Energy Dynamics Notes

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1 Summary of Papers on this Topic

Camargo [1]: Nonlinear transfer functions are calculated for the Hasegawa-Wakatani model revealing an inverse cascade for the vorticity nonlinearity and a direct cascade for the pressure nonlinearity and transfer is predominantly local. The adiabaticity parameter is varied.

Holland [2]: ETG and ITG type fluid models are simulated and energy dynamics are computed and compared. Also compared with gyrokinetic simulations. The nonlinear transfer functions are studied with a focus on zonal flow interactions. Reynold's stress drive of zonal flows, scattering of drift waves in k_r and drive of drift waves by zonal flows were all found.

Holland [3]: Experimental measurement of the cross bispectrum shows that GAMs in the edge of DIII-D modulate the intensity of density fluctuations and drive a forward transfer of internal fluctuations energy. Gyrokinetic simulations show similar interactions between ZMF zonal flows and turbulence in the core.

Drake [4]: The nonlinear instability mechanism is present even when there are no linearly unstable eigenmodes. The mechanism is independent of magnetic shear and collisionality, so it can be relevant in many environments. The physical mechanism is explained. The instability is still present when the $k_z \neq 0$ components are removed from the $\mathbf{v_E} \cdot \nabla N_0$ term in the continuity equation, which eliminates the linear drive in any drift-wave system.

Biskamp [5]: First explanation of the nonlinear instability mechanism for 3D HW model. Find that $\phi_k >> n_k$ for $k_z, k_\perp << 1$ but $\phi_k \sim n_k$ for $k_z >> 1, k_\perp << 1$. This presents a possible experimental test of the instability mechanism if it holds in our case. They also find $k_z = 0$ modes drive strong zonal flows which quench the transport in the case of a certain box size.

Scott [6, 7]: Nonlinear instability can drive turbulence in the absence of linear instability. Find that $\phi_k >> n_k$ only when temperature fluctuations are included.

Nordman [8]: Subcritical instability is found in the case of 2D ITG turbulence.

Waltz [9]: Subcritical instability is found in 2D MHD ballooning turbulence.

Korsholm [10]: Damping of fluctuations at the radial boundaries rather than using triply periodic boundaries causes a change in the steady state tubulence. The zonal flow is decreased and the radial flux remains active throughout.

Zeiler [11]: Magnetic shear can stabilize the linear drift wave instability so drift-resistive ballooning models are used to drive the system with interchange instability. When the diamagnetic effects are larger than the ballooning effects, the resistive ballooning modes are stabilized and turbulence is self-sustained by nonlinear drift wave instability.

Zeiler [12]: Temperature fluctuations are added and are shown to partially suppress the nonlinear drive mechanism. This works through parallel heat conduction, which dissipates electron temperature energy which is fed by the density gradient and the adiabatic response.

Scott [13]: A drift-Alfven sheared slab model is studied for nonlinear instability. The linear adiabatic response is not strong enough to cause instability. Nonlinear instability or self-sustained turbulence dominates in a linearly stable drift wave system by changing the mode structure of the disturbances enhancing nonadiabaticity. The nonlinearity broadens the parallel current divergence, allowing a greater range of phase shifts between p_e and ϕ and the phase shifts that cuase instability are preferrentially selected by the free energy source. The vorticity nonlinearity is necessary and sufficient to drive the nonlinear instability, while the pressure nonlinearity saturates the turbulence through cascade to subgrid scales. Magnetic shear and field line connection eliminate the $k_{\|} = 0$ dominance. Phase diagrams of Im $ln(n^*_{k_y}\phi_{k_y})$ show the location and spread of the energy drive/transport.

Scott [14]: Similar to [13] but with added gyrofluid model and calculations. No main difference in results.

Scott [15]: Similar to [13] but not in slab, so it focuses on when the interchange drive dominates. The non-adiabaticity is dominated by the nonlinear polarization current for most tokamak regimes and the ballooning mechanism isn't important.
In the conventional mixing picture of turbulence driven by a linear instability, the amplitude grows until the rms vorticity is comparible to the linear growth rate γ_L. The eddies comprising the turbulence are said to be driven by the instability and decorrelated nonlinearly by their vorticity, leading to the estimate of linear mixing length models,

$$\gamma_L \sim k_\perp v_{k_\perp}, D_{mix} \sim \frac{v_{k_\perp}}{k_\perp} \sim \frac{\gamma_L}{k_\perp^2} \tag{1}$$

In his case, he finds that the RMS vorticity is much larger than γ_L and also that the overall drive rate in the turbulent stage is lower than the linear growth stage. (This also seems to be true in our case. See Fig. **??**) This means that the turbulence is producing its own vorticity through the process of nonlinear self-sustainment, and the reason that the linear instability at small scale has so little effect on the turbulence mode structure is that small scale structures are scattered apart by the vorticity before they can grow. The dynamics does not feel the linear instability in these scales. This is a generalization of the ideas of linear mixing length models as well as of the linear $E \times B$ shear-suppression scenario, but instead of the instability being suppressed by background $E \times B$ vorticity, it is suppressed by the turbulent $E \times B$ vorticity.
The scattering rate of the nonlinear vorticity decreases from $c_s/L_\perp$ towards large scale roughly as does ω_*, so a reasonable rule of thumb for linear instabilities is that they should become relevant when $\gamma_L > \omega_*$.

Baver [16]: A two field 2D TEM model exibits nonlinear instability through the excitation of eigenmodes on the stable branch. Where the nonlinear instability is greater than the linear instability, this is called supercritical instability.

2 Time Integration

Extra Documentation Equation **??** describes the instantaneous energy transfer dynamics for each (m, n) Fourier structure. While the instantaneous energy transfer is useful, the energy transfer summed over time is perhaps more statistically significant. A consequence of integrating over time is the implicit enforcement of three-wave frequency matching requirements. In other words, even if a wave with particular k' transfers energy to another wave with wavenumber k at a given time, there may be no net transfer of energy over time if the two waves have a random phase relationship. The extent of the relationship between the frequency and wavenumber matching requirements is discussed thoroughly in [3].

3 Zonal Flow Effects

Extra Documentation Up to this point, this study neglected Fourier analysis in the radial direction. The reason is that the equilibrium density profile is a function of radius and cylindrical differential vector operators contain factors of r. The dependences on radius in the spectrally decomposed equations make analytic expressions for the energy and energy transfer rate unnecessarily complicated. The nonlinear transfer function, however, has a simple form when the fields are Fourier decomposed in all spatial directions. The result is:

$$T_{tot}(k, k') = Re\left\{(mk'_r - m'k_r)\mathbf{F}_{\mathbf{1}k'}\phi_{k-k'} \cdot \mathbf{F}^*_{\mathbf{2}k}\right\}, \tag{2}$$

where the subscript k now represents (k_r, m, n). Of particular interest is the energy transfer of radial modes due to zonal flow convection. Typically, zonal flows are thought to cause a scattering of drift wave energy to high k_r due to radial shearing of the zonal flow. This particular interaction is represented by the transfer function in equation 2 when $m = m'$ and $n = n'$ because the third mode is the zonal mode with $m = 0, n = 0$ and finite k_r. Then,

$$T_{tot}(m, n, k_r, k'_r) = Re\left\{m(k'_r - k_r)\mathbf{F}_{\mathbf{1}k'}\phi^{ZF}_{k_r - k'_r} \cdot \mathbf{F}^*_{\mathbf{2}k}\right\}. \tag{3}$$

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