Laboratory Manual

PHSC 12710 Galaxies

The University of Chicago

Winter 2020

Labs

LAB 1

Observing with small optical telescopes

In this lab, we will be using a reflecting optical telescope to observe objects in the KPTC lab. This will give you practice using the CCD camera to take images. More importantly, these activities aim to give you a sense of scale for what the combined telescope and CCD are capable of, and why this is useful for astronomical observations. The goal is for you to build some understanding and appreciation for how telecopes of this size, equipped with a digitial camera and filters, can be used to take images, and how those images relate to the capabilities of your most familiar imaging system — i.e. your eye.

1.1 Reflection Telescopes

The primary utility of a telescope is its ability to gather light, thereby enabling visualization and analysis of the faint astronomical objects we are trying to observe. This requires focusing light incident on a large surface area. We will be using a **reflecting** telescope, which means that light rays from observation targets are focused into an eyepiece or onto a detector with reflecting mirrors. This is in contrast to refracting telescopes, which use refracting lenses to focus light rays. Figure 1.1 shows schematically how this kind of telescope works.

To take images in this lab, we make use of a **Charge Coupled Device (CCD)**. CCDs are the standard detectors for taking astronomical images at wavelengths blueward of approximately 1 micron. Think of the CCD we are using as a very advanced low-noise digital detector, not wholly dissimilar from the digital detector in your smartphone. Every time a photon within a certain energy range hits the detector, an electron is knocked off of the incident pixel, charging that pixel's capacitor. Thus, for each pixel, more photons $\implies$ more electrons $\implies$ more charge, and the charge can be read off into a digital signal that is then processed as an image.

The main thing to note is that the CCD material is not sensitive to all wavelengths of light uniformly. Photons of certain energies are more likely to excite electrons in the detector and thus contribute to the output image. Consequently, the observed image intensity will be weighted by the response function of the detector.

Filters

Light is composed of photons with energies that determine their wavelengths (sorter wavelength $\implies$ higher energy). Thus every light source exhibits a textbfspectrum of energies based on its energetic components, determined by the physics of the light emission process. Thus, observing the energetic constituents of light from astronomical objects - a.k.a. observing the spectrum of emitted radiation

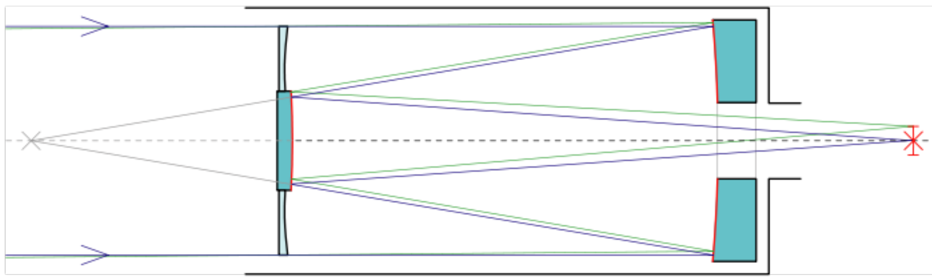


Figure 1.1: Schematic for a reflecting telescope of a Schmidt-Cassegrain design. Light rays from astronomical objects enter the telescope in parallel because their source is effectively at infinity. They are then reflected by a parabolic primary mirror onto a secondary mirror that again reflects the light to a focus. An eyepiece or a camera is placed at the focal plane of the resulting image. Image source: https://en.wikipedia.org/wiki/Cassegrain_reflector#/media/File:Schmidt-Cassegrain-Telescope.svg

- is a fundamental tool in observational astrophysics. However, obtaining the specific intensity of radiation as a function of energy from an astronomical source is challenging. An easier way to asses the electromagnetic energies observed is to image them in different filters: materials that are transparent to a known range of wavelengths and opaque to all others. Thus, one can image the same object with multiple different filters to get a sense of the wavelength regimes that make the strongest contributions to the overall electromagnetic output.

A filter is characterized by its **transmission function**: a function that characterizes the amount of light that is transmitted by the filter at each wavelength. Figure 1.2 shows the transmission functions for some standard astronomical filters (similar to the ones you'll be using in this class).

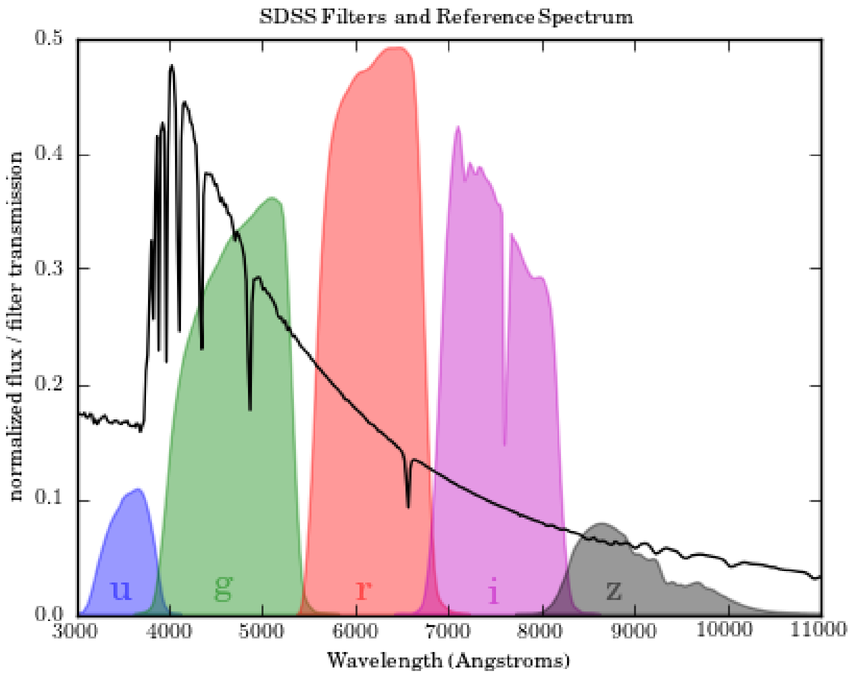


Figure 1.2: Filter Transmission Functions for the Sloan Digital Sky Survey, overlaid on a stellar spectrum (in this case the spectrum is probably an A-type star). The magnitude observed by each filter will be proportional to the integrated spectrum multiplied by the filter transmission. It's clear in this image that the underlying spectrum of the star will cause the different filters to have different magnitudes. How would these magnitudes change if the spectrum was, say, of an M-star instead? (Image source: http://www.astroml.org/_images/fig_sdss_filters_1.png)

1.2 Lab Tasks: Introduction

The telescope and camera assembly is an expensive and relatively fragile piece of hardware, so move carefully when interacting with the system. We have only one setup, and there are three main observing tasks, so before starting divide into three groups. Each group will set up and control the telescope and camera for observing in one of the sections below, assisted by others in the lab, and then share the output images with everyone in the lab group.

1.3 Telescope Set Up, Preliminary "Observations"

First we will be observing an object at the opposite end of the lab room with the lights turned off. During this exercise we will 1) check that the telescope is focused (which should already be the case), 2) gain practice positioning the telescope and targets, and 3) demonstrate the light gathering and angular resolution capabilities of the telescope with a physical object for which you have an direct sense of scale. **NOTE: For safety purposes, do not walk around the lab while the lights are off; instead, turn the lights off only as needed to gather data or make observations, and when the lights are off, don't move about significantly.**

1. The telescope should already be set up, with the camera on and cooled, and be approximately in focus and pointed at the target at the other end of the lab bench. If not, your TA will work with you to get the setup roughly correct.

2. Your first task will be to tweak the centering of the image target. This doesn't have to be perfect, but you should be able to see the mm scale marks. The telescope can be moved in elevation (up-down). The fine control of elevation is achieved using a knob on the mount - your TA will show you details. To shift the image left or right, move the target instead.

3. Next we want to precisely focus the telescope. Do this with most of the lights off - otherwise the image has too much light. Use the 4th filter of the five available; this can be selected from the 'Filter' menu. Then, use the **Focus** task from the camera control system toolbar, and take images of a few seconds (at most). If the image is completely dark, and the peak intensity number in the focus box is reading around 65000, then the CCD is saturated, and the light in the room should be reduced, or the exposure time shortened.

4. Start the focus sequence - the camera will just take and display images repeatedly, of the exposure duration you specified. Now, adjust the focus knob (on the back of the telescope, off center) until you achieve an optimal focus. Note that you can (and should) zoom the magnification of the image so you can see the imaging target in detail. Do this by adjusting the magnification, and adjusting the image x,y centering so you're looking at an appropriate zoomed location in the image. Note that you'll need to not be touching the telescope or mount to get a sharp image, so you'll have to adjust, then move away, wait, and repeat. Your classmates walking around the lab will cause image shake that will make focusing difficult, so enforce a still classroom while you do this.

5. Time to demonstrate one helpful aspect of telescopes by taking an image in the dark. Once the telescope is focused to the sharpest possible images, turn off the lights and acquire an image using the **Grab** button on the toolbar. An exposure time of 40–60 seconds is usually good with all of the lab lights off and the blinds closed.

6. Save this image in 'fits' format; the 'ds9' tool available on all of the lab computers can be used to examine the image in detail.

7. With the lights back on, gather around the target on the lab bench and then turn the light off again. Observe the target. Can you see it? Can you see the details? Does it matter how close you are?

8. Compute and report the ratio of the light gathering power of the telescope relative to your eye, and comment on how that relates to what you do (or do not) observe by eye compared to the telescope.

9. In the acquired and saved image you can see a mm scale, which correponds to some number of pixels. Compute and report the 'plate scale' in mm/pixel. Given the distance between the telescope and the target, what is the angular plate scale in arcseconds/pixel?

   To find this, note that for an arc (segment) of a circle, the length of that arc s is related to the radius of circle r, and the angle (in radians) subtended by that arc θ by

   $$s = r\theta \tag{1.1}$$

10. What do you think is the smallest feature (in arcsec) that you could resolve using this setup? Would we expect to see images of stars that sharp if we turned the telescope and camera skyward? Also, what is the field of view (arcseconds $\times$ arcseconds) of this camera+telescope setup? Finally, observe the target from the vantage point of the telescope by eye. What detail can you see? How does that compare to what you can see with the telescope?

1.4 Grating Spectrum and Filters

The next portion of the lab will involve observing a grating spectrum projected onto a blank white target. We will use this to understand how the filters work. In addition, you will combine the outputs of this and the third portion of the lab to unambiguously identify the 5 filters in the filter wheel.

11. Replace the reticle target and box with the blank white target mounted in a stand. Ensure the stand is placed so the target occupies the same plane as the reticle target did (otherwise, you'll need to refocus the telescope; The paper has a pen mark on it - use this a reference if you do need to tweak the focus.). Next, place the light box with the attached diffraction grating on the same wooden stand using in the previous section, pointed toward the white target, and plug the light box in. This should generate a rainbow on the target screen. You may need to turn the lab lights off to see the rainbow easily. This rainbow is the spectrum of the incandescent lamp inside the light box.

12. Once you've configured the setup, turn the lab lights off, and grab an exposure of 1 second in the 4th filter (if this section follows the above, that filter should already be selected). The bright band in the image should be positioned in the center of the image. If not, adjust the position of the spectrum by rotating the stand holding the lightbox+grating around the vertical axis to move spectrum slightly left or right on the screen. Adjust until the light band of the image is centered left/right. Also, to center vertically so that the light fully covers the image along the vertical axis, you may adjust the telescope elevation. To do these adjustments, you may find it easiest to put the camera in focus mode again. Centering vertically is less important - so long as a significant portion of the image contains light on the vertical axis there is no particular need to fine-tune the vertical alignment.

13. Once you have the telescope aligned and focused on the reflected grating spectrum, grab an image of 1 second for each filter available in the CCD filter wheel. Save each image and note the differences. Comparing the different images, where are the bands located? How constant is the intensity of light across the band in the horizontal direction? How broad/narrow are the bands? What is the physical explanation for these observations (see also the end of the next section)?

1.5 Indirect H-lamp observations

14. Now we will make some observations to demonstrate the utility of the narrow-band filters. Turn off the lamp with the grating attached and remove it from the lab bench. Remove the white screen that was used to reflect the grating spectrum, and again place the reticle target on the box (same as the activities for Section 1.3) at that location and refocus on the target if necessary.

15. If the Hydrogen lamp is not already in place and set up, place the Hydrogen lamp in front off to side, plug the tube of Hydrogen gas into the lamp stand, and plug this into an electrical outlet. Use the pedal to turn it on and make sure it's working: it will be a bright and somewhat odd (to your eye) magenta color (why?). Also, have a look at the Hydrogen lamp by eye, using the unmounted gratings — note what you see, and explain what you see in your lab report. How many emission lines are visible?

16. Make sure the reticle target is in the field of view of the telescope, similar to Part 1. Turn off the lab room lights, leave the Hydrogen lamp off, and grab a 10 second image in each of the 5 filters. Save each image as a fits file, with a name that indicates the filter and lamp condition (for example '`filter1nolight.fits`' etc.) The image should be dark but the target should be visible at least in some cases. Now, turn on the Hydrogen lamp using the small attached pedal, and grab another set of images (one per filter, ten seconds in each case) and similarly save them. You may find it easier to take each filter sequentially, turning the lamp on and off, rather than doing all filters lamp off and then lamp on. Do what works best for you.

17. One of the two narrowband filters in the wheel - see Figure 1.2 to know which filters are narrow in wavelength - is an H-alpha filter that isolates emission at the wavelength of the H-alpha Balmer line. Which filter is this? Filter1? Filter2? Filter3? Explain, using the data taken.

The filter wheel contains two narrow band and three broad band filters. As already indicated above, one of the narrow band filters is H-alpha, and the other is OIII-5007 (an emission line at 500.7nm from star forming regions that is strong in star-forming galaxies, such as the Milky Way). The broad band filters are the g-band ('g'=green) r-band ('r'=red) i-band ('i'=infrared) ; see Figure 2 for details.

18. Based on the data taken in Sections 1.4 and 1.5, deduce the identification of each filter; provide your final list in your lab report - i.e. filter 1 as 'this', filter 2 is 'that' etc. Describe step-by-step how you solved this problem.

1.6 Optional Section: See yourself with a telescope!

Time permitting, you can also take images of yourself with the telescope. Try placing your hand where the imaging target are (use the wooden box a rest stand). Take images at different wavelengths (if you stayed carefully still, you could make a color composite), or even see what your fingers look like in H-alpha light). Or, place a stool nearby, take a seat, and snap a partial portrait. Your face is far too large to fit in the field of view at that distance, but your eye would fit! (refocusing will be necessary in this case...)

1.7 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation. See Appendix B for guidance on writing the report and formatting tables and graphs.

- Image of the target that you captured and qualitative observations (Steps 6–7)
- Ratio of light-gathering power (Step 8) with comments.
- Calculation of pixel scale, questions regarding resolution and field of view. (Steps 9–10)
- Images of spectrum with each filter with analysis (Step 13).
- Qualitative observations of the hydrogen emission with diffraction gratings (Step 15).
- Images of hydrogen emission with each filter and identification of H-alpha filter (Steps 16–17).
- Identification of each filter in the filter wheel, with justification (Step 18).

LAB 2

Measuring distant objects with parallax

2.1 Introduction

Since it takes time for light to travel to us from objects in the universe, the further out an object is, the further back in time we see it. So for us to have an accurate picture of how the universe was in the past, we need to know how far away things are. For things that are nearby on Earth, we can travel there and see how far we went, or how long we took to get there. For things further away like the moon, we can use Kepler's laws, or we can bounce a beam of light off of it and see how long it takes to get back. For objects outside of our solar system, it would take too long, and the light would disperse too much, for us to use this last technique. For those objects that are still relatively nearby, we can use the parallax technique as the first rung on our distance ladder.

2.2 Procedure

First, complete the worksheet "The Parsec" that is printed out in the lab. Each member of the group will turn in their own worksheet, and you can work in groups to complete it.

Go to the fifth floor of Eckhardt Research Center. Near the elevators on the north side, there are two telescopes

You will now use the telescope apparatus to measure the parallax of a "foreground star" (actually a nearby building), with the Chicago skyline serving as "background stars." Make sure that it is positioned so that sightlines from both telescopes pass through both the "foreground star" and the "background stars," (an overlapping background of the skyline). With your smartphone, take an image of the "foreground" and "background" stars through each telescope. Also measure the distance between the two telescopes using a measuring tape and record this value in your lab notebook. See Figure 2.1 for example data.

You have now finished collecting data, so now it's time to analyze it. Upload your photographs to a computer. The simplest way to do this may be to email the images to yourself from your phone. Give each file a descriptive name (e.g. "`parallax_left_telescope`").

You will now convert the images from their default format (likely .png or .jpeg) into .fits files, a format commonly used by astronomers. This format will be readable by SAO Image DS9, an astronomical image analysis tool. Open the first file in GIMP ("GNU Image Manipulation Program"). From the FILE menu, select EXPORT AS, change the file extension to ".fits," and then click EXPORT. Repeat this procedure for each of your images.

Figure 2.1: Example images. My foreground "star" was the point defined by the left intersection of the lower cable and the white pillar on top of the gym on campus. One of my reference "stars" was the top right corner of the building in the background. Note that the images produced by this telescope are upside down.

Open a saved .fits image of the star field in DS9. Your first task is to measure the size of the field of view in pixels. Adjust the contrast so you can clearly see the field of view. From the menu at the top of the screen, select REGION, SHAPE, LINE. On the first row of buttons in the DS9 window, click EDIT then on the second row click REGION. Draw a line across the field of view. On the first row of buttons, click REGION then on the second row click INFORMATION. A window should pop up that will give you the length of the line in physical units, that is, in pixels. Record this value in your lab notebook.

Now open the first parallax image. Measure the distance from the foreground star to several reference background stars and record these values in your lab notebook. Make sure to record both the X- and Y- offsets. Repeat these measurements for the second parallax image using the same background stars.

2.3 Calculations

Given that the field of view of the Galileoscope is 1.5°calculate the *plate scale* of your images, in arcsec/pixel.

Select a reference star from your parallax measurements. Using your plate scale, determine the angular separation between the reference star and the target star. Record the value for the total separation, r, and for both the horizontal (x) and vertical (y) components. You can check your measurements against each other by inputting these values into the Pythagorean formula: $r^2 = x^2 + y^2$. Do your measurements agree?

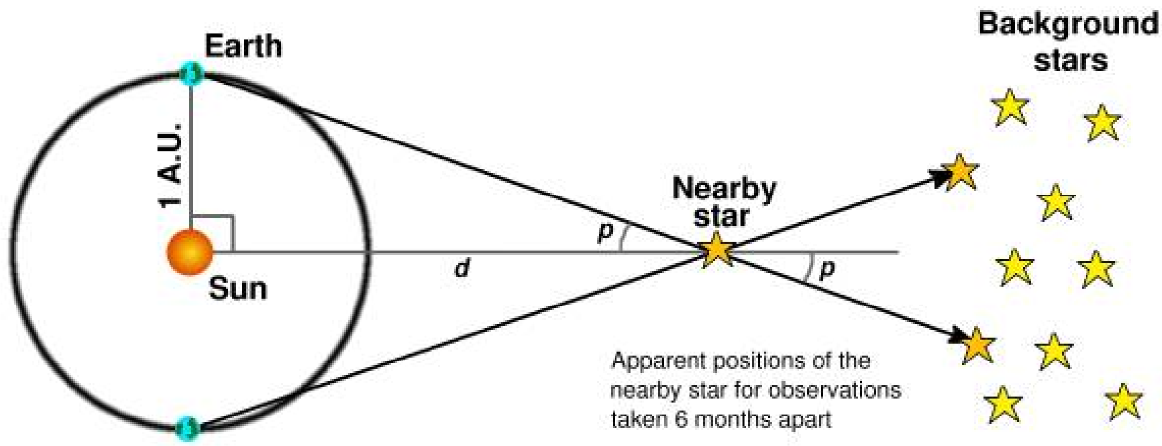


Figure 2.2: Illustration of the geometry involved in a parallax measurement to determine d, the distance to a nearby star.

Repeat the above calculations for all reference stars in both of your parallax images. You can now calculate the distance to the foreground star. The distance to a star can be found using the triangle formed by the line segments Sun-Earth, Earth-Star, and Star-Sun (see Figure 2.2). Trigonometry relates these lengths to each other according to

$$\tan p = \frac{a}{d}, \tag{2.1}$$

where p is the parallax angle in radians, d is the distance to the star, and a is half of the distance between the two measurement positions. Since the length d is much greater than a, the angle p is very small, and so we can use the small angle approximation $\tan u \approx u$, and therefore

$$p = \frac{a}{d}, \tag{2.2}$$

Use each reference star to calculate an independent measurement of parallax. To do this, for each reference star, you'll need to find how far (in radians) the target star has moved between the two observations. Using vector arithmetic[1], subtract the two displacement vectors from each other by components and find the magnitude of that difference vector. Divide by 2 and convert to radians to find p. After finding p from each reference star, average these values together and estimate your uncertainty by finding the standard deviation of these measurements. You can perform this calculations in Excel using the functions AVERAGE() and STDEV(). Report your measurements in a table in your lab report.

Also use a map to find the distance from you to your foreground star (with uncertainty). Compare these quantities with their uncertainties using the procedure found in Appendix A.3, to see the degree to which they agree.

2.4 Questions

These should be included in your lab report.

1. Your parallax measurements depend on an incorrect implicit assumption. What is this assumption, and how will it bias your results? How would you change the procedure in order to minimize this bias?

[1] See https://www.mathsisfun.com/algebra/vectors.html for a short tutorial

2. Quantify the uncertainty of your measurements. What were the primary sources of uncertainty? How would you improve the procedure for future measurements?

2.5 Report checklist

Include the following in your lab report. See Appendix B for formatting details. Each item below is worth 10 points, with an additional 10 points given for attendance and participation.

1. The completed worksheet "The Parsec".
2. A figure with your two images.
3. The table of reference stars and displacement vectors.
4. A statement of your final determined value of the distance, with uncertainty, and comparison with the distance found with a map. Show your work (see Appendix B).
5. Answers to the questions in Section 2.4, with justification.
6. Your reflection on the experiment, as detailed in Appendix B.

LAB 3

Radio Astronomy and the Rotation of the Galaxy (Small Radio Telescope)

3.1 Assessment

Your lab report will be assessed based on answering lab questions correctly and with justification (showing work or giving reasoning) and the following rubric rows found in Appendix **??**: A11, F2, G4, G5.

3.2 Changes to the lab write-up

The lab write-up is included below, and the following changes should be made to it.

Operating the small radio telescope (page 3)

The telescope is mounted 3° azimuth out of alignment. Each time the SRT program is loaded, the following command must be run to compensate for this, before any useful observations can be made.

- Click on the "`offset`" button in the top row.
- Type "`3.0 0.0`" and press enter.

3.3 Lab write-up

PHSC12710
Radio Astronomy and the Rotation of the Galaxy
with the Small Radio Telescope (SRT)

Introduction

Observational astronomy using light at radio wavelengths, i.e., meter through millimeter wavelengths, is a relatively new field, starting with Karl Jansky's discovery of radio wavelength emission from the sky in 1933. Jansky worked at Bell Labs and was trying to understand the excess noise in trans-Atlantic radio communications. It was a remarkable discovery, making headlines in the New York Times and leading to the birth of new field. However, Bell Labs did not pursue radio astronomy further at that time. An enthusiast Grote Weber did, however. As a hobby, he built in 1933 the first modern radio telescope in his backyard in Wheaton, IL and used it to map the sky, identifying many interesting astrophysical sources. He was detecting continuum emission, most of the sources were later found to be highly energetic active galactic nuclei emitting a type of radiation called synchrotron emission. In 1944, Hendrik van de Hulst worked out the theory of the hydrogen spin-flip transition, often simply called 21 cm hydrogen emission, which soon became a powerful tool for astronomy. It still is today. The 21 cm line (1420.5 MHz frequency) is used to map the cold neutral hydrogen gas in the Galaxy and other galaxies. It was used to show unambiguously the spiral arms in the Milky Way. It also allows astronomers to measure the rotation curves of galaxies and solve for the enclosed mass. Such measurements have led to the discovery that large halos of an invisible, "dark" matter dominate the mass of galaxies. In this lab you will make such a measurement. In so doing you will learn the basics of radio astronomical observations and use them to measure the rotation of the Milky Way.

The Small Radio Telescope (SRT)

The Small Radio Telescope (SRT) is built using a commercial satellite dish and a specialized radio astronomy receiver. It consists of a 10ft diameter parabolic reflector, which concentrates the radio waves onto a short rod (a dipole) tuned to a frequency of around 1420 MHz. The radio waves from the sky generate small currents in the dipole that are then amplified electronically. The parabolic reflector increases the sensitivity of the telescope and defines its field of view. We can therefore look at individual objects or scan over regions in the sky to obtain a map. Unlike a digital camera that records many pixels of the image simultaneously, the radio telescope measures only one pixel at a time, i.e., it can only measure the flux from one direction in the sky at one time. The telescope is pointed from one position to another in the sky to build up a map of the region.

The radio telescope also provides highly accurate spectroscopic data of the emission from every point. The frequency range of the telescope allows for the detection of the hydrogen line at a frequency of 1420.4 MHz, which will allow us to map the location of hydrogen in the galaxy. Note that radio waves at these frequencies are practically unattenuated by Earth's atmosphere, so we can make our measurements if the sky is cloudy, even if it is raining!

Resolution

The diameter of the telescope and the wavelength of the received radiation (light) determine the resolution of the observations. A double object is said to be *resolved* if we can see that we are looking at two objects rather a single object. If the objects are closer than this limit we will see only one object. The resolution of a telescope is not a totally precise quantity, because we can sometimes obtain better resolution by careful computer processing of the data, but is given approximately by the angular diameter of the *Point Spread Function* (PSF). The PSF is obtained experimentally by observing an object that is small compared to the theoretical resolution of the telescope (a *point source*) and measuring the angular size of the image.

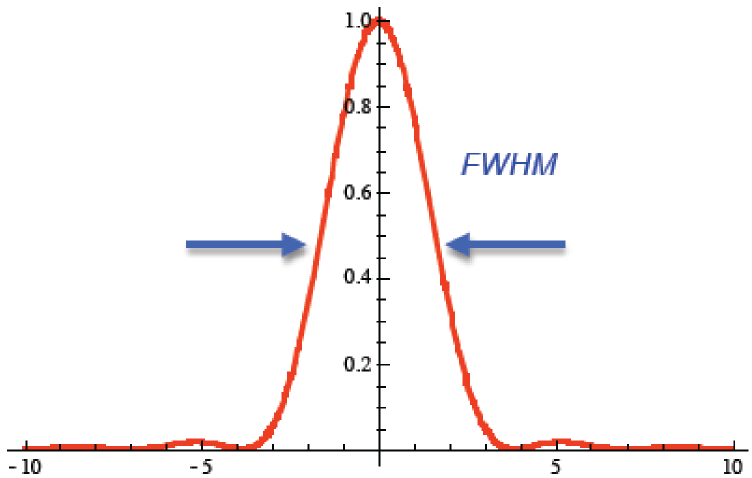


Figure 1: Point Spread Function of a diffraction limited Tel

The theoretical angular diameter of the PSF is given by:

$$\Delta\theta = \lambda/D$$

where

$\Delta\theta$ is the Full Width diameter of the PSF at Half Power (or Full Width at Half Max, FWHM), in radians
λ is the wavelength of the radiation
D is the diameter of the telescope.

Resolution is an important concept in astronomy, especially radio astronomy. The TA will go over this topic with you in the lab.

Questions for your lab report:

> *(1) What is the expected Full Width Half Maximum (FHWM) beam diameter for the SRT in degrees, given that the frequency of the radiation is 1420 MHz and the telescope is 3.0 meters in diameter?*
> *(2) How does this compare with the angular diameter of the sun?*

Present calculations and answers to these questions in your lab report.

Operating the small radio telescope

To begin operating the telescope, first switch on the grey box next to the designated lab computer, connecting it to the SRT. To start the SRT software, click on the icon with a picture of the telescope called "SRT" on the desktop of the right computer.

Control Buttons

The control panel display has a line of control buttons at the top that set up a command for the telescope. For instance, if you want to change the receiver frequency, you click on the "freq" button. Instructions and help information are then displayed in the help panel below the map while the cursor is pointed to the button (This field is blank when you move the cursor away). If you want to change the frequency, then type data into the data line (e.g. "1420.4 4"), click "enter" or "return". Be sure you leave a space between 1420.4 (which is the frequency) and the number 4 (which sets the bandwidth of the system). The current observing parameters will be updated in the appropriate box.

Control Display & Sky Map

The control panel shows a map of the current sky in Azimuth and Elevation units. 0 degrees azimuth points North, 90 degrees points East, 180 degrees points South and 270 degrees points West. The horizon is at 0 degrees elevation and Zenith is at 90 degrees. The telescope can point to about 85 degrees in elevation.

The map displays objects visible at the current time. The software tracks an object or a given azimuth and altitude position, including corrections for the rotation of the earth. Also displayed on the map are various individual astronomical sources and a track of dots that show the plane of the Milky Way. The Longitude of points along the equator of the Milky Way are shown as Gxxx, where xxx is the Galactic longitude. We have an unobstructed view of all objects above about 20 degrees.

Pointing the Telescope

You can point to any given position in the sky by clicking on the "AzEl" button and then typing the desired Azimuth and Elevation in the command line at the bottom of the display. (e.g., 30 45 "enter" will send the telescope to a position of 30 degrees azimuth and 45 degrees elevation). When you enter a position you will see a yellow cross and the CMD numbers will change to these numbers.

When “track” shows up in green at the top of the display, the telescope has acquired the position and is tracking it, this holds for sources. If the telescope does not move, then you probably have to click on “track”.

Occasionally the telescope motor will get stuck and stall. Normally it fixes itself automatically by going back to the “Stow” position (where it is stowed after every observation), but if it remains stalled for several minutes click “Stow” to do so manually. This can be a nuisance and time-consuming but you should still be able to obtain the necessary data if this happens.

Calibration in Theory

In radio astronomy, it is usual to interpret the measured antenna signal in terms of temperature, as if the measured source is a blackbody, and fills the full field of view of the telescope. This isn’t necessarily the case, but it’s often a good approximation and gives a physical parameterization of the measured power. For large wavelengths (i.e., for radio waves as in our case), the expected blackbody emission spectrum is well approximated by the Rayleigh-Jeans law:

$$B_\lambda(T) \approx \frac{2c}{\lambda^4} kT$$

so the emitted power (and thus the power measured by the antenna) is simply proportional to the blackbody temperature. Therefore, if the antenna is pointed at a blackbody source, the measured power signal received, P, will be proportional to T, and we can write:

$$P_{received} = \beta \; T_{source}$$

where β is a constant that takes care of all the physical constants, as well as the efficiency of the antenna and amplifiers in the receiver, and so on. The Universe, which is filled by the Cosmic Microwave Background (CMB) radiation (see PHSC12620 – The Big Bang, if you want the details), is effectively an oven of temperature T_{CMB} = 3K, and this is the temperature the antenna will measure if the antenna beam is pointed to a direction in the sky where no radio sources are present.

In practice, the radio telescope receiver has an intrinsic noise, which produces a power signal even in the absence of an astronomical source. To this power, P_{SYS} , we define an equivalent system temperature, T_{SYS} , according to

$$P_{\mathrm{sys}} = \boldsymbol{\beta} \, T_{\mathrm{sys}}$$

Since this noise is present in all our measurements, we need to know it so we can correct our measurements accordingly, i.e., subtract it from our measurements. The power P_{SYS} can be measured by pointing the SRT toward a direction in the sky without radio sources. In this configuration, the measured power is dominated by the receiver noise, since the contribution from the CMB signal is negligible.

To get rid of the unknown coefficient of proportionality β, we make use of a calibrated radio source of known temperature. For this purpose, the SRT is equipped with a small electronic source of radio waves at the vertex of the telescope dish to allow remote calibration. The equivalent temperature of the calibration source is T_{cal} ~ 200 K. When we switch on the calibration source, we introduce an additional noise temperature of 200 K, and the software notes the new power reading (P_{sys}+P_{cal}). The ratio of these two measurements cancels out the factor β and T_{sys} can be derived.

The SRT software does all of these calculations for you and result is displayed in the middle right window in the SRT console.

Calibration in practice

It is important to first set the frequency of the radiation that will be analyzed by the software. We will start with continuum radiation so we will avoid the Hydrogen line, and instead set the frequency to 1416 MHz.

The control panel display has a line of control buttons at the top that set up a command for the telescope. To change the receiver frequency, you left click on the “freq” button. Instructions and help information are then displayed in the help panel below the map while the cursor is pointed to the button (This field is blank when you move the cursor away).

Lab tasks

- *For continuum radiation, click on the Freq button, then type “1424 1”* in the command line and hit return (the “1” is to set the spectrometer mode – more about that later). **You will not get reasonable or consistent results for the calibration if the frequency has not been set.**

- Click on the AzEl button and type an Az and El for a blank part of the sky (e.g., azimuth and elevation: 10 35; check on the display to make sure you select coordinates away from the Galactic plane and from the Sun) into the data input line at the bottom of the control panel (see Fig. 2) and then hit “enter”. You should see the telescope in the left hand display start to move in elevation. The telescope position is printed out in the top right panel of the control display. The top line gives the command position (which should reflect the position you typed in for AzEl) and the next line gives the actual position.

  Note: the system uses a single cable to transmit both control and measurement data so while the telescope is moving to a new command position there will be no data (and possibly no display) during the slew (this can be frustrating, so try to be patient!).

- When the telescope position is settled (i.e., the cross on the sky display is red and the “Track” button at the top of the display is green), click “Clear” and write down the raw output power and temperature (see figure 1). Click on Cal button and watch these numbers. The system will record the output power itself and then switch on the noise source. The computer will then calculate the temperature of the sky at that point, which includes the system noise temperature. *Note that you should "Clear" before each new observation, for both calibration and data, since the telescope is continually integrating and thus keeps all the previous data since it was last cleared.*

- Repeat this calibration a few times, waiting about 1 minute between observations. The first calibration of the run requires a few cycles. Once you have a stable system temperature, record the sky counts temperature (it is shown in units of degrees Kelvin in the text above the cmd line) and system temperature from the middle right panel of the console. Note that the sky temperature will still be fluctuating some: estimate a range and average value.

Background Noise

The telescope is sensitive to all sources on the sky, as well as buildings, etc. Although the telescope is most sensitive over a small area of sky it does have some low sensitivity to sources at much larger angles from its pointing position. Radio power generated by cell phone towers and wireless devices within buildings can cause significant addition signals even though the 1420 MHz band that we are using is protected for radio astronomy by law against such interference. Communication devices operate near our frequency and do not always suppress power in our frequency bands; although the fraction of the radio power in these sidebands is tiny, our telescope is very sensitive and we can detect this interference. Man made interference can be distinguished from astronomical sources because it is usually fixed in position while astronomical sources move across the sky.

Is the radio sky uniform in Hyde Park?

Given that there are background sources that can interfere with our measurements of real sky signal, before we proceed to make any measurements, it is instructive to explore just how uniform the sky is in Hyde Park.

<u>Lab tasks:</u>

- Make sure the frequency is set to 1424 MHz by clicking on freq and typing in 1424 1

- From the telescope pointing that you used in the previous exercise (say [az,el]=[10,35]) point the telescope 15 degrees lower in elevation (at the same azimuth) and 20, and 40 degrees higher in elevation. Repeat the calibration at each of these positions 2-3 times and record azimuth and elevation of the pointings and values of the sky temperature and system temperature displayed by the SRT console. Avoid labeled astronomical objects.

- After you are done with measurements at different elevation, return to your original elevation and change azimuth with step of 50 or 75 degrees (at constant elevation) and repeat calibrations 2-3 times at each pointing. Record azimuth and elevation of the pointings and values of the sky temperature and system temperature displayed by the SRT console.

Lab report question:

(3) How uniform is the radio sky around Hyde Park? In your lab report, discuss the uniformity of the sky signal and system temperature based on your measurements. Include a table of your measurements and your interpretation and conclusions.

Measuring the velocity of Hydrogen clouds in the galaxy

Hydrogen is the most common element in the universe. It exists in interstellar space as individual atoms, each atom consisting of a proton and an electron. Both particles have a property called *spin* and a hydrogen atom can exist either with the spins of the proton and electron parallel or anti-parallel. Sometimes the atom changes its spin state and, in doing this, emits a radio wave at a precisely known frequency of 1420.4 MHz corresponding to the wavelength of 21cm. By tuning the SRT receiver to this frequency, we can directly measure the amount of hydrogen in that direction and, importantly, its velocity.

We can measure its velocity through the effect known as *Doppler Shift*. When an object is moving towards or away from an observer, the wavelengths of the light observed from the object get compressed or stretched. Since the wavelength and frequency of light are inversely related, frequencies are respectively increased or decreased. This is called Doppler Shift. Thus if we know the intrinsic frequency at which an object emits – in this case from the fundamental physics of the Hydrogen spin-flip transition – then we can calculate the velocity of that object with respect to our observational frame of reference – in this case the Earth. Your TA will go over this important concept with you in more detail during the first lab section. (Make sure you understand what's going on and ask questions if you are confused!)

To do this, the SRT receiver measures the flux at a number of very finely spaced frequencies, typically 7.5 kHz wide and spaced 7.5 kHz apart, and so can produce a spectrum of the hydrogen line. This allows us to measure the Doppler shift of the atomic hydrogen clouds in the interstellar space of the Milky Way emitting the radio waves and allows us to measure the rotation velocity of that gas around the center of our Galaxy.

Lab report question:

(4) What is the smallest difference in Doppler shift in km/sec you would expect to measure, assuming the resolution is set by the channel bandwidth of 7.5 kHz? Provide the equation and your calculation to

support your answer in your lab report.

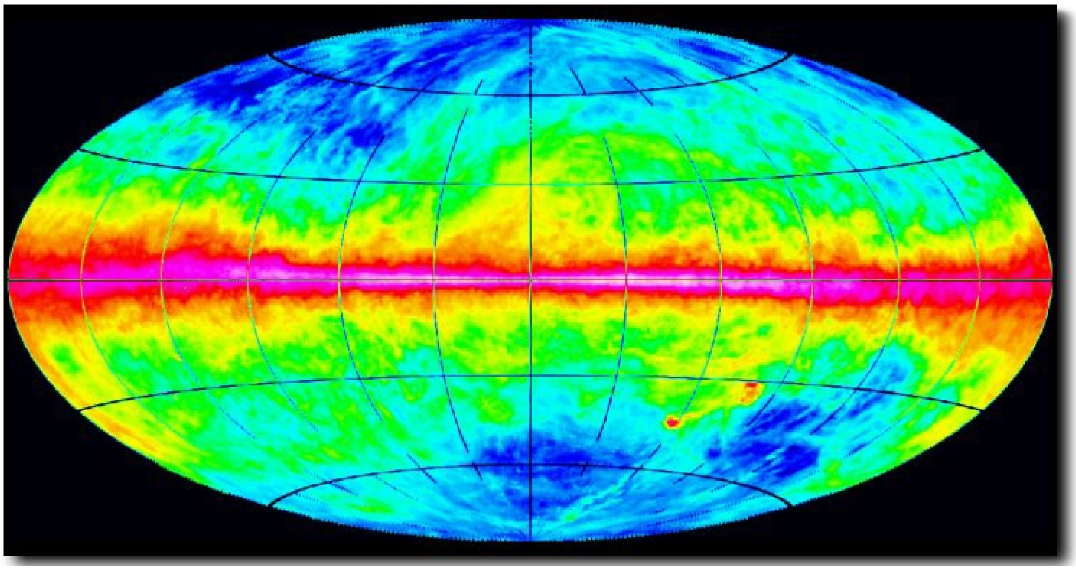


Figure 3: The diffuse Hydrogen 21 cm emission over the entire sky shown in Galactic coordinates with the Galactic center in the middle. Red indicates intense emission, while blue indicates very weak emission.

Rotation of the Galaxy and Dark Matter

New stars are formed in our Galaxy in huge clouds of gas and dust. When we look at the sky at optical wavelengths we see nearby stars, but the dust acts like a fog in interstellar space and this limits the distance to which we can see stars in our Galaxy; it was this effect that made early astronomers think that we were near the center of our Galaxy. At radio wavelengths there is almost no absorption and flux at 21 cm is large due to large amount of atomic hydrogen in diffuse gas clouds between stars. This means that we can see these clouds right across the Galaxy even with a small radio telescope. By making various assumptions we can measure the rotation of the Galaxy and map out its structure. We first have to assume that the clouds move round the galaxy in approximately circular orbits with a velocity that depends only on the distance from the center of the Galaxy. This is shown in Figure 4a. The reasoning behind such assumption is that we see gas moving in such circular motion in other galaxies, which we can view from the outside.

The Sun is moving around the center of the Milky Way in an orbit close to circle. The radius of this circle is measured independently and is about 8.5 kpc (i.e., the distance from the Sun to the center of our Galaxy is 8.5 kpc). If we look through the Galaxy we will see clouds also moving around the center of the Milky Way at different distances from the center and with different velocities. The Doppler shift that can be measured with SRT can only measure the radial component of velocity (i.e., the velocity along our line of sight) and not the total velocity of the gas. The velocity we measure with SRT is the difference in velocity between us and the cloud measured along the line of sight (indicated by the red vectors in Figure 4a). In any given direction we will see a number of clouds moving at different velocities to our line of sight so that the radio spectrum will look like Figure 4b.

The radio flux from any cloud depends on the number of hydrogen atoms in the cloud itself. Atoms within the cloud have their own random motions so that we see a spread in frequencies from each cloud (see Figure 4b). The mean frequency depends on the difference in circular velocity between the cloud and the Sun and on a geometric effect introduced because we are only measuring the velocity along the line of sight.

We should note that, initially, the line of sight velocity gets larger as we look at clouds rotating nearer to the center of the galaxy. However, at some distance (in this case cloud 4) there is a maximum line of site velocity. This cloud is going away from us directly along our line of sight. Clouds further away appear to have a smaller line of sight velocity. We can determine this maximum velocity by measuring in the radio spectrum the maximum Doppler shifted frequency (see Figure 4b).

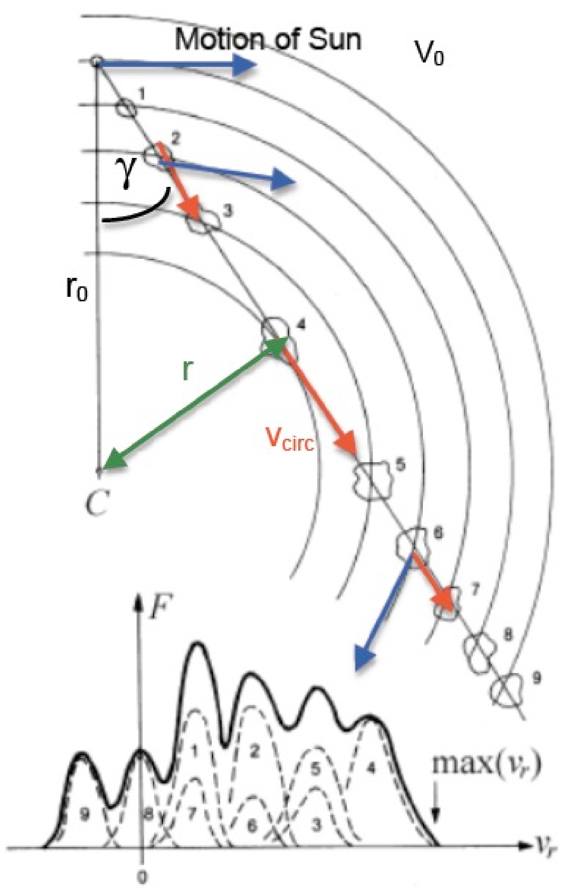


Figure 4a: Geometry of measuring lines of sight in our galaxy. The Sun, all the billions of other stars in the galactic disk, and all of the emitting hydrogen gas between the stars orbit in the same direction, in circular orbits confined to a plane.

Figure 4b: Corresponding frequency spectrum. The hydrogen lines corresponding to individual clouds are indicated by dashed lines.

We measure Galactic coordinates in a heliocentric frame of reference. The center of the galaxy is at 0 degrees longitude and the angle above the plane, as viewed from the Earth is given as a latitude. When we point at a given longitude, in the galactic plane, we measure the difference between the line of sight velocities of all the different clouds relative to the Sun's line of sight velocity to that direction.

What we are hoping to measure is the actual velocity of clouds as they circle the galaxy and for this we have to know the velocity of the Sun going round the Galaxy. It turns out that we cannot measure this velocity from 21 cm line observations because we need to measure the solar motion using a frame of reference outside the disk of the galaxy that does not rotate. One such reference frame is the halo of globular clusters that surround the Galaxy, another is the reference frame of distant galaxies. By measuring their Doppler shifts using the spectra of their stars, we can measure a mean velocity of the sun. The best estimate is that the objects at the same distance as the Sun from the center of the Galaxy rotate in a circular orbit at an average circular velocity of 220 km/sec.

There is additional motion due to the Earth's rotation around the Sun and the Sun's own motion. Working out all these extra velocities is tedious, but fortunately the SRT software calculates them for us, and provides velocity relative to the Local Standard of Rest (VLSR). The VLSR is given in km/sec in the control panel and the measured velocities are referred to this VLSR. You will also see that the spectrum of radio emission, which you can get by clicking on the spectrum window also shows radio flux as a function of both frequency and VSLR.

From the geometry of Figure 4a, the velocity of the Sun in the line of sight for a given galactic longitude γ is given by:

$$V_{sun}(\gamma) = 220 \sin(\gamma)$$

As discussed above, in any one direction of galactic longitude we can measure the maximum velocity of the spectrum, V_{max} (the rightmost edge of the emission for cloud 4 in Fig. 4b). By geometry from Figure 4a, for a given longitude γ the distance of the cloud to the center of the Galaxy is

$$r = r_0 \sin(\gamma)$$

where r_0 = 8.5 kpc is the distance of the sun to the center of the Galaxy. The corresponding circular velocity is given by:

$$V_c(r) = V_{max}(\gamma) + V_{sun}(\gamma)$$

To measure rotation curve of the Galaxy we need to measure V_{max} for various distances r, that is for different Galactic longitudes γ. We are now ready to perform such a measurement. Before we proceed, pause and think about what you are about to do: you are about to detect and analyze radio emission from Hydrogen atoms in interstellar space thousands of light years away (i.e., it travelled to us for periods of time comparable to the history of human civilization). Note that depending on the time of observation, not all galactic longitudes are visible in the sky. How much we could measure with the SRT directly, depends on which portion of the Galactic disk plane is accessible to observation (i.e., up in the Chicago sky at the time of your lab session). Sometimes, only sources between 90 and 270 degrees are visible during the lab session: in such case, you can still get the spectrum of 21 cm emission from interstellar atomic hydrogen, but you cannot use your SRT data to measure the rotation velocity of the galaxy. Instead we will use data from an online all-sky 21 cm hydrogen survey. No matter what time of the year the lab is performed, we need the online data for galactic longitudes between -10 and -90 degrees as this portion of the sky is never visible from Chicago.

Lab report question:

(5) In your lab report please explain why we cannot use longitudes larger than 90 degrees (or more negative than -90 degrees) to obtain rotation curve of the Galaxy. Use Fig. 4a in in your reasoning.

You might wonder why we care about the rotation of the galaxy in the first place. Ultimately, the goal of this lab is to map out the *rotation curve*– the profile of stellar orbital velocities as a function of radius from the galactic center – of the galaxy, which tells us about the *mass distribution* of the galaxy. The gravitational field caused by the mass distribution of the galaxy determines the velocities of stellar orbits. Assuming spherical symmetry (which is not accurate in detail but a good first approximation), we can relate the velocities of stars at a radius r to the enclosed mass distribution by

$$M(< r) = \frac{v(r)^2 r}{G}$$

Therefore, by measuring v as a function of r we can estimate the enclosed mass distribution. You will see that the distribution of mass cannot be accounted for by the observable galactic disk alone, thus providing evidence for an “invisible” matter component termed *Dark Matter.*

Setting the receiver frequency for Hydrogen and calibration

First set the frequency of the receiver to the Hydrogen at 1420.4 MHz. To change the frequency and set the spectrometer, first click on “freq” then 1420.4 4 on cmd line and hit “enter”. Be sure you leave a space between that 1420.4 (which is the frequency) and the number 4 (which sets the bandwidth of the system for the largest velocity coverage).

You now calibrate the system but we now have to worry about Hydrogen emission from the sky messing up the calibration. We can avoid this either by moving the telescope to a position far off source (which may still be contaminated by another source of Hydrogen emission) or by changing the frequency just for the calibration and then switching it back afterwards. To calibrate with a frequency offset when the system is on the target, first change the frequency to

1423 4, which is sufficiently far away from the hydrogen line that you are measuring only the receiver noise and thermal background.

Note that if you have spectral features showing up in your calibration spectrum, then the sky position you used for your calibration probably had strong Hydrogen emission. Try to calibrate at another position, or at an offset frequency as described above.

Lab tasks:

- Perform the calibration procedure, as described above. The offset frequency method is recommended.

- Bring up an Excel window to record your work. Before each observation enter the galactic longitude of your pointing and then copy the spectrum you obtain below this.

- In the control panel, the positions of different Galactic longitudes along the equator are indicated by Gxxx (where xxx is in degrees). If you see longitude of 90 degrees and smaller, start at the longitude of 90 degrees and work your way down to the smallest longitude you can observe down to zero if the center of our Galaxy (coincident with the source named Sgr A). If you cannot observe longitudes smaller than 90 degrees observe one or two longitudes that are up in the sky. This will give you an idea about the spectrum of clouds in the Galaxy, but we will use other data to construct rotation curves (see below). Note that galactic coordinates labeled in the SRT display can be pointed to by clicking on them – for the rest you have to estimate (or look up) the Az El of the desired longitude on the galactic plane.

- Move to your first pointing and press Clear button to clear the spectrum accumulated by the SRT.

- Set the frequency to 1420.4 4 and obtain the spectrum for the galactic longitudes visible in the sky by integrating for 10-20 seconds (or longer) along the same pointing.

- After integration click on the spectrum window to get a detailed plot of the spectrum in a separate window. You will see emission flux as a function of frequency and VSLR. Estimate that maximum velocity of the clouds visible in the spectrum (similar to cloud 4 in Fig. 4b).

- Record the spectrum by first clicking on the spectrum in the viewer to bring up the detailed plot, then use Alt-PrintScreen to capture the plot in the clipboard and then press Ctrl-V to paste the clipboard in the Excel window. Be sure to maximize the window size to make your plot as legible as possible.

- Proceed to the next Galactic longitude available for observation. Press Clear button to clear the spectrum before you make observation at each new longitude. Record longitude and spectrum in the Excel file along with your estimate of the maximum velocity for each pointing.

If some or all longitudes between 0 and 90 degrees are not observable at the time of your lab, we will use the all-sky survey of 21 cm hydrogen emission performed by astronomers and available online. We will also use it to obtain data at galactic longitudes from -10 to -90, which are always below the horizon from Chicago. The all-sky survey of radio emission (see Fig. 3) was obtained by a collaboration of astronomers at Leiden, Argentine and Bonn (LAB survey). The web site is at https://www.astro.uni-bonn.de/hisurvey/AllSky_profiles/.

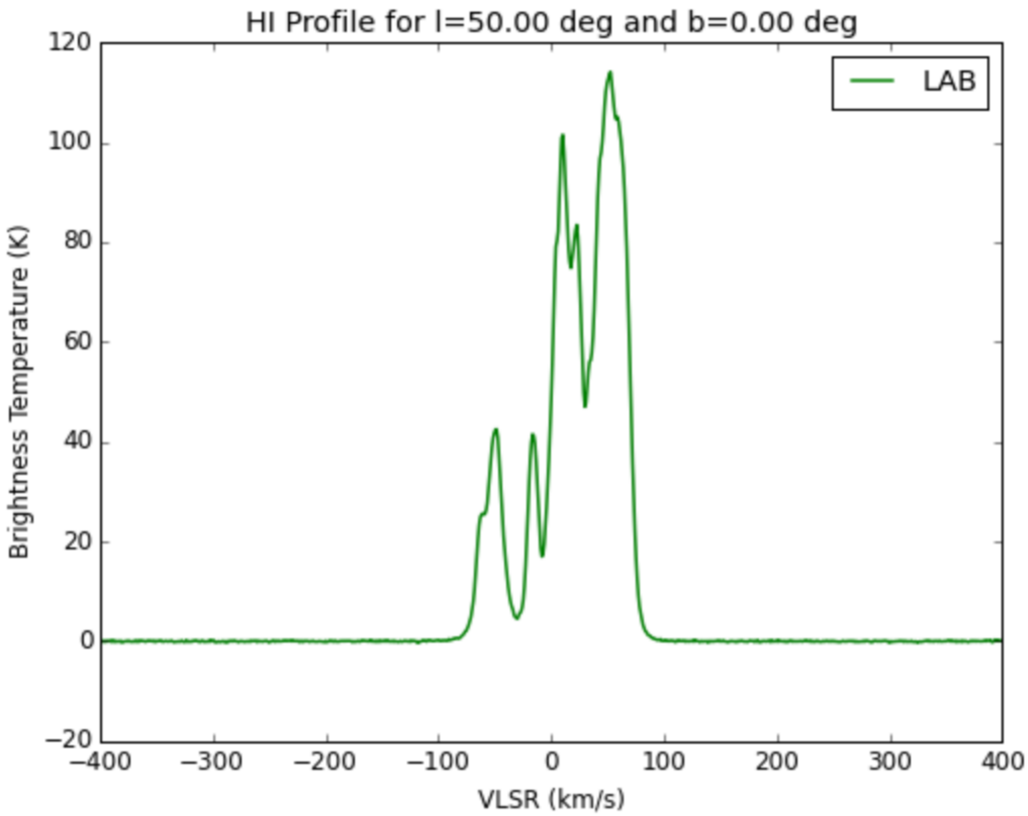
The LAB survey input form is shown in Figure 5. To obtain a spectrum from the LAB survey for a position along the galactic equator, select Galactic coordinates longitude and altitude = (l,b) for the coordinate system, input your desired l and b (l here is the same as γ used in Fig. 4). For all your pointings b=0. Select LAB Survey only. You can also select the “effective” beam FWHM. The angular resolution of the LAB survey is 40 arcminutes (0.67 degrees). In Figure 5 and 6 we have used 0.2 degrees, in this case the program returns the average spectra for a 0.2 degree patch of the sky for comparison with the measurements you made with the SRT.

Search Position		
Coordinate system	Galactic coordinates (l, b)	
Center	RA [h m s]/ l [°]	50
	Dec [±° ' "]/ b [°]	0
Effective beamsize FWHM [°] (must be < 1°)		0.2
Surveys	EBHIS (δ > -4°)	☐
	GASS III (δ < 1°)	☐
	LAB	☑
Search		

Figure 5: The input form for accessing the LAB spectra. The resulting output is shown in Figure 6 below (note that the form resets after each search).

Lab tasks:

- Obtain spectra from the LAB survey along the galactic equator for the longitudes that you could not observe with SRT. Measure the maximum radial velocity, V_{max} (note these will be max positive for longitudes 10 to 90 deg. and max negative for longitudes -10 to -90 deg.). You will note that the spectrum does not provide a clean maximum velocity. Try to think about the best way to measure maximum velocity. Discussion possible options with your TA. Once you decided on the particular algorithm to estimate maximum velocity stick with the same algorithm for all longitudes. If you want more precision, you can download the spectra in digital form from the site.
- Re-observe one of the longitudes that you have observed with the SRT and compare the spectrum you get from the LAB survey to that you have obtained with the SRT. Discuss any similarities and differences that you may find. You can also substitute any source on the sky for making the comparison of the SRT and the LAB survey spectra.
- Assuming a distance from the Sun to the galactic center of 8.5 kpc and a circular velocity of 220 km/sec at this radius, fill in Table 1 with the results of your calculations using your measurements of V_{max} and equations above.
- Plot the circular velocity you obtain with eq. 3 against tangential distance for each longitude. You can make your plot either in Excel or any other plotting program you are comfortable with.



Position		
Requested	l [°]	50.00
	b [°]	0.00
	RA [°]	290.83
	Dec [°]	15.14
Column Densities		
EBHIS	0.200° Beam	0.135E+23
GASS III	0.266° Beam	0.00
LAB	Interpolated	0.142E+23
	Nearest gridpoint	0.142E+23

Figure 6: Resulting HI spectra and the LAB query.

Galactic Longitude (degrees)	Tangential distance (kpc) r	Maximum LSR velocity V_{max}(km/sec)	Line of sight solar velocity V_{sun} (km/sec)	Circular velocity V_C (km/sec)
10				
20				
30				
40				
50				
60				
70				
80				
90				
-10				
-20				
-30				
-40				
-50				
-60				
-70				
-80				
-90				

Table 1: Data table for measurement of the rotation velocity of the Galaxy

Lab report questions:

(6) Compare a spectra from the LAB with FWHM beam of 0.267 degrees to that with 9 degrees (like the SRT). Why might these plots look different? Compare these both to the spectra you measure with the SRT for any source on the sky. Present the spectra and your thoughts in your lab report.

(7) Include the filled out table (or printed from excel) and the plot of your measured circular velocity vs longitude in your lab report. What does this say about the mass distribution of the Galaxy? Include your answer in your lab report.

Stowing the telescope

When you have finished with the SRT don't forget to stow it. Put the cursor on the STOW button, click and watch the television camera to see that the telescope does stow. When it is stationary in this position, turn off the control box power and exit the control program.

APPENDIX A

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, "$5 \pm 1\,\mathrm{m}$" means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters[1]. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

A.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument's scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment's manual often gives the instrument's resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

[1] The phrase "most likely" can mean different things depending on who is writing. If a physicist gives the value and does not given a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet, or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}}, \tag{A.1}$$

where $\{x_1, x_2, \ldots, x_N\}$ are the measured values, $\bar{x}$ is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called "standard deviation of the mean", which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}. \tag{A.2}$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as 52 ± 3 m.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn't change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

A.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction)}, \ \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{A.3}$$

$$\text{For } c = ab \text{ (or for division)}, \ \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \tag{A.4}$$

$$\text{For } c = a^n, \ \frac{\delta c}{c} = n\frac{\delta a}{a} \tag{A.5}$$

For other calculations, there is a more general formula not discussed here.

Expression	Implied uncertainty
12	0.5
12.0	0.05
120	5
120.	0.5

Table A.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π, is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table A.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value "`0.0322857142857143`". Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as "`9.65683578099600e-04`", where the '`e`' stands for "times ten to the". I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as "0.032 ± 0.001 km/s". You may also see the equivalent, more succinct notation "0.032(1) km/s".

A.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced "tee prime"). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as[2]

$$t' = \frac{|a - b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \tag{A.6}$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

[2]Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.

APPENDIX B

Lab Report Format

B.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.

B.2 Organizing the report

The report should follow the sequence of the lab manual. Answers to questions and inclusion of tables and figures should appear in the order they are referenced in the manual. In general, include the following:

- Any procedure that you performed that is different from what is described in the lab manual.
- Any data that you've collected: tables, figures, measured values, sketches. Whenever possible, include an estimate of the uncertainty of measured values.
- Any calculations that you perform using your data, and the final results of your calculation. Note that you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92\,\mathrm{m/s^2})t + 0.2\,\mathrm{m/s}$.
- Answers to any questions that appear in the lab handout. Each answer requires providing justification for your answer.
- At the end of each experiment, you should discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

B.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like "distance (pc)". This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like "distance (pc)".
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called "error bars", along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.

Bibliography

[1] E. Etkina, A. Van Heuvelen, S. White-Brahmia, D. T. Brookes, M. Gentile, S. Murthy, D. Rosengrant, and A. Warren, “Scientific abilities and their assessment”, Physical Review Special Topics - Physics Education Research **2** (2006) 10.1103/PhysRevSTPER.2.020103.