Brett Scroggins

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1. $(A^{-1})^T = (A^T)^{-1}$

$A^T(A^{-1})^T = A^T(A^T)^{-1}$

$(A^{-1}A)^T = I$

$I^T = I$

$I = I$ ■

2. a = First Mort., b = second mort., c = home improv., d = personal overdraft

max: $.14a + .26b + .2c + .1d$

① $a + b + c + d = 1.00$

② $a = .55(a+b) \longrightarrow .45a - .55b = 0$

③ $b = .25$

④ $.14a + .26b + .2c + .1d = .15$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ .45 & -.55 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ .14 & .2 & .2 & .1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ .25 \\ .15 \end{bmatrix}$$

a = 30.56%
b = 25%
c = 12.78%
d = 31.67%

First Mortgage = 30.56% ($76.39 M) Home Improvement = 12.76% ($31.94 M)
Second Mortgage = 25% ($62.5 M) Personal Overdraft = 31.67% ($79.17 M)

Problem 2

```
# Initialize A and b
b = matrix(c(1,0,.25,.15),4,1)
A = matrix(c(1,.45,0,.14,1,-.55,1,.2,1,0,0,.2,1,0,0,.1),4,4)

# Solve for percents and amounts
percents = solve(A) %*% b
print(percents)
```

```
##           [,1]
## [1,] 0.3055556
## [2,] 0.2500000
## [3,] 0.1277778
## [4,] 0.3166667
```

```
amounts = percents*100
print(amounts)
```

```
##          [,1]
## [1,] 30.55556
## [2,] 25.00000
## [3,] 12.77778
## [4,] 31.66667
```

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3. a = var 1, b = var 2, c = var 3, d = var 4

maximize: $1.5a + 2.5b + 3.0c + 4.5d$

$2a + 4b + 3c + 7d \leq 100000$

$3a + 2b + 3c + 4d \leq 50000$

$2a + 3b + 2c + 5d \leq 60000$

$a, b, c, d \geq 0 \rightarrow -a, -b, -c, -d \leq 0$

$$\begin{bmatrix} 2 & 4 & 3 & 7 \\ 3 & 2 & 3 & 4 \\ 2 & 3 & 2 & 5 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \leq \begin{bmatrix} 100000 \\ 50000 \\ 60000 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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4.

$$A \quad r \quad b$$

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{pmatrix} = \begin{pmatrix} -45 \\ -3 \\ -31 \\ -45 \\ 18 \\ 8 \\ 20 \\ 2 \\ -27 \\ -38 \\ 0 \end{pmatrix}$$

$$\hat{A} \quad r \quad = \quad \hat{b}$$

$$\begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{bmatrix} = \begin{bmatrix} -124 \\ 91 \\ -40 \\ -17 \\ 90 \end{bmatrix}$$

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{bmatrix} = \begin{bmatrix} -24.8 \\ 18.2 \\ -8 \\ -3.4 \\ 18 \end{bmatrix}$$

team 1 = rank 5
team 2 = rank 1
team 3 = rank 4
team 4 = rank 3
team 5 = rank 2

Problem 4

```
# Initialize A and b
b = matrix(c(-45,-3,-31,-45,18,8,20,2,-27,-38,0),11,1)
A = matrix(c(1,1,1,1,0,0,0,0,0,0,1,-1,0,0,0,1,1,1,0,0,0,1,0,-1,0,0,-1,0,0,1,1,0,1,0,0,-1,0,0,-1,0,-1,0,
# Solve for A_hat and b_hat
A_hat = t(A) %*% A
b_hat = t(A) %*% b

# Solve for ranks with A_hat and b_hat
ranks = solve(A_hat) %*% b_hat
print(ranks)
```

```
##       [,1]
## [1,] -24.8
## [2,]  18.2
## [3,]  -8.0
## [4,]  -3.4
## [5,]  18.0
```