Spring Semester 2023

AIRCRAFT AERODYNAMICS & FLIGHT MECHANICS

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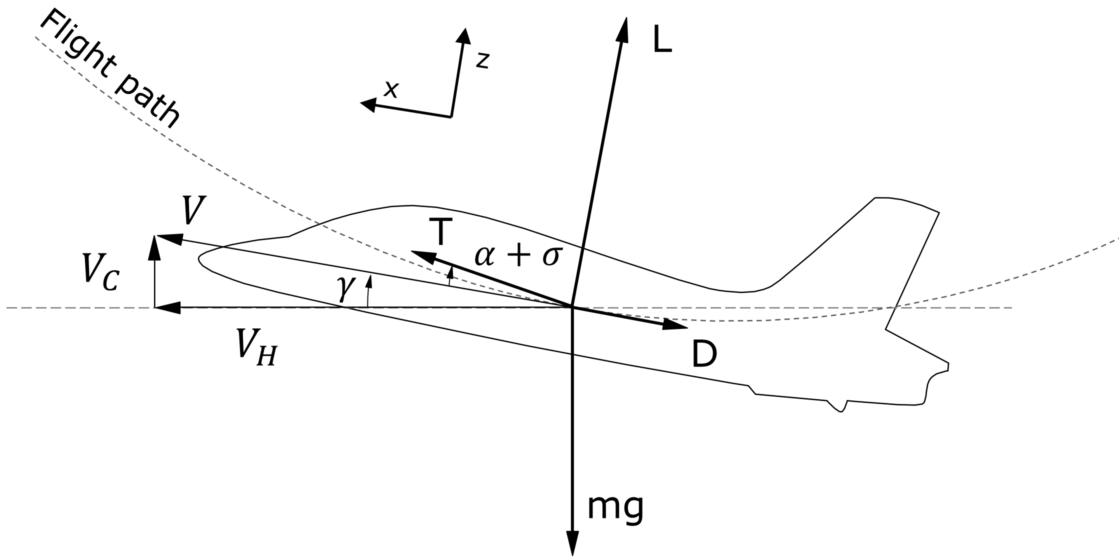
This lecture is adapted with permission from the lecture “Ausgewählte Kapitel der Flugtechnik” by Dr. Jürg Wildi

Recap - Equations of Motion

Velocity vector coordinate frame



In flight path direction

$$T cos(\alpha + \sigma) - D - mg\ sin(\gamma) = m\frac{dV}{dt}$$

Normal to flight path

$$L + T sin(\alpha + \sigma) - mg\ cos(\gamma) = mV\frac{d\gamma}{dt}$$

Kinematic relationships

$$\frac{dh}{dt} = V_C = V sin(\gamma)$$

$$\frac{dx}{dt} = V_H = V cos(\gamma)$$

α: angle of attack
σ: thrust incidence angle

Note: often small angle approximations are used

Recap - SEP

Specific Excess Power (SEP), denoted P_S

$$P_S = V\frac{(T-D)}{mg}$$

Where is SEP coming from?

- Excess thrust / excess power

$$P_S = V_C + \frac{V}{g}\frac{dV}{dt}$$

What can I do with positive SEP?

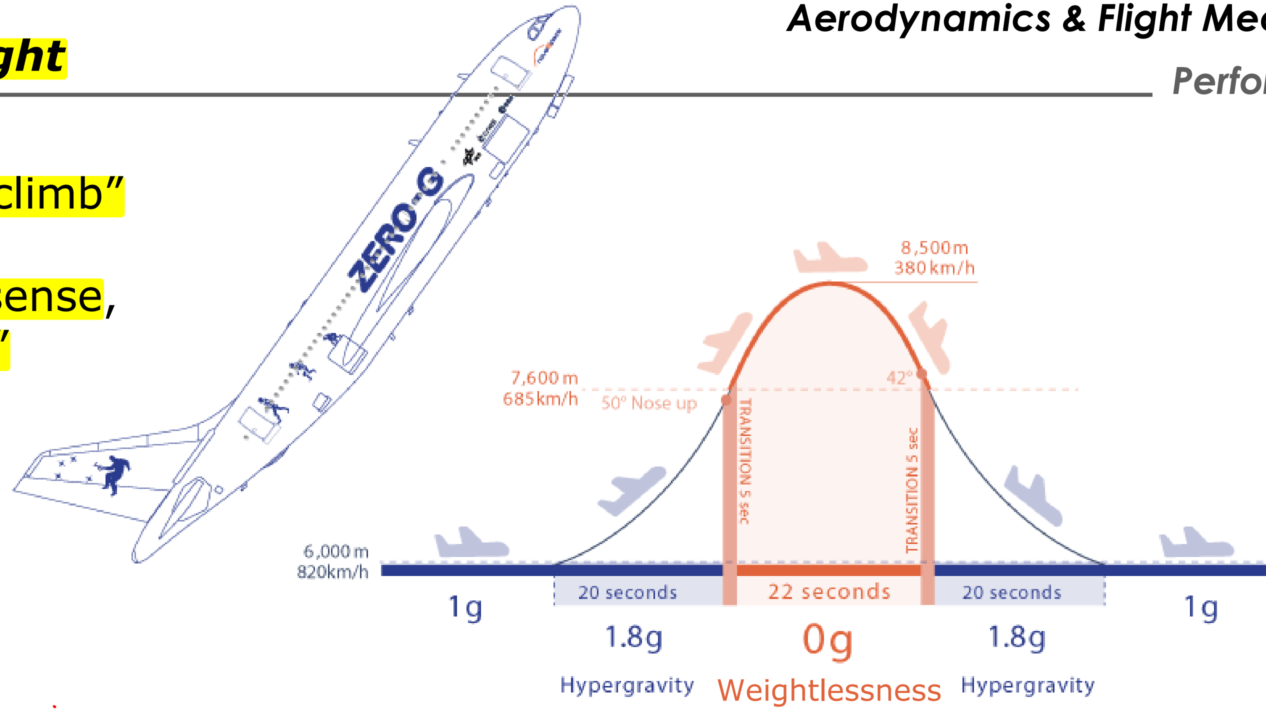
- Climb
- Accelerate

Since SEP depends on the available thrust and the aerodynamic drag, SEP is a function of the velocity ("drag bucket") and the altitude (engine power loss with altitude)

Climb

Parabolic Flight

This is not a "climb" in the flight performance sense, but a "pull-up"



no change in flight path angle

Climb: $L - mg\,cos(\gamma) = 0$

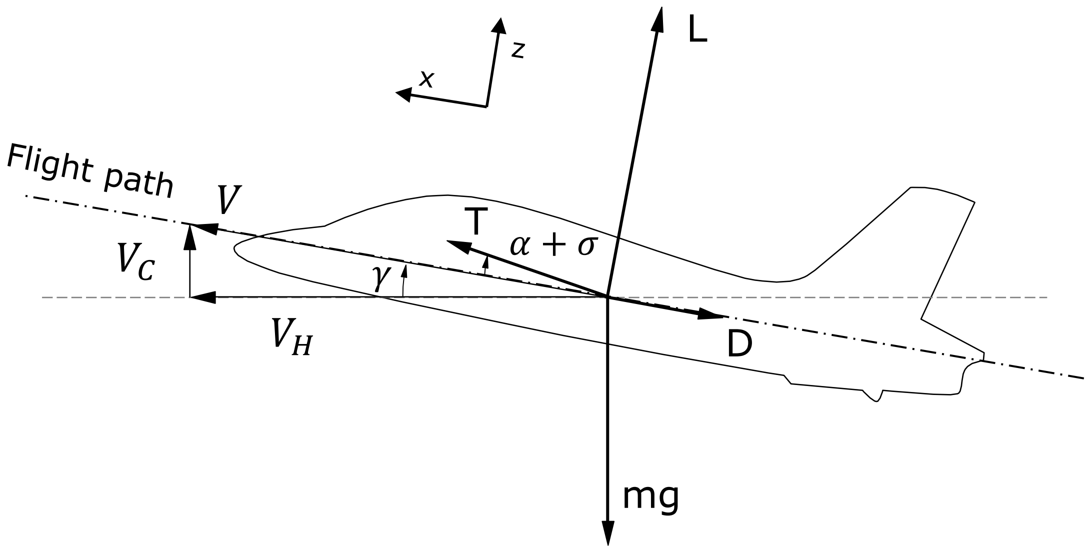
Pull-up: $L - mg\,cos(\gamma) = mV\dfrac{d\gamma}{dt}$

Load factor $n = \dfrac{L}{mg}$

example: n=1.8

For a climb: $n = \dfrac{L}{mg} = \dfrac{mgcos(\gamma)}{mg} = \cos(\gamma) \Rightarrow n < 1$

Stationary Climb Performance



Stationary climb

$$\frac{d\gamma}{dt} = 0 \qquad \frac{dV}{dt} = 0$$

$$T\cos(\alpha + \sigma) = T$$

$$T\cos(\alpha + \sigma) - D - mg\,\sin(\gamma) = m\frac{dV}{dt}$$

$$L + T\sin(\alpha + \sigma) - mg\,\cos(\gamma) = mV\frac{d\gamma}{dt}$$

$$\Rightarrow$$

$$T - D - mg\,\sin(\gamma) = 0$$

$$L - mg\,\cos(\gamma) = 0$$

$$V_C = V\sin(\gamma) \qquad T - D - mg\frac{V_C}{V} = 0 \quad \Rightarrow \quad V\frac{(T - D)}{mg} = V_C$$

This is the definition of SEP

$$\boxed{\sin(\gamma) = \gamma = \frac{(T - D)}{mg}}$$

Climb Performance with Acceleration

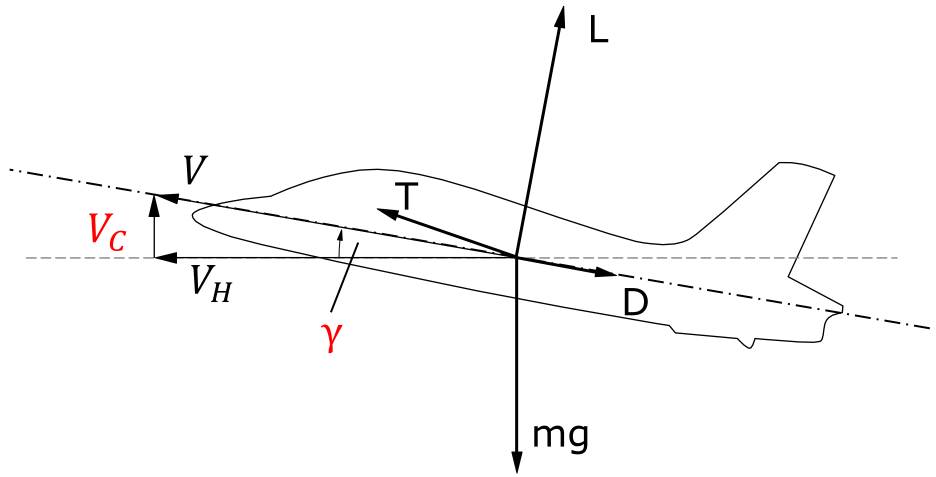
$$\frac{d\gamma}{dt} = 0 \qquad \frac{dV}{dt} \neq 0$$

$$T - D - mg\,sin(\gamma) = m\frac{dV}{dt} \qquad \text{with} \qquad V_C = V sin(\gamma)$$

$$T - D - mg\frac{V_C}{V} = m\frac{dV}{dt}$$

$$V_C = V\underbrace{\frac{(T-D)}{mg}}_{P_S} - \frac{V}{g}\frac{dV}{dt} \qquad\qquad P_S = V_C + \frac{V}{g}\frac{dV}{dt}$$

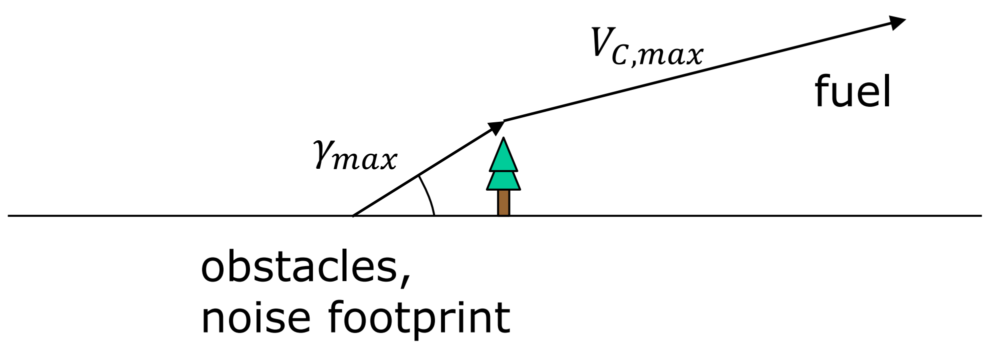
Stationary Climb Speeds



Two flight speeds V are of interest

- Speed for best climb rate: minimize time to climb
- Speed for best climb angle: minimize horizontal distance

relevant for takeoff



Climb Performance Chart

1. From c_{D0} and c_{Di} calculate the drag force [N] over a range of flight speeds (cruise condition)

$P_{required}$

2. Compute the required power from the drag force

$P_{available}$

3. Add the engine data (thrust available or power available)

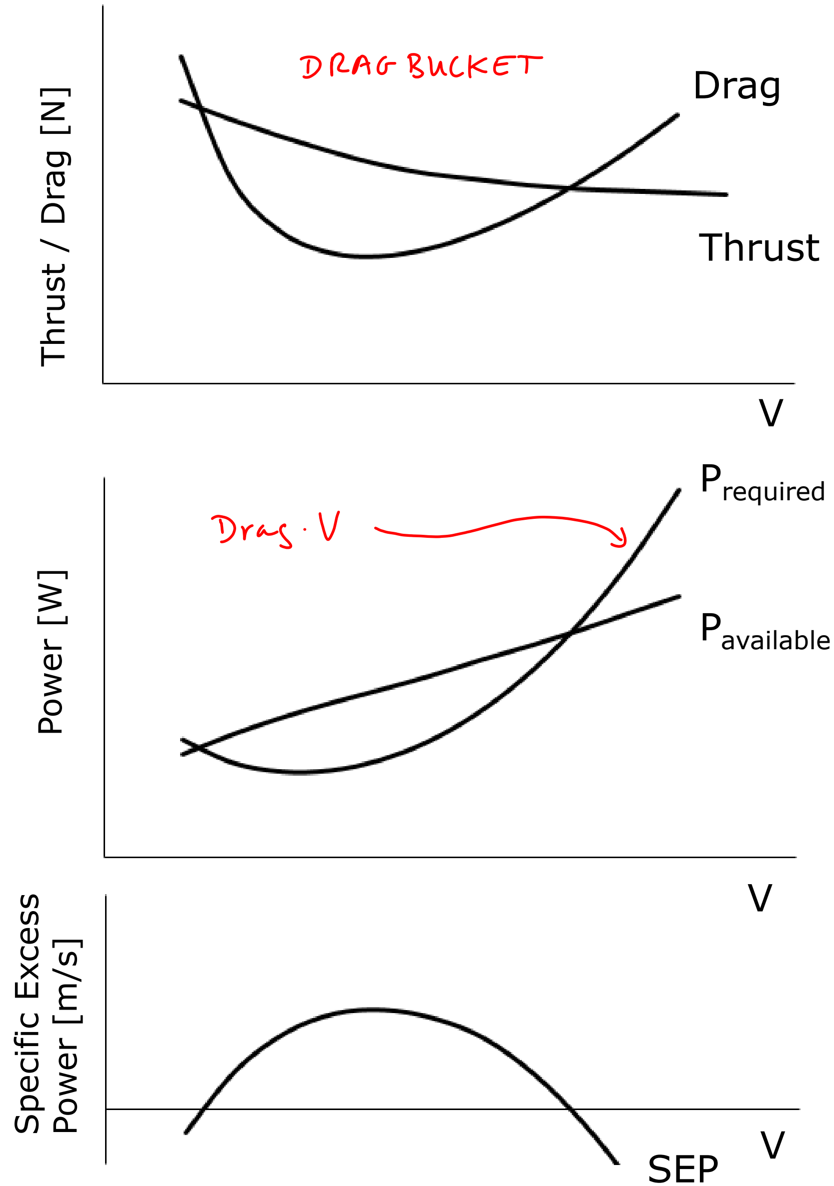
4. Compute SEP

$$P_S = V\frac{(T-D)}{mg} = \frac{P_{avail} - P_{req}}{mg}$$

$$\Rightarrow \boxed{P_S = V_C}$$

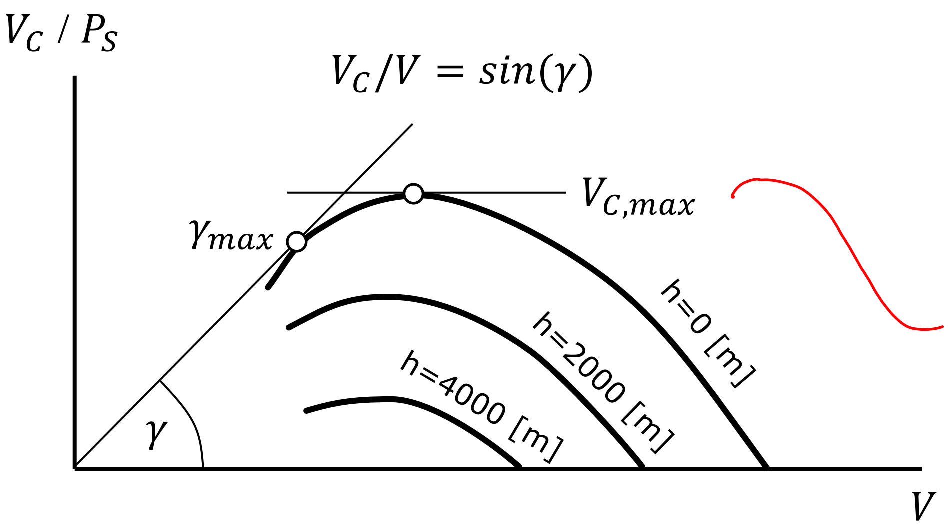
For stationary climb

5. Repeat for different altitudes



Climb Performance – Angle and Rate

The SEP-Altitude chart shows the climb speed, the airspeed and the climb angle



$$\sin(\gamma) = \gamma = \frac{(T - D)}{mg}$$

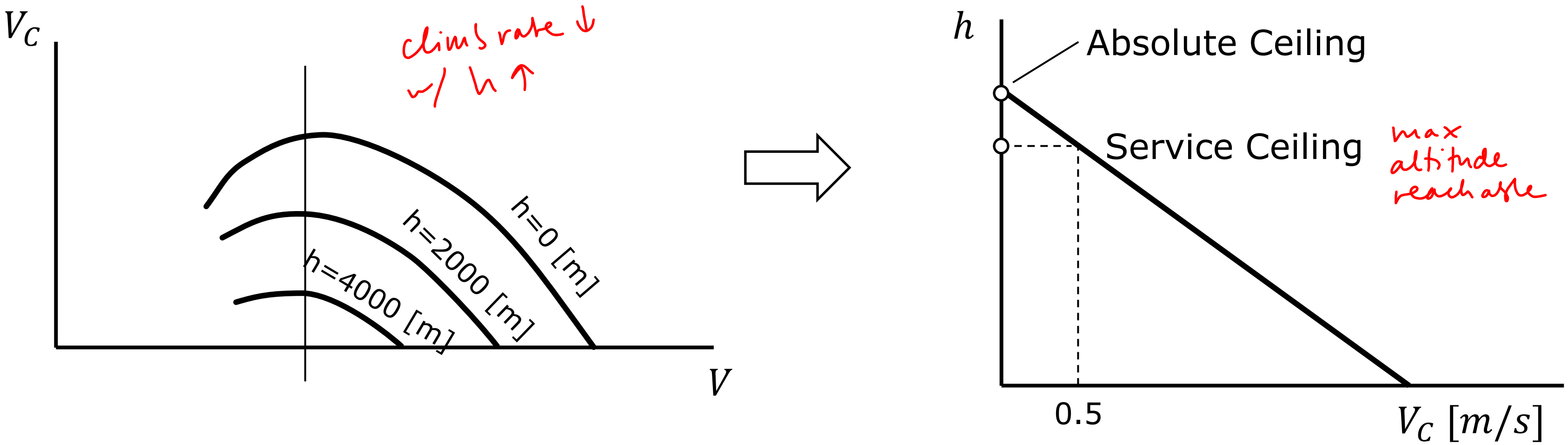
$$\gamma_{max} \Rightarrow (T - D)_{max}$$

$$V_{C,max} = V\frac{(T - D)}{mg} = \frac{\left(P_{avail} - P_{req}\right)_{max}}{mg}$$

This procedure works independent of the aircraft type / propulsion type

Note: some assumptions were made on the way,
such as $V = V_H$ and small AoA for the computation of SEP

Climb Performance – Altitude & Ceiling



Max. flight altitude (ceiling) can be defined using SEP:

		climb rate (SEP)	
Absolute Ceiling:	0 ft/min	0 m/s	
Service Ceiling:	100 ft/min	0.508 m/s	
Cruise Ceiling:	300 ft/min	1.524 m/s	Maneuver reserve
Combat Ceiling:	500 ft/min	2.54 m/s	Maneuver reserve

This is also easy to use in flight ops, e.g.:

- Climb is stopped when the climb rate reaches 300 ft/min

Source: Airbus, Getting to grips with Aircraft Performance

Time to Climb Calculation

$$t = \int_{h0}^{h1} \frac{1}{V_C(h)} dh$$

Can be integrated by assuming a climb rate that depends linearly on altitude:

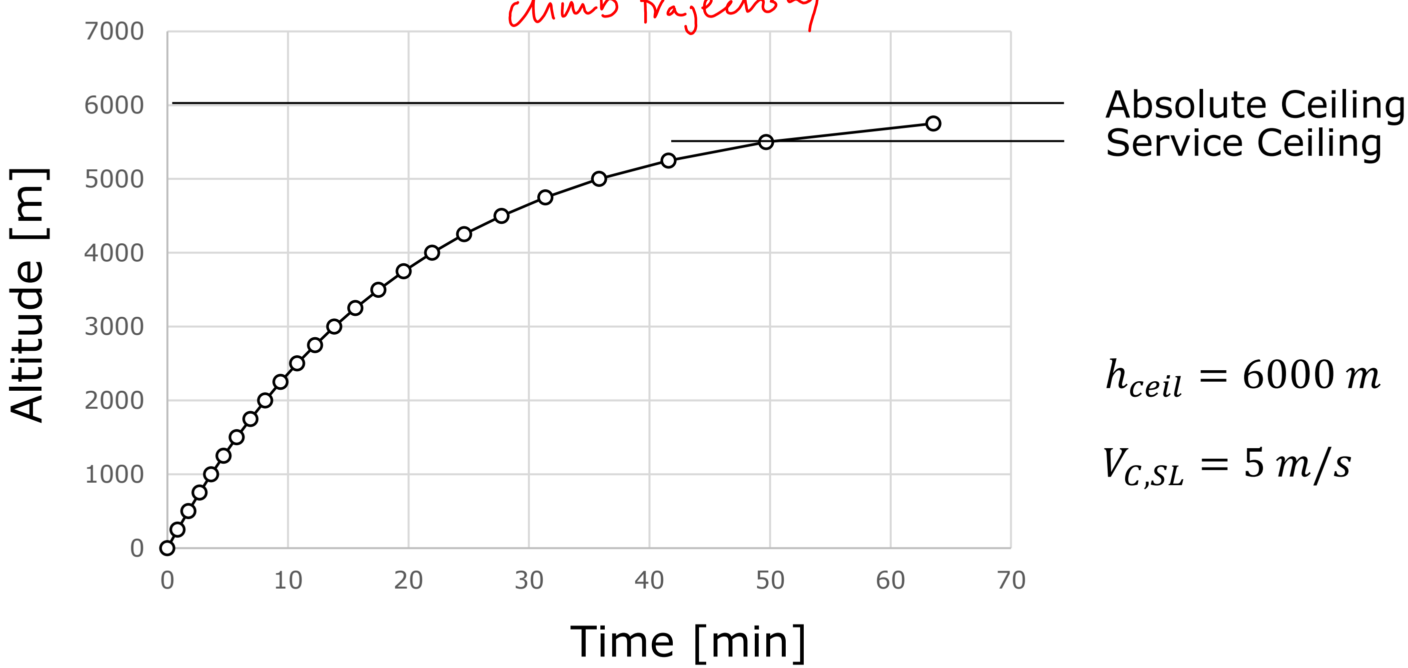
$$V_C(h) = V_{C,SL}\left[1 - \frac{h}{h_{ceil}}\right]$$

from linear graph on p11

$$t = \int_{h0}^{h1} \frac{1}{V_C(h)} dh = \frac{1}{V_{C,SL}} \int_{h0}^{h1} \frac{1}{1 - \frac{h}{h_{ceil}}} dh \quad \Rightarrow \quad t = \frac{h_{ceil}}{V_{C,SL}} \ln \frac{1}{1 - \frac{h}{h_{ceil}}}$$

Example chart

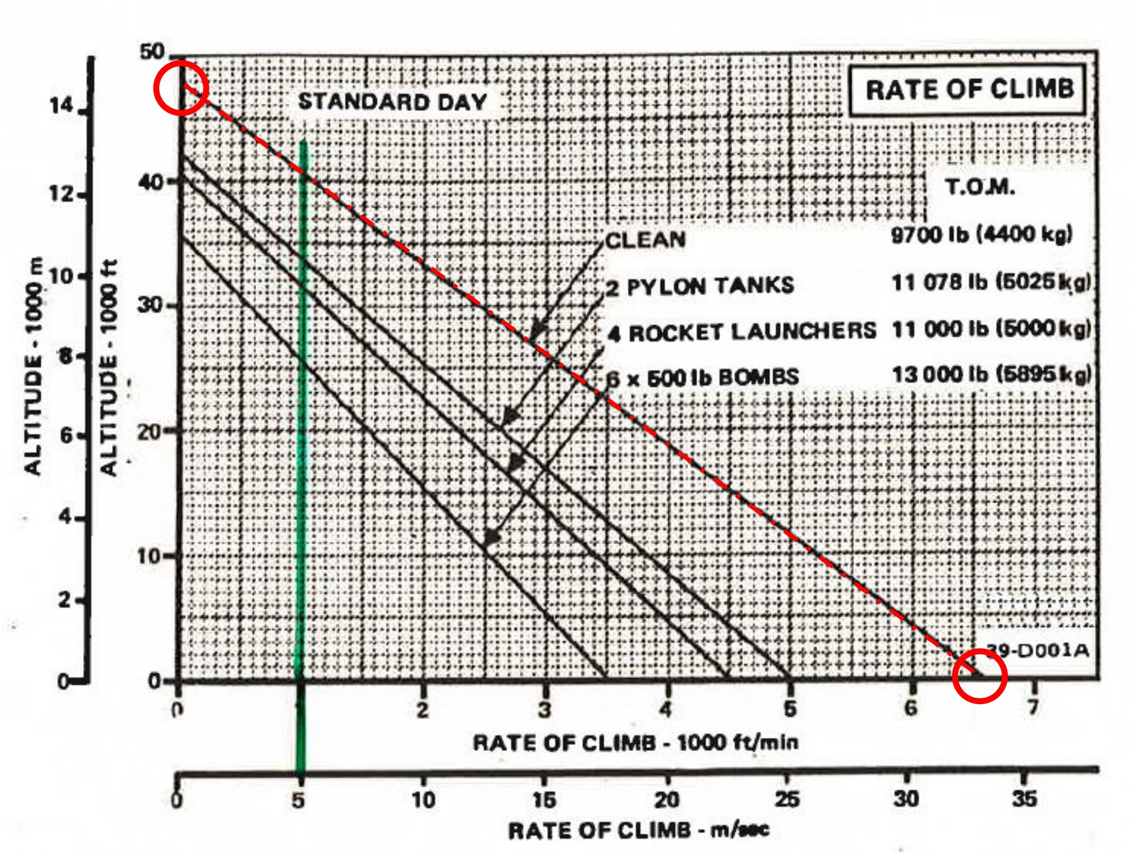
climb trajectory



$h_{ceil} = 6000\, m$

$V_{C,SL} = 5\, m/s \quad \approx 980\, ft/min$

Time to Climb - Example MB-339

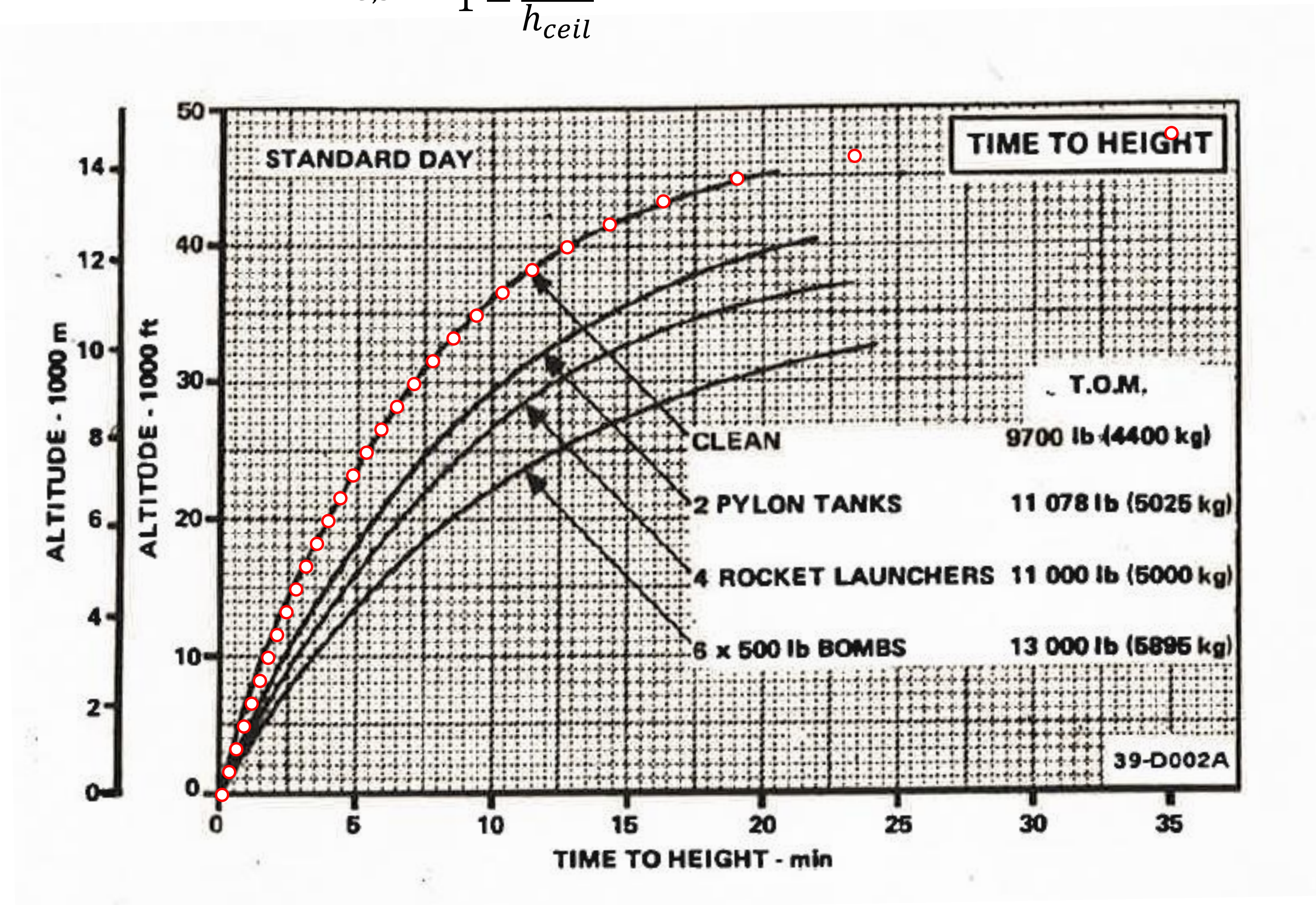


$$h_{ceil} = 48'000\ ft = 14630\ m$$

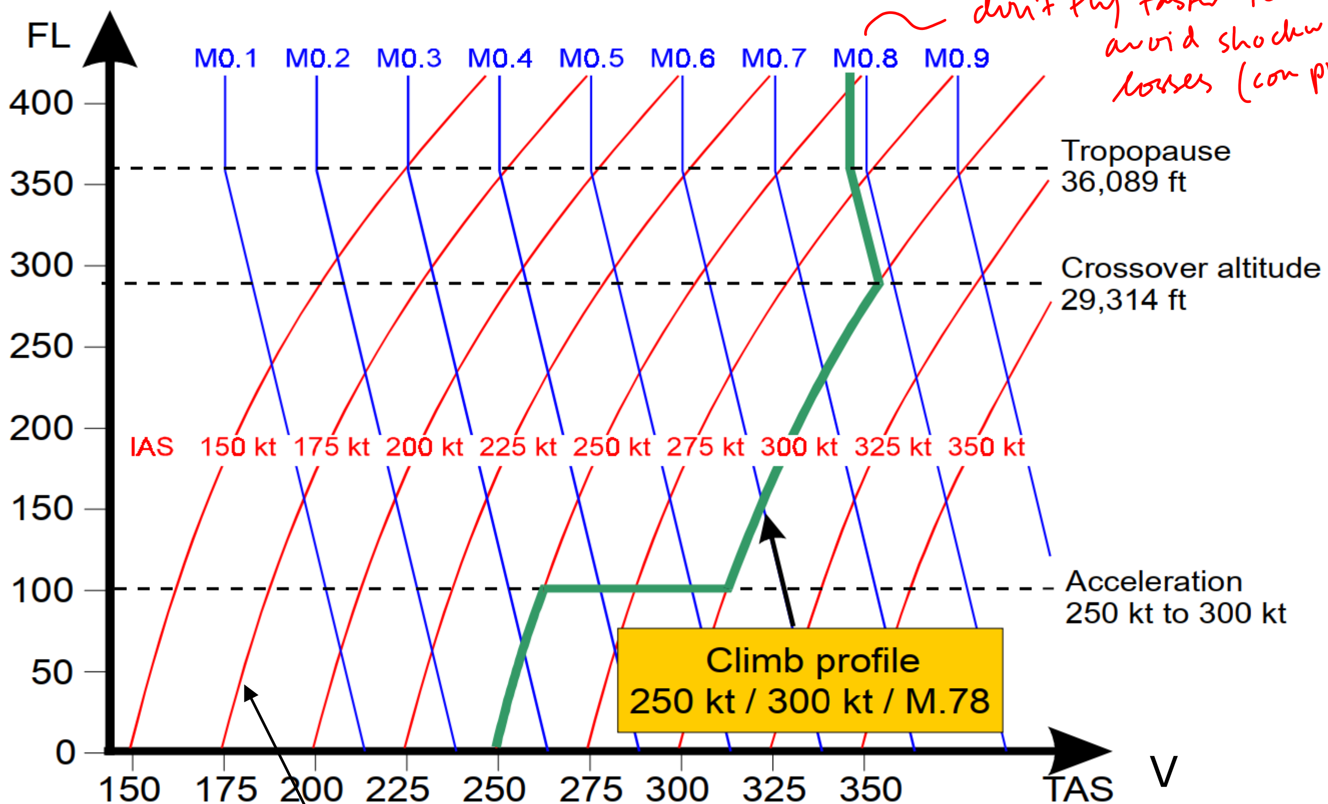
$$V_{C,SL} = 6'600 ft/min = 33.5 m/s$$

Time to Climb - Example MB-339

Result of $$t = \frac{h_{ceil}}{V_{C,SL}} \ln \frac{1}{1 - \frac{h}{h_{ceil}}}$$



CAS-MACH Climb Schedule



~ don't fly faster to avoid shockwave losses (compressible drag))

true airspeed

~ lines of constant dynamic pressure

IAS: Indicated airspeed ~ dynamic pressure q

$$q = \frac{1}{2}\rho V^2$$

$\xrightarrow{\quad}$ q

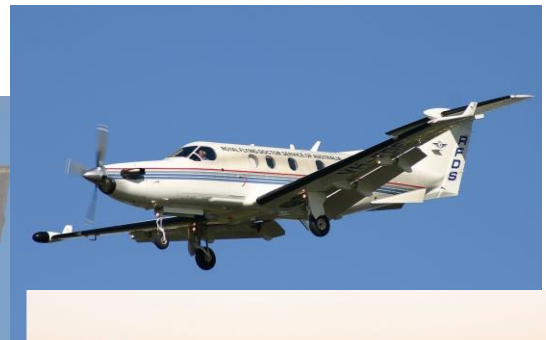
Pitot tube can only measure dynamic pressure.
We don't know ρ exactly (assumed constant) @ sea level ρ
$\Rightarrow$ V is only approx.

Take-off

Certification

Certification Specification (CS)

EASA CS-23
FAA FAR Part 23



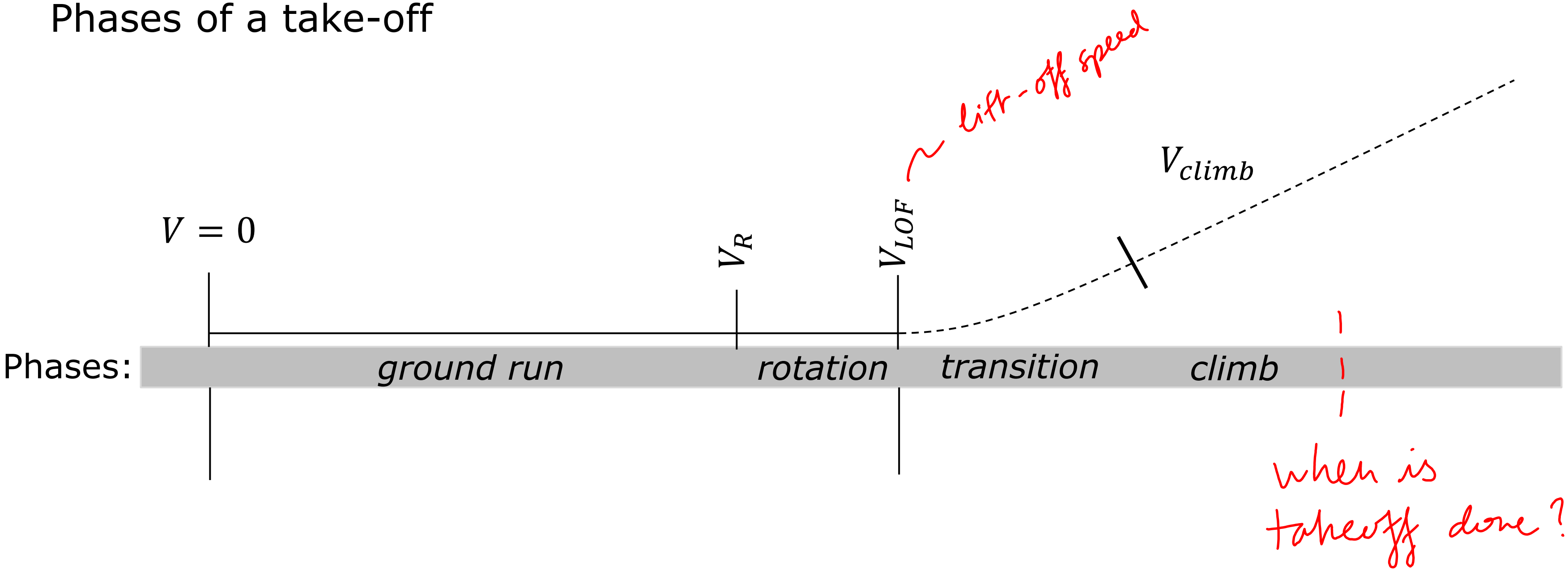
Airplanes
max 9 PAX
MTOW 5670 kg

Twin-engine propeller-driven
airplanes max 19 PAX
MTOW 8618 kg

CS-25 / FAR 25: Large Airplanes

Take-off

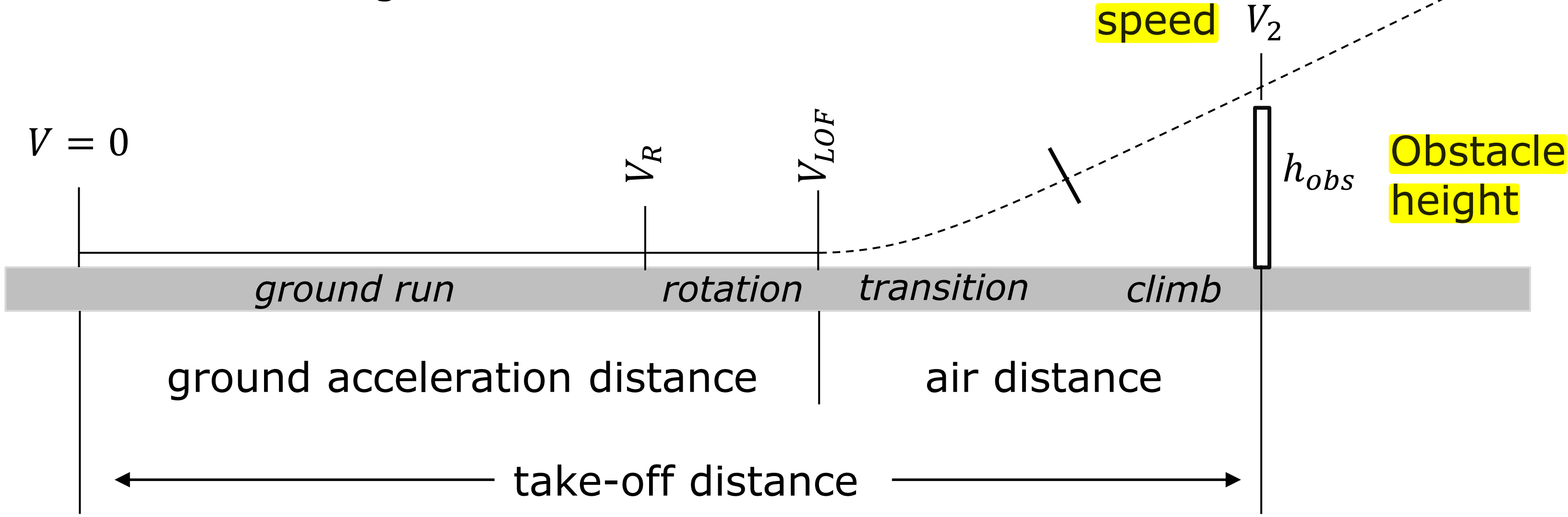
Phases of a take-off



V_R	Rotation speed
V_{LOF}	Liftoff speed

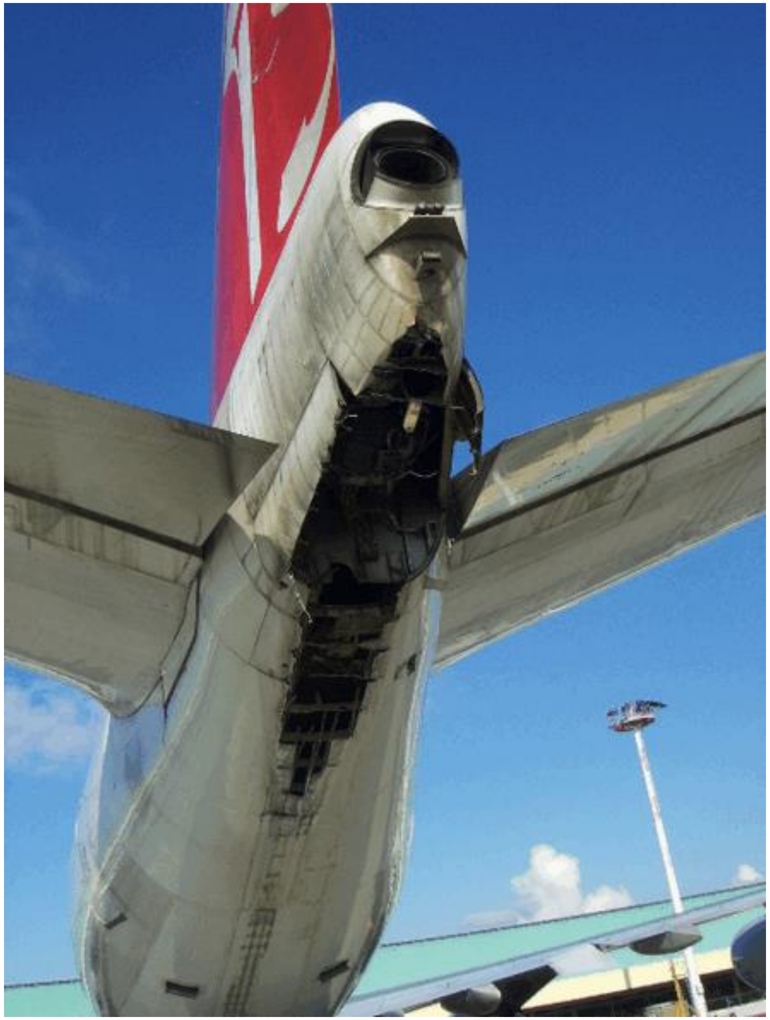
Take-off - Regulations

Take-off distance: ground distance + air distance



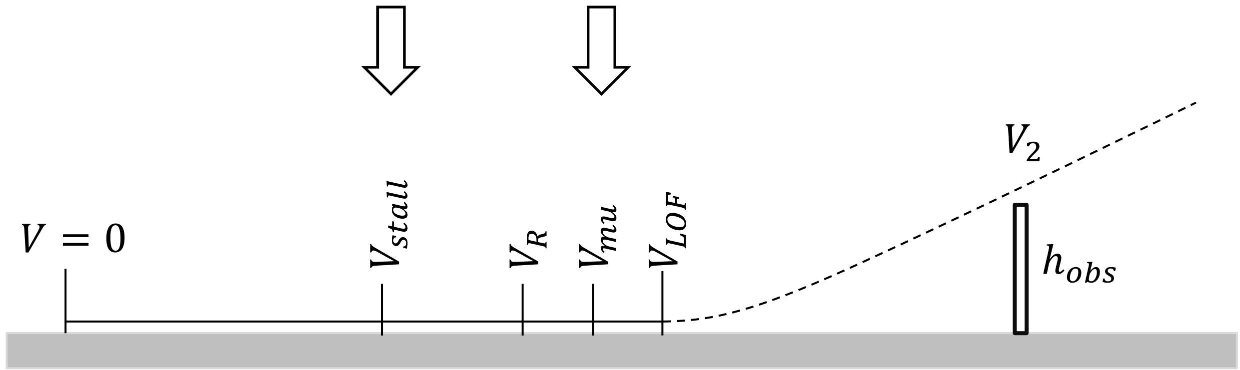
	Obstacle height	V_2	
CS-23	15 m (50 ft)	$1.2\,V_{stall}$	
CS-23 Commuter	10.668 m (35 ft)	$1.2\,V_{stall}$	
CS-25	10.668 m (35 ft)	$1.08\text{-}1.13\,V_{stall}$	(depends on engine type and #)
MIL-STD-3013	15 m (50 ft)	$1.2\,V_{stall}$	

Tail Strike

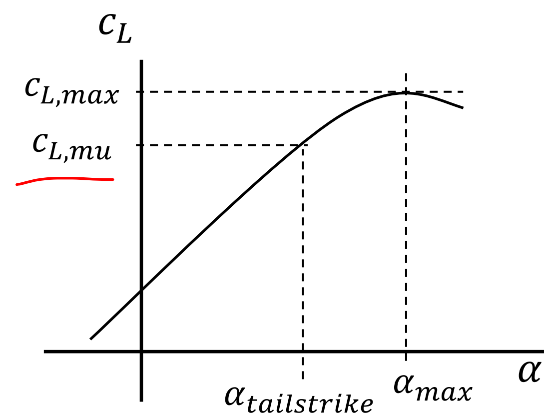


protection against tail strike

Tail Strike and Minimum Unstick Speed



– wheel should be close to CG so we can start rotating plane
– fuselage should be long to accommodate more passengers
=> α can't be very large

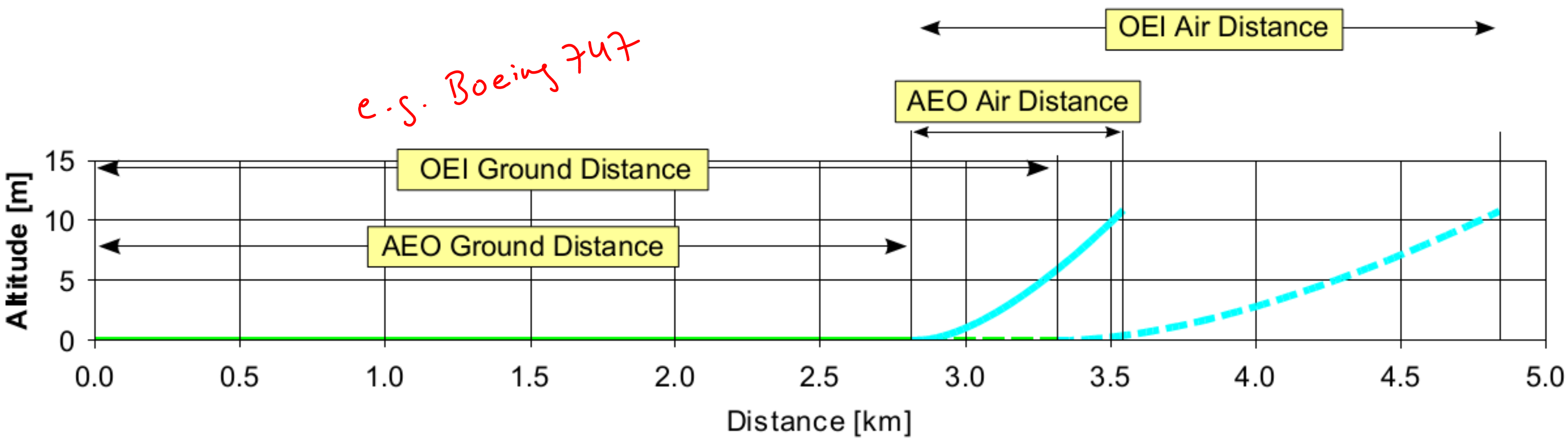


using $L = \frac{1}{2}\rho V^2 S_{ref}\, c_L \overset{!}{=} mg$

$$V_{stall} = \sqrt{\frac{2mg}{\rho S_{ref} c_{L,max}}} \qquad V_{mu} = \sqrt{\frac{2mg}{\rho S_{ref} c_{L,mu}}} \qquad c_{L,mu} < c_{L,max} \implies V_{mu} > V_{stall}$$

Engine Failure on Take-off

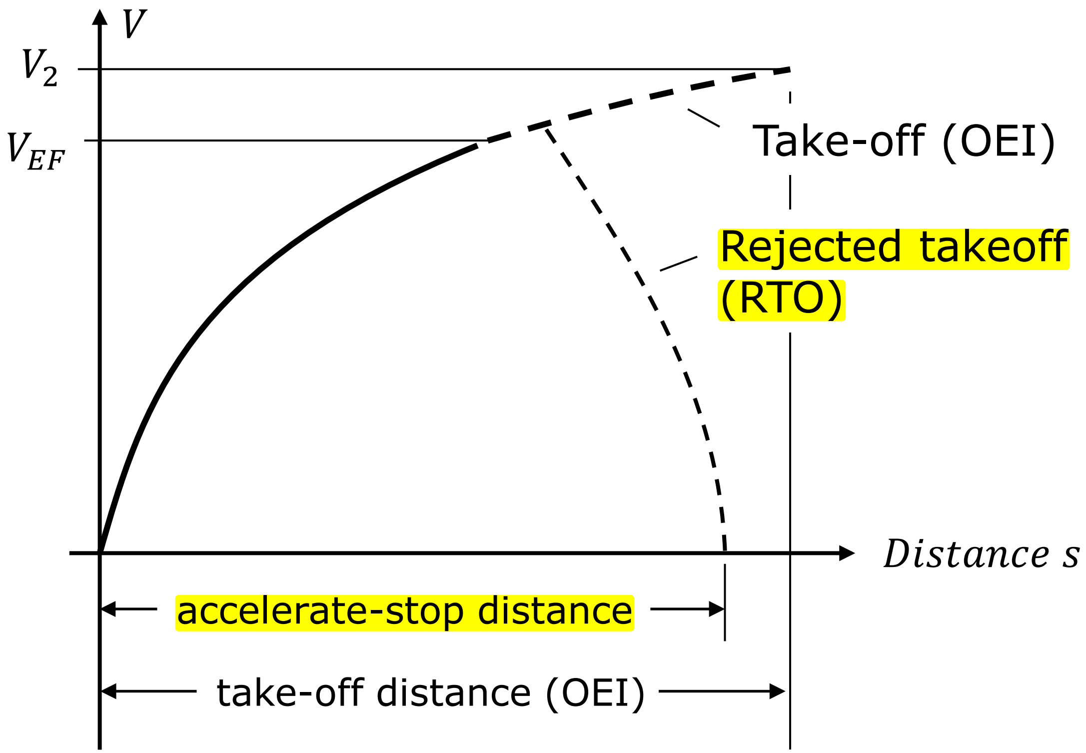
e.g. Boeing 747



AEO: All Engines Operative
OEI: One Engine Inoperative

Engine Failure on Take-off

Engine failure speed



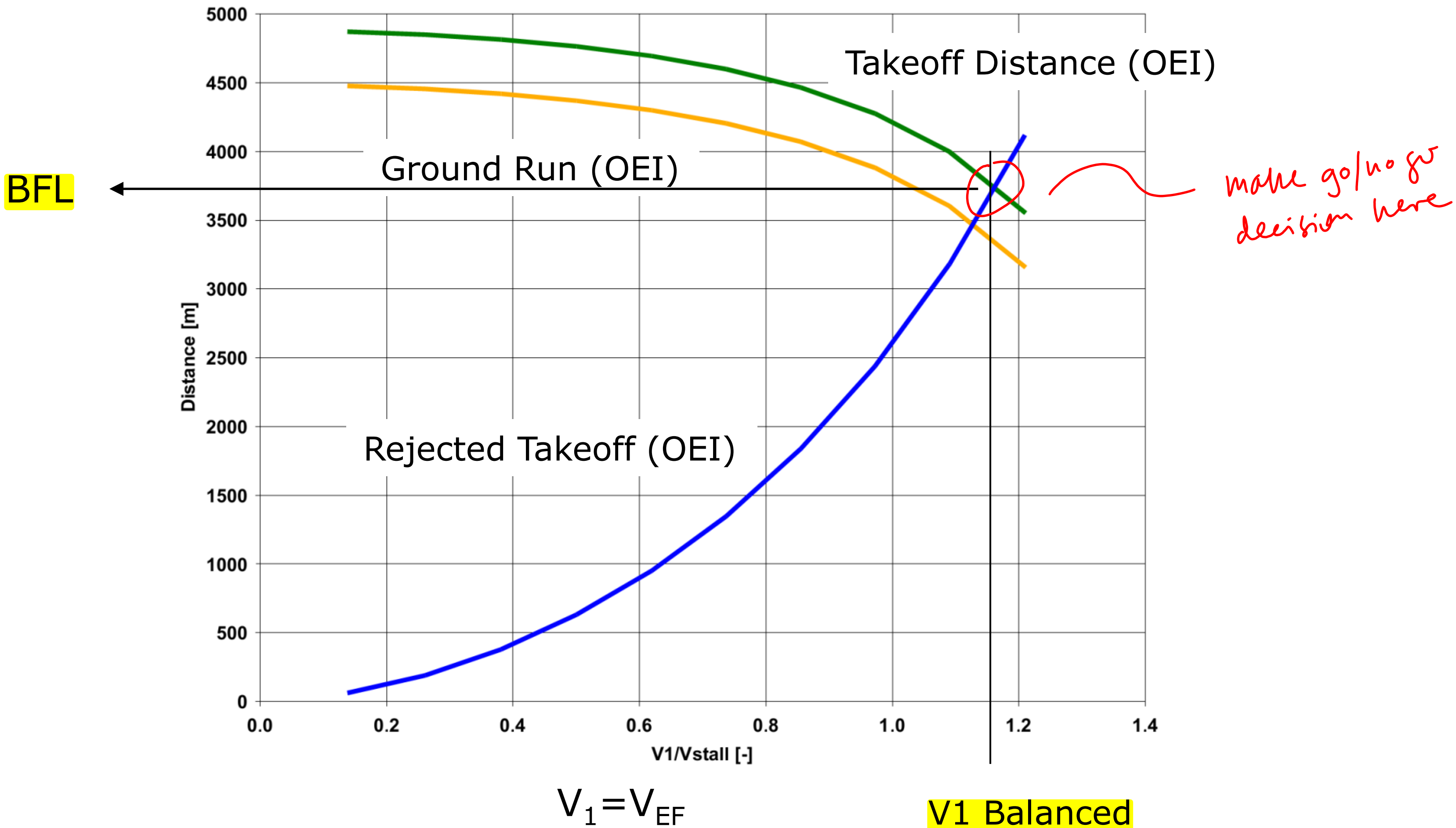
Go / No-Go Decision:
In case of an engine failure, is there enough runway available to perform and RTO?

⇨ **Takeoff decision speed V1**

V<V1: abort take-off
V>V1: continue take-off (OEI)

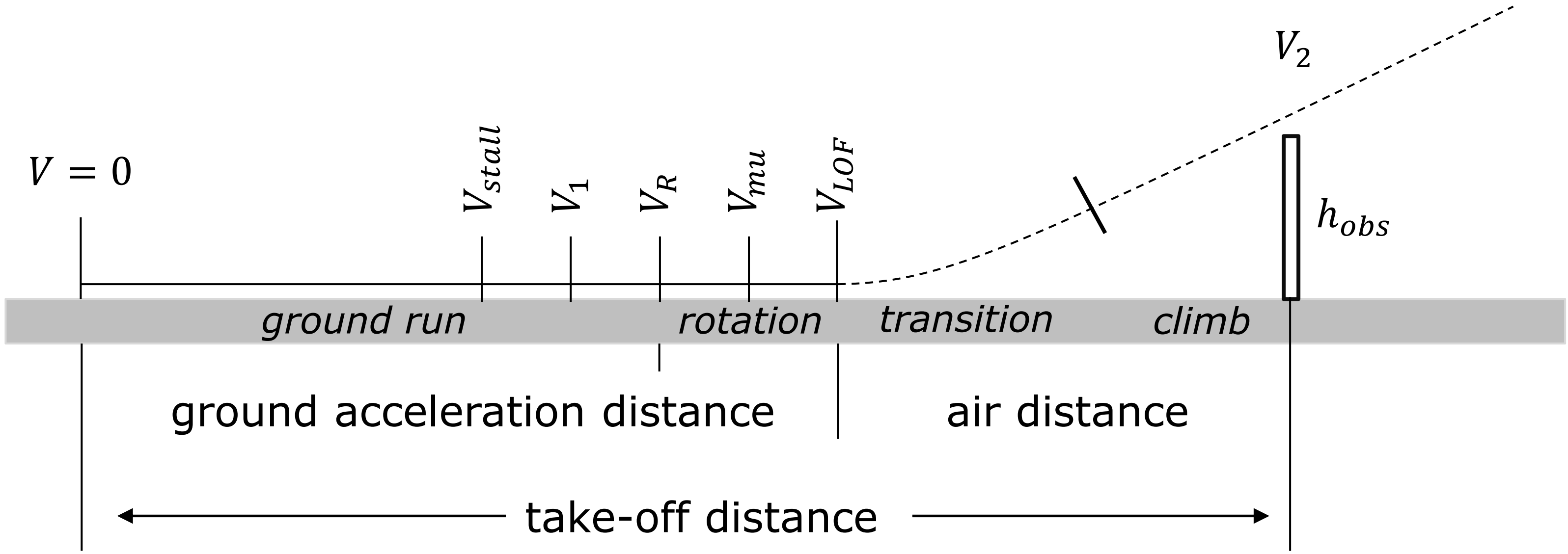
This avoids a runway overrun

Calculation of V1 / Engine Failure



Balanced Field Length (BFL) ⇨ Accelerate-Stop Distance = Takeoff Distance (OEI)

Take-off speeds - Summary



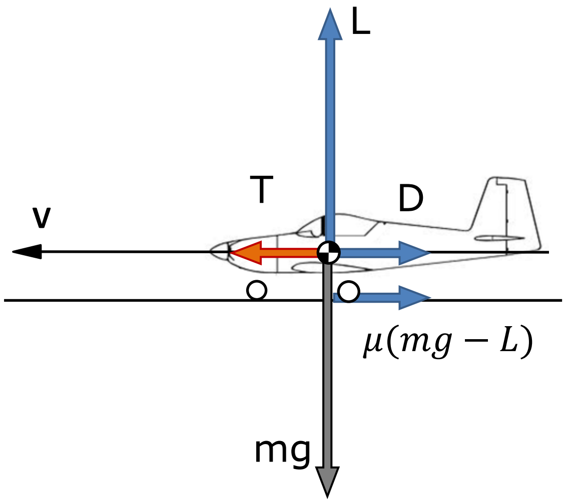
V_{stall}	Stall speed
V_1	Takeoff decision speed
V_R	Rotation speed
V_{mu}	Minimum unstick speed
V_{LOF}	Liftoff speed
V_2	Takeoff safety speed

Take-off Flight Mechanics

Ground-roll

ground reaction force

$$T - D - \mu(mg - L) = F_{res} = m\frac{dV}{dt}$$



Take-off roll: time and distance

$$t_r = \int_0^{t_R} dt = m\int_0^{V_{LOF}} \frac{1}{F_{res}}dV$$

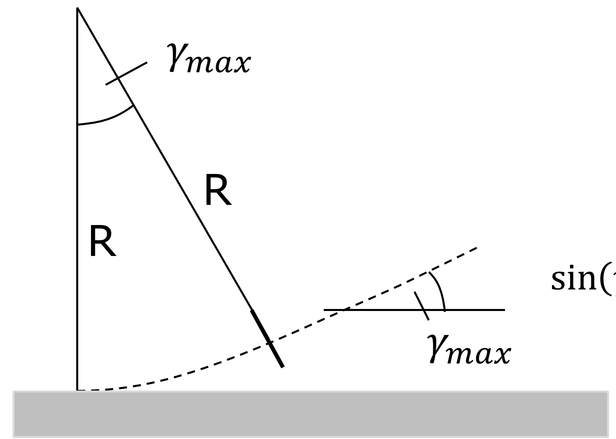
$$s_R = \int_0^{t_R} V dt = m\int_0^{V_{LOF}} \frac{V}{F_{res}}dV$$

Closed from solution requires assumptions/simplifications for T=f(V), D=f(V), and L=f(V)

Transition and Climb

$$T cos(\alpha + \sigma) - D - mg\ sin(\gamma) = m\frac{dV}{dt}$$

$$L + T sin(\alpha + \sigma) - mg\ cos(\gamma) = mV\frac{d\gamma}{dt}$$



$$\sin(\gamma_{max}) = \frac{T - D}{mg}$$

$$a_n = V\frac{d\gamma}{dt} = V\dot{\gamma} \qquad a_n = \frac{V^2}{R}$$

Calculation procedures:

- Hale, F. J. (2003). Aircraft Performance and Design.
- Raymer, D. (2012). Aircraft design: a conceptual approach. AIAA
- Roskam, J. (1985). Airplane design Series. DARcorporation.

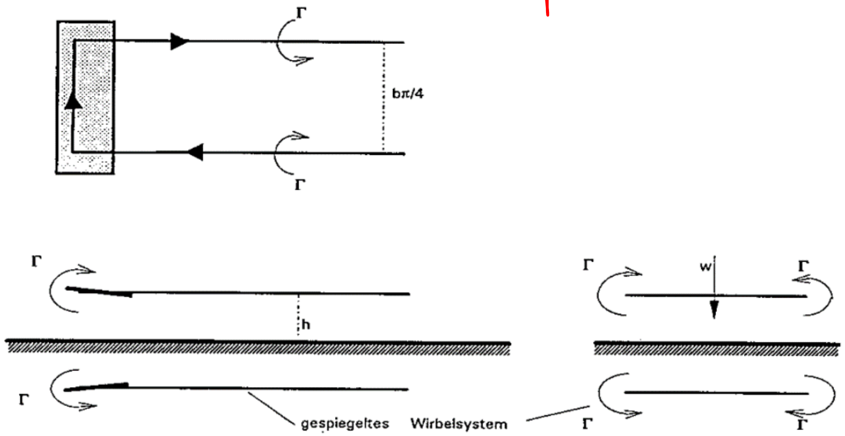
Ground Effect

when you are w/in 1/2 wingspan to the ground

Reduction of induced drag of a wing close to the ground

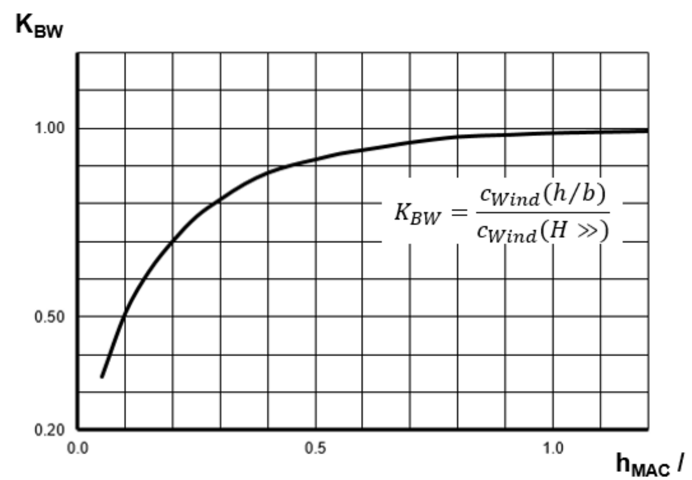
(IGE = in ground-effect)

problematic: e.g. @ hot, high altitude take off location (lower thrust)
once above 1/2 wingspan the ind. drag ↑ & plane can't climb



Vortex-model of a wing close to the ground

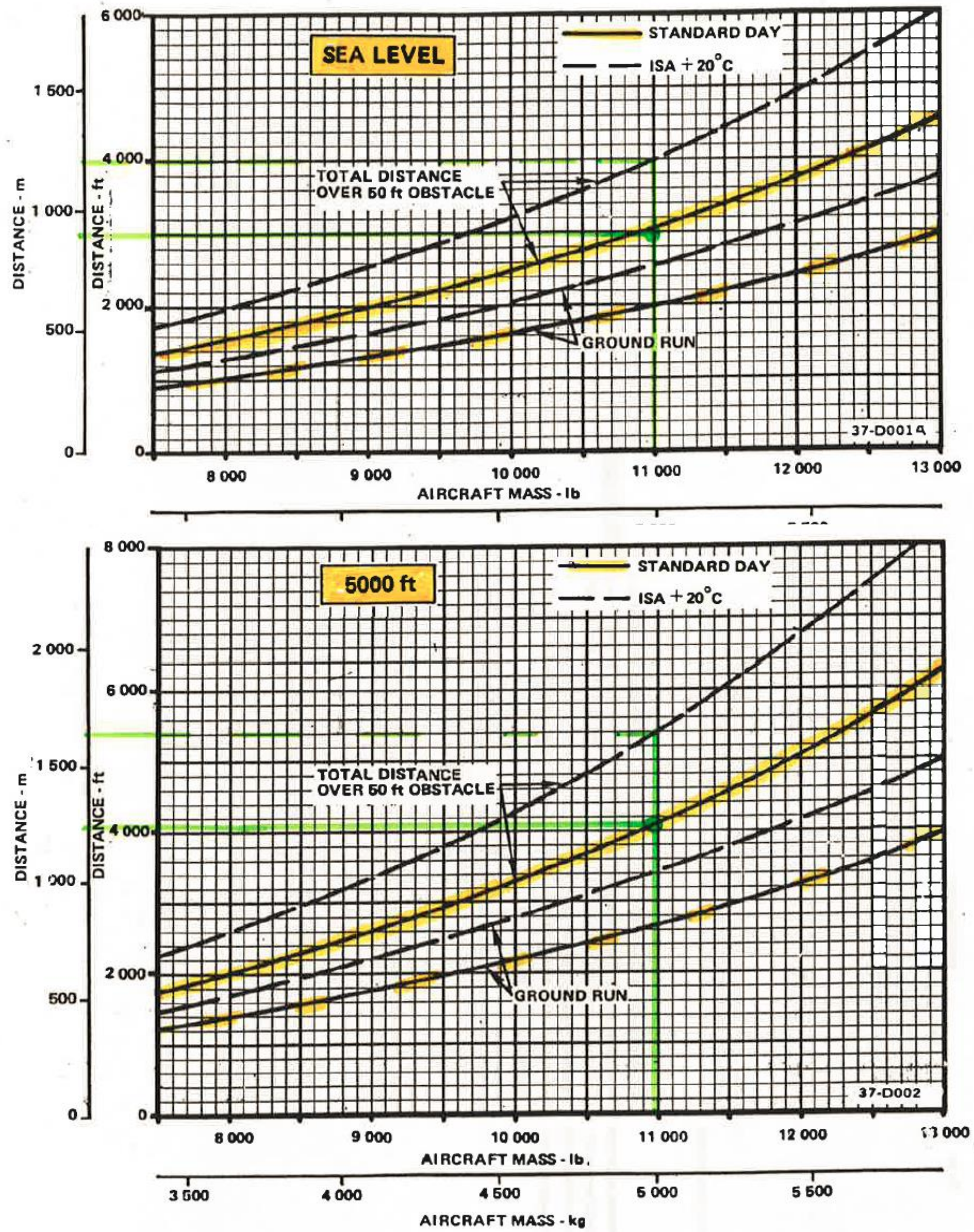
$$\frac{c_{Di,IGE}}{c_{Di}} = \frac{\left(16\frac{h}{b}\right)^2}{1+\left(16\frac{h}{b}\right)^2}$$



Take-off

Example

takeoff
dist. ↑
w/ alt ↑
& temp ↑ ⟶

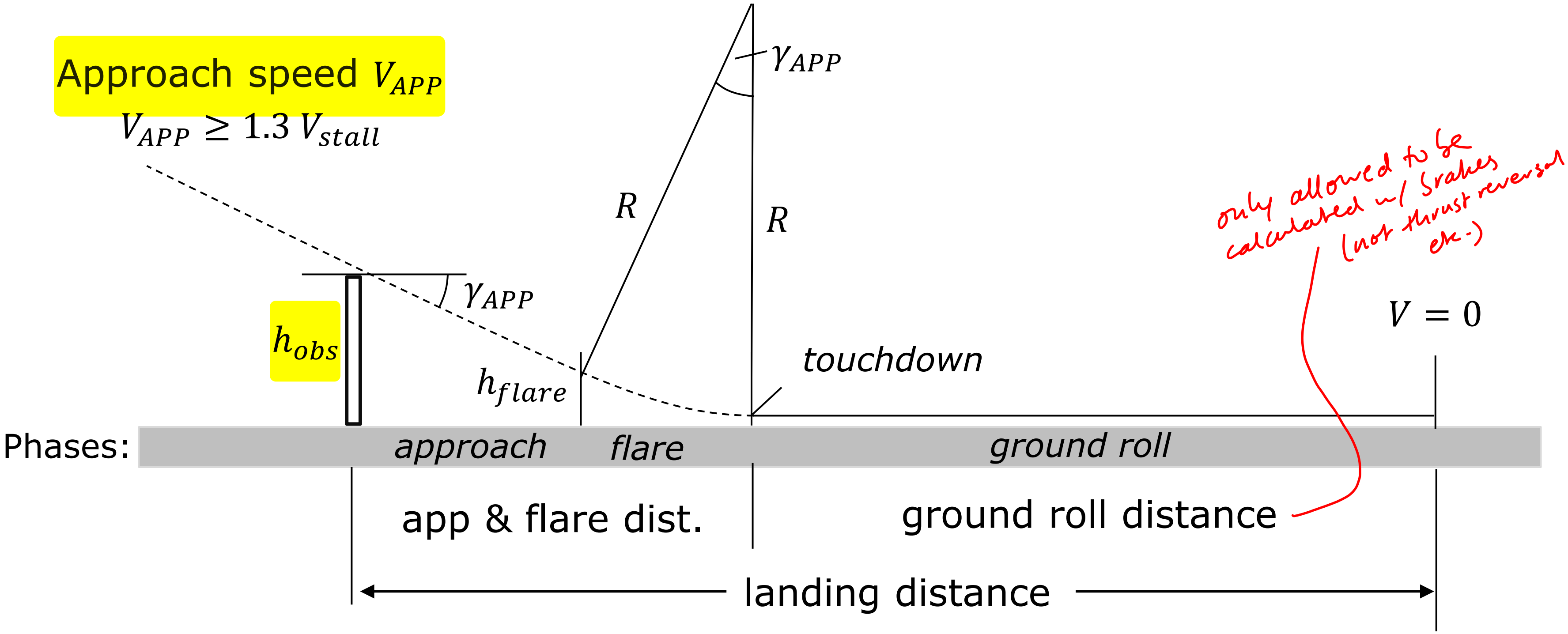


Landing

Landing

Approach speed V_{APP}

$V_{APP} \geq 1.3\, V_{stall}$

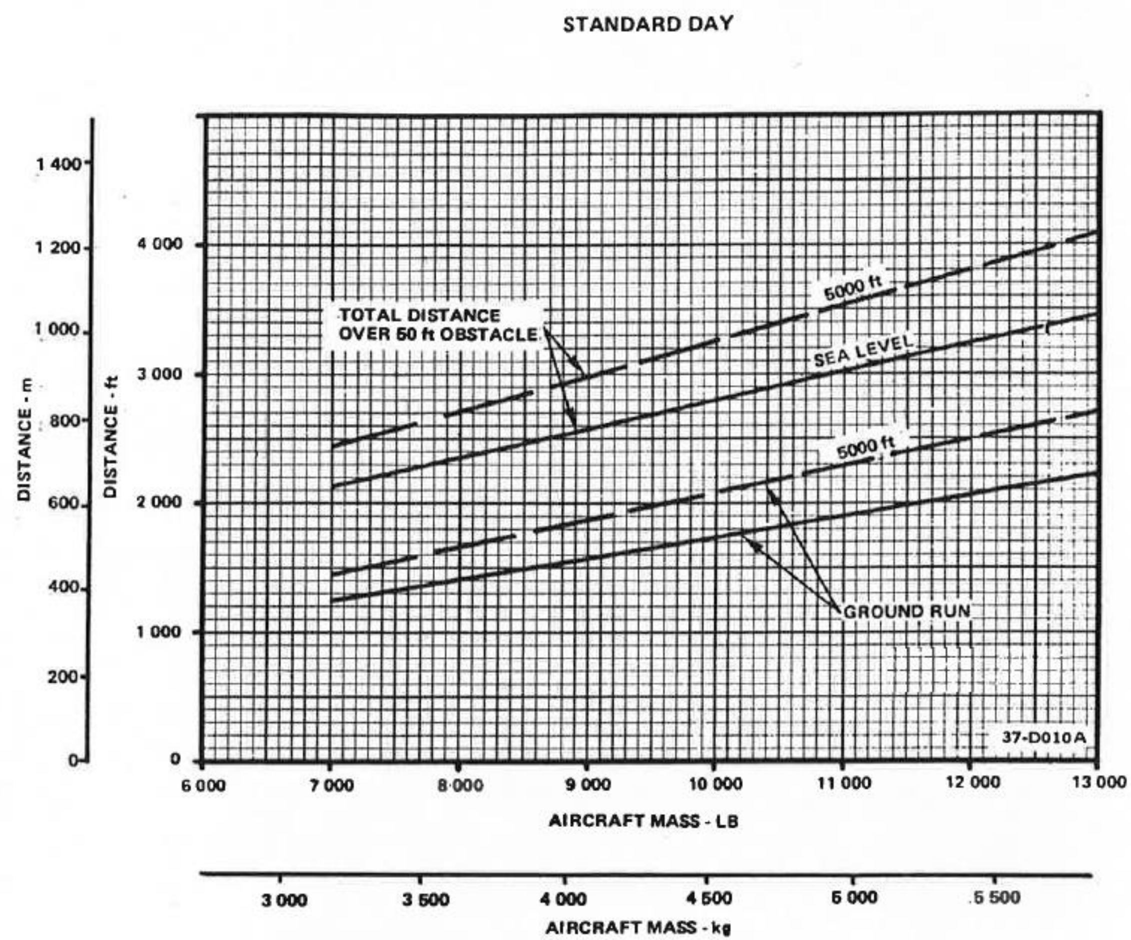


The landing starts from $h = h_{obs}$ with a stationary descent at $V = V_{APP}$ and corresponding glide path angle γ_{APP}

The ideal flare is a circular arc that minimizes touchdown rate and speed

The actual flare distance depends on the pilot technique

Example



linear dep. on mass (mass ↑ => longer distance to come to stop)

STOL

STOL: Short Take-off and Landing
VTOL: Vertical Take-off and Landing
STOVL Short Take-off and Vertical Landing
V/STOL: Vertical and/or Short Take-off and Landing

STOL

V/STOL

JATO (Jet-Assisted Take-off)

C-130 «Credible Sport», 1980

https://www.youtube.com/watch?v=fSFjhWw4DNo