Math and Logic Behind In-exchange Arbitrage

Yifan Wang

July 17, 2018

1 Arbitrage Detection

This section gives detail about how to decide if there's an arbitrage opportunity.

1.1 Definition

$BC \Longrightarrow$ Base Currency
$QC1 \Longrightarrow$ Quote Currency or Fee Currency 1
$QC2 \Longrightarrow$ Quote Currency or Fee Currency 2, such that "QC2QC1" exists, instead of QC1QC2
$F \Longrightarrow$ Fee rate for a trade, 0.001 for HitBTC. The fee is charged by feeCurrency, which can be found in the API, equivalently quoteCurrency.
$P_X \Longrightarrow$ is the ask price of BC in terms of QC1
$P_Y \Longrightarrow$ is the bid price of BC in terms of QC2
$P_Z \Longrightarrow$ is the bid price of QC2 in terms of QC1
$V_X \Longrightarrow$ is the ask volume of BC corresponding to P_X
$V_Y \Longrightarrow$ is the bid volume of BC corresponding to P_Y
$V_Z \Longrightarrow$ is the bid volume of QC2 corresponding to P_Z
$M \Longrightarrow$ quantity of QC1 that can maximize the profit of arbitrage

1.2 Procedure

For a triangle defined by virtex as $BC - QC1 - QC2$, or defined by edge as $BCQC1 - BCQC2 - QC2QC1$, the criterion of arbitrage is:

$$\frac{P_Y \cdot P_Z}{P_X} \cdot (1 - F)^3 > 1 \tag{1}$$

which can be approximately reduced to

$$\frac{P_Y \cdot P_Z}{P_X} \geq 1.004 \tag{2}$$

When the criterion is met, there's not necessary actual profit. We have to take a look at the volume in the orderbook before trading, with the following decision logic.

Algorithm 1: Arbitrage Initial Volume Calculation

```
Input: P_X, V_X, P_Y, V_Y, P_Z, V_Z
Output: M
  if P_X · V_X > P_Z · V_Z then
    if V_Z > P_Y · V_Y then
      M = P_Y · V_Y · P_Z
    else
      M = P_Z · V_Z
    end if
  else
    if V_X > V_Y then
      M = P_Y · V_Y · P_Z
    else
      M = P_X · V_X
    end if
  end if
```

The output M is the minimal tradable value in terms of $QC1$ on all edges in the triangle. However, the actual amount to be traded has to depend on some other factors like quantity increment.

2 Trading Calculation

2.1 Definition

M $\Longrightarrow$ quantity of QC1 from arbitrage detection procedure in Section 1 that can maximize the profit of arbitrage

$Q_{BC-QC} \Longrightarrow$ Quantity Increment for $BCQC$. For any order to be posted on the orderbook, the bid/ask amount of a BC must be a multiple of Q

$A^b_{BC} \Longrightarrow$ Amount of BC to buy
$A^s_{BC} \Longrightarrow$ Amount of BC to sell
$A^s_{QC2} \Longrightarrow$ Amount of QC2 to sell

$f_Q[X] = Q \cdot int(\frac{X}{Q})$, where $int()$ is to get integer part of a number.

2.2 Procedure

When a triangle-arbitrage $BCQC1 - BCQC2 - QC2QC1$ is identified with acceptable potential profit, we have M, from previous section, which is maximal input QC1,

$$A^b_{BC} = f_{Q_{BC-QC1}}[\frac{M \cdot (1-F)}{P_X}] \tag{3}$$

$$A^s_{BC} = f_{Q_{BC-QC2}}[A^b_{BC}] \tag{4}$$

$$A^s_{QC2} = f_{Q_{QC2-QC1}}[A^s_{BC} \cdot P_Y \cdot (1-F)] \tag{5}$$

The Amount of QC1 got after three trades is

$$A_{QC1} = f_{T_{QC2-QC1}} A^s_{QC2} \cdot P_Z \cdot (1-F) \tag{6}$$

The maximal amount of QC1 we calculate as the input of a triangular arbitrage can not necessarily be fully used for trading, because of the Q. We need to "calibrate" the amount after each trade.