Spring Semester 2023

AIRCRAFT AERODYNAMICS & FLIGHT MECHANICS

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This lecture is adapted with permission from the lecture “Ausgewählte Kapitel der Flugtechnik” by Dr. Jürg Wildi

Repe - 6-DoF Equations of Motion

Forces

$$m(\dot{U} - VR + WQ) = -mg\sin\theta + F_{A_x} + F_{T_x}$$

$$m(\dot{V} + UR - WP) = mg\sin\phi\cos\theta + F_{A_y} + F_{T_y}$$

$$m(\dot{W} - UQ + VP) = mg\cos\phi\cos\theta + F_{A_z} + F_{T_z}$$

dynamics come from aerodynamic forces & moments acting on aircraft.

But what aerodyn. charact. is responsible for dynamic behaviour observed?
→ this lecture.

Moments

$$I_{xx}\dot{P} - I_{xz}\dot{R} - I_{xz}PQ + (I_{zz} - I_{yy})RQ = L_A + L_T$$

$$I_{yy}\dot{Q} + (I_{xx} - I_{zz})PR + I_{xz}(P^2 - R^2) = M_A + M_T$$

$$I_{zz}\dot{R} - I_{xz}\dot{P} + (I_{yy} - I_{xx})PQ + I_{xz}QR = N_A + N_T$$

Kinematics

$$P = \dot{\phi} - \dot{\psi}\sin\theta$$

$$Q = \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi$$

$$R = \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi$$

Repe - Solution Procedure (I)

Small Disturbance Theory: linearized EoM

1) Define a stationary flight state

2) Superposition with small disturbances, e.g.

$$U = U_1 + u(t)$$

$$P = P_1 + p(t)$$

$$Y = Y_1 + y(t)$$

…

3) Simplify and use the stability axis system

Repe - Linearized EoM for Small Disturbances

Forces

$$m(\dot{u} + W_1 q) = -mg\theta \cos\Theta_1 + f_{A_x} + f_{T_x}$$

$$m(\dot{v} + U_1 r - W_1 p) = mg\phi \cos\Theta_1 + f_{A_y} + f_{T_y}$$

$$m(\dot{w} - U_1 q) = -mg\theta \sin\Theta_1 + f_{A_z} + f_{T_z}$$

Moments

$$I_{xx}\dot{p} - I_{xz}\dot{r} = l_A + l_T$$

$$I_{yy}\dot{q} = m_A + m_T$$

$$I_{zz}\dot{r} - I_{xz}\dot{p} = n_A + n_T$$

Kinematics

$$p = \dot{\phi} - \dot{\psi}\sin\Theta_1$$

$$q = \dot{\theta}$$

$$r = \dot{\psi}\cos\Theta_1$$

Repe - Solution Procedure (II)

Small Disturbance Theory: linearized EoM

deciding which derivatives to consider (last lecture) *

4) Model the forces as partial derivatives

$$f_x = \frac{\partial F_x}{\partial V} dV + \frac{\partial F_x}{\partial \alpha} d\alpha + \frac{\partial F_x}{\partial \theta} d\theta \; \ldots$$

(Note: this is a Taylor series expansion neglecting the higher order terms)

5) Write the EoM in non-dimensional from
Note: Step 5 is optional. It makes the derivation of the equations more complex, but allows to use the well-known aerodynamic coefficients. However, the results will be non-dimensional and will have to be converted

* some partial derivatives are very small for normal airframe configs

! => for exotic designs you need to reevaluate the Taylor expansions!!!

Repe - EoM in Non-Dimensional Form

For longitudinal dynamic stability

x-Force

$$2\mu\dot{\tilde{u}} - c_{x_u}\tilde{u} - c_{x_\alpha}\alpha + c_{L_1}\theta = 0$$

z-Force

$$2c_{L_1}\tilde{u} + 2\mu\dot{\alpha} - c_{z_\alpha}\alpha - c_{z_{\dot{\alpha}}}\dot{\alpha} - \left(2\mu + c_{z_q}\right)\dot{\theta} + c_{L_1}\theta\tan\Theta_1 = c_{z_\delta}\delta$$

y-Moment (pitching)

$$-c_{M_{\dot{\alpha}}}\dot{\alpha} - c_{M_\alpha}\alpha + i_y\ddot{\theta} - c_{M_q}\dot{\theta} = c_{M_\delta}\delta$$

where

$$c_{x_u}, c_{x_\alpha}, c_{z_\alpha}, c_{z_{\dot{\alpha}}}, c_{z_q}, c_{z_\delta}, c_{M_\alpha}, c_{M_q}, c_{M_{\dot{\alpha}}}, c_{M_\delta}$$

are the stability derivatives

Repe - Stability Derivatives

longitudinal dynamic stability of a conventional configuration

x-Force

$$2\mu\dot{\tilde{u}} - c_{x_u}\tilde{u} - c_{x_\alpha}\alpha + c_{L_1}\theta = 0$$

$$c_{x_u} = -2c_{D_1}$$

$$c_{x_\alpha} = c_{L_1} - c_{D_\alpha}$$

z-Force

$$2c_{L_1}\tilde{u} + 2\mu\dot{\alpha} - c_{z_\alpha}\alpha - c_{z_{\dot{\alpha}}}\dot{\alpha} - \left(2\mu + c_{z_q}\right)\dot{\theta} + c_{L_1}\theta\tan\Theta_1 = c_{z_\delta}\delta$$

$$c_{Z_\alpha} = -c_{L_\alpha}$$

y-Moment

$$-c_{M_{\dot{\alpha}}}\dot{\alpha} - c_{M_\alpha}\alpha + i_y\ddot{\theta} - c_{M_q}\dot{\theta} = c_{M_\delta}\delta$$

$$c_{M_\alpha} = c_{L_\alpha}\frac{\tilde{x}_{CG} - \tilde{x}_{\mathrm{NP}}}{l_{ref}}$$

$$c_{D_\alpha} = \frac{2c_{L_1}c_{L_\alpha}}{\pi ARe}$$

$$c_{z_q} = 2c_{L_{\alpha,H}}\eta_H V_H$$

$$c_{z_{\dot{\alpha}}} = -2\eta_H V_H\left(\frac{d\varepsilon}{d\alpha}\right)c_{L_{\alpha,H}}$$

$$c_{z_\delta} = -\eta_H\frac{S_H}{S_{ref}}c_{L_{\alpha,H}}\tau$$

$$c_{M_q} = -2c_{L_{\alpha,H}}\eta_H V_H\frac{l_H}{l_\mu}$$

$$c_{M_{\dot{\alpha}}} = -2\eta_H V_H\left(\frac{d\varepsilon}{d\alpha}\right)\frac{l_H}{l_{ref}}c_{L_{\alpha,H}}$$

$$c_{M_\delta} = -\eta_H\frac{S_H}{S_{ref}}\frac{l_H}{l_{ref}}c_{L_{\alpha,H}}\tau$$

Note: Thrust = const.

Repe - Longitudinal Dynamic Stability

In non-dimensional from

x-Force

$$2\mu\dot{\tilde{u}} - c_{x_u}\tilde{u} - c_{x_\alpha}\alpha + c_{L_1}\theta = 0$$

z-Force

$$2c_{L_1}\tilde{u} + 2\mu\dot{\alpha} - c_{z_\alpha}\alpha - c_{z_{\dot{\alpha}}}\dot{\alpha} - \left(2\mu + c_{z_q}\right)\dot{\theta} + c_{L_1}\theta\tan\Theta_1 = c_{z_\delta}\delta$$

y-Moment

$$-c_{M_{\dot{\alpha}}}\dot{\alpha} - c_{M_\alpha}\alpha + i_y\ddot{\theta} - c_{M_q}\dot{\theta} = c_{M_\delta}\delta$$

x-Force

$$C_1\tilde{u} + C_2\dot{\tilde{u}} - C_3\alpha + C_4\theta = 0$$

z-Force

$$D_1\tilde{u} + D_2\alpha - D_3\dot{\alpha} + D_4\theta = -D_5\delta$$

y-Moment

$$E_1\alpha + E_2\dot{\alpha} + E_3\dot{\theta} + E_4\ddot{\theta} = -E_5\delta$$

Remark - State Space Form

Linear time invariant (LTI) system / System of linear differential equations

$$\dot{\vec{x}}(t) = \mathbf{A}\vec{x}(t) + \mathbf{B}\vec{u}(t)$$

$\vec{x}(t)$: state vector $\mathbf{A}$: state matrix

$\vec{u}(t)$: input vector $\mathbf{B}$: input matrix

Longitudinal dynamics:

x-Force

$$C_1\tilde{u} + C_2\dot{\tilde{u}} - C_3\alpha + C_4\theta = 0$$

z-Force

$$D_1\tilde{u} + D_2\alpha - D_3\dot{\alpha} + D_4\theta = -D_5\delta$$

y-Moment

$$E_1\alpha + E_2\dot{\alpha} + E_3\dot{\theta} + E_4\ddot{\theta} = -E_5\delta$$

$$\vec{x}(t) = \begin{bmatrix}\tilde{u}\\ \alpha\\ \theta\\ \dot{\theta}\end{bmatrix} \quad \dot{\vec{x}}(t) = \begin{bmatrix}\dot{\tilde{u}}\\ \dot{\alpha}\\ \dot{\theta}\\ \ddot{\theta}\end{bmatrix} \quad \vec{u}(t) = [\delta]$$

δ: Elevator deflection

$$\mathbf{A_1}\dot{\vec{x}}(t) = \mathbf{A_2}\vec{x}(t) + \mathbf{B_1}\vec{u}(t)$$

$$\mathbf{A} = \mathbf{A}_1^{-1}\mathbf{A_2} \qquad \mathbf{B} = \mathbf{A}_1^{-1}\mathbf{B_1}$$

$$C_2\dot{\tilde{u}} = -C_1\tilde{u} + C_3\alpha - C_4\theta$$

$$D_3\dot{\alpha} = D_1\tilde{u} + D_2\alpha + D_4\theta + D_5\delta$$

$$E_4\ddot{\theta} = -E_1\alpha - E_2\dot{\alpha} - E_3\dot{\theta} - E_5\delta$$

Analytical Solution to the EoM

Longitudinal dynamics

$$C_1\tilde{u} + C_2\dot{\tilde{u}} - C_3\alpha + C_4\theta = 0$$

$$D_1\tilde{u} + D_2\alpha - D_3\dot{\alpha} + D_4\theta = -D_5\delta$$

$$E_1\alpha + E_2\dot{\alpha} + E_3\dot{\theta} + E_4\ddot{\theta} = -E_5\delta$$

$$\dot{\vec{x}}(t) = \mathbf{A}\vec{x}(t)$$

$$\mathrm{x}_i = x_{0_i}e^{\sigma t}$$

Ansatz for the solution

$$\tilde{u} = u_1 e^{\sigma t} \qquad \dot{\tilde{u}} = u_1\sigma e^{\sigma t}$$

$$\alpha = \alpha_1 e^{\sigma t} \quad \Rightarrow \quad \dot{\alpha} = \alpha_1\sigma e^{\sigma t}$$

$$\theta = \theta_1 e^{\sigma t} \qquad \dot{\theta} = \theta_1\sigma e^{\sigma t} \qquad \ddot{\theta} = \theta_1\sigma^2 e^{\sigma t}$$

Insert the Ansatz and divide by $e^{\sigma t}$

$$(C_1 + C_2\sigma)_2 u_1 - C_3\alpha_1 + C_4\theta_1 = 0$$

$$D_1 u_1 + (D_2 + D_3\sigma)\alpha_1 - D_4\theta_1 = 0$$

$$(E_1 + E_2\sigma)\alpha_1 + (E_3\sigma + E_4\sigma^2)\theta_1 = 0$$

Note: Ansatz = trial solution

Solution for the Longitudinal Dynamics

$$(C_1 + C_2\sigma)_2 u_1 - C_3\alpha_1 + C_4\theta_1 = 0$$

$$D_1 u_1 + (D_2 + D_3\sigma)\alpha_1 - D_4\theta_1 = 0$$

$$(E_1 + E_2\sigma)\alpha_1 + (E_3\sigma + E_4\sigma^2)\theta_1 = 0$$

from

$\det[A - \sigma I] = 0$

Characteristic Equation (fourth-order)

$$a_4\sigma^4 + a_3\sigma^3 + a_2\sigma^2 + a_1\sigma + a_0 = 0 \qquad \text{with} \qquad a_4 = 1$$

The possible solutions (roots) for σ can be:

1. Four real roots
2. Two real roots and one complex conjugate pair
3. Two complex conjugate pairs

Solution to the EoM (Example)

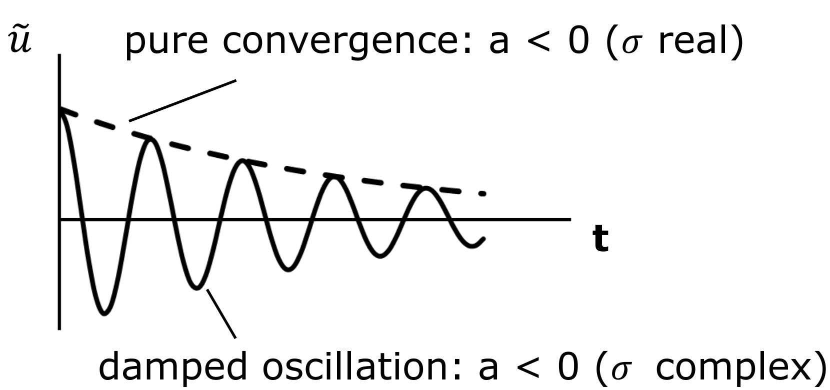
$\mathrm{x}_i = x_{0_i} e^{\sigma t}$ For example: $\tilde{u} = u_1 e^{\sigma t}$

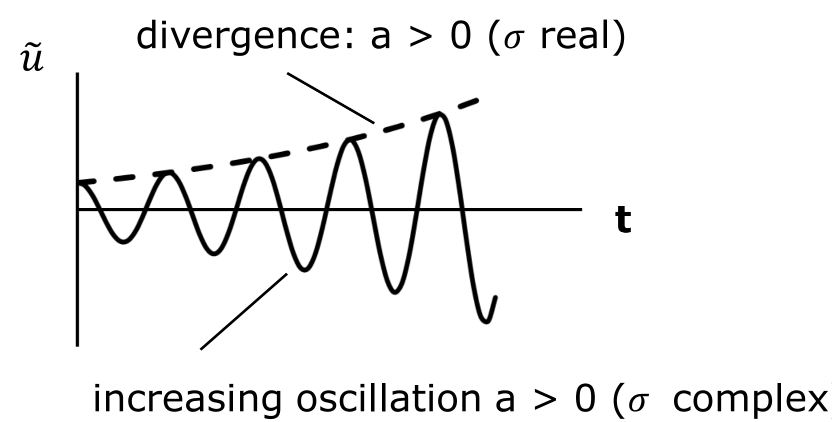
σ complex $\sigma = \mathrm{a} \pm ib$

$\mathrm{x}_i = x_{0_i} e^{at} e^{ibt}$ where a, b real

Amplitude

Oscillation





Solution to the EoM

Characteristic Equation

$$a_4\sigma^4 + a_3\sigma^3 + a_2\sigma^2 + a_1\sigma + a_0 = 0$$

factored

$$(\sigma^2 + B\sigma + C)(\sigma^2 + D\sigma + E) = 0 \quad \Longleftrightarrow \quad \begin{aligned} a_4 &= 1 \\ a_3 &= B + D \\ a_2 &= C + BD + E \\ a_1 &= DC + BE \\ a_0 &= CE \end{aligned}$$

Solutions

$$\sigma_{1,2} = -n_{1,2} \pm i\omega_{1,2}$$

$$\sigma_{3,4} = -n_{3,4} \pm i\omega_{3,4}$$

Example solution

$$\tilde{u} = e^{-n_{1,2}\tau}\left(u_1 e^{\omega_{1,2}i\tau} + u_2 e^{-\omega_{1,2}i\tau}\right) + e^{-n_{3,4}\tau}\left(u_3 e^{\omega_{3,4}i\tau} + u_4 e^{-\omega_{3,4}i\tau}\right)$$

Solution to the EoM

For the longitudinal dynamics, there are two distinct modes:

$$\tilde{u} = e^{-n_{1,2}\tau}\left(u_1 e^{\omega_{1,2} i\tau} + u_2 e^{-\omega_{1,2} i\tau}\right) + e^{-n_{3,4}\tau}\left(u_3 e^{\omega_{3,4} i\tau} + u_4 e^{-\omega_{3,4} i\tau}\right)$$

Both modes are oscillations with

Oscillation Period

~ dimensionalizing

$$T = \frac{2\pi}{\omega_{1,2}} \frac{l_{ref}}{2U_1} = \frac{\pi}{\omega_{1,2}} \frac{l_{ref}}{U_1}$$

non-dimensional (→ "air second")

Half-Life (damping)

time for oscillation to diminish by 1/2

$$T_{\frac{1}{2}} = \frac{l_{ref}}{2n_{1,2}U_1} \ln 2$$

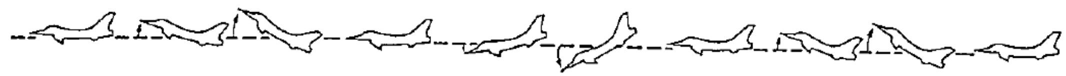
The two modes are called

- Short Period Motion
- Phugoid

Solution to the EoM

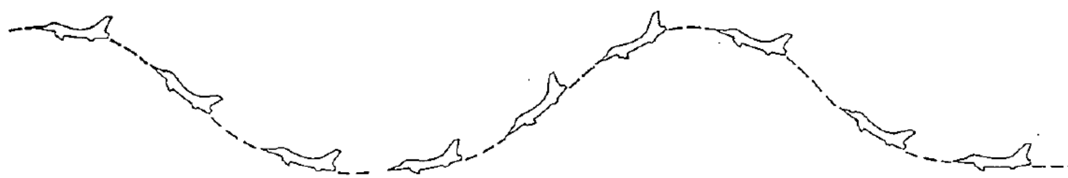
There are two distinct longitudinal modes:

Short Period Motion ~ rapid change in pitch (& AoA)



- Fast
- Rotation around the CG
- Usually strongly damped

Phugoid



- Slow
- Movement of the CG
- Weakly damped
- Exchange of kinetic and potential energy

We can derive approximate solutions for the two modes in order to relate the characteristics of the dynamics (period of the oscillation, damping of the amplitude) to the airplane parameters (velocity, mass, moment of inertia and aerodynamics?)

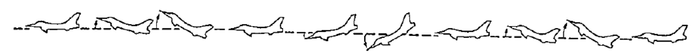
Short Period Motion

For a fast longitudinal pitching motion, the following approximations are made:

$u = const$

$c_{z_{\dot{\alpha}}} \ll \mu$ dimensionless mass

$c_{z_q} \ll \mu$



This reduces the EoM to

(x - Force eqn. disappears w/ u = const.)

z-Force

$$2\mu\dot{\alpha} - c_{z_\alpha}\alpha - 2\mu\dot{\theta} + c_{L_1}\theta\tan\Theta_1 = 0$$

y-Moment

$$-c_{M_{\dot{\alpha}}}\dot{\alpha} - c_{M_\alpha}\alpha + i_y\ddot{\theta} - c_{M_q}\dot{\theta} = 0$$

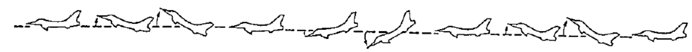
Note: since $u = const$ we don't have a x-force equation

Characteristic Equation

$$2\mu i_y\sigma^2 - \left[c_{z_\alpha}i_y + 2\mu\left(c_{M_q} + c_{M_{\dot{\alpha}}}\right)\right]\sigma + c_{z_\alpha}c_{M_q} - 2\mu c_{M_\alpha} = 0$$

Short Period Motion

Approximate solution for the short period mode



Characteristic Equation

$$2\mu i_y \sigma^2 - \left[c_{z_\alpha} i_y + 2\mu\left(c_{M_q} + c_{M_{\dot{\alpha}}}\right)\right]\sigma + c_{z_\alpha} c_{M_q} - 2\mu c_{M_\alpha} = 0$$

With the solution $\sigma = \mathrm{a} \pm i\omega$

$$\Rightarrow \quad a = \frac{c_{z_\alpha}}{4\mu} + \frac{c_{M_q} + c_{M_{\dot{\alpha}}}}{2 i_y}$$

non-dimensional moment of inertia

$$\Rightarrow \quad i\omega = \left[-\left(\frac{c_{z_\alpha}}{4\mu} + \frac{c_{M_q} + c_{M_{\dot{\alpha}}}}{2 i_y}\right)^2 - \left(\frac{2\mu c_{M_\alpha} - c_{z_\alpha} c_{M_q}}{2\mu i_y}\right)\right]^{\frac{1}{2}}$$

Damping of the Short Period Motion

Revisit lecture 8

Disturbance		Velocity			Incidence Angles		Angular Velocity		
Forces/Moments		u	v	w	β = v / V	α = w / V	p	q	r
X-Force	$F_{A_x} + F_{T_x}$	$\frac{\partial}{\partial u}(F_{A_x}+F_{T_x})<0$							
		$c_{T_{x_u}} - c_{D_u} < 0$							
Y-Force	$F_{A_y} + F_{T_y}$		$\frac{\partial}{\partial v}(F_{A_y}+F_{T_y})<0$						
			$c_{y_\beta} < 0$						
Z-Force	$F_{A_z} + F_{T_z}$			$\frac{\partial}{\partial w}(F_{A_z}+F_{T_z})<0$					
				$c_{L_\alpha} > 0$					
Roll-Moment	$L_A + L_T$				$\frac{\partial}{\partial \beta}(L_A+L_T)<0$		$\frac{\partial}{\partial p}(L_A+L_T)<0$		
					$c_{l_\beta} < 0$		$c_{l_p} < 0$		
Pitch-Moment	$M_A + M_T$	$\frac{\partial}{\partial u}(M_A+M_T)>0$				$\frac{\partial}{\partial \alpha}(M_A+M_T)<0$		$\frac{\partial}{\partial q}(M_A+M_T)<0$	
		$c_{m_u} > 0$				$c_{m_\alpha} < 0$		$c_{m_q} < 0$	
Yaw-Moment	$N_A + N_T$				$\frac{\partial}{\partial \beta}(N_A+N_T)>0$				$\frac{\partial}{\partial r}(N_A+N_T)<0$
					$c_{n_\beta} > 0$				$c_{n_r} < 0$

Dynamic stability: $a < 0$

$$a = \frac{c_{z_\alpha}}{4\mu} + \frac{c_{M_q} + c_{M_{\dot{\alpha}}}}{2 i_y}$$

$$a = \frac{-c_{L_\alpha}}{4\mu} + \frac{c_{M_q} + c_{M_{\dot{\alpha}}}}{2 i_y}$$

Aerodynamics & Flight Mechanics
Stability & Control

Speed Stability - w

A vertical velocity disturbance should result in an opposing vertical aerodynamic force.
Note: positive z-direction is downwards

$$\frac{\partial}{\partial w}(F_{A_z} + F_{T_z}) < 0$$

$$c_{L_\alpha} > 0$$

$$w = \alpha \mathbf{V}$$

$$c_{L_\alpha} = \frac{dc_L}{d\alpha} > 0$$

Note: positive lift is upwards (in negative z direction)

$c_{L\alpha}$ is usually positive within the normal flight regime ($c_L < c_{L_{max}}$)

Relates to the gust load factor and the damping of the short period mode

Aerodynamics & Flight Mechanics
Stability & Control

Static Stability – Pitching Motion

The increase of the pitch-rate q should result in an opposing pitching moment M

$$\frac{\partial}{\partial q}(M_A + M_T) < 0$$

$$c_{m_q} < 0$$

$c_{m_q} < 0$

This is also called the pitch damping derivative.

This stability condition is usually required and generally fulfilled by conventional aircraft configurations

Approximate Solution for the Phugoid

Approximation $\alpha = \text{const} \Rightarrow c_L = const$

this way we can isolate Phugoid mode

The equations of motion are reduced to

(4 eqn. = 0)

x-Force

$$2\mu\dot{\tilde{u}} - c_{x_u}\tilde{u} + c_{L_1}\theta = 0$$

z-Force

$$2c_{L_1}\tilde{u} - 2\mu\dot{\theta} = c_{z_\delta}\delta$$

Note: since $\alpha = \text{const}$ and therefore $\dot{\alpha} = 0$,we don't have a y-moment equation

Approximate Solution for the Phugoid

Assuming

$$\tilde{u} = u_1 e^{\sigma t} \qquad \theta = \theta_1 e^{\sigma t}$$

$$\dot{\tilde{u}} = u_1 \sigma e^{\sigma t} \qquad \dot{\theta} = \theta_1 \sigma e^{\sigma t}$$

~~z-Force~~ x Force

$$2\mu\dot{\tilde{u}} - c_{x_u}\tilde{u} + c_{L_1}\theta = 0$$

~~y-Moment~~ z Force

$$2c_{L_1}\tilde{u} - 2\mu\dot{\theta} = c_{z_\delta}\delta$$

The equations of motion become

$$\left(2\mu\sigma - c_{x_u}\right)u_1 + c_{L_1}\theta_1 = 0$$

$$2c_{L_1}u_1 - 2\mu\sigma\theta_1 = c_{z_\delta}\delta$$

Characteristic equation

$$-\left(2\mu\sigma - c_{x_u}\right)2\mu\sigma - 2c_{L_1}^2 = 0$$

$$\sigma^2 + \frac{c_{x_u}}{2\mu}\sigma + \frac{c_{L_1}^2}{2\mu^2} = 0$$

Solution: $$\sigma_{1,2} = -\frac{c_{x_u}}{4\mu} \pm \sqrt{\frac{{c_{x_u}}^2}{16\mu^2} - \frac{c_A^2}{2\mu^2}}$$

Approximate Solution for the Phugoid

Solution: $\sigma_{1,2} = -\frac{c_{x_u}}{4\mu} \pm \sqrt{\frac{{c_{x_u}}^2}{16\mu^2} - \frac{c_A^2}{2\mu^2}}$

$\sigma = a \pm i\omega$

$a = -\frac{c_{x_u}}{4\mu}$

$i\omega = \sqrt{\frac{{c_{x_u}}^2}{16\mu^2} - \frac{c_L^2}{2\mu^2}}$

For gliders and jet aircraft:

$c_{x_u} = -2c_{D_1}$

$\Rightarrow$ $a = \frac{c_{D_1}}{2\mu}$ $\quad$ $i\omega = \sqrt{\frac{{c_{D_1}}^2}{4\mu^2} - \frac{c_{L_1}^2}{2\mu^2}}$

Angular Frequency

$$\sigma = a \pm i\omega$$

$$i\omega = \sqrt{\frac{{c_{D_1}}^2}{4\mu^2} - \frac{c_{L_1}^2}{2\mu^2}} \qquad a = \frac{c_{D_1}}{2\mu}$$

with $\frac{c_{L_1}^2}{2} \gg \frac{c_{D_1}^2}{4}$

non-dimensional

$$\Rightarrow \quad \omega = \frac{c_{L_1}}{\sqrt{2}\mu} \quad \text{[rad/airsec]}$$

air second: $\tau = \frac{t}{\hat{t}}$

This is a dimensionless quantity. We can to convert back to the angular frequency [rad/s] using

$$c_L = \frac{2mg}{\rho U_1^2 S_{ref}} \quad \text{and} \quad \mu = \frac{2m}{\rho S_{ref} l_{ref}}$$

Time:

$$\omega = \sqrt{2}\frac{mg}{\rho U_1^2 S_{ref}} \frac{\rho S_{ref} l_{ref}}{2m} = \frac{1}{\sqrt{2}} \frac{l_{ref}}{U_1} \frac{g}{U_1} \quad \text{[rad/airsec]} \quad \Big| \; \frac{1}{\hat{t}}$$

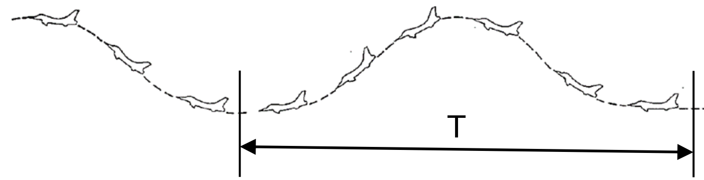
$$\hat{t} = \frac{l_{ref}}{2U_1} \quad \text{[s]}$$

$$\Rightarrow \quad \boxed{\omega = \sqrt{2}\frac{g}{U_1} \quad \text{[rad/s]}}$$

Period and Damping

Phugoid

Period



$$T = \frac{2\pi}{\omega} \qquad \text{with} \qquad \omega = \sqrt{2}\,\frac{g}{U_1} \quad \text{[rad/s]}$$

$$\Rightarrow \quad \boxed{T = \pi\sqrt{2}\,\frac{U_1}{g}} \quad \text{[s]} \qquad \boxed{\approx \frac{1}{2}U_1}$$

Half-life time (damping)

$$T_{1/2} = \frac{1}{a}\ln 2 \qquad \text{with} \qquad a = \frac{c_{D_1}}{2\mu}$$

$$T_{1/2} = \frac{1}{\frac{2c_{D_1}}{4\mu}}\ln 2 = \frac{1}{\frac{1}{2}c_{D_1}\frac{\rho S_{ref} l_{ref}}{2m}}\ln 2 = \frac{1}{\frac{1}{2}\frac{c_{D_1}}{c_{L_1}}\frac{l_{ref}\,g}{U_1^2}}\ln 2 \quad \text{[airsec]}$$

$$\Rightarrow \quad \boxed{T_{1/2} = \frac{c_{L_1}}{c_{D_1}}\frac{U_1}{g}\ln 2} \quad \text{[s]}$$

smaller L/D better for phugoid

The period of the phugoid is proportional to the velocity
The damping is a function of the glide ratio (L/D).

Lateral-Directional Dynamic Stability

The characteristic equation for lateral-directional stability is of 4th order.

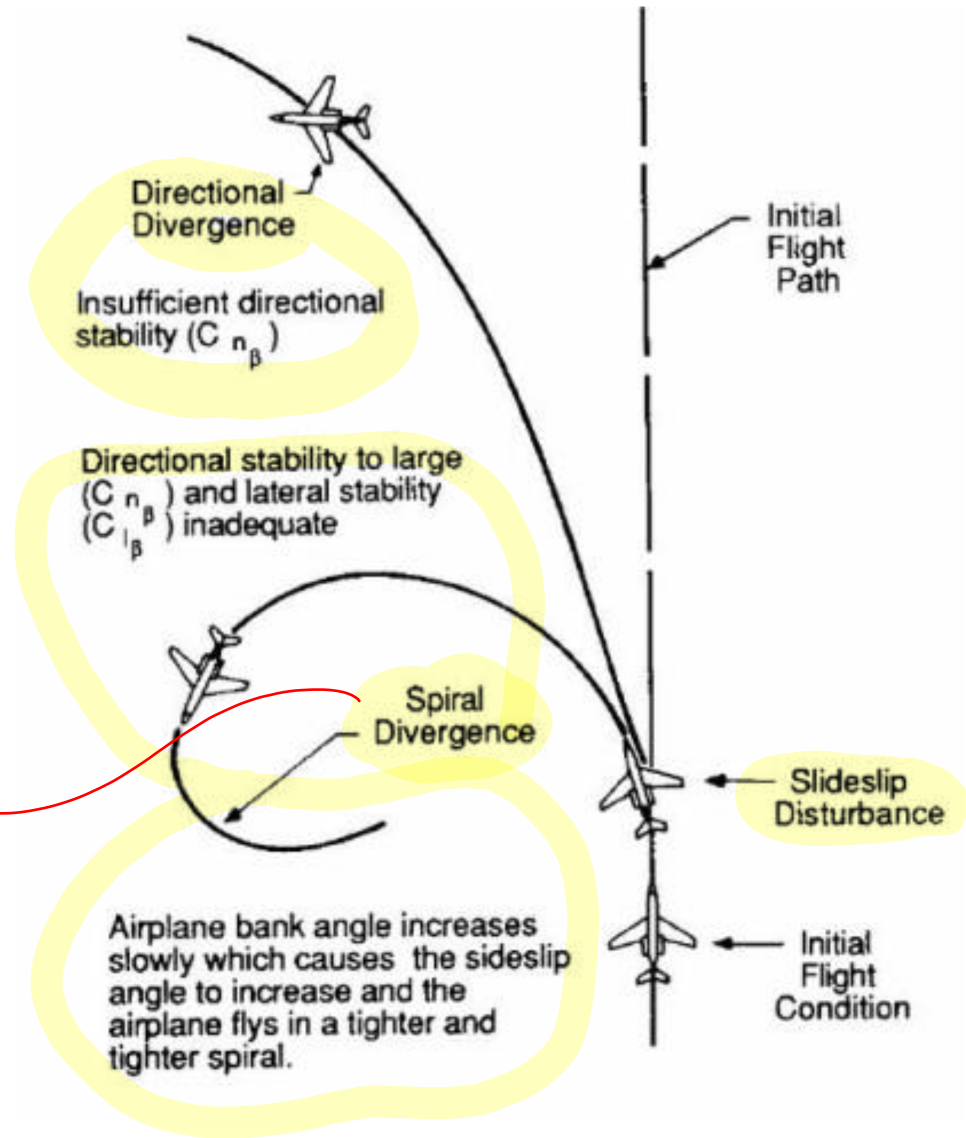
There are the following solutions (modes)

⇨ Two real roots and one conjugate complex pair

- The **Spiral** mode
- The **Roll** mode (roll subsidence mode)
- The **Dutch-Roll** mode

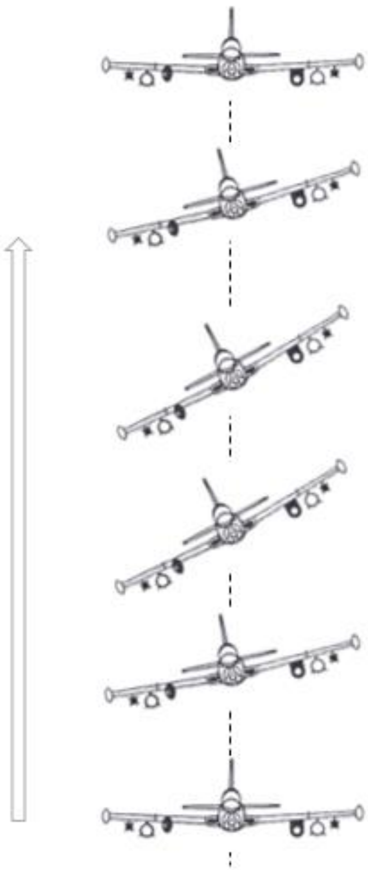
Spiral Mode

Non-oscillatory



most air planes have this

Roll Mode



(3) Recovery: pilot or FCS action required to roll aircraft to level attitude

(2) Stable response: the roll rate is damped (always positively). Half-time usually fast.
Note: roll angle remains

(1) Disturbance in roll-rate (p)

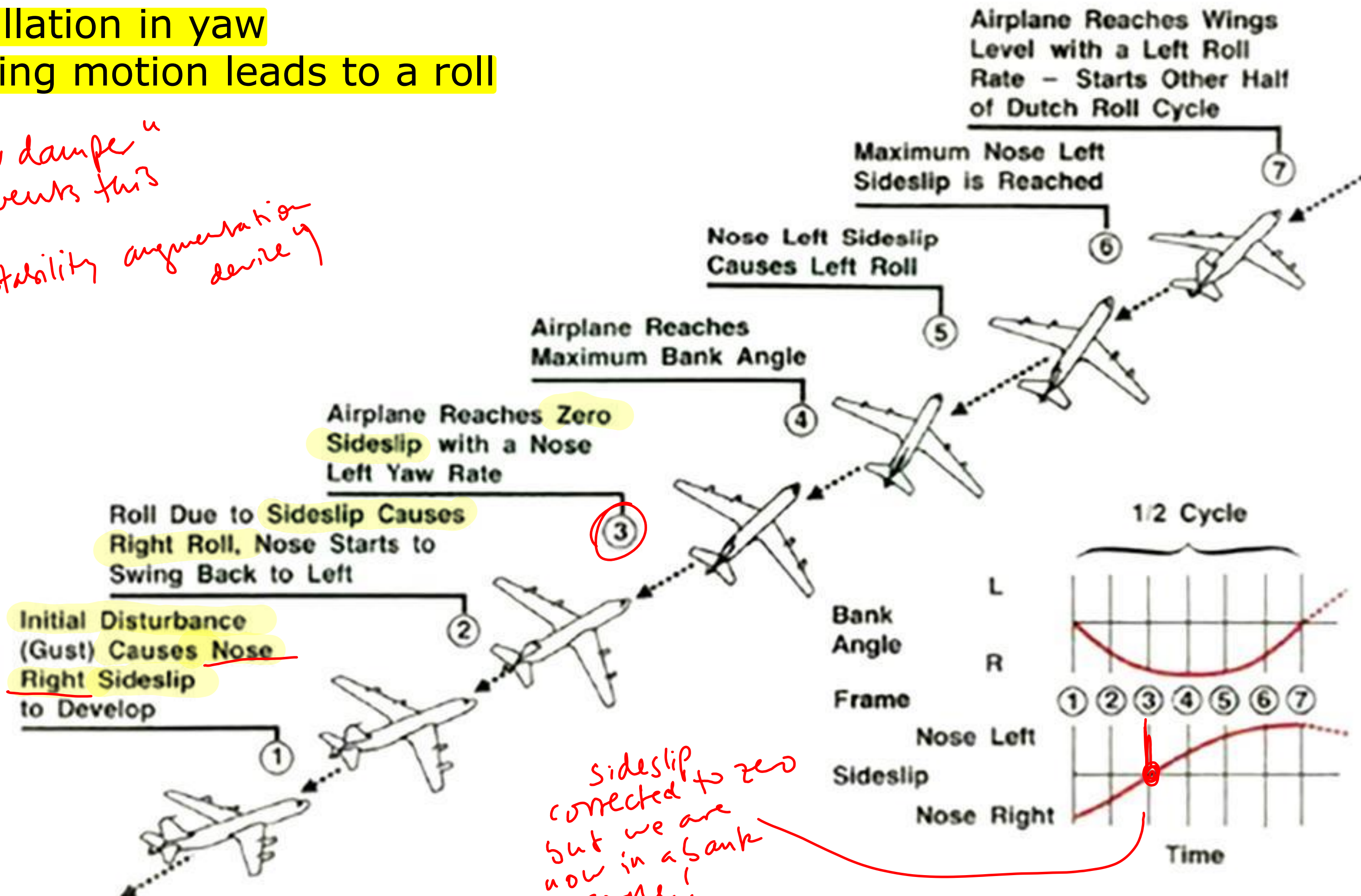
Initial state: level flight

Dutch Roll

The Dutch-Roll is a coupling of sideslip, roll and yaw.

Oscillation in yaw
Yawing motion leads to a roll

"yaw damper" prevents this
("stability augmentation device")



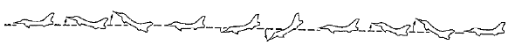
sideslip corrected to zero but we are now in a bank angle!

Source: USAF Aircraft Accident Investigation Board Report, KC-135R, 63-8877, 3 May 2013

Modes

We assume a de-coupling of the longitudinal and the lateral-directional motion

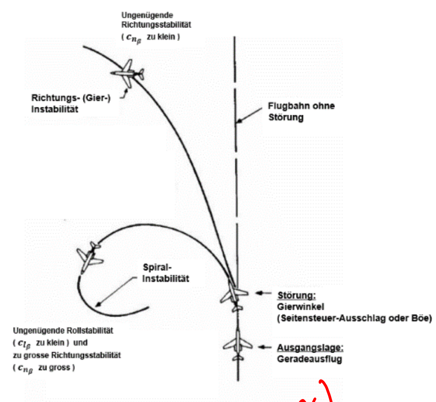
short period mode



Phugoid

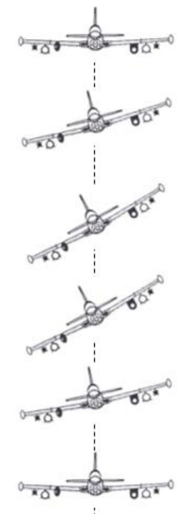
2 longitudinal modes

spiral mode/ divergence

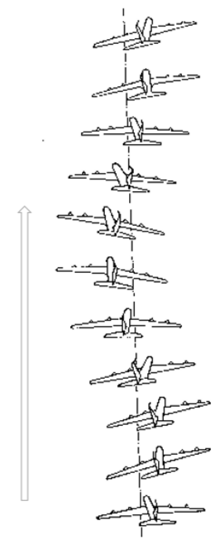


(real root)

roll damping



(real root)



(compl. root → oscill.)

3 lateral-directional modes

Examples Longitudinal Dynamics

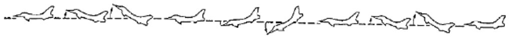
Piper Cherokee PA-28

V = 50 m/s
H = 1’500 m

Lear Jet

M = 0.7 V_e = 105 m/s
H = 12’000 m

Short Period



	Piper Cherokee PA-28	Lear Jet
T	= 1.8 s	= 2.4 s
$T_{½}$	= 0.3 s	= 0.7 s

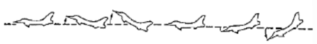
Phugoid

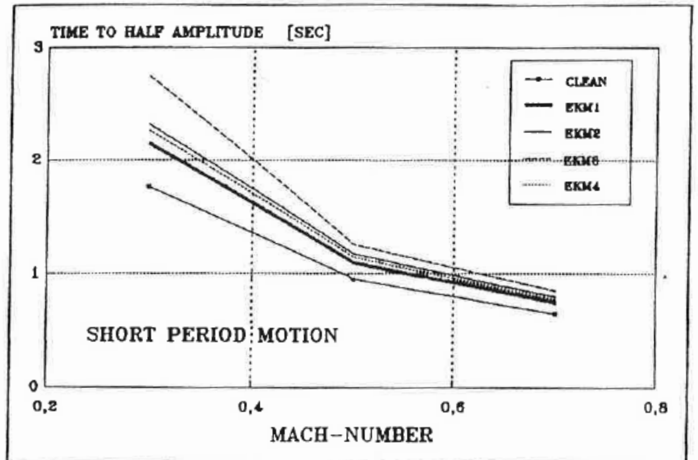
	Piper Cherokee PA-28	Lear Jet
T	= 25 s	= 69 s
$T_{½}$	= 26 s	= 99 s

Example F-5E Tiger

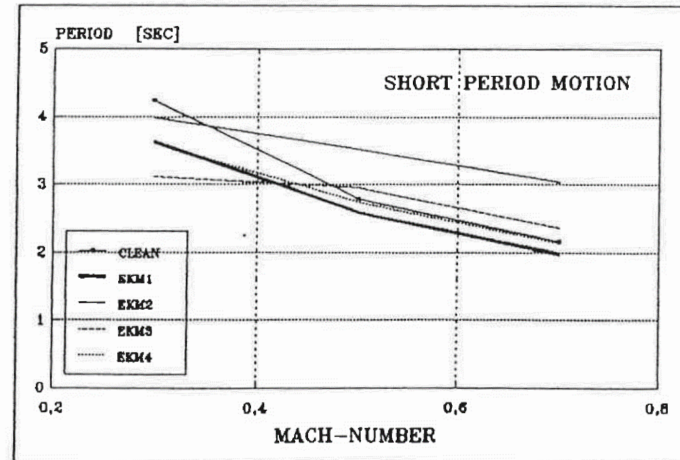
Longitudinal Motion

Short Period



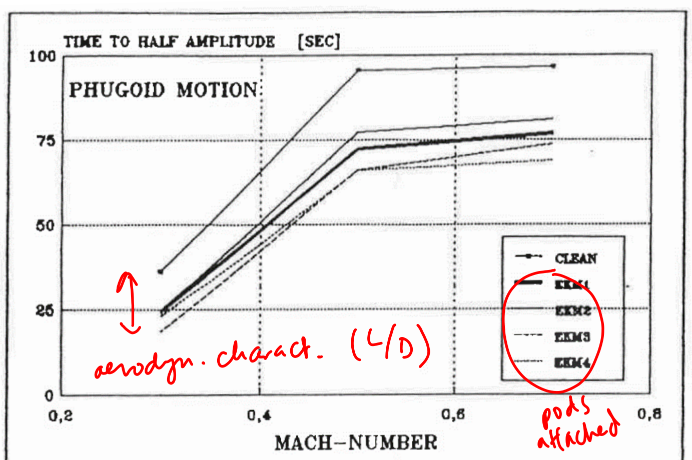


Dämpfung der hochfrequenten Längsschwingung

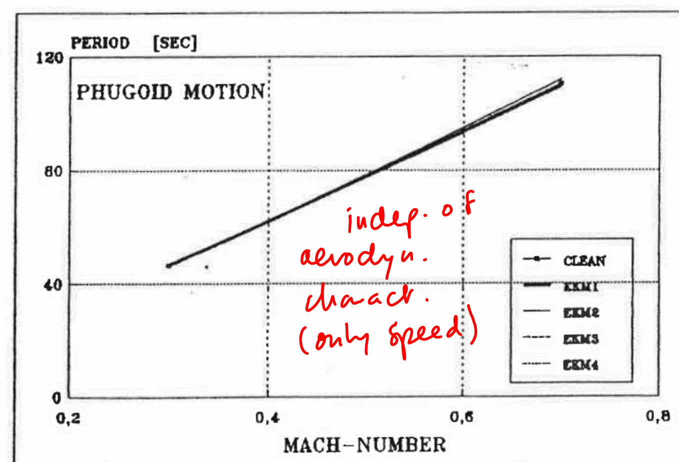


Periode der hochfrequenten Längsschwingung

Phugoid



Dämpfung der Phugoide

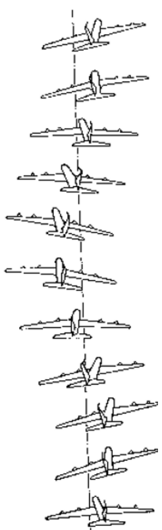


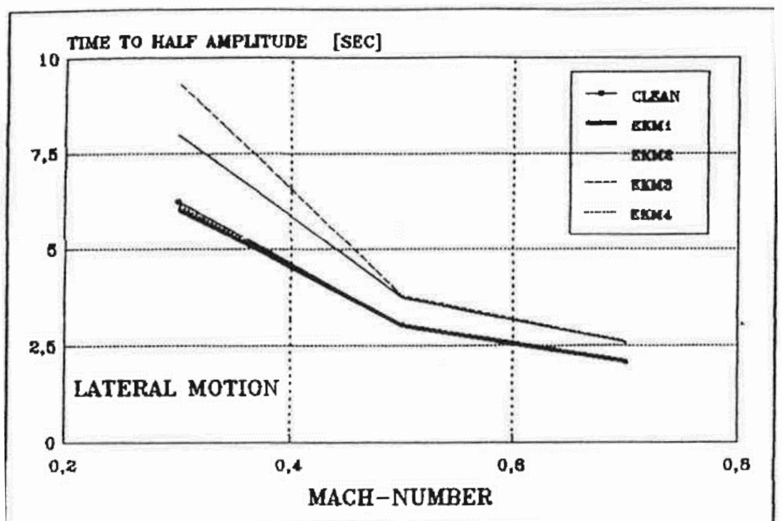
Periode der Phugoide

Example F-5E Tiger

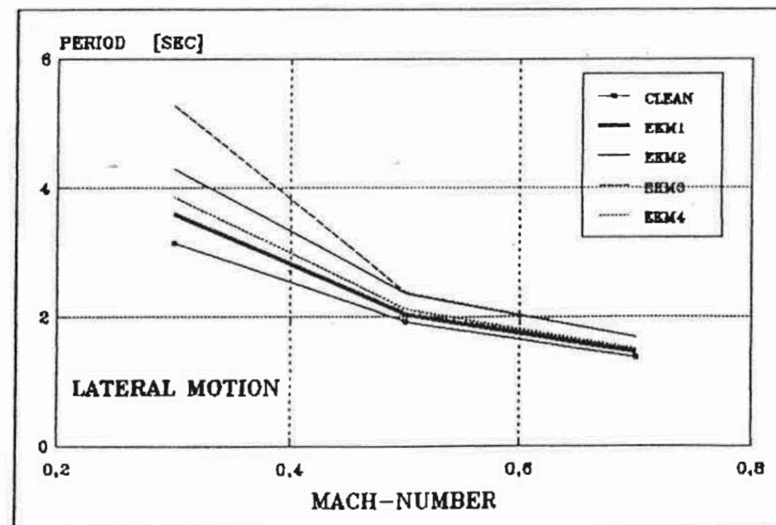
Lateral Motion

Dutch Roll





Dämpfung der Lateralbewegung

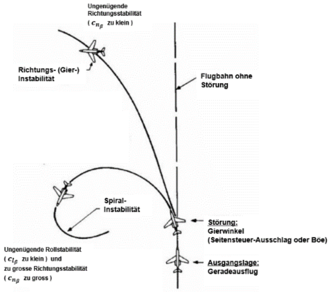


Periode der Lateralbewegung

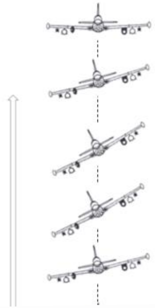
Example F-5E Tiger

Lateral Motion

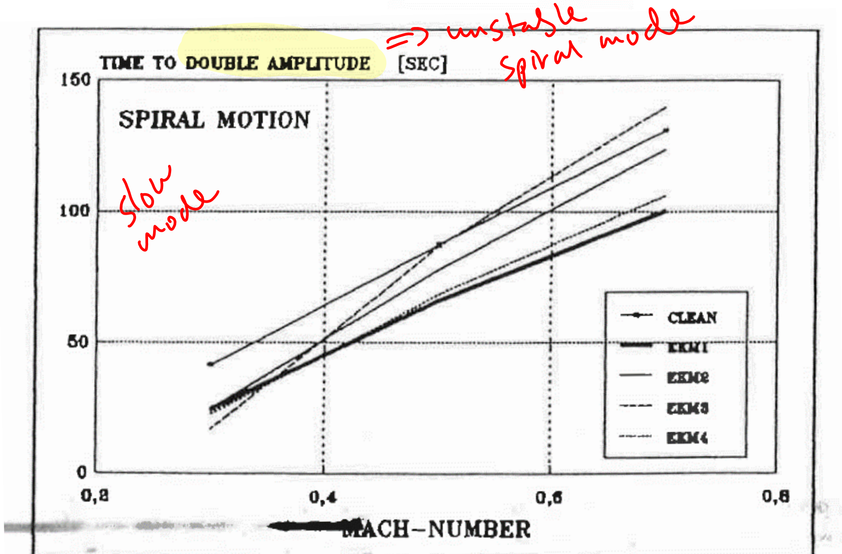
Spiral Mode



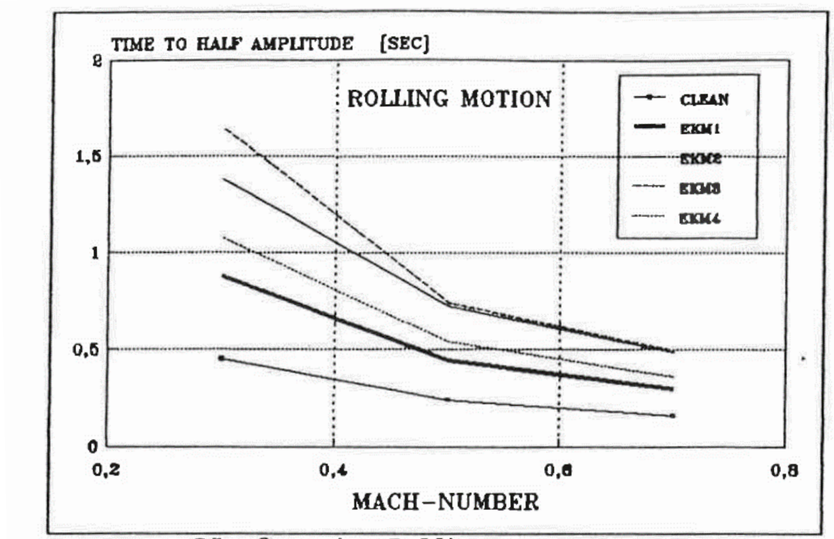
Roll Mode



=> unstable spiral mode



Negative Dämpfung der Spiralbewegung



Dämpfung der Rollbewegung

Summary

Forces

$$m(\dot{U} - VR + WQ) = -mg\sin\theta + F_{A_x} + F_{T_x}$$

$$m(\dot{V} + UR - WP) = mg\sin\phi\cos\theta + F_{A_y} + F_{T_y}$$

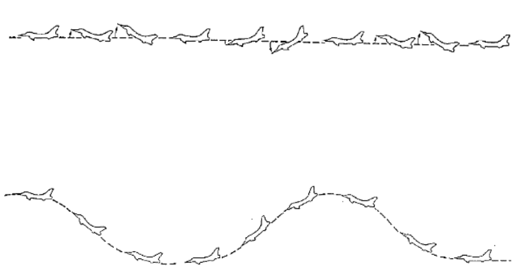
$$m(\dot{W} - UQ + VP) = mg\cos\phi\cos\theta + F_{A_z} + F_{T_z}$$

Moments

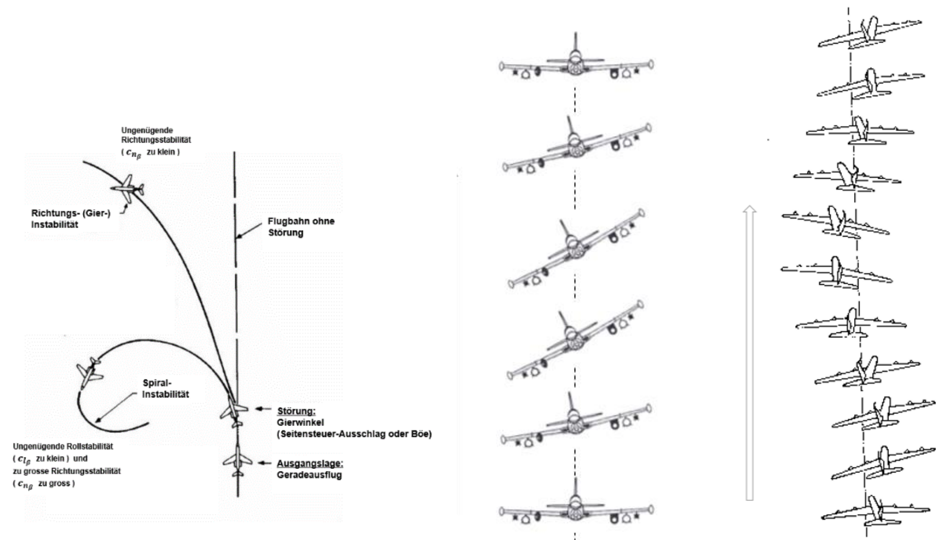
$$I_{xx}\dot{P} - I_{xz}\dot{R} - I_{xz}PQ + (I_{zz} - I_{yy})RQ = L_A + L_T$$

$$I_{yy}\dot{Q} + (I_{xx} - I_{zz})PR + I_{xz}(P^2 - R^2) = M_A + M_T$$

$$I_{zz}\dot{R} - I_{xz}\dot{P} + (I_{yy} - I_{xx})PQ + I_{xz}QR = N_A + N_T$$

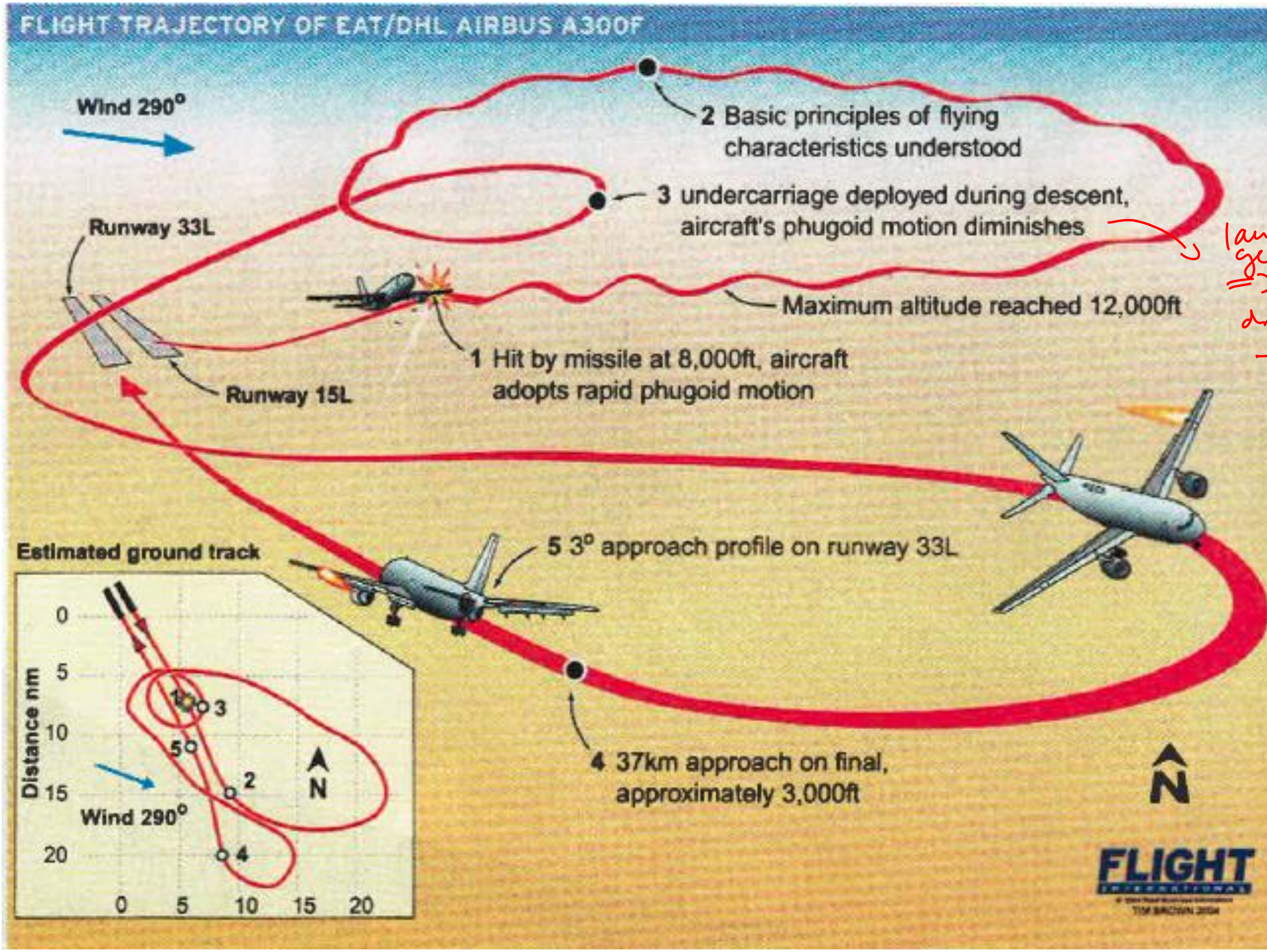


2 Longitudinal Modes



3 Lateral-Directional Modes

DHL Airbus A300F



landing gear → lots of drag → fast half life

Approximation for the Phugoid: $\alpha = \text{const}$

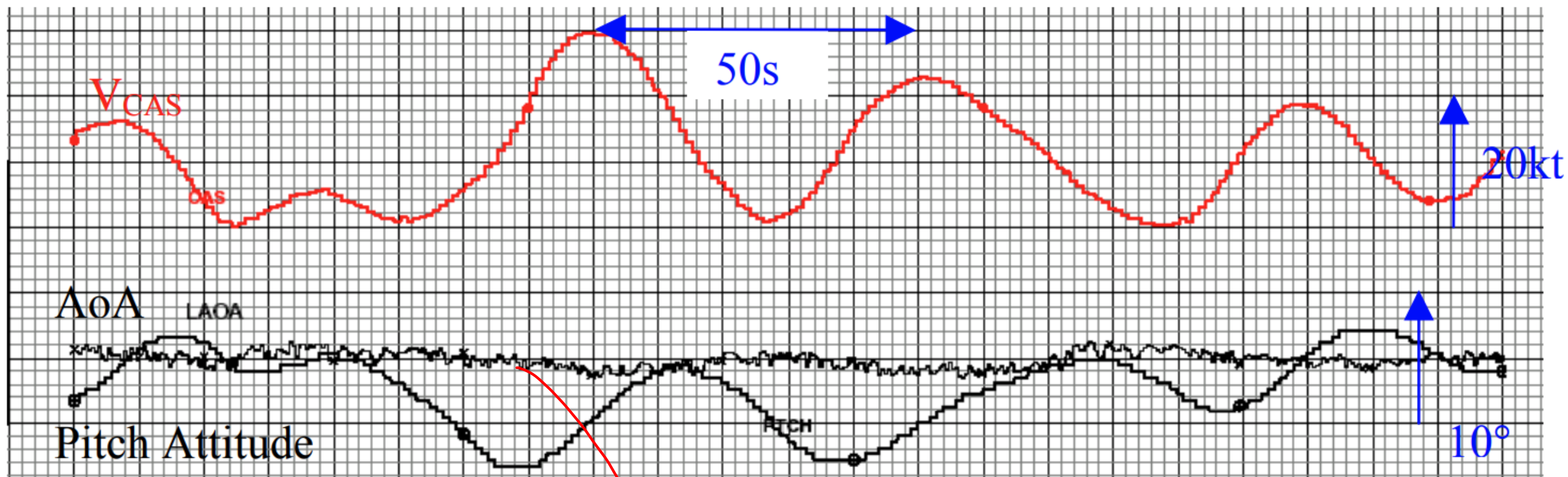


Figure 7. Phugoïde during Baghdad flight. DFDR Data.

Source: Lemaignan, B. (2005, August). Flying with no flight controls: Handling qualities analyses of the baghdad event. In *AIAA Atmospheric Flight Mechanics Conference and Exhibit*

AoA ($\approx$ constant)
=> our assumption of α= const. for phugoid ✓

$$T = \frac{2\pi}{\omega} = \pi\sqrt{2}\frac{U_1}{g} \approx \frac{U_1}{2}$$

$$T_{1/2} = \frac{c_{L_1}}{c_{D_1}}\frac{U_1}{g} \ln 2$$

Gear down: L/D from 11 to 8

(A300F was @ ~ 100 m/s
=> 50s as in graph ✓)