Non-modal Growth in LAPD Turbulence

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Large Plasma Device (LAPD) [W. Gekelman *et al.*, Rev. Sci. Inst. **62**, 2875 (1991)]

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I. INTRODUCTION

Since the early 1990's, the hydrodynamics community has known that non-normality of the linear operator describing the system dynamics is a necessary condition for subcritical turbulence[1]. Non-normality is the condition that a linear operator or matrix does not commute with its adjoint. One consequence of non-normality is that the linear eigenvectors of the operator are nonorthogonal. In non-normal systems, even when all of the eigenvalues are stable (have negative growth rates), certain types of fluctuations in a system can access the free energy in the equilibrium gradients and transiently amplify the fluctuation energy in the system. A paradigmatic illustration of this transient growth process may be seen if Figure 2 of a review by Schmid[2]. Such behavior is obscured by traditional normal mode analysis, which in non-normal systems, only effectively describes the long time asymptotic behavior of fluctuations under the action of the linear operator. Transient events, such as those that can dominate nonlinear turbulent evolution, require initial-value (non-modal) calculations rather than normal mode calculations.

While the hydrodynamics community has published dozens of papers on non-modal analysis and transient growth over the past two decades, the plasma community has published only a few. The plasma community still heavily relies on normal mode analysis to inform turbulent predictions and observations. In particular, quasi-linear theory uses modal growth rates and cross-phases to predict turbulent saturation levels and turbulent transport.

II. THE SIMULATION MODEL

A Braginskii-based fluid model[3] is used to simulate global drift wave turbulence in LAPD using the BOUT++ code[4]. The evolved variables in the model are the plasma density, N, the electron fluid parallel velocity $v_{\|e}$, the potential vorticity $\varpi \equiv \nabla_\perp \cdot (N_0 \nabla_\perp \phi)$, and the electron temperature T_e. The ions are assumed cold in the model ($T_i = 0$), and sound wave effects are neglected ($v_i = 0$). Details of the simulation code, derivations of the model, grid convergence studies, and analyses of simplified models may be found in previously published LAPD simulation studies[5–9].

The equations are developed with Bohm normalizations: lengths are normalized to the ion sound gyroradius, times to the ion cyclotron time, velocities to the sound speed, densities

to the equilibrium peak density, and electron temperatures and potentials to the equilibrium peak electron temperature. These normalizations are constants (not functions of radius) and are calculated from these reference values: the magnetic field is 1 kG, the ion unit mass is 4, the peak density is 2.86×10^{12} cm^{-3}, and the peak electron temperature is 6 eV. The equations are:

$$\partial_t N = -\mathbf{v}_E \cdot \nabla N_0 - N_0 \nabla_\parallel v_{\parallel e} + \mu_N \nabla_\perp^2 N + S_N + \{\phi, N\}, \tag{1}$$

$$\partial_t v_{\parallel e} = -\frac{m_i}{m_e}\frac{T_{e0}}{N_0}\nabla_\parallel N - 1.71\frac{m_i}{m_e}\nabla_\parallel T_e + \frac{m_i}{m_e}\nabla_\parallel \phi - \nu_e v_{\parallel e} + \{\phi, v_{\parallel e}\}, \tag{2}$$

$$\partial_t \varpi = -N_0 \nabla_\parallel v_{\parallel e} - \nu_{in}\varpi + \mu_\phi \nabla_\perp^2 \varpi + \{\phi, \varpi\}, \tag{3}$$

$$\partial_t T_e = -\mathbf{v}_E \cdot \nabla T_{e0} - 1.71\frac{2}{3}T_{e0}\nabla_\parallel v_{\parallel e} + \frac{2}{3N_0}\kappa_{\parallel e}\nabla_\parallel^2 T_e$$
$$-\frac{2m_e}{m_i}\nu_e T_e + \mu_T \nabla_\perp^2 T_e + S_T + \{\phi, T_e\}. \tag{4}$$

In these equations, μ_N, μ_T, and μ_ϕ are artificial diffusion and viscosity coefficients used for subgrid dissipation. They are large enough to allow saturation and grid convergence[8], but small enough to allow for turbulence to develop. In the simulations, they are all given the same value of 1.25×10^{-3} in Bohm-normalized units. This is the only free parameter in the simulations. All other parameters such as the electron collisionality ν_e, ion-neutral collisionality ν_{in}, parallel electron thermal conductivity $\kappa_{\parallel e}$, and mass ratio $\frac{m_i}{m_e}$ are calculated from experimental quantities. There are two sources of free energy: the radial density gradient due to the equilibrium density profile N_0, and the equilibrium radial electron temperature gradient in T_{e0}, both of which are taken from experimental fits. The mean potential profile ϕ_0 is set to zero in the model, and terms involving ϕ_0 are not included in Eqs. 1-4. The justification for this is that biasable azimuthal limiters in LAPD allow for the mean $E \times B$ flow and flow shear to be varied with high precision, even allowing the flow to be nulled out[10]. The simulations in this paper use the N_0, T_{e0}, and ϕ_0 profiles from the nulled out flow experiment, justifying setting the mean potential profile to zero in the simulations.

Simulations also use density and temperature sources (S_n and S_T) in order to keep the equilibrium profiles from relaxing away from their experimental shapes. These sources suppress the azimuthal averages ($m = 0$ component of the density and temperature fluctuations) at each time step. The azimuthal average of the potential ϕ is allowed to evolve in the simulation, allowing zonal flows to form, although they are relatively unimportant to the turbulent

dynamics[9].

The terms in Poisson brackets are the $E \times B$ advective nonlinearities, which are the only nonlinearities used in the simulations. The numerical simulations use finite differences in all three dimensions and use cylindrical annular geometry ($12 < r < 40$ cm). The radial extent used in the simulation encompasses the region where fluctuations are above a few percent in the experiment. Therefore, the radial boundaries are fixed to zero value.

III. CONCLUSION

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