

Homework #2

#1

1. a) Maximize: $4x + 5y$ where $x = \text{tort}$, $y = \text{pie}$

Constraint: $2x + 3y \leq 60$

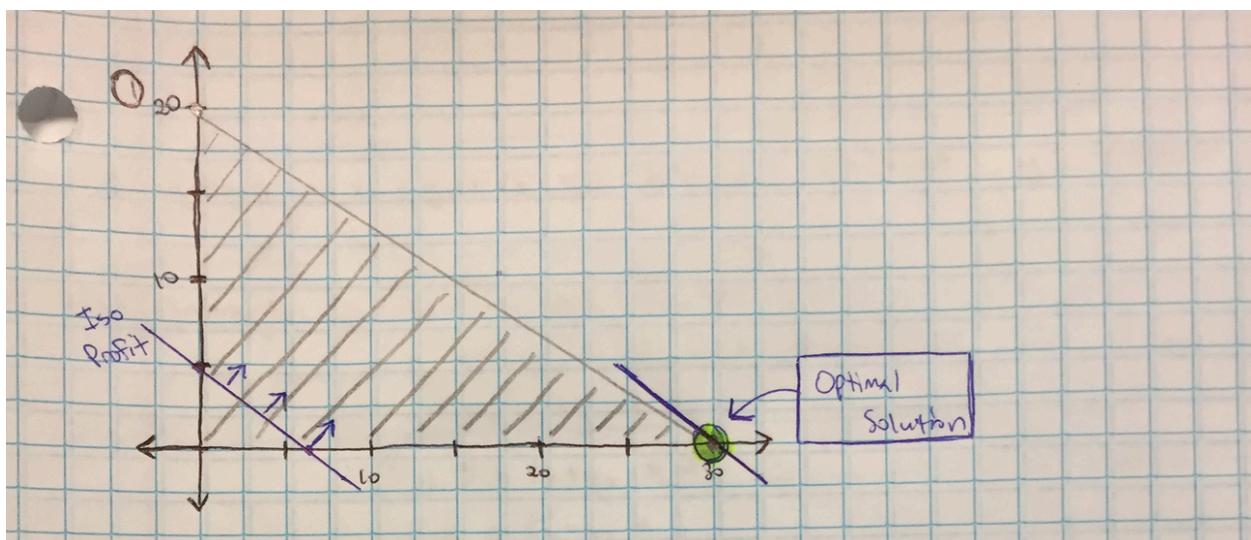
$x, y \geq 0$

Solution: 120 points scored at $(30, 0) \rightarrow 30 \text{ torts, no pies}$

b) Extra constraint: $y \geq x \rightarrow x - y \leq 0$

Solution: 108 points scored at $(12, 12) \rightarrow 12 \text{ pies + 12 torts}$

- This strategy cost 12 points



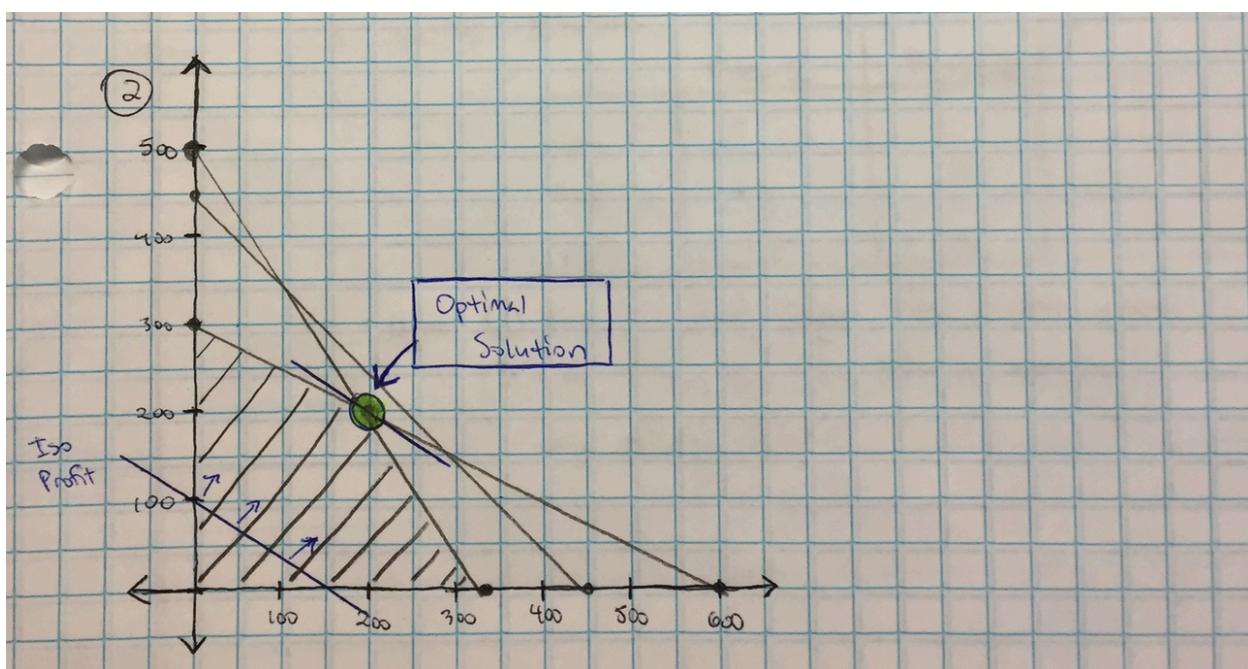
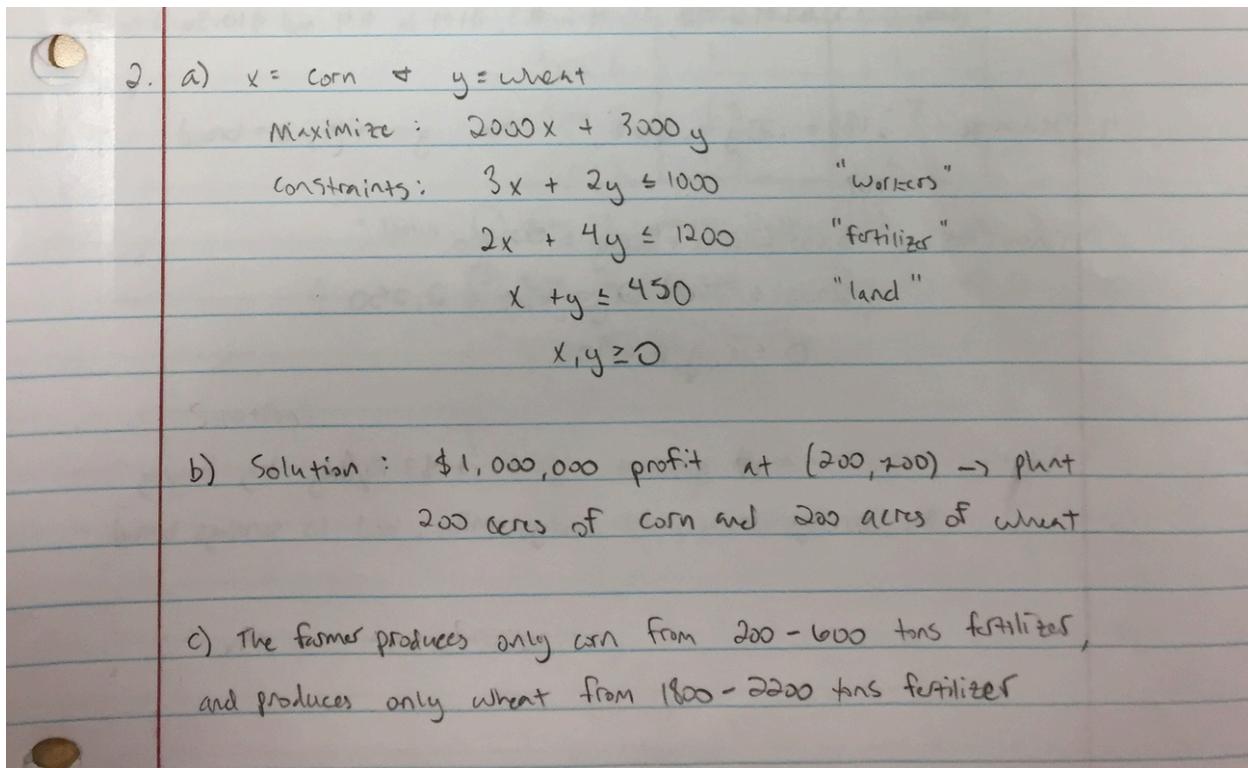
1.

A

b

$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \leq \begin{bmatrix} 60 \\ 0 \end{bmatrix}$$

#2



$$2. \quad \begin{array}{c|ccccc} & A & & & b \\ \left[\begin{array}{cc} 3 & 2 \\ 2 & 4 \\ 1 & 1 \end{array} \right] & \left[\begin{array}{c} x \\ y \end{array} \right] & \leq & \left[\begin{array}{c} 1000 \\ 1200 \\ 450 \end{array} \right] \end{array}$$

#3

$$3. \quad \text{Maximize: } 13a_0 + 16b_0 + 16c_0 + 14d_0 + 39e_0 + 13a_1 + 16b_1 + 16c_1 + 14d_1 + 39e_1$$

Constraints:

$$a_0 + b_0 + c_0 + d_0 + e_0 \leq 1, \quad a_1 + b_1 + c_1 + d_1 + e_1 \leq 1$$

$$11a_0 + 53b_0 + 5c_0 + 5d_0 + 29e_0 \leq 40$$

$$3a_1 + 6b_1 + 5c_1 + d_1 + 34e_1 \leq 20$$

$$a_0, b_0, c_0, d_0, e_0 \geq 0, \quad a_1, b_1, c_1, d_1, e_1 \geq 0$$

$$a_0 \leq 11, \quad b_0 \leq 53, \quad c_0 \leq 5, \quad d_0 \leq 5, \quad e_0 \leq 29$$

$$a_1 \leq 3, \quad b_1 \leq 6, \quad c_1 \leq 5, \quad d_1 \leq 1, \quad e_1 \leq 34$$

Solution: A NPV of \$60.49 Million is created by investing:

- Time 0 = \$6M in #2, \$5M in #3, and \$2M in #5
- Time 1 = \$3.64M in #2, \$5M in #3, \$1M in #4, and \$10.36M in #5

$$3. \quad \begin{array}{c|ccccc|c|ccccc} & A & & & & & b \\ \left[\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & & 1 \\ 11 & 53 & 5 & 5 & 29 & & 1 \\ 3 & 6 & 5 & 1 & 34 & & 40 \\ 40 & & & & & & 20 \\ 40 & & & & & & 20 \\ 40 & & & & & & 20 \\ 20 & & & & & & 20 \\ 20 & & & & & & 20 \\ 20 & & & & & & 20 \end{array} \right] & \left[\begin{array}{c} a_0 \\ b_0 \\ c_0 \\ d_0 \\ e_0 \\ a_1 \\ b_1 \\ c_1 \\ d_1 \\ e_1 \end{array} \right] & \leq & \left[\begin{array}{c} 1 \\ 1 \\ 40 \\ 20 \\ 11 \\ 53 \\ 5 \\ 5 \\ 29 \\ 3 \\ 6 \\ 5 \\ 1 \\ 34 \end{array} \right] \end{array}$$

#4

4. Minimize: $.18x + .23y + .05z$, $x = \text{corn}$, $y = \text{milk}$, $z = \text{bread}$

Constraints:

$$5000 \leq 107x + 500y \leq 50,000$$

$$2000 \leq 72x + 121y + 65z \leq 2,250$$

$$0 \leq x, y, z \leq 10$$

Solutions: The minimum amount spent is \$3.15/day by buying 1.94 servings corn, 10 servings milk, and 10 servings bread

4.

	A	b
	$\begin{bmatrix} 107 & 500 & 0 \\ -107 & -500 & 0 \\ 72 & 121 & 65 \\ -72 & -121 & -65 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \leq \begin{bmatrix} 50000 \\ -5000 \\ 2250 \\ -2000 \\ 10 \\ 10 \\ 10 \end{bmatrix}$

#5

5. Maximize: $x_1 + y_1 + x_2 + y_2 + x_3 + y_3$, $x^i = \text{Forest 1}$, $y^i = \text{Forest 2}$

$$\text{Constraints: } x_1 \leq 1, x_2 \leq 1.3, x_3 \leq 1.4$$

$$y_1 \leq 1, y_2 \leq 1.2, y_3 \leq 1.6$$

$$1.2 \leq x_1 + y_1 \leq 2$$

$$1.5 \leq x_2 + y_2 \leq 2$$

$$2 \leq x_3 + y_3 \leq 3$$

Solution: To obtain the max of 7 tons produced, here is

a cutting schedule

	Forest 1	Forest 2
time 1	1	1
time 2	1.3	.7
time 3	1.4	1.6

- Notes: ① time 1 must happen like it does above
- ② time 2 and time 3, this is not the only optimal solution.
- ③ there can be different answers as long as the

A

$$5. \quad \begin{bmatrix} 1 & 0 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

b

$$\leq \begin{bmatrix} 2 \\ -1.2 \\ 2 \\ -1.5 \\ 3 \\ -2 \end{bmatrix}$$

 \leq

$$\begin{bmatrix} 1 \\ 1.3 \\ 1.4 \\ 1 \\ 1.2 \\ 1.6 \end{bmatrix}$$