The Dynamics of Adverse Selection

Learning, Competition, and Liquidity

Brett Green
WashU Olin and FTG

2025 Finance Theory Group Summer School
University of Washington
June 28, 2025

Overview

Information asymmetries are ubiquitous

- Both in the real world (we think) and in the literature (undoubtedly)
- Used-cars, health insurance, labor markets, financial markets, real estate,...

Adverse selection can arise when the private information affects both parties value from transacting

- Riskier patients have a higher value for insurance and are also costlier to insure
- Startups with good projects are worth more to both VC and entrepreneur

Why do we care?

- Markets fail to realize (all) gains from trade

The Dynamics of Adverse Selection

- If you don't trade today, you *could* trade tomorrow.
  - Admati-Perry-Weiss critique–commitment power is the problem here
  - Can welfare increase with repeated opportunities to trade?
  - Lower types will be more anxious to trade (skimming property)
  - Can delay can serve as a useful screening device?
- Important considerations in dynamic models:
  - Delay itself inefficient
  - The information structure matters
  - If trade is delayed, new information may come to light
  - Buyers willingness to pay may depend on their ability to sell in the future

Plan for Today

Illustrate novel dynamics and welfare implications emerging from models with adverse selection

1. Review of Static Models
   - The Admati-Perry-Weiss Critique
   - Resolution(s) to the critique
2. Learning + Competition
   - A unification of Akerlof (1970) and Spence (1973)
   - More learning $\not\Rightarrow$ more efficient
   - Generalization of Coase (1972)
3. Demand for Liquidity
   - Endogenous source of volatility
   - Feedback effects

Some Related Literature

Static Benchmarks

- Akerlof (1970), Spence (1973), Rothschild and Sitglitz (1983), Leland Pyle (1977), Wilson (1980), Myers and Majluf (1984)

Static Models with News

- Levin (2001), Feltovich et al (2002), Alos Ferrer and Prat (2012), Daley Green (2014)

Dynamic Models

- Bargaining: FLT (1985), GSW (1986), Admati Perry (1987), Deneckere and Liang (2006), Fuchs Skrzypacz (2013)
- Competitive: Noldeke and Van Damme (1990), Swinkels (1999), Jannssen and Roy (2002), Horner Vielle (2009), Fuchs and Skrzypacz (2015), Fuchs et al (2015, 2016), Kim (2017)

Dynamic Models with Learning

- Bar-Isaac (2003), Kremer Skrzypacz (2007), Fuchs Skrzypacz (2010), Daley Green (2012, 2015, 2020), Kolb (2016, 2017), Strebulaev et al (2016), Geelen (2017), Kaya and Kim (2018), Asriyan Fuchs Green (2017)

Other Interesting Topics

Search/Segmentation Models

- Guerrieri, Shimer, and Wright (2010), Shimer Guerreri (2014), Kim (2015), Lester et al (2023), Lauermann Wolinsky (2016), Chiu Koeppel (2016), ...

Government Interventions and Market Design

- Tirole (2012) Phillipon and Skreta (2012) Carmargo Kim Lester (2015), Fuchs Skrzypacz (2016, 2019)

Asset Divisibility

- Gerardi Maestri Monzon (2022), Fuchs Gottardi Moreira (2023), Lee (2023)

Information Aggregation and Design

- Ostrovsky (2012), Golosov et al. (2014), Lauermann and Wolinsky (2017), Axelson and Makarov (2017), Babus and Kondor (2016), and Siga and Mihm (2018), Asriyan Fuchs Green (2022)

Amplification in Macro Eisfeldt (2004), Kurlat (2013), Bigio (2015), Li and Whited, 2015), Fuchs et al (2016)

The Basic Setting

Single seller with an indivisible asset of type $\theta \in \{L, H\}$, $\Pr(\theta = H) = p_0$

- Seller privately knows θ, buyers do not
- Seller flow value k_θ, buyers flow value v_θ, with $v_H > v_L$.
  - CKGT: $k_\theta < v_\theta$
  - Single-crossing: $k_H \geq k_L$
- Universal risk nuetrality, buyers have deep pockets, common discount rate r

We use this setup to review Akerlof and Spence. Then investigate their various dynamic counterparts.

Preliminaries

- Define: $K_\theta = \frac{k_\theta}{r}$ and $V_\theta = \frac{v_\theta}{r}$
- Payoffs:

$$\begin{aligned} \text{Seller:} \quad & e^{-rt}(w - K_\theta) \\ \text{Buyer:} \quad & e^{-rt}(V_\theta - w) \end{aligned}$$

- Buyer's maximal willingness to pay given belief $p_t = \Pr_t(\theta = H)$

$$V(p_t) \equiv \mathbb{E}_t(V_\theta) = p_t V_H + (1 - p_t) V_L$$

Akerlof's Market for Lemons

The extensive form (enhanced):

- There is a single date at which trade can occur.
- Multiple (> 2) buyers arrive and make-take-it-or-leave-it offers.

Akerlof's Market for Lemons

The extensive form (enhanced):

- There is a single date at which trade can occur.
- Multiple (> 2) buyers arrive and make-take-it-or-leave-it offers.

Typical parametric assumption: $K_H > V(p_0)$

- H won't sell for less than K_H.
- Any $w \geq K_H$ will lose money on average.
- Hence $w < K_H$. Zero profit implies $w = V_L$.

Prediction

- L sells for V_L w.p.1.
- H doesn't trade

$\implies$ Outcome is inefficient

Spence's Market Signaling

The extensive form:

- Seller can *commit* to any amount of costly delay before entertaining offers.
- Buyers observe seller's action ($t \in \mathbb{R}_+$).
- Buyers simultaneously make offers at date t.
- Seller decides which offer to accept (if any).

Spence's Market Signaling

Typical parametric assumptions: $K_H < V_L$ (no adverse selection problem)

Result

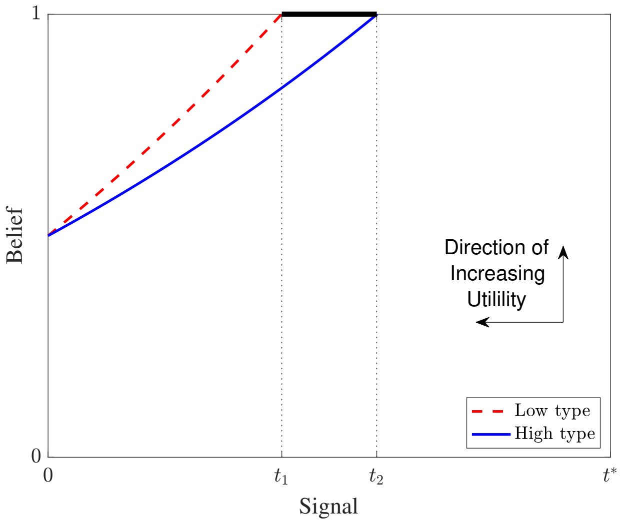
Under these conditions, the unique equilibrium outcome satisfying standard refinements is the least-cost-separating equilibrium.

- L *chooses* $t = 0$, $w(0) = V_L$
- H *chooses* t^*, $w(t^*) = V_H$, *where*

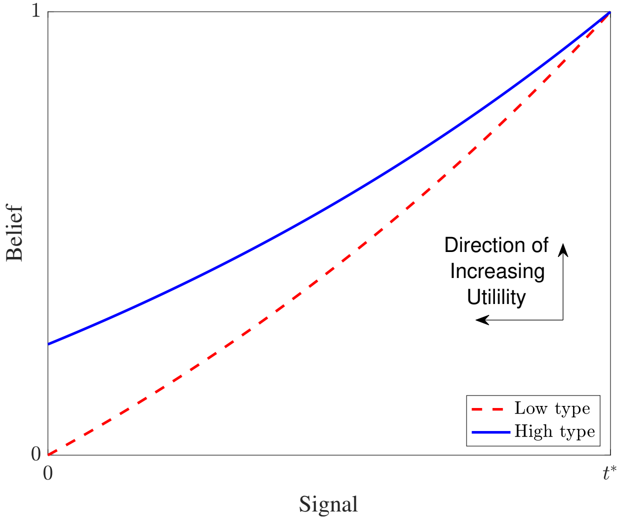
$$V_L = \int_0^{t^*} e^{-rs} k_L ds + e^{-rt^*} V_H$$

Intuition: from indifference curves

Why Refinements Select the LCSE



Why Refinements Select the LCSE



Akerlof vs Spence

The extensive form and the predictions of the two models are different:

- Akerlof - seller chooses whether to trade now or never
  - What prevents the seller from trading in the next period?
- Spence - seller can commit to any amount of costly delay
  - What stops buyers from making preemptive offers?

Akerlof vs Spence

The extensive form and the predictions of the two models are different:

- Akerlof - seller chooses whether to trade now or never
    - What prevents the seller from trading in the next period?
- Spence - seller can commit to any amount of costly delay
    - What stops buyers from making preemptive offers?

Observation

In a dynamic market (i.e., without commitment), these two settings are strategically equivalent

The Admati-Perry-Weiss Critique

Question: Are the predictions robust to a dynamic setting?

The Admati-Perry-Weiss Critique

Question: Are the predictions robust to a dynamic setting?

- Type is revealed by not selling at $t = 0$...
  - Buyers should snatch up any seller that remains for $V_H - \varepsilon$
  - L regrets decision to sell at $t = 0$
  - Equilibrium unravels in a dynamic setting

The Admati-Perry-Weiss Critique

Question: Are the predictions robust to a dynamic setting?

- Type is revealed by not selling at $t = 0$...
  - Buyers should snatch up any seller that remains for $V_H - \varepsilon$
  - L regrets decision to sell at $t = 0$
  - Equilibrium unravels in a dynamic setting

A related critique applies to

- Leland and Pyle (1977): type revealed by quantity sold at $t = 0$, why not sell more at $t = 1$?
- Meyers and Majluf (1984): decision to invest at $t = 0$ is a negative signal of assets in place, firms with high assets in place should invest at $t = 1$

A Dynamic Market

A reformulated version:

- Agent is interested in selling her asset but she is not forced to do so on any particular day
- If she does not sell today, she enjoys the flow value (or cost) retaining the asset
- And can entertain more offers tomorrow

The Details Matter

Some important details of our reformulated version are left unspecified:

- Nature of Competition among Buyers
  - Single or multiple buyers each period
  - Long or short-lived buyers
- Information Structure
  - Time on the market
  - Observability of offers
  - Other private or public signals

Nöldeke and van Damme (1990, ReStud)

- Discrete-time model $t \in \{0, \Delta, 2\Delta, ...\}$, let $\delta = e^{-r\Delta}$
- Multiple long-lived buyers make public offers in each period
- Signaling Environment: $K_H < V_L$
- Focus on sequential equilibrium satisfying the NWBR refinement

Nöldeke and van Damme (1990, ReStud)

For small Δ, NVD's equilibrium looks as follows:

- Buyers offer V_L at $t = 0$:
- H rejects, L accepts w prob σ such that posterior, p satisfies

$$V(p) = (1 - \delta)K_H + \delta V_H$$

- There is delay lasting in expectation N periods where

$$V_L = (1 - \delta^N)K_L + \delta^N \mathbb{E}_p[V_\theta]$$

  during which buyers make non-serious offers.
- After the delay, buyers offer $V(p)$ and the seller accepts.
- As $\Delta \to 0$, $\sigma, p \to 1$, and $N\Delta \to t^*$. We recover the LCSE!

Public offers + NWBR prevents preemption

After the initial rejection, the expected value of the asset is higher than V_L.

- Admati-Perry-Weiss critique applies: why don't buyers offer $w = \mathbb{E}_p(V_\theta) - \varepsilon$?

Public offers + NWBR prevents preemption

After the initial rejection, the expected value of the asset is higher than V_L.

- Admati-Perry-Weiss critique applies: why don't buyers offer $w = \mathbb{E}_p(V_\theta) - \varepsilon$?

Answer:

- Since $w > (1-\delta)K_L + \delta V_H$, only H could possibly benefit by rejecting w
- NWBR requires buyers to believe that only the high type will reject
- Rejecting such an offer is profitable for H (offer will be V_H tomorrow)
- But then making such an offer of w today is a losing proposition for buyers

Public offers + NWBR prevents preemption

After the initial rejection, the expected value of the asset is higher than V_L.

- Admati-Perry-Weiss critique applies: why don't buyers offer $w = \mathbb{E}_p(V_\theta) - \varepsilon$?

Answer:

- Since $w > (1-\delta)K_L + \delta V_H$, only H could possibly benefit by rejecting w
- NWBR requires buyers to believe that only the high type will reject
- Rejecting such an offer is profitable for H (offer will be V_H tomorrow)
- But then making such an offer of w today is a losing proposition for buyers

Key: *Because buyer deviations are observable, rejecting them can signal information.*

- Which in turn, inhibits buyer's willingness to make preemptive offers.

Swinkels (1999, ReStud)

- Discrete-time model $t \in \{0, \Delta, 2\Delta ...\}$
- Multiple short-lived buyers make private offers in each period
- Signaling Environment: $K_H < V_L$

Main Result

Suppose Δ is small enough. Then in any sequential equilibrium, buyers offer $w = \mathbb{E}_0(V_\theta)$ at $t = 0$ and the seller accepts w.p.1.

- Takeaway: preemptive private offers eliminate costly delay in signaling environments!

Proof Sketch

1. Seller's strategy characterized by reservation price: $r_t(\theta)$
   - Offers are private $\implies$ continuation value only depends on t
2. Along the equilibrium path, p_t is increasing (skimming)
   - For any distribution of offers in period t, H is more likely to reject.
3. Bertrand competition $\implies$ buyers make zero expected profits
4. Suppose $\tau > 0$ is the last period reached in equilibrium. Then $p_\tau = p_{\tau-\Delta}$.
   - If only L accepts at $\tau - \Delta$, $w_{\tau-\Delta} = V_L \ll w_\tau = V(p_\tau)$.
   - If H accepts with positive probability, buyer could incease $w_{\tau-\Delta}$ by ε and make positive profit.
5. $\tau = 0$.
   - If not, all workers reject at $\tau - \Delta$ with positive probability.
   - Offering $V(p_{\tau-\Delta}) - \epsilon$ attracts all sellers with positive probability and is profitable.

Public vs Private Offers in the Market for Lemons

Hörner and Vielle (2008, ECMA) compare public versus private offers with a sequence of short-lived buyers.

- With public offers, they show there is positive probability of trade in the first period and no trade thereafter.
- With private offers, trade happens with probability one but with delay.
- Equilibrium characterization is not trivial and the welfare ranking is not established within their continuous type space.
    - But quite simple in our two-type setting!

Hörner and Vielle (2008)

1. Public Offers
   - Initial offer is K_L. H rejects. L mixes so posterior is $\underline{p}$, where $V(\underline{p}) = K_H$.
   - All future buyers make non-serious offers $\implies$ impasse!
   - Sellers get their outside option. First buyer makes positive profit.

$$\mathcal{W}^{\text{public}} = (1-\mu)(V_L - K_L)\mathbb{P}(L \text{ trades})$$

Hörner and Vielle (2008)

1. Public Offers
   - Initial offer is K_L. H rejects. L mixes so posterior is $\underline{p}$, where $V(\underline{p}) = K_H$.
   - All future buyers make non-serious offers $\implies$ impasse!
   - Sellers get their outside option. First buyer makes positive profit.

$$\mathcal{W}^{\text{public}} = (1-\mu)(V_L - K_L)\mathbb{P}(L \text{ trades})$$

2. Private Offers
   - Initial offer is V_L. H rejects. L mixes so posterior is $\underline{p}$.
   - Future buyers mix between non-serious offers and K_H.
   - Expected time before K_H is offered makes L indifferent in the initial period.
   - $u_L = V_L$, $u_H = K_H$. Buyers make zero profit.

$$\mathcal{W}^{\text{private}} = (1-\mu)(V_L - K_L) > \mathcal{W}^{\text{public}}$$

Hörner and Vielle (2008)

1. Public Offers
   - Initial offer is K_L. H rejects. L mixes so posterior is $\underline{p}$, where $V(\underline{p}) = K_H$.
   - All future buyers make non-serious offers $\implies$ impasse!
   - Sellers get their outside option. First buyer makes positive profit.

$$\mathcal{W}^{\text{public}} = (1-\mu)(V_L - K_L)\mathbb{P}(L \text{ trades})$$

2. Private Offers
   - Initial offer is V_L. H rejects. L mixes so posterior is $\underline{p}$.
   - Future buyers mix between non-serious offers and K_H.
   - Expected time before K_H is offered makes L indifferent in the initial period.
   - $u_L = V_L$, $u_H = K_H$. Buyers make zero profit.

$$\mathcal{W}^{\text{private}} = (1-\mu)(V_L - K_L) > \mathcal{W}^{\text{public}}$$

Similar result in Fuchs, Öry, and Skrzypacz (2016)

Other Sources of Information

Thus far, we have focused on information revealed by

- Time on the market
- Past rejected offers (when public)

Other Sources of Information

Thus far, we have focused on information revealed by

- Time on the market
- Past rejected offers (when public)

In modern-day markets, there are many other channels through which uninformed agents can learn

- Home buyers: inspections, recent transactions, current listings (e.g., Zillow)
- Lenders: credit history, borrower performance (e.g., Square)
- Investors: firm performance, real-time news, trades, analysts reports, credit ratings...

Other Sources of Information

Thus far, we have focused on information revealed by

- Time on the market
- Past rejected offers (when public)

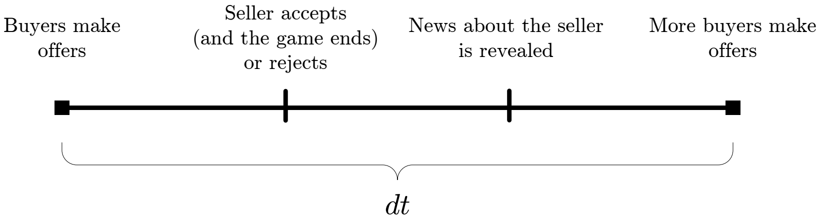
In modern-day markets, there are many other channels through which uninformed agents can learn

- Home buyers: inspections, recent transactions, current listings (e.g., Zillow)
- Lenders: credit history, borrower performance (e.g., Square)
- Investors: firm performance, real-time news, trades, analysts reports, credit ratings...

How do these other sources of information effect trade dynamics? What are the welfare implications?

Daley and Green (2012)

- Infinite-horizon, continuous-time setting
- At every t:
    - Competing buyers arrive and make (private) offers.
    - Owner decides which offer to accept (if any).
    - News is revealed about the asset.



Model: News

- Represented by a publicly observable process:

$$X_t(\omega) = \mu_\theta t + \sigma B_t(\omega)$$

  defined on $\{\Omega, \mathcal{H}, \mathcal{P}\}$ where B is standard B.M. and $\mu_H > \mu_L$

- The quality of the news is captured by the signal-to-noise ratio:

$$\phi \equiv \frac{\mu_H - \mu_L}{\sigma}$$

- Alternative: Poisson arrivals
  - Log-odds have linear drift
  - No news is good news vs bad news

Equilibrium objects

1. Offer process, $W = \{W_t : 0 \le t < \infty\}$
   - Alternative to modeling strategic buyers
2. Seller stopping times: τ^θ for each $\theta \in \{L, H\}$
   - Access to private randomization for mixing
   - Endows CDF over acceptance times: $\{S_t^\theta : 0 \le t \le \infty\}$
3. Buyers' belief process, $P = \{P_t : 0 \le t < \infty\}$

We construct an equilibrium that is stationary in the buyers' beliefs, i.e.,

- P is a time-homogeneous Markov process
  - Let $p = P_t$ denote the state
- Offer is a function that depends only on the state, $W_t = w(P_t)$

Equilibrium: Buyer and Seller Optimality

Seller Optimality: The seller faces a stopping problem:

$$F_\theta(p) \equiv \sup_\tau \mathbb{E}_p^\theta \left[e^{-r\tau} \left(w\left(p_t\right) - K_\theta \right) \right] \tag{SP$_\theta$}$$

Any $\tau^\theta \in \operatorname{supp}(S_\theta)$ must solve (SP$_\theta$).

Equilibrium: Buyer and Seller Optimality

Seller Optimality: The seller faces a stopping problem:

$$F_\theta(p) \equiv \sup_\tau \mathbb{E}_p^\theta \left[e^{-r\tau} \left(w\left(p_t\right) - K_\theta \right) \right] \tag{SP$_\theta$}$$

Any $\tau^\theta \in \text{supp}(S_\theta)$ must solve (SP$_\theta$).

Buyer Optimality: Motivated by "competitive" buyers we require

1. Zero profit. if a trade happens at date t, buyers earn zero profit.

$$w(p) = \mathbb{E}_p[V_\theta | \tau^\theta = t]$$

2. No deals. if trade does not happen at date t, there is no offer a buyer could make in which they would profit.

$$F_\theta(p) \geq \mathbb{E}_p(V_{\theta'} | V_{\theta'} \leq V_\theta)$$

- Both conditions hold in any PBE of a DT model with two buyers each period making private offers

Equilibrium: Belief Consistency

- At time t, buyers conditions on
  - (i) the path of the news,
  - (ii) seller rejected all past offers

- On path, we can use Bayes Rule to compute:

$$p_t = \frac{p_0 f_t^H(X_t)(1 - S_{t^-}^H)}{p_0 f_t^H(X_t)(1 - S_{t^-}^H) + (1 - p_0) f_t^L(X_t)(1 - S_{t^-}^L)}$$

- Off path, we impose belief monotonicity: beliefs do not decrease following an unexpected rejection.
  - Necessarily true on path (skimming property)

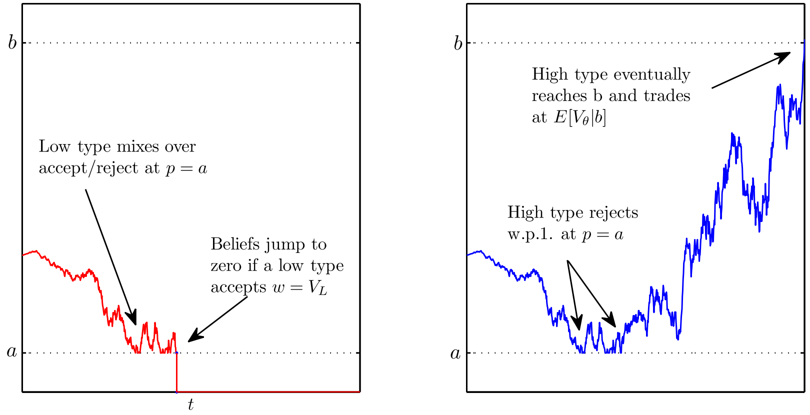
Equilibrium

Result

Assume $K_H > V_L$ (SLC). Then, there is a unique stationary equilibrium satisfying belief monotonicity. In it,

- *For $p_t \geq b$: buyers offer $V(p_t)$ and the seller accepts*
- *For $p_t < a$: buyers offer V_L, H rejects, and L mixes such that the posterior jumps to a*
- *For $p_t \in (a, b)$: there is no trade, both sides wait for news.*

Sample path of equilibrium play



A low type may eventually sell for V_L at $\mu = a$ (left), a high type never does (right)

Intuition for equilibrium play

1. H can get $V(p_t)$ whenever she wants it. For $p_t < b$, she does better by waiting for news.

2. For high enough p_t, H has little to gain by waiting, so exercises the option to trade at $V(p_t)$. The low type (happily) pools.

3. L can always get V_L. But for $p_t \in (a, b)$, he does better to mimic H.

4. L's prospects of reaching b decrease as p_t falls.
   - At $p_t = a$, she is indifferent $\implies$ willing to mix.

Useful Transformation

Define

$$Z_t = \underbrace{\ln\left(\frac{p_0}{1-p_0}\right) + \ln\left(\frac{f_t^H(X_t)}{f_t^L(X_t)}\right)}_{\hat{Z}_t} + \underbrace{\ln\left(\frac{1-S_{t^-}^H}{1-S_{t^-}^L}\right)}_{Q_t}$$

- By working with log-odds, we have linearized Bayesian updating (very convenient)
- Moreover, $\hat{Z}$ is linear in X

$$d\hat{Z}_t = \frac{\phi}{\sigma}\left(dX_t - \frac{\mu_H + \mu_L}{2}dt\right)$$

- The high type expects good news: $d\hat{Z}_t^H = \frac{\phi^2}{2} + \phi dB_t$ (submartingale)
- The low type expects bad news: $d\hat{Z}_t^L = -\frac{\phi^2}{2} + \phi dB_t$ (supermartingale)

Equilibrium Construction

To construct the equilibrium, consider the pair of stopping problems:

SP_H

- Z evolves according to $\hat{Z}$ with a reflecting barrier at some arbitrary α
- $w(z) = V(z)$
- For what values of z should the high type accept?

Equilibrium Construction

To construct the equilibrium, consider the pair of stopping problems:

SP_H

- Z evolves according to $\hat{Z}$ with a reflecting barrier at some arbitrary α
- $w(z) = V(z)$
- For what values of z should the high type accept?

SP_L

- Z evolves according to $\hat{Z}$
- The low type gets payoff $V(b)$ for some b.
- $w(z) = V_L$ for all $z < b$.
- For what values of z should the low type accept?

Illustration of Solution to $\mathbf{SP_H}$

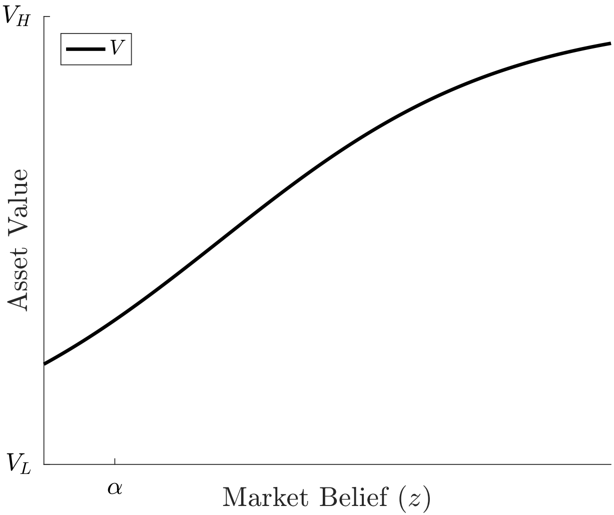


Illustration of Solution to $\mathbf{SP_H}$

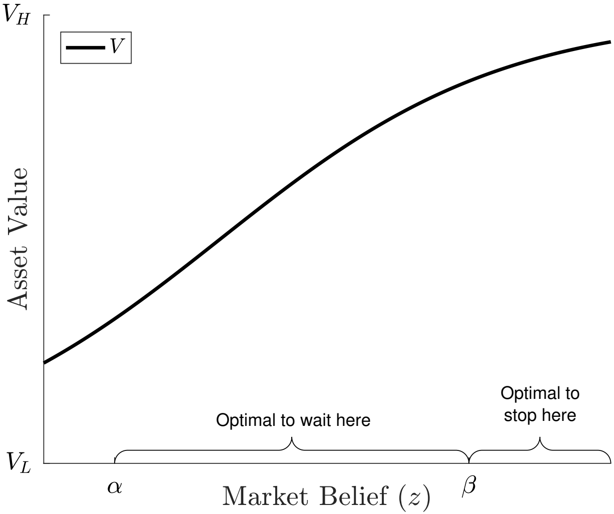
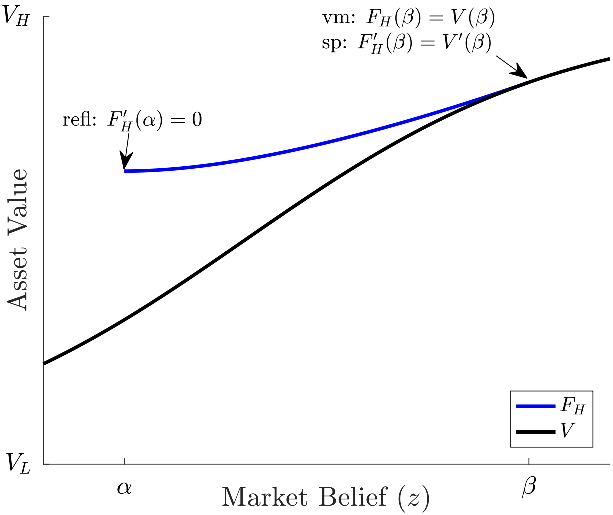


Illustration of Solution to SP_H



Analytic Solution to $\mathbf{SP_H}$

In the waiting region:

$$F_H(z) = E_z^H(e^{-\tau(\beta)}(V(\beta) - K_H)) \tag{1}$$

and $dZ = d\hat{Z}$, which means that $dZ_t^H = \frac{\phi^2}{2}dt + \phi dB_t$.

Analytic Solution to $\mathbf{SP_H}$

In the waiting region:

$$F_H(z) = E_z^H(e^{-\tau(\beta)}(V(\beta) - K_H)) \tag{1}$$

and $dZ = d\hat{Z}$, which means that $dZ_t^H = \frac{\phi^2}{2}dt + \phi dB_t$.

Apply Dynkin's formula to get that

$$rF_H(z) = \frac{\phi^2}{2}F_H'(z) + \frac{1}{2}\phi^2 F_H''(z)$$

which has solution of the form

$$F_H(z) = c_1 e^{q_1 z} + c_2 e^{q_2 z}$$

where $q_1, q_2 = \frac{1}{2}(-1 \pm \sqrt{1 + 8r/\phi^2})$.

- 3 unknowns $\{c_1, c_2, b\}$
- 3 boundary conditions

Solution to Stopping Problems

Lemma (Solution to $\mathrm{SP_H}$)

For any α, there exists a $\beta > \alpha$ such that the solution to SP_H is

$$T(b) = \inf\{t : Z_t \geq b\}$$

Solution to Stopping Problems

Lemma (Solution to $\mathrm{SP_H}$)

For any α, there exists a $\beta > \alpha$ such that the solution to SP_H is

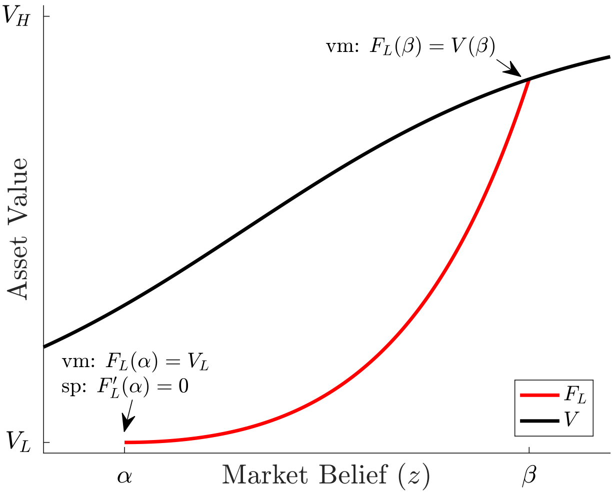
$$T(b) = \inf\{t : Z_t \geq b\}$$

Lemma (Solution to $\mathrm{SP_L}$)

For any b, there exists an $\alpha < \beta$ such that the solution to SP_L is

$$\tau(\alpha) = \inf\{t : Z_t \leq \alpha\}.$$

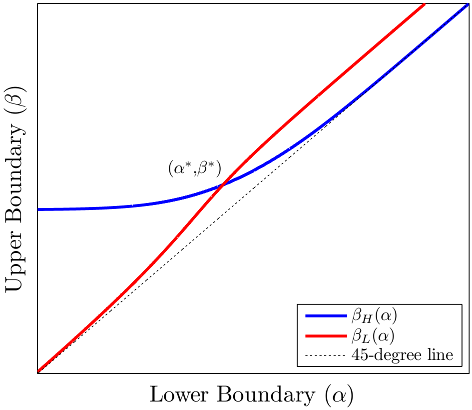
Illustration of Solution to $\mathrm{SP_L}$



Fixed Point

Result

There exists a unique (α^*, b^*) that simultaneously solves SP_H and SP_L.



Effect of News Quality

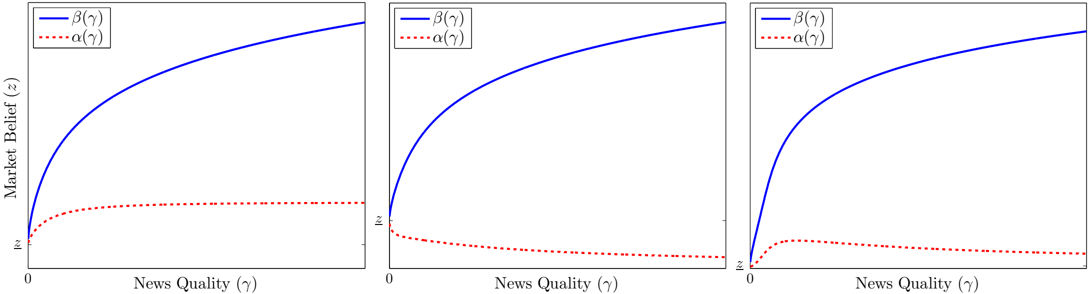
Result

As the quality of news (ϕ) increases

- *b increases*
- *$b - \alpha$ increases*
- *α may increase or decrease*
- *Efficiency increases for low beliefs but decreases for high beliefs*

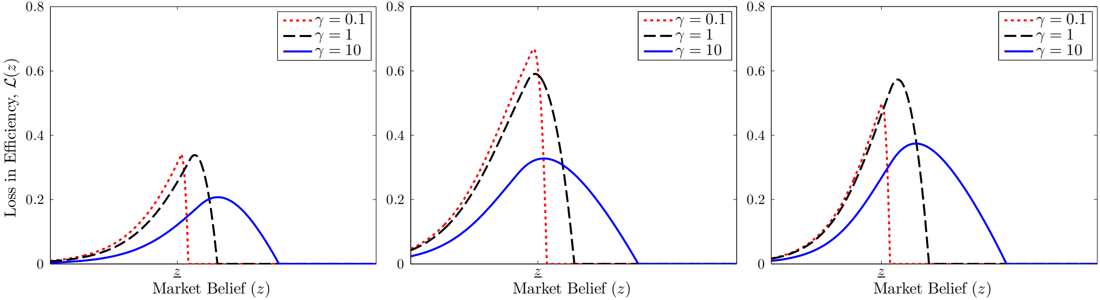
- More news $\not\Rightarrow$ more efficient
- Gives H more incentive to delay

Comparative Statics



(a) $K_L = -\frac{1}{2}$, $K_H = \frac{1}{4}$ (b) $K_L = -\frac{1}{10}$, $K_H = \frac{1}{4}$ (c) $K_L = -\frac{1}{20}$, $K_H = \frac{1}{20}$

Welfare Losses



(d) (a,ii) (e) (b,ii) (f) (c,ii)

Illustrates welfare losses for three different levels of news quality.

Without the SLC

Result

When the Static Lemons Condition does not hold (i.e., $K_H < V_L$), the equilibrium depends on the quality of the news:

Without the SLC

Result

When the Static Lemons Condition does not hold (i.e., $K_H < V_L$), the equilibrium depends on the quality of the news:

1. *When the news is sufficiently informative, there exists an equilibrium of the same (three-region) form.*

Without the SLC

Result

When the Static Lemons Condition does not hold (i.e., $K_H < V_L$), the equilibrium depends on the quality of the news:

1. *When the news is sufficiently informative, there exists an equilibrium of the same (three-region) form.*
    - *It is the unique eq. in which value functions are non-decreasing.*

Without the SLC

Result

When the Static Lemons Condition does not hold (i.e., $K_H < V_L$), the equilibrium depends on the quality of the news:

1. *When the news is sufficiently informative, there exists an equilibrium of the same (three-region) form.*
   - *It is the unique eq. in which value functions are non-decreasing.*

2. *When news is sufficiently uninformative, the unique equilibrium involves immediate trade for all p.*

Without the SLC

Result

When the Static Lemons Condition does not hold (i.e., $K_H < V_L$), the equilibrium depends on the quality of the news:

1. *When the news is sufficiently informative, there exists an equilibrium of the same (three-region) form.*
   - *It is the unique eq. in which value functions are non-decreasing.*

2. *When news is sufficiently uninformative, the unique equilibrium involves immediate trade for all p.*
   - *Same as with no news (Swinkels, 1999)*

Remarks

- Introduced gradual information revelation into a dynamic trading model with adverse selection
- Equilibrium involves three distinct regions
- More news is not necessarily better
- Accommodates both "lemons" and "signaling" environments.
    - In a dynamic market with news, Akerlof and Spence look the same!
- Next step
    - Analyze role of competition among buyers

Bargaining

To this point, we have focused on settings with competition among buyers.

- Suppose now that there is a single long-lived buyer

Starting Point: Coase (1972)

Absent commitment power ($\delta \to 1$), competition with ones future self is sufficient to stimulate the competitive outcome.

Bargaining

To this point, we have focused on settings with competition among buyers.

- Suppose now that there is a single long-lived buyer

Starting Point: Coase (1972)

Absent commitment power ($\delta \to 1$), competition with ones future self is sufficient to stimulate the competitive outcome.

Formalized by: Fudenberg, Levine and Tirole (1985), Gul, Sonenschein, and Wilson (1986), Deneckere and Liang (2006)

Bargaining

To this point, we have focused on settings with competition among buyers.

- Suppose now that there is a single long-lived buyer

Starting Point: Coase (1972)

Absent commitment power ($\delta \to 1$), competition with ones future self is sufficient to stimulate the competitive outcome.

Formalized by: Fudenberg, Levine and Tirole (1985), Gul, Sonenschein, and Wilson (1986), Deneckere and Liang (2006)

Result

Suppose that $\mathbb{E}_{p_0}(V_\theta) \geq K_H$, then as $\delta \to 1$, trade takes place immediately.

Bargaining for Lemons

Deneckere and Liang (2006) show that when $\mathbb{E}_{p_0}(V_\theta) < K_H$, there is real delay in equilibrium.

Result

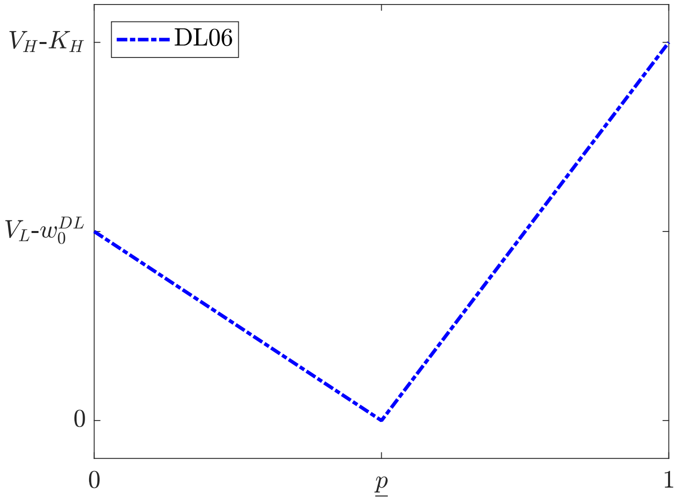
Suppose that $\mathbb{E}_{p_0}(V_\theta) < K_H$, then as $\delta \to 1$, the equilibrium converges to the following:

- *Initial offer is $w_0 = V_L^2/K_H < V_L$.*
- *Low type mixes such that posterior is μ^**
- *Buyer offers K_H after a period of delay τ satisfying $w_0 = \mathbb{E}(e^{-r\tau})K_H$*

Buyer's belief gets stuck at $\underline{p}$, which eliminates her temptation to speed up trade.

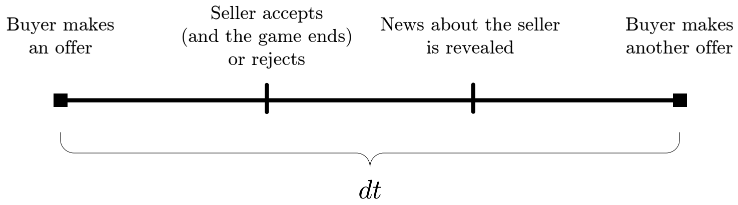
- This allows her to extract rents from low type sellers at $t = 0$.

Buyer Payoffs in DL06



Bargaining and News

Let's explore how the predictions change when we re-introduce "news"



Application: Due Diligence

Large transactions typically involve a due diligence period:

- Public M&A
- Private Equity Deals
- Commercial Real Estate

This information gathering stage is inherently dynamic.

Application: Due Diligence

Large transactions typically involve a due diligence period:

- Public M&A
- Private Equity Deals
- Commercial Real Estate

This information gathering stage is inherently dynamic.

Questions: How does the acquirer's ability to conduct due diligence and renegotiate the initial terms of sale influence

- Initial terms of sale? Eventual terms of sale?
- Likelihood of deal completion?
- Profitability of acquisition?

Samesies

- Seller problem (same as DG12)

$$F_{\theta)}(z) = \sup_{\tau} E_z^{\theta}\left[e^{-r\tau}\left(w\left(Z_\tau\right) - K_\theta\right)\right]$$

- Belief consistency (same as DG12)

$$Z_t = \underbrace{\ln\left(\frac{p_0}{1-p_0}\right) + \ln\left(\frac{f_t^H(X_t)}{f_t^L(X_t)}\right)}_{\hat{Z}_t} + \underbrace{\ln\left(\frac{1-S_{t^-}^H}{1-S_{t^-}^L}\right)}_{Q_t}$$

Buyer's problem

In any state z, the buyer essentially has three options:

1. Wait for news: Make a non-serious offer that is rejected
2. Screen: Make an offer $w < K_H$ that only the low type accepts with positive probability
3. Transact: Offer $w = K_H$ and buy w.p.1.

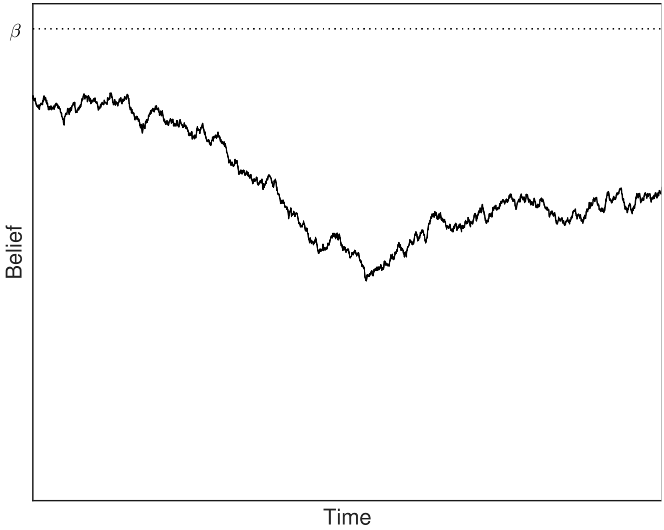
Equilibrium Characterization

Result

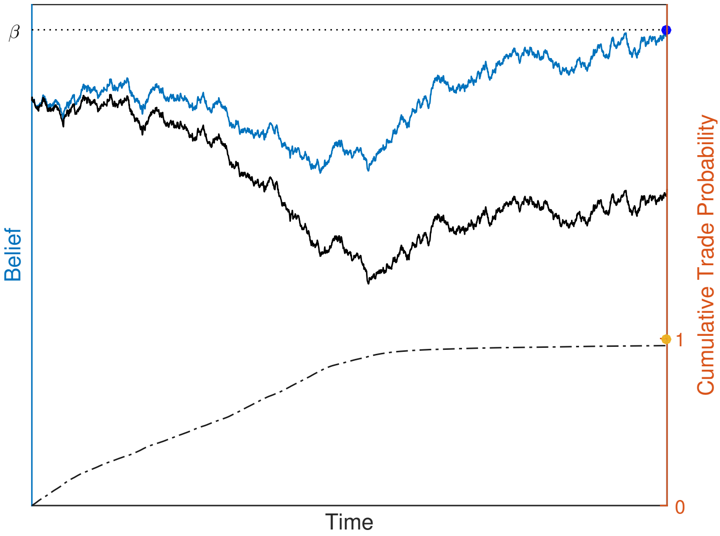
There exists a unique equilibrium.

- *For $p_t \geq b$, trade happens immediately: buyer offers K_H and both type sellers accept*
- *For $p_t < b$, trade happens "smoothly": only the low-type seller trades and with probability that is proportional to dt.*
    - *i.e., $dQ_t = \dot{q}(Z_t)dt$, where recall that $Z_t = \hat{Z}_t + Q_t$*

Equilibrium: sample path



Equilibrium: sample path



Equilibrium construction: sketch

1. Buyer's problem is linear in the rate of trade: $\dot{q}$
   - Derive F_B (independent of F_L)

Equilibrium construction: sketch

1. Buyer's problem is linear in the rate of trade: $\dot{q}$
   - Derive F_B (independent of F_L)

2. Given F_B, what must be true about F_L for smooth trade to be optimal?
   - Derive F_L, which implies w

Equilibrium construction: sketch

1. Buyer's problem is linear in the rate of trade: $\dot{q}$
   - Derive F_B (independent of F_L)
2. Given F_B, what must be true about F_L for smooth trade to be optimal?
   - Derive F_L, which implies w
3. Low type must be indifferent between waiting and accepting
   - Indifference condition implies $\dot{q}$ and therefore low-type acceptance rate.

Equilibrium construction: sketch

1. Buyer's problem is linear in the rate of trade: $\dot{q}$
   - Derive F_B (independent of F_L)
2. Given F_B, what must be true about F_L for smooth trade to be optimal?
   - Derive F_L, which implies w
3. Low type must be indifferent between waiting and accepting
   - Indifference condition implies $\dot{q}$ and therefore low-type acceptance rate.

Summary: Smooth $\implies F_B \implies F_L \implies \dot{q}$

A bit more about Step 1

For $z < b$,

$$rF_B(z) = \underbrace{\frac{\phi^2}{2}\left(2p(z) - 1\right)F_B'(z) + \frac{\phi^2}{2}F_B''(z)}_{\text{Evolution due to news}}$$

$$+ \dot{q}(z)\underbrace{\Big((1 - p(z))\big(V_L - F_L(z) - F_B(z)\big) + F_B'(z)\Big)}_{\Gamma(z)\text{=net-benefit of screening at } z}$$

A bit more about Step 1

For $z < b$,

$$rF_B(z) = \underbrace{\frac{\phi^2}{2}\left(2p(z)-1\right)F_B'(z) + \frac{\phi^2}{2}F_B''(z)}_{\text{Evolution due to news}}$$

$$+ \dot{q}(z)\underbrace{\Big((1-p(z))\big(V_L - F_L(z) - F_B(z)\big) + F_B'(z)\Big)}_{\Gamma(z)=\text{net-benefit of screening at } z}$$

- Buyer's value is linear in $\dot{q}$
- For "smooth" trade to be optimal, it must be that $\Gamma(z) = 0$
  $\rightarrow$ F_B does not depend on $\dot{q}$ (and has simple closed-form solution)

A bit more about Step 1

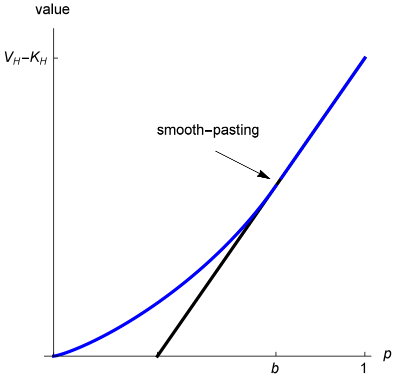
For $z < b$,

$$rF_B(z) = \underbrace{\frac{\phi^2}{2}\left(2p(z) - 1\right) F_B'(z) + \frac{\phi^2}{2} F_B''(z)}_{\text{Evolution due to news}}$$

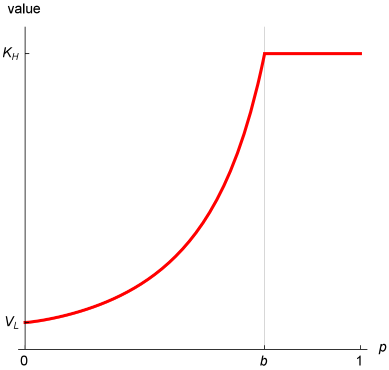
$$+ \dot{q}(z) \underbrace{\Big((1 - p(z))\big(V_L - F_L(z) - F_B(z)\big) + F_B'(z)\Big)}_{\Gamma(z)=\text{net-benefit of screening at } z}$$

- Buyer's value is linear in $\dot{q}$
- For "smooth" trade to be optimal, it must be that $\Gamma(z) = 0$
  - $\rightarrow$ F_B does not depend on $\dot{q}$ (and has simple closed-form solution)
- Therefore, buyer does not benefit from screening!
  - $\rightarrow$ Otherwise, she would want to trade "faster"
  - $\rightarrow$ Pins down exactly how expensive it must be to buy L: $F_L(z)$

Equilibrium payoffs

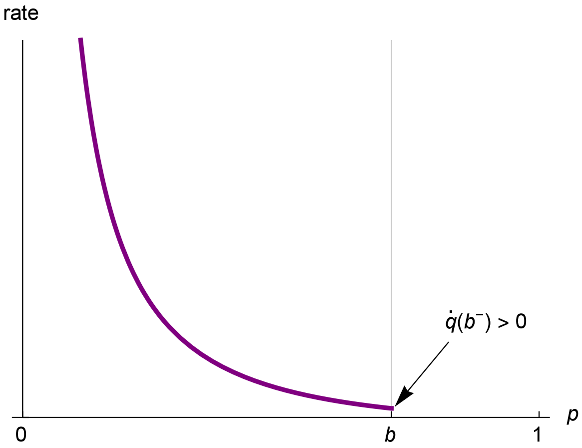


(a) Buyer value, F_B



(b) Low-type value, F_L

Equilibrium rate of trade



Interesting Predictions?

1. Buyer does not benefit from the ability to negotiate the price.
   - Though she *must* negotiate in equilibrium.
   - Her payoffs are the same as if the price was exogenously fixed at K_H.
   - Analogous to DeMarzo and He (2021) and DeMarzo and Urosevic (2006) (trading by a large shareholder)

Interesting Predictions?

1. Buyer does not benefit from the ability to negotiate the price.
   - Though she *must* negotiate in equilibrium.
   - Her payoffs are the same as if the price was exogenously fixed at K_H.
   - Analogous to DeMarzo and He (2021) and DeMarzo and Urosevic (2006) (trading by a large shareholder)
2. The buyer is guaranteed to lose money on any offer below K_H that is accepted.
   - A form of costly experimentation.

Who Benefits from the Negotiation?

Suppose the price is exogenously fixed at the lowest price that the seller will accept: K_H (e.g., initial terms of sale).

- The buyer conducts due diligence (observes $\hat{Z}$) and decides when and whether to actually complete the deal.
- Buyer's strategy is simply a stopping rule, where the expected payoff upon stopping in state z is

$$E_z[V_\theta] - K_H$$

- Call this the due diligence game.
  - NB: it is not hard to endogenize the initial terms.

Who Benefits from the Negotiation?

Result

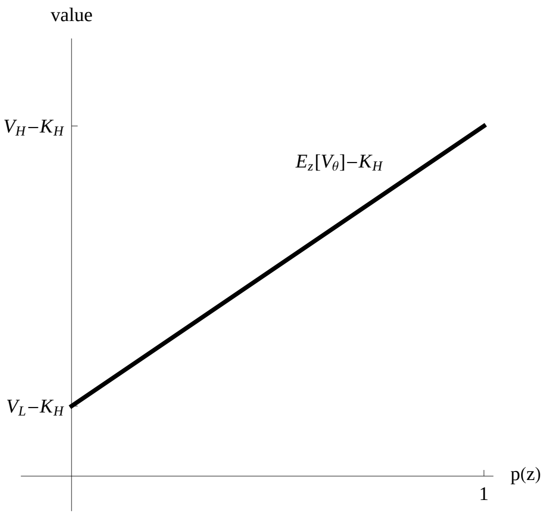
In the equilibrium of the bargaining game:

1. The buyer's payoff is identical to the due diligence game.
2. The (L-type) seller's payoff is higher than in the due diligence game.

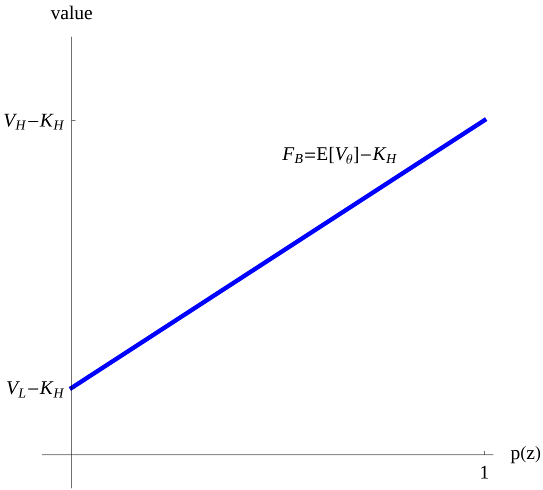
Total surplus higher with bargaining, but fully captured by seller.

- Despite the fact that the buyer makes all the offers.
- A manifestation of the Coasian force

No Lemons $\Longrightarrow$ No Learning



No Lemons $\implies$ No Learning



No Lemons $\implies$ No Learning

Result

When $V_L \geq K_H$, unique equilibrium is immediate trade at price K_H.

- Buyer's payoff is linear in p and her belief is a martingale.
- Absent a lemons condition, the Coasian force overwhelms the buyer's incentive to learn.

Effect of competition

Bilateral

Trade is efficient for $p \geq b_b$.

For $p < b_b$

- probability of trade is proportional to dt.
- rate is decreasing in p.

Competitive

Trade is efficient for $p \geq b_c$.

For $p < b_c$

- $p \in (a_c, b_c)$, complete trade breakdown.
- $p < a_c$, atom of trade

Effect of competition

Bilateral

Trade is efficient for $p \geq b_b$.

For $p < b_b$

- probability of trade is proportional to dt.
- rate is decreasing in p.

Competitive

Trade is efficient for $p \geq b_c$.

For $p < b_c$

- $p \in (a_c, b_c)$, complete trade breakdown.
- $p < a_c$, atom of trade

Result

Efficient trade requires higher belief in the competitive market: $b_b < b_c$

Difference in the efficient-trade threshold

Intuition?

- Agents differ in their expectations about the realization of future news.
- With competition, H decides when to "stop" and net $E_z[V_\theta] - K_H$.
- With bargaining, B decides when to "stop" and net $E_z[V_\theta] - K_H$.

Difference in the efficient-trade threshold

Intuition?

- Agents differ in their expectations about the realization of future news.
- With competition, H decides when to "stop" and net $E_z[V_\theta] - K_H$.
- With bargaining, B decides when to "stop" and net $E_z[V_\theta] - K_H$.
- *But H expects good news, while B does not.*

Difference in the efficient-trade threshold

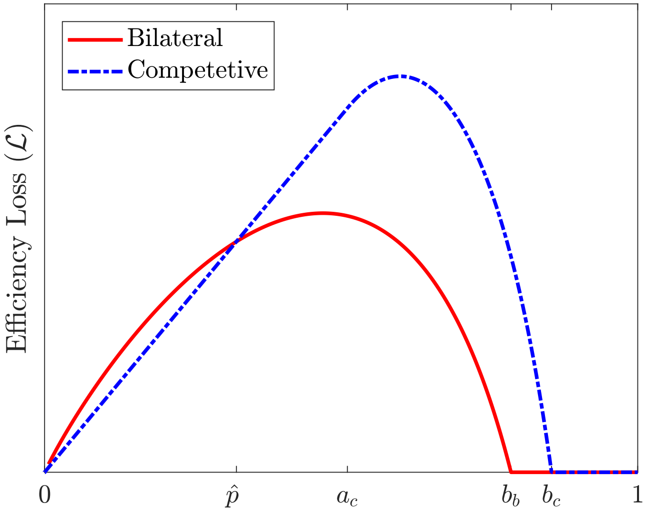
Intuition?

- Agents differ in their expectations about the realization of future news.
- With competition, H decides when to "stop" and net $E_z[V_\theta] - K_H$.
- With bargaining, B decides when to "stop" and net $E_z[V_\theta] - K_H$.
- *But H expects good news, while B does not.*

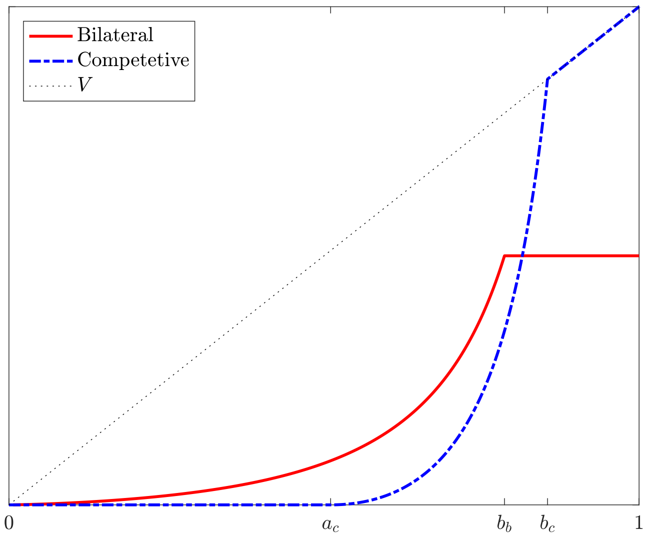
More generally: competition does not necessarily lead to more efficient outcomes in dynamic models with adverse selection

- Pushes prices up in the future $\implies$ more incentive to wait

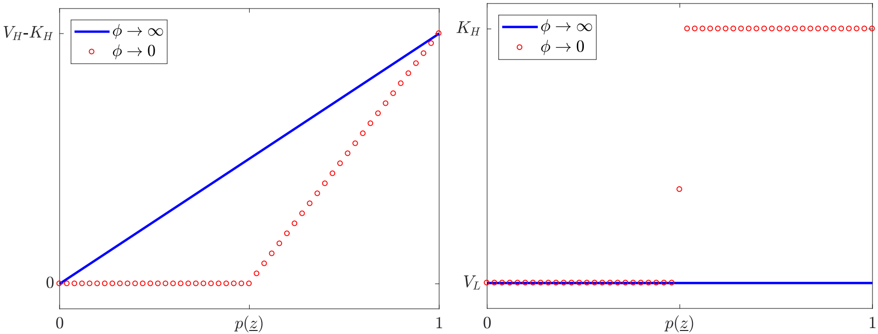
Efficiency: bilateral vs competitive



F_L: bilateral vs. competitive



Limiting payoffs



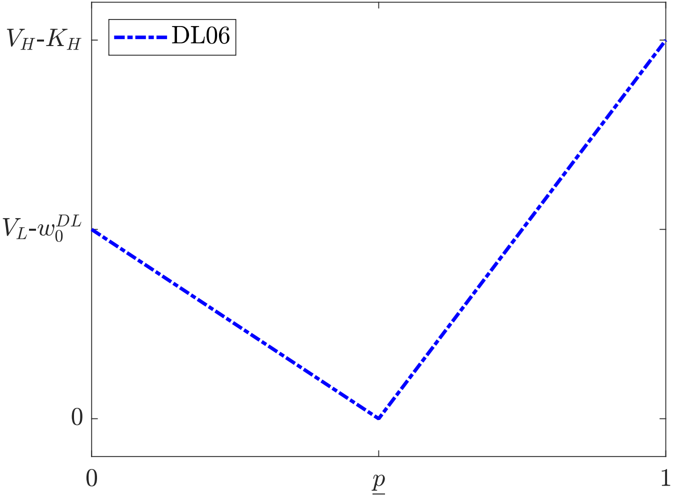
(c) Buyer payoff (d) Low type payoff

Effect of news

Our $\phi \to 0$ limit differs from Deneckere and Liang (2006)

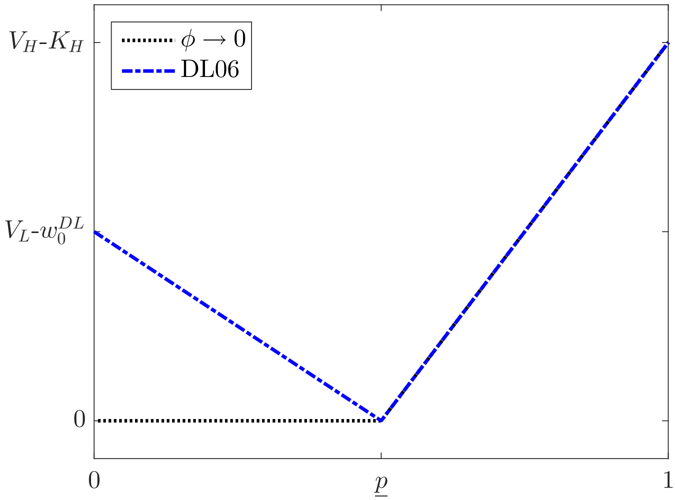
Effect of news

Our $\phi \to 0$ limit differs from Deneckere and Liang (2006)

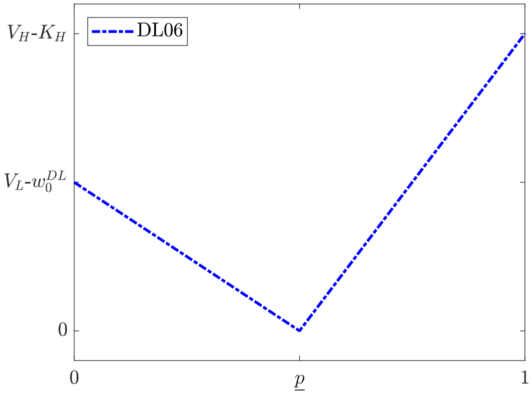


Effect of news

Our $\phi \to 0$ limit differs from Deneckere and Liang (2006)



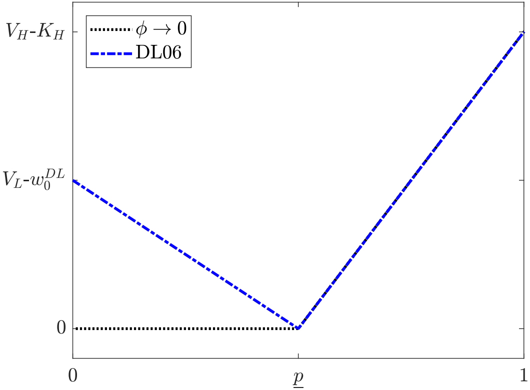
Effect of news



Intuition for DL06:

- Coasian force disappears at precisely $Z_t = \underline{z}$
- Buyer leverages this to extract concessions from low type at $z < \underline{z}$

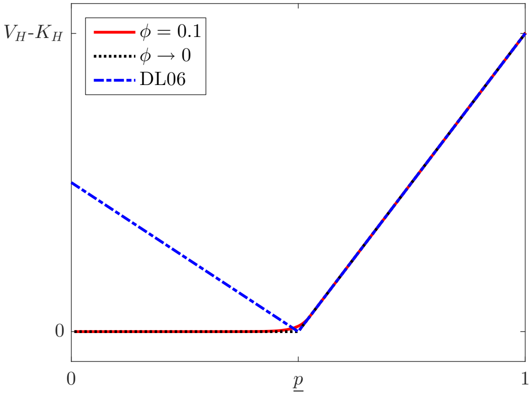
Effect of news



With news, his belief <u>cannot</u> just "sit at $\underline{z}$", so this power evaporates.

- Even with arbitrarily slow learning!

Effect of news



With news, his belief <u>cannot</u> just "sit at $\underline{z}$", so this power evaporates.

- Even with arbitrarily slow learning!

Uniqueness

Suppose there is some z_0 such that:

- Buyer makes offer w_0
- Low type accepts with atom

Let α denote the buyer's belief conditional on a rejection. Then

1. $F_L(z_0) = F_L(\alpha) = w_0$, by seller optimality
2. $F_L(z) = w_0$ for all $z \in (z_0, \alpha)$, by buyer optimality

Uniqueness

Suppose there is some z_0 such that:

- Buyer makes offer w_0
- Low type accepts with atom

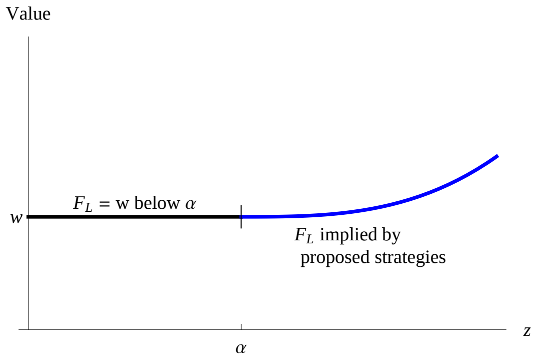
Let α denote the buyer's belief conditional on a rejection. Then

1. $F_L(z_0) = F_L(\alpha) = w_0$, by seller optimality
2. $F_L(z) = w_0$ for all $z \in (z_0, \alpha)$, by buyer optimality

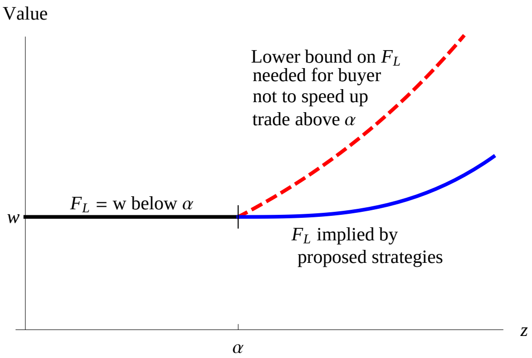
Therefore, starting from any $z \in (z_0, \alpha)$, the belief conditional on a rejection jumps to α.

- If there is an atom, the behavior must resemble the competitive-buyer model...

Why trade must be smooth with a single buyer



Why trade must be smooth with a single buyer



Intuitively,

- L is no more expensive to trade with at $z = \alpha + \epsilon$ than at $z = \alpha$.
- If the buyer wants to trade with L at price w below $z = \alpha$, he will want to extend this behavior above $z = \alpha$ as well.

Summary

We explore the effect of learning in a canonical bargaining environment with interdependent values

- Construct the equilibrium (in closed form).
- Buyer's ability to leverage news to extract surplus is remarkably limited.
    - Coasian force renders attempt to extract surplus through the price futile.
    - The robust implication of the Coasian force
- Relation to the competitive outcome
    - Competition eliminates the Coasian force, may reduce both total surplus and seller payoff.

Motivation: Liqidity Sentiments

Asset markets exhibit time variation in liquidity

- E.g., real estate, MBS, repo, merger waves, "physical" capital
- Liquidity/volume is procyclical, positively correlated with prices
- Often appears unrelated to new information or shocks to fundamentals
  - Usually interpreted as a 'behavioral' phenomenon: irrational exuberance, animal spirits, overconfidence, sentiments...

Question: Is there a fundamental link between prices and liquidity within a rational framework? Is there a role for sentiments?

Liquidity Sentiments

Model with asymmetric information and resale considerations

- Buyers worry about:
  1. Quality of assets for which they compete, and
  2. Liquidity they will face when trying to resell in the future.
- We show that **intertemporal complementarities** emerge
  - If buyers expect a liquid market tomorrow
    - They are willing to bid more aggressively for the assets today
    - Quality of assets that sellers willing to trade improves
    - Which leads to high liquidity and high prices today

Model

Discrete time, infinite horizon, $t = 0, 1, 2, ...$

Assets: Unit mass of assets indexed by $i \in [0, 1]$

- Asset i has (fixed) quality $\theta_i \in \{L, H\}$
- Fraction π of assets are high quality

Model

Discrete time, infinite horizon, $t = 0, 1, 2, ...$

Assets: Unit mass of assets indexed by $i \in [0, 1]$

- Asset i has (fixed) quality $\theta_i \in \{L, H\}$
- Fraction π of assets are high quality

Agents: Mass $M > 1$ of agents, indexed by $j \in [0, M]$

- Agents are risk-neutral with common discount factor δ
- Each agent can hold at most one unit of the asset
- Agent j at time t has private value or productivity $\omega_{j,t} \in \{l, h\}$
- Productivity is iid with $\lambda = P(\omega_{jt} = l)$

Flow Payoffs

If agent j owns asset i at date t:

- She receives a flow payoff $x_{ijt} = u(\theta_i, \omega_{jt})$
- High quality assets deliver higher payoff, $u(H, \omega) > u(L, \omega)$
- More productive agents generate higher payoff

$$\underbrace{v_\theta \equiv u(\theta, h) > c_\theta \equiv u(\theta, l)}_{\text{Gains from trade exist}}$$

Markets

Asset markets are competitive and decentralized. In each period:

- Multiple productive buyers bid for each asset à la Bertrand.
- Seller can accept an offer or reject and wait until the next period.
  - Buyer whose offer is accepted becomes asset owner
  - Owner who sells an asset becomes a buyer next period
- Trading is anonymous
  - History of asset or owner transactions is not observable
  - Rules out signaling through delay

Information friction

Absent frictions, outcome is efficient.

- Markets would reallocate assets from unproductive owners to productive non-owners (buyers).

Information friction

Absent frictions, outcome is efficient.

- Markets would reallocate assets from unproductive owners to productive non-owners (buyers).

But there is asymmetric information:

- Owner privately observes asset quality and productivity, (θ, ω).

Characterization of Stationary Equilibria

First characterize stationary equilibria in which the price is constant, p^*.

Result

In any stationary equilibrium,

$$V^*(L,l) = V^*(L,h) = p^* \leq V^*(H,l) < V^*(H,h).$$

Thus, (L,l)-owners always trade, whereas (H,h)-owners never do.

- Two candidate stationary equilibria, depending on whether (H,l)-owner trades.

Candidate stationary equilibria

Efficient trade equilibrium: (H, l)-owner trades

- All gains from trade are realized, prices and total output are:

$$p^{ET} = V^{ET}(H, l) \qquad Y^{ET} = E\{v_\theta\}$$

Candidate stationary equilibria

Efficient trade equilibrium: (H, l)-owner trades

- All gains from trade are realized, prices and total output are:

$$p^{ET} = V^{ET}(H, l) \qquad Y^{ET} = E\{v_\theta\}$$

Inefficient trade equilibrium: (H, l)-owner does not trade

- Some gains from trade are unrealized, prices and total output are:

$$p^{IT} < V^{IT}(H, l) \qquad Y^{IT} = E\{v_\theta\} - \underbrace{\lambda\pi(v_H - c_H)}_{\text{loss from misallocation}}$$

Multiplicity

Theorem

There exists two thresholds $\underline{\pi} < \bar{\pi}$ *such that:*

1. *Efficient trade is an equilibrium iff* $\pi \geq \underline{\pi}$*,*
2. *Inefficient trade is an equilibrium iff* $\pi \leq \bar{\pi}$*.*

Notably, both equilibria exist for $\pi \in (\underline{\pi}, \bar{\pi})$*.*

Multiplicity

Theorem

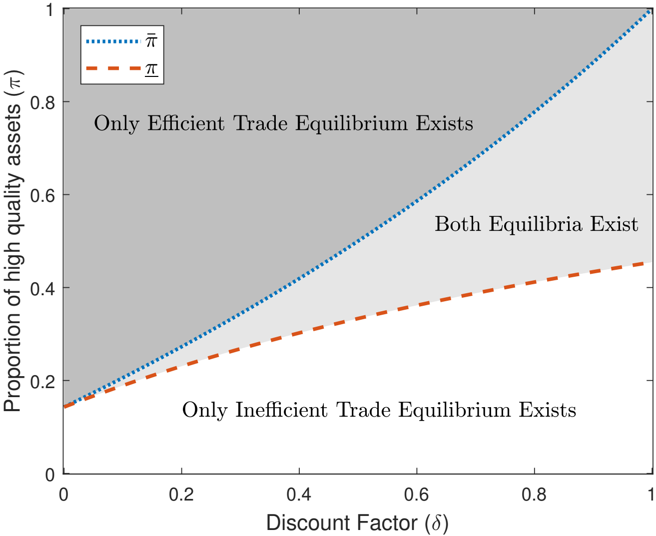
There exists two thresholds $\underline{\pi} < \bar{\pi}$ such that:

1. *Efficient trade is an equilibrium iff $\pi \geq \underline{\pi}$,*
2. *Inefficient trade is an equilibrium iff $\pi \leq \bar{\pi}$.*

Notably, both equilibria exist for $\pi \in (\underline{\pi}, \bar{\pi})$.

- Dynamic considerations are crucial for multiplicity.

Multiplicity and the role of dynamics



What is the source of multiplicity?

An intertemporal coordination problem:

- If buyers today expect future markets to be illiquid.
  - Their unconditional value today for an asset is low.
  - Hence the highest (pooling) price they are willing to offer is low.
  - At this low offer, the (H, l)-owners prefer to hold.

- Conversely, if buyers today expect future markets to be liquid.
  - Their unconditional value today for an asset is high.
  - Hence they are willing offer a high (pooling) price.
  - At this high price, the (H, l)-owners are willing to sell.

What is the source of multiplicity?

Efficient trade. Must be that (H, l)-owner does not want to reject:

$$V^{ET}(H, l) = p^{ET} \geq c_H + \delta E\{V^{ET}(H, \omega')\}$$

$$\underbrace{\hat{\pi} v_H + (1 - \hat{\pi}) v_L - c_H}_{\text{today's gain from selling}} \geq \underbrace{\delta (1 - \hat{\pi}) \overbrace{E\{V^{ET}(H, \omega) - V^{ET}(L, \omega)\}}^{\Delta^{ET}}}_{\text{future loss from selling at low price}}$$

What is the source of multiplicity?

Efficient trade. Must be that (H,l)-owner does not want to reject:

$$V^{ET}(H,l) = p^{ET} \geq c_H + \delta E\{V^{ET}(H,\omega')\}$$

$$\underbrace{\hat{\pi}v_H + (1-\hat{\pi})v_L - c_H}_{\text{today's gain from selling}} \geq \underbrace{\delta(1-\hat{\pi})\,\overbrace{E\{V^{ET}(H,\omega) - V^{ET}(L,\omega)\}}^{\Delta^{ET}}}_{\text{future loss from selling at low price}}$$

Inefficient trade. Sufficient to check that buyers do not want to deviate:

$$V^{IT}(H,l) \geq \hat{\pi}V^{IT}(H,h) + (1-\hat{\pi})\left(v_L + \delta E\{V^{IT}(L,\omega')\}\right)$$

$$\underbrace{\hat{\pi}v_H + (1-\hat{\pi})v_L - c_H}_{\text{today's gain from buying}} \leq \underbrace{\delta(1-\hat{\pi})\,\overbrace{E\{V^{IT}(H,\omega) - V^{IT}(L,\omega)\}}^{\Delta^{IT}}}_{\text{future loss from buying at high price}}$$

What is the source of multiplicity?

You might have noticed that it is actually the same condition, with the inequality reversed

What is the source of multiplicity?

You might have noticed that it is actually the same condition, with the inequality reversed...but

$$\Delta^{IT} > \Delta^{ET}$$

- High quality assets are relatively more valuable when assets are harder to trade.

Are there other equilibria?

Result

An equilibrium with deterministic transitions between efficient trade and inefficient trade generically does not exist.

Are there other equilibria?

Result

An equilibrium with deterministic transitions between efficient trade and inefficient trade generically does not exist.

Intuition?

- Suppose trade efficient at $t+1$ but inefficient at t

Are there other equilibria?

Result

An equilibrium with deterministic transitions between efficient trade and inefficient trade generically does not exist.

Intuition?

- Suppose trade efficient at $t+1$ but inefficient at t
- Then future market conditions are weakly better at t than at $t+1$
- Hence trade must also be efficient at t

Sentiment equilibrium

An equilibrium is a **sentiment equilibrium** with sunspot z_t if prices and allocations depend on its realization.

- Begin with a simple Markov family
  - Binary: $z_t \in \{B, G\}$.
  - Symmetric: $\rho = \mathbb{P}(z_{t+1} = B | z_t = B) = \mathbb{P}(z_{t+1} = G | z_t = G)$.
  - Candidate Equilibrium: play efficient trade iff $z_t = G$.

Sentiment equilibrium

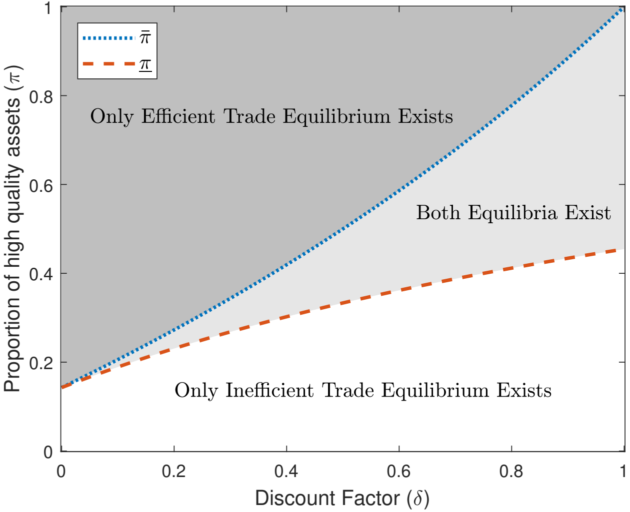
An equilibrium is a **sentiment equilibrium** with sunspot z_t if prices and allocations depend on its realization.

- Begin with a simple Markov family
  - Binary: $z_t \in \{B, G\}$.
  - Symmetric: $\rho = \mathbb{P}(z_{t+1} = B | z_t = B) = \mathbb{P}(z_{t+1} = G | z_t = G)$.
  - Candidate Equilibrium: play efficient trade iff $z_t = G$.

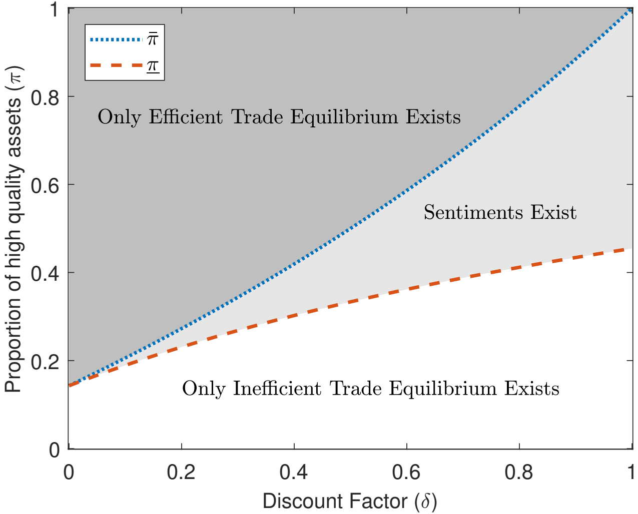
Result

A sentiment equilibrium with a binary-symmetric first-order Markov sentiment process z_t exists if and only if $\pi \in (\underline{\pi}, \bar{\pi})$ and $\rho \geq \bar{\rho}$, where $\bar{\rho}$ depends on parameters.

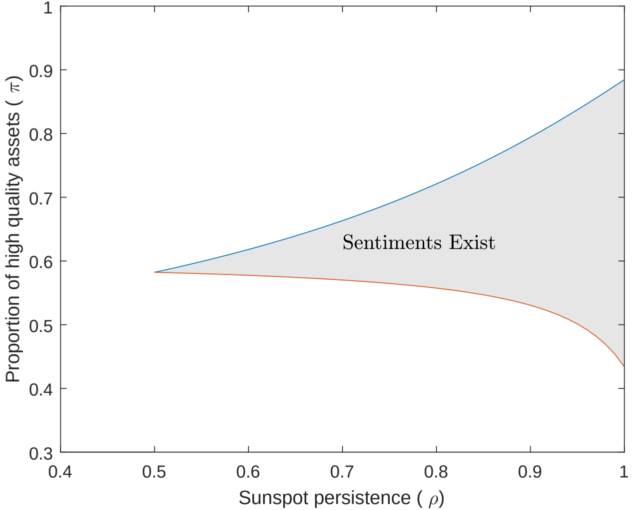
When do Sentiment equilibria exist?



When do Sentiment equilibria exist?



When do Sentiment equilibria exist?



Sentiments can be richer...

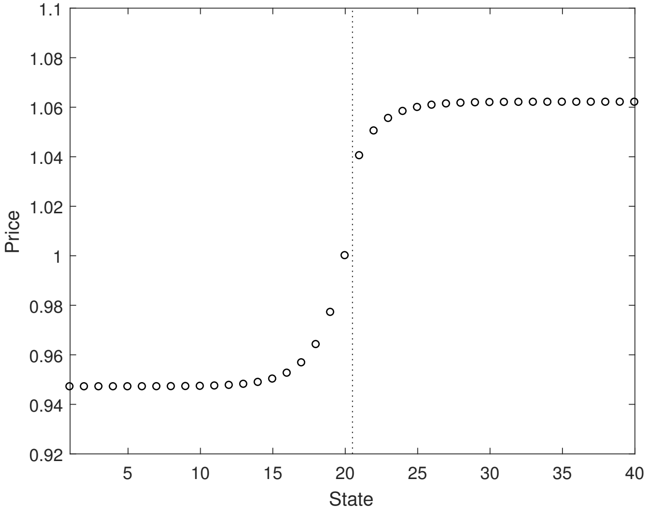
Example

- Sunspot process: Markov chain $z_t \in \{1, ..., N\}$
- Transition matrix: Q

$$Q = \left\{ \begin{matrix} \rho & 1-\rho & 0 & \dots & 0 \\ \frac{1-\rho}{2} & \rho & \frac{1-\rho}{2} & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & \frac{1-\rho}{2} & \rho & \frac{1-\rho}{2} \\ 0 & 0 & \dots & 1-\rho & \rho \end{matrix} \right\}$$

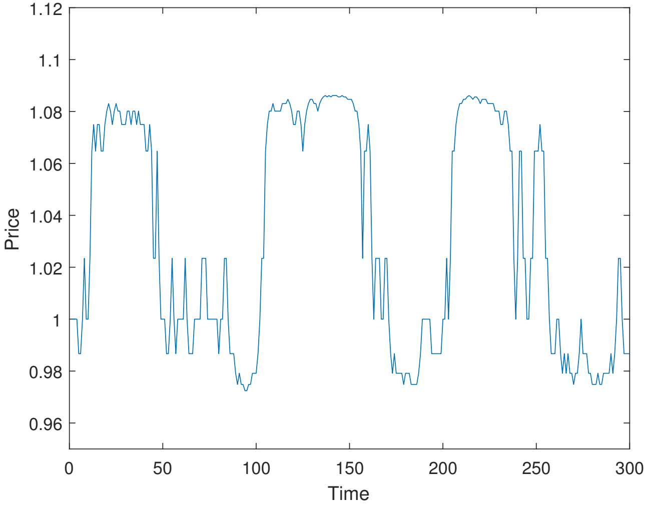
- Candidate Equilibrium: play efficient trade iff $z_t \geq n^* \in \{1, N\}$

Sentiments can be richer...



$N = 40,\ n^* = 20,\ \rho = 0.4$

Sentiments can be richer...



$$N = 40,\ n^* = 20,\ \rho = 0.4$$

Endogenous Production

Endogenous Production

- Producer choose how much to invest in capital quality q, at cost $c(q)$

The FOC for investment at time t is

$$c'(q_t) = \delta \underbrace{\left(E_t\{V^*_{t+1}(H,\omega) - V^*_{t+1}(L,\omega)\}\right)}_{\Delta^*_{t+1}}$$

Endogenous Production

Endogenous Production

- Producer choose how much to invest in capital quality q, at cost $c(q)$

The FOC for investment at time t is

$$c'(q_t) = \delta \underbrace{\left(E_t\{V^*_{t+1}(H,\omega) - V^*_{t+1}(L,\omega)\}\right)}_{\Delta^*_{t+1}}$$

And Δ^* is lower when liquidity sentiments are higher (e.g., $z_t = G$)

- Implication: If a sentiment equilibrium exists, then lower quality assets will be produced in "good" times.

Sentiments with endogenous production?

Result

When asset production is endogenous:

- *Efficient trade is an equilibrium* $\iff c'(\underline{\pi}) \leq \underline{c} \equiv \Delta_{ET}(\underline{\pi})$
- *Inefficient trade is an equilibrium* $\iff c'(\bar{\pi}) \geq \bar{c} \equiv \Delta_{IT}(\bar{\pi})$

Sentiments with endogenous production?

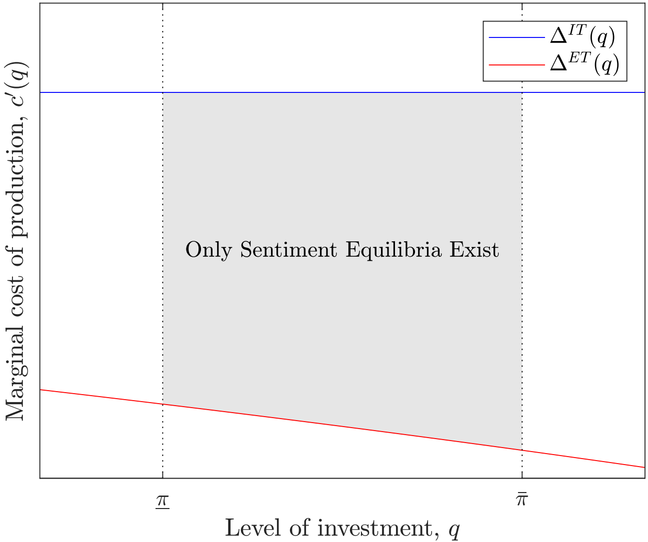
Result

When asset production is endogenous:

- *Efficient trade is an equilibrium* $\iff c'(\underline{\pi}) \leq \underline{c} \equiv \Delta_{ET}(\underline{\pi})$
- *Inefficient trade is an equilibrium* $\iff c'(\bar{\pi}) \geq \bar{c} \equiv \Delta_{IT}(\bar{\pi})$

Otherwise, any equilibrium must involve sentiments.

Illustrating the Result



Applications

Capital Reallocation

- Good times ($z_t = G$): higher output and productivity, only efficient firms operate capital, higher rates of capital reallocation.
- Bad times ($z_t = B$): lower output and productivity, some inefficient firms operate, lower rate of capital reallocation.

Real Estate

- Boom ($z_t = G$): high prices and volume, low time on the market.
- Bust ($z_t = B$): low prices and volume, high time on the market.

Remarks

- Adverse selection + resale considerations leads to an inter-temporal coordination problem:
  - Multiple self-fulfilling equilibria exist.
- Sentiments: expectations about future market conditions, generate endogenous volatility in prices, liquidity, output, etc.
  - The model disciplines set of possible sentiment dynamics.
  - Must be stochastic and sufficiently persistent.
- With endogenous asset production:
  - Sentiments are necessary for intermediate production costs.
  - Quality of assets produced is better in “bad” times.
- Application to capital reallocation and real estate markets.

Conclusion

Dynamic adverse selection $\implies$ rich dynamics

- Static predictions are not robust to repeated trade environments
- Liquidity can dry up after bad news
- Endogenous source of volatility
- Much more work to be done
  - Endogenous information acquisition
  - Empirical Identification
  - Optimal interventions/regulation

Dynkin's Formula for Discounted Hitting Times

Setup:

- Let X_t solve the SDE:
$$dX_t = \mu(X_t)\,dt + \sigma(X_t)\,dW_t$$
- Let $u(x)$ be twice continuously differentiable ($u \in C^2$)
- Let τ be a stopping time, and $r \geq 0$ a discount rate

Dynkin's Formula:

$$\mathbb{E}_x\left[e^{-r\tau}u(X_\tau)\right] = u(x) + \mathbb{E}_x\left[\int_0^\tau e^{-rs}\left(\mathcal{L}u(X_s) - ru(X_s)\right)ds\right]$$

where the generator is:

$$\mathcal{L}u(x) = \mu(x)u'(x) + \frac{1}{2}\sigma^2(x)u''(x)$$

Use cases:

- Derive ODEs for expected discounted hitting times
- Solve optimal stopping and control problems