ASTR 24100, Winter 2020
Spectrum of black body radiation

1 Introduction

In this lab you will study the continuous emission of bodies heated to a particular temperature. Such emission can be compared to that of an idealized blackbody," a term introduced by physicist Gustav Kirchhoff in 1860. A blackbody perfectly absorbs all incident radiation, and thus would appear perfectly black. At the same time, it emits continuous radiation with a spectrum that peaks at a wavelength inversely proportional to the bodys temperature.

Although the term may sound mysterious, you see sources of such radiation every day all around you. Incandescent light bulbs that are still commonly used for lighting emit a spectrum very close to that of an ideal blackbody, as you will see in this lab. The Sun light that we enjoy every day also has a spectrum that can be approximated by the blackbody spectrum. Likewise, stars that you see in the night sky emit light with spectra that can be approximated by blackbody spectra. You will check this in this lab for spectra of a few representative stars. This lab also illustrates a very important concept. Physical sciences in general, and astronomy in particular, achieved significant progress because physical laws discovered here and now in the lab turn out to be universally applicable. Thus, for example, you can study Balmer lines of hydrogen from a discharge tube in the lab and then find similar lines in spectra of stars or galaxies. It is this universality that makes science tick. It would be very difficult for us to make sense of astronomical objects, if they worked according to their own celestial laws, not accessible to study in our terrestrial labs. (Yet, this was the accepted view until the scientific revolution of 16th-17th centuries).

In this lab you will employ this powerful concept by first learning how to measure the temperature of a tungsten filament in an incandescent bulb: you will measure the bulb blackbody-like light spectrum with the Red Tide spectrometer, and then apply the same technique to calculate the surface temperature of stars from their spectra.

2 Black body spectrum and the Planck formula

The blackbody spectrum has a shape characterized by a broad peak. The peak wavelength position is inversely proportional to the temperature of a blackbody emitting the radiation. This is known as Wein's Law:

$$\lambda_{\max} = 2.9 \times 10^6\,\mathrm{nm/T} \tag{1}$$

Lab tasks.

2.1. We are all blackbody emitters! Estimate the peak emission wavelength for a normal body temperature of 310 K. Which part of the electromagnetic spectrum does this correspond to?

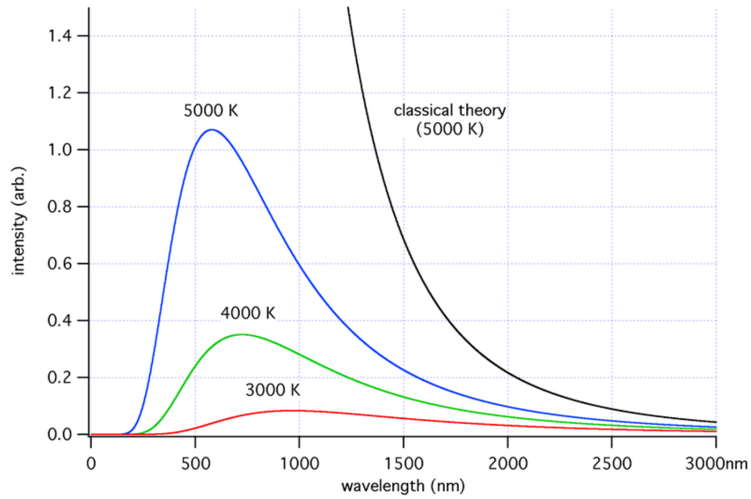

Figure 1: The spectrum of radiation emitted by a perfect blackbody at temperatures of 3000K, 4000K, and 5000K according to the Planck formula is shown by red, green, and blue lines, respectively. The solid black line shows prediction of classical (non-quantum) thermodynamics for emission at 5000K(Figure source: Wikipedia.org).

The shape of the blackbody spectrum was measured in laboratory at the end of 19th century and was a source of much puzzlement to the physicists of that time because it contradicted the expectations of classical thermodynamics (the branch of physics studying heat and radiation). Attempts to understand the shape of the blackbody spectrum led to development of quantum mechanics. The formula accurately describing the shape of the measured spectrum was proposed by the German physicist Max Planck in 1900. Planck also showed how this shape can be understood using theoretical calculations based on concepts of quanta of energy and thermodynamic equilibrium. The Planck formula describing the shape of the blackbody spectrum as a function of wavelength λ is given by the following formula:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp(\frac{hc}{\lambda kT}) - 1} \tag{2}$$

where $h = 6.63 \times 10^{-34}\,\mathrm{J/s}$ is the Planck constant, $c = 2.99 \times 10^8\,\mathrm{m/s}$ is the speed of light in vacuum, and $k = 1.38 \times 10^{-23}\,\mathrm{J/K}$ is the Boltzmann constant. The Planck spectrum shape for different temperatures along with predictions of the classical theory is shown in Fig. 1.

Fig. 2 shows the spectrum of the CMB radiation – a ubiquitous relic radiation from the first few thousand years in the existence of our universe. This radiation was emitted when the universe was very hot and dense and almost in thermodynamic equilibrium. The spectrum is thus very accurately approximated by the Planck formula. Note that this is yet another example of the amazing universality of physical laws in nature. You can use the same formula to describe the spectrum of a regular incandescent bulb and spectrum of relic radiation from the early era of evolution of our universe!

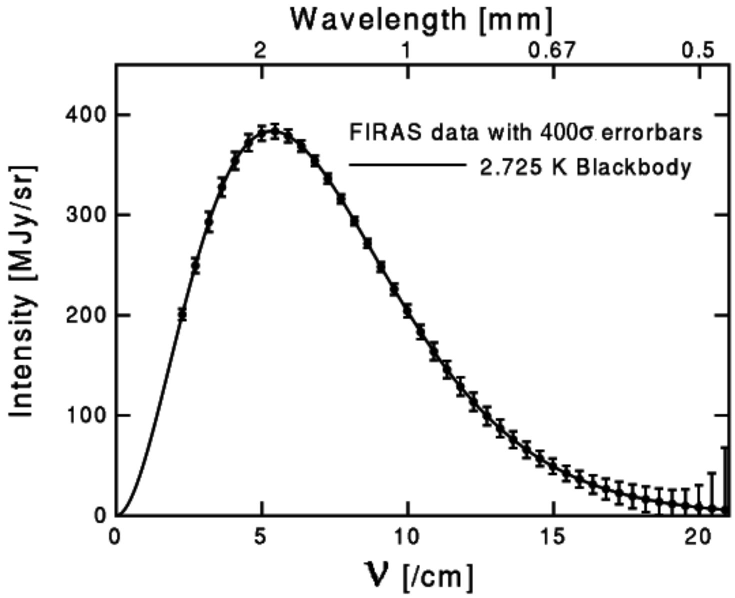

Figure 2: Spectrum of CMB as measured by the FIRAS instrument on COBE satellite in the 1990s. Note that the measured spectrum is described almost perfectly by the Planck formula with an effective temperature of 2.7K; the error bars shown are 400σ!

3 Experimental setup

3.1 Spectrometer hardware

In this lab we will use the the Ocean Optics digital spectrometer, which measures light in the range 350 to 1000 nm with a resolution of about 1 nm ($1\,\mathrm{nm} = 10^{-9}\,\mathrm{m}$.) Light from the source under study is collected by an optical fiber. **IMPORTANT: the fiber is fragile, handle with care**. The fiber will be fixed in a holder, which can be adjusted to point to the light bulb. The spectrometer is connected to the computer through USB, and dedicated software (Spectra Suite) is used to operate the spectrometer and save the data. For reference, Fig. 3 shows the main controls of the SpectraSuite software.

You will use a regular 25W incandescent light bulb setup in a holder, with a voltage regulator to change the light intensity.

3.2 Suggested procedure for saving spectra

For the following experiments, you can work in groups of 2 or 3. You'll need to operate the voltage control, run the Spectrum Suite software and copy the data and images into Excel and Word documents, and take notes. Please create a personal folder into which files can be saved, and delete it at the end of the day.

You're welcome to use the software of your choice when completing these assignments; e.g., Libre-

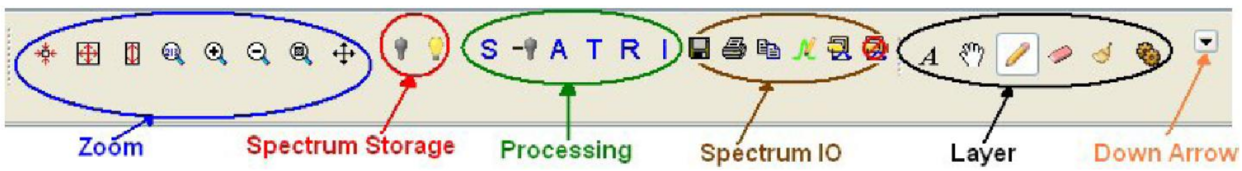

Figure 3: The Spectra Suite software controls: zoom controls can be used to zoom in and out the graph. The gray bulb records background (dark) spectrum, while the yellow bulb records processed signal spectrum

Office, Python, or Matlab. But, in the past most students have found copying and pasting into Word and Excel to be easiest. To do this, we suggest open a Word and an Excel file in which you save your measured spectra at the beginning of your work.

To save an image of your graph, click on the third from right icon in the Spectrum IO controls (Fig. 3). This will copy image of the graph to the clipboard. Then, paste the image in the Word document by pressing Ctrl-V. To save the spectrum you measured in digital form as two columns of numbers, press the third icon from the left in the Spectrum IO controls (the icon showing two sheets of paper to the right of the printer icon), which will copy the digital spectrum into the clipboard. Now go to an Excel file and choose the column into which you want to paste your spectrum (usually column A, first row) and press Ctrl-V, which will paste the measured spectrum as two columns of numbers (wavelength in nm and counts for your spectrum) into the Excel worksheet.

At the end of the lab, save the Word and Excel files with images of graphs and numerical spectra and their analyses done during the lab on a USB stick or email copies to yourself.

An alternative, but slower, way to save the data is to click on the floppy disk icon in the Spectrum IO controls. The format must be Tab delimiter, no header. The writing directory must be specified (Browse button). The spectrum is saved in a text file (.txt) as two columns, the first column giving the wavelength in nm, the second column the corresponding intensity. This file can be imported into an Excel worksheet using Open File within the File menu of Excel.

3.3 Spectrometer calibration

In order to accurately measure the black body spectrum of a source, we have to make sure that the amplitude measured by spectrometer is correct. The light measured by the spectrometer has to travel through fiber optics experiencing multiple reflections, as well as through the optics within spectrometer. Light may be lost in those reflections and such losses are generally different at different wavelengths. The CCD that records the digital spectrum also records light at different wavelengths with different efficiency. The end result is that the digital spectrum recorded by the spectrometer is not a faithful representation of the true spectrum of light. This is unavoidable in all experimental instruments.

Astronomers deal with this issue by calibrating the spectral response of the instrument using a standard source of light with known spectrum. In this lab, you will perform a simplified version of such calibration to correct for the wavelength dependent spectral response of the Red Tide spectrometer.

As a standard source we will use the 25W soft white bulb itself. The light bulb contains a tungsten filament that is heated by the passage of the electrical current and becomes incandescent, emitting light with a spectrum which is very close to a blackbody spectrum. We know from the manufacturer that the emitted light spectrum at the nominal voltage of 120 V is characterized by a color temperature of about 2800K. The color temperature is defined as the temperature of a blackbody emitter which would appear to have the same color as the actual emitter. We calibrate the Red Tide spectrometer spectral response by assuming that at 120 V the light bulb emits the spectrum of a 2800k blackbody.

To calibrate the spectrometer, turn on the light bulb, set the voltage to 120 V and align the optical fiber so that it points towards the bulb at a distance that does not saturate the spectrum measured in SpectraSuite (you can also adjust the integration time in the top left corner of SpectraSuite to deal with saturation). The spectrum should not be saturated but the counts shown in the vertical axis of the spectrum on your computer screen should be sufficiently high at the level of 3500-3800.

First, specify the color temperature of the measured spectrum by clicking on Processing, and then on Set Color Temperature, where you will enter 2800K. Then take a reference spectrum by clicking on the yellow bulb (see Fig. 3). Finally, shield the fiber from the light by placing your hand in front of the fiber and take a background spectrum clicking on the gray bulb (see Fig. 3). You can now click on the I (the first button on the right in the processing section, Fig. 3) and will see the calibrated spectrum. (If you're unsure about this part, please check with your TA that you're on the right track.)

Save the spectrum in digital form into Excel for future reference. You should notice that the calibrated spectrum is very different from the uncorrected one (you can move from one to the other by clicking on S and I), which explains the importance of this calibration procedure to perform quantitative measurements.

Important: Once you measure a reference spectrum at 120V, do not change the position of the light bulb and fiber! Do not change the integration time or anything in the SpectraSuite setup or calibration during these measurements. It is important for the success of the experiment that measurements at different voltages are done with identical configuration of the bulb, the optical fiber and the software setup.

4 Experiment 1: Spectrum of incandescent bulb at different temperatures

After having calibrated the spectral response at 120V, you will measure the light bulb spectrum at lower voltages, to which correspond lower temperatures of the tungsten filament. However, we will first perform some simple experiments with colored filters. You will be given a couple of colored glass filters. Put each of these color filters in front of the fiber and observe its effect on the spectrum you measure with SpectraSuite. Save the image of the graph for each of the filters.

The red plastic you will use has an effect similar to the filters used in astronomy to measure flux of stars only in a certain range of wavelengths. The red piece of plastic could have been used as a filter to measure the flux of the bulb only in the red and infrared parts of the spectrum.

Lab tasks.

4.1. Include plots of these filtered spectra in your lab report and discuss the effect that each filter has on the spectrum.

4.2. In what range of the spectrum does the incandescent bulb spectrum you observe at 120V emits most of its radiation?

4.3. Comment on why incandescent bulbs are considered non-green (in the energy conservation sense, not the color sense) based on the spectrum you measure. (Note that the radiation visible to the human eye is limited to 380-740 nm.) What is effect of radiation emitted by the bulb outside of the range of wavelengths visible to human eye?

We will now measure the emission spectra of the light bulb when its filament is heated to different temperatures. Change the voltage to 110V and examine how the acquired spectrum changes. Save the image of the spectrum into your Word work file and save the digital spectrum into an Excel worksheet into the first two columns (A and B). Decrease the voltage in steps of 10-20V down to a minimum value of 40 to 60 V (until the spectrum you measure becomes too noisy). At each voltage, save the spectrum image and the digital spectrum into Word and Excel files, respectively. Save the spectrum for each voltage into a separate Excel work sheet (into columns A and B in each sheet) and rename these sheets to reflect the voltage at which the spectrum was taken (e.g. the sheets can be named V120, V110, V090, etc.).

Once you have spectra at different voltages measured and saved, you are ready to perform measurements of temperatures. This measurement will be done using a common scientific procedure: fitting a model to data. The model in this case is the Planck formula for the spectrum, and data is your measured spectrum at each voltage. Sophisticated fitting algorithms are used in practice, but here we will use a simplified, but sufficiently accurate and illustrative procedure in which you will fit the spectrum shape by eye. In this procedure you will fix the predicted Planck spectrum to match the measured spectrum at a particular wavelength and then adjust the temperature until you find the value for which the Planck formula best matches the shape of the measured spectrum.

Use the example Excel sheet supplied with this manual (see Fig. 4) as an example of how such fit is done. In the example sheet, the columns A and B are the measured spectrum (wavelength in nm and counts). Column C encodes the Planck formula, given by eq. 2 (click on cell C1 to see formula displayed in the f x field above the columns). The formula uses h, c, and k constants in the cells T1, T2, T3 and temperature in degrees Kelvin in the cell P1. Note that constants in Excel are written as P$1, while references to particular data cells are written as A1. The Planck formula is normalized to match the counts of the measured spectrum at the reference wavelength corresponding to row 251 (cell P2), which corresponds to the reference wavelength 600 nm (cell P3). The unnormalized amplitude of the Planck formula at the reference wavelength is computed in cell P4, while the actual measured counts at the reference wavelengths is in cell P5. Column D contains the squared difference between the measured intensity (column B) and the intensity predicted by the Planck formula (column C) for the temperature set in cell P1 and normalized at

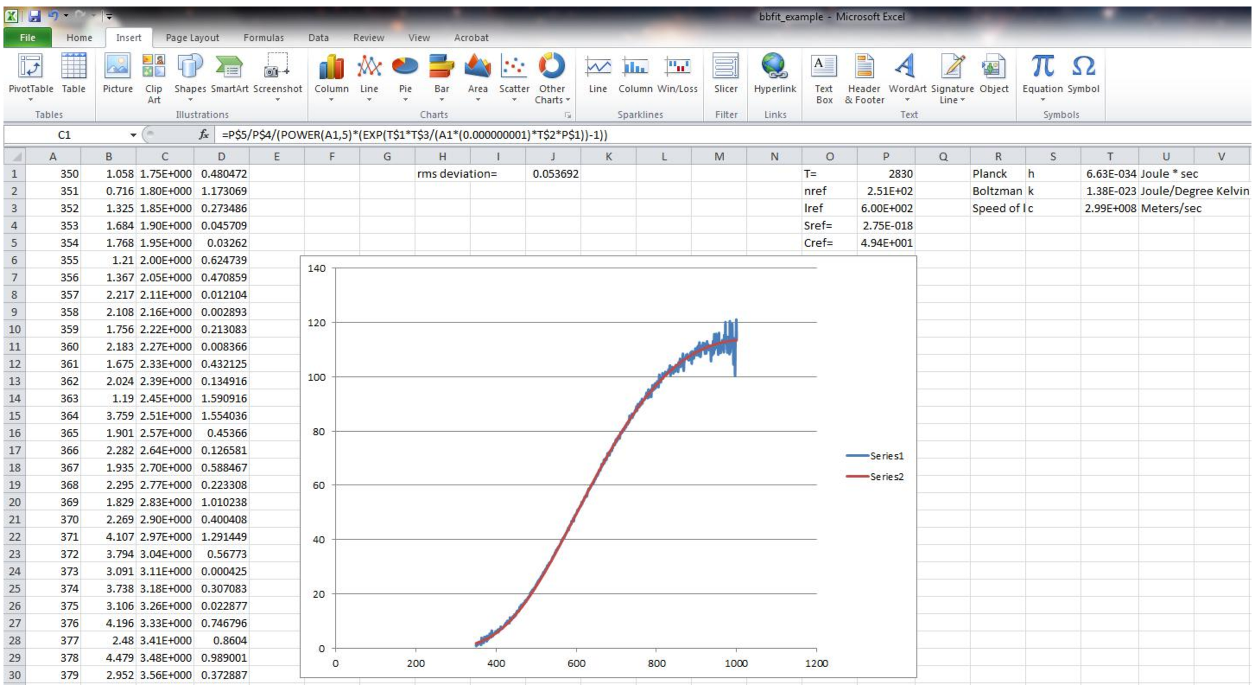

Figure 4: Example Excel worksheet, which shows the digital spectrum of the incandescent bulb in columns A and B, the Planck spectrum normalized to the measured intensity at the reference wavelength in column C (see the corresponding formula in the fx field above the columns) and the squared difference between numbers in column B and C in column D. Cell J1 contains rms deviation between measured spectrum and the Planck spectrum in column C predicted for the temperature given in cell P1 in degrees K. The measured and predicted spectra are compared in the graph in the middle of the worksheet. Best fit temperature is obtained by adjusting the temperature in cell P1 so that the rms deviation in cell J1 is smallest and the spectra in the graph match best visually.

the reference wavelength. This column is used to compute the so-called root mean square (rms) deviation of the measured spectrum, which is contained in the cell J1, from the Planck model.

The plot shows the measured spectrum in blue and the Planck formula prediction in red. You can change the temperature in cell P1 and see how the Planck prediction changes in the figure. Try temperatures from 2400 to 2900 and find the temperature for which the Planck spectrum best matches the measured spectrum by comparing them in the graph and also monitoring the rms deviation in cell J1. The best fit temperature value corresponds to the smallest value of the rms deviation. Such minimization is called least squares fitting.

You can use the example sheet to carry out similar estimates of the best fit temperature for your measured spectra. To do this, copy columns C and D from the example sheet into your Excel worksheet with your measured spectrum in columns A and B. You can do this by clicking on the header of column C to highlight this column, then pressing Shift and right arrow key to add column D to selection. Copy also numbers and constants in columns H through U into your worksheet, but do not copy the graph (or if you copied it, delete it in your worksheet). Make sure that you copy these columns into the same columns as the original worksheet file. After you copy these columns and make sure that numbers look fine, create your own graph in your worksheet by selecting columns A, B, and C, going to Insert part of Excel Menu, choosing Scatter Graph, and choosing the icon which shows graph with two continuous lines (the type of plot is called Scatter with Continuous Lines). This will create a graph with the measured and Planck spectra shown by two lines of different color (the Planck formula is the smoother curve).

Estimate the temperature by changing the temperature value until the shape of the measured spectrum is best matched by the Planck spectrum and the rms deviation in cell J1 is minimized. Repeat this procedure for each of the voltages you measured in separate worksheets. Record the best fit temperature you estimate for each voltage in the table below. Calculate the predicted wavelength at which the Planck spectrum should peak according to the Wiens law (eq. 1). Is the peak of the spectrum within the spectral range of spectrometer at any of the temperatures?

Note: for lower voltages, where the spectrum can be quite noisy, you may need to change the reference wavelength at which the Planck spectrum is normalized in order to get a meaningful fit. You can do this by changing the corresponding numbers in cells P2, P3, and P5 (P4 will be updated automatically after you change P2, P3, P5). Consult with your TA if you are unsure how to do this or whether the fit you get is optimal.

Voltage	**Est. Temperature (K)**	**Predicted peak wavelength (nm)**

Lab tasks.

4.4. In your report, present graphs showing the Planck spectrum compared to the measured spectrum for each voltage along with your best fit estimate of the bulb filament temperature.

4.5. Also include a table such as the one above.

4.6. Comment on how well the measured spectrum is matched by the Planck formula for different voltages.

5 Experiment 2: The scaling of intensity with temperature

note: You're welcome to carry out this part of the exercise at home, using Excel, the free and open source LibreOffice Spreadsheets, or the data analysis and plotting software of your choice. But, before leaving, please make sure you understand how to do it.

In the previous experiment you have checked that the Planck formula describes the shape of the light bulb spectrum measured for a given voltage (i.e. for a fixed temperature of the cord). In this experiment, you will test the scaling of intensity at a fixed wavelength as a function of temperature. From the Planck formula given by equation 2 on p. 2 of the manual, we expect the intensity at a particular wavelength to scale as

$$B(T, \lambda_{\text{ref}}) \propto \frac{1}{\text{Exp}\left(\frac{hc}{\lambda_{\text{ref}}}\right) - 1} \tag{3}$$

where λ_{ref} is is the reference wavelength at which you will choose to check scaling (such scaling should of course work for any value of λ_{ref} for a spectrum close to the blackbody spectrum).

In Excel, or the software of your choice, make a scatter plot of temperature vs. intensity at a specific wavelength λ_{ref} of your choice somewhere near the middle of the spectrum, e.g. $\sim 700\,\text{nm}$. Include the 2800K and 120V data point. Use a log scale for the intensity axis.

Overplot the predicted scaling from equation 3, normalized to the 2800K measurement.[1]

Lab tasks.

5.1. Include this plot with both the data and scaling curve in your report.

5.2. Comment on how close your data fit the expected scaling law, and likely sources of error.

6 Experiment 3: estimating the surface temperatures of stars

Note: This part can also be completed at home. But, before leaving, please make sure you understand how to do it.

[1] For example, if you're working in Excel, with temperatures in column A and constants h, k, and c in cells T1, T2, T3, then you might use the following formula in cell C1: "=K /(EXP(T$1T$3/((7001.E-9)A1T$2)-1))" where K is an adjustable constant used to normalize the reference value.

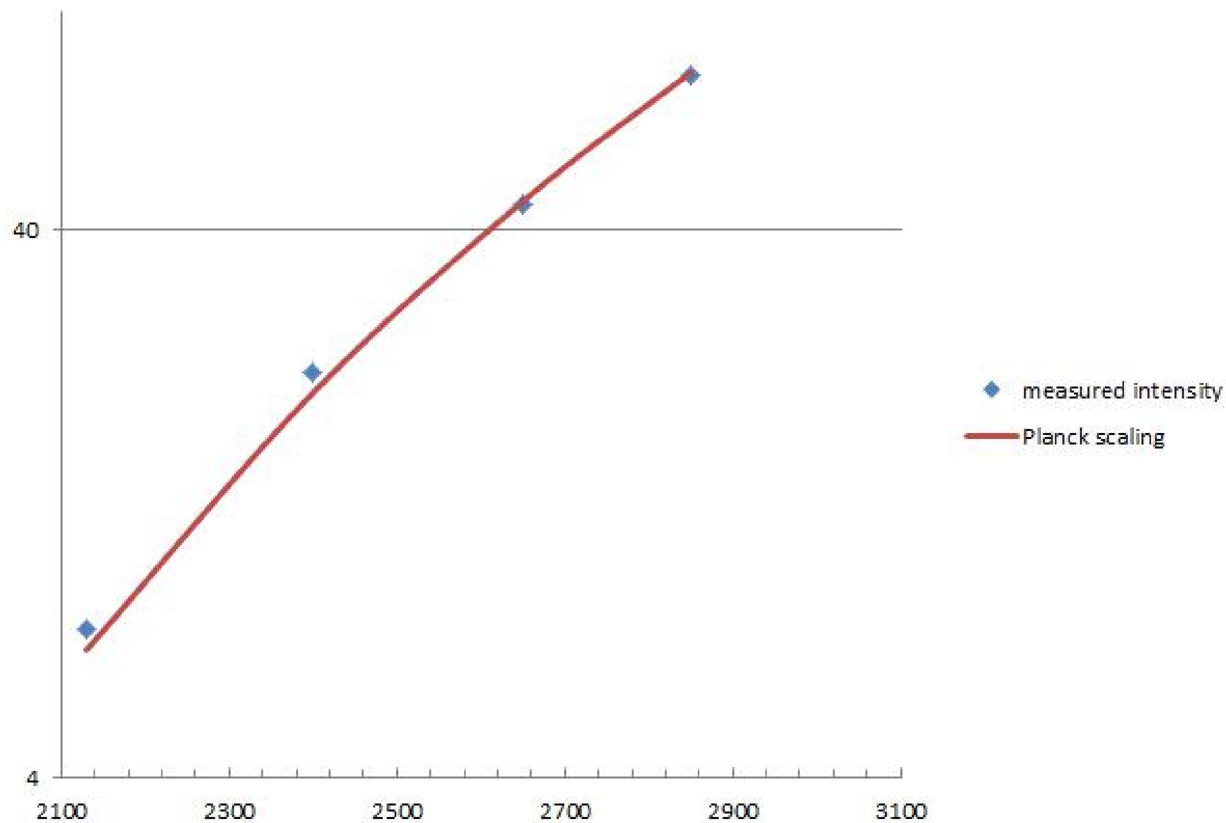

Figure 5: Example of graph comparing the expected Planck scaling (red line) of the spectrum intensity at a fixed wavelength as a function of temperature, estimated for spectra taken at different voltages, with the actual measured intensity from spectra at the reference wavelength (diamonds). Note that the vertical axis is logarithmic, while the horizontal axis is linear. Note that the intensity numbers in the y axis shown here depend on distance between fiber and bulb and may be different in your measurements.

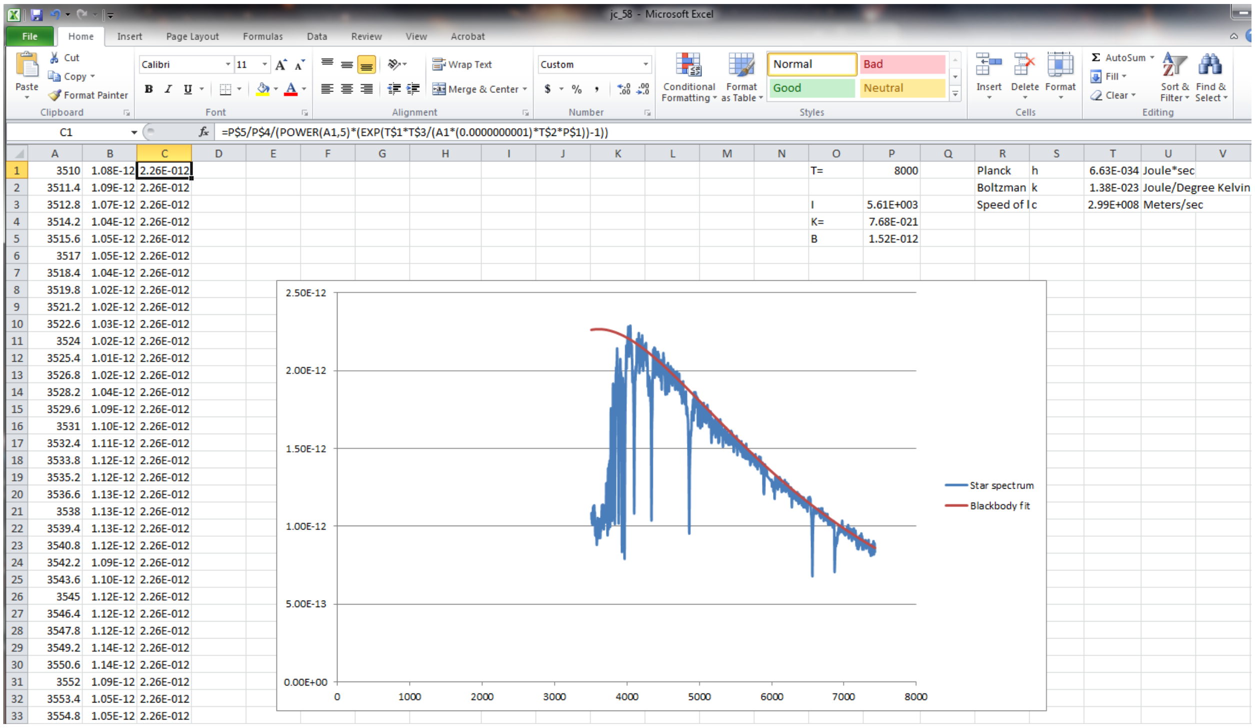

Figure 6: Example worksheet with stellar spectrum in columns A and B, the Planck spectrum in column C computed for temperature in cell P1 (8000K in this case). In this case finding temperature and best match Planck spectrum requires more care because spectrum may not be well described by Planck spectrum at some wavelength due to absorption lines and absorption bands. For this reason rms deviation is not helpful in this fit because good overall fit may not correspond to minimal rms deviation due to deviations caused by absorption lines. Try your best to match the spectrum as well as possible. The example in this figure shows what can be considered to be a good match for this spectrum.

In this experiment you will apply a technique similar to the one you used in Experiment 1 to estimate the temperature of the bulb filament in order to estimate the surface temperature of stars from their spectra. For this experiment we will use spectra from the library of stellar spectra of Jacoby et al. 1984 (the original paper will be on chalk along with other lab materials if you are curious to check it out). The spectra for stars of spectral classes O, B, A, F, G, K, M will be provided to you on chalk along with other lab materials as Excel spreadsheets (wavelength and intensity in columns A and B).

note: if you are curious to try a spectrum different from those provided to you, ascii files with spectra are available at this site (files with ascii at the end of their name): ftp://ftp.stsci.edu/cdbs/grid/jacobi/

File names at the above site are jc_NN.ascii, where NN corresponds to different spectral classes as follows:

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O stars: NN=1-10, B stars: NN=11-18; A stars: NN= 19-30; F stars: NN=31-42;
G stars: NN= 43-51; K stars: NN=52-54; M stars: NN=55-57
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These ascii files can be imported into Excel using space as delimiter. Each of the file with spectra contains two columns of numbers: wavelength in Angstroms (1 Å$= 10 \times 10^{-10}$ m) and intensity.

note: The wavelength are in Angstroms, not nm as in previous parts of this lab!

You will be provided with example Excel worksheet, which computes the Planck spectrum for a specified temperature normalized at a given wavelength. The worksheet and fitting technique are similar to the ones you used to estimate temperature of the bulb in Experiment 1. There are, however, important differences. First, stellar spectra generally have strong absorption lines and bands, which are of course strong deviations from Planck spectrum. These lines arise in cooler outer layers of stellar atmospheres, where atoms scatter blackbody radiation of denser, hotter lower layers at wavelengths corresponding to their atomic energy levels. In the fit we will ignore this and just fit the Planck spectrum visually to the overall shape of the stellar spectrum (see Fig. 5). For this reason, minimizing rms deviation is not useful and you will simply rely on the visual match to estimate the best fit temperature. Second, you must be careful in choosing at which wavelength to fix the Planck spectrum, this wavelength should not coincide with any of the strong absorption lines. Play with fitting procedure and examine critically where the Planck spectrum is matched.

Estimate the surface temperatures for stars of seven different spectral types, enter them in a table and present it in your report.

Spectral Class	**Est. Surface Temperature**	**Approx. Visual color**
O		
B		
A		
F		
G		
K		
M		

Lab tasks.

6.1. Present the table above, and plots of your stellar spectra and best fit Planck spectrum for each star.

6.2. Discuss in your report how well spectrum of stars of different types is described by the blackbody spectrum and at what wavelengths stars of different types emit most of their radiation.

6.3. Based on the spectra, estimate the visual color of stars of each spectral type (note that we perceive wavelengths < 4800 Å as blue, wavelengths of $4800 - 5800$ Å as green, and > 6000 Å as orange and red).