

Laboratory Manual

PHSC 12700 Stars

The University of Chicago

Autumn 2019

Labs

1	Bending light to see into space	1
2	Answering questions with astronomical images	7
3	Inventing Color	13
4	How old is a star clusters	19
5	Self-Directed Experiment	25
A	Analysis of Uncertainty	31
B	Rubrics	35
C	Lab Report Format	45
	Bibliography	47

Bending light to see into space

Optical telescopes are one way that astronomers use to better observe the cosmos. In this lab, you will build up your understanding about how telescopes help us do this.

1.1 Learning Goals

- Learn how light behaves when traveling between mediums.
- Create an image with a lens.
- Configure a refracting telescope and explain how it helps for observing the sky.

1.2 But first! Observing with Stone Edge Observatory

Throughout the quarter, you will be taking data (images) with a robotic telescope that is located in Sonoma, California. This week, you will select a target and queue some images to be taken, at queue.stoneedgeobservatory.com. Please choose from one of the targets in Table 1.1. The camera is a monochrome camera, broadly sensitive to all visible wavelengths ($\sim 400\text{--}700\text{ nm}$). For now, let's collect light in all of those wavelengths by selecting the “Clear” filter, as well as leaving “Dark” checked.

So that the queue is not overwhelmed, please limit your group's observations to one queue submission per student per week.

target	RA (hh mm ss)	Dec (\pm dd mm ss)	V-band magnitude	Suggested exposure time (s)	Notes
51 Pegasi	22 57 27.9804	+20 46 07.7822	5.46	15	first Main Sequence star found to host an exoplanet
Altair	19 50 46.9986	+08 52 05.9563	0.76	5	rapidly rotating star
Betelgeuse	05 55 10.3054	+07 24 25.4304	0.42	5	red supergiant star and one of the largest stars visible with the naked eye
Deneb	20 41 25.91514	+45 16 49.2197	1.25	10	forms the Summer Triangle with Altair and Vega
Kelt-9	20 31 26.3534	+39 56 19.7652	7.56	30	hosts the hottest known gas giant exoplanet
Epsilon Eridani	03 32 55.8450	-09 27 29.7312	3.73	15	one of the nearest Sun-like stars with a planet and very young
KIC 8462852 (aka Tabby's Star)	20 06 15.4527	+44 27 24.7914	12.01	90	has very irregular changes in brightness (ask Prof. Fabrycky about what causes the dips!)
Mira	02 19 20.7921	-02 58 39.4956	6.53	25	variable binary system with a red giant and a white dwarf
Polaris	02 31 49.0946	+89 15 50.7923	2.02	10	lines up almost exactly with Earth's rotational axis to the north and is often called the North Star
Sirius	06 45 08.9173	-16 42 58.0171	-1.46	2	brightest star in the sky
UY Scuti	18 27 36.5286	-12 27 58.8933	11.20	90	largest known star by radius
Vega	18 36 56.3364	+38 47 01.2802	0.03	5	one of the most well studied stars in the sky and has been used as a reference for calibrating photometric brightness scale for stars

Table 1.1: List of targets for you to choose from.

1.3 How do we use materials to bend light?

We use lenses all the time to shape the path that light takes, either with eyeglasses or with optical telescopes. These activities are intended to help you understand how we use materials to bend light.

1. Find the laser and a trapezoidal prism in your kit. Set the prism on a white sheet of paper and adjust the laser so that it is shining into the prism from the side. Play with the angle at which the beam strikes the prism and observe what happens to its direction as it enters the prism (ignore the reflections or transmission through the other side, for now). How does the path of the laser beam change when it enters the prism? **Record your observations. Be specific about any patterns you notice.**
2. It's hard to see the light in the prism, so let's use a simulation. Go to <https://phet.colorado.edu/en/simulation/bending-light> and select the play button to launch the simulation. Select the leftmost "Intro" box. This is a side view of the interface between two materials, currently air on the top half of the screen and water on the bottom. A laser is positioned above the interface.
3. Play with the controls on this screen until you have an idea of how to move the laser, turn it on and off, and adjust the materials on the top and bottom. To reset the simulation, select the orange circular button on the lower right.
4. Observe what happens to the laser beam when you change the a) angle, b) type of material on top and bottom, and c) indices of refraction. In each case, does the beam get deflected from its original straight-line path more, less, or the same amount? **Record your observations for your lab report in a table format.**
5. Open the second screen at the bottom of the sim. Lenses are wider in the middle and thinner at the top and bottom, and we can model this with a circular prism. Drag a circle up and shine a ray through it. Set the laser to output several parallel rays and aim it so they hit the middle of the circle. **Record your observations of what the rays do when they exit the other side of the prism.**
6. Go back to your physical ray box and optics set. Adjust the ray box to emit 5 parallel rays. From the Pasco Optics Set, set up the convex lens (the one that is thinner on the ends and thicker in the middle) so that the rays are hitting it from the side. Notice where the rays go after they go through the lens. **Record your observations.**

This setup of parallel rays is convenient for seeing precisely what the lens does. It also happens to be the situation when we observe things that are very far away compared to length scales of the lens. Consider two stakes driven into the ground next to each other, both perpendicular to the ground (and thus pointing directly at the center of the Earth). Since the Earth is a sphere, those stakes can't actually be both pointing directly toward the center of the Earth and also parallel to each other. For the former to be true, they must be angled slightly away from each other. But since the distance between them is so short compared to the distance away from the Earth's center, they are effectively parallel. *This is the same with light arriving from distant objects like stars.*

7. Let's try putting some parallel rays from a distant object on a lens and see what happens where those rays intersect. Take one of the round lenses that are mounted in a black plastic frame and shine those parallel rays from the light box on it to find the distance where the rays intersect with each other. Hold a sheet of paper or hand in the path of the rays so you can see them.
8. Select a distant bright object with sharply contrasting edges. A good choice is the florescent ceiling lights in the hallway outside the lab. Hold the lens between the object and a white sheet of paper (or the floor if you are using the hallway ceiling lights). The white sheet of paper should be placed about the same distance away as the intersection distance from the previous step. **Record your observations.**

If the object is a long distance away from the lens, compared to this intersection distance, then this intersection distance is called the focal length of the lens. The focal length is a property of the lens, based on its material and curvature. If an object is a long distance away compared to the focal length, then its image is formed at the focal length (if the object is closer, the image is formed further away than the focal length).

This principle works in reverse too — if an object is placed at the focal length of the lens, the rays come out parallel on the other side. The image is effectively formed an infinite distance away.

9. Design and conduct an experiment to measure the focal length of the lens you just used. Decide as a group how to measure, and how to estimate an uncertainty for your measurement. See Appendix A for detailed information about estimating uncertainty. **Record a sketch of your setup, a description of your procedure for gathering and analyzing your data, and the data itself, including your value of focal length with its uncertainty.**
10. Compare: how does this focal length compare to the focal length that is printed on the lens holder? See Appendix A.3 for how to compare two values, taking into account their uncertainties. **Record this comparison calculation and what you conclude about how close they are. Is the printed value correct?**

This lens setup is great for producing images, for example to record onto photographic film or a digital camera's image sensor. It's less good for looking through the lens to magnify and gather more light the way we want to with a telescope. For that, we'll need at least two lenses.

1.4 Your first telescope

Telescopes come in many different configurations. Here you'll construct one that is simple by comparison, a refracting telescope, using just 2 lenses.

11. Here's the principle for building this telescope: the image created by the first lens, called the objective lens, is the object for the second lens, which is called the eyepiece lens. The thing we are wanting to look at (the object for the objective lens) is far away. We want the image created by the eyepiece lens to be an infinite distance away on the near side (just trust me on this). **Given these design goals and the information about lenses above, where should the two lenses be positioned with respect to each other? Sketch your proposal, labeling each lens and drawing the focal lengths of each lens.**
12. Using the optical bench, construct and test your telescope. Look at a distant (across the room) object through it. If you don't get a clean image of it by looking through the eyepiece lens, iterate on your design until you get it.
13. Find the magnification of your telescope. Hint: if you have two identical objects, how close does one need to be to look the same size/distance as the one you see through the telescope? Experimentally determine this magnification factor (1 is no magnification, 2 means it looks twice as close, 0.5 means that it looks twice as far away, etc), and estimate the uncertainty of your magnification. **Record this.**
14. Magnification should be related to the properties of the two lenses. Make up a formula that relates the focal lengths of your lenses to the magnification of your telescope. You may need to switch the lenses around or use different ones to test your formula. **Record this.**

1.5 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation. See Appendix C for guidance on writing the report and formatting tables and graphs.

- Detailed observations from Steps 1–8.
- Sketch of your setup and procedure for finding the focal length in Step 9, as well as value, with uncertainty, of the focal length.
- Comparison of your measured focal length to the manufacturer’s stated focal length, from Step 10.
- Detailed sketch of your working telescope design, from Steps 11–12.
- Experimental determination of the magnification of your telescope, with uncertainty.
- Formula relating the focal lengths to the magnification.
- Discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment (“What did the experiment tell us about the world?”) and in the frame of a student (“What skills or mindsets did I learn?”).

Answering questions with astronomical images

Last week, you queued up some images to be taken by the robotic telescope and learned how telescopes work to create images. This week, you'll learn how we record these images, and you'll learn one way to analyze those images to create knowledge about the distance between stars.

2.1 Learning goals

- Answer questions by analyzing data that you took yourself.
- Experience and analyze the light gathering and angular resolution benefits of telescopes.
- Use pixel scale and length measurements to determine the angular separation of objects in a digital image.
- Gain insight of and appreciation for the scale of astronomical distances.

2.2 How big is your face?

This is a simple question, so that you can learn some astronomical concepts and tools while answering it and still keeping the subject matter intuitive.

Background: reflecting telescopes and CCDs

The primary utility of a telescope is its ability to gather light, thereby enabling visualization and analysis of the faint astronomical objects we are trying to observe. This requires focusing light incident on a large surface area. We will be using a **reflecting** telescope, which means that light rays from observation targets are focused into an eyepiece or onto a detector with reflecting mirrors. This is in contrast to refracting telescopes, which use refracting lenses to focus light rays. Figure 2.1 shows schematically how this kind of telescope works.

To take images in this lab, and to observe astronomical objects using the SEO, we make use of a **Charge Coupled Device (CCD)**. CCDs are the standard detectors for taking astronomical images at wavelengths blueward of approximately 1 micron. Think of the CCD we are using as a very advanced low-noise digital detector, not wholly dissimilar from the digital detector in your smartphone. Every time a photon within a certain energy range hits the detector, an electron is knocked off of the incident

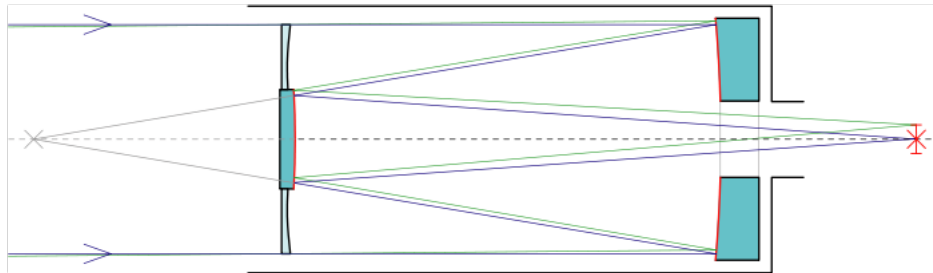


Figure 2.1: Schematic for a reflecting telescope of a Schmidt-Cassegrain design. Light rays from astronomical objects enter the telescope in parallel because their source is effectively at infinity. They are then reflected by a parabolic primary mirror onto a secondary mirror that again reflects the light to a focus. An eyepiece or a camera is placed at the focal plane of the resulting image. Image source: https://en.wikipedia.org/wiki/Cassegrain_reflector#/media/File:Schmidt-Cassegrain-Telescope.svg

pixel, charging that pixel's capacitor. Thus, for each pixel, more photons \Rightarrow more electrons \Rightarrow more charge, and the charge can be read off into a digital signal that is then processed as an image.

The main thing to note is that the CCD material is not sensitive to all wavelengths of light uniformly. Photons of certain energies are more likely to excite electrons in the detector and thus contribute to the output image. Consequently, the observed image intensity will be weighted by the response function of the detector.

Telescope Set Up, Preliminary Observations

The telescope and camera assembly is an expensive and relatively fragile piece of hardware, so move carefully when interacting with the system. We have only two setups, so the class will be split into two groups for using the classroom telescope.

First we will be observing an object at the opposite end of the lab room with the lights turned off. During this exercise we will 1) check that the telescope is focused (which should already be the case), 2) gain practice positioning the telescope and targets, and 3) demonstrate the light gathering and angular resolution capabilities of the telescope with a physical object for which you have a direct sense of scale. **NOTE: For safety purposes, do not walk around the lab while the lights are off; instead, turn the lights off only as needed to gather data or make observations, and when the lights are off, don't move about significantly.**

1. The telescope should already be set up, with the camera on and cooled, and be approximately in focus and pointed at the target at the other end of the lab bench. If not, your TA will work with you to get the setup roughly correct.
2. Your first task will be to tweak the centering of the image target. This doesn't have to be perfect, but you should be able to see the mm scale marks. The telescope can be moved in elevation (up-down). The fine control of elevation is achieved using a knob on the mount - your TA will show you details. To shift the image left or right, move the target instead.

3. Next we want to precisely focus the telescope. Do this with most of the lights off — otherwise the image has too much light. Use the 4th filter of the five available; this can be selected from the ‘Filter’ menu. Then, use the **Focus** task from the camera control system toolbar, and take images of a few seconds (at most). If the image is completely dark, and the peak intensity number in the focus box is reading around 65000, then the CCD is saturated, and the light in the room should be reduced, or the exposure time shortened.
4. Start the focus sequence - the camera will just take and display images repeatedly, of the exposure duration you specified. Now, adjust the focus knob (on the back of the telescope, off center) until you achieve an optimal focus. Note that you can (and should) zoom the magnification of the image so you can see the imaging target in detail. Do this by adjusting the magnification, and adjusting the image x,y centering so you’re looking at an appropriate zoomed location in the image. Note that you’ll need to not be touching the telescope or mount to get a sharp image, so you’ll have to adjust, then move away, wait, and repeat. Your classmates walking around the lab will cause image shake that will make focusing difficult, so enforce a still classroom while you do this.
5. Once the telescope is focused, **grab a 1-second exposure** using the Grab command.
6. Save this image in ‘fits’ format; the ‘ds9’ tool available on all of the lab computers can be used to examine the image in detail (you can also install this free software on your computer). You can use the university supported ‘Box’ system to share data with your lab classmates (or any other filesharing app that you prefer). See uchicago.account.box.com/login for details.
7. Load the image in DS9. You can adjust the contrast of the image by holding down the right-click and dragging left, right, up, and down. In the upper right window, you can drag the frame around to view different parts of the image. To measure a length, select Region > Shape > Ruler from the drop-down menu. Then select Edit > Region. Now if you click and drag on the image, it will draw a line and display the distance in pixels, or in angular separation if the image file has been calibrated with the pixel scale already.
8. In the acquired and saved image you can see a mm scale, which corresponds to some number of pixels. Compute and report the ‘pixel scale’ in mm/pixel. Given the distance between the telescope and the target, what is the angular pixel scale in arcseconds/pixel?
 To find this, note that for an arc (segment) of a circle, the length of that arc s is related to the radius of circle r , and the angle (in radians) subtended by that arc θ by

$$s = r\theta \tag{2.1}$$
9. Based on the pixel scale alone, how far apart in angle would two stars need to be in order to be distinguished as two objects instead of one?
10. Observe the target from the vantage point of the telescope by eye. What detail can you see? How does that compare to what you can see with the telescope?
11. Now take an image of part of one of your groupmates with the telescope. Centering on their eye can make for a fun picture. Use the pixel scale that you determined, along with that image, to find the angular separation (or “angular distance”) between two points in the image — for example, what is the angle subtended by their eye? Use the distance from the telescope to the person to also determine the distance between those two points, for example in millimeters.

2.3 How far apart are these stars?

Now that you have experience measuring distances for objects you can directly measure, you'll try your hand at measuring distant objects that none of us can measure directly — other stars in your image from SEO! The goal here is to pick two stars in your image and find the angular separation between them, as well as physical distance. To get the physical distance, you'll need to know their distances from Earth. To get that, you'll need to match up those stars in your image to known stars. You can use Stellarium for this, a free virtual observatory app. Then you can select those stars in the app and easily find information about them.

Note that since we are using a real robotic telescope, many things can go wrong that may result in you not having an image that you've taken. There could have been cloudy nights all week, or smoke from wildfires, or planned power outages to protect from wildfires, or part of the observatory could have broken down. We have archival images in the case that your images are not taken, so that you can still complete the lab.

12. Log into queue.stoneedgeobservatory.com, find your completed observation, and click “go to image”. If you don't have a completed observation, find a classmate's or TA's image URL to visit to get an image to work with. If the image that appears is very fuzzy or has no stars in it, choose a different option from the “Pipe Step” drop-down menu.
13. Select the link “Download Selected FITS File” to do so. Open it in DS9.
14. Also open Stellarium on a computer. It is a free open source virtual observatory, so you can install it on your computer if you'd like. Use Stellarium to find your target (hover over the left edge of the screen to find the menu that includes the Search function). Toggle the Ground and Atmosphere in the bottom menu, so you can see the star clearly. Zoom in by selecting the rectangle icon third from the right in the upper-right-hand corner. This simulates a field of view that is similar to our telescope.
15. Use the Stellarium view to identify two stars from your image. The two pictures might be rotated relative to each other.
16. For those two stars, use DS9 to measure the angular separation (angular distance) between them. You may need to use the pixel scale of the Stone Edge Observatory telescope, which is $0.76''/\text{pix}$ (arcseconds per pixel). **Record this and your uncertainty for it.**
17. In Stellarium, select each of the two stars in turn and record their equatorial coordinates (RA/Dec J2000.0). Calculate the angular separation using a web form, for example http://hea.iki.rssi.ru/AZT22/ENG/cgi-bin/c_dist.htm.
18. Use the t' statistic (see Appendix A.3) to compare these two values of angular separation.

2.4 But wait! Next set of observations with SEO

In the next lab, we will be exploring the use of color in astronomical images. The CCD image sensor is monochrome. That is, it has a relatively flat sensitivity to different wavelengths in the visible spectrum. To gain information about color, or the relative contributions of different wavelengths, you'll use filters that can be placed in front of the sensor. So for next week, **queue another observation or several** (no more than 1 per student per week). Choose the same target from last week. Select the r, g, and b filters (red, green, and blue, respectively), and leave the “dark” checked as well. For the exposure time, you can adjust it if you think the image was over- or under-exposed, but be careful — the color filters cut down the intensity of light, even in the color that it transmits the most, by at least a factor of two.

2.5 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation. See Appendix C for guidance on writing the report and formatting tables and graphs.

- Image of the target that you captured, with the calculation of pixel scale. (Step 8)
- Observation from Step 10.
- Image of a person, with calculation of the angular separation and linear separation. (Step 11)
- Image from SEO, with your two stars marked, along with the calculation of angular separation. (Step 16)
- Calculation of angular separation from Stellarium's coordinates, along with the comparison using the t' statistic.

Inventing Color

Everything glows (gives off electromagnetic radiation) when it has a temperature — particles are wiggling around randomly (faster if it's hotter) and giving off energy as they wiggle. This **thermal radiation**, also called blackbody radiation, is the same for any object at the same temperature. This means we can take something's temperature by looking at the spectrum of light it gives off. This is great for learning about something's temperature at a distance, like stars.

Incandescent lightbulbs are so warm that they glow in the visible spectrum (like stars). In this lab, you'll investigate the radiation that is given off by a lightbulb at various temperatures, then use filters to mimic the situation astronomers are in when they observe astronomical objects with different color filters. You'll invent a metric to numerically state the temperature of the lightbulb, even without knowing the actual temperature. Finally, you'll use the various images that you took with SEO with different color filters and combine them to form a color image.

3.1 Lightbulbs

Light can be made of many different wavelengths at the same time. It combines to form a color that we perceive. To see what colors make up light, we can split them again with a triangular prism, or with a *diffraction grating*.

1. Use the diffraction grating to view the lightbulb while the lightbulb is at full voltage (120 V). Hold up the grating in front of your eye, with the text on the frame upright. Start by looking at the lightbulb through it, then turn your feet to the left or right about 30 degrees, letting your whole body-arm-grating system rotate with your feet. You should see a rainbow. This is all the colors that make up the light coming from the lightbulb, split into different wavelengths.

Orientation to the digital spectrometer

Warning! Fragile Equipment! If the fiber optic cable is bent into a circle of less than 9 cm (6 inch) diameter, then the fiber inside may break.

Also, the blue end cap should be replaced on the end of the fiber optic cable when you are done using it, to protect from dust and debris entering.

A digital spectrometer also uses a diffraction grating, but instead of collecting the dispersed light on a screen to be viewed by people, it collects the light with a charge-coupled device (CCD), an

array of light-sensitive pixels much like a digital camera. It then translates the position on the CCD to individual wavelengths and displays a plot of intensity vs. wavelength on a computer. Another difference is that we use an optical fiber to collect the light.

Here are a few guidelines:

- **Background subtraction.** When you are observing the spectrum of something, there may be stray light entering that is not coming from the thing you are studying. To remove this *background* from the plot, select the gray lightbulb icon in the toolbar. This records the current plot as the background. To display the plot while subtracting the background, select the icon with the minus-lightbulb. To go back to viewing the full spectrum including the background, select the blue S icon.
- **Oversaturation.** If, when you zoom out all the way, the plot includes a flat line near the top of the plot, this means that the pixels of the image sensor are recording their maximum value, and they cannot tell you the actual intensity. Reduce the intensity by either reducing the integration time (upper-left corner) or moving the fiber optic cable off-center, so that it receives less light. Note that if you change either of these, then the background frame is no longer correct and must be remeasured.
- **Low signal.** If the target is dim, then you might find when you zoom in, the plot is not smooth, but instead jagged and hard to find patterns in. In this case, increase the integration time using the field in the upper-left of the program window. This increases the exposure time, equivalently. Note that if you change the integration time, then the background frame is no longer correct and must be remeasured.
- **Saving data.**
 - Save the images of spectra and numeric files that you generated with SpectraSuite during your experiments on a USB stick, so that you can use them at home during preparation of your report (you can also email them as attachments from your computer at the end of the lab).
 - You should open a word processor document and a spreadsheet document in which you can save your measured spectra at the beginning of your work. To save an image of your graph, click on the fourth icon from the left in the Spectrum IO controls. This will copy an image of the graph to the clipboard. Then in your word processor, paste the image by pressing Ctrl-V.
 - To save spectrum in the digital form, click on the third from the left icon in Spectrum IO controls (to the right of print icon). This copies it to clipboard. In Excel file make sure you are in a new sheet and press Ctrl-V. This should create two columns of numbers: wavelength (in nm) and counts for your spectrum.
 - An alternative way to save the data is to click on the floppy disk icon in the Spectrum IO controls. The format must be “Tab delimiter, no header”. The writing directory must be specified (“Browse” button). The spectrum is saved in a text file (.txt) as two columns, the first column giving the wavelength in nm, the second column the corresponding intensity. This file can be imported into a spreadsheet or plotting program.

Observing a lightbulb

2. Using the spectrometer, observe the spectrum created by a lightbulb at various brightnesses / voltages. What’s the overall shape? **Record your answer.**
3. How does the shape change when the brightness increases? **Record your answer.**
4. How does the peak frequency change? **Record your answer.**

5. How does the visual color of the bulb change as it gets brighter? **Record your answer.**
6. As the lightbulb gets brighter, the filament inside gets hotter — the temperature increases. If all objects act like the tungsten filament as it gets hotter, what general pattern can you state about the light given off by an object as a function of its temperature? **Record your answer.**

Cameras have come a long way since the days of developing film, but sensors are not as smart you might think. They register light, but they can't usually tell the color (i.e. wavelength) of the incoming photons. So then how do you get a color image? In astronomy (and in your cell phone!) color images are generated by measuring the amount of light at specific colors and then combining these measurements to create a colorful image. A filter is used to select bands of color to allow through. In consumer cameras and phone cameras, these filters are permanently attached to the front of individual pixels. In astronomical imaging, there is no permanent filter, and different filters are moved into place.

7. View the lightbulb's spectrum with the diffraction grating again, this time alternately holding the red and green filters between the grating and the bulb. Observe and **record** the effect of the filter on the spectrum.
8. Use the red and green filters in front of the fiber optic input to the spectrometer to see how they affect the spectrum that is received. **Save a graph of the spectrum with each filter and include in your report.**

In a regular astronomical image, each pixel gives just one value — the number of counts detected in that pixel, regardless of the wavelength of the photon detected. We can treat the fiber optic as a single-pixel camera if we count up the total number of counts detected. To find that total number, you can find the area under the curve in the plot.

9. With no filter, add up the total number of counts detected by calculating the area under the curve. To do this in an approximate way, count the number of boxes underneath the curve, then multiply this by the height of one box (in counts per nanometer) and by the width of one box (in nanometers). **Record this value.**
10. Do the same for the spectrum using the red and green filters separately. Note that the values are different for different filters. This difference (for example, the green value minus the red value) can tell us a quantitative number for what color this object is. **Record the red, green, and $g - r$ values.**

Now you have the value of our single-pixel camera for the case of clear (no filter), red, and green filters. Time to revisit the pattern found above in Step 6.

11. If that pattern from Step 6 is true, what should happen to the relative values of red and green as the voltage is increased? Should one increase more than the other? What should happen to the quantitative color $g - r$? **Record your answers.**
12. Perform an experiment to test whether this prediction is supported. **Record your procedure, analysis, and results.**

3.2 Creating a color image

To create the beautiful color astronomy images we like to see, one must manually combine images taken with different filters. There are always choices to be made that will change how the combined image looks.

Now you'll create a color image from the three images you took last week, of the same target with r (red), g (green), and i (infrared) filters. Since one of the filters is infrared, and we only have red, green, and blue colors available in DS9, then you will assign the image taken in the infrared to the color blue.

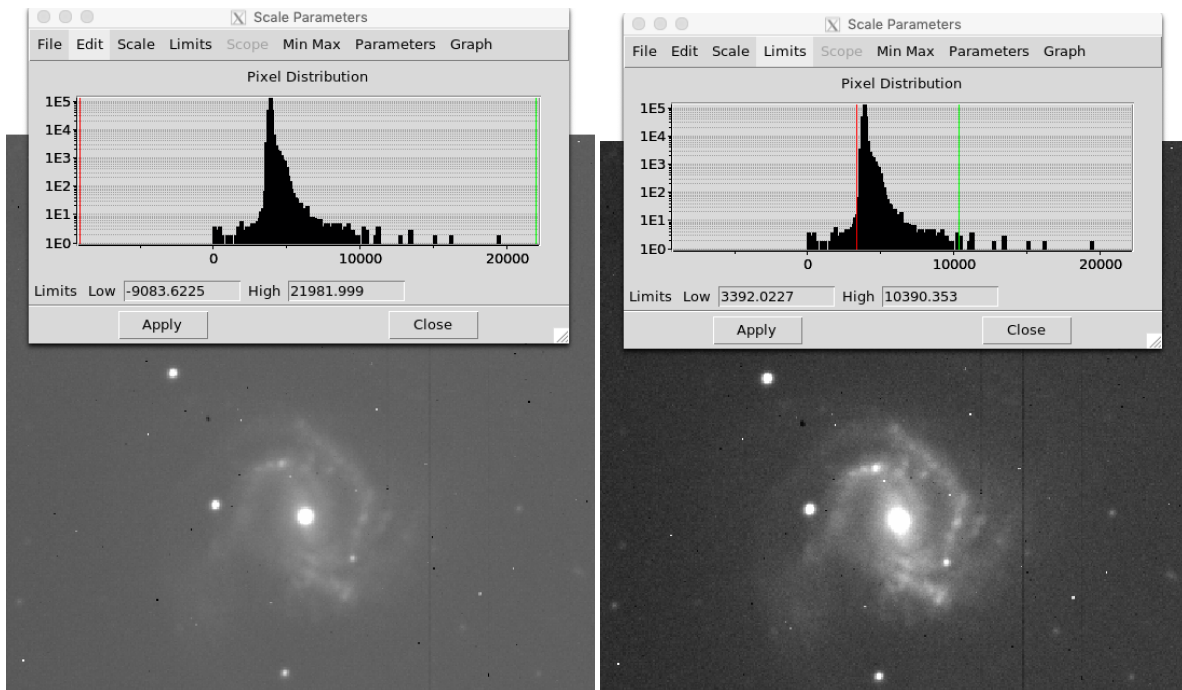


Figure 3.1: Proper choice of data ranges is important. The default for ds9 is often the min/max values in the image, which can be a poor choice if there are outlier pixels, as shown here on the left. The red line shows the lower limit z_1 , which is mapped to no color (black here), and the green line is the upper limit z_2 , mapped to full color (white here). Pixel values between these are shown in various brightnesses of the color. On the right, z_1 and z_2 are more tuned to the distribution of pixel values, which more effectively uses the dynamic range of the display for pixel values where there are significant amounts of data.

This makes this image a *false color image*, since the blue we will see in it does not correspond to the actual wavelength of the light captured.

Loading and manipulating images in ds9 consists of:

- loading an image (file > open)
- setting lower, upper limits (z_1, z_2) on an image (scale > various algorithms; use scale > scale parameters for full control). See Figures 3.1–3.2 for examples.
- controlling the intensity mapping within those bounds (mouse right click-and-hold and drag)

You can change the zoom and center location in an image by

- moving around in image (mouse middle click if edit>point is set[the default], or edit>pan and mouse left click)
- zooming in and out (mouse wheel, zoom> +,- etc.)

To build a color image

- Identify and download from SEO your three different filter image FITS files.
- Open a color (rather than monochrome) frame: Frame > new rgb
- Open the red, green, infrared files using the rgb subwindow to select which channel you are working in, and then scale and control intensities on each one.

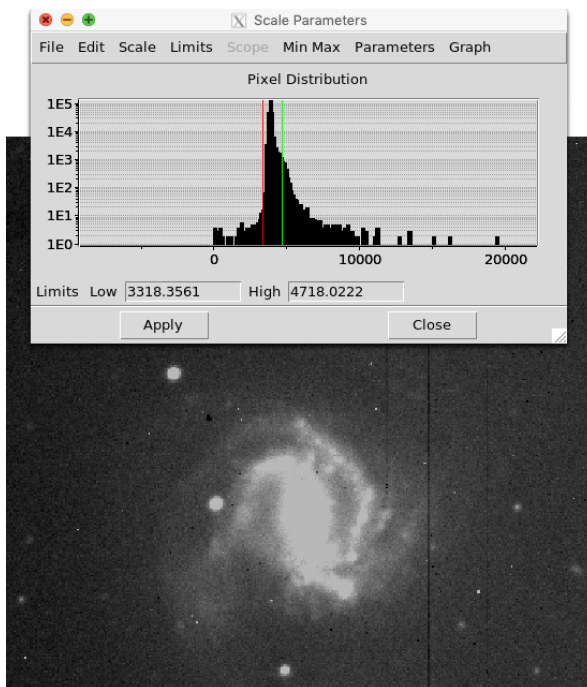


Figure 3.2: Smaller values of z_2 will emphasize fainter values in the target. Compare the image to the left to the right-hand image above.

- There are many possible ways to scale the images. Some testing suggests that choosing Scale > ASINH (or Linear or Square Root as other choices) and Scale > 99.5% (or maybe 99% or 98%) produces reasonable results. Experiment!
- One thing to note: the rgb subwindow allows you to control how the images are aligned spatially via the “align” menu at the top. There are three relevant choices: “WCS”, “Image” or “Physical”. The latter two should give the same result in this instance. “WCS” alignment uses information in the image header that has been added by the processing pipeline, that establishes a World Coordinate System (this tells ds9 and other programs how pixel x,y values are mapped into the sky coordinates - typically Right Ascension [east/west] and Declination [north/south]). The default is WCS and should work fine, if the images processed correctly. If that doesn’t look good, you could try the others. If that still doesn’t look good, note that you tried your best, and give an example of how it didn’t work will in either mode. Examples of good and bad image alignment are shown in Figure 3.3.

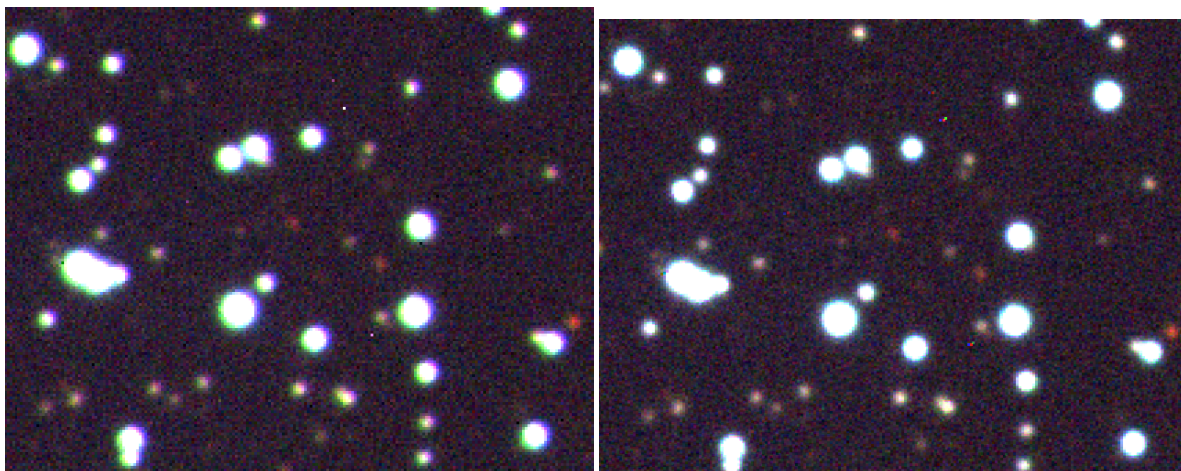


Figure 3.3: Zoom-in on a color image, showing poor (left) and good (right) image alignment across filters. Note how objects are shifted between different color channels in the poorly aligned image.

3.3 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation. See Appendix C for guidance on writing the report and formatting tables and graphs.

1. Qualitative observations with the digital spectrometer (Steps 2–5).
2. The pattern / relationship between temperature and spectrum (Step 6).
3. Effect of filters on spectrum (Steps 7–8).
4. Total intensity calculations and plots of spectra with clear, red, and green filters. (Steps 9–10).
5. Prediction of what should happen to $g - r$ with temperature increase, and procedure, analysis, results of testing that prediction (Steps 11–12).
6. Your beautiful color image (Section 3.2).

How old is a star clusters

4.1 Introduction

In this lab you will use a Hertzsprung-Russell (H-R) Diagram to learn about a star cluster. This will give us insight into the stellar populations in the clusters we observe, and allow us to estimate the age of their constituent stars.

4.2 Learning goals

- Gain an understanding of astronomical observation, image analysis and photometry
- Gain practice performing basic calculations on datasets and making informative scientific figures from those data
- Retrieve data from astronomical databases.
- Learn where different stellar populations lie on the HR diagram, and understand the physical reasons behind these localizations
- Estimate stellar cluster ages using the predictions of stellar evolutionary models.

Rubric rows to focus on: D1, D4, F1, F2, G2, G4, G5

4.3 Scientific background

Stars evolve (i.e., change their properties) over millions and often billions of years – too slow for us to see the evolution over a human lifespan. Such impressive longevity is due to the fact that stars are powered by thermonuclear reactions, which are very efficient in generating abundant energy and have quite a bit of fuel to last for a long time. Stars like our Sun last about 10 billion years (so the Sun is in its middle age). The long timescale of evolution also means that we have to develop a different way to study stellar evolution.

Astronomers explore evolution of stars by observing large populations of stars where different stars are in different stages of evolution. Of course, in order to do this we need to be able to tell which star is in what stage. This is done by a combination of observations – which measure luminosity and surface temperatures – and theoretical models – which predict how luminosity and surface temperature change as stars evolve. The key is that luminosity and temperature at a certain age are determined by star's mass, chemical composition, and details of thermonuclear reactions (which elements are burning, over what fraction of star's volume, etc.).

Luminosity and temperature of stars are related because they are both determined by their internal structure, which, in turn, is determined by the basic physical properties (mass, chemical composition, age). Therefore, stars are not scattered randomly in the luminosity and temperature space but follow well-defined sequences, which reflect the ranges of the controlling parameters in a given stellar population.

The surface temperatures of stars can be deduced by fitting a blackbody radiation spectrum to their spectra. Even for stars that do not have spectra measured, their temperatures can be deduced from their colors (Recall: does bluer color correspond to cooler or hotter temperature?). Our eyes and brain perceive color by analyzing spectral composition of the incoming light. In astronomy, a star's color is defined as the difference between its magnitudes measured through two different filters that block out all light except light within a fairly narrow range of wavelengths.

In order to interpret evolutionary states, we look at physical groupings of stars called stellar clusters, which are located at the same distance from us and were born at the same time from the same cloud of dense gas. The spread in their properties will thus not be due to different ages or initial compositions, but mainly due to different masses. As you will see, stars occupy distinct regions of the observable equivalent of the luminosity-temperature space — the magnitude-color space called the Hertzsprung-Russell (H-R) diagram. We will make this diagram for a star cluster.

Filters

Light is composed of energy-packets termed **photons** with energies that determine their wavelengths (shorter wavelength \implies higher energy). Thus every light source exhibits a **spectrum** of energies based on its components, which are determined by the physics of the light emission process. Observing the spectrum of radiation emitted from astronomical objects is a fundamental tool in observational astrophysics. However, obtaining the specific intensity of radiation as a function of energy for many dim sources is challenging. An easier way to assess the electromagnetic energies observed is to image them in different **filters**: materials placed at the opening of a telescope that are transparent to a known range of wavelengths and opaque to all others (thus “filtering” the light). Thus, one can image the same object with multiple different filters to get a sense of the wavelength regimes that dominate the light from a source.

A filter is characterized by its **transmission function**: a function that characterizes the amount of light that is transmitted by the filter at each wavelength. Figure 4.3 shows the transmission functions for some standard astronomical filters (similar to the one's you'll be using in this class).

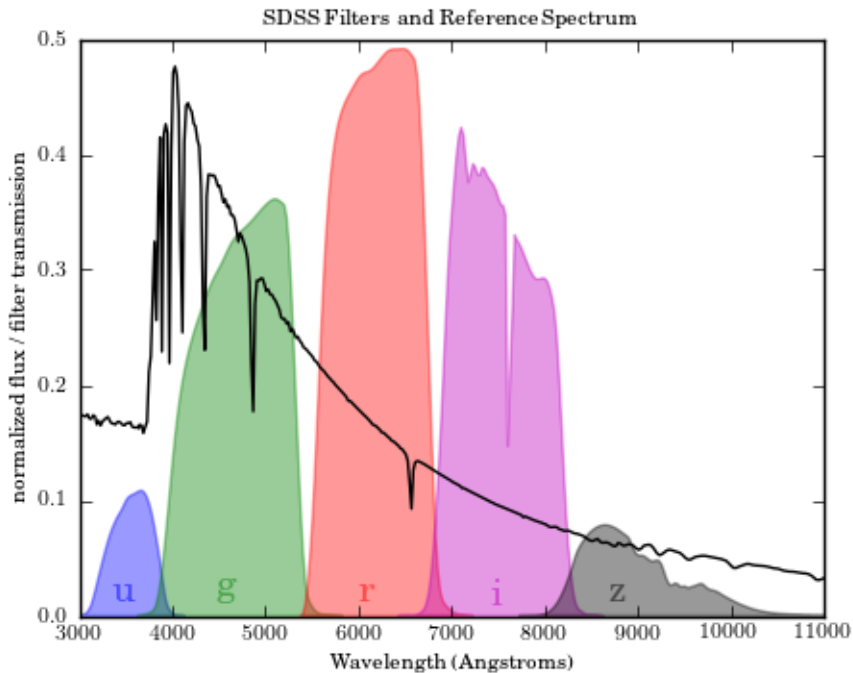


Figure 4.1: Filter Transmission Functions for the Sloan Digital Sky Survey, overlaid on a stellar spectrum. The magnitude observed by each filter will be proportional to the integrated spectrum multiplied by the filter transmission. It's clear in this image that the underlying spectrum of the star will cause the different filters to have different magnitudes. Image source: http://www.astroml.org/_images/fig_sdss_filters_1.png

4.4 Defining color

Since different bands measure brightness in different wavelengths, the ratio of flux in two bands is a measure of an object's color. Therefore, the difference between astronomical magnitudes of an object in different bands is a measure of its color, since the magnitude scale is logarithmic:

$$m_A - m_B = -2.5 \log \left(\frac{F_A}{F_B} \right). \quad (4.1)$$

An H-R diagram is a plot of stellar magnitude vs. color - aka a "color magnitude diagram". A star's color is an observational indication of its surface temperature. Since all of the stars in a star cluster are at approximately the same distance, their apparent magnitude gives a good relative indicator of luminosity. Therefore, a color-magnitude diagram of stars at roughly constant distance is effectively a temperature-luminosity diagram, and stars fall in characteristic regions of this parameter space based on their mass, age, and metallicity.

We will have data in two wavelength bands, so we can subtract those magnitudes to get a color index. With the software or coding language of your choice, you will plot g on the vertical axis and $g - r$ on the horizontal axis.

4.5 Making an HR Diagram

To learn about a star cluster, you'll make a color-magnitude diagram of the stars in that cluster. You'll be analyzing M15, a globular star cluster in the Pegasus constellation. To make the diagram, you need

magnitudes in at least two different filters for many stars in the cluster. Since this would be incredibly time-consuming to do by hand, you will retrieve this information from an online database.

1. Open the SkyServer Search Form in a browser: <http://cas.sdss.org/dr7/en/tools/search/form/Default.aspx>.
2. Select "Show me [stars] in the region [around:]" . Note that you will need to know the celestial coordinates of the part of the sky that you want retrieve data about, as well as the radius of a circle that captures all the objects you want.
3. Look up M15 in Stellarium and record its RA and Dec (J2000 values). It may be helpful to turn off the ground and atmosphere to get a better view. Then, determine a radius in arcminutes that includes all of the cluster. You can either use the Field Of View (FOV) listed on the screen, or you can turn on the Equatorial Grid by selecting that option in the bottom toolbar or pressing 'e' and use the contour lines.
4. The Search Form wants the RA and Dec in decimal degrees. So convert the RA and Dec that you recorded to decimal degrees and enter this (for example, an RA of 15h30m20s converts to 232.5833 degrees). You can use this website to assist: <https://www.swift.psu.edu/secure/toop/convert.htm>.
5. In the Form, tell it to output 1000 objects, and the magnitudes of the objects.
6. Select "Generate" or "Update Query" and it will convert all the options into a database query that you should then "Submit". This opens a new tab with the output from the query.
7. If the outcome from the query has much fewer than 1000 objects, then increase the radius (and/or check to ensure you have the correct RA and Dec entered).
8. Once you have lots of objects, change the form to output a CSV file.
9. Plot the data from the CSV file using your favorite spreadsheet or programming environment. Specifically plot 'g' vs. 'g-r', with 'g' on the vertical axis, as a scatter plot.
10. Label the different stages of stellar evolution on your plot.

4.6 Comparing to stellar evolution models

The last portion of this lab involves estimating the age of these stellar populations by comparing their color magnitude diagrams with predictions from a stellar evolution model. Stellar physics is sufficiently well-understood that accurate evolutionary models have been constructed to calculate the observable properties of stars across their lifetimes. Because stars evolve in their position on the HR diagram, we can estimate the age of our observed cluster by comparing these models to our observations.

Several files containing model predictions for the magnitudes of stars at different ages are contained in files on the course website. These have been calculated using the observed metallicity of the star cluster to predict the positions of stars at ages 10^7 , 10^8 , 10^9 , and 10^{10} years. These are called isochrones - stellar properties as a function of stellar mass for a fixed age (and metallicity). Note that these magnitudes were calculated using prior knowledge of the star cluster metallicity, and have been corrected for cluster distance and extinction due to intervening galactic dust (taken from [3] for M15 and [4] for NGC 869).

11. Plot these isochrones on your H-R diagrams and compare them to your data to estimate the cluster age. Be sure to estimate an error on this value and explain your method in your lab report.
12. What changes in stellar properties are represented by the different locations of isochrones of different ages?
13. What physical processes occurring inside the star underlie those changes?

4.7 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation. See Appendix C for guidance on writing the report and formatting tables and graphs.

1. The coordinates of M15 and the radius you choose, along with how you found these (Step 3).
2. The plot of g vs. $g - r$ with labels of different stages of stellar evolution (Steps 9–10).
3. Plot of H-R diagram with isochrones (Step 11).
4. Analysis and determination of cluster age, with uncertainty (Step 11).
5. Answers to questions in Steps 12–13.

Self-Directed Experiment

You've spent the last 4 weeks learning various techniques for collecting and analyzing astronomical data. Now it's your turn to choose a project and use those techniques more independently to answer a scientific question that you choose. At least, you can choose from a list, so it is more likely you'll find a project that appeals to you.

So in the next two weeks, you will conduct the experiment / project, and in the following week, you will present your findings in class (as well as submitting a written report). Our goal is for you to work more independently than in past weeks, and also use your TA, professor, and teaching support manager for assistance when needed.

5.1 Picking a Project

1. Individually, read through the available projects and pick your favorite, writing it on the board.
2. Once everyone has written their preference, if any project has more than the max number of people listed for the project, people will be chosen at random to be in that project.
3. The remaining people without projects pick from the understaffed projects that remain and the random selection happens again, until all students have a project, and each project has the minimum group size.

5.2 Project A: When did the Crab Nebula star explode?

Group size: 2

The Crab Nebula (M1) is a supernova remnant that has been observed by humans for nearly 1000 years. The force of the supernova propels gas outward, expanding into space. But just how fast is it expanding? Calculate the velocity (with uncertainties) of the expansion of the Crab Nebula in km/s. Work backwards and calculate when the Crab Nebula originally exploded, giving your uncertainty for that year.

Materials: archival Crab Nebula images (SEO + https://archive.stsci.edu/cgi-bin/dss_form)

- The date an image was taken is found in the header of FITS files. This can be viewed by loading the image in DS9 and selecting File > Display Header...

Table 5.1: Historical Data for 61 Cyg AB

Date	Separation r (arcsec)	Position Angle θ ($^\circ$)	Reference
1753	19.6	35	J. Bradley, obtained via WVDSC
1838	16.204	95.325	Bessel 1938, AN, 16, 65
1914	23.3 ± 1.4	136.4 ± 3.5	Yerkes Plates
1951	26.3 ± 3.0	141 ± 7	Digitized Sky Survey, Blue I
2019			your measurement

5.3 Project B: How heavy is the binary star system 61 Cygni AB?

Group size: 2

Binary stars afford us an opportunity to determine the mass of the star system using Kepler's laws, since we can view the star's movements over time and calculate orbits. Your mission is to use historical measurements in Table 5.1, and add your own using SEO, to determine the mass of the 61 Cygni AB binary star system.

- For two bodies of mass M_A and M_B that are orbiting each other, the period P of the orbit and the semi-major axis of the ellipse swept out by the difference in the positions between bodies A and B., a , are related by Newton's version of Kepler's third law,

$$P^2 = \frac{4\pi^2}{G(M_A + M_B)} a^3, \quad (5.1)$$

where G is the Newtonian constant of gravitation.

- To add your values to the historical table, you will need to convert your RA (α) and Dec (δ) to separation angle r and position angle θ . You can use the following formulas,

$$r = \sqrt{[\cos(\delta_{\text{avg}}) \Delta\alpha]^2 + (\Delta\delta)^2} \quad (5.2)$$

and

$$\theta = \arctan \frac{\cos(\delta_{\text{avg}}) \Delta\alpha}{\Delta\delta}, \quad (5.3)$$

where δ_{avg} is the average declination expressed in degrees, $\Delta\alpha$ is the difference in RA of the two stars in arcseconds, and $\Delta\delta$ is the Dec difference in arcseconds. The difference should always be the fainter star minus the brighter star.

- To get the orbital parameters P and a , you'll probably want to plot the ellipse visually and estimate from there. To do that easily, you may want to convert the data from polar coordinates (r, θ) to rectangular (x, y) values. See <https://www.mathsisfun.com/polar-cartesian-coordinates.html> for assistance with this. You can use these formulas:

$$x = r \cos \theta \quad y = r \sin \theta \quad (5.4)$$

To put x and y in physical units, divide these values by the measured parallax of the system, 0.314 arcsec, which gives the distances in AU.

- Report your result in both kilograms and number of solar masses. Include estimation of uncertainty.

Helium (nm)	Argon (nm)	Neon (nm)	Mercury (nm)
389	697	585	365
447	707	594	404
469	738	614	435
492	751	627	546
502	764	640	579
588	772	651	
668	795	660	
707	801	668	
727	811	693	
	826	703	
	841	717	
	852	744	
	866		
	912		

Table 5.2: Some known emission lines of various elements.

5.4 Project C: What elements are in the Sun?

Group size: 2–3

Determine what elements in the atmosphere of the Sun by comparing the spectrum of the Sun to the spectrum of light given off by those elements when they have high voltage applied to them. First, familiarize yourself with analyzing spectra by using gas discharge tubes and the digital spectrometer in the lab and identifying unknown elements (a list of prominent wavelengths is found in Table 5.2). Then, sign up with Prof. Fabrycky to use the telescope on the roof of Kersten to take a spectrograph of the Sun and identify hydrogen and sodium by comparing to the spectra taken with gas discharge lamps. Finally, identify at least two other elements by comparing your spectra to the spectral atlas found at http://bass2000.obspm.fr/download/solar_spect.pdf.

- You must be available to take data with Prof. Fabrycky using the Dunham telescope, during one of these two windows of time: Nov 12 9:00a–12:00p; Nov 14 1:00–4:00p. Email him to arrange when to meet during that time.

5.5 Project D: How fast are stars moving?

Group size: 2

Humans have been looking at the sky for thousands of years, and the sky is ever changing. Here is a tiny fraction (a few decades) worth of archival imagery for you to peruse (https://archive.stsci.edu/cgi-bin/dss_form). Select your favorite patch of the sky, and see how it changes over time. In particular, find a star that moves significantly over the course of these decades, and tell us just how quickly this star is moving in angular velocity (for example, in milliarcseconds/year). Check your answer using Stellarium. Use Stellarium to find the distance to this star and convert the angular speed to a physical speed in m/s. Compare this to human-scale speeds to gain perspective (e.g. a running human, plane, Elon Musk’s Hyperloop).

- Do this for 4 stars and include uncertainty estimates for your speeds.
- The date an image was taken is found in the header of FITS files. This can be viewed by loading the image in DS9 and selecting File > Display Header...

- To compare two images of the same part of the sky, you can open both in DS9 and “blink” between them. To do so, first open one image normally. Then select Frame > New Frame, and then use File > Open... to open the second image. Next, align the two images together by selecting Frame > Match > Frame > WCS. Then you can blink between them with Frame > Blink Frames.

5.6 Project E: Fun with GAIA

Group size: 3

GAIA is an ongoing mission that measures stellar proper motion — the motion of the star perpendicular to the observer’s line of sight. We’ve downloaded some data for you to play with in Excel (or programming language of your choice). Please make the following plots and answer the associated questions. Be very specific, even though the questions seem quite general:

1. **RA vs. Dec.** What structure do you see, if any?
2. **Color vs. Temperature.** What does this trend tell you?
3. **Luminosity vs. Temperature (aka H-R Diagram).** Make sure the axes are scaled properly and go the standard direction for H-R diagrams.
4. **Luminosity vs. Color.** Why does this look better than the typical HR diagram in the last plot?
5. **Color vs. Parallax.** What is the structure? Describe what you see and why things look that way.

Notes

- The description of the headers of the file can be found here: https://gea.esac.esa.int/archive/documentation/GDR2/Gaia_archive/chap_datamodel/sec_dm_main_tables/ssc_dm_gaia_source.html
- The various filter passbands (what wavelengths the filter lets through) can be seen here: https://www.cosmos.esa.int/web/gaia/iow_20180316

5.7 Report checklist and grading

Each group will submit one (1) report for the group. The report must list specifically how each group member contributed to the report.

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation. See Appendix C for guidance on writing the report and formatting tables and graphs.

1. Introduction (at least one paragraph for each item)
 - a) Historical background — when was this phenomenon first studied, why
 - b) Theoretical background — explain the underlying science
2. Procedure
 - a) Provide enough detail that someone else in the class who did not do this project, could do it the way that you did.
 - b) Labeled pictures or diagrams of apparatus, instruments, telescopes used, even if from archival data or surveys.

3. Data and Analysis

- a) All astronomical images used.
- b) Table of measured and calculated data. Do not include things like 1000-item lists of star data from surveys.
- c) Graphs, if specified in project description or helpful for displaying data.
- d) Description of analysis of data, including relevant formulas.

4. Results

- a) Final determination of any quantities, including uncertainties.
- b) Comparison to accepted or other values, if they exist.

5. Discussion (at least one short paragraph for each item)

- a) What you would change to make it better, either changing what you would do, or changing the project description.
- b) What are other interesting similar things to study, or next steps for this line of research?
- c) Discuss the findings and reflect deeply on the quality and importance of the findings. This should be both in the frame of a scientist conducting the experiment (“What did the experiment tell us about the world?”) and in the frame of a student (“What skills or mindsets did I learn?”).

5.8 Presentation parameters

Each group will give a 10 minute presentation, with a 5-minute question period following. Students are encouraged to ask questions of each other. During the questions, the TA will ask each group member a question that they need to answer themselves, which should be answerable if they know everything that went into the report. The presentation grade will include both a group portion and an individual portion, with the individual portion based on the accuracy of that group member’s answer (10 points), as well as demonstrating being an engaged audience member. This is demonstrated by paying attention and having no phones or laptops out during the other presentations (5 points), and at the end of each talk, writing down one thing you learned and one thing you are still curious about, to be turned in at the end of the session (5 points).

Things to include in presentation (5 points for each section)**1. Introduction**

- a) Historical background — when was this phenomenon first studied, why
- b) Theoretical background — explain the underlying science

2. Procedure

- a) Summary of procedure so people can understand generally what you did.
- b) Labeled pictures or diagrams of apparatus, instruments, telescopes used, even if from archival data or surveys.

3. Data and Analysis

- a) All astronomical images used.
- b) Graphs, if specified in project description or helpful for displaying data.

- c) *Brief* description of analysis of data, including relevant formulas. For every formula displayed, explain each variable.

4. Results

- a) Final determination of any quantities, including uncertainties.
- b) Comparison to accepted or other values, if they exist.

5. Discussion

- a) What you would change to make it better, either changing what you would do, or changing the project description.
- b) What are other interesting similar things to study, or next steps for this line of research?
- c) Discuss the findings and reflect deeply on the quality and importance of the findings. This should be both in the frame of a scientist conducting the experiment (“What did the experiment tell us about the world?”) and in the frame of a student (“What skills or mindsets did I learn?”).

5.9 Tips for slide presentations

- General tips with slides:
 - Font and color choices should stress clarity and ease of viewing.
 - Avoid large blocks of text. Instead, use bullet points and short phrases.
 - Avoid simply reading the slides.
 - The slides should be your visual notes, not your script.
 - Annotate pictures and graphs to explain or highlight features
- All relevant equations should be stated in variable form. For example, use $x = x_0 + v_0t + \frac{1}{2}at^2$, rather than $x=x_0+v_0*t+1/2at^2$ or $x=12+0+4.9*t*t$.
- For graphs:
 - Be sure to label both axes with titles and units.
 - Legends and trend-line equations should be legible on slides (At least as big as slide text).
 - Use an appropriate scale for your graphs without excessive unused white-space.

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, “ 5 ± 1 m” means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters¹. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

A.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument’s scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment’s manual often gives the instrument’s resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

¹The phrase “most likely” can mean different things depending on who is writing. If a physicist gives the value and does not give a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet, or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^N \frac{(x_i - \bar{x})^2}{N-1}}, \quad (\text{A.1})$$

where $\{x_1, x_2, \dots, x_N\}$ are the measured values, \bar{x} is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called “standard deviation of the mean”, which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}. \quad (\text{A.2})$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as 52 ± 3 m.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn’t change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

A.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction), } \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \quad (\text{A.3})$$

$$\text{For } c = ab \text{ (or for division), } \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \quad (\text{A.4})$$

$$\text{For } c = a^n, \frac{\delta c}{c} = n \frac{\delta a}{a} \quad (\text{A.5})$$

For other calculations, there is a more general formula not discussed here.

Expression	Implied uncertainty
12	0.5
12.0	0.05
120	5
120.	0.5

Table A.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π , is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table A.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value “0.0322857142857143”. Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as “9.65683578099600e-04”, where the ‘e’ stands for “times ten to the”. I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as “ 0.032 ± 0.001 km/s”. You may also see the equivalent, more succinct notation “0.032(1) km/s”.

A.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced “tee prime”). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as²

$$t' = \frac{|a - b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \quad (\text{A.6})$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

²Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

Rubrics

Each scientific ability rubric row assessed is worth a possible 1 point, with “Missing” being 0 points, “Inadequate” 1/3 points, “Needs Improvement” 2/3 points, and “Adequate” 1 point.

The scientific abilities rubrics are found on the following pages.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
A11	Graph	No graph is present.	A graph is present but the axes are not labeled. There is no scale on the axes.	The graph is present and axes are correctly labeled, but the axes do not correspond to the independent and dependent variables, or the scale is not accurate.	The graph has correctly labeled axes, independent variable is along the horizontal axis and the scale is accurate.

Table B.1: Rubric A: Ability to represent information in multiple ways

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B1	Is able to identify the phenomenon to be investigated	No phenomenon is mentioned	The description of the phenomenon to be investigated is confusing, or it is not the phenomenon of interest.	The description of the phenomenon is vague or incomplete.	The phenomenon to be investigated is clearly stated.
B2	Is able to design a reliable experiment that investigates the phenomenon	The experiment does not investigate the phenomenon.	The experiment may not yield any interesting patterns.	Some important aspects of the phenomenon will not be observable.	The experiment might yield interesting patterns relevant to the investigation of the phenomenon.
B3	Is able to decide what physical quantities are to be measured and identify independent and dependent variables	The physical quantities are irrelevant.	Only some of physical quantities are relevant.	The physical quantities are relevant. However, independent and dependent variables are not identified.	The physical quantities are relevant and independent and dependent variables are identified.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B4	Is able to describe how to use available equipment to make measurements	At least one of the chosen measurements cannot be made with the available equipment.	All chosen measurements can be made, but no details are given about how it is done.	All chosen measurements can be made, but the details of how it is done are vague or incomplete.	All chosen measurements can be made and all details of how it is done are clearly provided.
B5	Is able to describe what is observed without trying to explain, both in words and by means of a picture of the experimental setup.	No description is mentioned.	A description is incomplete. No labeled sketch is present. Or, observations are adjusted to fit expectations.	A description is complete, but mixed up with explanations or pattern. Or the sketch is present but is difficult to understand.	Clearly describes what happens in the experiments both verbally and with a sketch. Provides other representations when necessary (tables and graphs).
B6	Is able to identify the shortcomings in an experiment and suggest improvements	No attempt is made to identify any shortcomings of the experiment.	The shortcomings are described vaguely and no suggestions for improvement are made.	Not all aspects of the design are considered in terms of shortcomings or improvements.	All major shortcomings of the experiment are identified and reasonable suggestions for improvement are made.
B7	Is able to identify a pattern in the data	No attempt is made to search for a pattern.	The pattern described is irrelevant or inconsistent with the data.	The pattern has minor errors or omissions. Terms like “proportional” used without clarity, e.g. is the proportionality linear, quadratic, etc.	The pattern represents the relevant trend in the data. When possible, the trend is described in words.
B8	Is able to represent a pattern mathematically (if applicable)	No attempt is made to represent a pattern mathematically.	The mathematical expression does not represent the trend.	No analysis of how well the expression agrees with the data is included, or some features of the pattern are missing.	The expression represents the trend completely and an analysis of how well it agrees with the data is included.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B9	Is able to devise an explanation for an observed pattern	No attempt is made to explain the observed pattern.	An explanation is vague, not testable, or contradicts the pattern.	An explanation contradicts previous knowledge or the reasoning is flawed.	A reasonable explanation is made. It is testable and it explains the observed pattern.

Table B.2: Rubric B: Ability to design and conduct an observational experiment [5].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C1	Is able to identify the hypothesis to be tested	No mention is made of a hypothesis.	An attempt is made to identify the hypothesis to be tested but it is described in a confusing manner.	The hypothesis to be tested is described but there are minor omissions or vague details.	The hypothesis is clearly, specifically, and thoroughly stated.
C2	Is able to design a reliable experiment that tests the hypothesis	The experiment does not test the hypothesis.	The experiment tests the hypothesis, but due to the nature of the design it is likely the data will lead to an incorrect judgment.	The experiment tests the hypothesis, but due to the nature of the design there is a moderate chance the data will lead to an inconclusive judgment.	The experiment tests the hypothesis and has a high likelihood of producing data that will lead to a conclusive judgment.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C4	Is able to make a reasonable prediction based on a hypothesis	No prediction is made. The experiment is not treated as a testing experiment.	A prediction is made, but it is identical to the hypothesis, OR prediction is made based on a source unrelated to the hypothesis being tested, or is completely inconsistent with hypothesis being tested, OR prediction is unrelated to the context of the designed experiment.	Prediction follows from hypothesis but is flawed because relevant assumptions are not considered, OR prediction is incomplete or somewhat inconsistent with hypothesis, OR prediction is somewhat inconsistent with the experiment.	A prediction is made that follows from hypothesis, is distinct from the hypothesis, accurately describes the expected outcome of the experiment, and incorporates relevant assumptions if needed.
C5	Is able to identify the assumptions made in making the prediction	No attempt is made to identify assumptions.	An attempt is made to identify assumptions, but the assumptions are irrelevant or are confused with the hypothesis.	Relevant assumptions are identified but are not significant for making the prediction.	Sufficient assumptions are correctly identified, and are significant for the prediction that is made.
C6	Is able to determine specifically the way in which assumptions might affect the prediction	No attempt is made to determine the effects of assumptions.	The effects of assumptions are mentioned but are described vaguely.	The effects of assumptions are determined, but no attempt is made to validate them.	The effects of assumptions are determined and the assumptions are validated.
C7	Is able to decide whether the prediction and the outcome agree/disagree	No mention of whether the prediction and outcome agree/disagree.	A decision about the agreement/disagreement is made but is not consistent with the results of the experiment.	A reasonable decision about the agreement/disagreement is made but experimental uncertainty is not taken into account.	A reasonable decision about the agreement/disagreement is made and experimental uncertainty is taken into account.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C8	Is able to make a reasonable judgment about the hypothesis	No judgment is made about the hypothesis.	A judgment is made but is not consistent with the outcome of the experiment.	A judgment is made, is consistent with the outcome of the experiment, but assumptions are not taken into account.	A judgment is made, is consistent with the outcome of the experiment, and assumptions are taken into account.

Table B.3: Rubric C: Ability to design and conduct a testing experiment [5].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D1	Is able to identify the problem to be solved	No mention is made of the problem to be solved.	An attempt is made to identify the problem to be solved but it is described in a confusing manner.	The problem to be solved is described but there are minor omissions or vague details.	The problem to be solved is clearly stated.
D2	Is able to design a reliable experiment that solves the problem.	The experiment does not solve the problem.	The experiment attempts to solve the problem but due to the nature of the design the data will not lead to a reliable solution.	The experiment attempts to solve the problem but due to the nature of the design there is a moderate chance the data will not lead to a reliable solution.	The experiment solves the problem and has a high likelihood of producing data that will lead to a reliable solution.
D3	Is able to use available equipment to make measurements	At least one of the chosen measurements cannot be made with the available equipment.	All of the chosen measurements can be made, but no details are given about how it is done.	All of the chosen measurements can be made, but the details about how they are done are vague or incomplete.	All of the chosen measurements can be made and all details about how they are done are provided and clear.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D4	Is able to make a judgment about the results of the experiment	No discussion is presented about the results of the experiment.	A judgment is made about the results, but it is not reasonable or coherent.	An acceptable judgment is made about the result, but the reasoning is incomplete, OR uncertainties are not taken into account, OR assumptions are not discussed, OR the result is written as a single number.	An acceptable judgment is made about the result, with clear reasoning. The effects of assumptions and experimental uncertainties are considered. The result is written as an interval.
D5	Is able to evaluate the results by means of an independent method	No attempt is made to evaluate the consistency of the result using an independent method.	A second independent method is used to evaluate the results. However there is little or no discussion about the differences in the results due to the two methods.	A second independent method is used to evaluate the results. The results of the two methods are compared correctly using experimental uncertainties. But there is little or no discussion of the possible reasons for the differences when the results are different.	A second independent method is used to evaluate the results and the evaluation is correctly done with the experimental uncertainties. The discrepancy between the results of the two methods, and possible reasons are discussed.
D7	Is able to choose a productive mathematical procedure for solving the experimental problem	Mathematical procedure is either missing, or the equations written down are irrelevant to the design.	A mathematical procedure is described, but is incorrect or incomplete, due to which the final answer cannot be calculated. Or units are inconsistent.	Correct and complete mathematical procedure is described but an error is made in the calculations. All units are consistent.	Mathematical procedure is fully consistent with the design. All quantities are calculated correctly with proper units. Final answer is meaningful.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D8	Is able to identify the assumptions made in using the mathematical procedure	No attempt is made to identify any assumptions.	An attempt is made to identify assumptions, but the assumptions are irrelevant or incorrect for the situation.	Relevant assumptions are identified but are not significant for solving the problem.	All relevant assumptions are correctly identified.
D9	Is able to determine specifically the way in which assumptions might affect the results	No attempt is made to determine the effects of assumptions.	The effects of assumptions are mentioned but are described vaguely.	The effects of assumptions are determined, but no attempt is made to validate them.	The effects of assumptions are determined and the assumptions are validated.

Table B.4: Rubric D: Ability to design and conduct an application experiment [5].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
F1	Is able to communicate the details of an experimental procedure clearly and completely	Diagrams are missing and/or experimental procedure is missing or extremely vague.	Diagrams are present but unclear and/or experimental procedure is present but important details are missing. It takes a lot of effort to comprehend.	Diagrams and/or experimental procedure are present and clearly labeled but with minor omissions or vague details. The procedure takes some effort to comprehend.	Diagrams and/or experimental procedure are clear and complete. It takes no effort to comprehend.
F2	Is able to communicate the point of the experiment clearly and completely	No discussion of the point of the experiment is present.	The experiment and findings are discussed but vaguely. There is no reflection on the quality and importance of the findings.	The experiment and findings are communicated but the reflection on their importance and quality is not present.	The experiment and findings are discussed clearly. There is deep reflection on the quality and importance of the findings.

Table B.5: Rubric F: Ability to communicate scientific ideas [5].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
G1	Is able to identify sources of experimental uncertainty	No attempt is made to identify experimental uncertainties.	An attempt is made to identify experimental uncertainties, but most are missing, described vaguely, or incorrect.	Most experimental uncertainties are correctly identified. But there is no distinction between random and instrumental uncertainty.	All experimental uncertainties are correctly identified. There is a distinction between instrumental and random uncertainty.
G2	Is able to evaluate specifically how identified experimental uncertainties affect the data	No attempt is made to evaluate experimental uncertainties.	An attempt is made to evaluate uncertainties, but most are missing, described vaguely, or incorrect. Or the final result does not take uncertainty into account.	The final result does take the identified uncertainties into account but is not correctly evaluated. Uncertainty propagation is not used or is used incorrectly.	The experimental uncertainty of the final result is correctly evaluated. Uncertainty propagation is used appropriately.
G3	Is able to describe how to minimize experimental uncertainty and actually do it	No attempt is made to describe how to minimize experimental uncertainty and no attempt to minimize is present.	A description of how to minimize experimental uncertainty is present, but there is no attempt to actually minimize it.	An attempt is made to minimize the uncertainty in the final result is made but the method is not very effective.	The uncertainty is minimized in an effective way.
G4	Is able to record and represent data in a meaningful way	Data are either absent or incomprehensible.	Some important data are absent or incomprehensible. They are not organized in tables or the tables are not labeled properly.	All important data are present, but recorded in a way that requires some effort to comprehend. The tables are labeled but labels are confusing.	All important data are present, organized, and recorded clearly. The tables are labeled and placed in a logical order.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
G5	Is able to analyze data appropriately	No attempt is made to analyze the data.	An attempt is made to analyze the data, but it is either seriously flawed or inappropriate.	The analysis is appropriate but it contains errors or omissions.	The analysis is appropriate, complete, and correct.

Table B.6: Rubric G: Ability to collect and analyze experimental data [5].

Lab Report Format

C.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.

C.2 Organizing the report

The report should follow the sequence of the lab manual. Answers to questions and inclusion of tables and figures should appear in the order they are referenced in the manual. In general, include the following:

- Any procedure that you performed that is different from what is described in the lab manual.
- Any data that you've collected: tables, figures, measured values, sketches. Whenever possible, include an estimate of the uncertainty of measured values.
- Any calculations that you perform using your data, and the final results of your calculation. Note that you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92 \text{ m/s}^2)t + 0.2 \text{ m/s}$."
- Answers to any questions that appear in the lab handout. Each answer requires providing justification for your answer.
- At the end of each experiment, you should discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

C.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like “distance (pc)”. This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like “distance (pc)”.
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called “error bars”, along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.

Bibliography

- [1] D. An, J. A. Johnson, J. L. Clem, B. Yanny, C. M. Rockosi, H. L. Morrison, P. Harding, J. E. Gunn, C. Allende Prieto, T. C. Beers, K. M. Cudworth, I. I. Ivans, Ž. Ivezić, Y. S. Lee, R. H. Lupton, D. Bizyaev, H. Brewington, E. Malanushenko, V. Malanushenko, D. Oravetz, K. Pan, A. Simmons, S. Snedden, S. Watters, and D. G. York, “Galactic Globular and Open Clusters in the Sloan Digital Sky Survey. I. Crowded-Field Photometry and Cluster Fiducial Sequences in ugriz”, *The Astrophysical Journal Supplement* **179**, 326 (2008).
- [2] C. L. Slesnick, L. A. Hillenbrand, and P. Massey, “The Star Formation History and Mass Function of the Double Cluster η and χ Persei”, *The Astrophysical Journal* **576**, 880 (2002).
- [3] P. R. Durrell and W. E. Harris, “A color-magnitude study of the globular cluster M15”, *Astronomical Journal* **105**, 1420 (1993).
- [4] T. Currie, J. Hernandez, J. Irwin, S. J. Kenyon, S. Tokarz, Z. Balog, A. Bragg, P. Berlind, and M. Calkins, “The Stellar Population of η and χ Persei: Cluster Properties, Membership, and the Intrinsic Colors and Temperatures of Stars”, *The Astrophysical Journal Supplement* **186**, 191 (2010).
- [5] E. Etkina, A. Van Heuvelen, S. White-Brahmia, D. T. Brookes, M. Gentile, S. Murthy, D. Rosengrant, and A. Warren, “Scientific abilities and their assessment”, *Physical Review Special Topics - Physics Education Research* **2** (2006) 10.1103/PhysRevSTPER.2.020103.