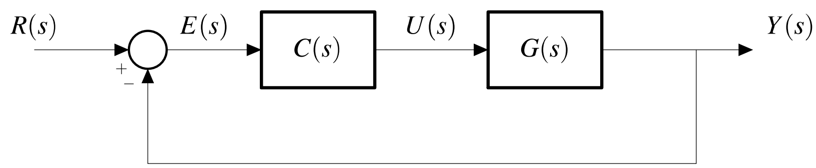
2022 MXEN3004/ETEN6000 Design Assignment

An industrial plant is governed by the the following differential equation

$$k\left(a\frac{du(t)}{dt}+bu(t)\right)=\frac{d^3y(t)}{dt^3}+c\frac{d^2y(t)}{dt^2}+d\frac{d^1y(t)}{dt^1},$$

where u is the input and y the output.



A feedback compensator $C(s)$ needs to be designed such that the closed-loop system in the figure above is asymptotically stable and meets the following design criteria:

1. the gain crossover frequency satisfies $\omega_c \in [\texttt{wl}, \texttt{wh}]$;
2. the phase margin is at least 55^o;
3. the velocity error is smaller than `ev`;
4. the steady-state error in response to step inputs is zero.

If the four performance criteria are met, further iteration of the controller may be undertaken (if you wish) to minimise the settling time of the step response from $r(t)$ to $y(t)$. If you cannot meet any of the design criteria, get as close as you can while ensuring closed-loop stability, and explain where and why compromises were needed

Use Matlab and the file *systems_generator.p* to obtain student-individualised parameters of the plant and of the control problem

`[k,a,b,c,d,wl,wh,ev]=systems_generator(N),`

where N **is your student number. Please ensure that you use the correct student number**.

1 Process transfer function (5 marks)

Derive the transfer function $G(s)$ from the input u to the output y.

2 Proportional controller (5 marks)

Consider first a proportional controller $C(s) = K_p$. Find a proportional gain K_{p1} such that the constraint on the gain crossover frequency is satisfied. Check, by using the Bode diagram, that such proportional gain does not satisfy the design requirements and give the corresponding phase margin. Find a different proportional gain K_{p2} that satisfies the constraints on the velocity error. Check, by using the Bode diagram, that such proportional gain does not satisfy the design requirements, and show the tracking error in response to a unit ramp input.

3 Phase-lead compensator (15 marks)

Consider a phase-lead compensator

$$K_p \frac{\tau s + 1}{\alpha \tau s + 1}.$$

Show that this controller cannot satisfy the design requirements (hint: select K_p to satisfy the velocity error requirement and show that the conditions to apply the inversion formulae do not hold). Find the phase-lead compensator that satisfies the constraints on the gain crossover frequency and on the phase margin while keeping the velocity error as small as possible (hint: the ramp tracking error is inversely proportional to the gain of the controller). Draw the bode plot of the resulting system, showing the required phase margin, and plot the closed-loop response to a unit step input. Show that the ramp error requirement is not yet met.

4 Lead-lag compensator (15 marks)

Consider a lead-lag compensator

$$K_p \frac{\tau s + 1}{\alpha \tau s + 1} \frac{\beta T s + 1}{T s + 1}.$$

Select the parameters of the controller such that all design constraints are satisfied (hint: the term $\frac{\beta T s+1}{T s+1}$ will decrease the phase margin. Select first τ and α using the procedure of the previous point, but assigning a phase margin slightly greater 55^o, and select K_p to satisfy the velocity error requirement). Plot the closed-loop response to a unit step input, to two sinusoids with frequencies one decade above and one decade below the gain crossover frequency. Also plot the error in response to the unit ramp input. Hence show that all of the required performance criteria are met, or else get as close as possible and explain where compromises have been made and why.

This is a controller design assignment, and should be documented as such. The use of MATLAB for relevant calculations and plots is encouraged. Plots **and calculations** should be explained in the context of the design task and the required performance of the closed loop system. A brief introduction and conclusion should be included, but do not use the text of the assignment document directly in your submission. **Up to 10 marks are awarded for the structure and presentation fo the report.**

Please ensure that a clear statement of your student Name and student ID is made in the front page. Your report should be limited to a **maximum of 5 pages, including graphs and parts of code that contains relevant calculations.** Do not include all your code. Include only the relevant lines of code, where (and if) you think it is appropriate in the context of you report. You can have extra pages exclusively for title, table of contents and bibliography, if any. No other content (e.g., appendices) is allowed in the extra pages.

Anything in the body of the report that goes beyond the five pages will not be marked.

If the design is carried out using a response obtained with the wrong student number, the final mark is reduce by 50%

The report is due in Turnitin by **11:59pm, Monday May 23rd**, 2022.