

University of California  
Los Angeles

# Numerical Studies of Turbulence in LAPD

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of the requirements for the degree  
Doctor of Philosophy in Physics

by

Brett Cory Friedman

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Abstract of the Dissertation

Numerical Studies of Turbulence in LAPD

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# CHAPTER 1

## Introduction

## CHAPTER 2

### Turbulence and Instability

- 2.1 The Kolmogorov Paradigm of Turbulence
- 2.2 The Standard Plasma Paradigm of Linear Instability
- 2.3 Nonlinear Stability Effects
- 2.4 Nonlinear Stability Effects in Plasma Physics

## CHAPTER 3

### The Braginskii Fluid Model and LAPD

#### 3.1 LAPD Suitability to the Braginskii Fluid Model

At a basic level, the state of a plasma is described by seven-dimensional distribution functions  $f_j(\mathbf{x}, \mathbf{v}, t)$  for each species  $j$ . The behavior of the plasma is described by the system of kinetic equations (Boltzmann equations), which evolve the distribution functions forward in time:

$$\frac{\partial f_j}{\partial t} + \mathbf{v} \cdot \nabla f_j + \frac{e_j}{m_j} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_j}{\partial \mathbf{v}} = \left( \frac{\partial f_j}{\partial t} \right)_C. \quad (3.1)$$

$\left( \frac{\partial f_j}{\partial t} \right)_C$  is the change in the distribution function due to collisions. For plasmas, the collisions are Coulomb collisions, and the collision term takes the form of the Fokker-Planck operator. With this operator, Eq. 3.1 is called the Fokker-Planck equation. Now it is well known that the Fokker-Planck equation cannot be solved numerically for problems that require time intervals much larger than the electron-cyclotron time due to computational and time limitations. The phase space is just too large. Therefore, reduced equations, such as gyrokinetic, drift kinetic, or fluid equations have been derived to produce numerically tractable equations. These equations are all derived under certain physical assumptions such as strong guiding magnetic fields, small fluctuation levels, or slow spatial and/or time variations such that these different equations are best applied to different physical situations.

The equations that are arguably most suitable to describe waves and turbulence in LAPD (and fastest to solve numerically) are the fluid equations, specif-



ically those derived by Braginskii [Bra65]. In deriving his equations, Braginskii approximates the solution as  $f_j = f_j^0 + f_j^1$  where the zero-order piece  $f_j^0$  is a Maxwellian and the first-order piece  $f_j^1$  is a perturbation on the zero-order distribution function:  $|f_j^1| \ll f_j^0$ . The equations are then derived by taking moments of the Fokker-Planck equation to create coupled equations of the independent variables,  $n_j$ ,  $\mathbf{v}_j$ , and  $T_j$ . Now certain requirements must hold to justify the Braginskii approximation, all of which have the flavor that macroscopic quantities must vary slowly in time and space. This is generally caused by strong relaxation processes such as collisions, which keep the distribution functions close to Maxwellians. In general, for the Braginskii equations to be applicable, processes of interest must occur on time intervals much greater than the collision time and quantities should vary slowly over distances traversed by the particles between collisions.

Specifically, the requirement that time variations must be slow can be written  $\frac{d}{dt} \ll \nu$ , where for electron drift wave turbulence, this is approximately  $\omega_* \ll \nu_e$ . Table 4.3, which displays typical LAPD operating parameters, shows that  $\omega_*/\nu_e \sim 0.01$ . The requirement that spatial quantities vary slowly compared to the collisional mean free path can be written simply for the direction parallel to the magnetic field as  $\lambda_{ei} \sim \lambda_{ee} \ll L_{\parallel}$ . For LAPD,  $\lambda_{ei}/L_{\parallel} \sim 0.01$ . For the direction perpendicular to the magnetic field, the same kind of relation  $\lambda_{mfp} \ll L_{\perp}$  must also hold. However, due to the cyclotron motion of particles around the magnetic field,  $\lambda_{mfp}$  is really the larmor radius, unless the collisional mean free path is less than the larmor radius. For electrons,  $\rho_e \ll \lambda_{ei}$  and  $\rho_e/L_{\perp} \sim 10^{-4}$  where  $L_{\perp} \sim 0.1$  m. For the ions, the ion cyclotron frequency is close to the ion collision frequency, meaning that either the ion larmor radius or the ion mean free path may be used. Using the larmor radius,  $\rho_i/L_{\perp} \sim 0.01$ . Therefore, the collisionality is high enough and the machine dimensions are large enough so that the Braginskii fluid model should be applicable to LAPD.

### 3.2 The Braginskii Equations

The Braginskii fluid equations are as follows: the continuity equation for species  $j$ , electrons or ions, is [Wes04, Bra65]

$$\frac{\partial n_j}{\partial t} = -\nabla \cdot (n_j \mathbf{v}_j). \quad (3.2)$$

The momentum balance equation is

$$n_j m_j \frac{d\mathbf{v}_j}{dt} = -\nabla p_j - \frac{\partial \Pi_{j\alpha\beta}}{\partial x_\beta} + n_j e_j (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}) + \mathbf{R}_j. \quad (3.3)$$

$p_j = n_j T_j$  is the pressure.  $\Pi_{j\alpha\beta}$  is the stress tensor, which involves the products of viscosity coefficients and rate-of-strain tensor components. The viscosity coefficients are some of the several terms that are called transport coefficients. The transport coefficients are calculated by the Braginskii procedure in terms of  $n$ ,  $\mathbf{v}$ , and  $T$ .  $\mathbf{R}_j$ , which involves several other transport coefficients, is the rate of collisional momentum transfer. The momentum transfer from ions to electrons is given by

$$\mathbf{R}_e = -m_e n_e \nu_e (0.51 u_{\parallel e} + \mathbf{u}_{\perp e}) - 0.71 n_e \nabla_{\parallel} T_e - \frac{3}{2} \frac{n_e \nu_e}{\omega_{ce}} \mathbf{b} \times \nabla T_e \quad (3.4)$$

where  $\mathbf{u} = \mathbf{v}_e - \mathbf{v}_i$  and  $\nu_e$  is the electron collision frequency with ions.  $\mathbf{R}_e$  includes both the friction force and the thermal force, which, like the friction force, is due to electron-ion collisions, but originates from the temperature dependence of the collisionality. The thermal force terms are those proportional to the gradients of temperature.  $\mathbf{R}_i = -\mathbf{R}_e$  in a fully ionized plasma with one ion species. However, LAPD has a significant neutral density. Collisions with neutrals are much more important for the ions [PUC10a]. So

$$\mathbf{R}_i = -\mathbf{R}_e - n_i m_i \nu_{in} \mathbf{v}_i. \quad (3.5)$$

The energy balance equation is

$$\frac{3}{2}n_j\frac{\partial T_j}{\partial t} = -n\mathbf{v}_j \cdot \nabla T_j - p_j \nabla \cdot \mathbf{v}_j - \nabla \cdot \mathbf{q}_j - \Pi_{j\alpha\beta} \frac{\partial v_{j\alpha}}{\partial x_\beta} + Q_j \quad (3.6)$$

where the term involving the stress tensor describes viscous heating. The electron heat flux (with more transport coefficients) is

$$q_e = n_e T_e \left( 0.71 u_{\parallel} + \frac{3\nu_e}{2\omega_{ce}} \mathbf{b} \times \mathbf{u} \right) + \frac{n_e T_e}{m_e \nu_e} \left( -3.16 \nabla_{\parallel} T_e - \frac{4.66 \nu_e^2}{\omega_{ce}^2} \nabla_{\perp} T_e - \frac{5\nu_e}{2\omega_{ce}} \mathbf{b} \times \nabla T_e \right) \quad (3.7)$$

where the first part of this expression constitutes convection, while the second part is conduction. The ion heat flux is

$$q_i = \frac{n_i T_i}{m_i \nu_i} \left( -3.9 \nabla_{\parallel} T_i - \frac{2\nu_i^2}{\omega_{ci}^2} \nabla_{\perp} T_i - \frac{5\nu_i}{2\omega_{ci}} \mathbf{b} \times \nabla T_i \right). \quad (3.8)$$

The last transport coefficients are in the heating  $Q$ . The ion heating due to collisional heat exchange between ions and electrons is

$$Q_i = \frac{3m_e}{m_i} n_e \nu_e (T_e - T_i) \quad (3.9)$$

while the electron heating is

$$Q_e = -\mathbf{R} \cdot \mathbf{u} - Q_i. \quad (3.10)$$

The electron heat exchange involves an ohmic heating contribution ( $\mathbf{R} \cdot \mathbf{u}$ ) that is absent from the ion heating because electrons colliding with ions transfer very little momentum to the ions.

### 3.3 The Vorticity Equation

Now the Braginskii equations in the previous section contain electric and magnetic fields which must be self-consistently determined by the charges and currents that are evolved by the equations. This is done with the inclusion of Maxwell's equations. Two of those equations are used to write the fields in terms of potentials:

$$\begin{aligned}\mathbf{E} &= -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t} \\ \mathbf{B} &= \nabla \times \mathbf{A}.\end{aligned}\tag{3.11}$$

The vector potential  $\mathbf{A}$  is strictly a fluctuating quantity, meaning it is not used to describe the guide field  $\mathbf{B}_0$ . The next equation,

$$\nabla \times \mathbf{B} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu_0 \mathbf{j}\tag{3.12}$$

is used to relate the vector potential to the current, where the displacement current is neglected as is generally done in plasmas. The Poisson equation is not that useful for the main part of the plasma, in which the quasineutrality relation,  $n_e = n_i \equiv n$ , holds. The useful equation that can be used instead is the conservation of charge (or ambipolarity condition),  $\nabla \cdot \mathbf{j} = 0$ . The vorticity equation is derived from this conservation of charge equation.

The current is  $\mathbf{j} = en(v_{\parallel i} - v_{\parallel e}) + en(\mathbf{v}_{\perp i} - \mathbf{v}_{\perp e})$ . In LAPD, the parallel current is carried primarily by the fast streaming electrons, while the perpendicular current is primarily carried by the ions, which have larger Larmor radii. So the conservation of charge equation can be simplified to

$$\nabla_{\parallel}(nv_{\parallel e}) = \nabla_{\perp} \cdot (n\mathbf{v}_{\perp i}).\tag{3.13}$$

The perpendicular ion component of this equation is derived from Eq. 3.3

for the ions. Neglecting terms that have finite ion temperature (pressure and stress tensor), and solving for the ion velocity in the Lorentz force term, the perpendicular ion velocity has three terms [PUC10a, SC03]:

$$\mathbf{v}_{\perp i} = \mathbf{v}_E + \mathbf{v}_{pi} + \mathbf{v}_{\nu i} \quad (3.14)$$

where the  $\mathbf{E} \times \mathbf{B}$  velocity is  $\mathbf{v}_E = \mathbf{E} \times \mathbf{B} / B^2 = -\nabla_{\perp} \phi \times \mathbf{B} / B^2$ , the polarization velocity is  $\mathbf{v}_{pi} = (1/\omega_{ci}) \mathbf{b} \times (\partial_t + \mathbf{v}_i \cdot \nabla) \mathbf{v}_i$ , and the Pedersen velocity is  $\mathbf{v}_{\nu i} = (\nu_{in}/\omega_{ci}) \mathbf{b} \times \mathbf{v}_i$ . The charge conservation equation then takes the form:

$$\nabla_{\parallel} (nv_{\parallel e}) = \frac{1}{\omega_{ci}} \nabla_{\perp} \cdot [n \mathbf{b} \times (\partial_t + \mathbf{v}_i \cdot \nabla + \nu_{in}) \mathbf{v}_i]. \quad (3.15)$$

Note that the  $\mathbf{E} \times \mathbf{B}$  velocity doesn't contribute to the current due to the electrons producing an equal and opposite  $\mathbf{E} \times \mathbf{B}$  current. We now employ the approximation  $\mathbf{v}_i \sim \mathbf{v}_E$  to Eq. 3.15. This approximation wasn't appropriate of course for Eq. 3.14 due to the fact that  $\mathbf{v}_E$  doesn't contribute to the current, but it is appropriate here. Then,

$$\begin{aligned} \nabla_{\parallel} (nv_{\parallel e}) &= \frac{1}{\omega_{ci}} \nabla_{\perp} \cdot [n \mathbf{b} \times (\partial_t + \mathbf{v}_E \cdot \nabla + \nu_{in}) \mathbf{v}_E] \rightarrow \\ \nabla_{\parallel} (nv_{\parallel e}) &= -\frac{m_i}{eB^2} \nabla_{\perp} \cdot [n \mathbf{b} \times (\partial_t + \mathbf{v}_E \cdot \nabla + \nu_{in}) \nabla_{\perp} \phi]. \end{aligned} \quad (3.16)$$

Next, defining the vorticity as  $\varpi \equiv \nabla_{\perp} \cdot (n \nabla_{\perp} \phi)$ , the vorticity equation reads,

$$\frac{\partial \varpi}{\partial t} = -\mathbf{v}_E \cdot \nabla_{\perp} \varpi - \nabla_{\perp} \mathbf{v}_E : \nabla_{\perp} (n \nabla_{\perp} \phi) - \frac{eB^2}{m_i} \nabla_{\parallel} (nv_{\parallel e}) - \nu_{in} \varpi. \quad (3.17)$$

Finally, the term with the tensor product can be rewritten in a different form [PUC10a]:

$$\frac{\partial \varpi}{\partial t} = -\mathbf{v}_E \cdot \nabla_{\perp} \varpi + \frac{1}{2} (\mathbf{b} \times \nabla_{\perp} n) \cdot \nabla_{\perp} \mathbf{v}_E^2 - \frac{eB^2}{m_i} \nabla_{\parallel} (nv_{\parallel e}) - \nu_{in} \varpi. \quad (3.18)$$

### 3.4 Minimizing the Equation Set for LAPD Parameters

#### 3.4.1 The Reduced Equations

The continuity equations 3.2 for electrons and ions do not have to both be used due to the quasineutrality condition  $n_e = n_i \equiv n$ . So, if one focuses on the electron continuity equation, then,

$$\frac{\partial n}{\partial t} = -\nabla \cdot (n\mathbf{v}_e). \quad (3.19)$$

Now,  $\mathbf{v}_e = \mathbf{v}_{\perp e} + v_{\parallel e}$ , where  $\mathbf{v}_{\perp e} = \mathbf{v}_E + \mathbf{v}_{de} + \mathbf{v}_{pe}$ , with the diamagnetic velocity  $\mathbf{v}_{de} = \frac{\mathbf{b} \times \nabla p_e}{en_e B}$ , which wasn't included for the ions in Eq. 3.14 due to the neglect of ion pressure. To a good approximation, the electron polarization velocity is smaller than the  $\mathbf{E} \times \mathbf{B}$  velocity, so that  $\nabla \cdot (n\mathbf{v}_{\perp e}) = \mathbf{v}_E \cdot \nabla n$  [PUC10a, SC03]. So, the continuity equation reads

$$\frac{\partial n}{\partial t} = -\mathbf{v}_E \cdot \nabla n - \nabla_{\parallel} (nv_{\parallel e}). \quad (3.20)$$

Next, the momentum equations (Eq. 3.3), of which there are six (three for electron velocity components and three for ion velocity components) are reduced to two here. The first is the vorticity equation (Eq. 3.18), in which we used the perpendicular momentum equations to derive it. The second is the equation for the parallel electron momentum. We neglect the parallel ion momentum equation since  $v_{\parallel e} \gg v_{\parallel i}$  for LAPD. The electron parallel momentum equation is then

$$nm_e \frac{\partial v_{\parallel e}}{\partial t} = -nm_e \mathbf{v}_E \cdot \nabla v_{\parallel e} - \nabla_{\parallel} p_e - enE_{\parallel} - 0.71n\nabla_{\parallel} T_e - 0.51m_e n\nu_e v_{\parallel e}, \quad (3.21)$$

where the viscous terms have been neglected. The conservation of energy equations (Eq. 3.6) are left. Since the ion temperature in LAPD is very low

( $T_i \leq 1$  eV), the ion energy equation is neglected. The electron energy equation is [SC03]

$$\begin{aligned} \frac{3}{2}n\frac{\partial T_e}{\partial t} = & -\frac{3}{2}n\mathbf{v}_E \cdot \nabla T_e - p_e \nabla_{\parallel} v_{\parallel e} \\ & + 0.71T_e \nabla \cdot (nv_{\parallel e}) + \nabla_{\parallel}(\kappa_{\parallel e} \nabla_{\parallel} T_e) + 0.51m_e n \nu_e v_{\parallel e}^2 - 3\frac{m_e}{m_i} n \nu_e T_e, \end{aligned} \quad (3.22)$$

where  $\kappa_{\parallel e} = 3.16 \frac{nT_e}{m_e \nu_e}$ .

### 3.4.2 The Electrostatic Justification

Plasma currents create magnetic fields in plasmas. Often times, analytic and numerical calculations of plasma waves and turbulence neglect the time dependent magnetic field perturbations, focusing only on the electrostatic contribution to the waves, turbulence, and transport. In the reduced fluid equations of the previous subsection, the magnetic perturbation enters in two important ways. First, it enters the electric field term of Eq. 3.21 because  $E_{\parallel} = -\nabla_{\parallel} \phi - \frac{\partial A_{\parallel}}{\partial t}$ , where  $A_{\parallel}$  is the parallel component of the vector potential. Second, it affects the parallel gradient operator,  $\nabla_{\parallel} = \mathbf{b} \cdot \nabla$  where  $\mathbf{b}$  is in the direction of the total magnetic field [SC03]. In the electrostatic limit,  $A_{\parallel} \rightarrow 0$ , so  $E_{\parallel} = -\nabla_{\parallel} \phi$  and  $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla$ . We take this limit in the remaining chapters, but there is the question of how justified we are to do so.

As a first step in answering this question, examine Eq. 3.21. The four independent variables,  $n, \phi, v_{\parallel e}$ , and  $T_e$ , which each have their own evolution equation, are all present in Eq. 3.21. Taking the parallel projection of Eq. 3.12 gives

$$\nabla_{\perp}^2 A_{\parallel} = -\mu_0 j_{\parallel} = \mu_0 n e v_{\parallel e}. \quad (3.23)$$

So  $A_{\parallel} \sim \mu_0 n e L_{\perp}^2 v_{\parallel e}$ , where  $\nabla_{\perp}^2 \sim 1/L_{\perp}^2$ . Then, Eq. 3.21 can be approximately rewritten as,

$$nm_e \frac{dv_{\parallel e}}{dt} \sim -T_e \nabla_{\parallel} n + en \nabla_{\parallel} \phi + \mu_0 e^2 n^2 L_{\perp}^2 \frac{\partial v_{\parallel e}}{\partial t} - 1.71 n \nabla_{\parallel} T_e - 0.51 m_e n \nu_e v_{\parallel e}. \quad (3.24)$$

The electromagnetic induction term, ( $EM = en \frac{\partial A_{\parallel}}{\partial t}$ ) is now written in terms of  $v_{\parallel e}$  as  $EM = \mu_0 e^2 n^2 L_{\perp}^2 \frac{\partial v_{\parallel e}}{\partial t}$ . It can therefore be directly compared to the other terms proportional to  $v_{\parallel e}$  to test for its importance. The other terms are the inertial term,  $M = nm_e \frac{dv_{\parallel e}}{dt}$  and the resistive term,  $R = 0.51 m_e n \nu_e v_{\parallel e}$ . A common way to compare these terms is to approximate the time derivative as the ion cyclotron frequency  $\frac{\partial}{\partial t} \sim \omega_{ci}$  and the perpendicular length scale as the ion sound gyroradius  $L_{\perp} \sim \rho_s$ , where  $\rho_s = c_s / \omega_{ci}$ . Then the ratio of the three terms (obtained by dividing each term by  $e B n v_{\parallel e}$ ) is:

$$M : EM : R = \frac{m_e}{m_i} : \beta : \frac{0.51 \nu_e}{\omega_{ce}}. \quad (3.25)$$

It can be seen from Table 4.3 that in LAPD, this ratio is 1 : 3.6 : 1.5. Thus, all three terms are of the same order with the electromagnetic term slightly larger than the other two. It seems then quite unjustified to use an electrostatic approximation.

However, estimating  $\frac{\partial}{\partial t} \sim \omega_{ci}$  isn't necessarily accurate. The equation set describes drift waves and so a more proper estimate might be  $\frac{\partial}{\partial t} \sim \omega_*$ . Under this approximations, the ratio is 1 : 3.6 : 70, meaning that the resistive term is more than an order of magnitude larger than the other two; however, the approximation  $\frac{\partial}{\partial t} \sim \omega_*$  is still rough and the numerical value of  $\omega_*$  in Table 4.3 is somewhat of a rough itself. Moreover, one could also argue with the approximation of the perpendicular length scale as the sound gyroradius. This is probably too small, in which case the electromagnetic inductance has been underestimated. While it's clear that the inertial term is probably unimportant, the inductive term could be important.



Similarly, the contribution of  $\tilde{\mathbf{b}} \sim A_{\parallel}$  in  $\nabla_{\parallel}$  can be approximated in a similar manner with similar inconclusive results. Without a clear separation between the resistive and inductive terms, the best way to determine the validity of the electrostatic approximation is by direct numerical calculation of the turbulence with and without the electromagnetic contributions. Therefore, we simulated an electrostatic and two electromagnetic versions of LAPD turbulence. The details of the electrostatic code are described in Chapter 4 and in Appendix A.

The only difference between the electrostatic and the first electromagnetic simulation is the exclusion/inclusion of the electromagnetic term  $en \frac{\partial A_{\parallel}}{\partial t}$  in the parallel electron momentum equation (Eq. 3.21). Of course the Maxwell equation (Eq. 3.12) must also be included for the electromagnetic simulation. The second electromagnetic simulation includes not only this term but also the  $A_{\parallel}$  contribution to  $\nabla_{\parallel}$  in the parallel electron momentum equation.

Now, turbulence is best characterized and compared in a statistical and often spectral manner. More details of turbulence characterization and comparison will be discussed later, but for now, we make a few statistical comparisons between the electrostatic and electromagnetic simulation results. Figure 3.1 shows the results of the three simulations as well as the experiment – namely, a comparison of the frequency spectra, the probability distribution function (pdf), and the rms level of the density fluctuations. The “Full Electromagnetic” curves are from the simulation including the  $A_{\parallel}$  contribution to  $\nabla_{\parallel}$ , while the “Electromagnetic” curves just include the  $A_{\parallel}$  contribution to  $E_{\parallel}$ . Clearly, the fluctuations are statistically similar in all cases and none of the simulations are inconsistent with the experiment. However, the electromagnetic effects are noticeable, and as we include more electromagnetic contributions in the simulations, the turbulent statistics more closely resemble those of the experiment. We make no quantitative comparison here, but rely only on a visual examination in making this conclusion.

Now, as mentioned above, we do not include any electromagnetic contribu-

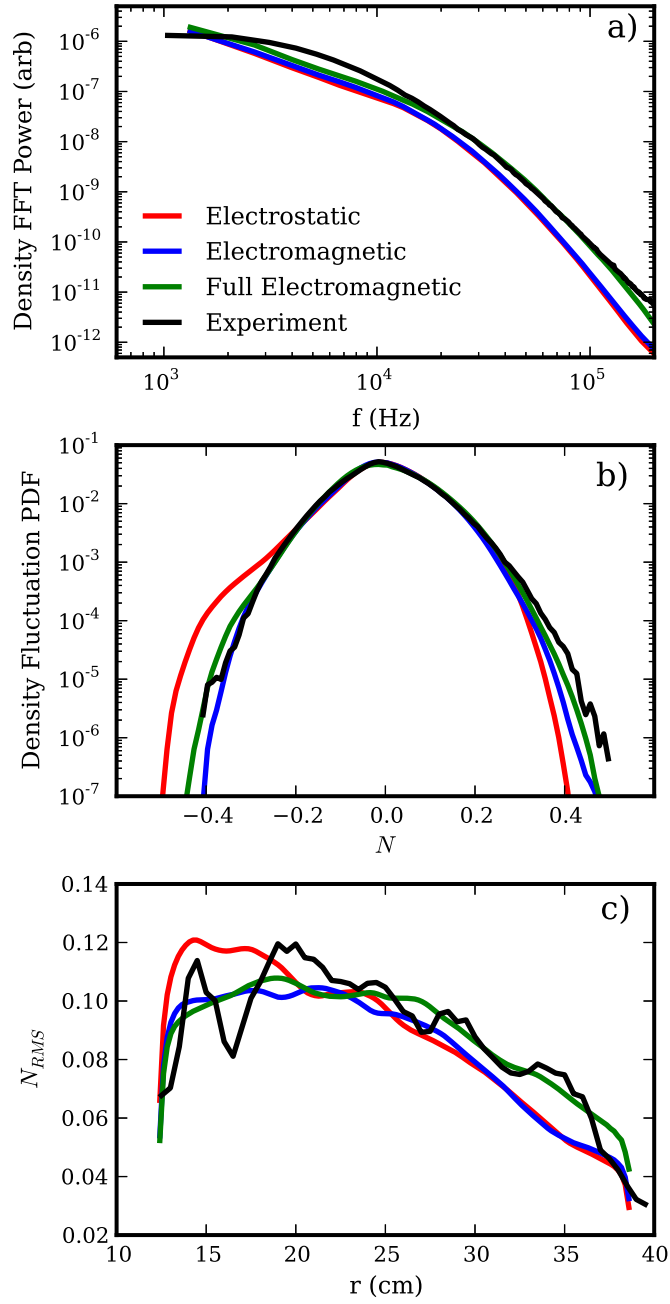


Figure 3.1: Electromagnetic statistics comparison

tions in the simulations used in the following chapters. It seems rather unjustified to do so since we are clearly able to run electromagnetic simulations and they seem to reproduce experimental turbulence with slightly better accuracy than the electrostatic ones. One justification for our abandonment of electromagnetic simulations, however, is that electromagnetic simulations take a bit longer than electrostatic ones due to the extra relation in Eq. 3.12 that is used to solve for  $A_{\parallel}$ , which requires an inversion of the Laplacian. This takes extra computation. Another justification is that the electromagnetic equations make the energy dynamics analysis in Chapter 6 a bit more complicated. Both of these factors are mitigated, however, if the inertial term  $nm_e \frac{dv_{\parallel e}}{dt}$  is dropped. Nevertheless, at the beginning of this work, we strived to find the simplest possible model to describe the turbulence in LAPD, and we determined that the electrostatic approximation was acceptable. At that time, we didn't have the results of Fig. 3.1. If we had the time, we would redo all of the simulations and analysis to include electromagnetic contributions, but drop the inertial term in Eq. 3.21. This is a clear route to take for future work. Nevertheless, we are confident that electromagnetics would not change any of our conclusions in this work. So for the remainder of this work, we will present theoretical calculations, simulation results, and conclusions using the electrostatic approximation.

# CHAPTER 4

## LAPD Simulation Details

In this chapter, we state and explain the equations, boundary conditions, parameters, and profiles that we use in the LAPD simulations. There are two main classes of simulations: those with a mean radial electric field and those without. The simulations with the mean radial electric field will be explored in Chapter 9. This chapter and all those up to Chapter 9 deal with the simulations with no mean radial electric field. There are only four different simulations that are used until Chapter 9, and they differ in only one way: their axial boundary conditions, which are explained in Sec. 4.2.

We solve the equations using the BOUT++ code [DUX09]. The code is discussed and partially displayed in Appendix A, and specific numerical routines and other details are described there as well.

### 4.1 The Equations

We use the Braginskii equations as shown in Chapter 3 to model LAPD turbulence. We separate all variables into time-independent equilibrium parts and time-dependent fluctuating parts. We do this in order to input experimental time-independent profiles. The alternative would be to solve the full equations with no equilibrium/fluctuation separation and no experimental profile input. The difficulty in this alternative technique is the need to specify sources, sinks, and boundary conditions, which can be difficult to measure or estimate. This alter-

native method has been undertaken by Rogers and Ricci [RR10]. Our approach is easier, and since the time-independent profiles are so important in driving the turbulence, we feel that inputting the experimentally measured profiles can help us produce physically realistic turbulence.

Now due to our equilibrium/fluctuation separation technique, we can linearize the equations, keeping only one nonlinearity in each equation: the advective nonlinearity. While this isn't necessary, it does simplify the energy dynamics as formulated in Chapter 6. The justification is practical rather than mathematical, and the partially linearized equations produce fluctuations that are quite statistically similar to experimental fluctuations, which is shown in Chapter 5, so we feel justified in doing this.

In the equations below, we normalize all quantities to give dimensionless variables. All times are normalized to the inverse ion cyclotron frequency  $\omega_{ci} = \frac{eB}{m_i}$ , velocities are normalized to the ion sound speed  $c_s = \sqrt{\frac{T_e}{m_i}}$ , lengths are normalized to the sound gyro-radius  $\rho_s = c_s/\omega_{ci}$ , potentials to  $T_e/e$ , densities to the density at the radial cylindrical axis, and temperatures to the temperature at the cylindrical axis. Quantities such as  $c_s$  that are typically functions of radius due to the radial dependence of the electron temperature are taken to be constant in these normalizations, where we use the values at the radial axis. The equations below don't reflect this, but the transport coefficients in the actual code do. So, the LAPD simulation equations are as follows:

$$\partial_t N = -\mathbf{v}_E \cdot \nabla N_0 - N_0 \nabla_{\parallel} v_{\parallel e} + \mu_N \nabla_{\perp}^2 N + S_N + \{\phi, N\}, \quad (4.1)$$

$$\partial_t v_{\parallel e} = -\frac{m_i}{m_e} \frac{T_{e0}}{N_0} \nabla_{\parallel} N - 1.71 \frac{m_i}{m_e} \nabla_{\parallel} T_e + \frac{m_i}{m_e} \nabla_{\parallel} \phi - \nu_e v_{\parallel e} + \{\phi, v_{\parallel e}\}, \quad (4.2)$$

$$\partial_t \varpi = -N_0 \nabla_{\parallel} v_{\parallel e} - \nu_{in} \varpi + \mu_{\phi} \nabla_{\perp}^2 \varpi + \{\phi, \varpi\}, \quad (4.3)$$

$$\begin{aligned} \partial_t T_e = & -\mathbf{v}_E \cdot \nabla T_{e0} - 1.71 \frac{2}{3} T_{e0} \nabla_{\parallel} v_{\parallel e} + \frac{2}{3 N_0} \kappa_{\parallel e} \nabla_{\parallel}^2 T_e \\ & - \frac{2 m_e}{m_i} \nu_e T_e + \mu_T \nabla_{\perp}^2 T_e + S_T + \{\phi, T_e\}. \end{aligned} \quad (4.4)$$

Note that the advective nonlinearities in each equation are written with Poisson brackets. Additionally, the only equilibrium profiles are  $N_0$  and  $T_{e0}$ , which are only functions of radius.  $\phi_0 = v_{\parallel e0} = 0$  in these equations. The linearized vorticity is  $\varpi = \nabla_{\perp} \cdot (N_0 \nabla_{\perp} \phi)$ .  $N$ ,  $v_{\parallel e}$ ,  $\phi$ , and  $T_e$  are fluctuating first-order quantities.

These equations have a few additional terms not included in the equations of Chapter 3. First, we have included density and temperature sources  $S_N$  and  $S_T$ . We have left out a momentum source as well as the contribution of the density source to changes of the momentum and temperature. Second, we have included diffusive ( $\mu_N \nabla_{\perp}^2 N$  and  $\mu_T \nabla_{\perp}^2 T_e$ ) and viscous ( $\mu_{\phi} \nabla_{\perp}^2 \varpi$ ) terms in Eqs. 4.1, 4.4, and 4.3 respectively.

#### 4.1.1 Sources

The density source is actually a source/sink. It models both the ionization of neutral atoms as well as the recombination of ions and electrons. The sink action in LAPD is dominated by parallel (along  $\mathbf{B}$ ) losses to materials at the machine ends because the magnetic field prevents rapid radial loss. It's also possible that a layer of neutral atoms near the end of the machine opposite the cathode cools the plasma enough so that recombination can be strong in this layer. The sink action occurs at all radii with finite plasma density, which constitutes regions both inside and outside of the cathode radius due to radial ion transport. If the sink action is primarily at the end plates, the sink can be calculated by  $2n_{se}c_s/L_{\parallel}$ , where  $n_{se}$  is the density at the sheath edge in front of the end plate,  $c_s$  is the sound speed at the sheath edge, and the factor of 2 accounts for the two plates.  $n_{se}$  and  $c_s$  are functions of radius such that the sink is strongest at the radial axis and decreases at larger radii. Calculation of the sink term requires knowledge of the density and temperature at the end of the machine, which is generally not measured experimentally.

The ionization source occurs primarily inside of the cathode radius. The source term is calculated with  $n_e n_n \langle \sigma v \rangle_{iz}$ , where  $n_n$  is the neutral Helium density and  $\langle \sigma v \rangle_{iz}$  is the ionization rate of Helium and is a strong function of temperature. The strong temperature dependence is the reason why ionization occurs only within the cathode radius. Ionization rates are readily available [Sta00], but the neutral density is not, making the source difficult to calculate. However, it is clear that if one were to sum up the source and sink and integrate axially, the region inside of the cathode radius must be a net source, while the region outside of it must be a net sink.

Now when we simulate the turbulence in LAPD without the source terms, turbulence drives radial transport such that the total flux-surface-averaged density gradient relaxes over time as seen in Fig. 4.1 a) until the radial transport ceases. It's interesting that  $\langle N_t \rangle_{fs} = \langle N_0 + N \rangle_{fs}$  doesn't become totally flat, but maintains a gradient. This is probably a result of the partial linearization of the equations, especially the diffusion term  $\mu_N \nabla_\perp^2 N$ , and possibly boundary conditions. Nevertheless, the strong relaxation is not physical due to the presence of the physical source/sink mechanism. (Note that the equilibrium density profile  $N_0$  used in the simulation to make Fig. 4.1 a) is not the same  $N_0$  shown in Fig. 4.2. We show the relaxation of the profile in Fig. 4.1 a) because it has a very steep gradient, which makes the relaxation effect more pronounced and easier to see.)

Now, rather than developing a first principle's source based on the theoretical source/sink expressions, we use ad hoc controlling sources. We estimate that  $\langle N_t \rangle_{fs}$  remains constant over time, and model the source using the integral portion of a PID controller. This means that we write an equation for the source:

$$\partial_t S_n = - \langle N \rangle_{fs}. \quad (4.5)$$

Therefore,

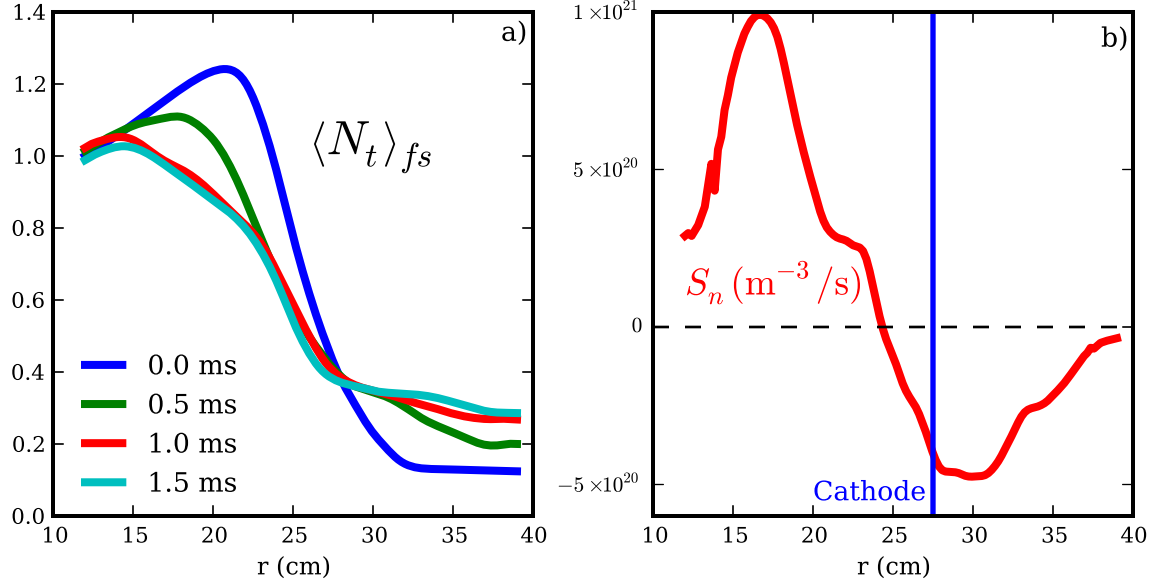


Figure 4.1: a) Profile relaxation and b) evolved density source

$$S_n(t) = - \int_0^t \langle N(\tau) \rangle_{fs} d\tau. \quad (4.6)$$

A typical time-averaged density source is shown in Fig. 4.1 b). The upshot of using such a source is that  $\langle N_t(t) \rangle_{fs} \simeq N_0$ . Notice that the source in Fig. 4.1 b) is net positive inside of the cathode radius and negative outside of it, just as one would expect. We use the same method for the temperature source. The temperature source ultimately comes from the hot electrons that are boiled off of the cathode, which transfer their energy to the plasma through ionizing collisions. This heat transfer is mostly to the electrons of the plasma. The temperature sink is caused by collisions with ions and neutrals which line radiate and by heat loss to the sheath and end walls.

We emphasize that the sources are not first principle sources. They are constructed based on the simulated radial transport. The alternative first principle's



approach was used by Rogers and Ricci for LAPD [RR10]. They use a stationary top-hat-like ionization source that models the physical density-producing process in LAPD. Furthermore, they do not separate equilibrium from fluctuations or input equilibrium profiles. Their source feeds the density, which then transports itself until it comes to a quasi-equilibrium state (a sink is also present). This method solves for the full plasma state with very little experimental input. They input the sources and derive the plasma state. We, on the other hand, input part of the plasma state and derive the sources. As stated before, our method has the advantage of using experimentally measured profiles. This experimental input allows us to more easily simulate turbulence that resembles that in the experiment, and therefore make conclusions on the fluctuation properties. We do not, however, evolve the equilibrium and gain the knowledge that comes from that.

#### **4.1.2 Artificial Diffusion and Viscosity**

Artificial diffusion or hyperdiffusion terms are ubiquitous in fluid simulations. They are generally intended to prevent high frequency or high wavenumber ringing caused by numerical advection schemes at steep interfaces. They can, however, cause unphysical smoothing in systems that are non-diffusive and non-viscous or cause over-smoothing if applied haphazardly. Some numerical advection schemes contain their own diffusion, called numerical diffusion. Other non-advective finite difference schemes also contain numerical diffusion or dispersion.

We use artificial diffusion and viscosity for several reasons. The first is to prevent artificial high-wavenumber oscillations due to the Arakawa advection scheme that we use [Ara66]. Second, it smooths out the solutions, preventing the total density and temperature from becoming negative at any point in space, which is obviously unphysical. Third, we can use it to prevent the need to go to very fine grid spacing at which physical diffusion and viscosity are important. Finally, we can use it to help saturate the turbulence at levels consistent with experiment.

These reasons are all somewhat related, and more discussion is left to Appendix B.

Diffusion and viscosity are real effects that are present in the non-reduced Braginskii equations. In Chapter 3, we made the approximation that  $\nabla \cdot (n\mathbf{v}_{\perp e}) = \mathbf{v}_E \cdot \nabla n$ , which neglected the polarization velocity part of  $\mathbf{v}_{\perp e}$ . Now the “full polarization velocity” [SC03] (from crossing Eq. 3.3 with  $\mathbf{b}$  and neglecting the stress tensor) is

$$\mathbf{v}_{pe} = (1/\omega_{ce}) \left[ \frac{d(\mathbf{b} \times \mathbf{v}_{\perp e})}{dt} + \nu_e \mathbf{b} \times \mathbf{v}_{\perp e} - \nu_e \mathbf{b} \times \mathbf{v}_{\perp i} - \frac{3}{2} \frac{\nu_e}{m_e \omega_{ce}} \nabla_{\perp} T_e \right]. \quad (4.7)$$

The part of this that causes collisional diffusive terms is  $(\nu_e/\omega_{ce})\mathbf{b} \times \mathbf{v}_{\perp e}$ . Now this contains  $\mathbf{v}_{\perp e}$  itself, which must be approximated as  $\mathbf{v}_{\perp e} = \mathbf{v}_E + \mathbf{v}_{de}$  to allow for closure. Only the diamagnetic drift part will be important for the collisional diffusion, so the part of the polarization velocity that we focus on is  $(\nu_e/\omega_{ce})\mathbf{b} \times \mathbf{v}_{de}$ . Recall that we want to use this in the continuity equation, so we are interested in the term  $\nabla \cdot (n_e v_e) \rightarrow \nabla \cdot (n_e (\nu_e/\omega_{ce})\mathbf{b} \times \mathbf{v}_{de}) = -\nabla \cdot \frac{\nu_e m_e}{e^2 B^2} \nabla_{\perp} p_e$ . Now defining  $D = \frac{\nu_e m_e T_e}{e^2 B^2}$ , we have  $\nabla \cdot (n_e v_e) = -\nabla \cdot (D \nabla_{\perp} n) + \text{lots of other terms}$ .  $D$  is the classical diffusion coefficient, which is about  $0.01 \text{m}^2/\text{s}$  for LAPD parameters. One of the terms in  $\nabla \cdot (D \nabla_{\perp} n)$  is  $D \nabla_{\perp}^2 n$ , which has the same form of the artificial diffusion term that we’ve added to Eq. 4.1. Of course, we have neglected many terms of the same order as this term in Eq. 4.1, but this shows that such a classical diffusion term is present in the Braginskii equations.

A similar treatment can be used for the energy conservation equation (Eq. 3.6), using the same procedure as for the continuity equation but with the  $p_e \nabla \cdot \mathbf{v}_e$  term in Eq. 3.6. The result is  $p_e \nabla \cdot \mathbf{v}_e = D n_e \nabla_{\perp}^2 T_e + \text{lots of other terms}$ . This has the same form as the temperature diffusion term in Eq. 4.4.

The viscosity in the vorticity equation comes from the ion stress tensor term  $\frac{\partial \Pi_{i\alpha\beta}}{\partial x_{\beta}}$  that we neglected when deriving the vorticity equation because we neglected everything with finite ion temperature. If this was included, a vortic-

ity diffusion term (aka a viscosity) would have been included in the vorticity equation [PUC10b] as it is in other equation sets like the well-known Hasagawa-Wakatani equations [HW83]. The magnetized Braginskii viscosity coefficient is  $\eta_1^i = \frac{3nT_i}{10\omega_{ci}^2\tau_i}$ , which is about  $2 \times 10^{-8} \text{kg/m} \cdot \text{s}$  for LAPD. Since LAPD's ions are not necessarily magnetized due to the fact that  $\omega_{ci}\tau_i \sim 1$ , the unmagnetized ion viscosity is  $\eta_0^i = 0.96nT_i\tau_i$  [Bra65] which is about  $4 \times 10^{-7} \text{kg/m} \cdot \text{s}$ .

For the artificial diffusion and viscosity coefficients in Eqs. 4.1-4.4, we use a single value of  $1.25 \times 10^{-3}$  in our normalized units, which is  $0.075 \text{m}^2/\text{s}$  in real units. We find that this value produces turbulent fluctuation levels consistent with experimental levels. We use this as a free parameter in this sense. This value is much larger than the real classical diffusion  $D$ , but is smaller than  $\frac{\eta_1^i}{nm_i} = 1 \text{m}^2/\text{s}$ . Nevertheless, we neglected a number of terms in Eqs. 4.1-4.4 such that there isn't justification to use the real diffusion and viscosity in these equations. Artificial diffusion and viscosity terms, however, serve a numerical purpose.

## 4.2 Boundary Conditions

Boundary conditions are often difficult to determine in plasma devices. While the properties of the boundaries are usually known, the way that the plasma interacts with them can be complex. Plasma boundary physics is one of the main elements of present day fusion research [Sta00]. Often times there is uncertainty in the equations that need to be used in simulations, and once the equations are found, they can be difficult to implement in codes.

The boundary conditions in LAPD are difficult to determine. LAPD contains at one end, a hot emitting cathode behind a biased mesh annode. In front of the annode are biasable azimuthal limiters with radius about equal to the cathode radius, though the limiter radius may be changed. The far end contains a floating mesh plate. The cylinder is conducting and has a radius about 20 cm larger than

the cathode radius.

#### 4.2.1 Simple Boundaries

In all simulations, we use an annulus rather than a cylinder. Although the inner radius of the annulus may be arbitrarily small, we take the inner radius to be 12 cm. We take the outer radius to be 39 cm. This is generally the radial extent of our experimental probe measurements. Furthermore, the plasma fluctuations are nearly zero (when normalized to values at the cylindrical axis) outside of this annular region, which is seen in Fig. 3.1 c). Therefore, we set the radial boundaries on all of the fluctuating variables ( $N$ ,  $\phi$ ,  $v_{\parallel e}$ , and  $T_e$ ) to zero. It would be nice in the future to take data spanning at least a few more cm and extend the simulation domain accordingly. But for now, the results use such an annular domain.

As for the axial boundaries, we use four different boundary conditions: periodic, zero-value (Dirichlet), zero-derivative (Neumann), and Bohm sheath. The only non-trivial one, Bohm sheath, is derived and described in the following subsection. The others are all trivial to implement and provide a test of the importance of the axial boundary conditions on the nature of the turbulence.

#### 4.2.2 Bohm Sheath Boundaries

Bohm sheath boundary conditions are applicable when a plasma terminates at a conducting plate. We note that this is not necessarily the case in LAPD. The cathode/annode system is obviously much different from a simple floating or biased conducting plate. Furthermore, the mesh wall at the far end is not a solid wall. Moreover, it's not clear if the plasma is even attached to the far end mesh wall or if it becomes detached in the neutrals in front of it, where the plasma cools and recombines before interacting with the wall. In any case, it is still instructive to apply such an idealized boundary condition to LAPD because it is

somewhat more realistic than the simpler boundary conditions, and it creates a new linear instability (see Sec. 5.1.2), which can be used to test the robustness of LAPD's nonlinear instability. Therefore, we proceed with the derivation of the Bohm sheath boundary conditions.

Now, it is known that to good approximation, a plasma bounded by a wall can be divided into two regions: the main plasma and the Debye sheath [Sta00]. The Debye sheath is a small region adjacent to the wall, generally several Debye lengths long. It has a net positive charge ( $n_i > n_e$ ) that shields the negative charge on the wall. The sheath does not completely shield the negative wall, however, and a small electric field penetrates into the main plasma (the ambipolar field), which mostly serves to accelerate the cold ions toward the wall, and slightly retard the electrons before entering the sheath. In the main plasma, the quasi-neutrality relation holds ( $n_i = n_e$ ).

The well-known Bohm criterion along with other considerations restricts the ions to move into the sheath entrance at the sound speed  $c_s = \sqrt{T_e/m_i}$ . We consider here the case where there is no external biasing; in other words, the end plates are electrically isolated and floating. The wall can be set to an arbitrary potential, say  $\phi_w = 0$ , while the potential at the sheath entrance is then the positive floating potential  $\phi_{sf}$ . This potential difference across the sheath reflects slow electrons that enter the sheath. The electrons approximately maintain a cutoff Maxwellian velocity distribution throughout the sheath, and at the wall, their velocity is retarded by a Boltzmann factor due to the floating potential. In total, the current to the wall is [BRT91, BCR93, XRD93]

$$J_{\parallel} = \pm en \left[ c_s - \frac{(T_e/m_e)^{1/2}}{2\sqrt{\pi}} e^{\left(-\frac{e\phi_{sf}}{T_e}\right)} \right], \quad (4.8)$$

where the  $\pm$  indicates that the plasma flux goes into the wall, which is in different directions for the different end plates. Note that there is a factor of

$\sqrt{2}$  discrepancy between different reports on the expression used for the thermal velocity, which should have only a minor consequence. In this expression, all values are total (equilibrium + fluctuations).

Now, this is not only the current to the wall, but also the current going into the sheath edge, since the sheath is too small for there to be appreciable radial current loss or an ionization source within the sheath. All values, in fact, are taken to be those at the sheath edge. Furthermore, since the wall is electrically isolated, the equilibrium current at the wall vanishes. This sets the value for the floating potential to be  $\phi_{sf} = \Lambda T_{e0}/e$  with  $\Lambda = \ln\left(\frac{1}{2\sqrt{\pi}}\sqrt{\frac{m_i}{m_e}}\right)$ . Note that  $T_{e0}$  is a function of radius, meaning  $\phi_{sf}$  is also a function of radius. Thus, a radial temperature gradient produces a radial electric field, at least at the sheath edge and likely penetrating axially into the main plasma. It is noted that  $J_{\parallel}$  need not vanish on every field line since the end plates are conducting and charges can move around on the plate, however, the vanishing equilibrium current is generally a fair approximation [BCR93].

On the other hand, the fluctuating component of the current is allowed to vary between field lines. The first order fluctuating component is obtained by linearizing Eq. 4.8, giving the result:

$$J_{\parallel} = \pm e N_0 c_{s0} \left[ \frac{e\phi}{T_{e0}} - \Lambda \frac{T_e}{T_{e0}} \right], \quad (4.9)$$

where now,  $J_{\parallel}$ ,  $\phi$  and  $T_e$  are fluctuating components, consistent with previous notation. This expression for the current sets the fluctuating axial boundary condition of the plasma and is often called the Bohm Sheath boundary condition. This current condition holds both at the wall and at the sheath entrance. So rather than taking the simulation domain all the way to the wall, simulations often end at the sheath entrance and employ this analytically derived boundary condition to the boundaries of the main plasma. Then one doesn't have to worry

about the small spatial scales and the non-quasineutrality of the sheath. The corresponding boundary conditions for the other fluid variables such as the density and temperature have recently been derived by Loizu et al. [LRH12]. However, we simply take them to have zero-gradient as most others have done, although this isn't wholly inconsistent with Loizu's calculations.

Now while one may set the parallel current (or equivalently  $v_{\parallel e}$ ) at the axial boundaries to the quantity on the right hand side of Eq. 4.9, we don't do that. We use Ohm's Law ( $-\nabla_{\parallel}\phi = \eta_{\parallel}J_{\parallel}$ ) to set the boundary condition for the gradient of  $\phi$ . We do this for practical reasons in the coding. Therefore, the boundary condition used in the code is (in our normalized units):

$$\nabla_{\parallel}\phi = \pm \frac{\nu_e m_e}{m_i} (\phi - \Lambda T_e). \quad (4.10)$$

### 4.3 Profiles and Parameters

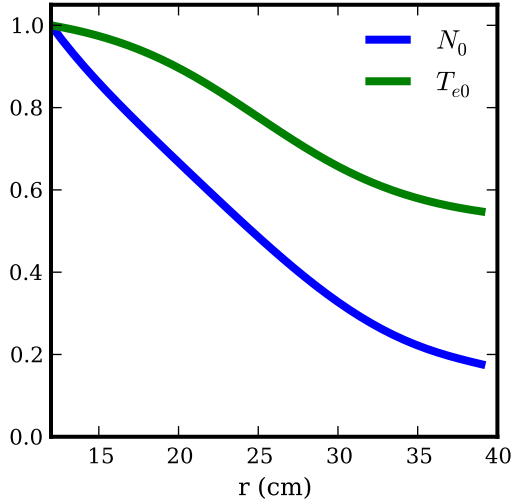


Figure 4.2: Equilibrium density and electron temperature profiles

As explained above, we take all equilibrium profiles and parameters from ex-

perimental measurements. All simulations before Chapter 9 use profiles and parameters from one particular experiment. This experiment used limiter biasing to essentially null out the mean radial electric field [SCR12]. We used this experiment so that we could neglect the mean potential profile in the equations (as is done in Eqs. 4.1-4.4), which simplifies our analysis. The normalized profiles that we use are shown in Fig. 4.2, and the parameters are shown in Table 4.3. The density profile is a polynomial fit to the experimental equilibrium density profile. The temperature profile is a tanh function that somewhat resembles typical LAPD temperature profiles. At the time of the first simulations, we didn't have reliable temperature profile measurements, so I was forced to estimate what the profile might look like. We note that the real temperature profile has a steeper gradient than the one we use. And again, we use  $\phi_0 = 0$ , which is a good approximation for the experimental nulled out potential profile.

The profiles that we use have no azimuthal or axial variation because we don't have the corresponding experimental measurements. So we assume that the equilibrium profiles are only functions of radius. It's likely, however, that there is some axial variation in the profiles and parameters. In LAPD,  $\nu^* \equiv L_{\parallel}/\lambda_{ei} \sim 100$ , generally indicating that a parallel temperature gradient can exist depending on the locations of the sources and sinks [Sta00]. Furthermore, if the Bohm sheath boundary condition is correct, the equilibrium potential must have a parallel gradient in order to accelerate the ions up to the sound speed at the sheath entrance. This ambipolar parallel electric field should exist between the location of the sheath entrance and an ion collision length into the main plasma. The parallel electric field generated by the condition of Eq. 4.10 is just the perturbed field that responds to electron temperature perturbations. It doesn't constitute the equilibrium electric field.

Moreover, recall that the Bohm sheath condition combined with a radial equilibrium electron temperature profile implies (at least near the sheath) a corre-



sponding equilibrium potential profile, since  $\phi_{sf} = \Lambda T_{e0}/e$ . Experimentally, this relation doesn't hold where we measure the plasma, meaning that either the ambipolar field doesn't penetrate far into the plasma or that the real LAPD boundary conditions are more complicated than simple floating conducting plates. So we don't use any equilibrium axial variation, leaving this to future work.

Species	${}^4\text{He}$
$Z$	1
$n$	$2.86 \times 10^{18} \text{ m}^{-3}$
$T_e$	6 eV
$T_i$	$\lesssim 1 \text{ eV}$
$B_0$	0.1 T
$L_{\parallel}$	17 m
$a$	0.4 m
$\lambda_D$	$10^{-5} \text{ m}$
$\omega_{ci}$	$2.4 \times 10^6 \text{ rad/s}$
$\omega_{ce}$	$1.8 \times 10^{10} \text{ rad/s}$
$\rho_e$	$5.3 \times 10^{-5} \text{ m}$
$\rho_i$	$\sim 1 \times 10^{-3} \text{ m}$
$\rho_s$	$5 \times 10^{-3} \text{ m}$
$v_{te}$	$9.4 \times 10^5 \text{ m/s}$
$c_s$	$1.1 \times 10^4 \text{ m/s}$
$v_A$	$7 \times 10^5 \text{ m/s}$
$\beta$	$5 \times 10^{-4}$
$m_e/m_i$	$1.4 \times 10^{-4}$
$\ln\Lambda$	11
$\nu_e$	$7.2 \times 10^6 \text{ Hz}$
$\lambda_{ei}$	0.13 m
$\nu_i$	$\sim 10^6 \text{ Hz}$
$\nu_{in}$	$3 \times 10^3 \text{ Hz}$
$\kappa_{\parallel}^e$	$9.8 \times 10^{23} \text{ eV/m}^2 \text{ s}$
$\eta_0^i$	$\sim 10^{12} \text{ eV s/m}^3$
$\omega_*$	$\sim 5 \times 10^4 \text{ rad/s}$

Table 4.1: LAPD simulation parameters

## CHAPTER 5

### The Nature of LAPD Turbulence

#### 5.1 LAPD Linear Instabilities

Linear instabilities are prevalent in plasma physics. They come from the linearization around an equilibrium of the plasma equations. Physically, if a plasma is in a time-independent steady state that is linearly unstable and a finite fluctuation of any size occurs, the fluctuation will grow exponentially. The LAPD equations, parameters, and profiles described in Chapter 4 give rise to a couple of linear instabilities. They are both drift wave type instabilities, but they have different pressure/potential coupling mechanisms. One type couples through the adiabatic response, while the other couples through the sheath boundary response.

##### 5.1.1 Drift Waves

Electron drift waves driven by an equilibrium density or pressure gradient that use the adiabatic response are generally referred to as just drift waves or the universal instability. The electron drift wave mechanism is the following: An electron pressure fluctuation in the plasma is linked with a potential fluctuation through the adiabatic response. The adiabatic response is simply a parallel force balance between the pressure force and the electrostatic force. A simplified version of Eq. 3.3 can be written:

$$\nabla_{\parallel} p_e = en \nabla_{\parallel} \phi + R v_{\parallel e}, \quad (5.1)$$

where the term  $Rv_{\parallel e}$  represents effects such as electron inertia, resistivity, and electromagnetic induction. If  $R = 0$ , the electrons are said to be adiabatic, meaning  $\nabla_{\parallel} p_e = en \nabla_{\parallel} \phi$ . When  $T_e$  fluctuations are neglected and  $\nabla_{\parallel} \neq 0$ , this integrates to the Boltzmann expression:

$$n = n_0 e^{e\phi/T_e}. \quad (5.2)$$

For any  $R$  and  $T_e$  fluctuations, the parallel electron dynamics couple the pressure to the potential as long as the parallel wavelength  $k_{\parallel}$  is finite. The perpendicular electric field associated with the potential fluctuation has a component in the azimuthal direction with  $k_{\perp} \gg k_{\parallel}$ . This creates a radial  $\mathbf{E} \times \mathbf{B}$  drift that advects the pressure in the radial direction. Because of the radial pressure gradient, the fluctuation propagates azimuthally as a wave at the drift speed  $v_{De} = \frac{T_e}{eB} \frac{\partial \ln N_0}{\partial r}$  [Che06] in the electron diamagnetic drift direction. If there is a small phase difference between the pressure and the potential of the wave (the result of  $R \neq 0$ ), the equilibrium pressure gradient will enhance the fluctuation, causing instability. Since  $p_e = n_e T_e$ , the pressure fluctuation may be due to either a density fluctuation, an electron temperature fluctuation, or both. The universal instability generally refers to the situation where an equilibrium density gradient drives a density fluctuating wave. But a temperature gradient driving a temperature fluctuation wave is also possible, and may be called a thermal drift wave [MTK11]. It's not necessary, however, to separate them, and we will just refer to both of these as drift waves.

The LAPD equation set (Eqs. 4.1-4.4) supports such drift waves, which are unstable with the parameters and profiles used in the simulations. The growth rate as a function of the azimuthal wavenumber  $m$  is shown in Fig. 5.1 a) for the LAPD parameters in Table 4.3 and profiles in Fig. 4.2. The growth rates are found by simulating the linearized version of Eqs. 4.1-4.4 in BOUT++ with the three different simple axial boundary conditions: periodic, zero-value (Dirichlet),

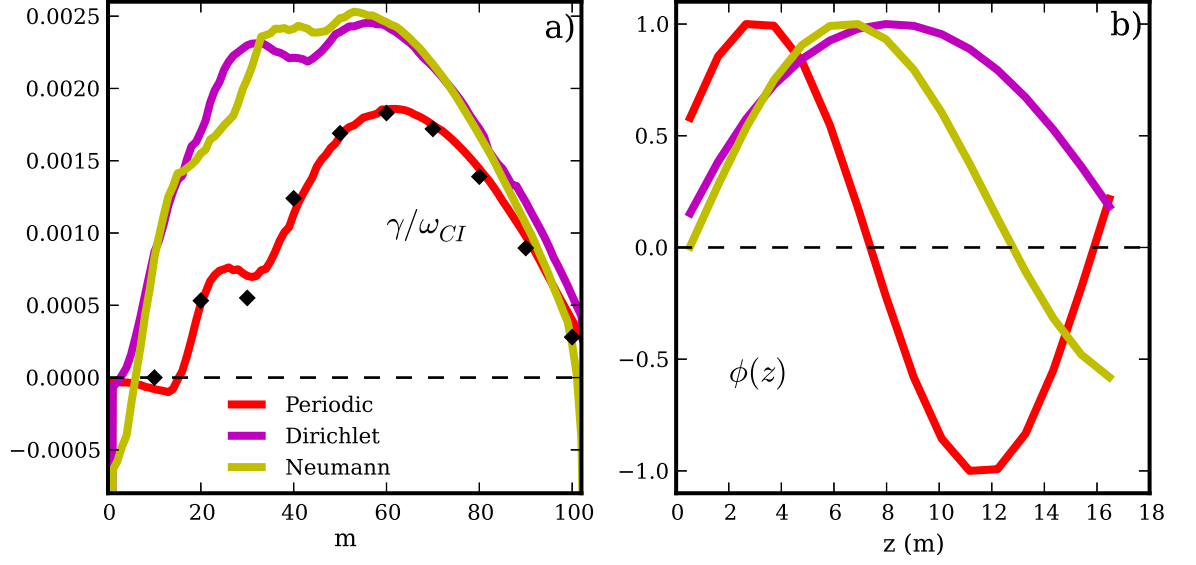


Figure 5.1: Linear drift wave a) growth rates and b) axial structures

and zero-derivative (Neumann). The linear equations simply omit the advective nonlinearities and the source terms, though the source terms have no effect on any of the linear modes besides  $m = 0$  modes. The simulations are run for long enough so that the fastest growing modes can dominate the dynamics.

The solid curves in Fig. 5.1 are calculated from the simulation results using the formula,  $\gamma_m = \frac{\partial E_m}{\partial t} / (2E_m)$  where  $E_m$  is the energy of the fastest growing linear eigenmode with azimuthal mode number  $m$ . The energy is defined in Chapter 6. The details of obtaining  $\gamma_m$  are explained in that chapter, but for now, it is sufficient to state that this procedure calculates  $\gamma_m$  at a particular time using only the structures of the fluctuating quantities:  $N, \phi, v_{\parallel e}$ , and  $T_e$ . An alternative way to calculate  $\gamma_m$  is to use BOUT++'s Fourier filtering capabilities and run many simulations where each one filters out a different azimuthal mode. Then, we take the log of the envelope of one or several of the fluctuating quantities and calculate the slope of the line, which gives the growth rate for each particular

simulation. This procedure uses the time signal of the fluctuations rather than their spatial structure to calculate the growth rate, thus providing a check on the first method. The results using this alternative method for the periodic case are shown with the black diamonds in Fig. 5.1 a), which agree well with the curve calculated using the alternative energetic structure-based calculation. We do this check with all of the simulations to ensure consistency. This second method is more time consuming, so we only sample a few values of  $m$ . Furthermore, it's difficult to get growth rates when  $\gamma_m < 0$  using this second method.

The difference in the growth rate curves with the different boundary conditions is due to the different  $k_{\parallel} = \frac{2\pi n}{L_{\parallel}}$  where  $n$  is the axial mode number. The periodic simulation restricts  $n$  to integer values, while the Dirichlet and Neumann simulations allow for any fractional  $n$ . The largest growth rate occurs for  $n \sim 1/2$ . The Dirichlet and Neumann axial structures for the most unstable  $m$  mode, shown in Fig. 5.1 b), reflect this. The periodic simulation, which has  $n = 1$  structure, has a smaller growth rate, especially at low  $m$ . Note that in Fig. 5.1 b), the axial boundaries are not plotted. For instance, the zero-valued boundaries for the Dirichlet simulation are not shown. Also, the axial structures are taken at one random point in the  $r - \theta$  plane and at one time point, and are normalized to their maximum value.

### 5.1.2 Conducting Wall Mode

We now consider the linear instability that can exist in a plasma bounded by two conducting walls on the boundaries where the magnetic field lines terminate (the axial boundaries) [BRT91, BCR93, XRD93]. The instability is actually of the drift wave variety, but unlike the drift waves discussed above, the pressure-potential coupling mechanism is through the sheath boundary response rather than through the adiabatic response. The Bohm sheath boundary conditions that were derived in Sec. 4.2.2 can provide this coupling. As already noted, these boundary condi-

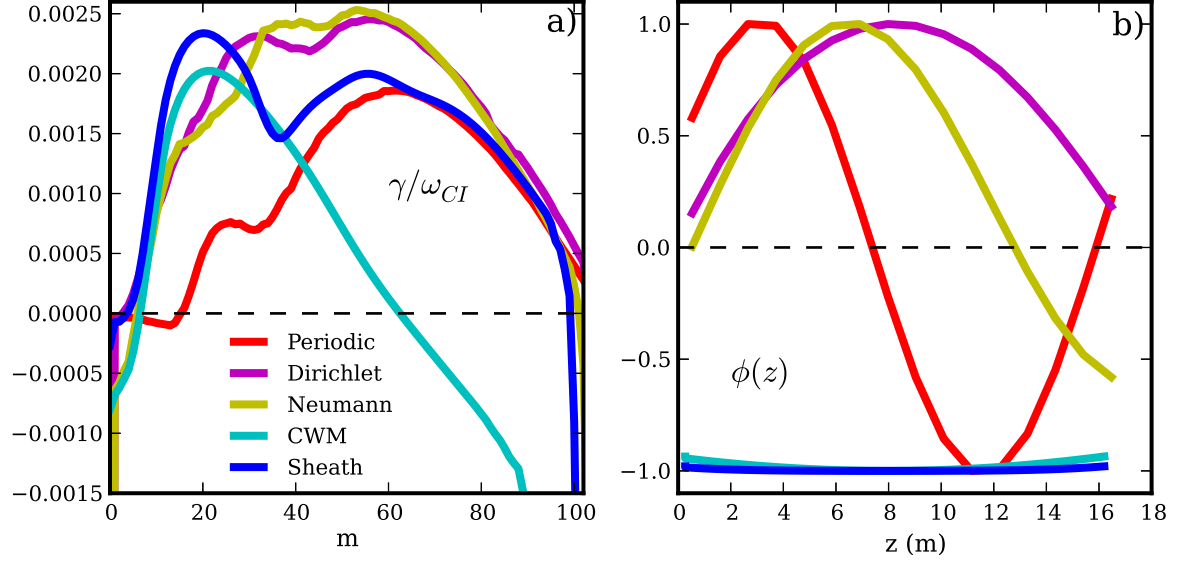


Figure 5.2: Linear conducting wall mode a) growth rates and b) axial structures

tions are not necessarily the correct ones for LAPD, but are somewhat idealized. Yet, it is still academically instructive to apply such an idealized boundary condition to LAPD because it creates this new linear instability, which can be used to test the robustness of LAPD's nonlinear instability.

The conducting wall mode (CWM) instability in the case considered here is purely an electron temperature gradient instability, although other types of gradients can cause it [BCR93]. Electron temperature fluctuations are advected by electrostatic potential fluctuations and feed off the equilibrium electron temperature gradient as in the case of the thermal drift waves. However, in contrast to the thermal drift waves, the coupling between the temperature and potential fluctuations comes through the sheath boundary condition rather than through the adiabatic response. Furthermore, the CWM can have (nearly)  $k_{\parallel} = 0$  flute-like behavior. The coupling mechanism is as follows: an electron temperature perturbation – say a positive constant fluctuation along a small flux tube – in-

creases the sound speed and the electron thermal speed on the flux tube. Since the ions must enter the Bohm sheath at the sound speed by being accelerated by a parallel electric field, the temperature increase must coincide with an increase in the parallel potential gradient as derived in Eq. 4.10. Additionally, the increased electron thermal speed causes an increase in the floating potential along the flux tube. These serve to couple the electron temperature to the potential.

The CWM can be isolated from the normal drift waves by removing the adiabatic response from the full LAPD equation set, and of course using the Bohm sheath boundary condition of Eq. 4.10. Removal of the adiabatic response in this case means removal of the  $\nabla_{\parallel} p_e$  and the  $0.71 \nabla_{\parallel} T_e$  terms in the parallel momentum equation (Eq. 4.2). This causes the density fluctuation  $N$  to become a passive scalar, so Eq. 4.1 can be removed as well with no consequence. So the isolated linear CWM equations are:

$$\partial_t v_{\parallel e} = \frac{m_i}{m_e} \nabla_{\parallel} \phi - \nu_e v_{\parallel e}, \quad (5.3)$$

$$\partial_t \varpi = -N_0 \nabla_{\parallel} v_{\parallel e} - \nu_{in} \varpi + \mu_{\phi} \nabla_{\perp}^2 \varpi, \quad (5.4)$$

$$\partial_t T_e = -\mathbf{v}_E \cdot \nabla T_{e0} + \frac{2}{3N_0} \kappa_{\parallel e} \nabla_{\parallel}^2 T_e - \frac{2m_e}{m_i} \nu_e T_e + \mu_T \nabla_{\perp}^2 T_e, \quad (5.5)$$

The CWM growth rate curve is shown in Fig. 5.2 a). The CWM is most unstable at values of  $m \sim 20$ , which is much lower than the  $m \sim 60$  values of the drift waves. Furthermore, the CWM maximum growth rate is about equal to the drift wave growth rates. And from Fig. 5.2 b), the CWM axial structure is flute-like ( $k_{\parallel} \simeq 0$ ). Finally, the growth rate curve of the full set of equations along with the sheath boundary condition is shown in this figure as the curve labeled “sheath.” This set of equations contains the drift wave and CWM instabilities. From both Figs. 5.2 a) and b), it is clear that the sheath simulation is dominated by the CWM at  $m \leq 20$ , which in fact is where the growth rate is maximum. At  $m \geq 40$ , the drift waves dominate.



## **5.2 LAPD Turbulence: A Visual Examination**

## **5.3 LAPD Turbulence: A Statistical Examination**

## CHAPTER 6

### Energy Dynamics Formalism

#### 6.1 Total Energy and Dynamics

#### 6.2 Spectral Energy Dynamics

## CHAPTER 7

### Nonlinear Instability for the Periodic Simulation

#### 7.1 The Energy Spectra

#### 7.2 Energy Dynamics Result

#### 7.3 $n=0$ Suppression

## CHAPTER 8

### Energy Dynamics for the Non-periodic Simulations

- 8.1 The Importance of Axial Boundary Conditions
- 8.2 Fourier Decomposing Non-periodic Functions
- 8.3 Energy Dynamics Results
- 8.4 Linear vs Nonlinear Structure Correlation

## CHAPTER 9

### Finite Mean Flow Simulations

9.1 The LAPD Biasing Experiment

9.2 New Linear Instabilities

9.3 Statistical Comparisons to Experiment

9.4 Energy Dynamics Results

## CHAPTER 10

### Conclusion

# APPENDIX A

## The BOUT++ Code

### A.1 The Object-Oriented Fluid Framework

### A.2 Explicit Finite Differences

### A.3 The Physics Inputs

## APPENDIX B

### Grid Convergence



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