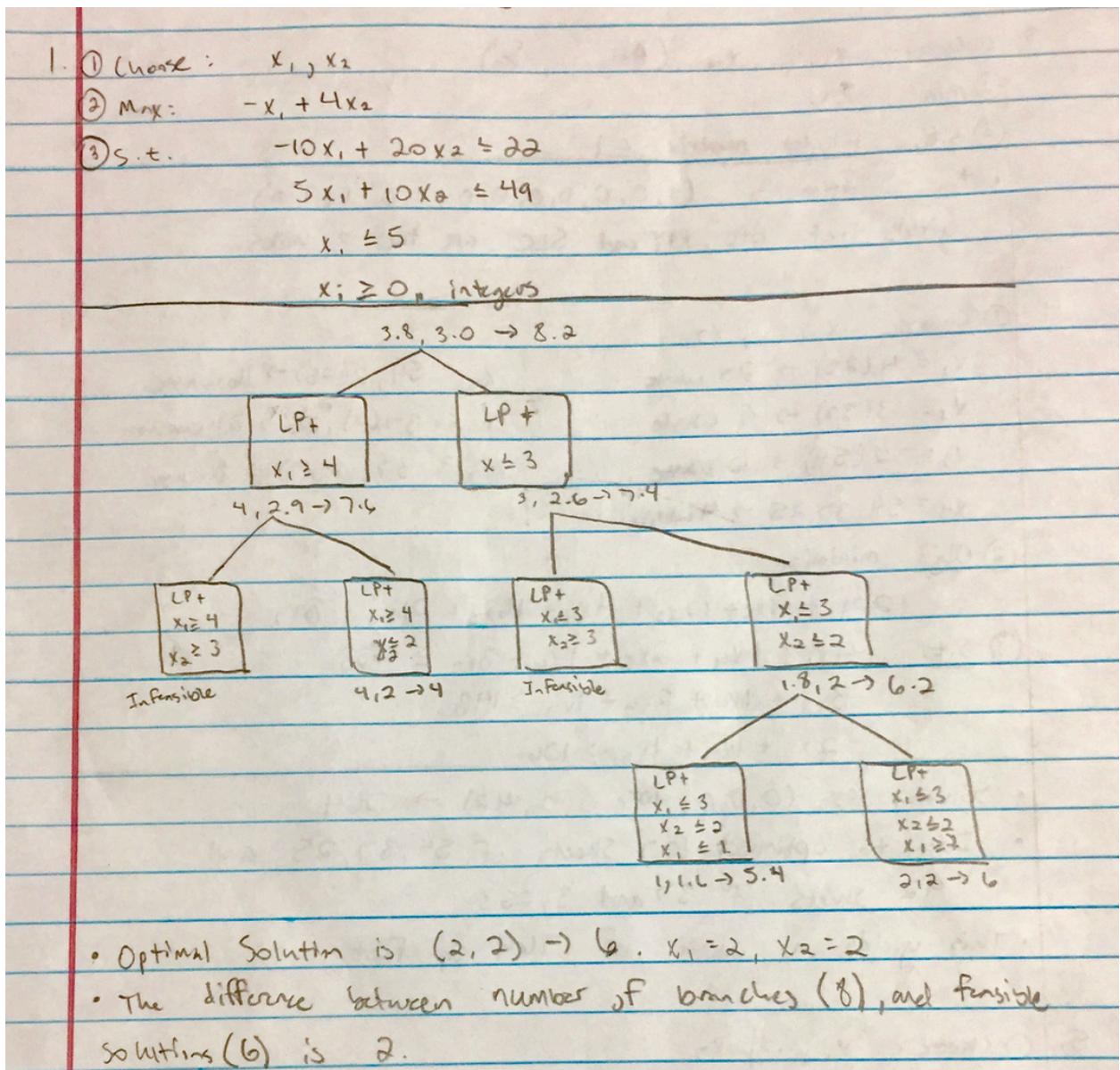


## Homework #3

## Question #1



### Question 1

```
library(lpSolve)

## Problem 1
# Original Setup
c = c(-1,4)
A = matrix(c(-10,5,1,20,10,0),3,2)
dir = rep('≤',3)
b = c(22,49,5)

sol = lp("max",c,A,dir,b, compute.sens = TRUE)
sol$solution

## [1] 3.8 3.0
sol$objval

## [1] 8.2
# x1 ≥ 4
A2 = matrix(c(-10,5,1,-1,20,10,0,0),4,2)
dir2 = rep('≤',4)
b2 = c(22,49,5,-4)

sol2 = lp("max",c,A2,dir2,b2, compute.sens = TRUE)
sol2$solution

## [1] 4.0 2.9
sol2$objval

## [1] 7.6
# x1 ≥ 4, x2 ≥ 3
A3 = matrix(c(-10,5,1,-1,0,20,10,0,0,-1),5,2)
dir3 = rep('≤',5)
b3 = c(22,49,5,-4, -3)

sol3 = lp("max",c,A3,dir3,b3, compute.sens = TRUE)
sol3$status

## [1] 2
# x1 ≥ 4, x2 ≤ 2
A4 = matrix(c(-10,5,1,-1,0,20,10,0,0,1),5,2)
dir4 = rep('≤',5)
b4 = c(22,49,5,-4, 2)
```

## Brett Scroggins

```
sol4 = lp("max",c,A4,dir4,b4, compute.sens = TRUE)
sol4$status

## [1] 0
sol4$solution

## [1] 4 2
sol4$objval

## [1] 4

# x1 <= 3
A5 = matrix(c(-10,5,1,1,20,10,0,0),4,2)
dir5 = rep('<=',4)
b5 = c(22,49,5,3)

sol5 = lp("max",c,A5,dir5,b5, compute.sens = TRUE)
sol5$solution

## [1] 3.0 2.6
sol5$objval

## [1] 7.4

# x1 <= 3, x2 >= 3
A6 = matrix(c(-10,5,1,1,0,20,10,0,0,-1),5,2)
dir6 = rep('<=',5)
b6 = c(22,49,5,3,-3)

sol6 = lp("max",c,A6,dir6,b6, compute.sens = TRUE)
sol6$status

## [1] 2

# x1 <= 3, x2 <= 2
A7 = matrix(c(-10,5,1,1,0,20,10,0,0,1),5,2)
dir7 = rep('<=',5)
b7 = c(22,49,5,3,2)

sol7 = lp("max",c,A7,dir7,b7, compute.sens = TRUE)
sol7$solution

## [1] 1.8 2.0
sol7$objval

## [1] 6.2

# x1 <= 3, x2 <= 2, x1 <= 1
A8 = matrix(c(-10,5,1,1,0,1,20,10,0,0,1,0),6,2)
dir8 = rep('<=',6)
b8 = c(22,49,5,3,2,1)

sol8 = lp("max",c,A8,dir8,b8, compute.sens = TRUE)
sol8$solution

## [1] 1.0 1.6
```

```

sol8$objval

## [1] 5.4

# x1 <= 3, x2 <= 2, x1 <= 1
A9 = matrix(c(-10,5,1,1,0,-1,20,10,0,0,1,0),6,2)
dir9 = rep('≤',6)
b9 = c(22,49,5,3,2,-2)

sol9 = lp("max",c,A9,dir9,b9, compute.sens = TRUE)
sol9$solution

## [1] 2 2

sol9$objval

## [1] 6

# Check with int.vec
c = c(-1,4)
A = matrix(c(-10,5,1,20,10,0),3,2)
dir = rep('≤',3)
b = c(22,49,5)

sol = lp("max",c,A,dir,b, int.vec = 1:2)
sol$solution

## [1] 2 2

sol$objval

## [1] 6

```

## Question #2

2. ① Choose:  $x_1, x_2 \rightarrow$  factory Austin, Dallas,  $y_1, y_2 \rightarrow$  warehouse Austin, Dallas

② Max:  $9x_1 + 5x_2 + 6y_1 + 4y_2$

③ s.t.

$$6x_1 + 3x_2 + 5y_1 + 2y_2 \leq 11$$

$$x_1 + x_2 + y_1 + y_2 \leq 4$$

$$x_1, x_2, y_1, y_2 \geq 0$$

$$x_1 + x_2 \geq 1, y_1 + y_2 \geq 1$$

• Optimal solution is  $(1, 1, 0, 1) \rightarrow$  factory in Austin, and factory and warehouse in Dallas. Yields a max expected profit of \$18 M.

### Question 2

```
## Problem 2
c = c(9,5,6,4)
A = matrix(0,8,4)
A[1,] = c(6,3,5,2)
A[2,] = c(rep(1,4))
A[3:6,] = diag(1,4)
A[7,] = c(1,1,0,0)
A[8,] = c(0,0,1,1)
dir = c(rep('<=' ,6),rep('>=' ,2))
b = c(11,4,1,1,1,1,1,1)

sol = lp("max",c,A,dir,b, int.vec = 1:4)
sol$solution

## [1] 1 1 0 1
sol$objval

## [1] 18
```

### Question #3

3. ① Choose:  $x_1, \dots, x_{12}$  ( $A+I_1, \dots, S_{12}$ )  
② Min:  $\sum x_i$   
③ s.t. Flight matrix  $\leq I$

The solution is  $(1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0)$   
yields that ATL, NY and SLC are the 3 hubs

### Question 3

```
## Problem 3
c = rep(1,12)
b = rep(1,12)
dir = rep('>=',12)

A = matrix(0,12,12)
A[1,] = c(1,0,1,0,1,0,1,1,1,0,0,0)
A[2,] = c(0,1,0,0,0,0,0,1,1,0,0,0)
A[3,] = c(1,0,1,0,0,0,1,1,1,0,0,0)
A[4,] = c(0,0,0,1,0,0,0,0,0,1,0,0)
A[5,] = c(1,0,0,0,1,0,1,0,0,0,0,0)
A[6,] = c(0,0,0,0,0,1,0,0,0,1,1,0)
A[7,] = c(1,0,1,0,1,0,1,0,0,0,0,0)
A[8,] = c(1,1,1,0,0,0,1,1,0,0,0)
A[9,] = c(1,1,1,0,0,0,0,1,1,0,0,0)
A[10,]= c(0,0,0,1,0,1,0,0,0,1,1,1)
A[11,]= c(0,0,0,0,0,1,0,0,0,1,1,1)
A[12,]= c(0,0,0,0,0,0,0,0,0,1,1,1)

sol = lp("min",c,A,dir,b, int.vec = 1:12)
sol$solution

## [1] 1 0 0 0 0 0 0 1 0 1 0 0
sol$objval

## [1] 3
```

## Question #4

4. ① choose:  $x_1, \dots, x_7$

$$x_1 = 4(25) \rightarrow 20 \text{ waste}$$

$$x_2 = 3(37) \rightarrow 9 \text{ waste}$$

$$x_3 = 2(54) \rightarrow 12 \text{ waste}$$

$$x_4 = 54, 37, 25 \rightarrow 4 \text{ waste}$$

$$x_5 = 54, 2(25) \rightarrow 16 \text{ waste}$$

$$x_6 = 37(2), 25 \rightarrow 21 \text{ waste}$$

$$x_7 = 37, 25(2) \rightarrow 8 \text{ waste}$$

② Obj: minimize

$$20x_1 + 9x_2 + 12x_3 + 4x_4 + 16x_5 + 21x_6 + 8x_7$$

③ s.t.

$$4x_1 + 1x_4 + 2x_5 + 1x_6 + 3x_7 \geq 233$$

$$3x_2 + 1x_4 + 2x_6 + 1x_7 \geq 148$$

$$2x_3 + 1x_4 + 1x_5 \geq 106$$

- Solution is  $(0, 0, 0, 107, 0, 0, 42) \rightarrow 764$
- Ideal to optimize 107 sheets of 54, 37, 25 and 42 sheets of 37 and 3, 25's
- This yields a waste of 764 sq. feet

## Question 4

```

## Problem 4
c = c(20, 9, 12, 4, 16, 21, 8)
b = c(233, 148, 106)
dir = c(rep('>=', 3))

A = matrix(0, 3, 7)
A[1,] = c(4, 0, 0, 1, 2, 1, 3)
A[2,] = c(0, 3, 0, 1, 0, 2, 1)
A[3,] = c(0, 0, 3, 1, 1, 0, 0)

sol = lp("min", c, A, dir, b, int.vec = 1:16)
sol$solution

## [1] 0 0 0 107 0 0 42
sol$objval

## [1] 764

```

## Question #5

S. ① Choose:  $x_1, \dots, x_7$   
as  $x_1 = \text{Sunday} - \text{Thursday}$ ,  $x_2 = \text{Monday} - \text{Friday}$ , ...

$$\begin{array}{ll} ③ x_1 + x_4 + x_5 + x_6 + x_7 \geq 5 & x_1 + x_2 + x_3 + x_4 + x_5 \geq 14 \\ x_1 + x_2 + x_5 + x_6 + x_7 \geq 13 & x_2 + x_3 + x_4 + x_5 + x_6 \geq 8 \\ x_1 + x_2 + x_3 + x_6 + x_7 \geq 12 & x_3 + x_4 + x_5 + x_6 + x_7 \geq 6 \\ x_1 + x_2 + x_3 + x_4 + x_7 \geq 10 & \end{array}$$

② Obj: minimize  
 $\underline{330x_1 + 300x_2 + 330x_3 + 360x_4 + 360x_5 + 360x_6 + 360x_7}$

- Solution  $(1, 8, 2, 0, 3, 0, 1) \rightarrow 4830$
- The minimum profit of \$4830/week can be achieved with the  $x$ 's chosen.

## Question 5

```

## Problem 5
c = c(330, 300, 330, rep(360, 4))
b = c(5, 13, 12, 10, 14, 8, 6)
dir = c(rep('>=', 7))

A = matrix(1, 7, 7)
A[1, 2:3] = 0
A[2, 3:4] = 0
A[3, 4:5] = 0
A[4, 5:6] = 0
A[5, 6:7] = 0
A[6, ] = c(0, 1, 1, 1, 1, 1, 0)
A[7, 1:2] = 0

sol = lp("min", c, A, dir, b, int.vec = 1:7)
sol$solution

## [1] 1 8 2 0 3 0 1
sol$objval

## [1] 4830

```