Spring Semester 2023

AIRCRAFT AERODYNAMICS & FLIGHT MECHANICS

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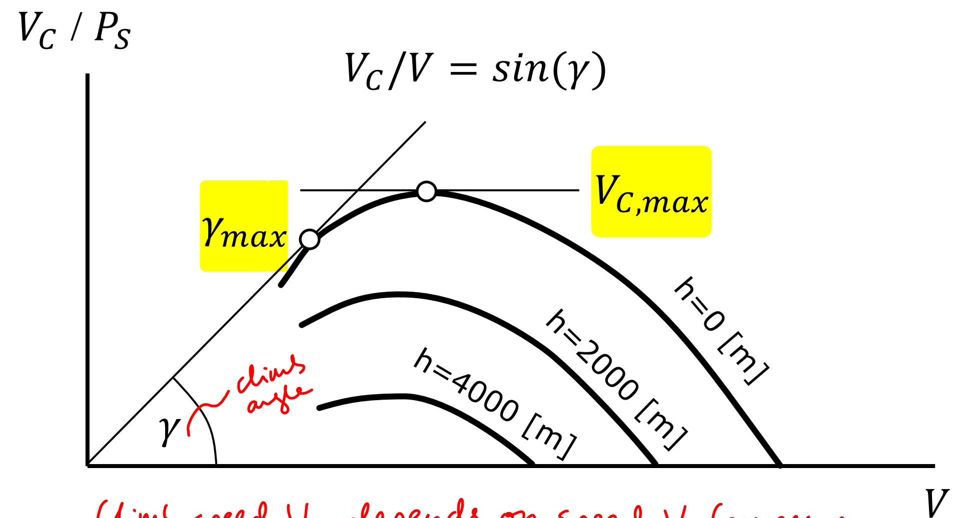
This lecture is adapted with permission from the lecture “Ausgewählte Kapitel der Flugtechnik” by Dr. Jürg Wildi

Recap - Climb



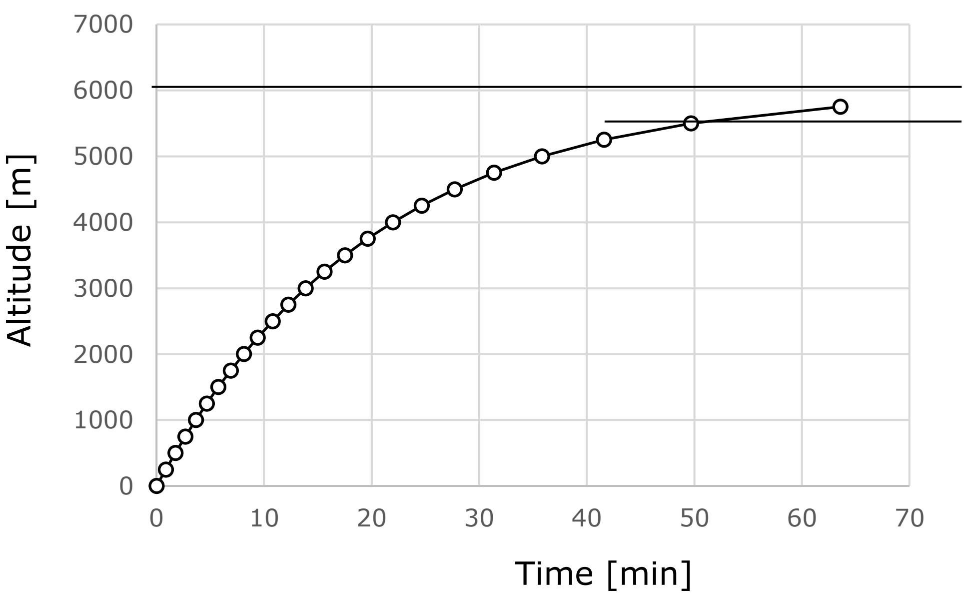
Climb speed V_C depends on speed V (excess power)

$$\sin(\gamma) \overset{\approx}{=} \gamma = \frac{(T-D)}{mg}$$

$$\gamma_{max} \Rightarrow (T-D)_{max}$$

$$\boxed{V_{C,max}} = V\frac{(T-D)}{mg} = \frac{(P_{avail} - P_{req})_{max}}{mg}$$

max engine thrust

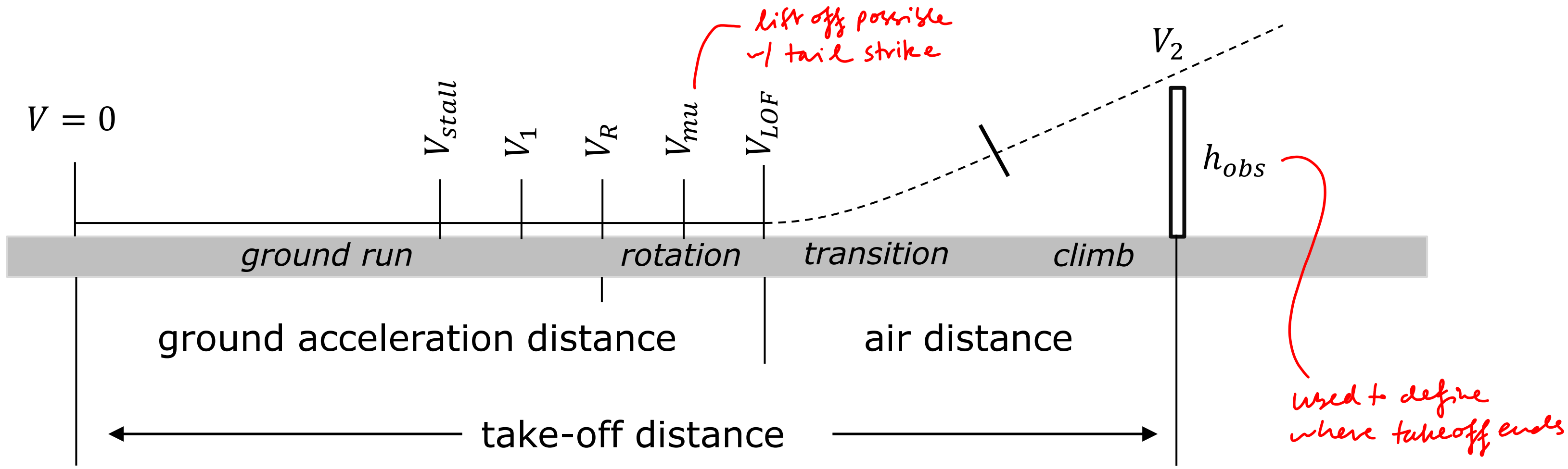


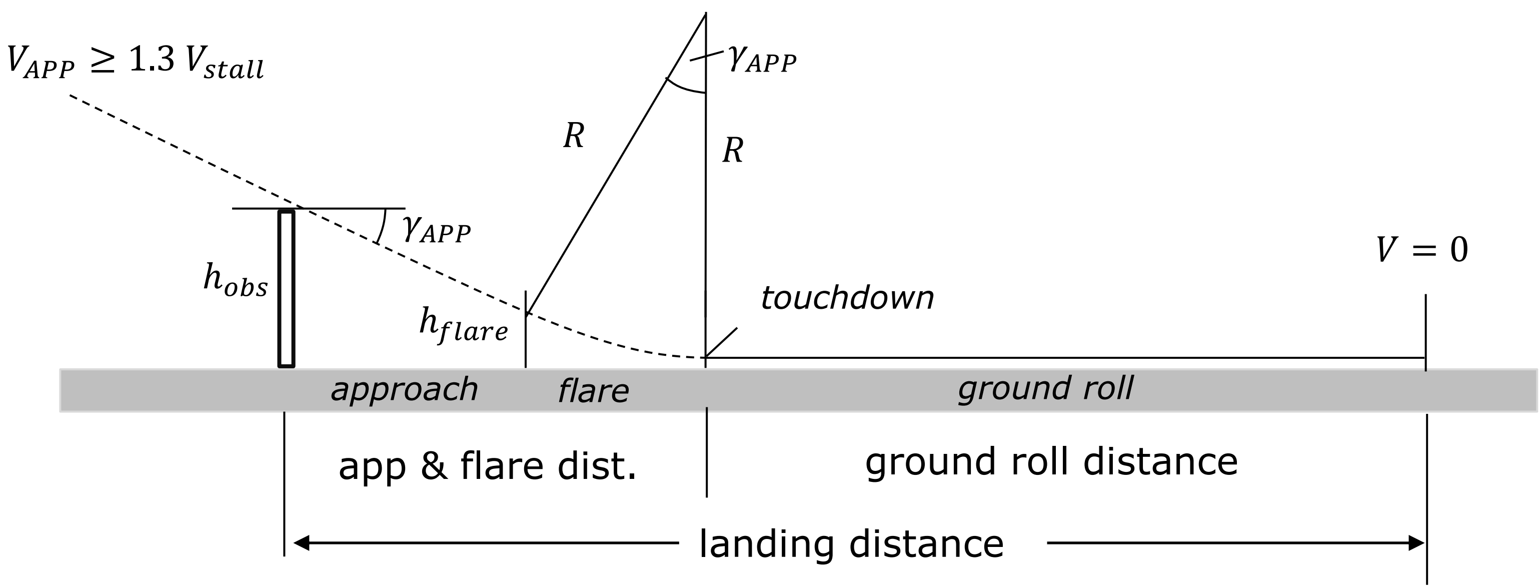
Absolute Ceiling
Service Ceiling

Max. useful flight altitude (ceiling) can be defined using SEP

Absolute Ceiling:	0 ft/min
Service Ceiling:	100 ft/min
Cruise Ceiling:	300 ft/min

Recap - TOL

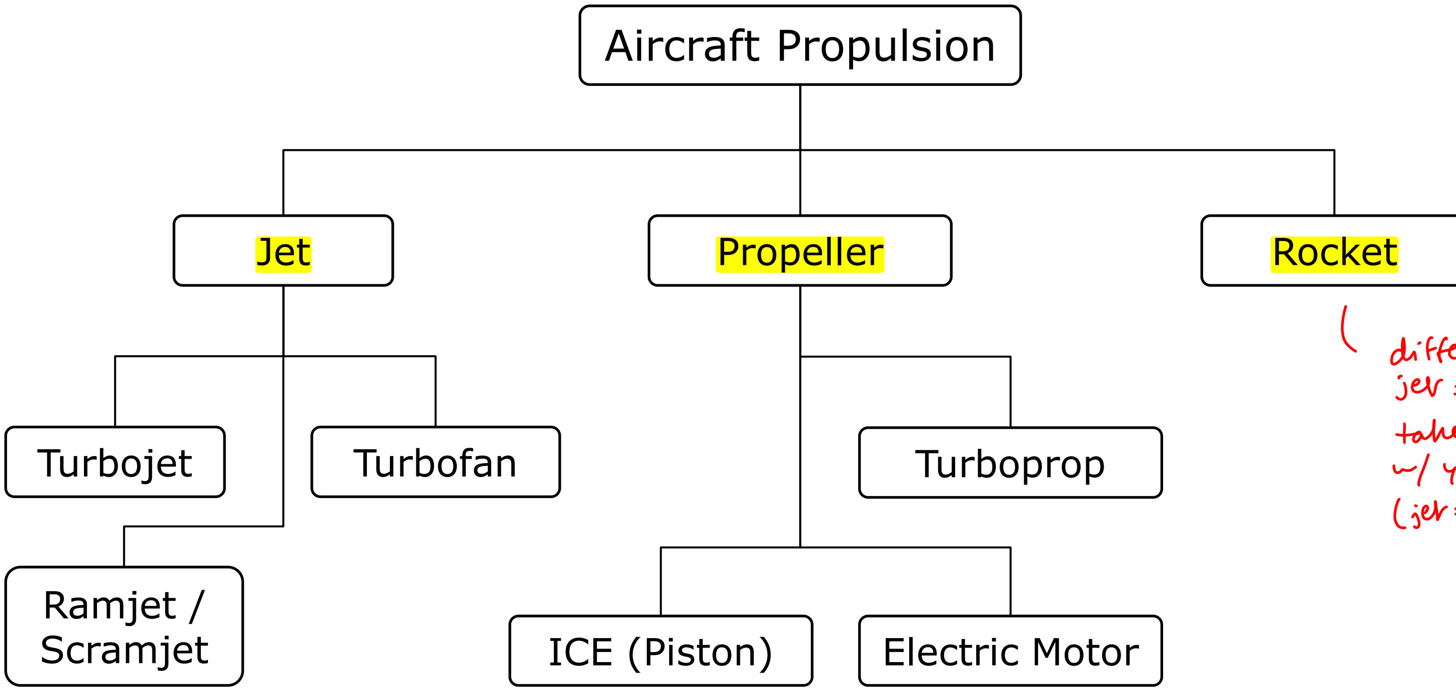




mass is main factor for take-off & landing distance

Aircraft Propulsion

Aircraft Propulsion

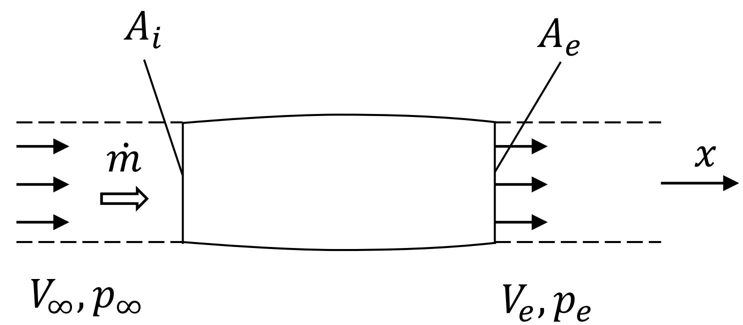


(difference to jet: take oxidizer w/ you (jet: use surrounding air as oxidizer))

Notes:
- many different combinations exist
- This chart does not show the fuel / energy source

Momentum Equation

External force on control volume
in x-direction:



$$F_x = \oiint_{CS} V_x\left(\rho\vec{V}d\vec{A}\right)$$

$$\oiint_{CS} V_x\left(\rho\vec{V}d\vec{A}\right) = -\rho_\infty V_\infty V_\infty A_i + \rho_e V_e V_e A_e$$

with $\quad \dot{m} = \rho_\infty V_\infty A_e = \rho_e V_e A_e = const \quad$ Continuity equation

$$F_x = -\dot{m}V_\infty + \dot{m}V_e = \dot{m}(V_e - V_\infty)$$

External forces acting on the control volume

$$F_x = T - (p_e - p_\infty)A_e$$

loss from not converting all pressure from combustion into Ek (usually mitigate by good nozzle design)

$$T = \dot{m}(V_e - V_\infty) + (p_e - p_\infty)A_e$$

If the pressure difference is small

$$\Longrightarrow \quad T = \dot{m}(V_e - V_\infty)$$

Propulsion Efficiency

The total efficiency is the thrust power divided by the rate of energy provided by the propellant

$$\eta_{tot} = \frac{TV}{\dot{m}_f HV}$$

~ thrust power

heating value

This can be split into a thermal efficiency and a propulsive efficiency

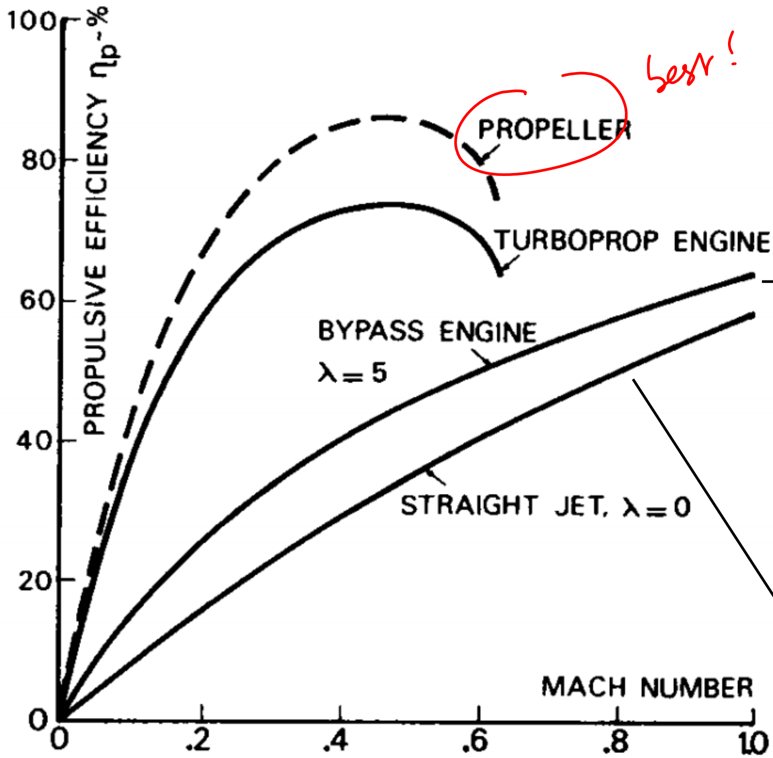
$$\eta_{tot} = \eta_t \eta_p$$

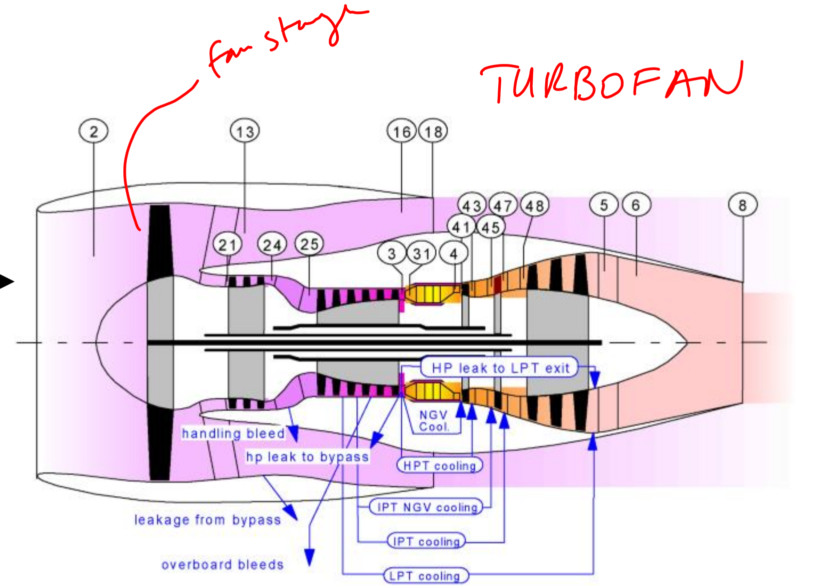
$$\eta_t = \frac{P_m}{\dot{m}_f HV} \qquad \eta_p = \frac{TV}{P_m}$$

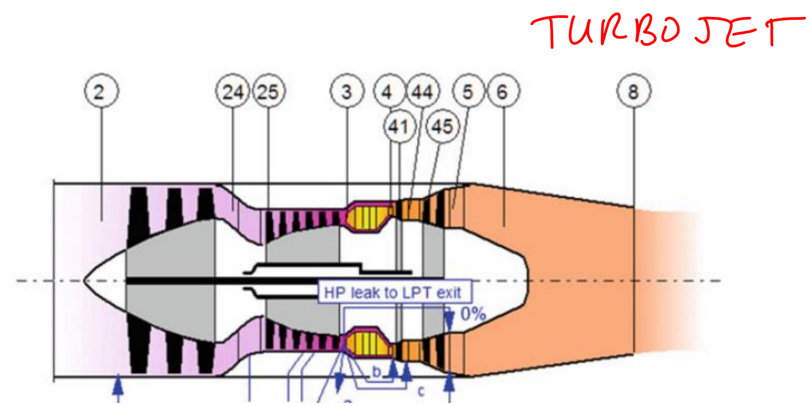
power output (e.g. brake shaft (horse) power)

P_m	Mechanical power output of the engine (e.g. shaft power)
$\dot{m}_f$	fuel flow
HV	heating value of the fuel

Comparison – Speed

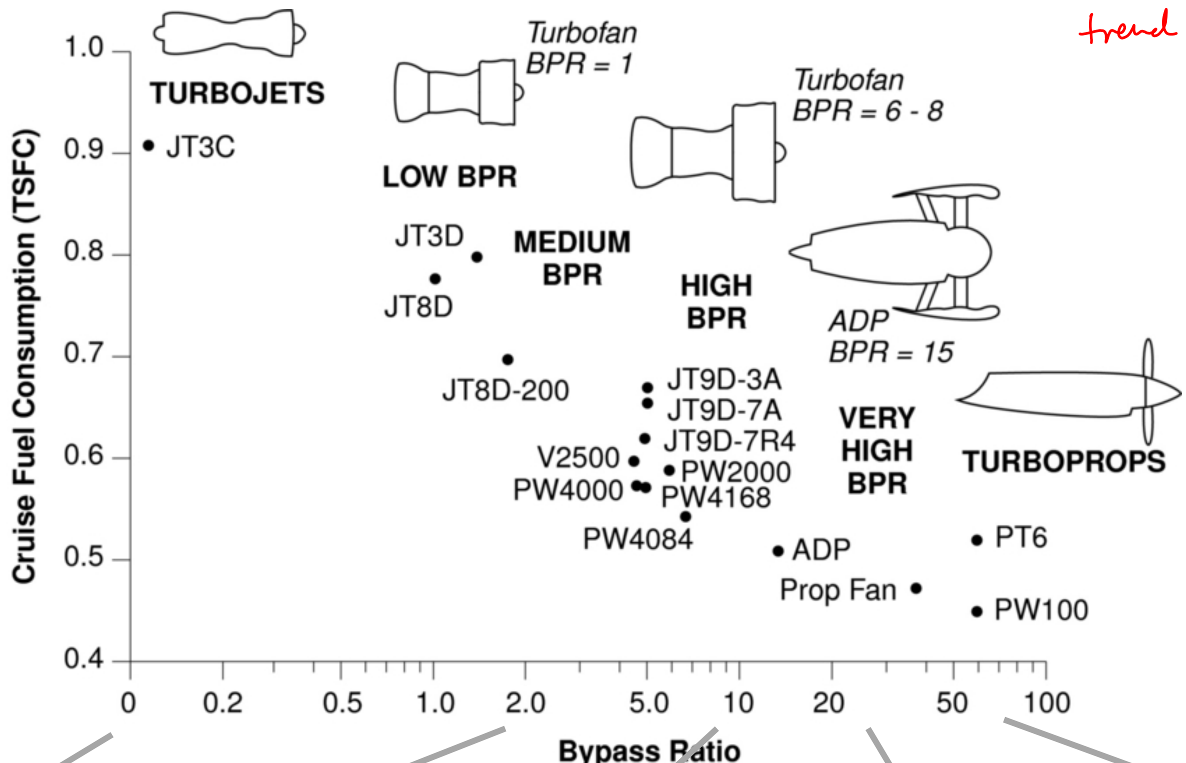


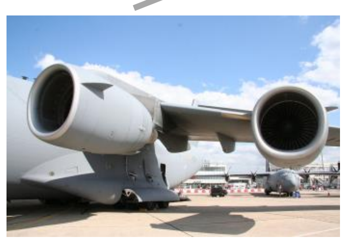


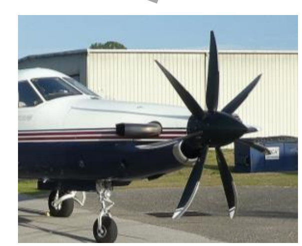


Comparison – SFC and BPR

trend: ↑ bypass ratio
problem: BIG



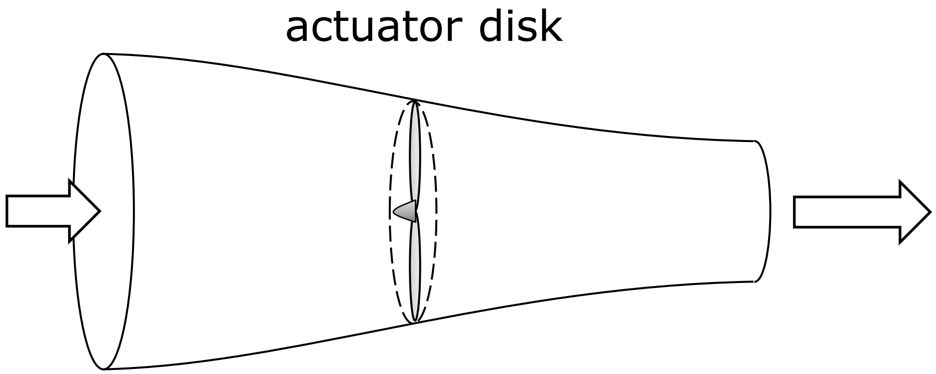




proportion of air that doesn't go into jet

Propeller – Momentum Theory

Also known as actuator disk theory



- Constant velocity over the propeller disk
- Uniform pressure over the propeller disk
- No rotational effects
- Incompressible flow

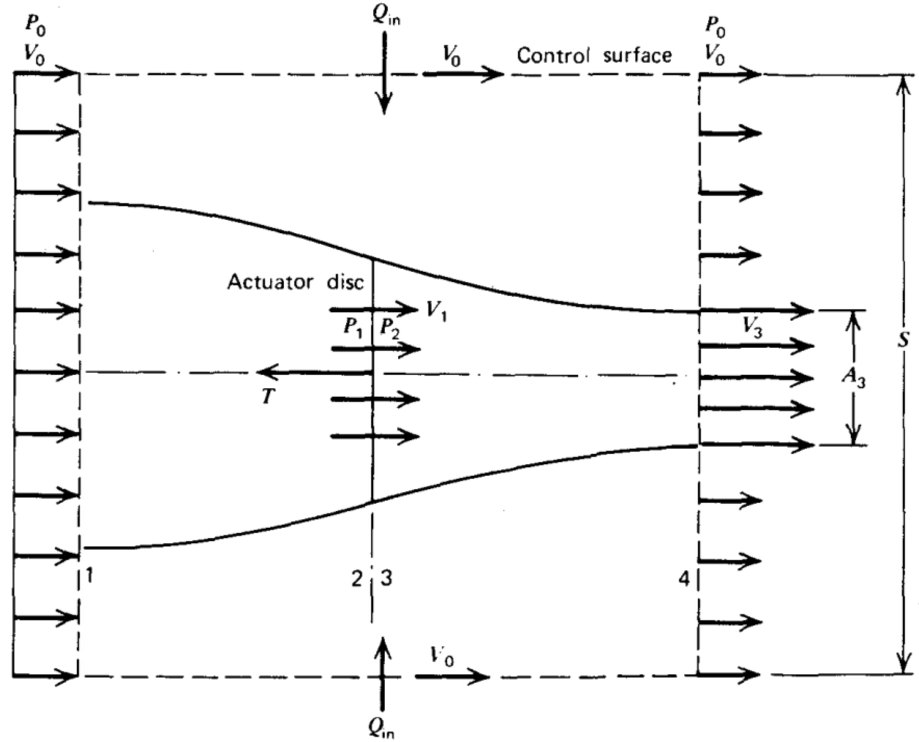
Propeller – Momentum Theory

Continuity

$$SV_0 + \Delta Q = A_3V_3 + (S - A_3)V_0$$

flux in flux out

$$\Delta Q = A_3(V_3 - V_0)$$



Momentum in x direction
(propeller axis)

$$\sum\vec{F} = \int \rho\vec{V}(\vec{V}d\vec{A})$$

$$-\rho SV_0^2 + \rho(S - A_3)V_0^2 + \rho A_3V_3^2 - \Delta QV_0 = T$$

$$T = \rho A_3V_3(V_3 - V_0)$$

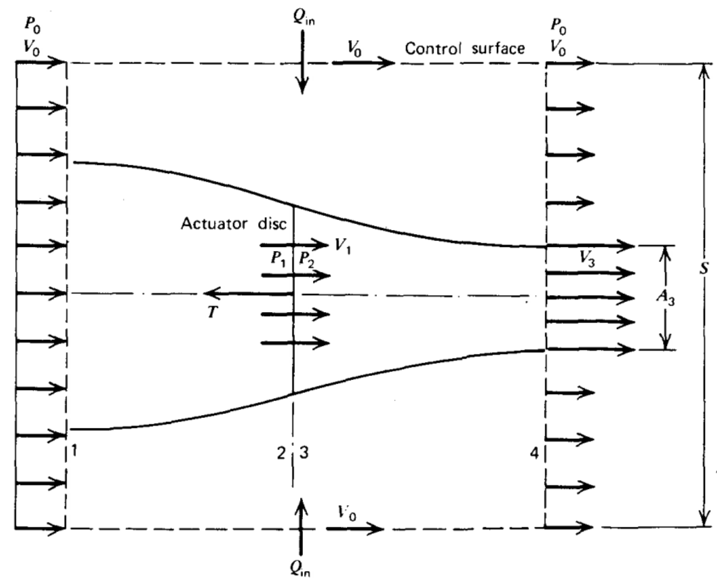
$\rho A_3V_3 = \rho AV_1$ (mass flow through disc)

$$\Rightarrow \quad T = \rho AV_1(V_3 - V_0)$$

Thrust

Thrust is the pressure difference over the propeller disc with area A

$$T = A(p_2 - p_1)$$



Bernoulli along a streamline

ahead of disc

$$p_0 + \frac{\rho}{2}V_0^2 = p_1 + \frac{\rho}{2}V_1^2$$

aft of disc

$$p_0 + \frac{\rho}{2}V_3^2 = p_2 + \frac{\rho}{2}V_1^2$$

Subtracting results in

$$p_2 - p_1 = \frac{\rho}{2}(V_3^2 - V_0^2)$$

$$\Rightarrow \quad T = A(p_2 - p_1) = \frac{\rho}{2}A(V_3^2 - V_0^2)$$

new expression found using Bernoulli approach

with $T = \rho A V_1(V_3 - V_0)$

(from last approach)

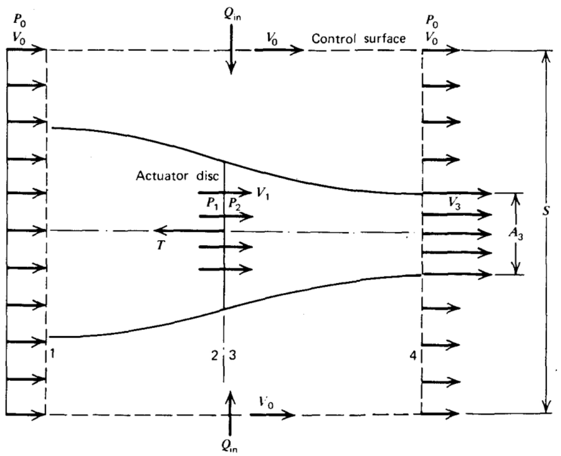
$$T = \frac{\rho}{2}A(V_3^2 - V_0^2) = \rho A V_1(V_3 - V_0)$$

Induced Velocity

$$T = \frac{\rho}{2} A\left(V_3^2 - V_0^2\right) = \rho A V_1 (V_3 - V_0)$$

$$V_1 = \frac{V_3 + V_0}{2}$$

$$V_3 = 2V_1 - V_0$$



V_1 and V_3 can be written in terms of the velocity w induced at the propeller disc:

$$V_1 = V_0 + w$$

$$V_3 = V_0 + 2w$$

$$T = \rho A (V_0 + w) 2w$$

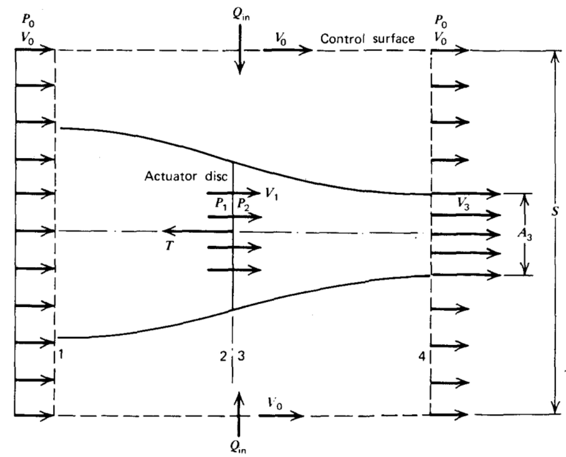
$$\Rightarrow \quad w = -\frac{V_0}{2} + \sqrt{\left(\frac{V_0}{2}\right)^2 + \frac{1}{2\rho}\frac{T}{A}}$$

$\frac{T}{A}$ Disc loading

→ Blade element momentum theory e.g. "JavaProp"

Power and Efficiency

Power (from the kinetic energies of the flow)



$$P = 2\rho A w (V_0 + w)^2 \qquad \left[\frac{kg}{m^3} m^2 \frac{m}{s} \frac{m^2}{s^2} = \frac{Nm}{s} = W\right]$$

with $\quad T = \rho A (V_0 + w) 2\mathrm{w}$

$$P = T(V_0 + w)$$ propulsive power (not only from prop)

$$P_{use} = TV_0 \qquad P_{ind} = Tw$$

useful power

induced power (power needed to turn propeller / induced drag (think turning wing: lift -> induced drag))

$$\eta_i = \frac{P_{use}}{P} = \frac{TV_0}{T(V_0 + w)} = \frac{1}{1 + \frac{w}{V_0}}$$

$$\eta_i = \frac{1}{\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{T}{2\rho V_0^2 A}}} = \frac{2}{1 + \sqrt{1 + \frac{T}{\frac{\rho}{2} V_0^2 A}}} = \frac{2}{1 + \sqrt{1 + T_c}}$$

Thrust coefficient

$$T_c = \frac{T}{\frac{\rho}{2} V_0^2 A}$$

$\Rightarrow$ Low disc loading for high efficiency

=> big prop for same thrust is more efficient