include the following in your lab report. See Appendix B for formatting details. Each item below is worth 10 points.

1. Calibration details - method used, sequence of Tsys and Trec observed.
2. Emission flux vs frequency graphs for your group's observations and description of maximum velocity determination
3. Completed data table with SRT observations marked, and your group observations marked
4. Flux vs frequency graph for one of the longitudes from the LAB survey
5. Data from LAB survey included in data table
6. Graph of your measured Milky Way rotation curve
7. Inference of matter distribution from light curve
8. Inference of matter distribution from rotation curve
9. Re-evaluation of assumptions and possible explanation for discrepancies between matter distributions.

4. HOW FAST IS THE GALAXY ROTATING AND WHAT DOES IT MEAN?

10. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

Measuring mass using Gravitational Lensing

“The observation of such a gravitational lens effect promises to furnish us with the simplest and most accurate determination of nebular [sic] masses. No thorough search for these effects has yet been undertaken.” — Fritz Zwicky, 1937

5.1 Introduction

Gravitational lensing is the bending of the paths of photons in the distortion of space-time caused by mass (i.e., by gravity). Gravitational lensing manifests in the universe on a range of mass and length scales. At the smallest scales — the Solar System — the mass of the Sun can cause an subtle shift in the apparent positions of background stars along lines of sight close to the Sun. This effect, only visible during an eclipse (otherwise the light from the Sun precludes observing the background stars) was first observed by Sir Arthur Eddington in 1919, and provided crucial early support to Einstein’s theory of General Relativity, because the observed effect agreed with his predictions.

In the 1930s the astronomer Fritz Zwicky showed that galaxies (then still referred to as nebulae, as you can see in the quote above) and clusters of galaxies might be *the* best place in the universe to search for gravitational lensing, and argued that observation of this effect would be the best and cleanest way to measure the mass of these objects.

In this lab, you will find examples of gravitational lenses on cosmological scales, using data from the Hubble Legacy Archive (HLA) database. Then you will use the image of a distant object created by the galaxy lens to find the mass of the galaxy or galaxy cluster that is acting as the lens.

5.2 Gravitational lensing theory

The basics of gravitational lensing can be understood from a simple diagram shown in Figure 5.1, coupled with the knowledge that mass bends the paths of photons. Consider an observer, O, a lensing mass, L, and a source of photons, S. We will consider the cosmological scale case here, so S might be a distant galaxy or quasar, and L a cluster of galaxies or a lone massive galaxy, situated between O and S.

Photons are being emitted by S in all directions, and normally in the absence of an intervening lens, O will see the source at a position corresponding to the direction from which photons are arriving from the source (case a). With an intervening lens L in place, other photons which normally would not arrive at O have their paths diverted as they pass by L, and end up being observed by O (cases

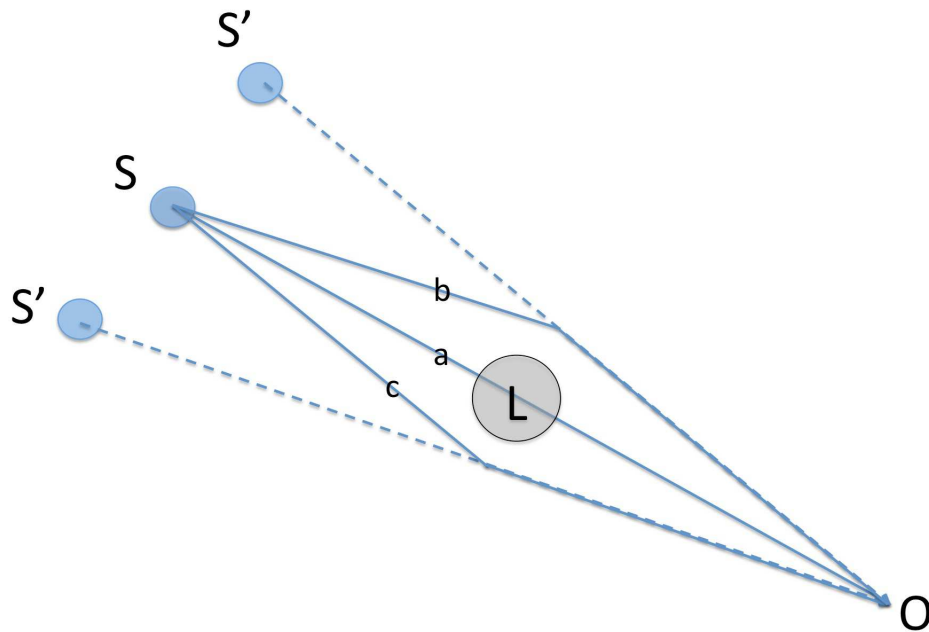


Figure 5.1: Diagram showing the geometry of source (S), observer (O), and gravitational lens (L). The source that would appear along direction OS without the lens, in the presence of lens appears along directions S' instead.

b,c). Under this circumstance the observer 'sees' images of S not at its actual location but at virtual locations corresponding to the direction from which the diverted photons are arriving (images S'). If the alignment between S, L and O is perfectly along a line, and L is circularly symmetric on the sky, then the image S' is a ring on the sky. This is called an Einstein Ring – a sketch of this from the observer's perspective, and actual examples from Hubble Space Telescope observations of lensing systems found in the Sloan Digital Sky survey (SDSS) are shown in Figure 5.2.

Note that the degree to which the photon's path is deviated is proportional to the mass of L, with a larger mass providing a greater deflection. The apparent radius of the Einstein ring on the sky thus provides a measure of the mass of the lens interior to the ring. The size of the ring is also related to the distance along the line of sight between S, L and O; to robustly determine the mass of the lens we must also know these distances.

Also note that the alignment between S, L and O is rarely perfect, and the lens L is rarely perfectly circular on the sky; the result of this is that complete Einstein rings are very rare and most lensed images S' are arcs which look like portions of a ring image.

Lab Tasks

1. To visualize how the lensing works, watch the animation at the following link:

<https://public.nrao.edu/gallery/animation-of-a-gravitational-lens-creating-an-einstein-ring/>

Note that real lenses move much more slowly and are actually stationary on the human timescale, and that only precise alignment yields full rings and that partial arcs are much more typical.

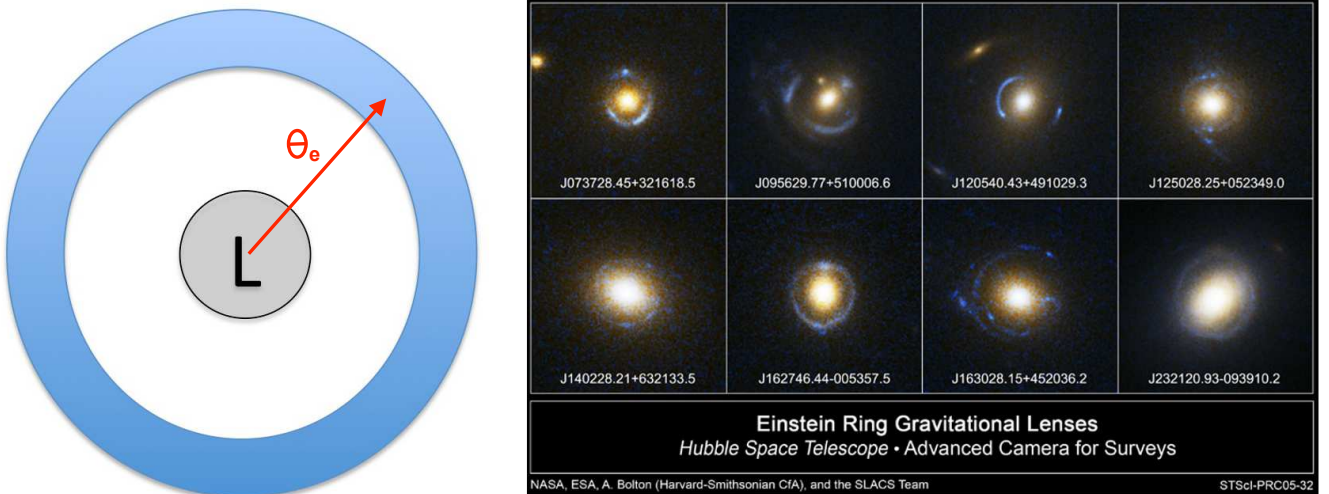


Figure 5.2: A sketch of the Einstein ring from the observer's perspective (left), and actual examples from Hubble Space Telescope observations of lensing systems found in the SDSS (right). The distance from the center of the ring to the ring itself is called *the Einstein radius* noted as θ_e in the sketch above, and in the equations below. It is this quantity which you will be estimating from the images, once you have identified several systems showing strong lensing.

2. Take a few minutes to explore some images of gravitational lensing recorded by instruments on the Hubble Space Telescope. An extensive gallery of images can be found at <http://hubblesite.org/images/news/18-gravitational-lensing>

The type of lensing we are exploring in this lab is called strong lensing, and is the most obvious manifestation of the bending of photon paths by gravity. Other types of lensing have also been observed.

When the lens involved has the mass of a star (apart from the specific case of the Sun, discussed above) rather than a galaxy or cluster of galaxies, then the radius of the Einstein ring is typically micro-arcseconds. In that case the lensing cannot be observed as a spatial distortion directly in the image (no telescope produces images with a resolution fine enough to see micro-arcsecond structures) however the focusing effect of lensing still produces an enhancement in brightness which can be observed. This type of lensing is often referred to as micro lensing.

A further manifestation of lensing is the weak distortion of the images of background objects which are well away from the observer to lens line-of-sight. In that case the background source is not imaged into a ring or arc or multiple images, but there is still a discernible pattern of distortion around the massive object. This pattern can be analyzed statistically, and provides another mass estimate for the lens. This effect, not surprisingly, is known as weak lensing.

Gravitational lensing as a phenomenon has a wide range of manifestations in the universe, and it is now an important technique for study mass in the universe on a range of scales, from extrasolar planets through to the largest structures. What began as a theoretical curiosity in the early 20th century is now one of the most powerful tools that we have to learn more about the cosmos. In the remainder of this lab you'll find a few astronomical lenses to work with and then apply some of the basics of strong lensing to measures the masses of galaxies and clusters of galaxies.

5.3 Measuring Masses

In this section of the lab, you will use gravitational lensing to measure masses of galaxies that bend the rays of light. Theory predicts that the angular size, θ_e , of the Einstein radius (i.e. angle on the sky from the center of the lens to the ring or arc) is related to the mass within this angular radius, $M_{<\theta_e}$, by the following equation:

$$\theta_e = \left(\frac{4GM_{<\theta_e}}{c^2} \frac{D_{LS}}{D_{OL}D_{OS}} \right)^{1/2}, \quad (5.1)$$

where D_{OL} is the distance from the observer to lens, D_{OS} is the distance from the observer to the lensed source galaxy, and D_{LS} is the distance from the lens to the source galaxy, $c = 2.999 \times 10^{10}$ cm/s is the speed of light in vacuum, and $G = 6.673 \times 10^{-8}$ cm³ g⁻¹ s⁻² is Newton's gravitational constant.

The source galaxies are too faint for the SDSS to have measured their redshift, so we do not know the redshifts of the lensed sources in these cases. However, galaxies are lensed most efficiently when the lens is half-way between source and observer. Therefore, in the absence of better information we will make a reasonable assumption that the sources are at twice the distance to the lens: i.e., $D_{OS} = 2D_{OL} = 2D_{LS}$, which is in agreement with the typical redshift of sources in known gravitational lenses. Using this assumption and solving for the mass we get:

$$M_{<\theta_e} = \frac{\theta_e^2 c^2 D_{OL}}{2G}. \quad (5.2)$$

Note that the angular size of the Einstein radius, θ_e , here should be in radians, not in arcseconds.

3. From the gallery of lenses in Figure 5.2, select two of them. They are identified by their name, for example "J073728.45+321618.5".

For each of the two selected lens systems, complete the following steps to find their masses by measuring their Einstein radius, looking up their redshifts, finding the distances to them using a calculator, then using the above equation.

4. Search for the system by name in the Hubble Legacy Archive at <https://hla.stsci.edu/hlaview.html>.

Immediately below the green tabs, it shows the system name followed by its equatorial coordinates RA and Dec (as well as RA and Dec in the alternate units [hour:arcminute:arcsecond degree:arcminute:arcsecond]). Each row in the table is a different observation of this system.

5. Pick one of the observations and select “Display”, which opens a new window.

This new window is a low-resolution view of the image taken by the Hubble Space Telescope. The box in the upper left gives the coordinates of the pixel that the cursor is hovering over. In order to more easily see different features, you can lighten or darken the image using the buttons in the top left of the window.

6. Find the lens system by clicking and dragging the image around, using the system’s coordinates as a guide.
7. Use the zoom, lighter, and darker buttons to create the best zoomed in view of the lens and Einstein ring. **Record a screenshot of this image of the ring.**
8. To measure the radius of the ring, move your cursor and record the coordinates of the center and the edge of the ring. Input these into the angular separation calculator at <https://cads.iiap.res.in/tools/angularSeparation> to get the angular radius. Record these values of θ_e for each lens in degrees. Convert the values in degrees to radians and **record the result.**
9. To find the distance to the lens systems, you will need to look up its redshift in the SIMBAD database. Go to <http://simbad.u-strasbg.fr/simbad/sim-fbasic> and search for your system’s name. The redshift is the number after “z(spectroscopic)”. **Record this number.**
10. Start a browser and go the Ned Wright cosmology calculator. You can google it or use this link: <http://www.astro.ucla.edu/~wright/CosmoCalc.html>. Use default values for all parameters, but enter the appropriate value of the redshift for each of your two lenses in the input field marked **z** and click on the button **General**. The window on the right will display a variety of information for the input redshifts and assumed cosmological parameters. **Record the “angular size distance” D_A in Mpc and and the scale, i.e. how many kiloparsecs correspond to 1 arcsecond (kpc/”).**
11. Compute mass within θ_e , $M_{<\theta_e}$, using the distance D_{OL} that you calculate from redshift with the cosmology calculator, i.e., D_A . Also convert the angular size θ_e into the physical size in kiloparsecs of the Einstein radius using information you get from the calculator. **Include the measured radii and masses in your report.** Express radii in kiloparsecs and masses M_{θ_e} in solar masses ($1M_{\odot} = 1.989 \times 10^{30}$ kg).
12. The mass of *stars* in the most massive galaxies known in the universe is about $10^{12} M_{\odot}$. **How do your measured masses compare to this value of the maximum mass of visible stars? What could cause the difference?**

5.4 Report Checklist

Include the following in your lab report. See Appendix B for formatting details. Each item below is worth 10 points.

1. For each lens selected, the image of the lens, a table of name, θ_e , redshift, and angular size distance D_A (Steps 7–10)
2. Calculation and final value for physical radii and masses (Step 11)

3. Comparison to known maximum galaxy mass (Step 12)
4. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, “ 5 ± 1 m” means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters¹. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

A.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument’s scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment’s manual often gives the instrument’s resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

¹The phrase “most likely” can mean different things depending on who is writing. If a physicist gives the value and does not give a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet, or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^N \frac{(x_i - \bar{x})^2}{N-1}}, \quad (\text{A.1})$$

where $\{x_1, x_2, \dots, x_N\}$ are the measured values, \bar{x} is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called “standard deviation of the mean”, which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}. \quad (\text{A.2})$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as 52 ± 3 m.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn’t change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

A.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction), } \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \quad (\text{A.3})$$

$$\text{For } c = ab \text{ (or for division), } \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \quad (\text{A.4})$$

$$\text{For } c = a^n, \frac{\delta c}{c} = n \frac{\delta a}{a} \quad (\text{A.5})$$

For other calculations, there is a more general formula not discussed here.

Expression	Implied uncertainty
12	0.5
12.0	0.05
120	5
120.	0.5

Table A.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π , is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table A.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value “0.0322857142857143”. Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as “9.65683578099600e-04”, where the ‘e’ stands for “times ten to the”. I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as “ 0.032 ± 0.001 km/s”. You may also see the equivalent, more succinct notation “0.032(1) km/s”.

A.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced “tee prime”). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as²

$$t' = \frac{|a - b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \quad (\text{A.6})$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

²Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.

Lab Report Format

B.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.

B.2 Organizing the report

The report should follow the sequence of the report checklist. Answers to questions and inclusion of tables and figures should appear in the order they are referenced in the manual. In general, include the following:

- For any calculations that you perform using your data, and the final results of your calculation, you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92 \text{ m/s}^2)t + 0.2 \text{ m/s}$."
- Answers to any questions that appear in the lab handout. Each answer requires providing justification for your answer.

B.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like “distance (pc)”. This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like “distance (pc)”.
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called “error bars”, along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.