Laboratory Manual

PHSC 12600 Matter, Energy, Space, & Time

The University of Chicago

Autumn 2019

Labs

LAB 1

Observing Falling Filters

The ability to observe without evaluating is the highest form of intelligence.
Jiddu Krishnamurti

While Mr. Krishnamuri may be making a stretch with his superlative, it remains true that observing without evaluating is essential for the creation of knowledge. In our lives, we have bias (conceptions, self-constructed mental models) that we use as our lens to view the world. These models are based on how each of us were socialized and on our subsequent experience. To learn and create new knowledge, we must develop skill in observation. In this lab, we will direct you to make detailed, careful quantitative observations, describe the patterns you find with mathematics, and finally make some wild guesses ("hypotheses") about a more universal principle that explains this pattern that one could use to make predictions. Due to time and brain constraints, we will not, in this lab, test those hypotheses.

1.1 Learning Goals

- Become familiar with measurement, uncertainty, and writing lab reports.
- Learn how to conduct an observational experiment, including collecting data and analyzing the data to find and describe a pattern quantitatively.
- Use measurement uncertainty to describe physical quantities meaningfully.
- Format a lab report in a helpful way.

1.2 The Scientific Cycle[1]

One way of describing science is the process of incrementally improving a shared model of how our universe works. In different fields of science, different methods and cycles are used, so there is no "One True Scientific Method." One can still create a model for the process of science, and we describe here one such cycle, summarized in Figure 1.1.

In this cycle, there are three types of experiments, each one representing a different stage of the scientific effort. One stage, often started when encountering a novel phenomenon, is the **observational experiment**. This is an experiment that consists of deciding what to observe and how to observe it, collecting data, finding a pattern, and brainstorming possible explanations for what is observed (also called "hypotheses").

[1] adapted from [1]

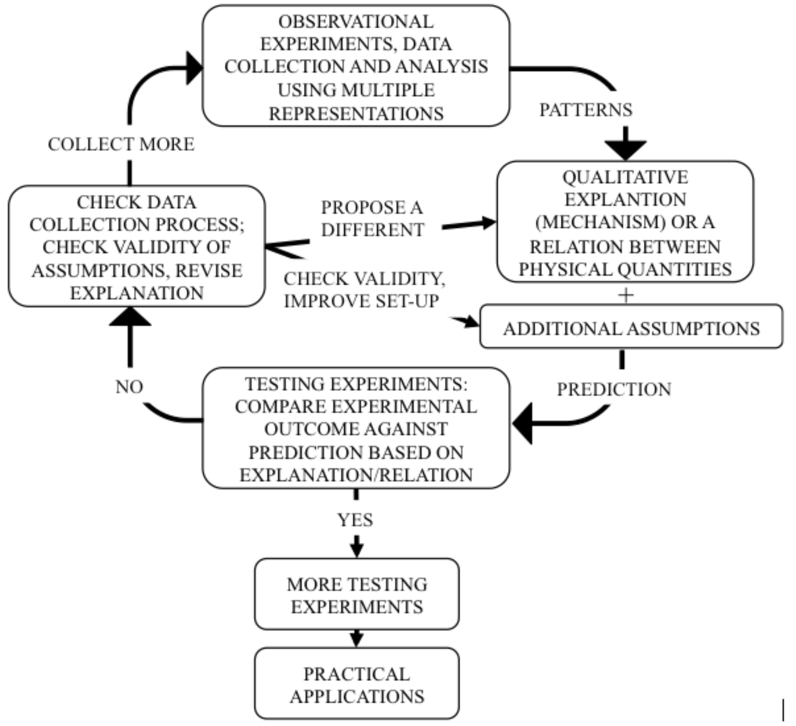

Figure 1.1: A model of the process some scientists go through to create knowledge.[2]

Once one has some trial explanations, one can test one or more of those with a **testing experiment**. Here, one designs a new experimental procedure and uses each hypothesis to predict what will happen. Then the prediction is compared to the procedure's outcome. If they are different, then the hypothesis is judged to be not a helpful explanation for that phenomenon. If they are the same, then it is still helpful. Throughout this stage, one may make various assumptions that would need to be validated, as they can effect the prediction or outcome.

Once a hypothesis has been tested enough for people to find it useful, then it can be applied to solve practical problems, or to determine properties of particular situations, in an "application experiment."

1.3 Observation experiment: Observing falling filters

In today's lab, you will investigate the relationship between the size of coffee filters and how long it takes them to fall. In the first section, you will determine the size of the coffee filters. In the second, you will determine how long each take to fall, controlling for other variables, and then find a mathematical pattern that describes the relationship. Note that this lab does not include any hypothesis testing.

> **Self-assessment:** To help you improve your scientific abilities, we provide you with self-assessment rubrics. A rubric is a method of aligning expectations for performance. Self-assessment is determining how well you performed a particular task. So, these self-assessment rubrics are designed to help you evaluate your performance while you are designing and performing your experiment.
>
> The complete set of rubrics is available in Appendix B.

Rubrics to focus on during this experiment: B5, B7, B8, F1, F2, G1, G2. See Appendix B for details.

Available equipment: several differently-sized coffee filters, meter stick, balance or scale, stopwatch, scratch paper

You may want to **decide on roles** for each group member. Example roles include Facilitator (ensures time and group focus are efficiently used), Scribe (ensures work is recorded), Technician (oversees apparatus assembly, usage), Skeptic (ensures group is questioning itself). Note that each role is responsible for ensuring that the thing happens, rather than necessarily doing it themselves.

How big are the filters?

Goal: Find the cross-sectional area of each coffee filter and make a determination of that area, including uncertainty in that area, for use in the next section.

1. Review Rubric G (Table B.6) and discuss any unclear expectations with your group and the instructor.
2. Brainstorm different methods you could use to determine the cross-sectional area. Feel free to play with the equipment as desired. Here are some things to consider:
   - Will you measure the area directly, or will you measure something else and use that to calculate the area?
   - With any method, you will probably make one or more assumptions about the shape of the filter. How valid are those assumptions?
   - For each method you consider, there may be different sources of uncertainty — the resolution of the measuring devices themselves, how you use them to measure, etc. If there is a source of random uncertainty, then you will need to take several measurements and use Appendix A.1 to determine the uncertainty. The decision of how many measurements to take is a trade-off between increasing precision (decreasing the uncertainty of the mean) and decreasing the time the measurement process takes.
   - If you make a measurement and use that measurement in an equation to find the area, you will need to propagate uncertainty as described in Appendix A.2.
3. Decide on your method and discuss it with an instructor before you begin. They will help increase the chances that your method will lead to successful results, or at least that the unhelpful path that you choose will take a short enough amount of time for you to change it when you discover it does not work. We want you to have productive failure that you have time to learn from.
4. Write down an outline of your intended procedure. You might end up changing this as you go, but it is helpful to start with a plan and then change it, rather than having no plan at all.
5. For your procedure, list the sources of uncertainty involved with each measurement. For each source, identify whether it is a random or instrumental uncertainty.
6. Execute your procedure, including setup, data collection, calculation of area, uncertainty estimation and propagation.

> At the end of this step, you should have a table of coffee filter cross-sectional areas, with uncertainties.

7. Once you are done collecting this data, review your written procedure and correct it to match what you actually did, and ensure you have sketched any measurement setups, so you can include it in the lab report. In particular, ensure that you have enough written so you can demonstrate Rubric Rows F1, G1 and G2 in your report (see Tables B.5 and B.6).

How fast do the filters fall?

Goal: Determine how long it takes each coffee filter to fall.

1. Review Rubric B (Table B.2) and discuss any unclear expectations with your group and the instructor.

2. Identify any variables (things that could change between measurements — either between measurements of the same filter, or among different filters) that could affect the fall time other than the coffee filter's cross-sectional area. If there is controversy in the group, feel free to test what variables might affect that fall time.

3. Since you are testing how the fall time is related to the filter's area, you should hold the other variables constant, so that they affect all the filters in the same way. For each variable identified in the previous step, decide how to keep that constant.

4. Brainstorm different methods you could use to determine the time it takes for the filter to fall. Feel free to play with the equipment as desired. Here are some things to consider:

   - Will you measure the fall time directly, or will you measure something else and use that to calculate the time?
   - For each method you consider, there may be different sources of uncertainty — the resolution of the measuring devices themselves, how you use them to measure, etc. If there is a source of random uncertainty, then you will need to take several measurements and use Appendix A.1 to determine the uncertainty.
   - If you make a measurement and use that measurement in an equation to find the time, you will need to propagate uncertainty as described in Appendix A.2.

5. Decide on your method and discuss it with an instructor before you begin. They will help increase the chances that your method will lead to successful results, or at least that the unhelpful path that you choose will take a short enough amount of time for you to change it when you discover it does not work. We want you to have productive failure that you have time to learn from.

6. Write down an outline of your intended procedure. You might end up changing this as you go, but it is helpful to start with a plan and then change it, rather than having no plan at all.

7. For your procedure, list the sources of uncertainty involved with each measurement. For each source, identify whether it is a random or instrumental uncertainty.

8. Execute your procedure, including setup, data collection, calculation of area, uncertainty estimation and propagation.

> At the end of this step, you should have a table of coffee filter cross-sectional areas, with uncertainties, with another column for fall time, with uncertainty in the fall time.

9. Once you are done collecting this data, review your written procedure and correct it to match what you actually did, and ensure you have sketched any measurement setups, so you can include it in the lab report. In particular, ensure that you have enough written so you can demonstrate Rubric Rows B5, F1, G1 and G2 in your report (see Tables B.2, B.5, and B.6).

Now that you have these measurements, it is time to find a pattern.

Finding a pattern

The penultimate step in an observational experiment is to find a pattern. Note that we are not explaining why this pattern is happening yet — we are focusing on describing it first.

Goal: Find a pattern in the data and describe it mathematically. **Available equipment:** Computer with spreadsheet software

1. Use a plotting program, for example LibreOffice Calc or Microsoft Excel, to plot a graph of fall time vs. filter area. The independent variable should be on the horizontal axis. The axes should each be labeled with the quantity name and the unit in parentheses. For example, if you measured fall time in seconds, then the axis label should be something like "fall time (s)".

2. In that graph, include also the uncertainty in each value. This usually involves right-clicking on a data point and selecting "error bars". Then you can highlight the column of cells that include the uncertainties.

3. Visually, discuss what shape the data points make. Speculate what kind of relationship you see. Is it proportional? Linear? Parabolic? Exponential? Logarithmic?

4. Create a line of best fit (or "trend line") in the graph using the software. Choose the equation type to match what your group guessed in the previous step. If the line obviously does not match the data, try again with a different equation type. Quantitatively, the goodness of fit of a line (how close the line is to your data points) can be represented by the correlation coefficient, given as r^2 in the software. If $r^2 \gtrsim 0.8$, then the equation that you found describes the data fairly well. Record that equation and the r^2 (or RMSE if given).

5. Make a final determination for describing in words the pattern found. If it applies, you should use one of the terms given in Step 3 in order to precisely describe the pattern.

6. Review Rubric Rows B7 and B8 in Table B.2 and ensure that you are demonstrating them here or have enough information to do so in your lab report.

1.4 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation. See Appendix C for guidance on writing the report and formatting tables and graphs.

- Your detailed procedure, with labeled sketch, of finding the area of the filters. This includes the experimental setup, how you collected data and estimated the uncertainty of your measurements.

- Same as above, but for finding the fall times. In particular, how did you hold the other variables, besides the area of the filter, constant?

- Plot of fall time vs. filter area (fall time on the vertical axis). Include a best fit line.

- Describe the pattern that you found, the mathematical relationship between fall time and filter area.

- Discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

LAB 2

Intro to Radioactivity

2.1 Introduction

Over the next two weeks, we will explore a few basic properties of radioactive decay: (1) counting statistics, (2) types of radioactive decay, and (3) half-life. In the first week, we'll use samples of radioactive materials — isotopes of cobalt, strontium, and cesium— to generate energetic particles. We'll then make measurements to explore the properties of these particles and the counting statistics related to their decays. In the second week, we'll produce our own short-lived isotope of silver and watch the new atoms decay by counting the number of particles they emit. From that, we will measure the so-called half-life of the silver isotope we produce.

Learning Goals

- Become familiar with the statistics of counting events.
- Describe the four types of radioactive decay products
- Identify sources of random and systematic error.
- Make careful measurements.
- Use a spreadsheet application to calculate and plot data.
- Relate small scale physics (studying atoms in the lab) to large scale astrophysics (dating the big bang, supernovae, and the solar system)

Scientific Background

Nuclear reactions play an important role in astronomy, geophysical sciences, archaeology, and physical anthropology. They explain energy generation in stars, the relative abundance of chemical elements, and provide a method for determining the age of things — from a piece of wood, to a meteorite, to the universe itself.

Most elements exist in a number of different forms, called isotopes, some of which are unstable and can change from one type of element to another. When this occurs, a high-energy particle is usually emitted from the nucleus of the element as it changes. By measuring the ratios of isotopes with differing decay rates, one can infer the age of an object.

2.2 Testing experiment: Do radioactive isotopes decay randomly?

Goal: Answer the question, "Given what we understand about the standard deviation of measurements of random processes, do radioactive isotopes decay randomly?" Or, in the frame of a testing experiment, test the following hypothesis: "Radioactive sources decay randomly."

Rubrics to focus on during this experiment: C1, C4, C7, C8, F1, F2, G2. See Appendix B for details.

Available equipment: Stopwatch, SpecTech Geiger counter, computer with spreadsheet software, radioactive source (^{137}Cs)

> **Warning: Radioactive Material!** The radiation levels are very low and they present no hazard for the short time that you are in the lab. We estimate that you will receive an additional 2–10 μSv dose of ionizing radiation for the time that you are in lab today, about the same as you receive every day normally. You can compare this to the example doses in Fig. 2.1. Here are tips to keep your exposure low:
>
> - **Do not have any food, drink, food containers, or make-up on the lab bench, and do not consume any food or drink, and do not apply cosmetics, in the lab.** If the sealed button source were to leak, your food, drink, food container, or cosmetics could become contaminated with low levels of radioactive material that could be ingested.
> - **Decrease time with and increase distance from sources.** Handle the sources only when you need to be for the lab, and return them to their container when not using them.

> **Caution: Fragile Equipment!** The Geiger tube (the upright cylinder sitting in the plastic stand and connected to a coaxial cable at the top) hold a gas under vacuum, with a thin, fragile window at the bottom of the tube. Do not touch it, as it breaks extremely easily.

Theory of counting statistics

If you've ever worked in a not-too-busy retail environment, you've probably had the experience of realizing (perhaps) that random and uniform are two quite different things. You no doubt have sat waiting for customers, with nobody around for long stretches of time, and then, as though they'd coordinated in advance, a half dozen people all show up within a few minutes of each other. Each customer is indeed independent (no, they didn't have a Twitter call to organize a flashmob![1]) yet their arrival times clearly appear clustered. They are, in fact. But randomly. Uniform – one customer each minute – is quite dramatically different than a random procedure that yields one customer per minute on average. Retail customers (to a great extent), and elementary particles, act randomly, not uniformly.

Each atom in a radioactive sample has, per unit time, some probability of undergoing radioactive decay. That probability is independent of all the other atoms in the sample. Each atom decays, or not, based on its own probability and no other. The ensemble of particle decay counts that one would measure in the sample (using a Geiger counter as we will do, for example) is described by the something called the Poisson distribution, which gives, in this instance, the probability of a integer number of events occurring in a fixed interval of time, given an average rate. For large count rates, like we have here, the Poisson distribution is indistinguishable from the normal distribution (that is, a simple Gaussian function).

[1] Yes, popular interest in flash mobs peaked in 2011. Bear with us here.

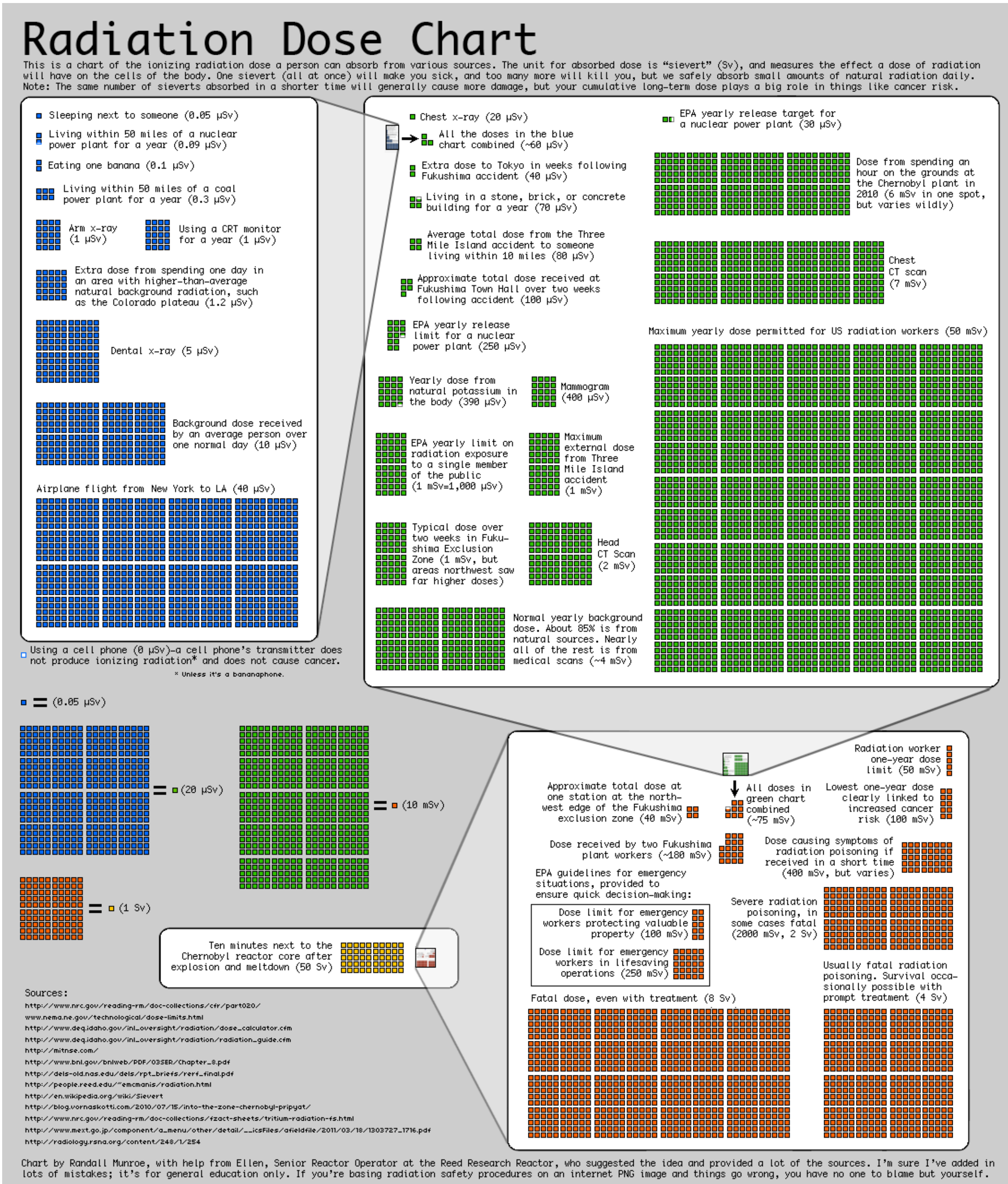

Figure 2.1: A chart of ionizing radiation dose from various sources. Source: https://xkcd.com/radiation/

Specifically, for a process that produces an average $\bar{N}$ counts in a certain time duration (in the case of large N), the expected measurement of that count for that time duration is a random draw from a normal distribution with mean $\bar{N}$ and a standard deviation of $\sqrt{N}$.

The Apparatus.

For this lab and the following one, you will be provided with a number of different radioactive materials packaged in small plastic disks. Note that each disk has the name of the isotope (e.g. ^{60}Co), the type of radiation, and the half-life printed on one side of the sample. Half lives can range from fractions of a second to billions of years, depending on the isotope.

The energetic particles produced in radioactive decay reactions can be detected by a device called a Geiger counter. This consists of a tube filled with inert gas with a wire running through it. A high voltage is applied between the wire and the tube. When a high- energy particle enters the tube, it can ionize the gas. The freed electrons produce a brief pulse of current at the output of the device. The pulses can then be counted.

The counter itself is controlled using buttons on the front panel. `COUNT` begins the count and starts the timer, `STOP` pauses the count and timer, and `RESET` sets the count and timer to zero. Use the dial on the counter to change the display between the count and the elapsed time. When you first turn on the Geiger counter you need to set the voltage to 1000 V — the TA will demonstrate how. **Caution:** Do not turn off the voltage during the experiment; use the `STOP` button to stop the count. **Do not touch the window on the bottom of the tube, as it breaks extremely easily.**

Procedure and analysis

1. To find the steps of a testing experiment, review Rubric C, found in Table B.3.
2. A prediction is what the hypothesis says will happen in the event of a particular experimental procedure. Given the hypothesis stated in the goal, and the theory described above, if you collect a series of counts of radioactive decay, what does the hypothesis predict the standard deviation of those counts should be equal to? **Record this prediction in your lab notebook.**
3. **Collecting data.** In order to find the standard deviation of a number of measurements, you will need to take those measurements. In the following steps, you will do 10 trials of measuring the count rate (counts per unit time) during periods of about 30 seconds, then take the standard deviation of those 10 trials. Since you will not be able to stop the counter after exactly 30 seconds, you will divide the displayed count by the displayed time to get a count rate (in units of "counts per second"), and then multiply this by 30 to get the count-rate per 30 seconds.

   a) **Take one of the ^{137}Cs disks and place it** printed side down in the plastic shelf shaped to hold it, and place that shelf in the slot second from the top in the stand under the Geiger counter. Note that if you have the Geiger counter on, that it is measuring a large count rate.

   b) Use the controls `COUNT`, `STOP`, and `RESET` to **measure the number of counts in 10 approximately 30-second intervals.** Use the dial on the display to access both the counts and the elapsed time. The provided stopwatch may also help you organize your effort — *but use the time from the Geiger counter apparatus directly* for your calculations.

   c) **Record the counts and elapsed time in each interval in a spreadsheet** and calculate the count rate in counts per half-minute.
4. **Analysis.** Note that the count rate is not identical in each trial, even though you accounted for the different measurement durations. Let's see how far away the standard deviation here is from

the prediction in Step 2. Calculate the standard deviation of these measurements. How close is it? Answer this quantitatively by calculating the *percent difference*,

$$\text{percent difference} = 200\% \times \frac{|A - B|}{A + B}\,, \tag{2.1}$$

where A and B are the two quantities that you are comparing. Without taking the time to find the uncertainty in the standard deviation, this is the best you can do to say how close it is.

5. Use this quantitative comparison to determine if the prediction and the outcome agree or not.

6. Based on that agreement, and taking into account the experimental conditions, make a judgment about the hypothesis. Is the radioactive decay happening randomly?

2.3 Observational experiment: types of radiation

Goal: Observe the effects of shielding on different types of radiation and generate some ideas about the patterns you see.

Rubrics to focus on: F1, F2, G2. See Appendix B for details. *Note that the C rubric rows from the previous experiment do not apply to an observational experiment.*

Available equipment: Stopwatch, SpecTech Geiger counter, computer with spreadsheet software, radioactive sources (^{60}Co, ^{137}Cs, ^{90}Sr), shielding of various thicknesses

Background: types of radiation

There are four basic types of radiation: alpha particles, which are the nuclei of helium atoms consisting of two protons and two neutrons; beta particles, which are energetic electrons; gamma rays, which are very energetic photons, *i.e.* particles of light; and finally neutrons. (The terms "alpha," "beta," and "gamma" were coined before anyone knew much about the actual particles involved.) We'll be working with gamma and beta radiation in what follows. Next week's lab includes the use of a neutron radiation source.

Different types of radiation of the same energy interact with matter in different ways. Alpha is stopped by a piece of paper, beta is stopped by a book, while gamma rays are stopped by a shelf of books, or many inches of steel or lead.

Procedure

7. We will begin by attempting to measure how much material it takes to stop the beta particles from ^{90}Sr. Take a sample of ^{90}Sr and put it under the Geiger counter in the second slot down from the top, printed side down, removing the ^{137}Cs source you used previously if you still have it in place.

8. Count the number of particles detected in 60 seconds from this source, and record this number.

9. Qualitative observe the effect of different shields by placing them between the sample and the Geiger counter and recording the count rate (for example, in counts per minute) for each shield. You should try a few different shields with different thicknesses (measured in mg/cm^2).

10. One way to quantity the penetrating ability of radiation is to find it's "stopping length", which is the thickness of shielding that reduces the detected count to half the unshielded amount. Find the stopping length of the radiation emitted by ^{90}Sr and include uncertainties in your reported count rates.

11. Now find the stopping length for ^{60}Co. This time, **place the sample holder in the third slot from the top instead of the second**. If you need to use a thicker shield than the thickest one, you can use two of them and add the thicknesses together.

12. How do the two stopping lengths compare, qualitatively, to each other. What does this tell you about the relative penetrating ability of beta and gamma radiation?

13. As before, note your measured count rates, with uncertainties, in your lab report, as well as recording the stopping length.

14. Now, finally, we will examine the radioactive properties of the ^{137}Cs source. Place it in the sample holder as before, and record the unshielded count rate in a 60 second interval. The sample is noted as having both gamma and beta radiation. Use the shields to attempt to establish this experimentally, by comparison to the results you got for the ^{60}Co and ^{90}Sr samples. *Note that this last step is not a standard part of an observational experiment, but rather an application experiment.*

Questions for discussion, to include in your lab report

15. How do the stopping lengths for beta particles compare to the stopping lengths for gamma rays? Can you come up with a physical explanation for the patterns you observe? Does this explanation extend to what you know about alpha particles?

2.4 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation.

- Prediction, data table from Steps 2–3.
- Analysis, percent difference, and conclusions from Steps 4–6.
- Procedure, relevant data (choose to use a data table or graph) for determining the stopping length of ^{90}Sr (from Step 10).
- Your determination of the stopping length of ^{90}Sr, with uncertainty estimate (Step 10).
- Procedure, data, analysis, and determination of ^{60}Co stopping length (Steps 11, 13).
- Procedure and explanation with data for how you showed that ^{137}Cs emits both beta and gamma radiation (Step 14).
- Question in Step 15.

LAB 3

Radioactive Half-Life

Building on our work from last week, we will continue to study the radioactive decay. The physical laws of radioactivity predict that the rate of decay (number of atoms decayed / time interval) is proportional to the number of radioactive nuclei present. This is due, as we saw in counting statistics previously, to the independence of the decay of each atom in the sample. The proportionality constant that describes the decay rate depends on the specific radioactive nucleus. A concise and suggestive way to characterize the nucleus is by its half-life, the time it takes for the number of radioactive nuclei to decrease to half of the initial value. You will obtain data to check the form of the law and to determine the half-life of one or more isotopes of silver.

We will use a device known as a neutron "Howitzer". It consists of a source of alpha particles and a material that absorbs alpha particles and immediately decays by emitting neutrons. (The neutrons shoot out from the barrel of the shielded volume, vaguely like shells from WWI artillery, hence the name.)

3.1 Background

We will let the neutrons bombard a small sample of the stable isotope of silver, ^{107}Ag, to produce ^{108}Ag, via the reaction

$$^{107}\mathrm{Ag} + \mathrm{n} \rightarrow\ ^{108}\mathrm{Ag} + \gamma\,, \tag{3.1}$$

where n represents a neutron and γ a gamma particle — that is, a high energy photon.

The radioactive isotope of silver, ^{108}Ag, spontaneously decays to an isotope of cadmium with the same mass number, ^{108}Cd, by the reaction

$$^{108}\mathrm{Ag} \rightarrow\ ^{108}\mathrm{Cd} + \mathrm{e}^- + \gamma\,, \tag{3.2}$$

where e^- is an electron (that is, a β particle, as we saw and measured last week).

Your TA will bombard silver foils with neutrons using the neutron howitzer. Some of the nuclei in the foil will have captured a neutron and transformed into a different isotope which is unstable and can be detected via their decay products. Each group will be given one of these silver foils.

3.2 The neutron source

The source is a mixture of plutonium and beryllium. The plutonium decays via alpha emission and the beryllium absorbs the alpha to become carbon + a free neutron. The neutron has an energy given by a very complicated distribution, but the energy distribution goes up to 11 MeV. The paraffin shielding (and the lucite in the plug) slows down neutrons, so that anything which escapes is thermalized such that E kT 1/40 eV. At the point where the foils are placed, the neutrons have been slowed some, but not completely... if they are a full 11 MeV still, they are too energetic to bind with the silver, so the foils are absorbing from the lower end of the spectrum or from neutrons that have scattered enough material to have less energy than they started with.

The activity of the Pu-Be core is an astounding 5 Ci (!!), but that's the alpha flux which doesn't penetrate out of the core. The neutron flux is considerably less. There is about 80 g of plutonium mixed with 41 g of beryllium and a listed, unshielded emission rate of 9×10^6 :n/sec.

3.3 Procedure

Rubric rows to focus on: D1, D4, F1, F2, G2, G3 (ignore actually doing it), G4

Your task is to measure the half-life of ^{108}Ag. We will use the Geiger tube to count decays. That is, we will count the β particles — the gamma rays make only a small contribution to the counts in this instance. You should attempt to carry out the counting fairly quickly after the silver foil is removed from the howitzer as the decay time is quite short.

1. Before the neutron irradiation begins you will want to record the background rate. Press `STOP` and `RESET` on the counter to set the display to zero.

2. Next, press `COUNT` with no sample below the Geiger tube and collect the total number of background counts, N_{bkg} , that accumulate in approximately 5 minutes. Once 5 minutes has elapsed press `STOP` to end the count.

3. Turn the dial to `TIME` and record a precise measurement of the elapsed time, t, in seconds. The background rate R_{bkg} is found with
$$R_{\text{bkg}} = N_{\text{bkg}}/t \tag{3.3}$$
with an uncertainty given by Poisson statistics. **Report both the background rate and uncertainty in your lab report.**

4. While the samples are being irradiated, set up your measurement apparatus.

5. Once the samples are ready, quickly place a silver foil sample in the tray below the Geiger tube. Using a stopwatch and the counter, record the number of counts and the time at 30 second intervals for about 10 minutes, continuously. Unlike last week's lab with the same apparatus, you will be recording data continuously, and so will need to use the watch to record times rather than timer built into the Geiger counter. **Record your data.** *You may want to take a video of the stopwatch and counter to get more precise readings of the 30-second intervals.*

3.4 Calculations

The experimental data will be used to determine a half-life (or half-lives). We know that the decay rate ($R = \Delta N/\Delta t$) of a radioactive nuclide is proportional to the number of nuclei present. The proportionality constant is called the decay constant λ, and the equation that describes what was just discussed is
$$R = \lambda N \,, \tag{3.4}$$
where N is the background-subtracted counts. Using integral calculus and the above equation, we find
$$\frac{N}{N_0} = e^{-\lambda t} \,, \tag{3.5}$$
where N_0 is the number of nuclei at the initial time $t = 0$. The half life $T_{1/2}$ is defined by the time it takes for $N = N_0/2$ and is related to the decay constant by $T_{1/2} = \ln(2)/\lambda$, where ln() is the natural logarithm function, and so $\ln(2) \approx 0.693$.

We can now write the radioactive decay equation as

$$R = \lambda N_0 e^{-t \ln(2)/T_{1/2}} \,. \tag{3.6}$$

Taking the logarithm of both sides and substituting for N gives

$$\ln(R) = -\left(\frac{\ln(2)}{T_{1/2}}\right) t + \text{const} \,. \tag{3.7}$$

Your TA will help you to understand the details of this derivation.

6. Make a plot showing $\ln(R)$ on the vertical axis and elapsed time, t, along the horizontal axis. Don't forget to subtract the background where appropriate.
7. Calculate the slope and use this value to solve for the half life using the above equations. **Report this value along with a table of your decay rate data and the plot described above.**

3.5 Questions (these should be included in your lab report)

8. Look up the half-lives of the various nuclides of silver. What is the published half-life of the nuclide you're observing? How does this compare with your calculated result? Calculate the percent difference in your result.
9. For the silver foil, how long would it take before you would expect to detect only one count per second, background corrected?
10. The detector only measures particles that travel up into the detector. The majority of particles traveling in the other directions escape detection. Will this short- coming affect the measured half-life? If so, how? If not, why not?
11. Describe one thing you could change in this experiment that could lead to a more accurate measurement of the half-life of the silver isotope.
12. The particular irradiated silver sample you used contained some unknown percentage of the unstable silver isotope, and was irradiated at some unmeasured time before you began your experiment. Does this matter to your results? Explain.

3.6 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation.

1. Background rate and uncertainty (Step 3)
2. Plot of $\ln(R)$ vs. t with the trendline and equation listed. (Step 6)
3. Calculation of slope of above plot and work of solving for the half-life, with the final half-life determination. (Step 7)
4. Answers to questions in Steps 8–9.
5. Answers to questions in Steps 10–12.

LAB 4

Local Gravitational Field

One of Newton's revelations was that physical laws that governed the movement of objects near Earth also predicted the movements of objects in the sky. The apocryphal story of an apple falling on Newton's head brings to mind the mechanism of gravity — the phenomenon of massive objects attracting each other. In this lab, you will measure the strength that gravity has where we are, near the Earth's surface. This measurement might also enable us to learn more about the mass of the Earth itself in a future lab.

4.1 Learning goals

- Understand Newton's law of universal gravitation and its linear approximation
- Identify sources of statistical and systematic error
- Demonstrate an ability to make careful measurements
- Demonstrate proficiency in basic calculations and plotting
- Explain the importance of repeated measurements and sufficiently large datasets

4.2 Scientific Background

The gravitational field strength and Newton's second law

The force of gravity, F, between two objects with mass m_1 and m_2 and whose centers are separated by a distance R is given by Newton's law,

$$F_{\text{gravity}} = G\frac{m_1\, m_2}{r^2}\,, \tag{4.1}$$

where the Newtonian constant of gravitation $G = 6.67408(31) \times 10^{-11}$ m^3 kg^{-1} s^{-2}. Astronomers apply Newton's law to infer fundamental information about astrophysical objects, for example the mass of binary stars. Indeed, this is one of the most common methods by which astronomers "weigh" astrophysical objects, including the Earth itself. For measuring the force acting on an object of mass m that is affected predominantly by the Earth's gravity, the force acting on it would be

$$F_{\text{Earth}} = \frac{GM_{\oplus}}{(R_{\oplus} + h)^2}m \tag{4.2}$$

where $M_\oplus$ and $R_\oplus$ are the mass and radius of the Earth, respectively, and h is the height above the Earth.

For objects near the Earth's surface, where h is much less than $R_\oplus$, h can be treated as zero, resulting in a constant gravitational force, with Equation 4.2 reducing to

$$F_{\text{Earth}} = \frac{GM_\oplus}{(R_\oplus)^2} m \,. \tag{4.3}$$

Notice that on the right-hand-side of this equation, the only variable is the mass. The others, together, constitute the *strength of the local gravitational field*, g (sometimes pronounced "little g"). So our simplified equation is

$$F_{\text{Earth}} = gm \,, \tag{4.4}$$

where we have made the substitution

$$g = \frac{GM_\oplus}{(R_\oplus)^2} \,. \tag{4.5}$$

Notice that we have taken a complicated inverse square equation (Equation 4.2) and converted it to a much simpler one (Equation 4.4). This process is called *linearization* and is a trick astronomers often use to make calculations more manageable. You will encounter this technique throughout this and other PHSC courses.

We see from Equation 4.5 that if we can make accurate measurements of g, G, and $R_\oplus$, we can calculate the mass of the Earth. We'll look up $R_\oplus$ online, and next week we will measure G. To find g, we note that Newton's second law of motion states that the acceleration a of an object is directly proportional to the net force F_{net} acting on it and inversely proportional to its mass, m, or, more succinctly and slightly rearranged,

$$F_{\text{net}} = ma \,. \tag{4.6}$$

If the Earth's gravity is the only force acting on our object, then $F_{\text{net}} = F_{\text{Earth}}$, and substituting Equation 4.4, we find that

$$gm = ma \,, \tag{4.7}$$

and thus, simplifying,

$$a = g \,. \tag{4.8}$$

So, the acceleration of an object that is subject only to the Earth's gravity is equal to the local gravitational field strength. If we can measure the acceleration, then we can find g, and get one step closer to determining the mass of the Earth.

Constantly accelerated motion

If an object is subject to a constant force, then according to Newton's second law, it undergoes constant acceleration. If an object undergoes constant acceleration a, and we know the object's initial position x_0 and velocity v_0, then after a time duration t, we can derive using calculus that the object's position x and velocity v are given by

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \tag{4.9}$$

and

$$v = v_0 + at \,. \tag{4.10}$$

4.3 Application experiment: determine g on the Earth's surface

Goal: Determine g near the Earth's surface by finding the acceleration of an object undergoing freefall (no substantial forces other than gravity) using two different methods:

Rubrics to focus on: D4, D5, F1, F2, G1, G2, G4

Available equipment: stopwatch, dense object to drop, meter stick, camera (including the one on your phone), computer with Tracker[1] installed.

[1] Open Source Physics Tracker can be downloaded from https://physlets.org/tracker and is also installed on the lab computers.

Method 1: freefall time

1. Drop the object from a known height and measure the time to fall with a stopwatch. Do this as many times as makes sense to you.
2. List the sources of uncertainty and determine whether each is a random uncertainty or an instrumental uncertainty.
3. Calculate the average fall time.
4. Calculate the standard deviation of the average fall time (using Equation A.2), and report the latter as the uncertainty in the average fall time.
5. Use the average fall time and the initial position and velocity of the object to calculate the acceleration.
6. Propagate the uncertainty in the time and position to find the uncertainty of your measured acceleration (see Section A.2)
7. Report the acceleration found by this method as "value ± uncertainty [units]". For example, $9.73 \pm 0.04\,\mathrm{m/s^2}$.

Method 2: Video tracking

It is helpful to use two methods to find the same quantity, so that mistakes or incorrect assumptions made in one method do not carry over to the other, and are thus more likely to be detected. In this method, you will record a video of an object falling, make a position vs. time plot, and fit the constant acceleration equation (Equation 4.9). You will use a computer program to make this analysis easier.

Record the video

8. Find a good object to drop. It should be dense enough to not be slowed down significantly by air resistance.
9. Using the camera on one of your group member's phones, record a video of the object falling.

   Here are some tips to get a quality video:

   - Include an object of known length in the shot, at the same distance from the camera as the falling object. This gives a reference length, so that you can find how each camera pixel scales to the physical situation.
   - Avoid parallax error by having the object be at about the same distance from the camera throughout the fall. Having the camera be farther away can help. Also, you can ensure that the top and the bottom of the fall are the same distance from the camera.
   - Hold the camera steady.
10. Record that video and transfer the video to a computer that has Tracker installed.

Importing the data into Tracker

In this part, you'll use Tracker to record the position of the object at each timestep. To do this, you'll need to tell it what direction "down" is in, what the scale of the image is, and when time $t = 0$ is. Then you'll record the positions, find out what parameters best fit the curve that is produced, and use those to find the acceleration.

11. Open Tracker on a computer. You can install it on your own computer by visiting `https://physlets.org/tracker`.

12. Optionally, watch this 3-minute tutorial on how to use Tracker: `https://www.youtube.com/watch?v=n4Eqy60yYUY`

13. In Tracker, open your video.

14. **Find frame when zero time is.** Move the slider below the video to the right to advance the frames until you find the first one in which the object is falling. Record that start frame number, which is found to the left of the slider bar in red.

15. **Find the last relevant frame.** Keep moving the slider to the right until you find the last frame before the object hits the floor. Record that end frame number.

16. To **tell Tracker about these frames**, click the 5th icon from the left on the toolbar above the video ("Clip settings") and enter the start frame and end frame.

17. **Tell Tracker how long things are.** In astronomy applications, this is known as the "pixel scale". Here we can just draw a line on the frame and tell Tracker how long that line is in real life. Click the 6th icon from the left (blue, with a "10") and select `New` → `Calibration Stick`. Shift-click to mark each end of your known length, and type in your known length, with units in the box that appears along the stick. Use "m" for meters.

18. **Align the coordinate system.** In the toolbar, click the 7th icon from the left (magenta crossed lines). Click and drag the coordinate system's origin (the intersection of long lines) to the location of the object in the start frame.

19. **Check to see if the camera was tilted.** Advance the video to see if the object moves along an axis. If it goes off at an angle, the camera was tilted compared to the direction of motion. In this case, rotate the coordinate system to align with the motion by clicking and dragging the small line that crosses one of the axes.

20. **Tell Tracker where the object is in every frame.**

    a) In the toolbar, click `Create` → `Point Mass`.
    b) Ensure the slider is at the start frame.
    c) Shift-click on the object. Notice that the frame advances to the next one automatically.
    d) Continue to shift-click to mark the object's position throughout the duration.

Analysis

21. **Ensure the correct axis is selected for analysis.** Look at the plot to the right of the video. If there is not a smooth-ish curved line, click on the axis label "x (m)" and choose instead "y (m)".

22. In the drop-down menu, select `View` → `Data Tool (Analyze...)`.

23. In the window that appears, above the plot, click `Analyze` → `Curve Fits`.

24. Notice that Eq. 4.9, which describes freefall, is a quadratic equation, which means the shape is a parabola. For "Fit Name", choose "Parabola" from the drop-down menu.

25. Use the Fit Equation and Parameter Values, comparing with Equation 4.9, to find the acceleration a, and thus the gravitational field strength g.

26. To get an uncertainty for this value, use the "rms dev" value, which describes the average deviation of the fit equation from the points, divide that by the average (mean) position, and multiply that by your value for the acceleration. You can find the mean position by selecting `Analyze` → `Statistics` and reading above the data table column.

Comparing the methods, final determination of g

27. Compare the values of g from the two methods using the t' statistic as described in Appendix A.3.

28. Use that comparison and your assessment of which method had fewer questionable assumptions to decide on your final answer for g (including an uncertainty). How close is it to the average g described, for example, on Wikipedia?

4.4 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation.

1. Procedure decisions, sources of uncertainty, and average fall time with uncertainty for method 1 (Steps 1–4).

2. Calculation of acceleration, calculation of uncertainty, and final report of both values (Steps 5–7).

3. Sketch or picture of setup for video tracking method, with written procedure for camera setup (Step 9).

4. Plot of distance vs. time (distance on vertical axis), with best-fit line and fit parameters (Step 24).

5. Description of analysis to find g, as well as the uncertainty, with final report of both values (Steps 25–26).

6. Quantitative comparison of g found with both methods, and decision about best value (Steps 27–28).

LAB 5

Cosmic Energy Transformations

5.1 Background: Energy Conservation

In every closed system, energy is conserved. Mass-energy, that is. Energy is never created or destroyed. This fact has been derived as a mathematical consequence of the fact that the laws of physics do not change over time (the relation between the two is a special case of Noether's theorem, discovered by Emmy Noether, a Jewish-German mathematician described by Einstein as the most important woman in the history of mathematics). Instead of being created or destroyed, energy is transformed from one kind to another.

There are two different kinds of energy, kinetic and potential. Kinetic is energy that something has by virtue of it moving (the faster it is going, the greater the kinetic energy), and potential is stored energy that something has because of its particular position in relation to something else. It is called "potential" because it gives the object the potential to do something. There are many different kinds of potential energy, depending on the situation. Examples include gravitational (being further away from massive bodies), chemical (sugar, batteries), electric (electric current), electromagnetic radiation (light, infrared, x-rays), elastic (springs, rubberbands), thermal (warmth), and nuclear (isotopes that can break down or combine to release energy).

We can use the principle of energy conservation to learn about physical processes. For example, if a car is traveling (kinetic energy) and is turned off and slows down and stops, the kinetic energy decreases. That energy did not disappear — it was either transformed into thermal energy through friction in the brake pads or road, or was stored up again as electrical energy (in the case of regenerative braking in electric vehicles). We even know how much — the amount of kinetic energy lost is the exact amount of energy gained in these other forms.

1. Since energy is transformed instead of being created or destroyed, start with the kinetic energy of your eyes moving to read this page and trace back the history of energy transformations as far as you can, in as much detail as you can, listing the type of energy in each case. **Record this for your report.**

> Now, you will investigate, quantitatively, several different kinds of energy transformations, qualitatively and quantitatively. You will rotate with other groups around to the different stations as you work, so you might not do the following sections in the order presented.

Helpful formulas:

- Kinetic energy $E_k = \frac{1}{2}mv^2$, where m is mass (in kilograms (kg)) and v is speed of the object (in meters per second (m/s)).
- Gravitational potential energy for small vertical displacements near the Earth's surface $U_{\text{grav}} = mgh$, where m is mass (in kg), g is the gravitational field strength ($9.8\,\text{m/s}^2$), and h is the height above some reference point (in m).
- The power P (in watts (W)) transferred onto a surface by light is equal to the intensity I (in W/m^2) of that light multiplied by the area of the surface A (in m^2), or $P = IA$.
- The power P (in W) transferred in an element of an electric circuit is equal to the current I (in amps (A)) going through the circuit element multiplied by the voltage V (in volts (V)), or $P = IV$.

5.2 Falling and picking up speed

There is a track and a toy car that can ride easily on the track.

2. Ensure the track starts and ends at different heights and is fixed in position.
3. Place the car at the top of the ramp and let it go. Notice what energy transformations take place. What types are changing?
4. Measure at least two of the types of energies that are changing, one near the top of the ramp, before the car is released, and one at the bottom of the ramp. You may want to use video tracking to measure speed.
5. Compare the total energy you can measure before and after. Is it the same, within uncertainties? If not, how do you account for the change? Is there an energy type that you are not measuring? What is that energy type? **Record your calculations and answers.**
6. In what ways is this similar to parts of the process of star formation, and how is it different? You may need to do some research. **Record your findings.**

5.3 Let there be light! The fire syringe

Inside the syringe, there is a tiny bit of cotton, surrounded by air at atmospheric pressure. As you push the plunger down, energy is added to the system as you push the air molecules and speed them up. Energy added is equal to $P\,\Delta V$, where P is the pressure of your pushing, and ΔV is the change in volume. In this system, there are no moving objects to have kinetic energy. If the plunger is pressed quickly, then there is no time for energy to go into heating up the sides of the syringe. Instead, all the added energy heats up the air inside and the cotton itself. If done fast enough, the cotton will ignite.

7. Ensure that there is, in fact, a tiny, wispy bit of unburnt cotton or tissue inside the syringe, and that the piston base is screwed on tightly.
8. Place the syringe on a stable surface and press down on the syringe with great and sudden force. If done properly, you will not damage yourself, and you will see the cotton ignite.
9. In what ways is this similar to star formation, and how is it different? You may need to do some research. **Record your findings.**

5.4 Merging black holes — or are they?

Two masses each hang from their own ~ 1 meter long string, and the strings are attached to the same overhead point. At rest, they are touching.

10. Move the masses about 10 cm apart from each other and push them in opposite directions so that they seem to orbit each other (for example, hold one in each hand the same distance away from you. Then push one away while pulling the other towards you.) Do this until you get a nice smooth, near-circular orbit.

11. As they spin, notice what energy transformations are happening. What kinds of energy is changing, and are each of those types increasing or decreasing? Remember that energy must be conserved, so if one energy type is decreasing, at least one other must be increasing. **Record your answers.**

12. Now quantify it: measure at least 2 types of energy shortly after you release the masses, and again as they touch each other and become still. Use the formulas above. You may want to use video tracking to measure speed. **Record your work and results.**

13. Compare the total energy you can measure before and after. Is it the same, within uncertainties? If not, how do you account for the change? Is there an energy type that you are not measuring? What is that energy type? **Record your calculations and answers.**

14. In what ways is this similar to black holes merging together, and in what ways is this different? You will need to research black hole mergers. **Record your answers.**

5.5 Absorbing light

The solar panel is connected to an light emitting diode (LED) that lights up when the panel is exposed to light. A multimeter can be connected to record the current through and voltage across the LED. There is also a light meter that can be used to measure the intensity of incoming light.

In the following steps, if the setup looks correct, but the **LED is not lighting up**, it might be because the current is going the wrong direction through the LED. Try switching the direction the current goes by switching the cables leading to it.

15. **Making the LED light.** Connect the solar panel to the LED as in Fig. 5.1 and see that it lights up. Try exposing the solar panel to less and more light and see what happens to the brightness of the LED.

Since energy is continuously flowing onto the solar panel and being output from the LED, you will compare the "power" instead of the energy. Power is defined as energy per time, so it is how much energy is flowing every second. You can use the last two equations in the Background section to find the input and output power.

16. **Measure the incoming power.** Turn on the light meter and hold it so that the white dot on top of it is positioned where the solar panel will be. Record the value from the meter. Repeat several times to get an uncertainty.

17. **Measure the voltage across the LED.** Ensure that the multimeter is set up to measure voltage (the 'V' setting), one end of the red cable is plugged into the red 'V' socket, and the black cable is plugged into 'COM'. Then plug the other ends of the cables into the two clips attached to the LED. This setup is shown in Fig. 5.2. This measures the voltage across the LED, which is like the amount of pressure the solar panel has to apply to get the electrons to go through it.

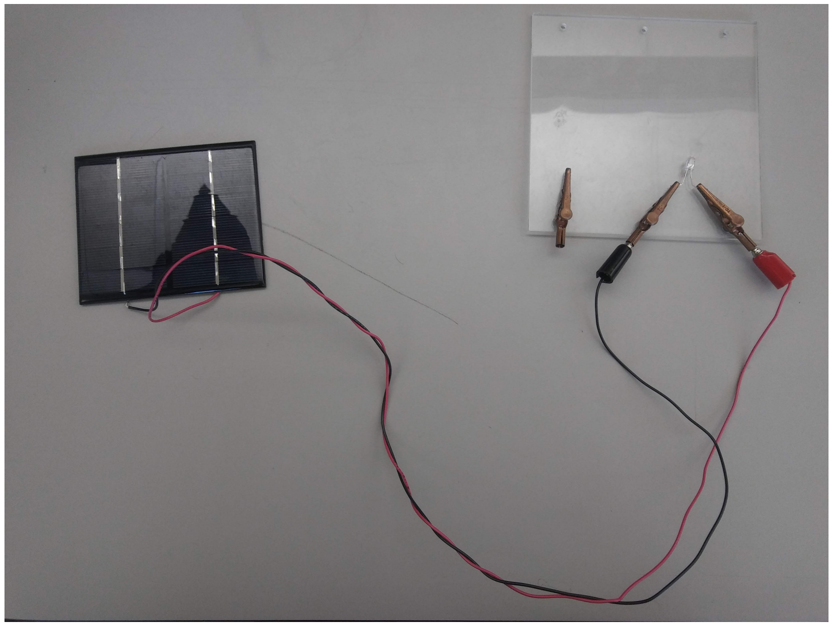

Figure 5.1: Solar panel connected to the LED.

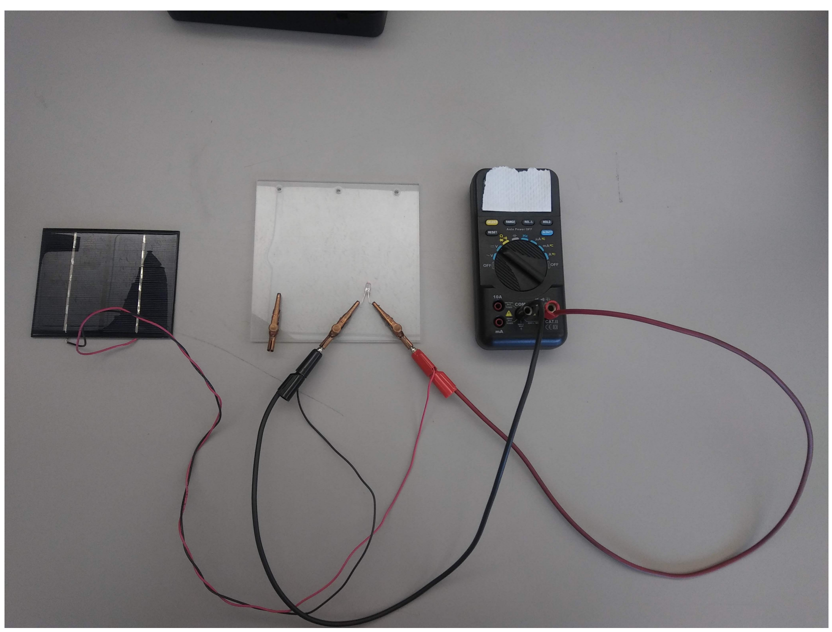

Figure 5.2: Solar panel connected to the LED, with the multimeter attached and set up to measure voltage.

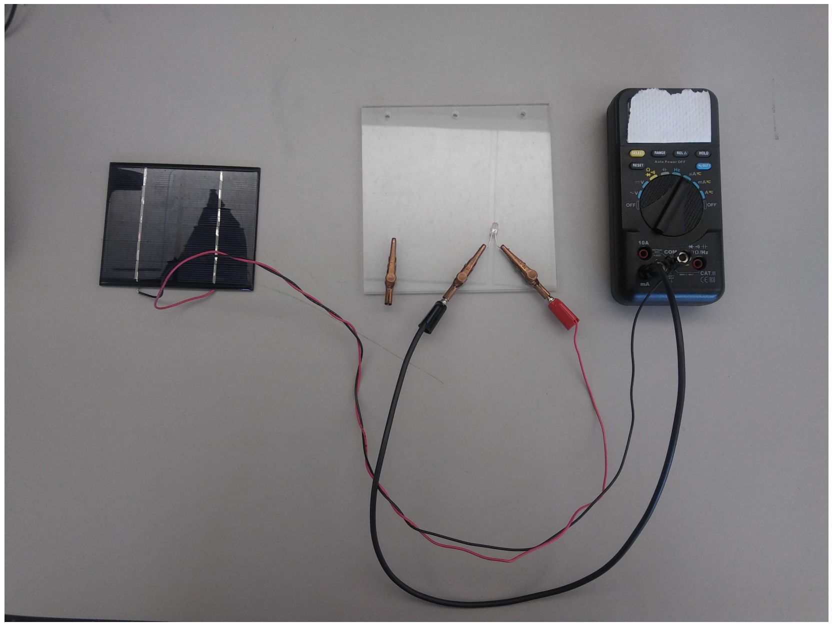

Figure 5.3: Solar panel connected to the LED, with the multimeter attached and set up to measure current.

18. **Measure current through the LED.** Now you will measure the current through the LED. This is a number that is proportional to the number of electrons passing through the wire every second. Ensure that the multimeter is set up to measure current (the 'μA' setting). Then, set up the cables so that the electric current goes from the solar panel's red wire, to the LED, from the other side of the LED to the 'mA' socket on the multimeter, and then from the black 'COM' socket on the multimeter back to the solar panel. This creates a connected loop (circuit) for the electricity to flow through. This setup is described in Figure 5.3. The LED should be lit up when this is correctly set up. **Record the current reading on the multimeter.**

19. Compare the total power input to the solar panel and output through the multimeter. Is it the same, within uncertainties? If not, how do you account for the change? Is there an energy type that you are not measuring? What is that energy type? **Record your calculations and answers.**

20. In what ways is this similar to light from our sun shining on plants and on water on Earth, and how is it different? **Record your answers.**

5.6 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation.

1. List of energy transformation history (Step 1).

2. For the car on the ramp: procedure (with sketch of setup), data, analysis, and results including types of energy measured before and after, and amounts of those energies, with uncertainties (Steps 2–4).

3. Car on the ramp: Quantitative comparison of energies before and after, with discussion of extra energy types in play (Step 5).

4. Analogy of car on ramp to star formation (Step 6).
5. Qualitative description of what happened when you pushed down on the fire syringe (Steps 7–8).
6. Analogy of fire syringe to star formation (Step 9).
7. Masses on strings: procedure (with sketch of setup), data, analysis, and results including types of energy measured before and after, and amounts of those energies, with uncertainties (Steps 10–12).
8. Masses on strings: Quantitative comparison of energies before and after, with discussion of extra energy types in play (Step 13).
9. Analogy of masses on strings to black holes merging (Step 14).
10. Data, analysis, and results for the solar panel and LED, including types of energy (per time) measured before and after, and amounts of those energies (per time), with uncertainties (Steps 16–19).
11. Analogy of solar planel to light impinging on Earth (Step 20).

APPENDIX A

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, "$5 \pm 1\,\mathrm{m}$" means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters[1]. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

A.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument's scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment's manual often gives the instrument's resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

[1] The phrase "most likely" can mean different things depending on who is writing. If a physicist gives the value and does not given a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet, or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}}, \tag{A.1}$$

where $\{x_1, x_2, \ldots, x_N\}$ are the measured values, $\bar{x}$ is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called "standard deviation of the mean", which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}. \tag{A.2}$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as $52 \pm 3\,\text{m}$.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn't change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

A.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction), } \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{A.3}$$

$$\text{For } c = ab \text{ (or for division), } \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \tag{A.4}$$

$$\text{For } c = a^n,\ \frac{\delta c}{c} = n\frac{\delta a}{a} \tag{A.5}$$

For other calculations, there is a more general formula not discussed here.

Expression	Implied uncertainty
12	0.5
12.0	0.05
120	5
120.	0.5

Table A.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π, is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table A.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value "`0.0322857142857143`". Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as "`9.65683578099600e-04`", where the '`e`' stands for "times ten to the". I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as "0.032 ± 0.001 km/s". You may also see the equivalent, more succinct notation "0.032(1) km/s".

A.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced "tee prime"). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as[2]

$$t' = \frac{|a - b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \tag{A.6}$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

[2]Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.

APPENDIX B

Rubrics

Each scientific ability rubric row assessed is worth a possible 1 point, with “Missing” being 0 points, “Inadequate” 1/3 points, “Needs Improvement” 2/3 points, and “Adequate” 1 point.

The scientific abilities rubrics are found on the following pages.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
A11	**Graph**	No graph is present.	A graph is present but the axes are not labeled. There is no scale on the axes.	The graph is present and axes are correctly labeled, but the axes do not correspond to the independent and dependent variables, or the scale is not accurate.	The graph has correctly labeled axes, independent variable is along the horizontal axis and the scale is accurate.

Table B.1: Rubric A: Ability to represent information in multiple ways

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B1	**Is able to identify the phenomenon to be investigated**	No phenomenon is mentioned	The description of the phenomenon to be investigated is confusing, or it is not the phenomenon of interest.	The description of the phenomenon is vague or incomplete.	The phenomenon to be investigated is clearly stated.
B2	**Is able to design a reliable experiment that investigates the phenomenon**	The experiment does not investigate the phenomenon.	The experiment may not yield any interesting patterns.	Some important aspects of the phenomenon will not be observable.	The experiment might yield interesting patterns relevant to the investigation of the phenomenon.
B3	**Is able to decide what physical quantities are to be measured and identify independent and dependent variables**	The physical quantities are irrelevant.	Only some of physical quantities are relevant.	The physical quantities are relevant. However, independent and dependent variables are not identified.	The physical quantities are relevant and independent and dependent variables are identified.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B4	**Is able to describe how to use available equipment to make measurements**	At least one of the chosen measurements cannot be made with the available equipment.	All chosen measurements can be made, but no details are given about how it is done.	All chosen measurements can be made, but the details of how it is done are vague or incomplete.	All chosen measurements can be made and all details of how it is done are clearly provided.
B5	**Is able to describe what is observed without trying to explain, both in words and by means of a picture of the experimental setup.**	No description is mentioned.	A description is incomplete. No labeled sketch is present. Or, observations are adjusted to fit expectations.	A description is complete, but mixed up with explanations or pattern. Or the sketch is present but is difficult to understand.	Clearly describes what happens in the experiments both verbally and with a sketch. Provides other representations when necessary (tables and graphs).
B6	**Is able to identify the shortcomings in an experiment and suggest improvements**	No attempt is made to identify any shortcomings of the experiment.	The shortcomings are described vaguely and no suggestions for improvement are made.	Not all aspects of the design are considered in terms of shortcomings or improvements.	All major shortcomings of the experiment are identified and reasonable suggestions for improvement are made.
B7	**Is able to identify a pattern in the data**	No attempt is made to search for a pattern.	The pattern described is irrelevant or inconsistent with the data.	The pattern has minor errors or omissions. Terms like "proportional" used without clarity, e.g. is the proportionality linear, quadratic, etc.	The pattern represents the relevant trend in the data. When possible, the trend is described in words.
B8	**Is able to represent a pattern mathematically (if applicable)**	No attempt is made to represent a pattern mathematically.	The mathematical expression does not represent the trend.	No analysis of how well the expression agrees with the data is included, or some features of the pattern are missing.	The expression represents the trend completely and an analysis of how well it agrees with the data is included.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B9	**Is able to devise an explanation for an observed pattern**	No attempt is made to explain the observed pattern.	An explanation is vague, not testable, or contradicts the pattern.	An explanation contradicts previous knowledge or the reasoning is flawed.	A reasonable explanation is made. It is testable and it explains the observed pattern.

Table B.2: Rubric B: Ability to design and conduct an observational experiment [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C1	**Is able to identify the hypothesis to be tested**	No mention is made of a hypothesis.	An attempt is made to identify the hypothesis to be tested but it is described in a confusing manner.	The hypothesis to be tested is described but there are minor omissions or vague details.	The hypothesis is clearly, specifically, and thoroughly stated.
C2	**Is able to design a reliable experiment that tests the hypothesis**	The experiment does not test the hypothesis.	The experiment tests the hypothesis, but due to the nature of the design it is likely the data will lead to an incorrect judgment.	The experiment tests the hypothesis, but due to the nature of the design there is a moderate chance the data will lead to an inconclusive judgment.	The experiment tests the hypothesis and has a high likelihood of producing data that will lead to a conclusive judgment.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C4	**Is able to make a reasonable prediction based on a hypothesis**	No prediction is made. The experiment is not treated as a testing experiment.	A prediction is made, but it is identical to the hypothesis, OR prediction is made based on a source unrelated to the hypothesis being tested, or is completely inconsistent with hypothesis being tested, OR prediction is unrelated to the context of the designed experiment.	Prediction follows from hypothesis but is flawed because relevant assumptions are not considered, OR prediction is incomplete or somewhat inconsistent with hypothesis, OR prediction is somewhat inconsistent with the experiment.	A prediction is made that follows from hypothesis, is distinct from the hypothesis, accurately describes the expected outcome of the experiment, and incorporates relevant assumptions if needed.
C5	**Is able to identify the assumptions made in making the prediction**	No attempt is made to identify assumptions.	An attempt is made to identify assumptions, but the assumptions are irrelevant or are confused with the hypothesis.	Relevant assumptions are identified but are not significant for making the prediction.	Sufficient assumptions are correctly identified, and are significant for the prediction that is made.
C6	**Is able to determine specifically the way in which assumptions might affect the prediction**	No attempt is made to determine the effects of assumptions.	The effects of assumptions are mentioned but are described vaguely.	The effects of assumptions are determined, but no attempt is made to validate them.	The effects of assumptions are determined and the assumptions are validated.
C7	**Is able to decide whether the prediction and the outcome agree/disagree**	No mention of whether the prediction and outcome agree/disagree.	A decision about the agreement/disagreement is made but is not consistent with the results of the experiment.	A reasonable decision about the agreement/disagreement is made but experimental uncertainty is not taken into account.	A reasonable decision about the agreement/disagreement is made and experimental uncertainty is taken into account.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C8	**Is able to make a reasonable judgment about the hypothesis**	No judgment is made about the hypothesis.	A judgment is made but is not consistent with the outcome of the experiment.	A judgment is made, is consistent with the outcome of the experiment, but assumptions are not taken into account.	A judgment is made, is consistent with the outcome of the experiment, and assumptions are taken into account.

Table B.3: Rubric C: Ability to design and conduct a testing experiment [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D1	**Is able to identify the problem to be solved**	No mention is made of the problem to be solved.	An attempt is made to identify the problem to be solved but it is described in a confusing manner.	The problem to be solved is described but there are minor omissions or vague details.	The problem to be solved is clearly stated.
D2	**Is able to design a reliable experiment that solves the problem.**	The experiment does not solve the problem.	The experiment attempts to solve the problem but due to the nature of the design the data will not lead to a reliable solution.	The experiment attempts to solve the problem but due to the nature of the design there is a moderate chance the data will not lead to a reliable solution.	The experiment solves the problem and has a high likelihood of producing data that will lead to a reliable solution.
D3	**Is able to use available equipment to make measurements**	At least one of the chosen measurements cannot be made with the available equipment.	All of the chosen measurements can be made, but no details are given about how it is done.	All of the chosen measurements can be made, but the details about how they are done are vague or incomplete.	All of the chosen measurements can be made and all details about how they are done are provided and clear.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D4	**Is able to make a judgment about the results of the experiment**	No discussion is presented about the results of the experiment.	A judgment is made about the results, but it is not reasonable or coherent.	An acceptable judgment is made about the result, but the reasoning is incomplete, OR uncertainties are not taken into account, OR assumptions are not discussed, OR the result is written as a single number.	An acceptable judgment is made about the result, with clear reasoning. The effects of assumptions and experimental uncertainties are considered. The result is written as an interval.
D5	**Is able to evaluate the results by means of an independent method**	No attempt is made to evaluate the consistency of the result using an independent method.	A second independent method is used to evaluate the results. However there is little or no discussion about the differences in the results due to the two methods.	A second independent method is used to evaluate the results. The results of the two methods are compared correctly using experimental uncertainties. But there is little or no discussion of the possible reasons for the differences when the results are different.	A second independent method is used to evaluate the results and the evaluation is correctly done with the experimental uncertainties. The discrepancy between the results of the two methods, and possible reasons are discussed.
D7	**Is able to choose a productive mathematical procedure for solving the experimental problem**	Mathematical procedure is either missing, or the equations written down are irrelevant to the design.	A mathematical procedure is described, but is incorrect or incomplete, due to which the final answer cannot be calculated. Or units are inconsistent.	Correct and complete mathematical procedure is described but an error is made in the calculations. All units are consistent.	Mathematical procedure is fully consistent with the design. All quantities are calculated correctly with proper units. Final answer is meaningful.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D8	**Is able to identify the assumptions made in using the mathematical procedure**	No attempt is made to identify any assumptions.	An attempt is made to identify assumptions, but the assumptions are irrelevent or incorrect for the situation.	Relevant assumptions are identified but are not significant for solving the problem.	All relevant assumptions are correctly identified.
D9	**Is able to determine specifically the way in which assumptions might affect the results**	No attempt is made to determine the effects of assumptions.	The effects of assumptions are mentioned but are described vaguely.	The effects of assumptions are determined, but no attempt is made to validate them.	The effects of assumptions are determined and the assumptions are validated.

Table B.4: Rubric D: Ability to design and conduct an application experiment [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
F1	**Is able to communicate the details of an experimental procedure clearly and completely**	Diagrams are missing and/or experimental procedure is missing or extremely vague.	Diagrams are present but unclear and/or experimental procedure is present but important details are missing. It takes a lot of effort to comprehend.	Diagrams and/or experimental procedure are present and clearly labeled but with minor omissions or vague details. The procedure takes some effort to comprehend.	Diagrams and/or experimental procedure are clear and complete. It takes no effort to comprehend.
F2	**Is able to communicate the point of the experiment clearly and completely**	No discussion of the point of the experiment is present.	The experiment and findings are discussed but vaguely. There is no reflection on the quality and importance of the findings.	The experiment and findings are communicated but the reflection on their importance and quality is not present.	The experiment and findings are discussed clearly. There is deep reflection on the quality and importance of the findings.

Table B.5: Rubric F: Ability to communicate scientific ideas [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
G1	**Is able to identify sources of experimental uncertainty**	No attempt is made to identify experimental uncertainties.	An attempt is made to identify experimental uncertainties, but most are missing, described vaguely, or incorrect.	Most experimental uncertainties are correctly identified. But there is no distinction between random and instrumental uncertainty.	All experimental uncertainties are correctly identified. There is a distinction between instrumental and random uncertainty.
G2	**Is able to evaluate specifically how identified experimental uncertainties affect the data**	No attempt is made to evaluate experimental uncertainties.	An attempt is made to evaluate uncertainties, but most are missing, described vaguely, or incorrect. Or the final result does not take uncertainty into account.	The final result does take the identified uncertainties into account but is not correctly evaluated. Uncertainty propagation is not used or is used incorrectly.	The experimental uncertainty of the final result is correctly evaluated. Uncertainty propagation is used appropriately.
G3	**Is able to describe how to minimize experimental uncertainty and actually do it**	No attempt is made to describe how to minimize experimental uncertainty and no attempt to minimize is present.	A description of how to minimize experimental uncertainty is present, but there is no attempt to actually minimize it.	An attempt is made to minimize the uncertainty in the final result is made but the method is not very effective.	The uncertainty is minimized in an effective way.
G4	**Is able to record and represent data in a meaningful way**	Data are either absent or incomprehensible.	Some important data are absent or incomprehensible. They are not organized in tables or the tables are not labeled properly.	All important data are present, but recorded in a way that requires some effort to comprehend. The tables are labeled but labels are confusing.	All important data are present, organized, and recorded clearly. The tables are labeled and placed in a logical order.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
G5	**Is able to analyze data appropriately**	No attempt is made to analyze the data.	An attempt is made to analyze the data, but it is either seriously flawed or inappropriate.	The analysis is appropriate but it contains errors or omissions.	The analysis is appropriate, complete, and correct.

Table B.6: Rubric G: Ability to collect and analyze experimental data [3].

APPENDIX C

Lab Report Format

C.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.

C.2 Organizing the report

The report should follow the sequence of the lab manual. Answers to questions and inclusion of tables and figures should appear in the order they are referenced in the manual. In general, include the following:

- Any procedure that you performed that is different from what is described in the lab manual.
- Any data that you've collected: tables, figures, measured values, sketches. Whenever possible, include an estimate of the uncertainty of measured values.
- Any calculations that you perform using your data, and the final results of your calculation. Note that you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92\,\mathrm{m/s^2})t + 0.2\,\mathrm{m/s}$.
- Answers to any questions that appear in the lab handout. Each answer requires providing justification for your answer.
- At the end of each experiment, you should discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

C.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like "distance (pc)". This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like "distance (pc)".
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called "error bars", along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.

Bibliography

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[2] E. Etkina, "Millikan award lecture: students of physics—listeners, observers, or collaborative participants in physics scientific practices?", American Journal of Physics **83**, 669 (2015).

[3] E. Etkina, A. Van Heuvelen, S. White-Brahmia, D. T. Brookes, M. Gentile, S. Murthy, D. Rosengrant, and A. Warren, "Scientific abilities and their assessment", Physical Review Special Topics - Physics Education Research **2** (2006) 10.1103/PhysRevSTPER.2.020103.