Spring Semester 2023

AIRCRAFT AERODYNAMICS & FLIGHT MECHANICS

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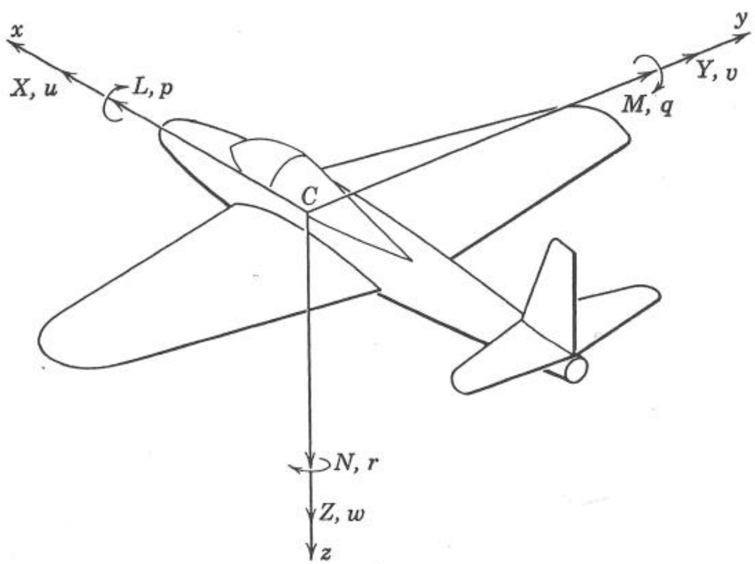
This lecture is adapted with permission from the lecture “Ausgewählte Kapitel der Flugtechnik” by Dr. Jürg Wildi

Repe – Body Axis System

Sign convention: right-handed coordinate system with the x-axis pointing forwards and y-axis pointing through the right wing



(x, y, z): body axis

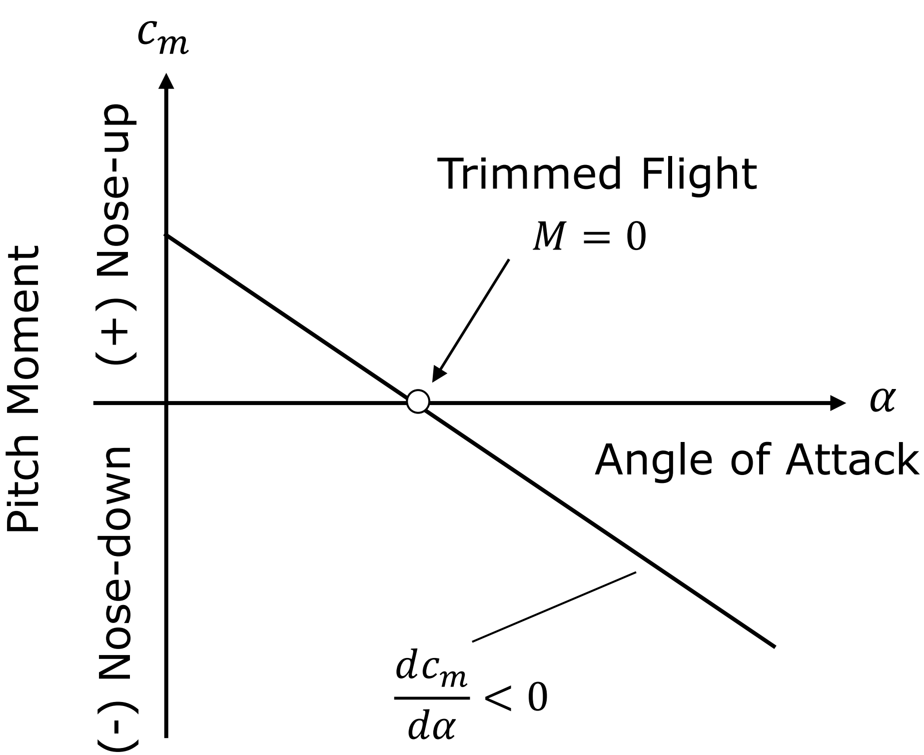
(u, v, w): velocity vector components

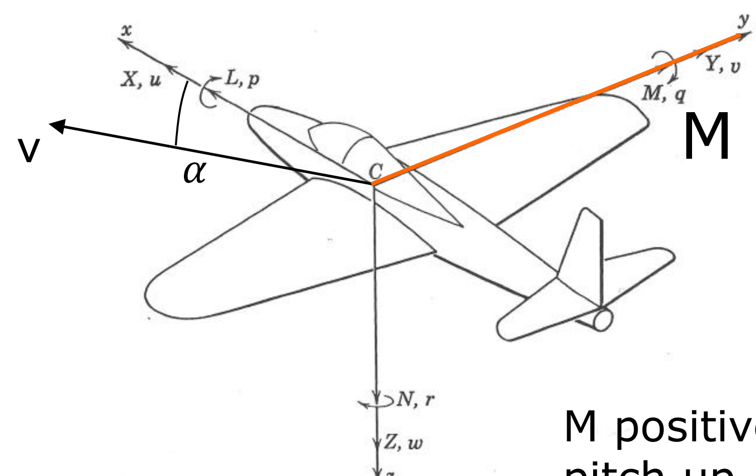
L: rolling moment	[Nm]
M: pitching moment	[Nm]
N: yawing moment	[Nm]
p: roll rate	[rad/s]
q: pitch rate	[rad/s]
r: yaw rate	[rad/s]

Repe – Static Stability

Stationary Flight State ⇒ Disturbance ⇒ Airplane Reaction ⇒ Derivative of the reaction w.r.t. disturbance

Stability derivative $\frac{dc_m}{d\alpha}$ c_{m_α}

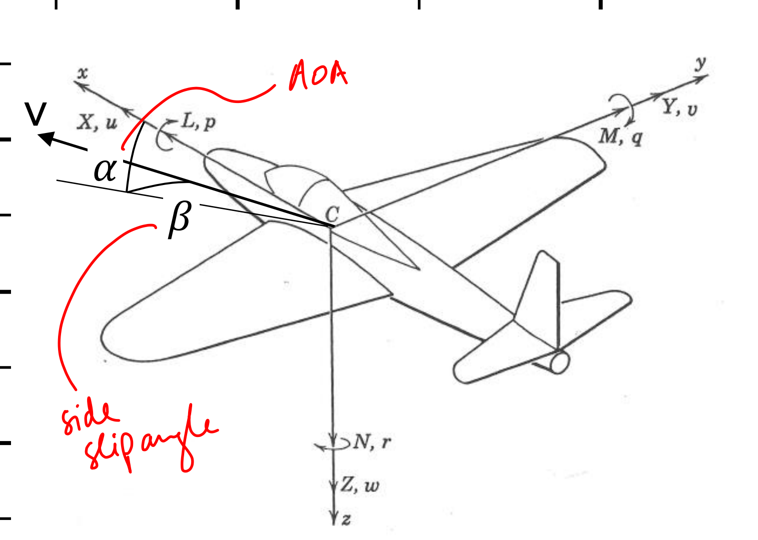




M positive: pitch up

Stability Derivatives

Disturbance		Velocity			Incidence Angles		Angular Velocity		
Forces/Moments		u	v	w	β = v / V	α = w / V	p	q	r
X-Force	$F_{A_x} + F_{T_x}$								
Y-Force	$F_{A_y} + F_{T_y}$								
Z-Force	$F_{A_z} + F_{T_z}$								
Roll-Moment	$L_A + L_T$								
Pitch-Moment	$M_A + M_T$								
Yaw-Moment	$N_A + N_T$								



Stability Derivatives - Overview

most important ones

Disturbance		Velocity			Incidence Angles		Angular Velocity		
Forces/Moments		u	v	w	β = v / V	α = w / V	p	q	r
X-Force	$F_{A_x} + F_{T_x}$	$\frac{\partial}{\partial u}(F_{A_x} + F_{T_x}) < 0$							
		$c_{T_{x_u}} - c_{D_u} < 0$							
Y-Force	$F_{A_y} + F_{T_y}$		$\frac{\partial}{\partial v}\left(F_{A_y} + F_{T_y}\right) < 0$						
			$c_{y_\beta} < 0$						
Z-Force	$F_{A_z} + F_{T_z}$			$\frac{\partial}{\partial w}(F_{A_z} + F_{T_z}) < 0$					
				$c_{L_\alpha} > 0$					
Roll-Moment	$L_A + L_T$				$\frac{\partial}{\partial \beta}(L_A + L_T) < 0$		$\frac{\partial}{\partial p}(L_A + L_T) < 0$		
					$c_{l_\beta} < 0$		$c_{l_p} < 0$		
Pitch-Moment	$M_A + M_T$	$\frac{\partial}{\partial u}(M_A + M_T) > 0$				$\frac{\partial}{\partial \alpha}(M_A + M_T) < 0$		$\frac{\partial}{\partial q}(M_A + M_T) < 0$	
		$c_{m_u} > 0$				$c_{m_\alpha} < 0$		$c_{m_q} < 0$	
Yaw-Moment	$N_A + N_T$				$\frac{\partial}{\partial \beta}(N_A + N_T) > 0$				$\frac{\partial}{\partial r}(N_A + N_T) < 0$
					$c_{n_\beta} > 0$				$c_{n_r} < 0$

by propulsion

by aerodynamics

Longitudinal Static Stability

Disturbance		Velocity			Incidence Angles		Angular Velocity		
Forces/Moments		u	v	w	β = v / V	α = w / V	p	q	r
X-Force	$F_{A_x} + F_{T_x}$	$\frac{\partial}{\partial u}(F_{A_x}+F_{T_x})<0$							
		$c_{T_{x_u}} - c_{D_u} < 0$							
Y-Force	$F_{A_y} + F_{T_y}$		$\frac{\partial}{\partial v}\left(F_{A_y}+F_{T_y}\right)<0$						
			$c_{y_\beta} < 0$						
Z-Force	$F_{A_z} + F_{T_z}$			$\frac{\partial}{\partial w}(F_{A_z}+F_{T_z})<0$					
				$c_{L_\alpha} > 0$					
Roll-Moment	$L_A + L_T$				$\frac{\partial}{\partial \beta}(L_A+L_T)<0$		$\frac{\partial}{\partial p}(L_A+L_T)<0$		
					$c_{l_\beta} < 0$		$c_{l_p} < 0$		
Pitch-Moment	$M_A + M_T$	$\frac{\partial}{\partial u}(M_A+M_T)>0$				$\frac{\partial}{\partial \alpha}(M_A+M_T)<0$		$\frac{\partial}{\partial q}(M_A+M_T)<0$	
		$c_{m_u} > 0$				$c_{m_\alpha} < 0$		$c_{m_q} < 0$	
Yaw-Moment	$N_A + N_T$				$\frac{\partial}{\partial \beta}(N_A+N_T)>0$				$\frac{\partial}{\partial r}(N_A+N_T)<0$
					$c_{n_\beta} > 0$				$c_{n_r} < 0$

see earlier slides

Lateral-Directional Static Stability

Disturbance		Velocity			Incidence Angles		Angular Velocity		
Forces/Moments		u	v	w	β = v / V	α = w / V	p	q	r
X-Force	$F_{A_x} + F_{T_x}$	$\frac{\partial}{\partial u}(F_{A_x} + F_{T_x}) < 0$							
		$c_{T_{x_u}} - c_{D_u} < 0$							
Y-Force	$F_{A_y} + F_{T_y}$		$\frac{\partial}{\partial v}\left(F_{A_y} + F_{T_y}\right) < 0$						
			$c_{y_\beta} < 0$						
Z-Force	$F_{A_z} + F_{T_z}$			$\frac{\partial}{\partial w}(F_{A_z} + F_{T_z}) < 0$					
				$c_{L_\alpha} > 0$					
Roll-Moment	$L_A + L_T$				$\frac{\partial}{\partial \beta}(L_A + L_T) < 0$		$\frac{\partial}{\partial p}(L_A + L_T) < 0$		
					$c_{l_\beta} < 0$		$c_{l_p} < 0$		
Pitch-Moment	$M_A + M_T$	$\frac{\partial}{\partial u}(M_A + M_T) > 0$				$\frac{\partial}{\partial \alpha}(M_A + M_T) < 0$		$\frac{\partial}{\partial q}(M_A + M_T) < 0$	
		$c_{m_u} > 0$				$c_{m_\alpha} < 0$		$c_{m_q} < 0$	
Yaw-Moment	$N_A + N_T$				$\frac{\partial}{\partial \beta}(N_A + N_T) > 0$				$\frac{\partial}{\partial r}(N_A + N_T) < 0$
					$c_{n_\beta} > 0$				$c_{n_r} < 0$

Directional Static Stability

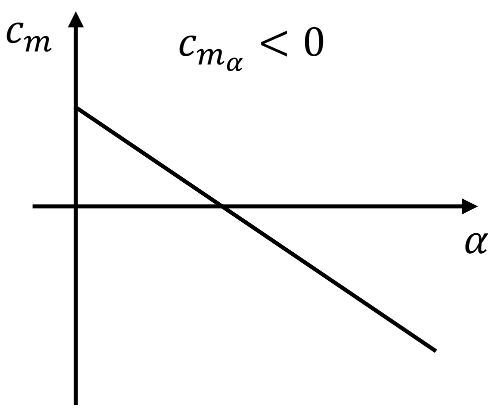
$\Longrightarrow \; c_{n_\beta}$

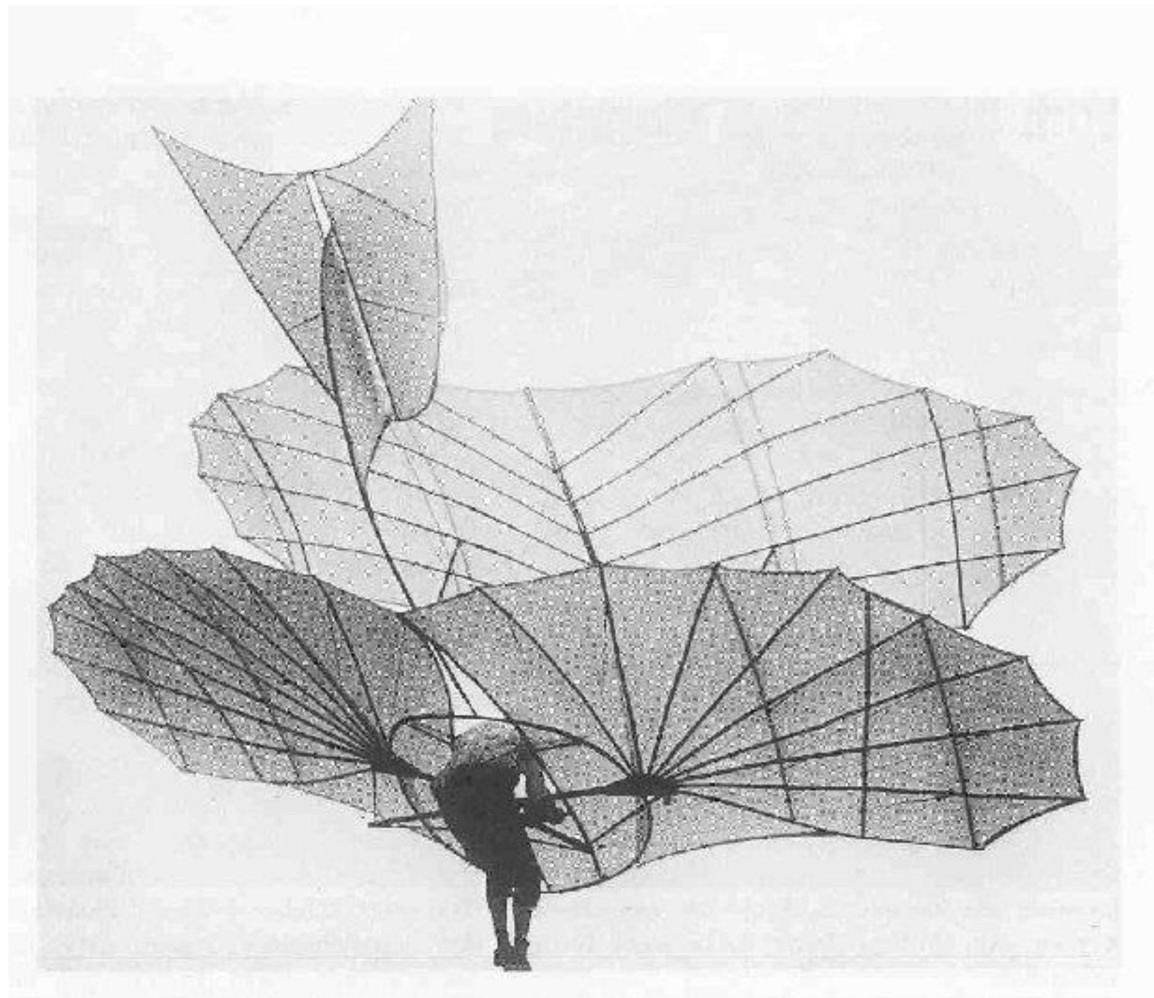
Longitudinal Static Stability

Disturbance		Velocity			Incidence Angles		Angular Velocity		
Forces/Moments		u	v	w	β = v / V	α = w / V	p	q	r
X-Force	$F_{A_x} + F_{T_x}$								
Y-Force	$F_{A_y} + F_{T_y}$								
Z-Force	$F_{A_z} + F_{T_z}$								
Roll-Moment	$L_A + L_T$								
Pitch-Moment	$M_A + M_T$					$\frac{\partial}{\partial \alpha}(M_A + M_T) < 0$ $c_{m_\alpha} < 0$			
Yaw-Moment	$N_A + N_T$								

Positive Longitudinal Static Stability

Otto Lilienthal
1848 - 1896

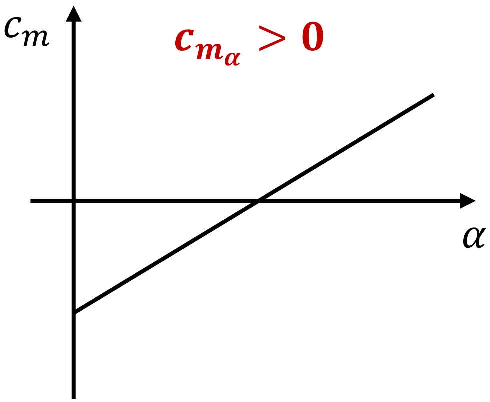




Negative Longitudinal Static Stability

Orville Wright
1871 – 1948

Wilbur Wright
1867 - 1912



Longitudinal Static Stability

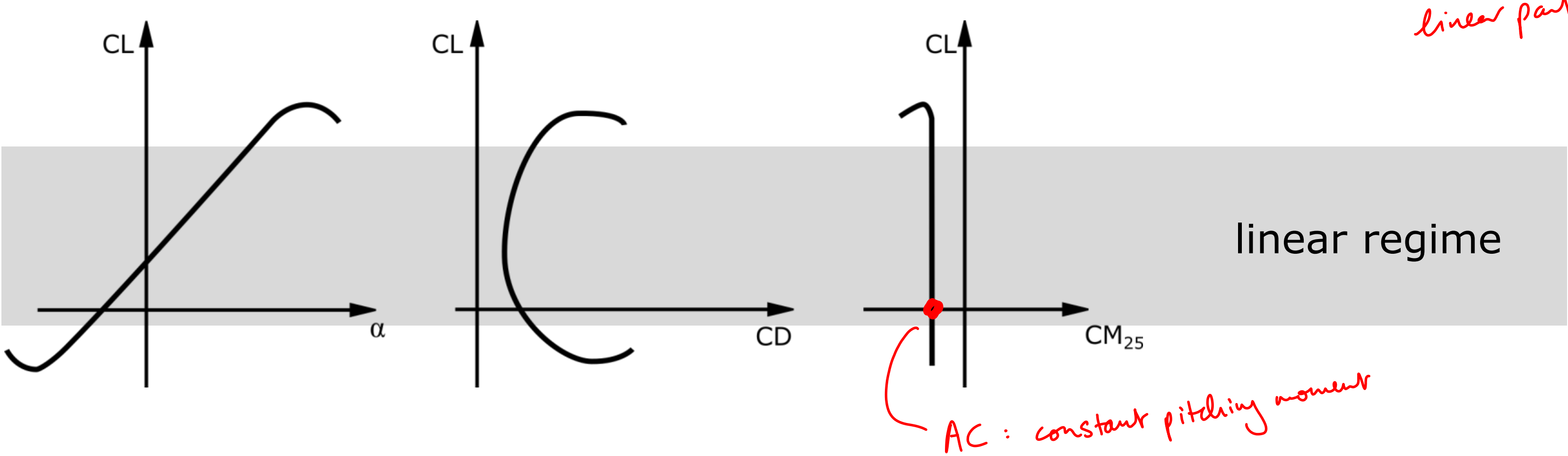
Positive Longitudinal Static Stability is required for certification

Longitudinal Static Stability – Assumptions

Assumption to determine longitudinal static stability

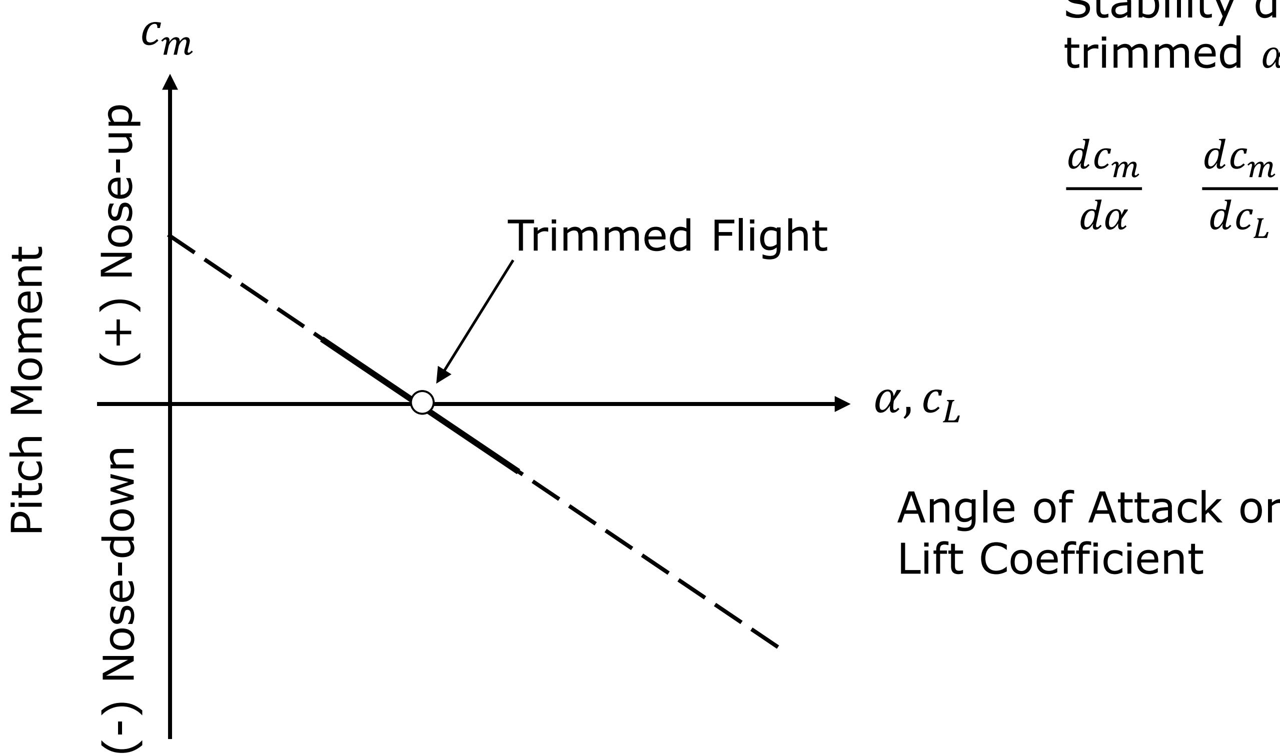
- Initial condition is stationary level flight (trimmed)
- Rigid body
- Focus on aerodynamic forces and moments (neglecting engine)
- Arbitrary body axis reference line (FRL)
- The aerodynamic forces of each component act through one point: the aerodynamic center (AC)
- In the AC act: lift force, drag force, pitching moment
- Lift- and drag force are a function of AoA (or c_L): aerodynamic polar
- Pitching moment is independent of AoA (=definition of AC)

only the linear part



Condition

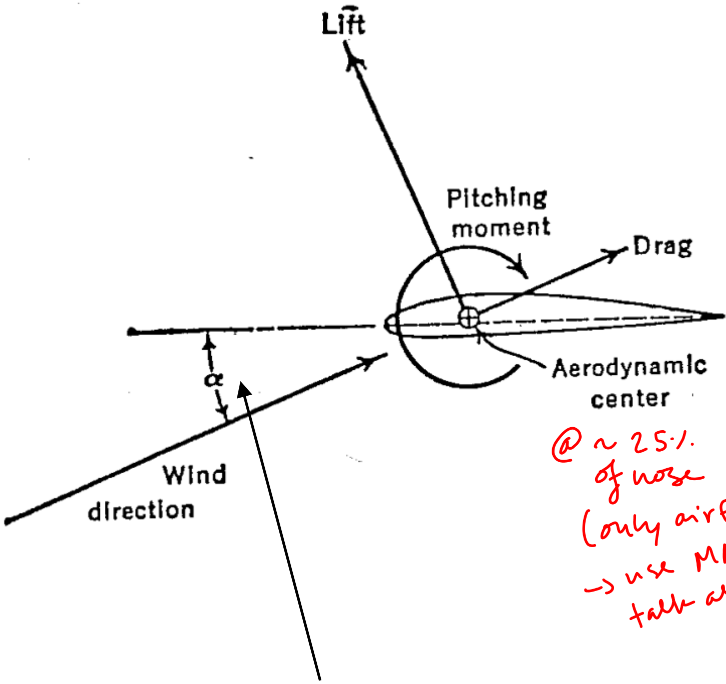
Trimmed Flight:
- Balance of forces: L=mg / T=D
- Balance of moments: M=0 (c_m=0)



Longitudinal static stability
Stability derivative at
trimmed α / c_L

$$\frac{dc_m}{d\alpha} \qquad \frac{dc_m}{dc_L}$$

Representation of a Lifting Surface



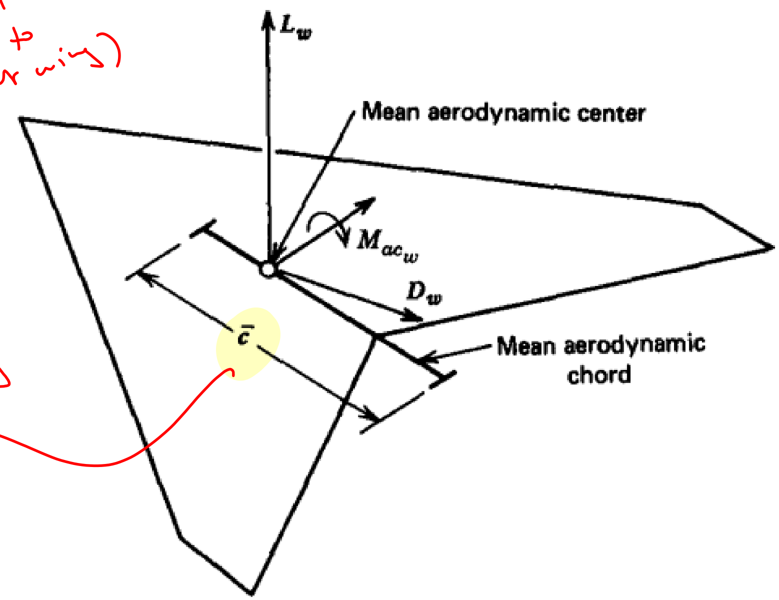
@ ~ 25% of nose
(only airfoil
-> use MAC to
talk about wing)

AC: Aerodynamic Center

MAC: Mean Aerodynamic Chord

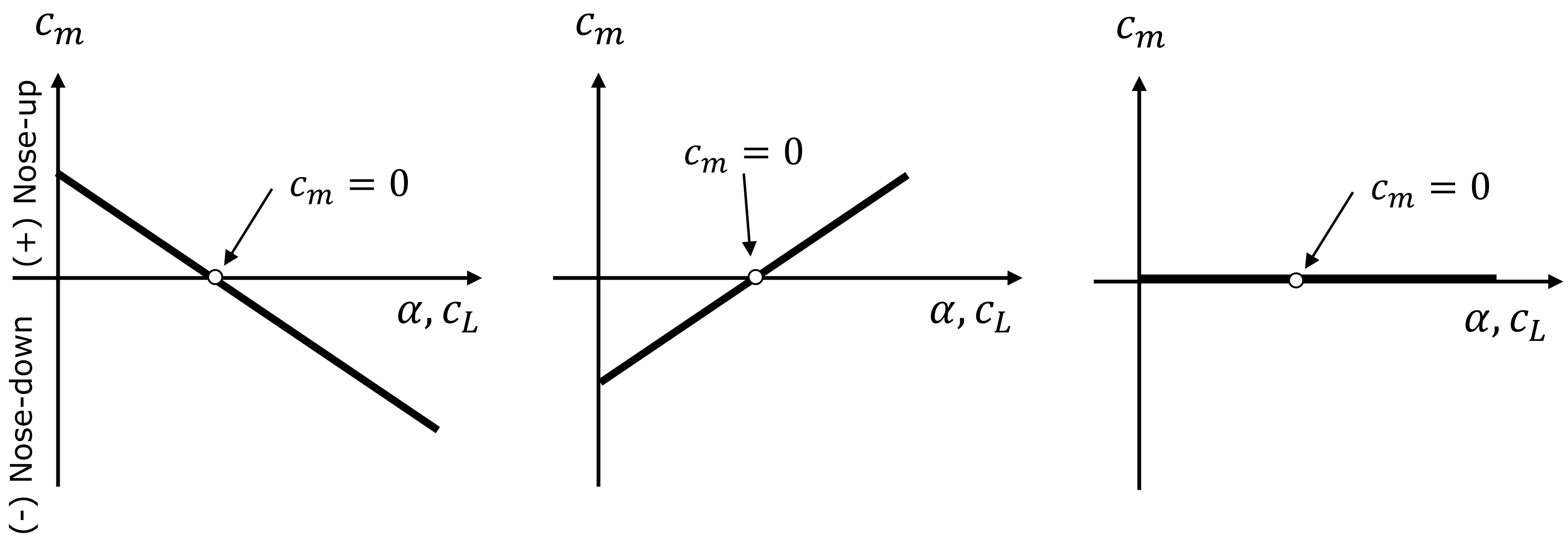
Moment reference point

AoA defined w.r.t the components reference line



treat 3D wing
as 2D
airfoil w/ $\bar{c}$

Presentation of the Results



stable $\frac{dc_m}{d\alpha} < 0$

unstable $\frac{dc_m}{d\alpha} > 0$

neutral $\frac{dc_m}{d\alpha} = 0$

Trim point: $c_m = 0$

["fixed stick stability"]
(no control input given)

Literature

Perkins, C. D., Hage, R. E., (1949).
Airplane performance, stability and control

Etkin, B., & Reid, L. D. (1959). *Dynamics of Flight Stability and Control*

McCormick, B. W. (1995). *Aerodynamics Aeronautics and Flight Mechanics*

Raymer, D. (2012). *Aircraft design: a conceptual approach*

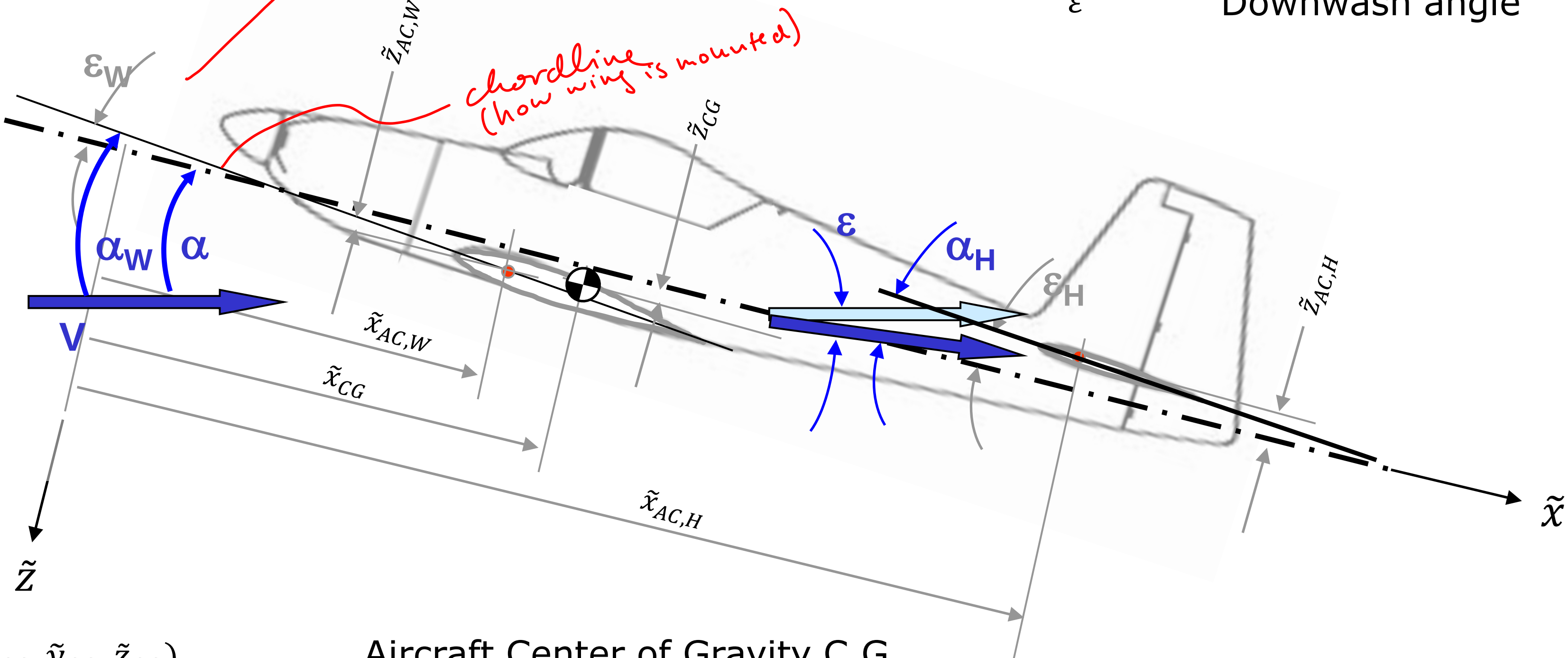
Note: a selection, in order of complexity, most to least

Geometry

place origin in front of plane => x values all positive

HTP: Horizontal Tail Plane
AoA: Angle of Attack

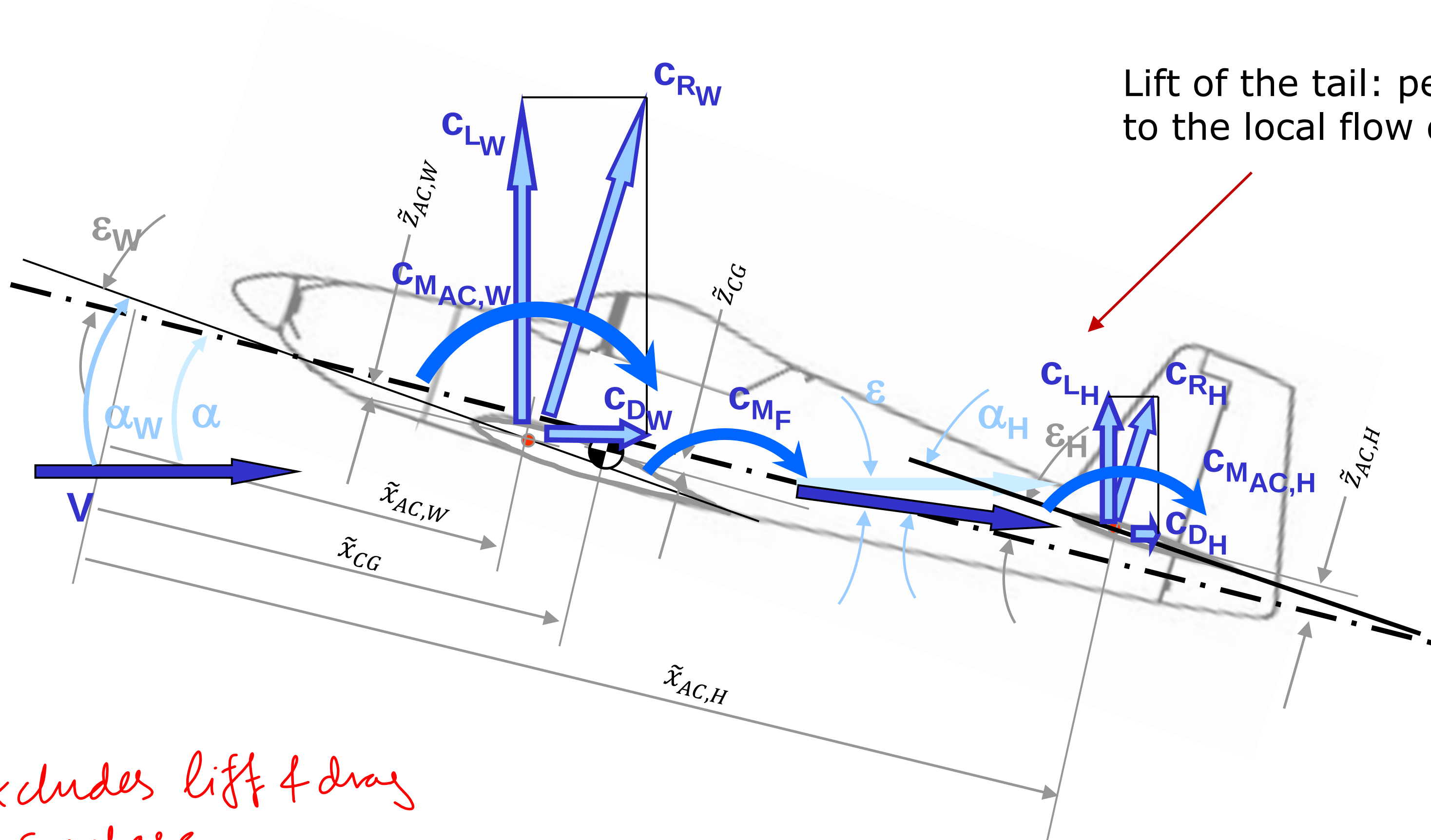
α	Aircraft AoA	ε_W	Wing angle of incidence
α_W	Wing AoA	ε_H	HTP angle of incidence
α_H	HTP AoA	$\varepsilon_H - \varepsilon_W$	Incidence difference
		ε	Downwash angle



$(\tilde{x}_{CG}, \tilde{y}_{CG}, \tilde{z}_{CG})$	Aircraft Center of Gravity C.G.
$(\tilde{x}_{NP}, \tilde{y}_{NP}, \tilde{z}_{NP})$	Aircraft Neutral Point (Aircraft Aerodynamic Center)
$(\tilde{x}_{AC,W}, \tilde{y}_{AC,W}, \tilde{z}_{AC,W})$	Aerodynamic Center of the Wing
$(\tilde{x}_{AC,H}, \tilde{y}_{AC,H}, \tilde{z}_{AC,H})$	Aerodynamic Center of the HTP

@ ~25% of MAC

Aerodynamic Forces and Moments



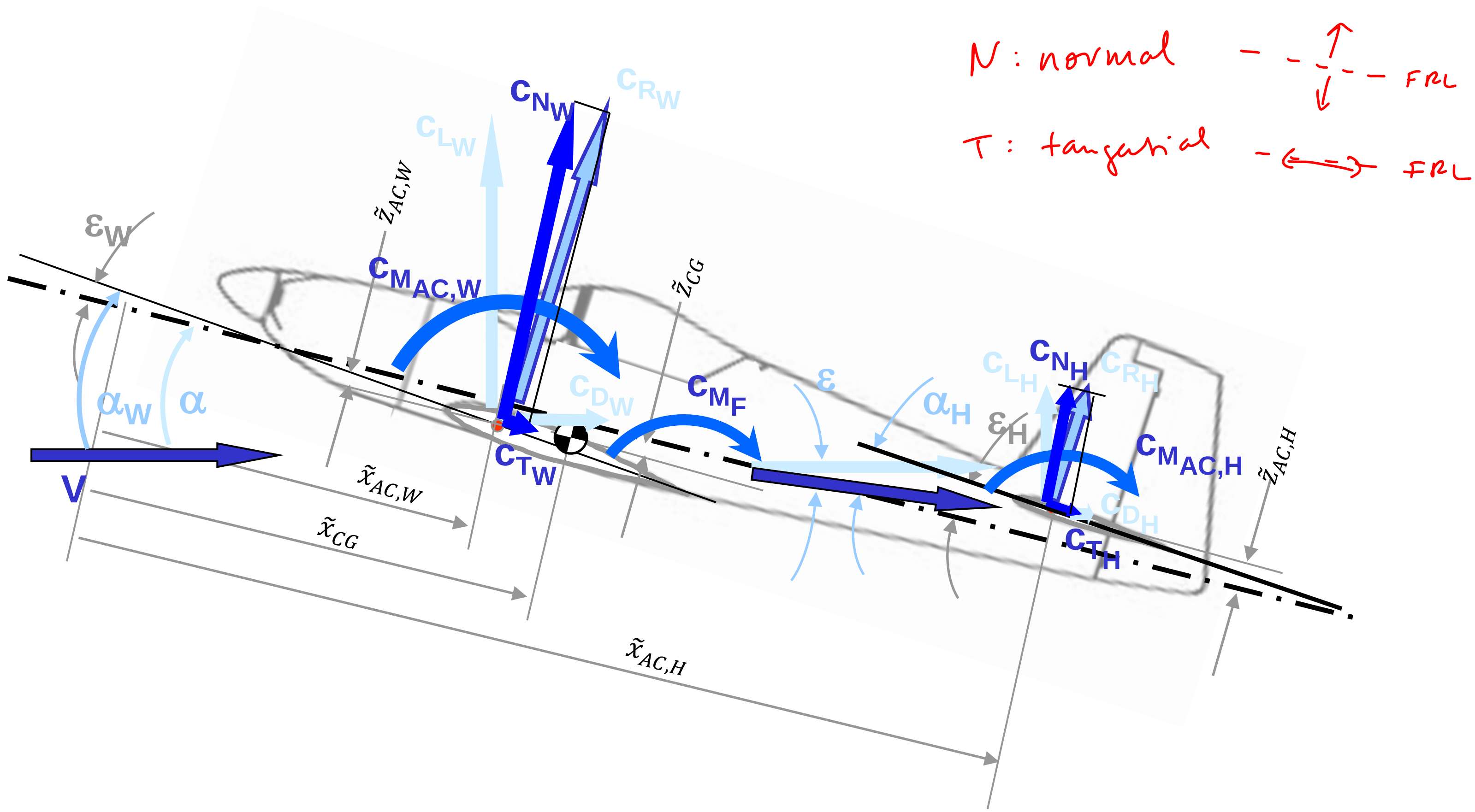
- excludes lift & drag of fuselage

c_L	Lift coefficient
c_D	Drag coefficient
c_R	Aerodynamic force coefficient
c_M	Pitching moment coefficient
$\bar{c}$	Mean Aerodynamic Chord (length)

Indices

w/o index	Aircraft
W	Wing
H	HTP
F	Fuselage
CG	Center of Gravity
AC	Aerodynamic Center

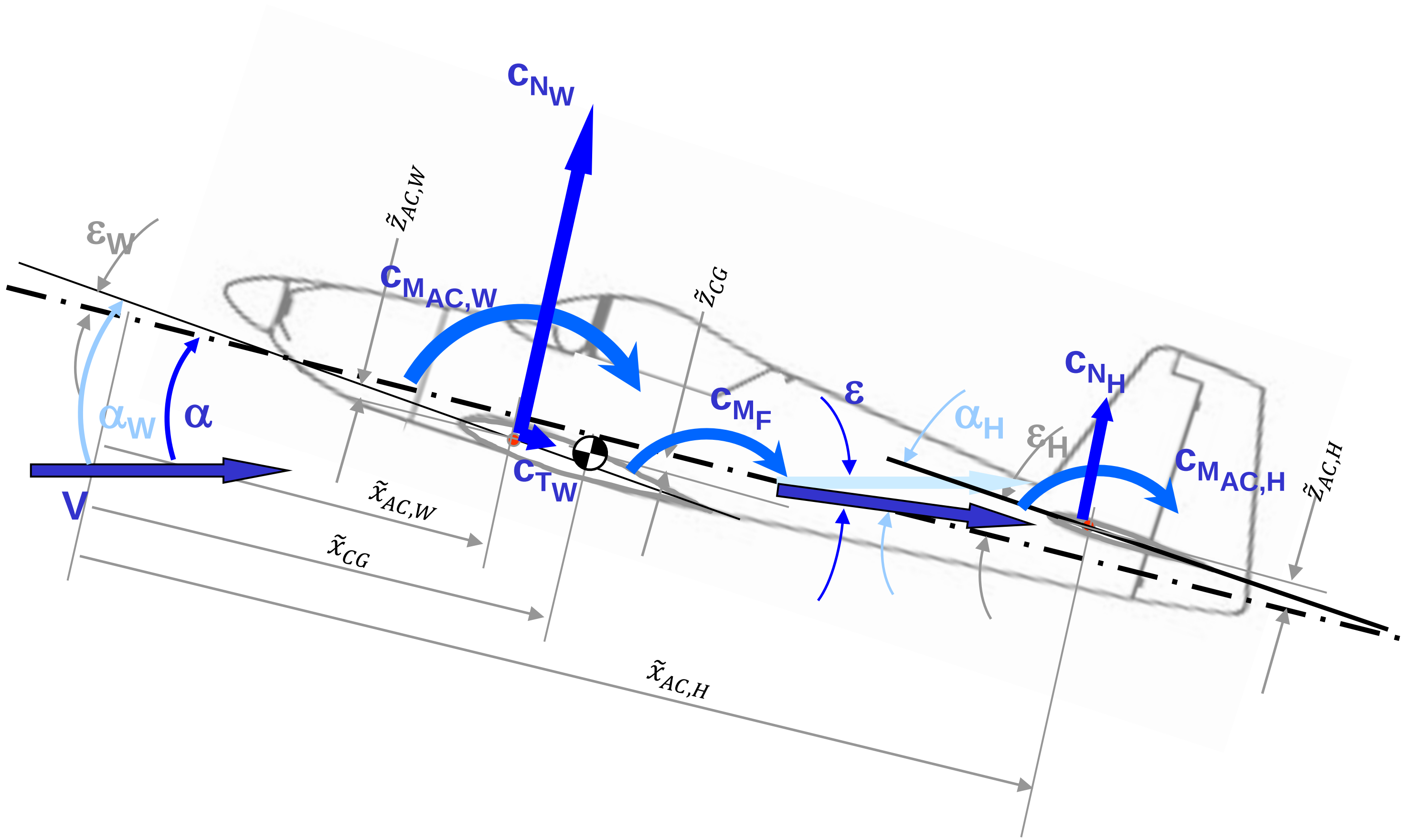
Aerodynamic Forces in Body Axis System



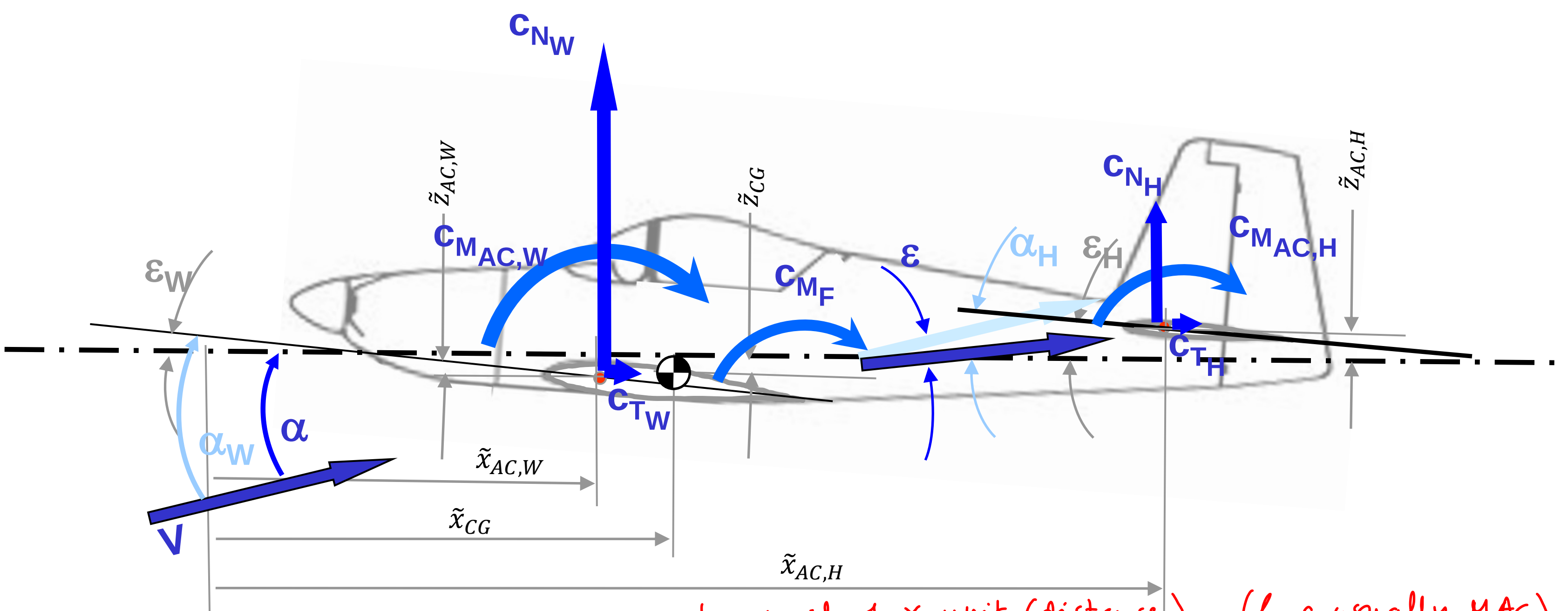
c_N Normal Force

c_T Tangential Force

Relevant Forces and Moments



Pitching Moment about the C.G.



to get rid of x unit (distance) (l_ref usually MAC)

$$c_{M_{CG}} = c_{N_W}\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}} + c_{T_W}\left(\tilde{z}_{CG} - \tilde{z}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}} + c_{M_{\mathrm{AC},W}}$$

moment coeff about airplane COG.

$$+\eta_H\left\{c_{N_H}\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},H}\right)\frac{1}{l_{ref}}\frac{S_H}{S_{ref}} + c_{T_H}\left(\tilde{z}_{CG} - \tilde{z}_{\mathrm{AC},H}\right)\frac{1}{l_{ref}}\frac{S_H}{S_{ref}} + c_{M_{\mathrm{AC},H}}\frac{\bar{c}_H}{l_{ref}}\frac{S_H}{S_{ref}}\right\}$$

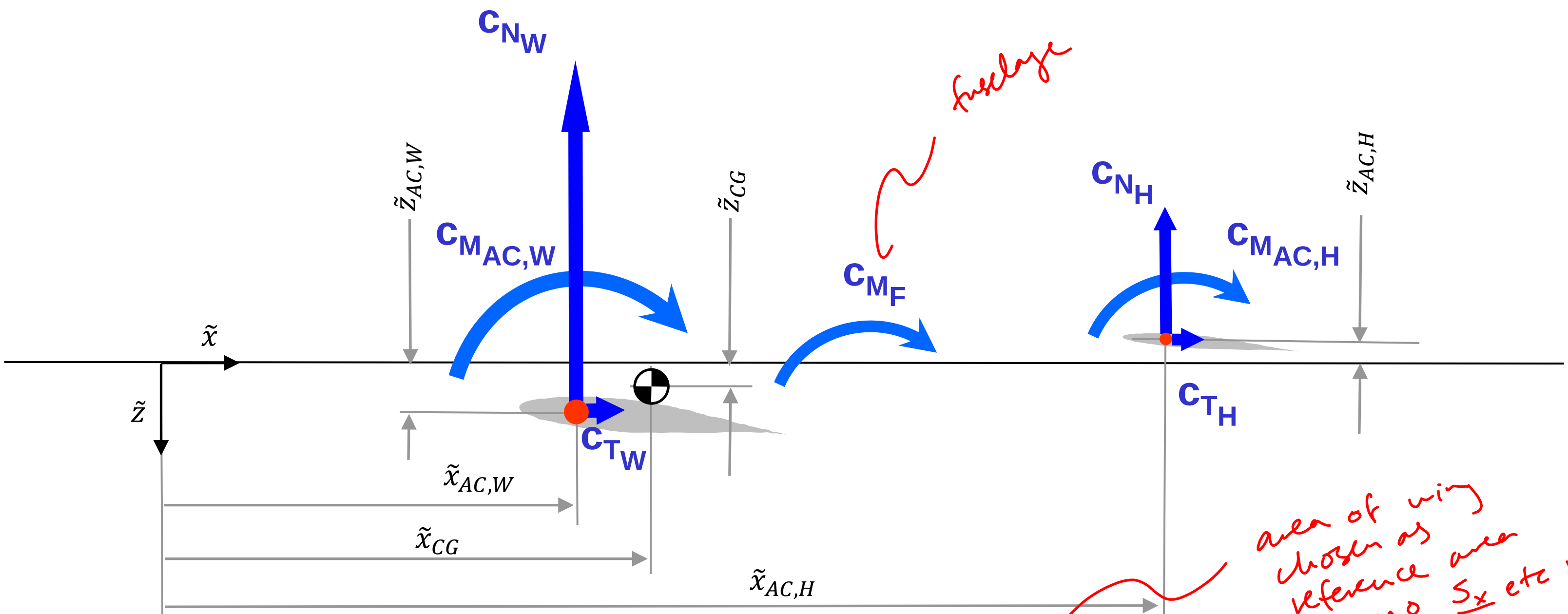
$$+c_{M_F}$$

dimensionalize $(C_{N_H} \cdot S_H)$ & then make dimensionless (wrt S_{ref}) to ensure all dimensionless coeff. summed up are in same reference system

with $\eta_H = \dfrac{q_H}{q_\infty}$ where q is the dynamic pressure

usually lower airspeed (dyn. pressure) @ tail !!!

Pitching Moment about the C.G.



area of wing chosen as reference area => no $\frac{S_x}{S_r}$ etc needed

$$c_{M_{CG}} = \boxed{c_{N_W}\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}} + c_{T_W}\left(\tilde{z}_{CG} - \tilde{z}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}} + c_{M_{\mathrm{AC},W}}}$$

contribution of the wing

$$+\eta_H\left\{c_{N_H}\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},H}\right)\frac{1}{l_{ref}}\frac{S_H}{S_{ref}} + c_{T_H}\left(\tilde{z}_{CG} - \tilde{z}_{\mathrm{AC},H}\right)\frac{1}{l_{ref}}\frac{S_H}{S_{ref}} + c_{M_{\mathrm{AC},H}}\frac{\bar{c}_H}{l_{ref}}\frac{S_H}{S_{ref}}\right\}$$

contribution of the HTP

for correct non-dimensionalization

$$+c_{M_F}$$

contribution of the fuselage

Longitudinal Static Stability Derivative

$$\frac{dc_{M_{CG}}}{dc_L}$$

Note: can be derived w.r.t. α or c_L

doesn't matter what we derive wrt (sign won't be different as we are in linear regime of lift polar)

$$\frac{dc_{M_{AC,W}}}{dc_L} := 0$$ *by definition of the AC*

$$c_{M_{CG}} = c_{N_W}\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}} + c_{T_W}\left(\tilde{z}_{CG} - \tilde{z}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}} + \cancel{c_{M_{\mathrm{AC},W}}}$$

$$+\eta_H\left\{c_{N_H}\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},H}\right)\frac{1}{l_{ref}}\frac{S_H}{S_{ref}} + \cancel{c_{T_H}\left(\tilde{z}_{CG} - \tilde{z}_{\mathrm{AC},H}\right)\frac{1}{l_{ref}}\frac{S_H}{S_{ref}}} + \cancel{c_{M_{\mathrm{AC},H}}\frac{\bar{c}_H}{l_{ref}}\frac{S_H}{S_{ref}}}\right\}$$

$$+c_{M_F}$$

small

$$\frac{dc_{M_{AC,H}}}{dc_L} := 0$$

Results in:

$$\frac{dc_{M_{CG}}}{dc_L} = \boxed{\frac{dc_{N_W}}{dc_L}\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}} + \frac{dc_{T_W}}{dc_L}\left(\tilde{z}_{CG} - \tilde{z}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}}}$$ Wing

$$\boxed{+\frac{dc_{N_H}}{dc_L}\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},H}\right)\frac{\eta_H}{l_{ref}}\frac{S_H}{S_{ref}}}$$ HTP

$$\boxed{+\frac{dc_{M_F}}{dc_L}}$$ Fuselage

Contribution of the Wing

$$\left(\frac{dc_{M_{CG}}}{dc_L}\right)_{Wing} = \frac{dc_{N_W}}{dc_L}\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}} + \frac{dc_{T_W}}{dc_L}\left(\tilde{z}_{CG} - \tilde{z}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}}$$

effective AoA of wing

$$c_N = c_L\cos(\alpha - \varepsilon_W) + c_D\sin(\alpha - \varepsilon_W)$$

$$c_T = c_D\cos(\alpha - \varepsilon_W) - c_L\sin(\alpha - \varepsilon_W)$$

For small α

$c_N = c_L$ as c_D usually 20 − 60 x smaller than c_L
(20: normal, 60: glider)

$c_T = c_D$ less accurate but ok (c_L not << c_D)

optional

1. Formulate $\frac{dc_T}{dc_L}$ (chain rule)
2. Simplify using cos=1 and sin=0 (small angles) and $c_D \ll 1$
3. get $\frac{dc_D}{dc_L}$ by using drag polar formulation $c_D = c_{D0} + \frac{1}{\pi ARe}c_L^2$

⇨ $$\frac{dc_T}{dc_L} = c_L\left(\frac{2}{\pi ARe} - \frac{1}{\frac{dc_L}{d\alpha}}\right)$$

$$\frac{dc_{M_{CG}}}{dc_L} = \frac{dc_{L_W}}{dc_L}\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}} + \overbrace{c_L\left(\frac{2}{\pi ARe} - \frac{1}{\frac{dc_L}{d\alpha}}\right)}\left(\tilde{z}_{CG} - \tilde{z}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}}$$

$\frac{dc_{L_W}}{dc_L} = 1$

Contribution of the Wing

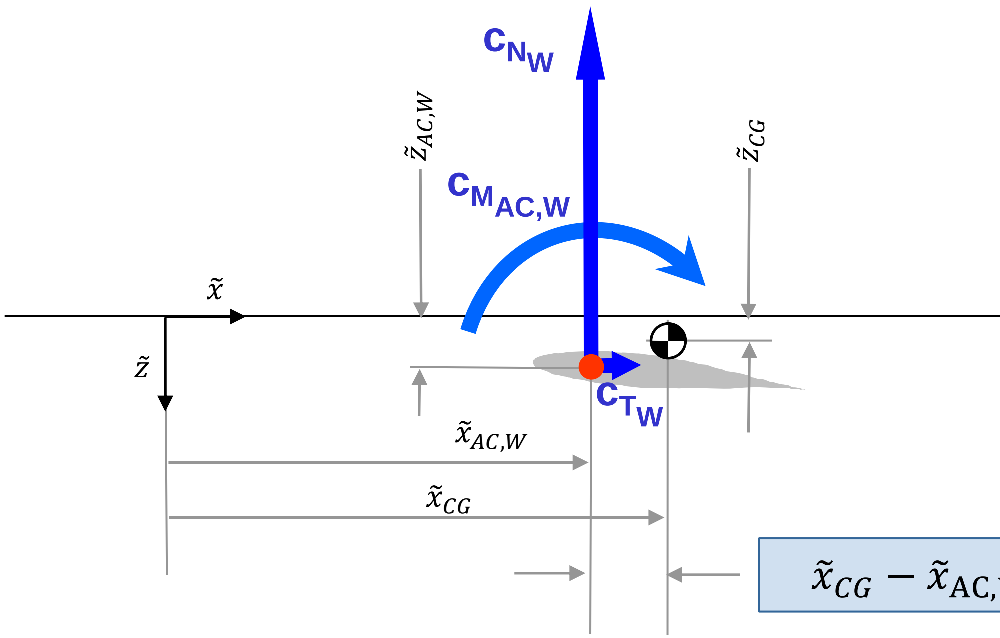
$$\left(\frac{dc_{M_{CG}}}{dc_L}\right)_{Wing} = (\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},W})\frac{1}{l_{ref}} + c_L\left(\frac{2}{\pi ARe} - \frac{1}{\frac{dc_L}{d\alpha}}\right)(\tilde{z}_{CG} - \tilde{z}_{\mathrm{AC},W})\frac{1}{l_{ref}}$$

The contribution of the wing to the stability derivative depends on:

- The distance of the wing AC to the CG $(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},W})$

In addition, a high wing has (small) stabilizing effect, but only at larger c_L

still in linear regime



$\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},W}$

most important contribution (of wing) to stability

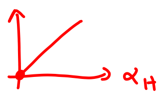
Contribution of the HTP

$$\left(\frac{dc_{M_{CG}}}{dc_L}\right)_{HTP} = \frac{dc_{N_H}}{dc_L}\left(\tilde{x}_{CG} - \tilde{x}_{AC,H}\right)\frac{\eta_H}{l_{ref}}\frac{S_H}{S_{ref}}$$

$$V_H = -\frac{\left(\tilde{x}_{CG} - \tilde{x}_{AC,H}\right)}{l_{ref}}\frac{S_H}{S_{ref}}$$

Horizontal tail volume coefficient

$\frac{dc_{N_H}}{dc_L}$ is not, like we had with the wing, equal to unity. This is because the horizontal tail is in the downwash of the main wing.



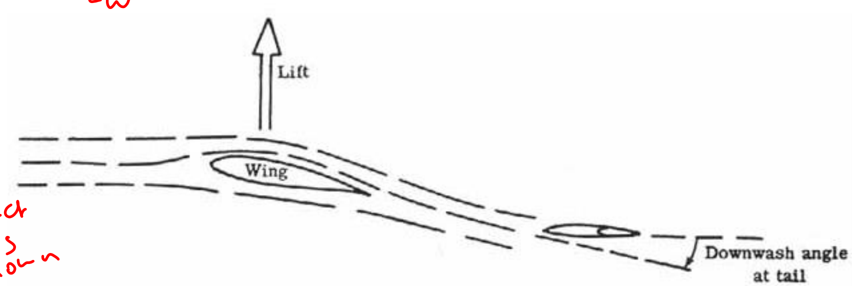
$$c_{N_H} = \frac{dc_{N_H}}{d\alpha_H}\alpha_H \quad \text{with} \quad \alpha_H = \alpha_W - \varepsilon + \varepsilon_H - \varepsilon_W$$

$$\frac{dc_{N_H}}{dc_L} = \frac{dc_{N_H}}{d\alpha_H}\underbrace{\left(\frac{d\alpha_W}{dc_L} - \frac{d\varepsilon}{dc_L}\right)}_{\frac{d\alpha_H}{dc_L}} \qquad \frac{dc_{N_H}}{dc_L} = \frac{\overset{c_{L_H}}{\boxed{(dc_{N_H})/(d\alpha_H)}}}{\underset{c_{L_W}}{\boxed{(dc_{N_W})/(d\alpha_W)}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$\Rightarrow \left(\frac{dc_{M_{CG}}}{dc_L}\right)_{HTP} = -\frac{c_{L_{\alpha},H}}{c_{L_{\alpha},W}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_H V_H$$

ratio of lift slopes

downwash effect (how much does airflow turn down w/ α ?)



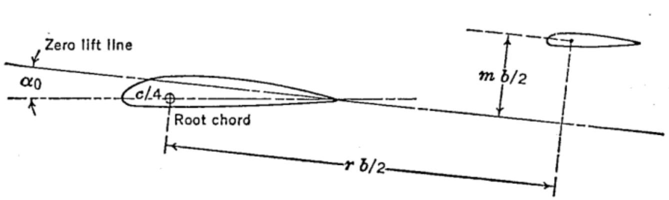
Source: NASA-SP-367

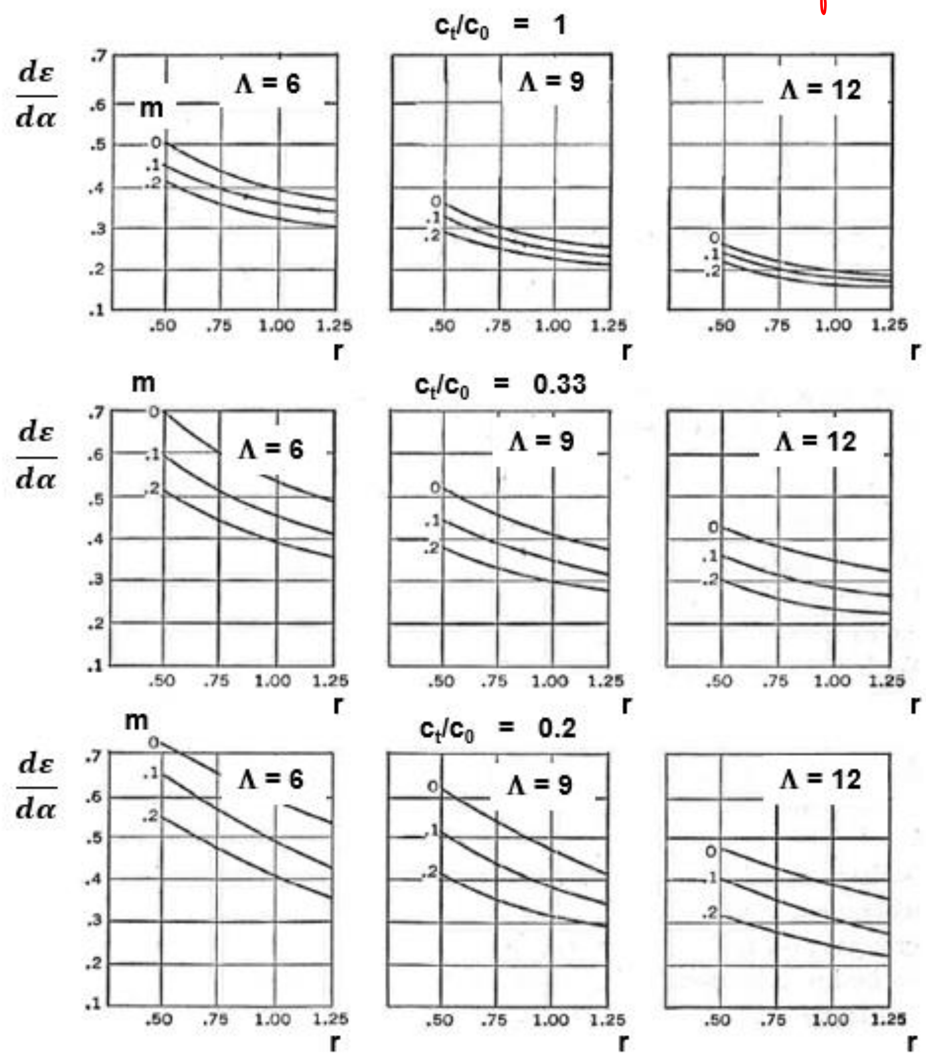
Downwash

Get downwash contribution from tables, or use a flow solver (e.g. VLM) vortex lattice solver

or open VSP

e.g. NTRS NASA technical report server





Remark
Using the theoretical result for the downwash of an ideal wing gives us:

$$\frac{d\varepsilon}{d\alpha} = \frac{2}{\pi AR}\frac{dc_L}{d\alpha}$$

Summary

Longitudinal static stability derivative

fix an airplane (make stable) using these two

$$\frac{dc_{M_{CG}}}{dc_L} = \underbrace{\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}}}_{\text{Wing}} - \underbrace{\frac{c_{L_\alpha,H}}{c_{L_\alpha,W}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_H V_H}_{\text{HTP}} + \underbrace{\frac{dc_{M_F}}{dc_L}}_{\text{Fuselage}}$$

(vortex lattice) solver gives this directly

not generalizeable

with $V_H = -\dfrac{\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},H}\right)}{l_{ref}} \dfrac{S_H}{S_{ref}}$ tail volume ratio

Pitching Moment about the C.G. (given without derivation)

$$c_{M_{CG}} = c_L\left(\tilde{x}_{CG} - \tilde{x}_{\mathrm{AC},W}\right)\frac{1}{l_{ref}} + c_{M_{\mathrm{AC},W}} - c_L \frac{c_{L_\alpha,H}}{c_{L_\alpha,W}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_H V_H$$
$$- c_{L_\alpha,H}\eta_H V_H\left(\alpha_0 - \varepsilon_W + \varepsilon_H\right) + c_{M_F}$$