Stochastic Simulation of Daily Precipitation, Temperature, and Solar Radiation

C. W. RICHARDSON
Grassland, Soil and Water Research Laboratory, U.S. Department of Agriculture
Temple, Texas 76501

Introduction:

This paper presents an approach that may be used to generate long samples of:

- daily precipitation
- maximum/minimum temperature
- solar radiation

Precipitation is generated independently of the other variables by using a Markov chain-exponential model.

Background Information:

The processes are time dependent within each variable and interdependent among the four variables.

For Example: radiation and temperature more likely to be below normal on rainy days than on dry days.

Maximum temperature will likely be low on a cloudy day with low solar radiation

Maximum and minimum temperatures on a given day will obviously be related because of heat storage in the soil and surrounding atmosphere.

General Approach:

The general approach followed in this study was to consider precipitation as the primary variable and then condition the other three variables for a given day on whether the day was wet or dry.

Let $Y_{p,i}$ be the daily precipitation amount (for year, p, and day, i):

-$X_{p,i}$ (1) the maximum temp

-$X_{p,i}$ (2) the minimum temp

-$X_{p,i}$ (3) the solar radiation

The Model:

(Precipitation)

For this study a first-order Markov chain with only two states, wet or dry, was used.

A day with total rainfall of 0.2 mm or more was considered a wet day.

Let $P_i(W|W)$ be the probability of a wet day on day, i, given a wet day on day, i - 1;

Let $P_i(W|D)$ be the probability of a wet day on day i given a dry day on day i- 1.

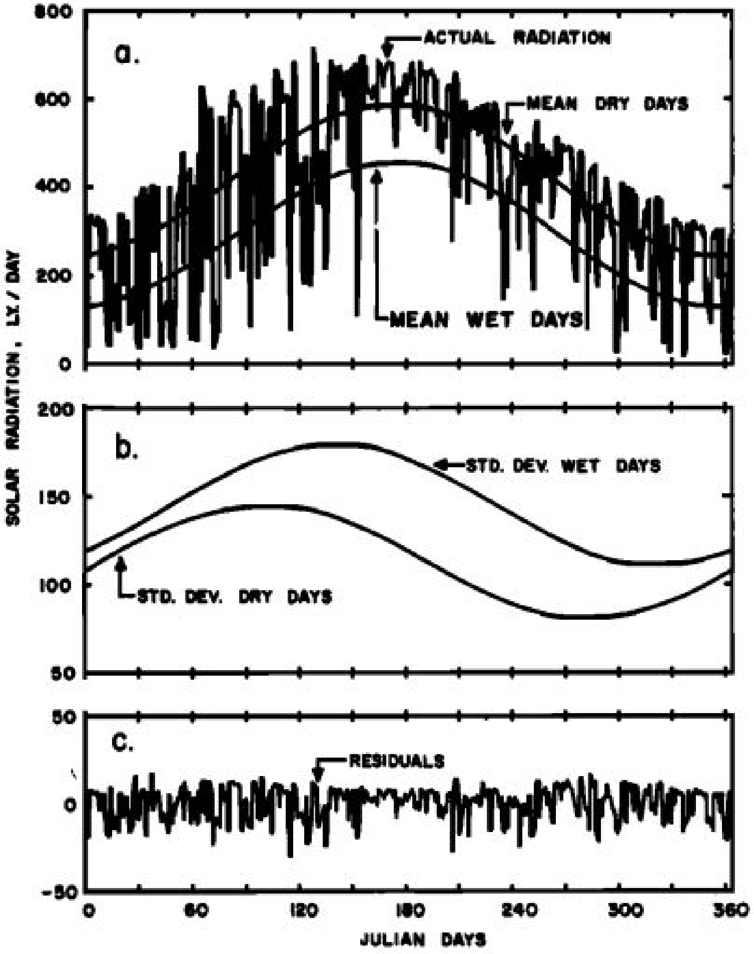

Fig. 1. Technique for reducing a daily solar radiation series to a series of residual elements, conditioned on the wet or dry status of the day.

The Model:

(Precipitation)

$$P_i(D/W) = 1 - P_i(W/W) \quad (1)$$

$$P_i(D/D) = 1 - P_i(W/D) \quad (2)$$

TABLE 1. Optimized Values of C_j and θ_j for Significant Harmonics of the Markov Chain–Exponential Precipitation Model Parameters

Parameter	C_0	C_1	θ_1	C_2	θ_2	C_3	θ_3
			Temple				
$P_i(W/W)$	0.445	0.058	−0.158	NS		NS	
$P_i(W/D)$	0.157	0.037	−1.060	NS		0.018	−0.836
λ_i, mm^{-1}	0.098	0.028	−0.339	0.021	−0.866	NS	
			Atlanta				
$P_i(W/W)$	0.467	NS		NS		NS	
$P_i(W/D)$	0.232	−0.051	1.435	0.052	−0.912	−0.028	−1.308
λ_i, mm^{-1}	0.094	−0.009	−1.097	0.009	−0.175	NS	
			Spokane				
$P_i(W/W)$	0.475	0.157	−0.326	NS		NS	
$P_i(W/D)$	0.226	0.107	−0.545	0.035	0.684	NS	
λ_i, mm^{-1}	0.282	−0.033	0.151	−0.029	0.873	NS	

NS means not significant at the 1% level by likelihood ratio test.

The Model:

(Min/Max Temp & Solar Radiation)

Variables: maximum temperature, minimum temperature, and solar radiation to be a continuous multivariate stochastic process

Daily means and standard deviations conditioned on the wet or dry state of the day.

The time series of each variable was reduced to a time series of residual elements by removing the periodic means and standard deviations [formula’s next slide].

These elements were analyzed to determine the time dependence (serial correlation) within each series and the cross correlation between each pair of variables.

The Model:

(Min/Max Temp & Solar Radiation)

Daily means and standard deviations of solar radiation were determined for wet and dry days using 5 years of data. Fourier series were used to smooth the seasonal means and standard deviations.

As expected, the mean was larger and standard deviation smaller on dry days than on wet days. The residual elements were calculated by removing the periodic mean and standard deviation:

$$\chi_{p,i}(j) = \frac{X_{p,i}(j) - \bar{X}_i^0(j)}{\sigma_i^0(j)} \qquad Y_{p,i} = 0 \qquad (4a)$$

or

$$\chi_{p,i}(j) = \frac{X_{p,i}(j) - \bar{X}_i^1(j)}{\sigma_i^1(j)} \qquad Y_{p,i} > 0 \qquad (4b)$$

The Model: Multivariate Generation Model

(Min/Max Temp & Solar Radiation)

The model that is proposed for generating residual series of maximum temperature, minimum temperature, and solar radiation is the weakly stationary generating process:

$$\chi_{p,i}(j) = A\chi_{p,i-1}(j) + B\epsilon_{p,i}(j) \qquad (5)$$

Implies the residuals of maximum temperature, minimum temperature, and solar radiation are:

-normally distributed

-serial correlation each variable described by a first-order linear autoregessive model.

The A and B matrices may be determined from the matrix equations:

$$M_0 = \begin{bmatrix} 1 & \rho_0(1,2) & \rho_0(1,3) \\ \rho_0(2,1) & 1 & \rho_0(2,3) \\ \rho_0(3,1) & \rho_0(3,2) & 1 \end{bmatrix} \quad (8)$$

$$M_1 = \begin{bmatrix} \rho_1(1) & \rho_1(1,2) & \rho_1(1,3) \\ \rho_1(2,1) & \rho_1(2) & \rho_1(2,3) \\ \rho_1(3,1) & \rho_1(3,2) & \rho_1(3) \end{bmatrix} \quad (9)$$

Testing the Model:

Daily weather data for Temple, Texas; Atlanta, Georgia; and Spokane, Washington

These three widely separated locations were chosen to include different climatic conditions.

Daily values of precipitation, maximum temperature, minimum temperature, and solar radiation data were obtained.

20 years of data used to estimate the parameters of the precipitation model

5 years of data used to estimate the parameters of the other three variables.

Summary:

A Markov chain-exponential model was used to describe precipitation

A multivariate model was used to describe maximum temperature, minimum temperature, and solar radiation.

Tests of the model showed that the model was capable of representing many of the characteristics that existed in the observed data.

The time dependence of each variable and the interdependence among the variables were closely described by the dependence structure that is inherent in (5).

Summary: (cont.)

Data generated with the model compared closely with the observed data in rainfall amounts, occurrence of wet days, mean daily temperatures and solar radiation, and annual extreme temperatures.

For all three locations used to test the model a set of about 40 coefficients was required to define the model completely.

The 12 coefficients used to describe the correlation structure of temperature and solar radiation were approximately the same for all locations.

Concluding Numbers:

the Three Test Locations

Variable	Annual Mean: Observed	Annual Mean: Generated
Temple		
Maximum temperature, °C	39.0	40.1
Minimum temperature, °C	−8.9	−7.8
Days ≥ 35°C	47.1	42.9
Days ≤ 0°C	29.1	20.1*
Atlanta		
Maximum temperature °C	35.9	36.5
Minimum temperature °C	−12.3	−13.6
Days ≥ 35°C	7.9	3.9
Days ≤ °C	51.4	68.7*
Spokane		
Maximum temperature °C	37.1	38.9*
Minimum temperature °C	−21.3	−17.9
Days ≥ 35°C	6.0	7.6
Days ≤ 0°C	139.4	142.7

* The observed mean and the generated mean are significantly different at the 1% level.

TABLE 10. Mean Daily Maximum Temperature, Minimum Temperature, and Solar Radiation by 28-Day Period and for the Year: Temple, Texas

Period	Maximum Temperature, °C		Minimum Temperature, °C		Solar Radiation, Ly	
	Observed	Generated	Observed	Generated	Observed	Generated
1	14.1	14.8	3.3	5.2	222	215
2	15.9	15.2	4.9	4.3	281	239
3	19.8	17.9	8.2	6.9	376	335
4	23.7	22.3	12.2	12.1	441	379
5	26.3	25.3	16.1	13.8	461	521
6	30.6	30.7	19.9	19.9	543	519
7	34.2	34.6	22.9	22.9	583	589
8	34.2	34.8	23.0	23.7	528	560
9	34.1	34.0	22.9	23.2	484	483
10	30.1	28.9	20.0	18.6	380	403
11	23.9	25.3	13.6	15.1	308	325
12	19.6	20.9	8.6	9.6	252	265
13	16.6	17.0	5.9	6.1	205	215
Year	24.9	24.7	14.0	14.0	390	388

TABLE 11. Mean Daily Maximum Temperature, Minimum Temperature, and Solar Radiation by 28-Day Period and for the Year: Atlanta, Georgia

Period	Maximum Temperature, °C		Minimum Temperature, °C		Solar Radiation, Ly	
	Observed	Generated	Observed	Generated	Observed	Generated
1	9.9	11.6	0.1	−0.4	202	218
2	10.8	8.9	−0.1	−2.7	270	235
3	16.0	14.2	3.4	2.0	394	320
4	22.4	20.4	9.9	7.7	451	426
5	24.9	23.6	13.0	11.6	508	482
6	26.8	27.1	16.4	15.8	518	528
7	29.3	30.7	19.3	19.6	510	530
8	29.9	30.6	19.8	19.7	487	507
9	28.9	28.9	18.7	18.3	451	451
10	25.7	24.3	14.6	13.1	391	406
11	20.7	20.6	8.6	8.9	310	319
12	15.9	15.8	4.5	4.1	234	234
13	12.3	11.6	1.2	0.3	202	195
Year	21.0	20.6	10.0	9.1	379	373

TABLE 12. Mean Daily Maximum Temperature, Minimum Temperature, and Solar Radiation by 28-Day Period and for the Year: Spokane, Washington

Period	Maximum Temperature, °C		Minimum Temperature, °C		Solar Radiation, Ly	
	Observed	Generated	Observed	Generated	Observed	Generated
1	0.3	0.6	−5.6	−6.2	99	90
2	3.8	−0.2	−4.0	−8.0	187	148
3	7.5	7.0	−2.5	−2.7	316	256
4	12.4	12.2	0.6	1.2	400	378
5	17.9	15.2	3.7	2.9	540	502
6	22.3	22.6	8.4	7.7	600	588
7	26.5	28.2	11.1	11.5	630	637
8	30.1	27.1	13.1	11.1	640	596
9	26.7	26.9	10.9	10.7	487	529
10	20.9	20.3	7.0	7.1	340	358
11	13.3	15.4	1.4	4.2	218	240
12	5.2	7.1	−1.4	−2.9	102	145
13	1.1	3.2	−4.4	−5.3	75	84
Year	14.5	14.3	2.9	2.4	356	350

Before Processes.

Gabriel and Neuman [1962] found that a first-order Markov chain provided a satisfactory model for daily precipitation occurrence at Tel Aviv, Israel.

A Markov chain was also used by Caskey [1963], Weiss [1964], and Hopkins and Robillard [1964] to describe the occurrence of sequences of wet or dry days.

Haan et al. [1976] used a first-order Markov chain with six rainfall states to model both occurrence and amounts of precipitation.

Smith and Schreiber [1973]. found a first-order Markov chain to be superior to a Bernoulli model (sequential independence) for describing the occurrence of wet or dry days in southeastern Arizona.

Nicks [1975] developed a model for generating daily values of maximum and minimum temperatures and solar radiation.