Valuing Lost Home Production of Dual Earner Couples

Christopher House, John Laitner, Dmitriy Stolyarov*
University of Michigan

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Abstract

This paper proposes a strategy for measuring the market value of forgone home production associated with increases in female labor force hours. We formulate a life-cycle model in which women divide their time between home and market work. The model implies a link between household retirement wealth and the value of forgone home production. We use recently available panel micro data from the Health and Retirement Study to estimate the model's parameters and adjust the growth rate of GDP to reflect recent reductions in non-market output.

We find that the value of forgone home production is modest, about 25 percent of women's measured earnings. On aggregate, due to the large transition of women into the labor force, the value of forgone home production relative to GDP has increased by only 2.5 percentage points over the decades 1959-1999.

1 Introduction

In the past fifty years there has been a dramatic increase in female labor force participation (e.g., Goldin 1990 and Table 5 below). Undoubtedly some of the measured increase in GDP during this time period reflects reallocations of women's time from unmeasured, home production to measured, market work. Failure to account for the associated reductions in home-production output will lead to biased estimates of the growth of economic well-being. Eisner (1988) captures this issue perfectly, as follows:

> "The difficulty with exclusion of nonmarket output is not merely that it results in lesser totals of GNP ... Most recently, the vast increases in conventional GNP associated with the major movement of women into the labor force may signify a much lesser gain in total output, as nonmarket child care gives way to the paid baby-sitter and nursery school, care of the aged to nursing homes, care of the sick to hospitals, and home cooking to McDonalds." [p.1613]

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The goal of the present paper is to derive consistent measurements of GDP during a time period when the balance of market production and home production shifted. To do this, we propose a strategy for measuring changes in home production. Specifically, we show that in a life–cycle model of dual earner households with home production, there is a connection between the quantity of home production and household saving. Using recently available micro–level data on household net worth and male and female earnings, we exploit this connection to estimate the model's parameters and to appraise changes in the value of forgone home production in the U.S. over the last four decades.

In our framework, households allocate their time between home production and market work. A Becker–style (1965) theory of optimal time allocation predicts that changes in home production will accompany decisions of household members to join or separate from the labor market. Accordingly, a household in which the wife is in the labor force may purchase goods and services that substitute for sacrificed home production. These substitutes include day care expenses, cleaning services, take-out meals, and so forth. We propose to measure the value of lost home production with expenditure on market substitutes for the home output, and we propose to estimate this expenditure indirectly from wealth accumulation behavior. Purchases of home production substitutes leave a household with fewer resources available for other expenditures and saving. Thus, the life-cycle model predicts that measured saving rates for households with two earners, for example, should be systematically lower than for households with the same income path but only a single earner. Likewise, households in which women work long hours may have to spend more on substitutes, leaving less of their market income for other consumption and saving, compared to households in which women work short market hours.

We use detailed micro data on lifetime earnings and net worth from the Health and Retirement Study (HRS) to compare retirement wealth for couples with different female earnings. In the context of our theoretical model, these comparisons allow us to infer a household's lifetime expenditure on home–production substitutes. We interpret the latter expenditure to be the value of forgone home production. Our analysis indicates that, while it is statistically significant, the value of forgone home production is surprisingly modest. In fact, on average, for every dollar a married women earns in the labor market, her household purchases roughly 25 cents of substitutes for lost home production. Thus, the private sector's net gain from married women's employment is roughly 75 percent of the women's market earnings.

The fraction of labor income earned by women increased from 20 percent in 1959 to over 35 percent in 1999 (see Section 5 below). If the increase in female labor force participation was accompanied by a commensurate reduction in home production activity, the measured increase in labor income overstates the net increase in the total economic activity. Because the National Income and Product Accounts (NIPA) omit home-production output (e.g., child care at home), but include its market substitutes (e.g., day care centers), measured growth in GDP likely overstates recent rises in living standards. This paper finds that the overstatement, while substantial, is not overwhelming. In particular, we estimate that the share of home production substitutes in GDP grew by about 2.5 percent between 1959 and 1999. That is, about 2.5 percent of 1999 GDP consisted of goods and services that would have been produced at home in 1959.

The life–cycle model that underlies our empirical results has a number of attractive features. First, the model allows many dimensions of household heterogeneity, including (unobserved) differences in home productivity, differences in wage and family–size profiles, and differences in retirement age. A second virtue of our approach is its flexibility. The model can be generalized in a number of ways that leave the substance of the analysis intact. These generalizations include allowing fixed costs of labor force participation and on–the–job human capital accumulation. The last features have been widely recognized as important determinants of female labor supply (e.g., Attanasio *et al.* 2004 and references therein). Fixed costs of labor force participation can cause labor–market hours to jump between zero and a substantial positive number; on-the-job human capital accumulation can induce women who want to work late in life to seek employment early in life as well.

There is a large literature devoted to the measurement and study of home production. Early approaches to the problem of measuring nonmarket output focused on valuing time spent at home at the market wage rate (Kendrick 1979, Nordhaus and Tobin 1973, Eisner 1988). Such estimates value every hour of home production at its marginal cost. This means that the net gain from female labor force participation is zero: each dollar earned in the market is offset by an equal loss in home production. More recently, Rosen (1996) echoes the same argument. He presents evidence suggesting that Swedish women who joined the labor force during 1963-1993 were overwhelmingly employed in sectors that supply home production substitutes. Rosen (1996, p. 735) writes:

> "If Swedish women take care of each other's parents in exchange for taking care of each other's children, how much additional real output comes of it?"

In contrast, our framework allows an upward sloping marginal cost curve for sacrificed home–production time; thus, marginal and average costs can differ. Our empirical results show that the average cost of forgone time is *significantly* lower than the marginal cost. This suggests that valuations of lost home-production output based on the marginal cost of time are biased upward.

Our work complements the literature that estimates the production function for home goods from aggregate time series (e.g., McGrattan *et al.* 1997), as well as estimation methodologies that require additional data on expenditure and time use (e.g., Rupert *et al.* 1995, 2000). Our methodology is different in that we exploit a relationship between retirement savings and the value of nonmarket output implied by the model. This indirect approach is similar in spirit to Hong (2005), who uses life–insurance data to make inferences about home production, and Guvenen (2007), who uses consumption and wealth observations to make inferences about households' learning about the stochastic process of their earnings.

The organization of this paper is as follows. Section 2 presents our model and derives the implications of optimizing household behavior for the value of lost home production. Section 3 generalizes the model to include male home production, fixed costs of labor–force participation, on–the–job human capital accumulation, and endogenous retirement age. Section 4 describes our data. Section 5 presents empirical results and discusses the quantitative implications of our estimates. Section 6 concludes.

2 Model

We analyze the life-cycle saving decisions of couples. Our baseline specification assumes that males work full time in the market until retirement and that male home production has negligible value at all ages. A woman, on the other hand, may work less than full time in the market, devoting her remaining hours to home production. After retirement, a woman engages exclusively in home production. Section 3 considers a generalized model in which male home production may have a substantial role.

To foreshadow our results, our baseline model implies the following relationship between male and female market earnings and net worth for a retired household:

$$\ln(NW_{it}) = \ln\left(\kappa_i(t,\sigma)\right) + \ln\left(Y_{it}^M + (1-\theta)\cdot Y_{it}^F\right) + \epsilon_{it}. \tag{1}$$

Here, t is the current age of the household and NW_{it}, Y_{it}^M, and Y_{it}^F are, respectively, net worth for household i, the present value (measured at household age t) of male lifetime earnings, and the present value of female lifetime earnings. The random variable ϵ_{it} reflects measurement error in NW_{it}. The function $\kappa(.)$ is implied by the model and is independent of NW_{it}, Y_{it}^M, and Y_{it}^F. Finally, $\theta \in [0, 1]$ and σ are parameters to be estimated. The parameter θ is of particular interest because it reflects the fraction of female measured market earnings that households seek to replace with market–produced substitutes.

Equation (1) can be used to estimate the cost of forgone home production for a female who devotes some of her time to market employment. If time at home is valuable, women in dual-earner households may compensate for their time away from home by purchasing market goods that substitute for their forgone home production. As a result, measured market earnings overstate the true net value of female labor market participation. In our model, the overstatement is captured by the parameter θ, which measures the average fraction of a woman's earnings used to purchase home–production substitutes. The net contribution of female earnings to the household resources is $(1-\theta) \cdot Y_{it}^F$. Our analysis shows that the sum of male earnings and net female earnings determines the amount of wealth that a household desires to carry into its retirement.

Notice that specification (1) allows for the extreme cases $\theta = 0$ and $\theta = 1$. If $\theta = 0$, the average value of a woman's forgone time at home is zero. In that case, each dollar earned in the market represents a real increase in household income. If $\theta = 1$, the opportunity cost of each hour at home is exactly offset by the woman's market wage, and changes in women's labor market earnings do not affect their household's real income at all.

We use household data on NW_{it}, Y_{it}^M, and Y_{it}^F to estimate θ. Our strategy rests on the relationship between households' retirement savings and women's measured market earnings. If our model is correct, dual-earner households should, *ceteris paribus*, have a lower ratio of net worth to their lifetime earnings than single-earner households or households in which the wife's market hours are short. In other words, cross-sectional variation in women's lifetime earnings, Y^F, provides the basis for our estimation.

2.1 Baseline model

Consider a household i that lives from age S_i to T. The household includes a man, a woman, and, possibly, their children. The size of a household of age t is N_{it} "equivalent adults" (see below). The man and woman both retire when the household reaches age R_i. Prior to retirement, the wife divides her time between market work and home production. After retirement, she does only home production. Historically, men have specialized in market work (e.g., Figure 1 below). This suggests, at least in the past, a strong comparative disadvantage in home production. Moreover, men's time allocations have not changed dramatically in the last fifty years. For both reasons, our baseline model simply assumes that husbands devote their entire workweek to market employment prior to retirement and provide no home production at any age. (Section 3 offers a more elaborate formulation that treats men and women symmetrically.)

In the baseline model, the adult male in household i supplies market hours inelastically at any age t prior to retirement and earns exogenous after-tax income y_{it}. The woman's market hours are h_{it}, the woman's after-tax wage rate in the market is w_{it}, and the after-tax real interest rate is r.

Household i solves the following dynamic optimization problem

$$\max_{\{c_{it} \geq 0,\, h_{it} \geq 0\}_{t=S_i}^{T}} \sum_{t=S_i}^{T} e^{-\rho t} \cdot \left[N_{it} \cdot u\left(\frac{c_{it}}{N_{it}} \right) + v_{it}(A_{it}) \right] \tag{2}$$

$$\text{s.t.} \quad c_{it} = x_{it} - A_{it} \cdot h_{it}^{\xi}, \quad \xi > 1, \quad A_{it} > 0, \tag{3}$$

(4) $$\sum_{t=S_i}^{T} e^{-rt}\left(y_{it} + h_{it} \cdot w_{it} - x_{it}\right) \geq 0,$$

$$y_{it} = 0 \quad \text{and} \quad h_{it} = 0 \quad \text{all} \quad t \geq R_i\,.$$

We make specific assumptions that enable us to use the model for structural estimation. Some of these assumptions merit additional discussion as follows.

The household derives flow utility $N_{it} \cdot u(c_{it}/N_{it})$ from consuming the market good and flow utility $v_{it}(A_{it})$ from consuming the home good. We think of the home good as a set of tasks that must be completed for the household to maintain its viability. Examples include child care, food preparation and cleaning. Let A_{it} measure the number of such tasks in household i. The woman can produce the home good with her time at home or purchase a perfect substitute on the market. As the woman begins to devote hours to market employment, her home production falls according to the function $A_{it} \cdot h_{it}^{\xi}$. This loss function is increasing in the number of tasks to be performed, A_{it}, and convex ($\xi > 1$), reflecting the fact that the least valuable home hours should be allocated to market work first. Because lost home production needs to be replaced, a household that spends x_{it} on market goods consumes $c_{it} = x_{it} - A_{it} \cdot h_{it}^{\xi}$ and pays $A_{it} \cdot h_{it}^{\xi}$ to replace forgone home production. This formulation automatically measures the cost of forgone home production in the same units as GDP.

The gain to the household from consumption of home goods in terms of utility is captured by the function $v_{it}\left(A_{it}\right)$, which simply depends on the number of home production tasks (i.e., the amount of the home goods consumed). A higher A_{it} may imply a greater well-being through $v_{it}(.)$, but it also entails a higher cost of substitutes for the wife's market hours. Since the term $v_{it}\left(A_{it}\right)$ does not affect the household's optimizing behavior, we suppress it in our analysis.

Time has two uses in our framework, market work and home production. After retirement, the household engages in home production only. This assumption of naturally greater levels of home production after retirement resembles, for instance, Aguiar and Hurst (2005), and is consistent with the observation that many households change their levels of measured consumption when they retire (see among others, Banks *et al.* 1998, Bernheim *et al.* 2001, Hurd and Rohwedder 2003, and Laitner and Silverman 2005). The last subsection of Section 3 discusses possible attributes of, and motives for, retirement. Our model allows a woman's efficiency at home production to vary with age and, therefore, with family composition.

Throughout the paper, we assume that the function u is isoelastic,

$$u(c) \equiv \frac{c^{\gamma}}{\gamma}\,, \quad \gamma < 1.$$

Define the age of the household, s, to be the age of its adult male. Let household net worth at age s be

$$a_{is} \equiv \sum_{t=S_i}^{s-1} e^{-r(t-s)}\left(y_{it} + w_{it}h_{it} - x_{it}\right),$$

and let the present value (at age s) of male and female lifetime earnings be

(5) $$Y_{is}^{M} \equiv \sum_{t=S_i}^{T} e^{-r(t-s)} y_{it}, \qquad Y_{is}^{F} \equiv \sum_{t=S_i}^{T} e^{-r(t-s)} w_{it}h_{it}.$$

The following proposition presents the implications of the model that form the basis of our empirical analysis.

Proposition 1: *Solution of (2)-(4) implies*

(6)
$$A_{it} \cdot h_{it}^{\xi} = \theta \cdot w_{it} \cdot h_{it}, \textit{ for any } t \le T.$$

For a given R_i *and for any* $s \ge R_i$,

(7)
$$\frac{a_{is}}{Y_{is}^M + (1-\theta) \cdot Y_{is}^F} = \frac{\sum_{t=s}^{T} N_{it} \cdot \sigma^t}{\sum_{t=S_i}^{T} N_{it} \cdot \sigma^t},$$

where

$$\theta = \frac{1}{\xi},$$

(8)
$$\sigma \equiv \exp\left(-r + \frac{r-\rho}{1-\gamma}\right).$$

Proof: See Appendix.

Equation (6) comes from the optimality condition that equates the marginal value of a woman's time at home and her market wage. Despite equality at the margin, the average value of time diverted from home production is less than the market wage because the marginal cost, in terms of forgone home production, of supplying market hours is increasing. Thus, an optimal time allocation implies that the current value of lost home production is a fraction, $\theta = \xi^{-1}$, of a woman's current earned income. The proposition's second result follows from accounting and isoelastic preferences. The present value (as of age $s \ge R_i$) of lifetime expenditure that offsets lost home production is θY_{is}^F. The remainder of a household's earnings, $Y_{is}^M + (1-\theta) \cdot Y_{is}^F$, determines the household's relative financial well-being. With isoelastic preferences, a household's post-retirement expenditure is proportional to its lifetime financial well-being. Condition (7) reflects this proportionality.

To derive equation (1), define the ratio of the present value of consumption from age s onward to the present value of total lifetime consumption as

$$\kappa_i(s\,,\,\sigma) \equiv \frac{\sum_{t=s}^{T} N_{it} \cdot \sigma^t}{\sum_{t=S_i}^{T} N_{it} \cdot \sigma^t}.$$

Taking logarithms of both sides of (7) and appending an error term reflecting difficulties of measuring net worth gives equation (1).

The following example illustrates our approach. Set $r = \rho = 0$ and consider three households whose income and asset data is summarized in the table below.

Household	Y^M	Y^F	$a_R = \frac{1}{2}\left(Y^M + (1-\theta) \cdot Y^F\right)$
I	10,000	0	5,000
II	20,000	0	10,000
III	10,000	10,000	$5,000 + (1-\theta) \cdot 5,000$

In household I, the male's present value of lifetime earnings is \$10,000 and the female's is zero; in household II, the male's lifetime earnings are \$20,000 and the female's zero; in household III, the male and female each have lifetime earnings of \$10,000. Since r and ρ are zero, all three households desire constant consumption profiles over their lifetime. That is, they want consumption (i.e., c_{it} as defined in equation (3)) before and after retirement to be the same. Under these conditions, the first and the second household should build net worth at retirement equal to one half of their household lifetime resources, \$5000 and \$10,000, respectively. Household III is different because it sacrifices some home production when the female works in the labor market. The third household's desired net worth at retirement will be $a_R = \left(Y^M + (1-\theta)\cdot Y^F\right)/2$. If the average value of sacrificed hours is the market wage (i.e., if $\theta = 1$ and $A_{it} = w_{it}$), our model predicts that household III will have the same net worth at retirement as household I. On the other hand, if the value of sacrificed hours is zero (i.e., if $\theta = 0$), household III will desire the same net worth at retirement as household II. Equation (1) allows intermediate cases $\theta \in (0, 1)$ as well. Given data on net worth a_R and lifetime earnings Y^M and Y^F, equation (1) provides an opportunity to estimate θ.

Section 5 estimates (1) using survey data from the Health and Retirement Study (HRS). Our model makes full use of the HRS: the data provides exceptionally complete earnings and family composition histories for men and women, it provides biennial inventories of each household's assets and debts, and it permits us to capitalize household pension and Social Security benefits once a household retires. Although the HRS has less complete information on hours of market work, equation (1) fortunately does not require data on h_{it}.

2.2 Discussion

We take the preference parameters ρ and γ, the technology parameter ξ, the market interest rate r, and the age of death T to be common to all households. At the same time, we should emphasize that Proposition 1 allows a great deal of heterogeneity among households. In particular, each household can have a different size profile N_{it}, its own home–production task profile A_{it}, its own male earnings profile y_{it}, and its own female wage profile w_{it}.

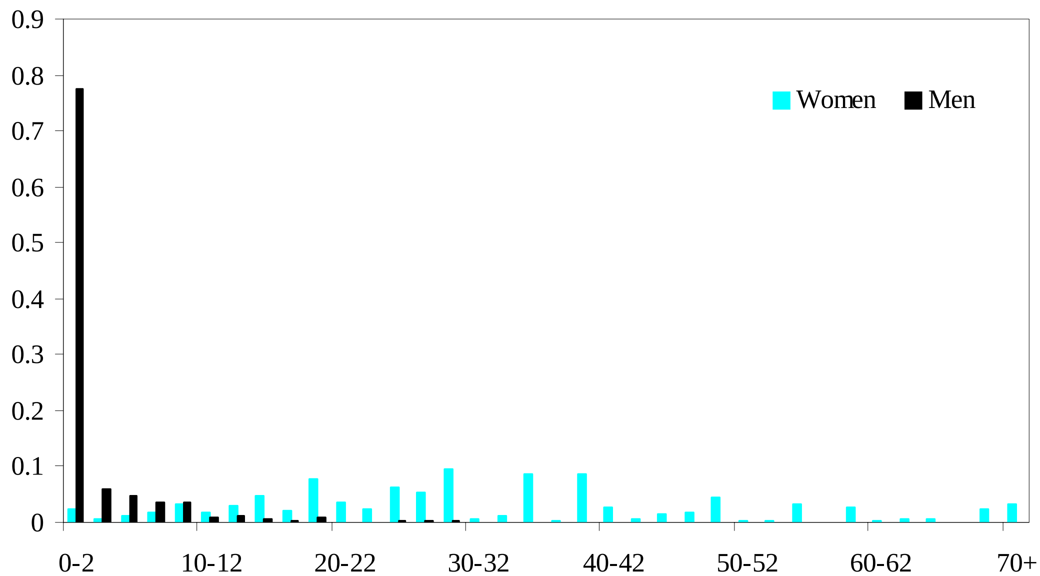

Figure 1. Hours per week spent on housework, 1970.

We believe that our baseline model provides, in practice, a useful framework for analysis. Though the model allows women to engage in home production as well as market work, it restricts men to the latter option alone. Traditionally at least, men have seemed to play a much smaller role in production at home. For example, Figure 1 shows a histogram of hours per week of housework (in 1970) for married men and women aged 20-60. The data come from the Panel

Study of Income Dynamics.[1] According to the data, in 1970 the median number of hours per week that men spent on housework was 0, compared with 30 for women. Data from the same source for men and women over the age of 65 shows the same pattern, with median male hours of housework remaining 0.

The next section, nevertheless, offers a more elaborate formulation with symmetric time-allocation options for both sexes.

3 Generalizations of the model

This section generalizes our framework to include home production by men, labor-force participation decisions for women, human capital accumulation from on-the-job experience, and endogenous retirement age. We show that equation (1) remains valid in each context, although in some cases the key parameter θ requires a new interpretation.

MALE HOME PRODUCTION. We can extend our basic model to incorporate male home production. Giving A_{it}, w_{it}, and h_{it} a superscript m for males and f for females, the household optimization problem becomes

$$\max_{\left\{c_{it}\geq 0,\, h_{it}^m\geq 0, h_{it}^f\geq 0\right\}_{t=S_i}^T} \sum_{t=S_i}^T e^{-\rho t}\cdot N_{it}\cdot u\left(\frac{c_{it}}{N_{it}}\right)$$

$$\text{s.t.} \quad c_{it} = x_{it} - A_{it}^f \cdot \left[h_{it}^f\right]^{\xi^f} - A_{it}^m \cdot [h_{it}^m]^{\xi^m} ,$$

$$\sum_{t=S_i}^T e^{-rt}\left(h_{it}^f \cdot w_{it}^f + h_{it}^m \cdot w_{it}^m - x_{it}\right) \geq 0,$$

with $\xi^f > 1$, $\xi^m > 1$, $A_{it}^f > 0$, $A_{it}^m > 0$, and $h_{it}^f = h_{it}^m = 0$ all $t \geq R_i$. In this formulation, we allow gender-specific convex loss functions for home production. As before, lost home production must be replaced with purchases of market substitutes. After retirement, both males and females engage in full-time home production so that home production losses disappear.[2] Proposition 1 then generalizes in the following straightforward manner.

Corollary to Proposition 1: *For a given R_i and for any $s \geq R_i$, solution of (2)-(4) implies*

$$\frac{a_{is}}{(1-\theta^m)\,Y_{is}^M + (1-\theta^f)\cdot Y_{is}^F} = \frac{\sum_{t=s}^T N_{it}\cdot\sigma^t}{\sum_{t=S_i}^T N_{it}\cdot\sigma^t}\,, \tag{9}$$

where

$$\theta^f = \frac{1}{\xi^f}, \quad \theta^m = \frac{1}{\xi^m}, \quad \sigma \equiv \exp\left(-r + \frac{r-\rho}{1-\gamma}\right).$$

Proof: The optimal time allocation conditions for males and females are strictly analogous to (6), with the corresponding gender-specific parameters. Repeating all the steps in the proof of Proposition 1 leads to (9). ■

[1] See question ER30053 in the PSID , which asks how much time the respondent spent on housework in an average week. Figure 1 is representative of the other years in the PSID data as well (though male home production activity has been gradually rising over time). We thank Claudia Olivetti for drawing our attention to this variable.

[2] This assumption is consistent with the observation that market expenditures systematically drop after retirement. In our model, a drop in expenditure at retirement will arise because individuals in retirement do not take time away from home production and therefore do not need to replace it with market substitutes. We hope to study this phenomenon further in future work.

Taking the logarithms of both sides of (9), letting

$$1 - \theta = \frac{1 - \theta^f}{1 - \theta^m}\,, \tag{10}$$

and adding a random term ε_{it} reflecting measurement error in net worth, we have the analogue of equation (1):

$$\ln(NW_{it}) = \ln\left(1 - \theta^m\right) + \ln\left(\kappa_i(t, \sigma)\right) + \ln\left(Y^M_{it} + (1 - \theta) \cdot Y^F_{it}\right) + \varepsilon_{it}. \tag{11}$$

The difference between (1) and (11) is the constant term $\ln\left(1 - \theta^m\right)$, which depends on the curvature of the male loss function. If (1) is estimated with an unrestricted constant, we can obtain a consistent estimate of θ from the coefficient on Y^F, we can recover an estimate of θ^m from the constant, and we can derive an estimate of θ^f from from

$$\theta^f = \theta^m + \theta \cdot \left(1 - \theta^m\right).$$

Notice that the interpretation of θ in (11) depends on the value of male home production. Even if one is interested exclusively in the home production of females, an estimate of θ^f in this case must be extracted from θ utilizing the separate estimate of θ^m. If θ is between zero and one, then $\theta^f > \theta$. Merely setting $\theta^f = \theta$ yields, in these circumstances, an estimate of θ^f which is biased downward.

Our estimation in Section 5 considers this generalization separately.

Labor Force Participation. The convex loss function of Section 2 implies that all women will choose to supply positive market hours at every age prior to retirement. Clearly this is, at best, an approximate description of actual behavior. Indeed, women's market participation decisions have received a great deal of attention in the labor–economics literature. To include labor–force participation choices for women, this subsection modifies our model to incorporate a fixed participation cost.

One strand of existing literature emphasizes tangible fixed costs such as commuting time (e.g., Cogan 1981, Hurd 1996, French 2005). There are also models with intangible fixed costs in which participation directly reduces household utility (e.g. Attanasio *et al.* 2004).[3] Examples of intangible fixed costs might include separation from children (Berger *et al.* 2005), changes of bargaining power within marriages (Goldin 1990), and costs of deviating from a social norm in which men provide for the family (Goldin 1990).

We can accommodate a non-trivial labor market participation decision with a model with intangible fixed costs. Suppose that household i of age t incurs a utility loss $\bar{u}_{it}$ during periods when the woman participates in the labor market. Let p_{it} be an indicator variable for whether the woman participates in the labor market at age t, with

$$p_{it}\left(h_{it}\right) = \begin{cases} 1, & h_{it} > 0 \\ 0, & h_{it} = 0 \end{cases}.$$

Returning to the baseline framework, a household's maximization problem becomes

$$\max_{\{c_{it} \geq 0,\, h_{it} \geq 0\}_{t=S_i}^{T}} \sum_{t=S_i}^{T} e^{-\rho t} \left[N_{it} \cdot u\left(\frac{c_{it}}{N_{it}}\right) - p_{it}\left(h_{it}\right) \cdot \bar{u}_{it}\right] \tag{12}$$

[3] The model of Attanasio *et al.* (2004) has tangible costs of participation in the form of child care expenditures as well. In our framework, such expenditures are part of the loss term Ah^{ξ}.

$$\text{s.t.} \quad c_{it} = x_{it} - A_{it} \cdot h_{it}^{\xi}, \quad \xi > 1,$$

$$\sum_{t=S_i}^{T} e^{-rt} \left(y_{it} + h_{it} \cdot w_{it} - x_{it} \right) \geq 0,$$

where $y_{it} = 0$ and $h_{it} = 0$ for all $t \geq R_i$. We can immediately prove the following proposition, which shows that equation (1) remains applicable.

Proposition 2: *The optimal solution of (12) implies conditions (6)-(7) from Proposition 1.*
Proof: Fix i and any lifetime participation profile $\{p_{it}\}$, and solve (12) with the constraint $h_{it} = 0$ when $p_{it} = 0$. In each such solution, Proposition 1 holds — in particular, (6) is automatically valid for all t, because $p_{it} = 0$ forces $h_{it} = 0$. Pick an optimal participation profile that maximizes the household's lifetime utility. Since Proposition 1 holds for any participation profile, it must also hold for the optimal profile. ■

Specification (12) allows a great deal of modeling latitude. The fixed cost $\bar{u}_{it}$ could vary with time and/or family size. For example, as more married women have entered the labor force in recent years, the possible stigma formerly associated with such employment may have diminished. Alternatively, the cost of separation from children may be higher when the children are younger.

Even if $\bar{u}$ is a constant common to all households, a substantial and potentially interesting heterogeneity of outcomes is possible. Women with young children, for example, may have a high A_{it}, which will tend to reduce participation in the labor force. If two women have the same A_{it} and the same w_{it}, the one whose husband earns the most will, according to (12), be the least likely to participate in the labor force herself. Likewise, a woman with a high w_{it} will, *ceteris paribus*, be more likely to seek market employment.

Tangible fixed costs would have similar implications in terms of concentrating market work into fewer periods. To take an illustration, one might argue that commuting costs compel a woman seeking part–time market work to accept a job closer to home at lower wages, so that, in effect, $w_{it} = w(h_{it})$ with $w'(.) > 0$. Provided the tangible costs make short hours unattractive, distinguishing between tangible and intangible costs may, however, be extremely difficult in practice. So, (12), and hence (1), may deliver a useful approximation. In any event, our data set provides few covariates for a separate analysis of specific tangible fixed costs.

Human Capital Accumulation. The basic earnings model of Mincer (1974) and others allows an individual's market earnings to increase with experience. Such a framework complicates a woman's time allocation problem because if she forgoes market work in favor of staying at home, she loses both current wages and investments in human capital that would lead to higher wages in the future (e.g., Attanasio *et al.* 2004 and Goldin 1990). Fortunately, we can extend our baseline model (12) to incorporate on-the-job human capital accumulation, while maintaining the validity of (1).

Let H_{it} be cumulative work experience for a woman of age t. We assume that H_{it} accumulates without depreciation (see, for example, Mincer and Ofek 1982 and Corcoran *et al.* 1983):

$$H_{i,t+1} = H_{it} + h_{it}.$$

We also assume that wages increase with human capital, though with diminishing returns:

$$w_{it} = B_{it} \cdot H_{it}^{\alpha}, \quad \alpha \in (0, 1),$$

where α and B_{it} are exogenous. B_{it} may capture aggregate wage growth as well as differences in individuals' abilities.

The household's (baseline) optimization problem becomes

$$\max_{\{c_{it}\geq 0,\, h_{it}\geq 0\}_{t=S_i}^{T}} \sum_{t=S_i}^{T} e^{-\rho t}\left[N_{it}\cdot u\left(\frac{c_{it}}{N_{it}}\right) - p_{it}\cdot \bar{u}_{it}\right] \tag{13}$$

$$\text{s.t.} \quad c_{it} = x_{it} - A_{it}\cdot h_{it}^{\xi}, \quad \xi > 1; \tag{14}$$

$$\sum_{t=S_i}^{T} e^{-rt}\left(y_{it} + h_{it}\cdot w_{it} - x_{it}\right) \geq 0, \tag{15}$$

$$H_{i,t+1} - H_{it} = h_{it}, \quad H_{S_i} = 0, \tag{16}$$

$$w_{it} = B_{it}\cdot H_{it}^{\alpha}, \quad t \geq S_i; \tag{17}$$

$$y_{it} = 0 \quad \text{and} \quad h_{it} = 0 \quad \text{all} \quad t \geq R_i\,.$$

The following proposition shows that (1) continues to hold, though the parameter θ has a new interpretation.

Proposition 3: *The solution to problem (13) implies*

$$\sum_{t=S_i}^{T} e^{-rt} A_{it}\cdot h_{it}^{\xi} = \theta Y_{i0}^{F} \tag{18}$$

where

$$\theta = \frac{1}{\xi}\left(1+\alpha\right).$$

For a given R_i and any $s \geq R_i$,

$$\frac{a_{is}}{Y_{is}^{M} + (1-\theta)\cdot Y_{is}^{F}} = \frac{\sum_{t=s}^{T} N_{it}\cdot \sigma^{t}}{\sum_{t=S_i}^{T} N_{it}\cdot\sigma^{t}}\,, \tag{19}$$

where σ, Y^M and Y^F are defined in Proposition 1.

Proof: See Appendix.

Proposition 3 shows that while the value of forgone home production is still proportional to lifetime earnings, the coefficient of proportionality θ now depends on α as well as ξ. The parameter α is the elasticity of the wage with respect to experiential human capital. Expressions (18)-(19) imply that regression equation (1) remains valid — with the caveat that the parameter θ must be interpreted with additional care.

Endogenous Retirement Decision. We can also allow for an endogenous choice of retirement age without disturbing our results. To see this, consider the following change to the utility function:

$$\hat{u}\left(c_{it}; R_i\right) = \begin{cases} \frac{1}{\gamma} c_{it}^{\gamma} & \text{for } t < R_i\,, \\ \frac{1}{\gamma} c_{it}^{\gamma} + \Gamma_i & \text{for } t \geq R_i\,, \end{cases}$$

where $\Gamma > 0$ is a flow benefit to being retired (e.g. Rust and Phelan 1997, Bound *et al.* 1999). In addition to choosing the flow allocations $\{c_{it} \geq 0,\, h_{it} \geq 0\}_{t=S_i}^{T}$, the household now chooses R_i when it maximizes its discounted lifetime utility. We have

Proposition 4: *The optimal solution of (2) with utility function $\hat{u}(\cdot)$ replacing $u(\cdot)$ and maximization over R_i, c_{it} and h_{it} implies condition (7) and equation (1).*

Proof: For any given R_i, (6)-(7) hold. Enumerating all possible retirement ages, choose the one yielding maximal lifetime utility. Fixing this R_i, we are done. ■

The straightforward nature of the proof of Proposition 4 shows that its results carry over to formulations (12) and (13) as well.

Summary. Propositions 2-4 show that, without compromising equation (1), we can generalize our framework to include female labor force participation decisions, experiential human capital accumulation, and endogenous retirement decisions. The Corollary to Proposition 1 shows that the inclusion of a constant term in our basic regression equation enables us to incorporate male home production as well. The next two sections discuss estimation. Estimation requires household data on NW_{it}, Y_{it}^{M}, Y_{it}^{F}, and N_{it}. Section 4 describes our data; Section 5 presents the empirical analysis.

4 Data

Overview. We use data from the Health and Retirement Study (HRS) to construct a comprehensive measure of household net worth and to calculate lifetime earnings of men and women. The implementation of our empirical strategy also requires data on, among other things, the age of the household, the age of retirement, and the number of equivalent adults in the household at each date. This section (and the appendix) presents a detailed description of our data inputs. A reader most interested in our parameter estimates could proceed directly to Section 5.

Sample Criteria. We use the original survey cohort from the HRS, consisting of households in which the respondent is age 51-61 in 1992 (Juster and Suzman 1995). Our analysis focuses on married couples. The survey waves 1992, 1994, 1996, 1998, 2000, and 2002 have 4663 married couples.[4] Because wealth at retirement may be significantly affected by marriage history (e.g. Guner and Knowles 2004), we limit attention to couples in which each spouse has been married only once. This reduces the sample to 3046 households. The number of couples with single–marriage spouses for which we can construct lifetime earnings for both spouses is 1582 (see below). We require birth dates for all children, which further limits the sample to 1581 households.

Our definition of "retirement" is as follows. The HRS asks individuals whether they are retired; it separately asks whether their retirement status is fully retired, partly retired, or not retired. In our analysis, an individual is retired if he (or she) answers yes to the first question, lists his/her retirement status as fully retired, or works less than 500 hours per year and does not list his/her retirement status as not retired. In addition, we require males to be collecting Social Security Benefits and to report the amount. We exclude from the sample males who never worked. Our procedure classifies each spouse as retired or not. We include in our analysis only those household observations for which both the man and the woman are retired by 2002.

We further restrict the sample as follows. As a protection against coding errors, we exclude any household observation with negative HRS net worth or comprehensive net worth above \$5 million. We exclude males who are disabled when they retire, males who retire but later return to work, males who retire before age 56 or after age 68, couples with age difference exceeding 6 years, and males or females less than 4 years short of the mean age of death (74 and 80, respectively).

[4] Unless otherwise noted, all HRS data is in public–use files — see http://hrsonline.isr.umich.edu/.

After restricting the sample to households in which both spouses are retired and making the other adjustments described in this paragraph and below, our final sample has 954 observations from 441 households.

4.1 Demographic variables

Household size. Our sample includes data from households with differing numbers of children and differing birth dates for children, age of marriage, etc. To incorporate this heterogeneity, we follow Tobin's (1967) procedure for aggregating numbers of family members into a single index of "equivalent adults." Specifically, for household i at age t, let the number of equivalent adults N_{it} be

$$N_{it} = 1 + \chi_i^S(t) \cdot \alpha^S + \chi_i^C(t) \cdot \alpha^C \,, \tag{20}$$

where $\chi_i^S(t)$ is an indicator for the presence of the spouse, $\chi_i^C(t)$ is number of children aged 0-20 present in household i at age t, and α^S and α^C are parameters. Our data set includes the birth date for each child. We assume that children leave their parent's household when they reach age 20.[5]

	Demographic variables					
Statistic	Male age of start work[c]	Male age of marriage	Number of children	Male age of retirement	Male age	Female age
Minimum	18	16	0	56	59	56
Median	19	23	3	62	66	64
Maximum	23	36	10	68	70	72
Mean	19.84	23.42	3.07	61.47	66.16	63.71
Coeff. Var.	0.1	0.14	0.47	0.04	0.04	0.04

a. Sample is households of "financial respondents" from original HRS with valid, nonnegative HRS net worth; married with age difference of 6 years or less; both spouses retired (see text); lifetime earnings are available for both spouses (see text); male retirement age is 56-68; male age is less than 71; female age is less than 77.
b. HRS household weight is applied to each observation.
c. The maximum of years of education plus 6 and 18.

Table 1. Demographic variables[a,b]

Household age. We define the age of the household to be the age of the adult male. The household begins when either the man or the woman becomes independent. We define independence as the maximum of age 18 or the individual's years of education plus 6. Prior to marriage, the man and the woman live apart and thus each contribute 1 to the household's equivalent adult total. Once they marry, prior to their first child, the number of household equivalents is 1.5. Formally, let S_i^M be the maximum of 18 and the adult male's years of education plus 6, let S_i^F be the same

[5] We experimented with different departure ages, and they make little difference to our estimates.

for the adult female, and let D_i be the female's birth date minus the male's. The household begins at age S_i with

$$S_i = \min\{S_i^M, S_i^F + D_i\}.$$

(Note that D_i serves only to restate the age of the household in terms of the man's age.) For ages s with $S_i \le s < \max\{S_i^M, S_i^F + D_i\}$, N_{is} is 1. If M_i is the man's age of marriage, for $\max\{S_i^M, S_i^F + D_i\} \le s < M_i$, N_{is} is 2. If C_i is the man's age at the birth of the couple's first child, then N_{is} is 1.5 for $M_i \le s < C_i$. We make adjustments for children as described above. When all children have left, until the first adult dies, N_{is} is 1.5. After the first adult's death, N_{is} drops to 1. Males die at the conclusion of age 74; females die at the conclusion of age 80.

Our framework implicitly assumes that upon reaching age S_i, the adults of household i choose their marriage date and the number and timing for their eventual children. In fact, given the model's nonstochastic environment, parents can plan at the same time, in detail, their life–cycle saving, retirement age, and pattern for female market work.

Table 1 presents demographic information on our sample.

4.2 Household Net Worth

In every survey wave, the HRS obtains an inventory of each household's assets. This measure includes the value of the household's home, other real estate, automobiles, bank accounts, stocks and bonds, equity in a family business, and equity in insurance. The HRS also collects information on the total value of household debts. While the difference between the assets and debts is a measure of net worth, it excludes the value of future pension flows, Social Security Benefits, and Medicare payments. Our preferred measure of net worth includes the capitalized value of the latter benefit flows.

Capitalized pensions. To capitalize pension, Social Security, and Medicare benefit flows (as well as lifetime earnings below), we assume a constant net-of-tax interest rate equaling a gross-of-tax interest rate of 5 percent per year,[6] less an income tax of 11.69 percent per year.[7] In other words, $r = .05 \cdot (1.0 - .1169) = 0.0442$. The HRS asks retired households about their pension and Social Security benefits. To approximate the income tax code, as we capitalize Social Security benefits, we subject only one-half to income taxes. (We use annual tax rates for all flows – see the preceding footnote.) For females under 62, we compute future Social Security benefits on the basis of statutory entitlements (including spousal benefit shares). Similarly, we project future household benefits on the basis of statutory rules and our ages of death. We subject private pension benefits to full (annual) income taxation. For the first two pensions, and the first two annuities, the HRS collects data on whether the flow is real or nominal, and on whether the flow carries survivorship rights. Our calculations assume a nominal interest rate three percent higher than our real rate.

[6] This is roughly the ratio of factor payments to capital to the market value of private net worth. For the numerator, NIPA Table 1.13 gives corporate business income, indirect taxes, and total labor compensation. The first less the other two is a measure of corporate profits (net of depreciation); the ratio of profits to profits plus labor remuneration is "profits share." Multiply the latter times corporate and noncorporate business income plus nonprofit–institution income, less indirect taxes. To this, add the income of the household sector (see NIPA Table 1.13) less indirect taxes and labor remuneration. Finally, reduce the numerator by personal business expenses (brokerage fees, etc. from NIPA Table 2.5.5, rows 61–64). For the denominator, use U.S. Flow of Funds household and private non–profit institution net worth (Table B.100, row 19), less government liabilities (Table L106c, row 20). Average the net sum at the beginning and end of each given year. The average ratio 1952–2003 of the numerator to the denominator is .0504.

[7] Our yearly tax rate comes from NIPA personal current taxes divided by personal income gross of contributions for social insurance but less one-half of transfer payments (see the treatment of Social Security benefits below). The average 1950-2002 is 0.1169.

(We assume that third pensions and annuities, if applicable, are nominal and have no survivorship rights).

Medicare benefits. We capitalize Medicare Benefits as follows: Tables 8.A1-2 from the 2001 *Annual Statistical Supplement to the Social Security Bulletin* provide annual Hospital Insurance and Supplementary Medical Insurance (SMI) expenditure as a ratio to premiums from aged participants. Multiplying the ratio minus one times the per capita SMI premium gives the approximate net annual benefit for people 65 and over.

Finally, because some retirees continue to work part time, we capitalize remaining lifetime earnings net of taxes.

Table 2 presents detailed information on net worth in our sample. The first column presents HRS assets less debts. Column 2 presents the capitalized value of present and future private pension flows. Column 3 presents the capitalized value of Social Security benefits. Column 4 presents capitalized Medicare benefits, and column 5 presents the capitalized value of any remaining lifetime earnings. Column 6 sums HRS assets less debts, capitalized remaining private pensions, Social Security Benefits, Medicare Benefits, and remaining earnings. The resulting "total net worth" is the *NW* variable used in our analysis. Clearly, private pensions, Social Security, and Medicare make substantial contributions to the table's last column. All figures are present values at respondent age 50, in 1984 dollars.

Statistic	HRS Net worth	Capitalized future				Total Net worth
		Private pension flows	Social Security Benefits	Medicare flows	Earnings	
Minimum	20	0	3,042	40,843	0	82,639
Lower Quartile	121,167	0	84,469	58,616	0	366,784
Median	220,215	51,827	110,737	62,322	0	497,393
Upper Quartile	415,188	137,720	129,110	65,750	8,560	686,404
Maximum	4,455,000	1,413,520	250,845	74,604	437,888	4,947,460
Mean	346,483	86,943	103,952	61,836	10,171	609,385
Coeff. Var.	1.21	1.27	0.37	0.09	0.26	0.73
Observations	954	954	954	954	954	954
Households	441	441	441	441	441	441

a. Sample is the same as Table 1.
b. HRS household weight is applied to each observation.

Table 2. Distribution of Household Net Worth: 1984 dollars.[a,b]

4.3 Lifetime earnings

For both men and women, the HRS provides annual earnings (as well as market hours) in each survey wave. If an HRS participant signs a permission waiver, we also have his or her Social Security

Administration (SSA) annual–earnings history for the years 1951-91.[8]

The SSA earning histories are a unique resource. They are not, however, without disadvantages that complicate analysis. (i) The Social Security System does not cover all jobs — the linked data provide no information on non-FICA employment. (ii) SSA earnings data are right–censored in some cases because they only track earnings up to the year's statutory maximum income level subject to the Social Security tax.[9] (iii) Social Security records do not include work hours or wage rates — though fortunately our analysis does not require that data.

Male earnings. We estimate a statistical model of earnings dynamics using a maximum likelihood procedure that utilizes censored as well as uncensored male data (see the Appendix). We utilize SSA earning histories that are available and full–time earning observations from the surveys. Using the observations for a male, we estimate his random effect. If he does not have any valid full–time observations, he is excluded from the sample. Although his SSA history prior to 1992 might have gaps from non–FICA jobs (or he might not even have signed the SSA waiver), we use the earnings dynamics model to impute his annual earnings for each year prior to 1992. After 1992, we rely on earnings data from the surveys. The details are in the Appendix.

Our data include only take–home pay. To account for non–wage compensation, we multiply measured earnings at each age by that year's ratio of NIPA total compensation to NIPA wage and salary accruals. We then subtract employee and employer payroll taxes (subject to year-specific Social Security earnings cap) and income taxes at a year-specific average rate. As with pensions, we then capitalize annual earnings with $r = 0.0442$.

[8] Access to this data is more restricted than the HRS survey data. See http://hrsonline.isr.umich.edu/.

[9] The SSA also provides linked W2 tax reports annually for1980-91. Although the W2 records are right–censored for confidentiality, the upper limit is substantially higher than the Social Security earnings cap — $125,000 for earnings under $250,000; $250,000 for earnings under $500,000, and $500,000 for earnings above that amount. In practice, we assume right–censoring at $125,000 for all W2 amounts at or above $125,000. The W2 amounts include non–FICA earnings — and separately identify the latter. They omit some tax deferred pension amounts. Although they also omit self–employment earnings, they identify Social Security measures of the latter. In practice, an individual may have multiple jobs, and we add the corresponding W2 amounts.

Statistic	Male earnings (Present value at male age 50)	Female earnings (Present value at female age 50)
Minimum	525,659	0
Lower Quartile	1,203,460	95,632
Median	1,479,490	280,911
Upper Quartile	1,907,780	542,388
Maximum	12,087,400	1,559,910
Mean	1,677,120	358,575
Coeff. Var.	0.56	0.90
Observations	954	954
Households	441	441

a. Male sample: once-married males with 9-24 years of education and at least one earnings observation. Female sample: once-married females with 9-24 years of education and either linked SSA earnings history or never worked prior to 1992 (see Table 1).
b. HRS household weight is applied to each observation.

Table 3. Distribution of male and female lifetime earnings: gross of benefits, net of taxes; 1984 dollars (NIPA PCE deflator).[a,b]

Female earnings. In our analysis of female lifetime earnings, we are much more concerned with the possibility of part–time work prior to 1992. We limit our sample, therefore, to women with linked SSA earning histories. As with men, we estimate an empirical earnings dynamics equation, which we use (in the case of women) only to impute right–censored observations. We take remaining observations prior to 1992 – including zeros – directly from the linked data. The Appendix provides details.

To deal with the possibility of non–FICA employment prior to 1992 (which appears as a zero in the linked data), we take additional steps as follows. We re–estimate an earnings dynamics equation using a very comprehensive sample. In the survey, women are asked how many years of non-FICA employment before 1992 that they had, and they are given the opportunity to report an interval of time when such employment occurred. If a women reports that she has previous years of non–FICA employment and reports the number of years, we use the earnings dynamics equation to impute such values, probabilistically imputing to years without SSA data. If she reports both the number of years and the time interval when that employment occurred, we impute for only years in the reported interval. If a women indicates that she has previous years of non–FICA employment but provides no information on number of years, we drop her household from the sample as having incomplete earnings data.

Finally, as in the case of males, we multiply earnings at each age by the year's ratio of NIPA total compensation to NIPA wage and salary accruals and subtract taxes.

Table 3 summarizes the distribution of lifetime earnings for males and females in our sample. Average female earnings are only about 20 percent as high as males, presumably reflecting lower hours and lower participation rates than males in this age cohort.

5 Estimation and Results

Our life-cycle model implies a simple relationship between a household's retirement wealth and the male/female composition of its lifetime earnings. Equation (1) — or, in the case of male home-production options, (11) — describes the relationship. Using HRS data, including linked Social Security earning histories, this section estimates the equation's coefficients. Throughout, we assume that all households have the same parameters σ, γ, and ρ, and that all face the same after-tax interest rate r.

Regression. We estimate

$$\ln(NW_{is}) = \beta_0 + \ln\left(Y_{is}^M + (1-\beta_1)\cdot Y_{is}^F\right) + \ln\left(\sum_{t=s}^{T} N_{it}\cdot[\beta_2]^t\right) -$$

$$- \ln\left(\sum_{t=S_i}^{T} N_{it}\cdot[\beta_2]^t\right) + \sum_{j=3}^{8}\beta_j X_{ij} + \mu_i + \eta_{is}, \tag{21}$$

$$\sum_{j=3}^{8}\beta_j = 0\,, \tag{22}$$

where NW_{is} is the comprehensive net worth (see Table 2) of household i at age $s \geq R_i$, Y_{is}^M is the present value of after–tax male lifetime earnings, Y_{is}^F is the present value of female lifetime earnings, T is the household's terminal age, and N_{it} comes from (20). Both Y_{is}^F and Y_{is}^M are present values at age s. We have data from up to six waves of the HRS (i.e., 1992, 1994, 1996, 1998, 2000, and 2002). We include year dummies to control for possible time-varying capital gains that are outside the scope of the analysis. X_{ij} is a dummy variable that is 1 if the data for NW_{is} comes from HRS wave $j = 3$ (1992),..., 8 (2002), and $X_{ij} = 0$ otherwise. To leave the possibility of a separate constant, we impose constraint (22). The error structure is the sum of a household random effect, μ_i, and measurement error in net worth, η_{is}. We estimate β_0, β_1,...,β_8 by non-linear least squares (NLLS). We use a two–stage procedure. The first stage estimates (21)-(22) without covariance restrictions. As in Greene (2003), we use the residuals to obtain a consistent estimate of $\sigma_\mu^2/\sigma_\eta^2$. The second stage generates FGLS estimates of β_j, $j = 0, 1, ..., 8$.

Equations (21)-(22) allow us to estimate the baseline model with no male home production, in which we impose $\beta_0 = 0$, as well as the extended model with both male and female home production. In either case, $\beta_1 = \theta$ and β_2 corresponds to σ, i.e.,

$$\beta_2 \equiv \sigma = \exp\left\{-r + \frac{r-\rho}{1-\gamma}\right\}..$$

On the basis of our theoretical specification, we expect $\beta_1 \in [0\,,\,1]$. We have $r = 0.0442$, and the existing literature usually sets $\rho \geq 0$ and $\gamma \leq 0$; hence, we anticipate $\beta_2 \in [e^{-r}, e^0] = [0.9568, 1.0]$. Because all households in our sample start with a husband and wife, the value of α^S makes little difference, its only impact coming at the very end of life when the different mortality rates for men and women affect household composition. The ratio $\alpha^C/(1+\alpha^S)$ matters more. Existing estimates of adult equivalency scales (e.g. Fernandez-Villaverde and Krueger 2006, Laitner and Silverman 2005) suggest ratios from 0.1 to 0.3. Accordingly, we consider variations in the ratio from 0 to 0.33.

Table 4 presents our estimates. The first three columns report estimates for our baseline model, which imposes $\theta^m = 0$. In this formulation, $\theta^f = \theta$. The estimates of θ are significantly different

Estimated Parameters[a,b]	Baseline case: $\theta^m = 0$.			Extended model: $\theta^m > 0$.		
	$\alpha^C = 0$	$\alpha^C = 0.15$	$\alpha^C = 0.5$	$\alpha^C = 0$	$\alpha^C = 0.15$	$\alpha^C = 0.5$
β_0	-	-	-	-0.1474 (0.2981)	-0.1100 (0.2947)	-0.0469 (0.2774)
θ	0.2645** (0.1220)	0.2630** (0.1221)	0.2608** (0.1223)	0.2576** (0.1220)	0.2567** (0.1221)	0.2567** (0.1223)
σ	0.9762*** (0.0012)	0.9762*** (0.0012)	0.9761*** (0.0012)	0.9812*** (0.0104)	0.9798*** (0.0101)	0.9777*** (0.0093)
DUM 1992	-0.1944* (0.1091)	-0.1949* (0.1091)	-0.1960* (0.1091)	-0.1873* (0.1103)	-0.1902** (0.1102)	-0.1951** (0.1099)
DUM 1994	-0.0573 (0.0602)	-0.0569 (0.0602)	-0.0563 (0.0602)	-0.0521 (0.0612)	-0.0533 (0.0610)	-0.0550 (0.0608)
DUM 1996	-0.0038 (0.0409)	-0.0036 (0.0409)	-0.0034 (0.0409)	-0.0015 (0.0411)	-0.0019 (0.0411)	-0.0026 (0.0411)
DUM 1998	0.0419 (0.0355)	0.0419 (0.0354)	0.0420 (0.0354)	0.0407* (0.0104)	0.0412* (0.0355)	0.0420* (0.0355)
DUM 2000	01158*** (0.0342)	0.1157*** (0.0359)	0.1157*** (0.0341)	01104*** (0.0361)	0.1120*** (0.0359)	0.1145*** (0.0355)
DUM 2002	0.0977*** (0.0346)	0.0978** (0.0346)	0.0981*** (0.0345)	0.0898** (0.0386)	0.0922** (0.0383)	0.0962** (0.0376)
$\frac{\sigma_\mu^2}{\sigma_\eta^2}$	0.2578	0.2577	0.2576	0.2589	0.2592	0.2594
Calculated parameters[c]						
$\theta^m = 1 - e^{\beta_0}$	0	0	0	0.1371 (0.2573)	0.1042 (0.2640)	0.0459 (0.2647)
$\theta^f = 1 - (1-\theta)e^{\beta_0}$	0.2645** (0.1220)	0.2630** (0.1221)	0.2608** (0.1223)	0.3594* (0.2086)	0.3341 (0.2133)	0.2908 (0.2146)
Summary statistics						
Observations	954	954	954	954	954	954
Households	441	441	441	441	441	441
Mean sq. err.	0.2263	0.2263	0.2262	0.2266	0.2266	0.2265
R^2	0.3097	0.3098	0.3099	0.3095	0.3096	0.3098

Significant at: * - 10 percent, ** - 5 percent, *** - 1 percent

a. Error structure $\mu_i + \eta_{is}$ (see text); consistent estimate of $\sigma_\mu^2 / \sigma_\eta^2$ reported.

b. Estimated dummy variable coefficients constrained to sum to zero.

c. Standard errors from delta method.

Table 4. Parameter estimates.

from zero (at the 5 percent significance level) and significantly different from one (at the 1 percent significance level). The point estimates are all roughly 0.26, with standard errors of approximately 0.12. The point estimates suggest that, on average, for every dollar that a woman earns in the market, her household must recover 26 cents of lost home production. Put differently, on average, every dollar that women earns in the market represents a net increase in total household income of roughly 74 cents. The estimate of θ suggests that as married women's pre-retirement time allocations have shifted toward market work in recent decades, the value of forgone home production has been small relative to their earnings.

The last three columns of Table 4 present estimates for the model with both male and female home production. We estimate (21)-(22) with an unrestricted constant β_0 and then recover an estimate of θ^m from $\beta_0 = \ln(1-\theta^m)$. Given estimates for θ and θ^m, equation (10) implies $\theta^f = \theta^m + \theta \cdot (1-\theta^m)$. (The so–called delta method yields standard error for θ^m and θ^f.) While the estimates of θ are insensitive to the value of α^C, the estimate of θ^m varies somewhat with the equivalency weight of children. Nevertheless, the implied point estimates for the female loss coefficient θ^f fall in the range 0.29 to 0.36, which seems quite close to the baseline model's estimate of 0.26. The standard errors of θ^f for the generalized model are relatively large compared to the point estimates, so that corresponding t-statistics are all roughly 1.5. Although the estimates for θ^f are not statistically significantly different from zero, they are significantly different from 1. The latter is consistent with a marginal cost function for forgone home production that increases with labor market hours. That is, the average cost of forgone home production is evidently less than the marginal cost, and the total net gain from female market employment is substantial — on the order of 64 to 71 cents per dollar of earned.

Since the estimated θ is clearly less than 1 in the last three columns of Table 4, θ^f is significantly greater than θ^m. This is consistent with the evidence on home production hours presented in Figure 1 of Section 2.

The point estimates in Table 4 for the male coefficient θ^m range from 0.046 to 0.137 and are never significantly different from zero. Overall, Table 4 seems broadly consistent with our baseline model, where the value of male home production is zero.

Finally, the time pattern of year-dummy coefficients throughout Table 4 seems to fit the behavior of stock market for 1992-2002: the dummy coefficients grow from 1992 to 2000, when the market was rising, and they fall in 2002, after the stock market had peaked.

The next subsection considers the implications of our estimates.

5.1 Quantitative Implications

Results in Table 4 indicate that the cost of forgone home production due to female labor force participation is relatively modest. On average, for each dollar a woman earns in the labor market, her household has a net gain of roughly 75 cents — after purchasing 25 cents of market substitutes for sacrificed home production. This subsection uses the table's parameter estimates to calculate aggregative changes in the level of home production in the recent past and implications for the elasticity of female labor–market hours.

The Net Contribution of Female Participation to GDP. As more women join the labor force, their earnings augment GDP. Because these women previously engaged in home production that the National Income and Product Accounts omitted, the true increase in domestic economic activity is presumably less than the measured increase. The gap corresponds to the value of forgone home production. With our estimates of θ^m and θ^f, we can calculate the difference between the measured GDP and GDP net of lost home production.

Measured GDP at time t is the sum of (gross of tax) male flow earnings $y^m(t)$ and female flow earnings $y^f(t)$ divided by labor's share, say ζ,

$$GDP(t) = \frac{1}{\zeta} \cdot [y^m(t) + y^f(t)].$$

On the other hand, the Corollary to Proposition 1 implies that GDP net of purchases of home–production substitutes is

$$GDP^*(t) = \frac{1}{\zeta} \cdot \left[y^m(t) + y^f(t)\right] - \theta^f \cdot y^f(t) - \theta^m \cdot y^m(t).$$

Let $f(t)$ denote the share of measured female earnings in total earnings

$$f(t) = \frac{y^f(t)}{y^m(t) + y^f(t)},$$

and define $\omega(t)$ to be the share of measured GDP devoted to recovering lost home production. Then

$$\omega(t) = \frac{GDP(t) - GDP^*(t)}{GDP(t)} = \frac{\theta^f \cdot y^f(t) + \theta^m \cdot y^m(t)}{\frac{1}{\zeta} \cdot [y^m(t) + y^f(t)]} = \zeta \left[\theta^f \cdot f(t) + \theta^m \cdot (1 - f(t))\right].$$

Changes in $\omega(t)$ over time have a direct interpretation: the changes indicate the value of goods, as a fraction of GDP, formerly produced at home but now part of measured GDP. As more women join the labor force, and as they choose to spend longer hours at market work, both $f(t)$ and $\omega(t)$ rise.

We can calculate $\omega(t)$ using our estimates of θ^m and θ^f. The share of female earnings $f(t)$ comes from the Census data in the Integrated Public Use Microdata Series (IPUMS) for the years 1959, 1979, and 1999. We calculate total male earnings and total female earnings. The ratio $f(t)$ follows.

Table 5 presents our results. We choose $\zeta = 0.7$ and calculate the implied change in $\omega(t)$ for pairs (θ^m, θ^f) of point estimates from Table 4. The change in $\omega(t)$ from 1959 to 1999 is in tight range 0.024-0.028 (see Table 5). This means that roughly 2.4-2.8 percent of measured GDP in 1999 consisted of goods and services that were produced at home in 1959. (Note that this assessment may be an upper bound because it assigns the same forgone home production to single women as it does to the married women of our sample on whom our estimation is based.)

Census year[b]	1960	1980	2000
Number of males	425,022	564,381	748,358
Number of females	445,426	594,511	771,042
Male average pre-tax wage earnings	$ 4,111	$ 14,300	$ 36,253
Female average pre-tax wage earnings	$ 989	$ 5,160	$ 19,554
Ratio of male to female earnings	3.9664	2.6307	1.7995
Ratio male to total earnings	0.7986	0.7246	0.6428
Ratio of female to total earnings, $f(t)$	0.2014	0.2754	0.3572
Changes in $\omega(t)$ from 1959 to 1999			
$\theta^m = 0, \theta^f = 0.26$	0.028		
$\theta^m = 0.14, \theta^f = 0.36$	0.024		
$\theta^m = 0.10, \theta^f = 0.33$	0.025		
$\theta^m = 0.05, \theta^f = 0.29$	0.027		

a. IPUMS (see text) are random samples of one percent of U.S. population. From the samples, we use all men and women age 22-62, including top-coded observations and zeros.
b. Data from each Census refer to earnings the years before.

Table 5. IPUMS Census male and female earnings over time: current dollars.[a]

Table 5 shows that the reduction in home production output that accompanied rising female labor force participation did not cause a large reduction in GDP growth net of home production substitutes. Use the notation $\widehat{x} \equiv \dot{x}/x$. Then for 1959-1999 the average correction to the annual growth rate of GDP is

$$\widehat{GPD^*} = \widehat{GDP} + \widehat{(1-\omega)} = \widehat{GDP} - 0.0007.$$

That is, correcting for goods formerly produced at home but now included in GDP requires an adjustment to the annual GDP growth rate of less than 0.1%.[10]

Labor supply elasticity. While the main focus of our analysis is the value of forgone home production, our model, and the parameter estimates, have implications that extend beyond the measurement of home production output. Among other things, the model implies a relationship between the value of forgone home production and the intensive labor supply elasticity for married women. First–order condition (6) implies a value for the uncompensated, intensive-margin labor supply elasticity of working, married women. Differentiating (6) shows that the elasticity of h^f_{it} with respect to w^f_t, holding A^f_{it} fixed, is

$$\frac{d\,\ln(h^f_{it})}{d\,\ln(w^f_t)} = \frac{\theta^f}{1-\theta^f}\,. \tag{23}$$

[10] Female labor-market participation costs might augment our model's cost of lost hours at home. If, as in Section 3, we assume, however, that participation costs take the form of decreases in utility, there is no need to make further adjustments to GDP.

The interval of estimates for θ^f is $[0.26, 0.36]$. The corresponding female labor supply elasticities in (23) vary from 0.35 to 0.56. The latter range includes the 0.43 estimate of the same elasticity for employed married women in Pencavel (1998) — based on a different modeling framework and different data sources. Our point estimates of male labor supply elasticity are much lower, 0 to 0.16.

The magnitude of male and female labor supply elasticities is potentially important for macroeconomic analyses at business–cycle frequencies (e.g., Benhabib *et al.* 1991, Greenwood and Hercowitz 1991). Our results reinforce the possible role of female labor-supply behavior in this regard. (See also, for example, Benhabib *et al.* 1991, McGrattan *et al.* 1997, and Rupert *et al.* 2000.) It should be emphasized, however, that our model's implications pertain only to the intensive labor supply elasticity (i.e., the choice of hours conditional on participation). In our specification, all pre-retirement women participate in both home production and market work. An extensive margin would emerge only with the inclusion of a fixed cost of market participation.

We should also point out that even the intensive labor supply elasticity might be different in a specification with human capital. For our baseline model, the immediate impact on labor supply of permanent and temporary wage–rate changes of the same magnitude are identical. For the model of Proposition 3 with human capital, a permanent change in the wage rate would have a larger effect on labor supply. The intuition is straightforward. In the model with human capital, working in the market has two distinct payoffs: (i) immediate wage earnings, and (ii) increased human capital formation, which augments future earnings. A permanent increase in the wage rate increases the current value of both components, while a temporary change affects only the first. Naturally, the intensive labor supply elasticity for a permanent change in wages is highest for young workers, who stand to benefit the most from human capital accumulation.

5.2 Potential Sources of Bias

This subsection briefly discusses two features of life–cycle behavior that are present in reality but omitted from our model, and hence possible sources of bias for our parameter estimates. Bequests and inheritances are the first; earnings uncertainty and precautionary saving are the second. We also discuss the potential endogeneity of the level of home production as a possible source of bias in our modelling specification.

Intergenerational Transfers. Our analysis does not include bequests and inheritances. To show their possible role, let the present value at age s of the inheritance of household i be I_{is}, and let the household's bequest (at death) be B_i — with the latter's present value at age s being

$$B_{is} = B_i \cdot e^{-r\cdot(T-s)}. \tag{24}$$

Then our baseline life–cycle model implies

$$\frac{a_{is} - B_{is}}{Y_{is}^M + (1-\theta)\cdot Y_{is}^F + I_{is} - B_{is}} = \kappa_i(s\,,\,\sigma)\,.$$

After some algebra,

$$\frac{a_{is}}{Y_s^M + (1-\theta)\,Y_s^F} = \kappa_i(s,\sigma) + \kappa_i(s,\sigma)\frac{I_{is}}{Y_s^M + (1-\theta)\,Y_s^F} + (1-\kappa_i(s,\sigma))\,\frac{B_{is}}{Y_s^M + (1-\theta)\,Y_s^F} \tag{25}$$

This expression closely resembles (7), but the two right–most terms are new.

We have several reasons for thinking that omitting bequests and inheritances in the present context may not significantly affect our results. First, the omitted terms seem to be small. Some authors suggest that private intergenerational transfers may play a significant role in aggregate wealth accumulation (e.g., Kotlikoff and Summers, 1981, Laitner 2002); however, to the extent that large–scale transfer activity is concentrated among the wealthiest family lines, it is unlikely to appear in middle class samples such as the HRS. (Indeed, Section 4 filters out the few very high asset figures as protection against coding errors.) Table 3 of Laitner and Ohlsson's (2001) analysis of the PSID shows household inheritances averaging about 2 percent of lifetime earnings. See also Hendricks (2001). Recent compilations of inheritances from the HRS (e.g. Table 8 of ch. 2 in Perry, 2007) imply similar percentages. In a steady state with 2 percent per capita long-term growth and a generational span of 20 years, if I_{is} is 2 percent of lifetime earnings, then B_i would average about 3 percent — though discounting would make B_{is} in (24) smaller. The last two terms of (25) present the ratio of a weighted average of I_{is} and B_{is} to household earnings net of lost home production. Hence, the two additional terms in (25) should lie within the range 0.02-0.03. Tables 2-3 of Section 4 show the left–hand side of (25) has an average value of roughly 0.30. So, the sum of the new terms is an order of magnitude smaller than the left hand side of (25).

Second, a share, perhaps a large share, of middle–class bequests may be accidental in the sense of arising from self–annuitization within family lines (e.g., Hurd 1987, Kotlikoff and Spivak 1981). Accidental bequests would tend not influence our results, because the B_{is} term of (25) would blend into our regression error term rather than being a part of household planning.

Precautionary Savings. This paper's model also abstracts from earnings risk. In particular, one might wonder if, in practice, dual earner households need less precautionary saving due to their diversified labor income. The answer does not seem clear-cut. The total labor income of a household consists of market earnings as well as home product. It is likely that women produce substantial output at home: indeed, households purchase life insurance for housewives that do not engage in market work (Hong and Rios-Rull, 2006). It is not clear *a priori* that dual-earner households have a better diversified labor income. In fact, households where the wife works at home may be better insured against certain risks, such as, for example, aggregate shocks to market wages.

Both family–line intergenerational transfers and behavior under uncertainty are interesting topics for future research. We would argue, however, that neither necessarily will have a critical effect on our estimates of the value of women's home production hours.

Endogeneity of Home Production. Our model specification assumes that any forgone home production must be replaced with market substitutes and that the overall level of home production is exogenous. (See the discussion in Section 2.1.) Thus, if a woman works in the market, the household must purchase day-care services or hire a nanny to take care of children. The assumption that households must recover all forgone home production allows us to identify the parameter θ through observed variation in household assets at retirement.

While analytically convenient, it is possible that, in reality, households may not recover all of their lost home production. Instead, households could choose to reduce consumption of home goods rather than replace them with market substitutes. Alternatively, it may be that market goods simply cannot be substituted for all forgone home production. In either case, if households recover only some of the lost home production, then our estimates will be biased downward.

In terms of our model, if we let φY^F be the value of lost home production and let ν be fraction home production recovered with market substitutes, then our estimates would give $\theta = \varphi\nu$. In our framework, we make assumptions that ensure $\nu = 1$ so that our estimate of θ gives us the value of lost home production as a fraction of measured female income. If $\nu < 1$ then $\varphi = \theta\nu^{-1} > \theta$. While this source of bias is possible, we think that households do in fact recover most if not all lost home production. In many cases, households simply have no choice but to replace lost cooking, laundry and child care with appropriate market alternatives.

6 Conclusion

We develop a life-cycle model that reveals a relationship between savings and the value of forgone home production from female labor force hours. We use this relationship to estimate parameters of the cost function for sacrificed hours at home. Our estimates indicate that the value of forgone home production is relatively modest. On average, for every dollar a woman earns in the labor market, her household forgoes only 25-35 cents of home production. Our estimates suggest that most of the measured increase in earnings of women entering the labor force in the past 50 years represents net gain. Unlike earlier papers that essentially assume a dollar-for-dollar trade-off between home and market production, our estimates suggest that only minor corrections to GDP are required to account for the reduction in home product.

Our analysis is a new application of the life-cycle framework. The life-cycle model has proved valuable for studying fiscal policy (e.g., Auerbach and Kotlikoff 1987), wage inequality (e.g., Rios-Rull 1993), the cost of children (e.g., Lee and Lapkoff 1988), retirement behavior (e.g., Gustman and Steinmeier 2000, French 2005, Laitner and Silverman 2005) and the importance of liquidity constraints (e.g., Mariger 1987, Deaton 1991). In virtually every case mentioned above, changes in female labor force participation on the scale of the changes observed in the past fifty years would have great influence on the analysis. Because of the scale of recent chnages in women's opportunities and economic behavior, we believe that studying specifications of the life-cycle model that explicitly incorporate female labor supply decisions is a priority.

Our results shed light on the elasticity of labor supply for women who participate in the labor force. Future analysis could estimate the fixed costs of participation outlined in Section 3 and should enable us to evaluate the elasticity of female participation. With more data, our framework can potentially allow one to measure households' relative efficiencies at home production as well.

Appendix

Proofs

We will first prove Proposition 3 and then establish Proposition 1 as a special case. Because there is no confusion in doing so, we drop the household subscript i for convenience.

Proof of Proposition 3: Consider the optimization problem (13)-(17). Eliminate x_t and w_t by substituting (14) and (17) into the lifetime resource constraint (15), which then becomes

$$\sum_{t=S}^{T} e^{-rt}\left(y_t + h_t \cdot B_t \cdot H_t^{\alpha} - A_t h_t^{\xi} - c_t\right) \geq 0 \tag{26}$$

Fix a participation profile $\{p_t\}$ such that the set $P = \{t : p_t = 1\}$ is non-empty. Let λ be the Lagrange multiplier on (26) and $\mu_S, ..., \mu_T$ be the Lagrange multipliers on (16). Define a Lagrangian

$$\mathcal{L} = \sum_{t=S}^{T} e^{-\rho t} N_t \cdot u\left(\frac{c_t}{N_t}\right) + \lambda \sum_{t=S}^{T} e^{-rt}\left(y_t + h_t \cdot B_t \cdot H_t^{\alpha} - A_t h_t^{\xi} - c_t\right) + \sum_{t=S}^{T} \mu_t \cdot (h_t + H_t - H_{t+1})\,.$$

The necessary conditions for optimality are

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Longleftrightarrow \left(\frac{c_t}{N_t}\right)^{\gamma-1} = \lambda e^{(\rho-r)t}, \quad S \leq t \leq T\,, \tag{27}$$

$$\frac{\partial \mathcal{L}}{\partial H_t} = \lambda e^{-rt} \alpha B_t \cdot H_t^{\alpha-1} \cdot h_t + \mu_t - \mu_{t-1} = 0\,, \quad S+1 \le t \le T. \tag{28}$$

$$\frac{\partial \mathcal{L}}{\partial H_{T+1}} = -\mu_T = 0\,. \tag{29}$$

Setting $h_t = 0$ for all $t \notin P$,

$$\frac{\partial \mathcal{L}}{\partial h_t} = 0 \Longleftrightarrow \lambda e^{-rt} B_t \cdot H_t^{\alpha} + \mu_t = \lambda e^{-rt} \cdot \xi \cdot A_t h_t^{\xi-1}\,, \quad t \in P\,. \tag{30}$$

Step I: Prove (18).
Let $\tau = \max P < R$ be the last period of participation and let h_t be the optimal solution to (13)-(17). By (29), $\mu_T = 0$. By definition of P, all $t > \tau$ have $h_t = 0$. By (28), $\mu_t = \mu_{t+1}$ for all $t \ge \tau$. Thus, $\mu_\tau = 0$.

Note that (28) implies that

$$\lambda e^{-rt} \alpha B_t \cdot H_t^{\alpha-1} \cdot h_t = \lambda e^{-rt} \alpha \frac{y_t^f}{H_t} = \mu_{t-1} - \mu_t$$

$$\Longleftrightarrow \lambda \alpha e^{-rt} y_t^f = H_t \mu_{t-1} - H_t \mu_t.$$

Summing from $t = S+1$ to τ and using the fact that $H_S = 0$ (which implies that $\sum_{t=S+1}^{\tau} e^{-rt} y_t^f = \sum_{t=S}^{\tau} e^{-rt} y_t^f$) gives

$$\lambda\alpha \sum_{t=S+1}^{\tau} e^{-rt} y_t^f \equiv \lambda \alpha Y_0^F = \sum_{t=S+1}^{\tau} \left[H_t \mu_{t-1} - H_t \mu_t \right].$$

The second summation can be written as

$$\sum_{t=S+1}^{\tau} \left[H_t \mu_{t-1} - H_t \mu_t \right] = H_{S+1}\mu_S + H_{S+2}\mu_{S+1} + H_{S+3}\mu_{S+2} + \ldots H_\tau \mu_{\tau-1}$$
$$- H_{S+1}\mu_{S+1} - H_{S+2}\mu_{S+2} - H_{S+3}\mu_{S+3} - \ldots - H_\tau \mu_\tau.$$

Collecting the terms with common multipliers,

$$\sum_{t=S+1}^{\tau} \left[H_t \mu_{t-1} - H_t \mu_t \right] = H_{S+1}\mu_S + \mu_{S+1}\left[H_{S+2} - H_{S+1}\right] + \ldots \mu_{\tau-1}\left[H_\tau - H_{\tau-1}\right] - H_\tau \mu_\tau.$$

Using $H_{S+1} = h_S$, $\mu_\tau = 0$, and $H_t - H_{t-1} = h_{t-1}$ for all $S+1 \le t \le \tau - 1$, we have

$$\sum_{t=S}^{\tau} \mu_t \cdot h_t = \alpha \lambda Y_0^F. \tag{31}$$

Rewrite first–order condition (30) as

$$\lambda e^{-rt} w_t \cdot h_t + \mu_t \cdot h_t = \lambda e^{-rt} \cdot \xi \cdot A_t h_t^{\xi}. \tag{32}$$

Notice that because $h_t = 0$ all $t \notin P$, (32) holds for all $S \leq t \leq T$. Summing (32) from $t = S$ to $t = \tau$ and using (31),

$$\lambda Y_0^F (1+\alpha) = \lambda \xi \sum_{t=S}^{\tau} e^{-rt} A_t h_t^{\xi}.$$

Rearranging and using the definition of τ establishes (18),

$$\sum_{t=S}^{T} e^{-rt} A_t h_t^{\xi} = \sum_{t=S}^{\tau} e^{-rt} A_t h_t^{\xi} = \frac{(1+\alpha)}{\xi} Y_0^F.$$

Finally, if the set P is empty, (18) trivially holds, because then $Y_0^F = 0$.
Step II: Prove (19).
From (26) and (18),

$$e^{rs} \sum_{t=S}^{T} e^{-rt} c_t = e^{rs} \left(Y_0^M + (1-\theta) Y_0^F \right) = Y_s^M + (1-\theta) Y_s^F. \tag{33}$$

On the other hand, the resource constraint at age $s \geq R$ and the fact that $c_t = x_t$ for all $t \geq R$ imply

$$e^{rs} \sum_{t=s}^{T} e^{-rt} c_t = e^{rs} \sum_{t=s}^{T} e^{-rt} x_t = e^{rs} \sum_{t=S}^{s-1} e^{-rt} \left(y_t + w_t h_t - x_t \right) \equiv a_s \quad \text{all} \quad s \geq R\,. \tag{34}$$

From (27),

$$c_t = \lambda^{\frac{1}{\gamma-1}} \cdot N_t \cdot e^{\frac{r-\rho}{1-\gamma} t}. \tag{35}$$

Taking the ratio of (34) to (33) and using (35) establishes (19): for all $s \geq R$, we have

$$\frac{a_s}{Y_s^M + (1-\theta) Y_s^F} = \frac{\sum_{t=s}^{T} e^{-rt} c_t}{\sum_{t=S}^{T} e^{-rt} c_t} = \frac{\sum_{t=s}^{T} N_t \cdot e^{\left(-r + \frac{r-\rho}{1-\gamma}\right) t}}{\sum_{t=S}^{T} N_t \cdot e^{\left(-r + \frac{r-\rho}{1-\gamma}\right) t}}.$$

■

Proof of Proposition 1: Set $\alpha = 0$, and let $B_t = w_t$. Repeating Step I and Step II gives the result. ■

Lifetime Earnings

As described in the text, we have annual earning observations from linked Social Security records for 1951-91, and we have HRS survey data for 1992, 1994, 1996, 1998, 2000, and 2002. To compute lifetime earnings for males and for females, we estimate standard earnings dynamics regressions and use them to impute missing or censored annual values (see text for a discussion of censored data). We assume that men work full time until they retire (see the definition of retirement in the text); we allow women to enter and exit the labor market throughout their working life spans.

Male Earnings: For male m of age s, let y_{ms} be real earnings (nominal earnings deflated with the GDP consumption deflator, normalized to 1 in 1984), and let X_{ms} be a vector including a constant, a quartic polynomial in years of work experience, and time dummies. We think of the polynomial as capturing the accumulation of human capital through work experience, and we think of the time dummies as registering the impact of macroeconomic forces of technological progress. We have one additional constant, a dummy for SSA observations. To economize on parameters for the computations, we employ a linear spline for our time dummies. Specifically, we assume that the adjustments to wages caused by aggregate variation is characterized by constant rates of growth for the periods 1951–60, 1961–65, 1966-70, 1971-75, etc.

We assume that underlying earnings are generated by an equation of the following form:

$$\ln(y_{ms}) = X_{ms} \cdot \beta + u_m + e_{ms}\,. \tag{A1}$$

The regression error term has two components: an individual–specific random effect u_m, and an independent, non–specific random error e_{ms}. In fact, we have one random error, e_{ms}, for observations prior to 1991 and another, $\bar{e}_{ms}$, for observations after 1991.

Let L_m be the likelihood of observing a sequence of data generated by (A1) for a given individual m. Each individual has three types of observations. Let I_m be the set of ages for individual m for which earnings figures from the SSA history are not right–censored; let J_m be the set of ages for which earnings are right–censored; and let K_m be the set of ages from the HRS public data sets 1992–2002 (which never subject to right–censoring). Assume that u_m, e_{ms}, and $\bar{e}_{ms}$ are independent normal random variables with precisions h_u, h_e, and $h_{\bar{e}}$. Let the normal density, say, for u, be $\phi(u, h_u)$, and let the corresponding normal cumulative distribution function be $\Phi(u, h_u)$. Define

$$z_{hs} = \ln(y_{hs}) - X_{hs} \cdot \beta\,.$$

Then the likelihood for a given individual m is

$$L_m\left(\beta, h_e, h_{\bar{e}}, h_u\right) = \int_{-\infty}^{\infty} \phi\left(u, h_u\right) \prod_{s\in I_m} \phi\left(z_{ms} - u, h_e\right) \prod_{s \in J_m} \left[1 - \Phi\left(z_{ms} - u, h_e\right)\right] \prod_{s\in K_m} \phi\left(z_{ms} - u, h_{\bar{e}}\right) du.$$

Our estimates of $(\beta, h_e, h_{\bar{e}}, h_u)$, which we call $(\hat{\beta}, \hat{h}_e, \hat{h}_{\bar{e}}, \hat{h}_u)$, are

$$(\hat{\beta}, \hat{h}_e, \hat{h}_{\bar{e}}, \hat{h}_u) = \arg \min_{\beta, h_e, h_{\bar{e}}, h_u} \left\{ - \sum_m \ln(L_m\left(\beta, h_e, h_{\bar{e}}, h_u\right)) \right\}.$$

For individuals with no right–censored observations, one can evaluate L_m in closed form. If an individual has right-censored observations, L_m must be evaluated numerically.[11]

Earnings Data: To minimize complications from mixing full–time and part–time work, our earnings dynamics equations omit observations from ages above 60 or past the man's retirement age. Similarly, we drop annual earnings amounts below 1500 *hours* $\times$ *statutory minimum wage*, and we drop SSA observations from years with less than four quarters of work. If earnings begin at age S, we drop observations from ages prior to $S + 3$. Technically the HRS asks earnings and hours for the preceding year, but we find it plausible that respondents report amounts for the current calendar year; thus, we assume survey earnings refer to 1992, 1994, etc., rather than 1991, 1993,.... As protection against coding errors, we drop earnings observations above \$1 million. Because we focus on couples, we omit single men. Since the resulting coefficient estimates are quite

[11] We minimize the log likelihood function with Newton's method. This paper's calculations use Compaq Visual Fortran 6.6. The minimization employs Newton's method (IMSL routine DUMIAH); we evaluate $\Phi(.)$ with IMSL function DNORDF, and we evaluate the integral for u with a 21–point Gauss–Kronrod rule (IMSL routine DQ2AGS) — truncating the bounds of integration at plus and minus six standard deviations from 0. Our version of Newton's method employs user-specified first and second derivatives.

conventional, we refer the reader to our earlier working paper House *et al.* (2005, Table A1) for details.

Male Lifetime Earnings: We predict male annual earnings from our regression equation, generating $(y_S, y_{S+1}, ..., y_R)$. We compute annual earnings from 1992 onward by linearly interpolating and extrapolating from y_{1991} and survey data. The last step is meant to capture possible part–time work in years immediately before retirement.

We predict missing or censored earnings prior to 1991 as follows. From our regression equation, we have

$$E[y_s \,|\, data] = e^{X_s \cdot \beta} \cdot E[e^u \,|\, data] \cdot E[e^{e_t} \,|\, data]\,.$$

Given the large sample size in our regression, we simply set $\beta = \hat{\beta}$. Since e_t is an independent random variable, assumed normally distributed, in the SSA sample we use $\sigma_e \equiv 1/\hat{h}_e$, etc., to set

$$E[e^{e_t} \,|\, data] = \begin{cases} e^{\sigma_e^2/2}, & \text{if year} \leq 1991, \\ e^{\sigma_{\bar{e}}^2/2}, & \text{otherwise.} \end{cases}$$

For a given individual, the data provide a vector of errors $\vec{z}_i$, a vector of intervals $\vec{Z}_j$ with $Z_j = [z_j, \infty)$, and a vector of errors $\vec{z}_k$. Letting $f(.)$ be a density function, we have

$$f(u \,|\, \vec{z}_i, \vec{Z}_j, \vec{z}_k) = \frac{f(u\,,\, \vec{z}_i, \vec{Z}_j, \vec{z}_k)}{f(\,\vec{z}_i, \vec{Z}_j, \vec{z}_k)}$$

The statistical model implies

$$f(u\,,\, \vec{z}_i, \vec{Z}_j, \vec{z}_k) = \phi\,(u, h_u) \prod_{i \in I} \phi\,(z_i - u, h_e) \prod_{i \in J} [1 - \Phi\,(z_j - u, h_e)] \prod_{k \in K} \phi\,(z_k - u, h_{\bar{e}})\, du.$$

Integrating with respect to u generates the marginal density $f(\vec{z}_i, \vec{Z}_j, \vec{z}_k)$; hence, we have

$$E\left[e^u \middle|\; data\right] = \frac{\int_{-\infty}^{\infty} e^u \cdot \phi\,(u, h_u) \prod_{i \in I} \phi\,(z_i - u, h_e) \prod_{i \in J} [1 - \Phi\,(z_j - u, h_e)] \prod_{k \in K} \phi\,(z_k - u, h_{\bar{e}})\, du}{\int_{-\infty}^{\infty} \phi\,(u, h_u) \prod_{i \in I} \phi\,(z_i - u, h_e) \prod_{i \in J} [1 - \Phi\,(z_j - u, h_e)] \prod_{k \in K} \phi\,(z_k - u, h_{\bar{e}})\, du}.$$

Table 3 in the text summarizes the distribution of male lifetime earnings.

Female Earnings. We are unwilling to assume that women necessarily work in the labor market full time, even prior to 1991; thus, our imputation procedure is more complicated than the one that we use for men.

We use the same earnings dynamics equation as for men, though for each education group we estimate the equation twice for women. First, we employ the same filters as in the case of males — calling this our "exclusive sample." For each earnings figure at the censoring limit, we use the maximum of the limit and the prediction from the first earnings dynamics equation. See House *et al.* (2005, Table A2) for the estimates of the earnings dynamics equation.

Second, we enlarge our data set to what we call our "inclusive sample." This includes ages from S up to retirement, earnings below 1500 hours $\times$ statutory minimum wage, and those with less than four SSA quarters of work. In 1996 the HRS inquired about number of years of non–FICA employment and the corresponding dates. We predict non–FICA earnings for jobs prior to 1980 (after 1980, our data includes non–FICA earnings) with the second earnings dynamics equation (and the prediction steps described above). If the survey reports, for instance, two years of non–FICA work 1955-58, we impute an annual amount for each of the four years but multiply predicted values by one half. If a non–FICA prediction overlaps an observation in the data, we sum the two. See House *et al.* (2005, Table A3) for the estimates of the second earnings dynamics equation.

Table 3 in the text summarizes the distribution of female lifetime earnings.