On the Design of Stable, High Performance Sigma Delta Modulators

by

Brett Christopher Hannigan

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The following individuals certify that they have read, and recommend to the Faculty of Graduate and Postdoctoral Studies for acceptance, the thesis entitled:

On the Design of Stable, High Performance Sigma Delta Modulators

submitted by **Brett Christopher Hannigan** in partial fulfillment of the requirements for the degree of **Master of Applied Science** in **Biomedical Engineering**.

Examining Committee:

Guy Dumont, Electrical and Computer Engineering Supervisor

Abstract

This document provides brief instructions for using the ubcdiss class to write a UBC!-conformant dissertation in LaTeX. This document is itself written using the ubcdiss class and is intended to serve as an example of writing a dissertation in LaTeX. This document has embedded URL!s (URL!s) and is intended to be viewed using a computer-based PDF! (PDF!) reader.

Note: Abstracts should generally try to avoid using acronyms.

Note: at **UBC!** (**UBC!**), both the **GPS!** (**GPS!**) Ph.D. defence programme and the Library's online submission system restricts abstracts to 350 words.

Lay Summary

The goal of this work was a method to better design analog-to-digital converters with special interest to recording weak bio-signals, such as those from electroencephalography and electrocardiography.

The sigma delta architecture of analog-to-digital converters is known for having high resolution for signals of this class while requiring fewer expensive analog circuit components. However, as its performance is increased, it tends to become unstable, a point at which the digitized signal no longer accurately represents the original.

To this end, a theory and set of software tools were developed that use mathematical optimization and control theory to design sigma delta circuits with varying degrees of performance and stability. It is even possible to generate a design that is guaranteed to be stable. The method is generalizable to any kind of signal, medical or otherwise. These developments were used to analyze and synthesize designs and will hopefully inspire future high-resolution analog-to-digital converters.

Preface

At **UBC!**, a preface may be required. Be sure to check the **GPS!** guidelines as they may have specific content to be included.

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List of Symbols

 \boldsymbol{a} The number of angels per unit area.

Glossary

A/D analog-to-digital.

AAF antialiasing filter.

CT continuous-time.

D/A digital-to-analog.

DT discrete-time.

ECG electrocardiography.

EEG electroencephalography.

OSR oversampling ratio.

PPG photoplethysmography.

Acknowledgments

I would like to thank my supervisor Prof. Guy Dumont for his support of my research and openness to help as well as the other members of the BC Children's Hospital Research Institute's Digital Health Innovation Lab team.

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Chapter 1

Introduction

The conversion of signals between analog and digital domanis is an often encountered problem in signal processing. For an analog signal to be represented digitally, it must undergo the processes of sampling and quantization. The former is the conversion from continuous-time (CT) to discrete-time (DT) and can be done without loss of information by the Nyquist-Shannon sampling theorem, given a sufficiently high sample rate. The latter is the mapping from an infinite set of possible values to a finite number of quantization levels. Unlike sampling, the process of quantization is non-injective and thus irreversible. The design of signal conversion circuits that minimize the error introduced by quantization is a major problem in mixed signal electronics.

Sigma delta modulation is a widely used technique for analog-to-digital (A/D) and digital-to-analog (D/A) conversion of signals that provides high resolution through the techniques of oversampling and noise shaping. Oversampling trades throughput for resolution, thus the sigma delta modulator generally lies between integrating converters, which are specialized for near-dc signals, and high-speed architectures, such as successive approximation and flash. The sigma delta quantization scheme is especially applicable to signals with low to moderate frequency content. Signals with these properties include most biosignals such as those recorded electrically (EEG, electrocardiography (ECG)) or through other means using transducers (photoplethysmography (PPG)), as well as audio signals.

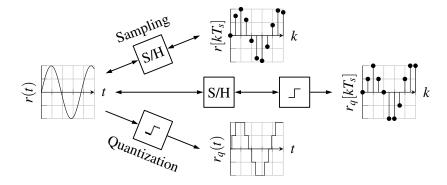


Figure 1.1: A continuous-time, continuous-value signal r(t) is sampled to produce a discrete-time, continuous-value signal $r[kT_s]$. r(t) independently undergoes quantization to yield a continuous-time, discrete-value signal $r_q(t)$. When both processes are applied in sequence, a discrete-time, discrete-value signal $r_q[kT_s]$ is the result.

1.1 Oversampling and Noise Shaping

Oversampling is simply the process where the analog signal is sampled at a rate higher than what the sampling theorem would dictate for perfect reconstruction, expressed as the oversampling ratio (OSR) relative to the Nyquist frequency. It may seem that this does not have a direct benefit *per se*, but it allows a less demanding analog antialiasing filter (AAF) to be used, saving circuit area. It also permits the quantization error to be spread across a larger bandwidth, reducing the in-band quantization noise power by a factor directly proportional to OSR. These two advantages — reducing analog circuit complexity and increasing resolution — are common goals in sigma delta modulator design.

It may appear that oversampling alone quickly becomes impractical because one must approach very high sampling frequencies to increase the signal-to-quantization-noise ratio (SQNR) substantially. However, this assumes that the quantization noise is evenly distributed across the spectrum. Noise shaping is the use of a filter operating on the oversampled signal to push quantization noise out of the signal band implemented by wrapping the quantizer in a feedback loop.

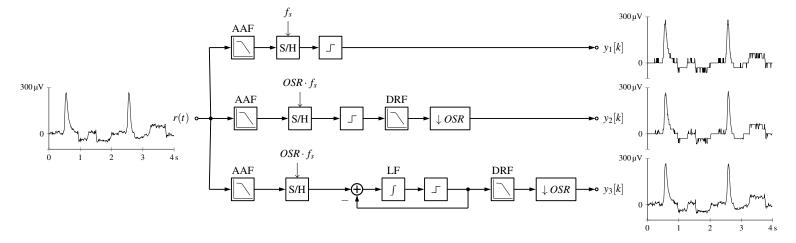


Figure 1.2: A comparison between naïve quantization (top), 10 times oversampled quantization (middle), and first order sigma delta modulation (bottom). The graphs on the right show the increasing quality of an EEG signal sampled to a final rate of 100 Hz and quantized to 5 bits by each scheme.

1.2 Related Works

1.3 Organization of this Thesis

a

This document provides a quick set of instructions for using the ubcdiss class to write a dissertation in LaTeX. Unfortunately this document cannot provide an introduction to using LaTeX. The classic reference for learning LaTeX is ? 's book [?]. There are also many freely-available tutorials online; Andy Roberts' online LaTeX tutorials seems to be excellent. The source code for this document, however, is intended to serve as an example for creating a LaTeX version of your dissertation.

We start by discussing organizational issues, such as splitting your dissertation into multiple files, in Section 1.4. We then cover the ease of managing cross-references in LATEX in Section 1.5. We cover managing and using bibliographies

¹ http://www.andy-roberts.net/misc/latex/

with BibTeX in Section 1.6. We briefly describe typesetting attractive tables in Section 1.7. We briefly describe including external figures in Section 1.8, and using special characters and symbols in Section 1.9. As it is often useful to track different versions of your dissertation, we discuss revision control further in Section 1.11. We conclude with pointers to additional sources of information in Section 1.13.

1.4 Suggested Thesis Organization

The specifies a particular arrangement of the components forming a thesis.² This template reflects that arrangement.

In terms of writing your thesis, the recommended best practice for organizing large documents in LATEX is to place each chapter in a separate file. These chapters are then included from the main file through the use of \include{file}. A thesis might be described as six files such as intro.tex, relwork.tex, model.tex, eval.tex, discuss.tex, and concl.tex.

We also encourage you to use macros for separating how something will be typeset (e.g., bold, or italics) from the meaning of that something. For example, if you look at intro.tex, you will see repeated uses of a macro \file{} to indicate file names. The \file{} macro is defined in the file macros.tex. The consistent use of \file{} throughout the text not only indicates that the argument to the macro represents a file (providing meaning or semantics), but also allows easily changing how file names are typeset simply by changing the definition of the \file{} macro. macros.tex contains other useful macros for properly typesetting things like the proper uses of the latinate *exempli gratiā* and *id est* (i.e., \eg and \ie), web references with a footnoted URL! (\webref{url}{text}), as well as definitions specific to this documentation (\latexpackage{}).

1.5 Making Cross-References

LATEX make managing cross-references easy, and the hyperref package's \autoref { } command³ makes it easier still.

²See http://www.grad.ubc.ca/current-students/dissertation-thesis-preparation/order-components

³The hyperref package is included by default in this template.

A thing to be cross-referenced, such as a section, figure, or equation, is *labelled* using a unique, user-provided identifier, defined using the \label{} command. The thing is referenced elsewhere using the \autoref{} command. For example, this section was defined using:

```
\section{Making Cross-References} \label{sec:CrossReferences}
```

References to this section are made as follows:

```
We then cover the ease of managing cross-references in \LaTeX\ in \autoref\{sec: CrossReferences\}.
```

\autoref{} takes care of determining the *type* of the thing being referenced, so the example above is rendered as

We then cover the ease of managing cross-references in LATEX in Section 1.5.

The label is any simple sequence of characters, numbers, digits, and some punctuation marks such as ":" and "-"; there should be no spaces. Try to use a consistent key format: this simplifies remembering how to make references. This document uses a prefix to indicate the type of the thing being referenced, such as sec for sections, fig for figures, tbl for tables, and eqn for equations.

For details on defining the text used to describe the type of *thing*, search diss.tex and the hyperref documentation for autorefname.

1.6 Managing Bibliographies with BibT_EX

One of the primary benefits of using LATEX is its companion program, BibTEX, for managing bibliographies and citations. Managing bibliographies has three parts: (i) describing references, (ii) citing references, and (iii) formatting cited references.

1.6.1 Describing References

BibTeX defines a standard format for recording details about a reference. These references are recorded in a file with a .bib extension. BibTeX supports a broad range of references, such as books, articles, items in a conference proceedings, chapters, technical reports, manuals, dissertations, and unpublished manuscripts.

A reference may include attributes such as the authors, the title, the page numbers, t. A reference can also be augmented with personal attributes, such as a rating, notes, or keywords.

Each reference must be described by a unique *key*.⁴ A key is a simple sequence of characters, numbers, digits, and some punctuation marks such as ":" and "-"; there should be no spaces. A consistent key format simiplifies remembering how to make references. For example:

where *last-name* represents the last name for the first author, and *contracted-title* is some meaningful contraction of the title. Then ? 's seminal article on aspect-oriented programming [?] (published in?) might be given the key kiczales-1997-aop.

An example of a BibTeX .bib file is included as biblio.bib. A description of the format a .bib file is beyond the scope of this document. We instead encourage you to use one of the several reference managers that support the BibTeX format such as JabRef⁵ (multiple platforms) or BibDesk⁶ (MacOS X only). These front ends are similar to reference manages such as EndNote or RefWorks.

1.6.2 Citing References

Having described some references, we then need to cite them. We do this using a form of the \cite command. For example:

```
\label{lem:citet} $$ \left\{ \text{kiczales} - 1997 - \text{aop} \right\}$ present examples of crosscutting from programs written in several languages}.
```

When processed, the \citet will cause the paper's authors and a standardized reference to the paper to be inserted in the document, and will also include a formatted citation for the paper in the bibliography. For example:

?] present examples of crosscutting from programs written in several languages.

⁴Note that the citation keys are different from the reference identifiers as described in Section 1.5.

⁵http://jabref.sourceforge.net

⁶http://bibdesk.sourceforge.net

Table 1.1: Available cite variants; the exact citation style depends on whether the bibliography style is numeric or author-year.

Variant	Result	
\cite	Parenthetical citation (e.g., "[?]" or "(? ?)")	
\citet	Textual citation: includes author (e.g., "?]" or or "?	
	(?)")	
\citet*	Textual citation with unabbreviated author list	
\citealt	Like \citet but without parentheses	
\citep	Parenthetical citation (e.g., "[?]" or "(??)")	
\citep*	Parenthetical citation with unabbreviated author list	
\citealp	Like \citep but without parentheses	
\citeauthor	Author only (e.g., "?")	
\citeauthor*	Unabbreviated authors list (e.g., "?")	
\citeyear	Year of citation (e.g., "?")	

There are several forms of the \cite command (provided by the natbib package), as demonstrated in Table 1.1. Note that the form of the citation (numeric or author-year) depends on the bibliography style (described in the next section). The \citet variant is used when the author names form an object in the sentence, whereas the \citep variant is used for parenthetic references, more like an end-note. Use \nocite to include a citation in the bibliography but without an actual reference.

1.6.3 Formatting Cited References

BibTeX separates the citing of a reference from how the cited reference is formatted for a bibliography, specified with the \bibliographystyle command. There are many varieties, such as plainnat, abbrvnat, unsrtnat, and vancouver. This document was formatted with abbrvnat. Look through your TeX distribution for .bst files. Note that use of some .bst files do not emit all the information necessary to properly use \citet{}, \citep{}, \citeyear{}, and \citeauthor{}.

There are also packages available to place citations on a per-chapter basis (bibunits), as footnotes (footbib), and inline (bibentry). Those who wish

to exert maximum control over their bibliography style should see the amazing custom-bib package.

1.7 Typesetting Tables

?] made one grievous mistake in LaTeX: his suggested manner for typesetting tables produces typographic abominations. These suggestions have unfortunately been replicated in most LaTeX tutorials. These abominations are easily avoided simply by ignoring his examples illustrating the use of horizontal and vertical rules (specifically the use of \hline and |) and using the booktabs package instead.

The booktabs package helps produce tables in the form used by most professionally-edited journals through the use of three new types of dividing lines, or *rules*. Tables 1.1 and 1.2 are two examples of tables typeset with the booktabs package. The booktabs package provides three new commands for producing rules: \toprule for the rule to appear at the top of the table, \midrule for the middle rule following the table header, and \bottomrule for the bottom-most at the end of the table. These rules differ by their weight (thickness) and the spacing before and after. A table is typeset in the following manner:

```
\begin{table}
\caption{The caption for the table}
\label{tbl:|abel}
\centering
\begin{tabular}{cc}
\toprule
Header & Elements \\
\midrule
Row 1 & Row 1 \\
Row 2 & Row 2 \\
% ... and on and on ...
Row N & Row N \\
\bottomrule
\end{tabular}
\end{tabular}
\end{table}
```

See the booktabs documentation for advice in dealing with special cases, such as subheading rules, introducing extra space for divisions, and interior rules.

LATEX Rocks!

Figure 1.3: Proof of LATEX's amazing abilities

1.8 Figures, Graphics, and Special Characters

Most LATEX beginners find figures to be one of the more challenging topics. In LATEX, a figure is a *floating element*, to be placed where it best fits. The user is not expected to concern him/herself with the placement of the figure. The figure should instead be labelled, and where the figure is used, the text should use \autoref to reference the figure's label. Figure 1.3 is an example of a figure. A figure is generally included as follows:

```
\begin{figure}
\centering
\includegraphics[width=3in]{file}
\caption{A useful caption}
\label{fig:fig-label} % label should change
\end{figure}
```

There are three items of note:

- 1. External files are included using the \includegraphics command. This command is defined by the graphicx package and can often natively import graphics from a variety of formats. The set of formats supported depends on your TeX command processor. Both pdflatex and xelatex, for example, can import GIF, JPG, and PDF. The plain version of latex only supports EPS files.
- 2. The \caption provides a caption to the figure. This caption is normally listed in the List of Figures; you can provide an alternative caption for the LoF by providing an optional argument to the \caption like so:

```
\caption[nice shortened caption for LoF]{% longer detailed caption used for the figure}
```

GPS! (**GPS!**) generally prefers shortened single-line captions in the LoF: multiple-line captions are a bit unwieldy.

Table 1.2: Useful LATEX symbols

LATEX	Result	IATEX	Result
\texttrademark	TM	\&	&
\textcopyright	©	\{ \}	{ }
\textregistered	R	\ %	%
\textsection	§	\verb!~!	~
\textdagger	†	\\$	\$
\textdaggerdbl	‡	\^{}	^
\textless	<	_	_
\textgreater	>		

3. The \label command provides for associating a unique, user-defined, and descriptive identifier to the figure. The figure can be can be referenced elsewhere in the text with this identifier as described in Section 1.5.

See Keith Reckdahls excellent guide for more details, *Using imported graphics in LaTeX2e*⁷.

1.9 Special Characters and Symbols

1.10 Changing Page Widths and Heights

The ubcdiss class is based on the standard LATEX book class [?] that selects a line-width to carry approximately 66 characters per line. This character density is claimed to have a pleasing appearance and also supports more rapid reading [?]. I would recommend that you not change the line-widths!

⁷http://www.ctan.org/tex-archive/info/epslatex.pdf

1.10.1 The geometry Package

Some students are unfortunately saddled with misguided supervisors or committee members whom believe that documents should have the narrowest margins possible. The geometry package is helpful in such cases. Using this package is as simple as:

```
\usepackage[margin=1.25in,top=1.25in,bottom=1.25in]{geometry}
```

You should check the package's documentation for more complex uses.

1.10.2 Changing Page Layout Values By Hand

There are some miserable students with requirements for page layouts that vary throughout the document. Unfortunately the <code>geometry</code> can only be specified once, in the document's preamble. Such miserable students must set LATEX's layout parameters by hand:

These settings necessarily require assuming a particular page height and width; in the above, the setting for \textwidth assumes a US Letter with an 8.5" width. The geometry package simply uses the page height and other specified values to derive the other layout values. The layout package provides a handy \layout command to show the current page layout parameters.

1.10.3 Making Temporary Changes to Page Layout

There are occasions where it becomes necessary to make temporary changes to the page width, such as to accommodate a larger formula. The chngpage package provides an adjustwidth environment that does just this. For example:

```
% Expand left and right margins by 0.75in \begin{adjustwidth}{-0.75in}{-0.75in} % Must adjust the perceived column width for LaTeX to get with it. \addtolength{\columnwidth}{1.5in} \[ an extra long math formula \] \end{adjustwidth}
```

1.11 Keeping Track of Versions with Revision Control

Software engineers have used **RCS!** (**RCS!**) to track changes to their software systems for decades. These systems record the changes to the source code along with context as to why the change was required. These systems also support examining and reverting to particular revisions from their system's past.

An **RCS!** can be used to keep track of changes to things other than source code, such as your dissertation. For example, it can be useful to know exactly which revision of your dissertation was sent to a particular committee member. Or to recover an accidentally deleted file, or a badly modified image. With a revision control system, you can tag or annotate the revision of your dissertation that was sent to your committee, or when you incorporated changes from your supervisor.

Unfortunately current revision control packages are not yet targetted to non-developers. But the Subversion project's TortoiseSVN⁸ has greatly simplified using the Subversion revision control system for Windows users. You should consult your local geek.

A simpler alternative strategy is to create a GoogleMail account and periodically mail yourself zipped copies of your dissertation.

1.12 Recommended Packages

The real strength to LATEX is found in the myriad of free add-on packages available for handling special formatting requirements. In this section we list some helpful packages.

1.12.1 Typesetting

enumitem: Supports pausing and resuming enumerate environments.

⁸http://tortoisesvn.net/docs/release/TortoiseSVN_en/

\usepackage[normalem,normalbf]{ulem}

to prevent the package from redefining the emphasis and bold fonts.

chngpage: Support changing the page widths on demand.

mhchem: Support for typesetting chemical formulae and reaction equations.

Although not a package, the latexdiff⁹ command is very useful for creating changebar'd versions of your dissertation.

1.12.2 Figures, Tables, and Document Extracts

pdfpages: Insert pages from other PDF files. Allows referencing the extracted pages in the list of figures, adding labels to reference the page from elsewhere, and add borders to the pages.

subfig: Provides for including subfigures within a figure, and includes being able to separately reference the subfigures. This is a replacement for the older subfigure environment.

rotating: Provides two environments, sidewaystable and sidewaysfigure, for typesetting tables and figures in landscape mode.

longtable: Support for long tables that span multiple pages.

tabularx: Provides an enhanced tabular environment with auto-sizing columns.

ragged2e: Provides several new commands for setting ragged text (e.g., forms of centered or flushed text) that can be used in tabular environments and that support hyphenation.

⁹http://www.ctan.org/tex-archive/support/latexdiff/

1.12.3 Bibliography Related Packages

bibunits: Support having per-chapter bibliographies.

footbib: Cause cited works to be rendered using footnotes.

bibentry: Support placing the details of a cited work in-line.

custom-bib: Generate a custom style for your bibliography.

1.13 Moving On

At this point, you should be ready to go. Other handy web resources:

- CTAN! (CTAN!)¹⁰ is *the* comprehensive archive site for all things related to T_EX and L^AT_EX. Should you have some particular requirement, somebody else is almost certainly to have had the same requirement before you, and the solution will be found on CTAN!. The links to various packages in this document are all to CTAN!.
- An online reference to LATEX commands¹¹ provides a handy quick-reference to the standard LATEX commands.
- The list of Frequently Asked Questions about TEX and LATEX¹² can save you a huge amount of time in finding solutions to common problems.
- The teTeX documentation guide¹³ features a very handy list of the most useful packages for LATeX as found in **CTAN!**.
- The color¹⁴ package, part of the graphics bundle, provides handy commands for changing text and background colours. Simply changing text to various levels of grey can have a very dramatic effect.
- If you're really keen, you might want to join the TEX Users Group 15.

 $^{^{10} {\}rm http://www.ctan.org}$

¹¹ http://www.ctan.org/get/info/latex2e-help-texinfo/latex2e.html

¹² http://www.tex.ac.uk/cgi-bin/texfag2html?label=interruptlist

¹³http://www.tug.org/tetex/tetex-texmfdist/doc/

¹⁴http://www.ctan.org/tex-archive/macros/latex/required/graphics/grfguide.pdf

¹⁵http://www.tug.org

Bibliography

Appendix A

Supporting Materials

This would be any supporting material not central to the dissertation. For example:

- additional details of methodology and/or data;
- diagrams of specialized equipment developed.;
- copies of questionnaires and survey instruments.