Homework in Macroeconomics: Household Production and Aggregate Fluctuations

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This paper explores some macroeconomic implications of including household production in an otherwise standard real business cycle model. We calibrate the model on the basis of microeconomic evidence and long-run considerations, simulate it, and examine its statistical properties. We find that introducing home production significantly improves the quantitative performance of the standard model along several dimensions. It also implies a very different interpretation of the nature of aggregate fluctuations.

I. Introduction

This project explores some of the implications of introducing home, or nonmarket, production into an otherwise standard model of aggregate fluctuations. This is of interest because the household sector is large, whether measured in terms of inputs or output. Data from the Michigan Time Use Survey indicate that an average married couple spends 33 percent of its discretionary time working for paid compensation and 28 percent, only slightly less, working in the home (see table 1 in Benhabib, Rogerson, and Wright [1990a], which is based on the data summarized in Hill [1985]; see also Juster and Stafford [1990] for a recent description of time-use data from other countries and periods).[1] Purchases of consumer durables and residential investment actually exceed purchases of producer durables and nonresidential investment in the U.S. data (Greenwood and Hercowitz, this issue). Studies that attempt to measure the value of household production indicate that it is also large; Eisner's (1988) recent summary of this literature suggests an estimate of home-produced output relative to measured gross national product in the range of 20–50 percent.

These facts lead us to conclude that home production is an empirically significant entity at the aggregate level. In light of this, why is it conspicuously absent from most models of aggregate economic activity? One possible conjecture is that the behavior of the home sector is approximately independent of the market. However, the evidence indicates that individuals employed in the market sector spend much less time working in the home than unemployed individuals and also that employed agents with higher wages substitute out of home and into market production (see Benhabib et al. 1990a, tables 2, 3; Rios-Rull 1990). This suggests not only that the home sector is large, but that there is a good deal of substitutability between it and the market, and leads us to believe that household production may be an important missing element in existing models of the aggregate economy.[2]

Our plan is to incorporate a home production sector into a simple real business cycle model of the type pioneered by Kydland and Prescott (1982), by assuming that households have access to home production functions that use time and capital to produce a nontradable

[1] Discretionary time includes market work, homework, and leisure, all three of which are measured directly in the time-use survey data. Equivalently, discretionary time is total time minus personal care, which is mainly sleep, and a few other miscellaneous activities.

[2] This has previously been argued by Becker (1988). He stresses *family behavior*, whereas we abstract from issues such as marriage, fertility, and so on in this paper. Rios-Rull (1990) and Greenwood and Hercowitz (this issue) have also recently emphasized the importance of home production for macroeconomics.

consumption good. We expect, a priori, that this will make a differ-
ence, for the following reasons. When individuals are able to substi-
tute between market and nonmarket production over time, volatility
in market activity can arise because of relative productivity differen-
tials between the two sectors, and not just absolute productivity
shocks, as is the case in one-sector models. Furthermore, the size of
the fluctuations induced by productivity shocks will depend on the
degree to which agents are willing to substitute between home and
market commodities (both time and goods) at a given date, and not
just the degree to which they are willing to substitute between these
commodities at different dates, as is the case in the standard real
business cycle model.

To facilitate comparison, we stay as close as possible to the specifi-
cation of the stochastic growth model described in Hansen (1985),
Prescott (1986), or Plosser (1989), for example.[3] It has been estab-
lished that this framework does fairly well at accounting for certain
salient aspects of the postwar U.S. data. Using functional forms and
parameter values that conform to microeconomic studies and long-
run observations, it accounts for a sizable fraction of observed fluctu-
ations in macroeconomic variables at cyclical frequencies, given rea-
sonable estimates of the process of technological change. Further,
the model is consistent with other phenomena, such as the fact that
consumption is less volatile than output and investment more volatile
than output, observed not only in the postwar U.S. economy but also
across many countries and time periods (see, e.g., Backus and Kehoe
1989). Nevertheless, it is apparent that this model does not do as well
along some dimensions as it does along others.

We intend to focus on the following five problems with the stan-
dard real business cycle model. In comparison with the data, in the
model, (1) output fluctuates too little; (2) relative to output, labor
hours fluctuate too little; (3) relative to output, consumption fluctu-
ates too little; (4) relative to output, investment fluctuates too much;
and (5) productivity's correlation with output or hours is far too high.
Most of these have been recognized before, of course. Also, various
extensions of the basic framework are known to help resolve some of
these problems. We shall demonstrate that introducing home produc-
tion improves the performance of the model along all these dimen-
sions simultaneously.

The rest of the paper is organized as follows. In Section II, we

[3] Our message is meant to be independent of whether or not market failures, taxes,
money, information problems, frictional unemployment, or other complications are
empirically important; nevertheless, we stick to a very basic version of the model in
order to isolate the impact of home production.

review the basic home production framework. In Section III, we introduce home production into the stochastic growth model and discuss our choice of functional forms and parameter values. In Section IV, we simulate numerical solutions to the model and compare its cyclical properties with those of the standard model and with the actual data. In Section V, we present our conclusions.

II. The Basic Home Production Model

Before we study the dynamic model, it is instructive to begin with a review of the static theory of home production.[4] Consider an individual with a utility function $U = U(c_m, c_n, h_m, h_n)$ defined over four objects—consumption of a market good c_m, consumption of a home-produced or nonmarket good c_n, hours of work in the market sector h_m, and hours of work in the home or nonmarket sector h_n—who solves the following problem:

$$\max U(c_m, c_n, h_m, h_n)$$

subject to $c_m \leq x + wh_m$, $\quad c_n \leq g(h_n)$, $\quad c_j \geq 0$, $\quad h_j \geq 0$, $\quad h_m + h_n \leq 1$.

Here, w denotes the real wage and x denotes exogenous income. What makes this a model with home production is the constraint $c_n \leq g(h_n)$, where $g(\cdot)$ is the home production function. We assume $U_1 > 0$, $U_2 > 0$, $U_3 < 0$, $U_4 < 0$, and $g' > 0$. Also, g is concave and U is strictly concave.

Substituting the home production constraint at equality into the utility function reduces the problem to

$$\max U[c_m, g(h_n), h_m, h_n] \quad \text{subject to } c_m \leq x + wh_m \tag{1}$$

(nonnegativity constraints on all variables, including leisure, $L = 1 - h_m - h_n$, are assumed to be nonbinding from now on). Define the homework function $h_n = h(c_m, h_m)$ to be the unique value of h_n that maximizes the objective function in (1), for given values of the market variables (c_m, h_m), and define the home consumption function by $c_n = c(c_m, h_m) = g \circ h(c_m, h_m)$. If we then define

$$V(c_m, h_m) = U[c_m, c(c_m, h_m), h_m, h(c_m, h_m)], \tag{2}$$

previous assumptions imply that $V(\cdot)$ is continuous, monotonic, and strictly concave (Benhabib et al. 1990*a*, theorem 1) and so is a well-behaved *reduced-form* utility function over market quantities only.

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[4] The fundamental reference is Becker (1965), although our particular specification is closer to that of Gronau (1977, 1985). See also Pollak and Wachter (1975) and Deaton and Muellbauer (1980).

It follows that problem (1) generates the same values of c_m and h_m as the problem that does not explicitly contain home production,

$$\max V(c_m, h_m) \quad \text{subject to } c_m \leq x + wh_m. \tag{3}$$

The point is that for any model with home production, there is a model without home production, but with different preferences, that generates the same outcome for market quantities; so there is a sense in which models with home production are observationally equivalent to those without. Yet precisely because preferences would have to be different if home production was not explicitly included, we find it a useful concept for understanding and interpreting economic phenomena.

As an example, consider the case in which technology shocks are added to the market and home production functions, with s_m and s_n the two shocks. When one follows the procedure outlined above, it will now appear in the reduced form that preferences are stochastic, even though true underlying preferences are deterministic. In this example, when s_m is relatively high, labor will flow into the market, resulting in a positive correlation between productivity and h_m; conversely, when s_n is relatively high, labor will flow the other way, raising productivity as h_m falls and therefore causing a negative correlation between market hours and productivity. With both shocks present, it becomes possible to reconcile the lack of a strong empirical correlation between employment and productivity with theories based on technology shocks, as we show in our real business cycle model in Section IV.[5]

We emphasize that many implications of introducing home production do *not* depend on its being stochastic. One example is that leisure in the reduced-form utility function can behave quite differently from leisure in the underlying utility function, and, in particular, the wealth effect is changed. Thus, whether or not it is stochastic,

[5] Clearly, one could also explain the employment-productivity observations by adding a preference shock to a model with a market productivity shock, as in Christiano and Eichenbaum (1988). As already explained, home production does not allow us to generate outcomes that cannot be generated without it; it simply provides an alternative interpretation. As Becker (1976, p. 5) writes, "The assumption of stable preferences provides a stable foundation for generating predictions about responses to various changes, and prevents the analyst from succumbing to the temptation of simply postulating the required shift in preferences to 'explain' all apparent contradictions to his predictions." However, he notes that "preferences that are assumed to be stable do not refer to market goods and services, like oranges, automobiles, or medical care, but to the underlying objects of choice that are produced by each household using market goods and services, their own time, and other inputs." Similarly, Juster and Stafford (1990, p. 22) write that "while it is common for economists to assume stable preferences, it is less common to assume stationary technology. From our perspective, technology is subject to change just as readily in the household sector as in the industrial sector."

introducing home production has an impact on labor supply elastic-
ities, as shown in Section IV. In fact, once home production is intro-
duced, the reduced-form utility function can imply that leisure is an
inferior good, even if the underlying utility function implies that it is
a normal good. Hence, the inclusion of home production not only
affects labor supply quantitatively but also impinges on a variety of
qualitative results in macroeconomics and labor economics that are
known to depend critically on this wealth effect.[6]

To close this section, we present a parametric example that illus-
trates some of the effects in the preceding discussion. Begin by letting

$$U = \ln(C) + A \cdot \ln(1 - h_m - h_n) + Bh_m, \tag{4}$$

where C is the composite consumption good:

$$C = (a_m c_m^e + a_n c_n^e)^{1/e}. \tag{5}$$

Assume $A > B \geq 0$ and $e \leq 1$. The elasticity of substitution between
market and nonmarket consumption is $1/(1 - e)$. If $B = 0$, then
market and nonmarket work are perfect substitutes, whereas if $B >
0$, then working in the market is preferred to working around the
house. For the time being, we also assume that home production is
linear, $c_n = s_n h_n$, which permits closed-form solutions to be obtained.

Consider first the case $e = 0$, so that (5) defines a Cobb-Douglas
function. If there is an interior solution, the homework function is

$$h_n = h(c_m, h_m) = \left(\frac{a_n}{A + a_n}\right)(1 - h_m). \tag{6}$$

If we substitute this into (4), the reduced-form utility function be-
comes

$$V = a_m \ln(c_m) + (A + a_n)\ln(1 - h_m) + Bh_m. \tag{7}$$

In this case, home production changes nothing, in the sense that if
we had ignored it or simply set c_n and h_n equal to the constants in (4),
then except for the constants in (7), nothing would have changed. To
get any substantive effects from introducing home production, in this
example, it is necessary to assume $e \neq 0$.

Consider next the case in which c_m and c_n are perfect substitutes: e

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[6] For instance, in indivisible labor models without home production, laid-off workers
are involuntarily unemployed if and only if leisure is inferior; with home production,
involuntary unemployment can occur if leisure is normal (see Benhabib et al. 1990a).
Similarly, in implicit contract models with asymmetric information, without home pro-
duction, there is underemployment if and only if leisure is inferior; with home produc-
tion, underemployment can occur if leisure is normal (see Nosal, Rogerson, and Wright
1990).

= 1. Also, for ease of notation set $a_n = a_m = 1$. If there is an interior solution, the homework function is now

$$h_n = h(c_m, h_m) = \frac{s_n(1 - h_m) - Ac_m}{s_n(1 + A)}, \tag{8}$$

and the reduced-form utility function becomes

$$V(c_m, h_m) = (1 + A) \cdot \ln[c_m + s_n(1 - h_m)] + Bh_m. \tag{9}$$

If $B = 0$, then (9) implies that the wealth effect on leisure is identically zero; if $B > 0$, then leisure is actually an inferior good in (9), even though it is normal in the underlying utility function. Further, in the reduced form described by (9), the marginal disutility of work varies with s_n.

In the dynamic analysis to follow, we shall not assume such simple preferences, nor shall we assume that the home production function is linear. Nevertheless, this example provides us with the following intuition. As the elasticity of substitution between c_m and c_n increases, the wealth effect in the reduced-form preferences is reduced and, therefore, the elasticity of labor supply increases (see Benhabib et al. [1990a, sec. 3] for a more detailed discussion). This leads to larger fluctuations in labor and output for a given stochastic process for technology shocks. Additionally, if we assume that the home production function is stochastic, the reduced-form preferences can also be stochastic.

III. The Dynamic Framework

Our starting point is the real business cycle model. There is a representative agent, with preferences over stochastic sequences of consumption and labor hours $\{c_t, h_t\}$ described by

$$w = E \sum_{t=0}^{\infty} \beta^t u(c_t, h_t), \tag{10}$$

where E denotes the expectation and $\beta \in (0, 1)$ is the discount factor. The agent has one unit of time to divide between leisure and labor each period. Labor and capital are used to produce output according to a constant returns to scale technology, $y_t = s_t \Gamma^t f(h_t, k_t)$, where s_t is stochastic and $\Gamma \geq 1$ is a deterministic growth term. Capital evolves according to $k_{t+1} = (1 - \delta)k_t + i_t$, where i_t is investment and $\delta \in (0, 1)$ is the depreciation rate; s_t evolves according to an autoregressive process. Feasibility requires $c_t + i_t \leq y_t$ for all t.

The first step is to extend this framework to include household variables. Preferences are now given by

$$W = E \sum_{t=0}^{\infty} \beta^t U(c_{mt}, c_{nt}, h_{mt}, h_{nt}), \tag{11}$$

where the arguments of $U(\cdot)$ at each date are as defined in the previous section. The market technology is now written $y_t = s_{mt}\Gamma^t f(h_{mt}, k_{mt})$, where h_{mt} and k_{mt} are hours and capital in market production; the nonmarket technology is $c_{nt} = s_{nt}\Gamma^t g(h_{nt}, k_{nt})$, where h_{nt} and k_{nt} are hours and capital in home production. The two technologies display constant returns to scale and common exogenous growth. Feasibility requires $c_{mt} + i_t \leq y_t$ plus $k_{mt} + k_{nt} \leq k_t$, where k_t is now the total capital stock, for all t. Notice that capital is produced exclusively in the market sector, even though it is an input to both market and home production. Total capital evolves according to $k_{t+1} = (1 - \delta)k_t + i_t$; s_{mt} and s_{nt} evolve according to a process that will be specified in detail later.[7]

It is standard in this literature to impose the condition that market hours should be constant along a balanced growth path (i.e., when s_{mt} and s_{nt} are constant). As discussed in King, Plosser, and Rebelo (1987), in models that do not explicitly include home production, this implies that the utility function must be of the form

$$u(c, h) = \ln(c) + v(h) \quad \text{or} \quad u(c, h) = \frac{c^{1-r}}{1 - r}v(h), \tag{12}$$

where $r > 0$, $r \neq 1$, and $v(\cdot)$ is a concave function. Since the properties of the model do not change significantly as r varies over a reasonable range (see Hansen 1988), the logarithmic case is often studied, and it is often further assumed that $v(h) = B \cdot \ln(1 - h)$. With a Cobb-Douglas production function and $s_t = \rho s_{t-1} + \epsilon_t$, where $\rho \in (0, 1)$ and ϵ_t is independently and identically distributed, this is exactly the specification in Hansen's (1985) base case (without indivisible labor). In the remainder of the paper, we refer to this specification as the *standard model*.

[7] We assume that capital is freely mobile between the home and market sectors. An alternative would be to assume that capital in a given sector cannot be transformed once it is in place. Theoretically, these two cases are polar extremes, but from a practical perspective, the difference is not substantial here. By choosing not to replace worn-out capital in one sector and putting all new investment in the other, the economy can reallocate a considerable amount of capital across sectors without actually moving the stuff that is in place. In the simulations conducted in this paper, only infrequently does any capital actually move between sectors, and then the amount that does move is quite small (rarely more than 0.5 percent of the stock in the declining sector).

In the home production model, one specification that delivers constant hours along a balanced growth path is

$$U = \frac{(C^b L^{1-b})^{1-r} - 1}{1 - r},$$ (13)

where $C = [ac_m^e + (1 - a)c_n^e]^{1/e}$ and $L = 1 - h_m - h_n$. If we set $r = 1$, this allows us to write

$$U = \left(\frac{b}{e}\right) \ln[ac_m^e + (1 - a)c_n^e] + (1 - b)\ln(1 - h_m - h_n).$$ (14)

We also assume Cobb-Douglas production functions in both sectors, $f(h_m, k_m) = k_m^\theta h_m^{1-\theta}$ and $g(h_n, k_n) = k_n^\eta h_n^{1-\eta}$. Our goal is to see how this model differs quantitatively from the model without home production. First, however, we show how to nest the standard model within our framework.

There is a general observational equivalence result for dynamic economies similar to the static result in the previous section (see Benhabib et al. 1990a), and in some special cases, we can solve for the reduced form explicitly. With $e = 0$, (14) becomes

$$U = ab \cdot \ln(c_m) + (1 - a)b \cdot \ln(c_n) + (1 - b)\ln(1 - h_m - h_n).$$ (15)

If we also set $\eta = 0$, so that $c_n = s_n \Gamma^t h_n$, we can solve for the homework function $h_n = \phi \cdot (1 - h_m)$, where $\phi = (1 - a)b/(1 - ab)$. If we insert this into (15) and simplify, the reduced-form utility function is of the form

$$V = \ln(c_m) + B \cdot \ln(1 - h_m),$$ (16)

which is identical to the utility function in the standard model without home production. Hence, the home production economy with $e = \eta = 0$ generates exactly the same time paths for all the market variables as the standard model.

We now turn to the parameter values we use in the experiments reported in the next section. We set the discount rate to $\beta = .99$. The parameters a and b are chosen (see the Appendix for details) so that the steady state of the model yields values for market work and homework that correspond to the numbers discussed in the Introduction, $h_m = .33$ and $h_n = .28$.[8] The remaining preference parameter

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[8] Greenwood and Hercowitz (this issue), in a closely related model, interpret *all* nonmarket time as home production. Following Gronau (1977), we prefer to divide discretionary time into three components—market work, homework, and leisure—rather than two. We think that it is useful, in principle, to differentiate between time spent in household production activities, which generates disutility, and time spent in leisure activities, which does not. Furthermore, in practice, we do have good cross-sectional measurements of all three components in the time-use data.

is e. Eichenbaum and Hansen (1990) use aggregate data to estimate a model in which individuals value both the services of market consumption goods and the flow of services from consumer durables; the latter can be thought of as the output of a home production process that uses durables as its only input. Although their results are sensitive to various assumptions, for one set of findings they report that there is "very little evidence against the hypothesis that the services from durable and non-durable goods are perfect substitutes" (p. 63), which would suggest setting $e = 1$.

   Cross-sectional data can also provide some information. Consider a static model, in which agent i has preferences described by

$$U_i = \ln[a_i c_{mi}^e + (1 - a_i)c_{ni}^e] + v_i(1 - h_{mi} - h_{ni}), \tag{17}$$

where a_i differs across the population according to some distribution. Suppose that all agents have the same home production technology, $c_{ni} = B \cdot h_{ni}$, but that each agent faces a potentially different market wage, w_i. Then the first-order condition for agent i implies

$$\ln\left(\frac{h_{mi}}{h_{ni}}\right) = \frac{1}{e - 1}\ln(B) - \frac{e}{e - 1}\ln(w_i) + \frac{1}{e - 1}\ln\left(\frac{1 - a_i}{a_i}\right). \tag{18}$$

This leads to a regression equation of the form

$$\ln\left(\frac{h_{mi}}{h_{ni}}\right) = \alpha_0 + \alpha_1 \ln(w_i) + \xi_i. \tag{19}$$

   This model describes the average time allocation decision as a function of the long-run wage. Using the pooled data from the Panel Study of Income Dynamics described in Rios-Rull (1990), we estimate equation (19) and derive a value of $e = .6$, somewhat lower than the value implied by Eichenbaum and Hansen's (1990) results.[9] The two methods are polar extremes: the first uses aggregate data and abstracts from the time allocation decision, whereas the second uses micro data and abstracts from savings or capital. While we emphasize the need for further empirical work along these lines, for the time being the mean value of these two numbers will be used: $e = .8$.

   Concerning technology, we set $\Gamma = 1$, $\delta = .025$, and $\theta = .36$, as in most of the related literature. Capital's share in the home production

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[9] We consider this estimate highly preliminary, for a variety of reasons. For one thing, Rios-Rull's sample selection criterion severely underreports low-wage workers (presumably with very low ratios of market to home hours). As a rough correction, we either adjusted home hours for the two lowest-wage groups so that their total work is the same as that of the other groups or simply ignored the lowest-wage groups. Either method results in a point estimate of about $e = .6$.

function is set to $\eta = .08$, producing a steady-state c_n/y ratio of $.26$.[10] At the same time, this yields a steady-state k_n/k_m ratio of $.14$. If one wanted to define household capital to include housing plus consumer durables, this ratio would certainly be too small; but in this framework, it is not clear that housing should be interpreted as part of k_n. This model is intended to capture the household's decision to combine its labor with machines, such as stoves or washing machines, to cook or do laundry, instead of purchasing meals or cleaning services in the market. Houses do not need to be combined with labor, at least not in the same way that stoves or washing machines do, in order to be useful. Because much of the time spent in one's home is leisure, sleep, and so on rather than homework per se, it seems appropriate that k_n does not include the housing stock in our framework.[11]

We still need to describe the shocks. The usual assumption is that $s_{mt+1} = .95 s_{mt} + \epsilon_{mt}$, where ϵ_{mt} is normal with a standard deviation of around $\sigma_m = .007$. In the absence of evidence concerning the home technology, we simply assume that it follows a similar process, $s_{nt+1} = .95 s_{nt} + \epsilon_{nt}$, where the standard deviation of ϵ_{nt} is σ_n. We have experimented with setting $\sigma_n = 0$ (so that the home technology is nonstochastic) and also with $\sigma_n = .007$ (so that it fluctuates as much as the market technology). Only the latter simulations are reported below, but the basic message turns out not to depend heavily on σ_n (see Benhabib et al. 1990b, app. B). The reason is that for most of our results, it is *relative* productivity variation that matters, and there will be relative productivity variation even if s_n is constant as long as s_m fluctuates. However, one exception to this is that the behavior of average productivity does depend critically on σ_n, as will be discussed below.[12]

This leaves one parameter, the correlation between the two shocks, $\gamma = \text{corr}(\epsilon_m, \epsilon_n)$. Our view is that γ is certainly positive but that it is also certainly less than one (because sometimes technological innovations affect productivity mainly in the market, like microcomputers, and sometimes they affect productivity mainly in the home, like microwave ovens). Smaller values of γ imply more frequent relative

[10] Recall that the range of estimates surveyed in Eisner (1988) is .20–.50. We prefer to be at the low end of this range since we have abstracted from taxation on market activity.

[11] A steady-state k_n/k_m ratio of .14 matches the data if we define k_n to include household equipment and furniture but not houses. Alternatively, Greenwood and Hercowitz (this issue) include housing plus durables in k_n. Combined with their assumption that h_n includes all time not sleeping or working for paid compensation, their model implies that c_n/y is 2.9 in the steady state, compared to our value of .25. In any case, including a realistic tax system in our model would considerably increase both c_n/y and k_n/k_m.

[12] It is also the case that the results reported below are not sensitive to the serial correlation properties of the home shock.

productivity differentials between the two sectors and, therefore, more frequent opportunities for short-run substitution between the market and the home. Intuitively, then, the smaller γ is, the greater is the extent to which home production should affect the cyclical behavior of the system. We set $\gamma = \frac{2}{3}$ for the simulations reported below, although the basic results are not affected too much if we choose $\gamma = \frac{1}{2}$ or $\frac{3}{4}$, for example (see Benhabib et al. 1990*b*, app. B).

IV.   Results

In this section, we compare certain statistics from simulations of the model with those from the data. All series are filtered using the Hodrick-Prescott technique before computing any statistics. Part A of table 1 summarizes the behavior of the U.S. quarterly data for the period 1954:1–1988:2, in terms of the following variables: y is output, c_m is market consumption, i is investment, h_m is market hours, k is capital, and w is average productivity (defined by $w = y/h_m$; we use the w notation because a Cobb-Douglas technology implies that the average product is proportional to the marginal product, which equals the real wage when the allocation is decentralized as a standard competitive equilibrium). The table provides the percentage standard deviation of output, and for each additional series x, it provides the percentage standard deviation of x relative to the percentage standard deviation of y and the correlation of x with y.[13]

Part B of table 1 provides a summary of the properties of the standard model without home production using the parameter values described in the previous section; these numbers are averages over 50 simulations, each of length 143 (the number of quarters in our data). The results are the same as those reported by Hansen (1985) for his base economy, except that some new statistics have been added. In particular, we disaggregate total market hours into those used to produce consumption goods, h_c, and those used to produce investment goods, h_i, where $h_c = (c/y)h_m$ and $h_i = (i/y)h_m$. Many authors have commented on how well this model captures certain aspects of the data. Nevertheless, we wish to draw attention to several dimensions along which there appears to be room for improvement.

First, the model economy is not as volatile as the actual economy: the model has a standard deviation of output equal to only 1.29 percent, compared to 1.74 in the data. Second, independent of the over-

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[13] Note that our output series is constructed as $y = c_m + i$. Also note that our consumption data correspond to expenditures on nondurables and services, while purchases of consumer durables have been included in investment. This results in a standard deviation of i relative to y of 2.82, lower than the number typically reported; e.g., Kydland and Prescott (1991) report 3.17.

TABLE 1

	c_m	i	w	k_m	h_m	h_t	h_c	h_n
				$x =$				
A. U.S. Data: std(y) = 1.74								
$\dfrac{\text{std}(x)}{\text{std}(y)}$	.49	2.82	.52	.38	.86			
corr(x, y)	.76	.96	.51	.28	.86			
B. Standard Model: std(y) = 1.29								
$\dfrac{\text{std}(x)}{\text{std}(y)}$	.30	3.14	.52	.26	.50	2.66	.25	
corr(x, y)	.90	.99	.99	.05	.98	.98	−.98	
C. Home Production Model: std(y) = 1.71								
$\dfrac{\text{std}(x)}{\text{std}(y)}$	.51	2.73	.39	.23	.75	2.40	.59	.70
corr(x, y)	.69	.94	.75	.09	.94	.95	.10	−.76

all volatility of the model, consumption is not volatile enough and investment is too volatile *relative* to output. Because output is the sum of consumption and investment and all three are highly correlated, the standard deviation of y is essentially a weighted average of the standard deviations of c_m and i; hence, insufficient volatility in consumption and excess volatility in investment tend to go together. A further difficulty is that total market hours do not fluctuate enough in the model. Further still, observe that although the correlations between output and most of the other variables are reasonable, the correlation between output and productivity is significantly off target. These are problems 1–5 listed in the Introduction.

These problems are fairly well known, and various embellishments of the basic framework have been shown to help each of them in isolation. There is, however, a feature of the model that has not been commented on before but that is closely related: the model implies an almost perfect negative correlation between output and the hours allocated to the production of consumption goods: corr(y_t, h_{ct}) = −.98. Although it is not shown in the table, h_{ct} is also highly negatively correlated with h_t and, a fortiori, h_{tt}. These predictions fly in the face of conventional wisdom concerning actual business cycles, which is that various sectors tend to move up and down together.[14] Further-

[14] Economists often define the cycle as the recurrent comovement of the *outputs* of various market sectors (see, e.g., Lucas 1977), but we doubt whether anyone would argue empirically that employment hours in different sectors move out of phase. We

more, we now show that these predictions will come out of *any* speci-
fication of the standard model that yields constant hours along a
balanced growth path and implies that labor's share of income is
constant (as it is with a Cobb-Douglas technology).

To see this, recall that the class of preferences that delivers constant
hours along a balanced growth path is described by (12). Consider
the case in which $u = \log(c_t) + v(h_t)$ (the other case is similar). At
each point in time, the efficiency condition equating the marginal
rate of substitution with the marginal product of labor in the produc-
tion of consumption goods is $c_t v'(h_t) = \text{MPL}_t$, or

$$h_{ct} v'(h_t) = \frac{\text{MPL}_t \cdot h_{ct}}{c_t}. \tag{20}$$

The right-hand side is labor's share of output in the consumption
sector, which is constant by assumption. Hence, as long as $v(\cdot)$ is
(strictly) concave, an increase in total hours h_t must be accompanied
by a (strict) decline in h_{ct}. Since h_{ct} and h_t move in opposite directions,
so do h_{ct} and h_{it}.[15]

The intuition is quite simple. Any specification that implies that
hours do not change along a balanced growth path also implies that
individuals never supply more labor in order to produce more output
for immediate consumption. In particular, in a model that is other-
wise standard except that it has no capital, employment is *constant*,
and consumption fluctuates one for one with the technology shock.
Even though agents have the opportunity to work harder when pro-
ductivity is high and increase consumption even more, they choose
not to work harder if the only reward is increased contemporaneous
consumption. When capital accumulation is reintroduced, labor does
vary with productivity because of intertemporal substitution opportu-
nities; but individuals still do not work more to increase current con-

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did not attempt to catalog various sectors as consumption or investment, but there is
no major sector of the U.S. economy that is known to have countercyclical employment
(see Murphy, Shleifer, and Vishny 1989). Using data provided by Donna Costello from
five countries each disaggregated into five sectors, we examined the correlations with
output of each sector's hours and the cross correlations between sectoral hours. Almost
all these correlations were positive, and none was significantly negative.

[15] We point out that these results are actually reinforced if labor's share is countercy-
clical rather than constant (it is slightly countercyclical in the data). Also, our argument
does not restrict the technologies in the investment and consumption sectors to be the
same or subject to the same shock, and so moving to a more general two-sector model
will not change things. The best one can do within the standard model is to make $v(\cdot)$
linear, in which case hours in the consumption sector will be *constant* over the cycle.
The indivisible labor economy studied by Rogerson (1984, 1988) and Hansen (1985)
is equivalent to an economy in which $v(\cdot)$ is linear. Hence, in that economy, although
h_{ct} and h_{it} do not move together, at least they do not move in opposite directions over
the cycle.

sumption. In fact, since consumption now moves less than one for one with output, individuals spend less time in the production of consumption goods when productivity is high.

We therefore have the following characterization of business cycles in the standard model: good times are periods in which resources flow from the production of consumption goods to the production of investment goods. We think that it is useful to focus on this because it sheds light on some of the other problems with the standard model. The fact that consumption is too smooth and investment too volatile relative to output is easy to understand given that labor is being moved out of the production of consumption goods and into the production of investment goods as the cycle moves from trough to peak. Similarly, the fact that total hours are too smooth relative to output is easy to understand given that h_{ct} is countercyclical; if h_{ct} did not decrease whenever h_{it} rose, the sum would be more volatile. Furthermore, if h_{ct} could be increased during upswings without decreasing h_{it}, total output would also be more volatile.

Hence, several discrepancies between theory and data can be traced back to h_{ct} being out of phase with y. What is needed is a mechanism that causes hours in the consumption sector to respond positively to an increase in market productivity. The addition of home production provides exactly this. In addition to the standard motive for increasing labor hours when market productivity is high (capital accumulation), in the home production economy, there is an additional motive to simultaneously substitute market- for home-produced consumption. The latter effect involves the transfer of hours from the home into the market consumption sector during upswings in the business cycle and, if it is large enough, could thereby produce a procyclical pattern to h_{ct}. Depending on the exact parameterization, the addition of a household sector may imply that upswings in aggregate market activity correspond to periods in which labor flows from the home into all market sectors rather than periods in which labor flows from the consumption into the investment sector.

Part C of table 1 reports the results for simulations of the home production economy using the parameter values discussed above. In comparison with the standard model, the volatility of investment relative to output has decreased, whereas that of consumption has increased. Additionally, the variability of market hours relative to output is greater than in the standard model. Output is also more volatile. All these improvements can be interpreted in light of the fact that h_{ct} is procyclical in the home production economy. Although the correlation between y and h_c is not large in part C of the table, as long as it is even slightly procyclical, the model improves along all these dimensions. Figure 1 plots the responses of hours to a 1 percent
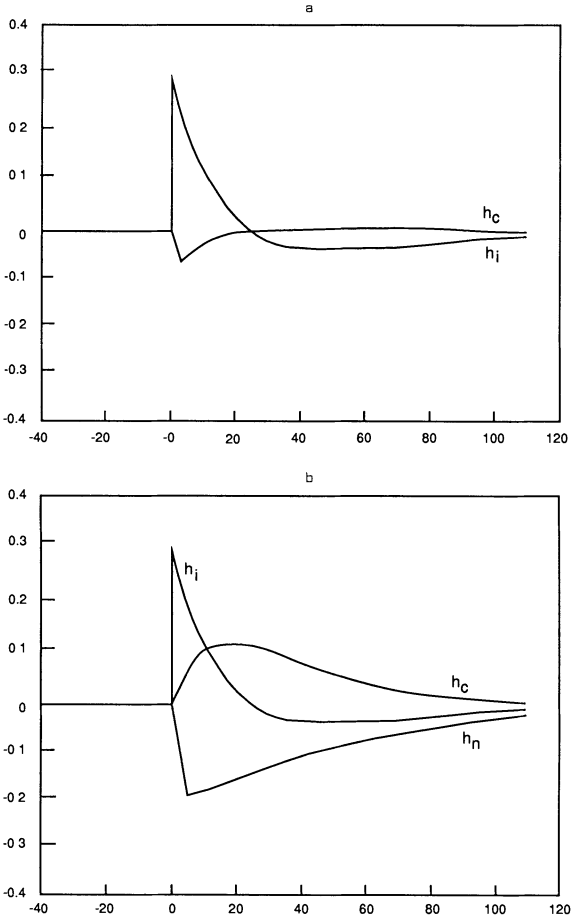
FIG. 1.—a, Standard model impulse response. b, Home production model impulse response.

shock in s_m, in the economies with and without home production. This shows clearly how h_c and h_i are out of phase in the standard model, whereas they move together in the home production model, with h_n taking up the slack.[16]

One prediction of the standard model that seems to be closer to

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[16] Although market consumption is more volatile here than in the standard model, it is really the composite good C that consumers care about, and that is actually quite smooth. Similarly, hours in home production act like a buffer against volatility in market labor, so that leisure L is also quite smooth. Hence, although market activity in the home production economy is more volatile, agents in the model actually do not mind.
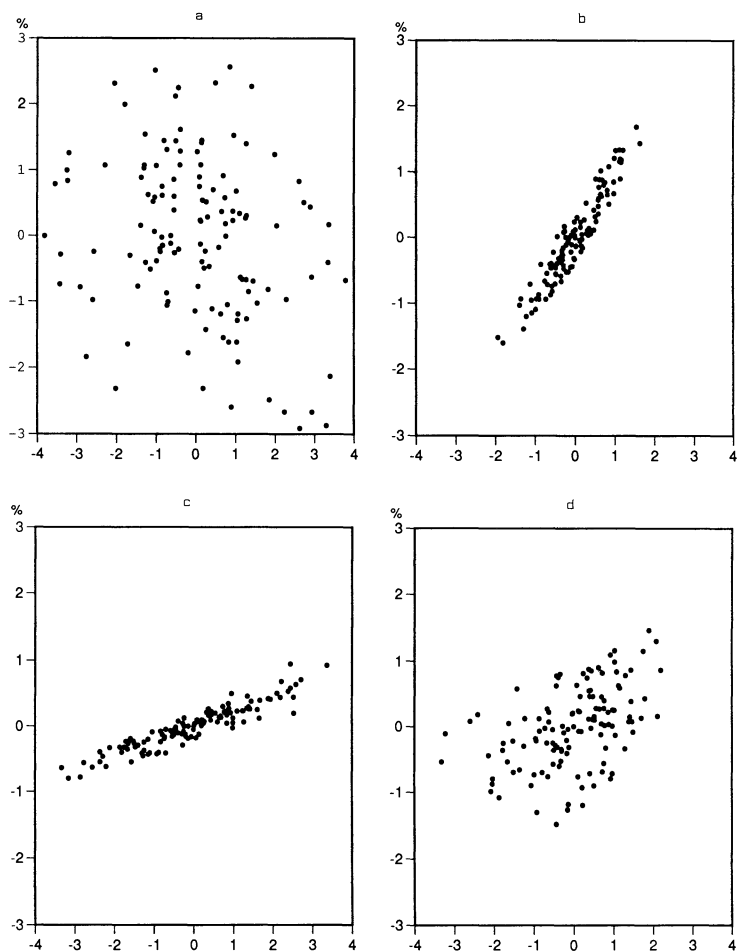
FIG. 2.—h_m vs. w (percentage deviations). a, U.S. data; b, standard model; c, home production model, zero home stock; d, home production model, both shocks positive.

the actual data is the volatility of productivity, but it misses so badly on the correlation between w_t and y_t that getting the standard deviation right seems to be of little consolation. Furthermore, the standard model also predicts that the correlation between w_t and h_{mt} is .99, and many commentators criticize the real business cycle paradigm for this. In the aggregate U.S. data, w_t and h_{mt} are in fact negatively correlated, as shown in figure 2a (which plots percentage deviations after filtering). For comparison, the data generated by the standard model are plotted in figure 2b. To say that these pictures are different would be an understatement. Now, it may be argued that there are problems

with the data, and correcting for measurement error suggests that the true correlation may actually be positive, perhaps even as high as .44 (see Christiano and Eichenbaum 1988, table A.3). But even under the most favorable assumptions, it is certainly not .99.

The feature of the standard model responsible for this inconsistency with the data is that it is driven by a single shock to technology, which implies a very tight relation between productivity and output or productivity and hours. Loosely speaking, shocks to the market technology shift labor demand and trace out a stable labor supply curve. The home production economy with only a single shock to the market technology (i.e., with $\sigma_n = 0$) also traces out a stable labor supply curve, as shown in figure 2c. Notice, however, that this curve is much more elastic than the one in figure 2b. In contrast to the standard model, which relies exclusively on intertemporal substitution, the home production model also includes intratemporal substitution between market work and homework at a given point in time. This makes the labor supply response in the home production economy more similar to that in the data, but still does not generate the cloud seen in figure 2a.

By including innovations to the home technology that are less than perfectly correlated with those in the market, we add a second shock. When both shocks are present, the net effect is as depicted in figure 2d. The correlation between h_{mt} and w_t in this case is .49, which is much better than in the standard model, although perhaps still high. However, this statistic is sensitive to the relative size of the shocks. If we increase σ_n from .007 to .010 and keep σ_m as well as all the other parameters the same, the correlation between h_{mt} and w_t is reduced to .08, which is well within the acceptable range. This change also implies that the standard deviation of productivity relative to output and the correlation between productivity and output become very close to the data, although market consumption becomes somewhat too volatile (see Benhabib et al. 1990b, table 2d). In any case, it is clear that there is no problem, in theory, accounting for the productivity-hours observations using models driven exclusively by technology shocks.[17]

To close this section, we compare our model with the closely related model of Greenwood and Hercowitz (this issue).[18] The focus of their

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[17] Similar shocks to the labor supply could be generated by assuming that preferences vary over time, which is the solution proposed by Christiano and Eichenbaum (1988) and by Bencivenga (in press).

[18] There are many technical differences in the two models; for instance, they calibrate to annual data, they assume that the technology shock follows a random walk, they assume that all nonmarket time is an input into home production instead of allowing for leisure, and their model includes taxation. These are not overly important for the following discussion.

paper is the behavior of investment in market and nonmarket capital over the cycle, which we denote i_m and i_n. Although this is not reported in table 1, our model obviously generates predictions for these variables; in particular, the correlation between y and i_m is .23 and the correlation between y and i_n is .35. Hence, i_m and i_n are both procyclical. However, our model is *not* consistent with one feature of the data emphasized by Greenwood and Hercowitz: the correlation between i_m and i_n in our simulations is negative (about $-.75$), whereas in the U.S. economy it is positive (about .75 in the quarterly data, if we measure i_n by purchases of consumer durables). This is not really a surprise since Greenwood and Hercowitz argue in some detail that any specification with a Cobb-Douglas home technology tends to make investment in the two types of capital negatively correlated.

Greenwood and Hercowitz also discuss how one can get around this by assuming a home production function of the form

$$c_n = [\eta k_n^\lambda + (1 - \eta)(s_n h_n)^\lambda]^{1/\lambda}, \tag{21}$$

where Cobb-Douglas is the case of $\lambda = 0$. With $\lambda < 0$, they show that i_m and i_n can become positively correlated in their model, at least when they assume that the two shocks s_m and s_n are perfectly correlated. With negative values of λ, we can also get i_m and i_n to be positively correlated in our model. However, there are two qualifications. First, the result is not very robust, in the sense that the correlation between s_m and s_n must be very close to one. Second, for parameter values that yield a positive correlation between i_m and i_n, we found that several of the previously noted improvements in the model were diminished and, in particular, h_c was countercyclical. Constructing a model that simultaneously accounts for hours and the two investments remains a topic for additional research.

V. Conclusion

The results in the previous section show that the existence of a household sector can have a large effect on the behavior of aggregate market variables. It is natural to inquire how sensitive these results are to the particular values of the parameters that we chose, especially since some of them are not especially well measured. Obviously, parameter values will matter somewhat; as discussed earlier, if $e = \eta = 0$, then the home production economy exactly reproduces the statistics of the standard model.[19] We have experimented with changing

[19] Even with η set at our preferred value of .08, when $e = 0$ the two models are remarkably similar. Hence, one way to interpret the standard model is that it contains a home sector implicitly but assumes that e is close to zero.

all the parameter values in a neighborhood of those used above. For example, changing e or γ changes agents' willingness or incentive to substitute between the market and nonmarket sectors: if e gets too low or γ too high, we approach the standard model; if e gets too high or γ too low, the effects in part C of table 1 become exaggerated. Nevertheless, including home production improves the performance of the model along several dimensions over a sizable region of parameter space (see Benhabib et al. [1990b, app. B] for details).

The fact that the household sector is large is incontrovertible. Models without home production implicitly make the assumption that the willingness or the incentive of individuals to substitute between market and nonmarket activity is small, but this does not seem to be the conclusion one would want to draw from the evidence. The fact that available evidence on some important variables is imperfect leads us to conclude that future research ought to subject parameters such as the elasticity of substitution and technological progress in the nonmarket sector to the same level of analysis that has been afforded variables such as the coefficient of risk aversion and the Solow residual in the market sector. If theory predicted that the choice of these parameters was of minor importance, then their values would not be of much interest to macroeconomists; but this is not what theory predicts.

Appendix

Here we analyze the deterministic steady state and demonstrate how the parameters a and b are chosen. Begin by setting the shocks to their unconditional means, $s_m = s_n = 1$, and substituting the constraints into the utility function, to yield the objective function

$$\sum \beta^t U[f(h_{mt}, k_{mt}) - k_{mt+1} - k_{nt+1} + (1 - \delta)(k_{mt} + k_{nt}), g(h_{nt}, k_{nt}), h_{mt}, h_{nt}].$$

The first-order conditions for maximizing this objective are

$$U_1(t)f_1(t) + U_3(t) = 0, \tag{A1}$$

$$U_2(t)g_1(t) + U_4(t) = 0, \tag{A2}$$

$$U_1(t)f_2(t) + (1 - \delta)U_1(t) = \beta^{-1}U_1(t + 1), \tag{A3}$$

and

$$U_2(t)g_2(t) + (1 - \delta)U_1(t) = \beta^{-1}U_1(t + 1), \tag{A4}$$

where the notation $F(t)$ indicates that a function $F(\cdot)$ is being evaluated at arguments as of date t. In the steady state, of course, these arguments do not depend on time.

We are given values for the parameters β, δ, θ, and η plus the steady-state time allocation h_m^* and h_n^*. With the functional forms described in the text, (A3) immediately implies $\theta(k_m/h_m)^{\theta-1} = \beta^{-1} - 1 + \delta$, and this can be solved

for k_m^*. The first-order conditions also imply the following relation between the capital/labor ratios in the market and the household:

$$\frac{k_m}{h_m} = \frac{\eta(1 - \theta)}{\theta(1 - \eta)} \frac{k_n}{h_n},$$

which can be solved for k_n^* given the solution for k_m^*. Now $c_n^* = g(h_n^*, k_n^*)$ and $c_m^* = f(h_m^*, k_m^*) - i^*$, where $i^* = \delta(k_m^* + k_n^*)$. Notice that we have solved for the steady-state allocation $(c_m^*, c_n^*, h_m^*, h_n^*)$ without using the instantaneous utility function at all. The strategy now is to determine the parameters a and b of this function so that this solution satisfies the marginal conditions (A1) and (A2). For the preference structure in the text there is a unique (a, b). Note that the elasticity parameter e affects the implied values of a and b, but none of the observable variables; the risk aversion parameter r does not affect the steady state at all.

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