Spring Semester 2023

AIRCRAFT AERODYNAMICS & FLIGHT MECHANICS

16.03.2023

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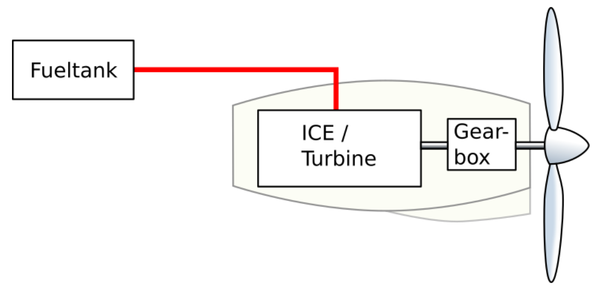
This lecture is adapted with permission from the lecture “Ausgewählte Kapitel der Flugtechnik” by Dr. Jürg Wildi

Recap

$$P_{avail} = (P_{engine,max} - \Delta P_{consumers})\,\eta_p \eta_{inst}$$



$$P_{use} = TV_0$$

Propeller momentum theory

⇨ Induced velocity and induced power

$$w = -\frac{V_0}{2} + \sqrt{\left(\frac{V_0}{2}\right)^2 + \frac{1}{2\rho}\frac{T}{A}}$$

$$P = T(V_0 + w)$$

$$P_{use} = TV_0 \qquad P_{ind} = Tw$$

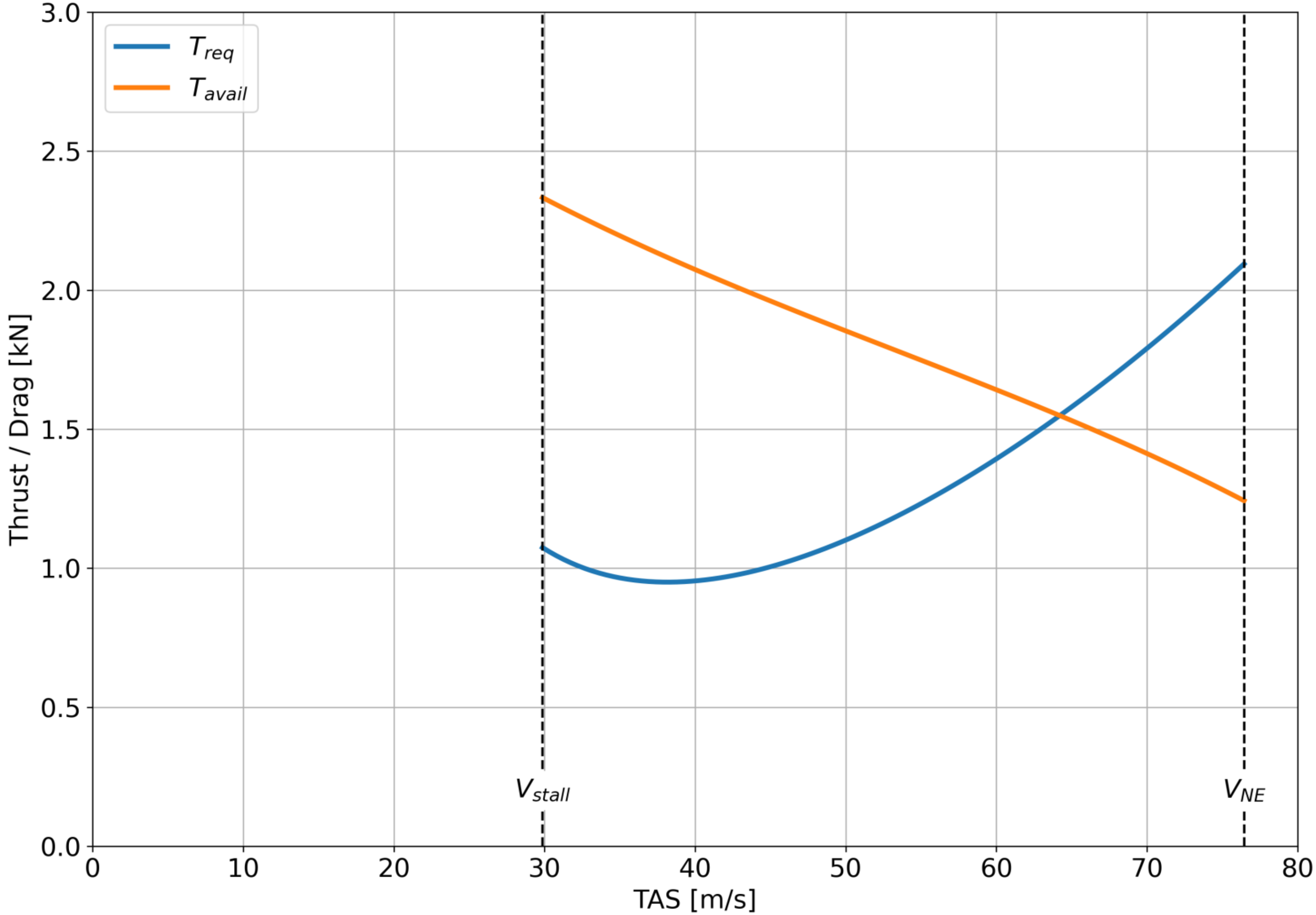
$$\eta_i = \frac{2}{1 + \sqrt{1 + \frac{T}{\frac{\rho}{2} V_0^2 A}}}$$

Universal Hydrogen Dash-8 - March 02, 2023

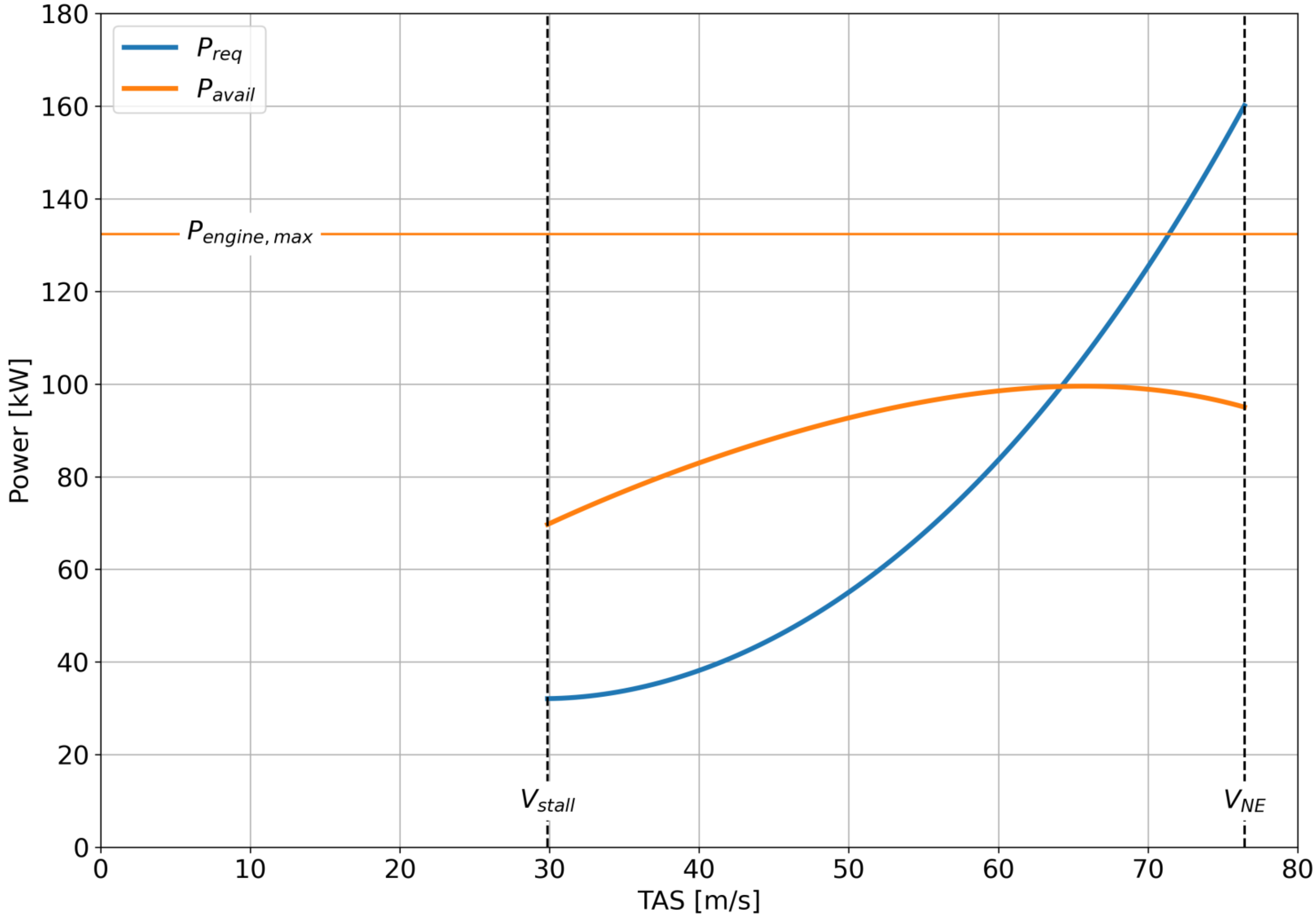
ZeroAvia Dornier228 - January 19, 2023

Exercise 3 – Sea Level Performance

Exercise 3 - Solution



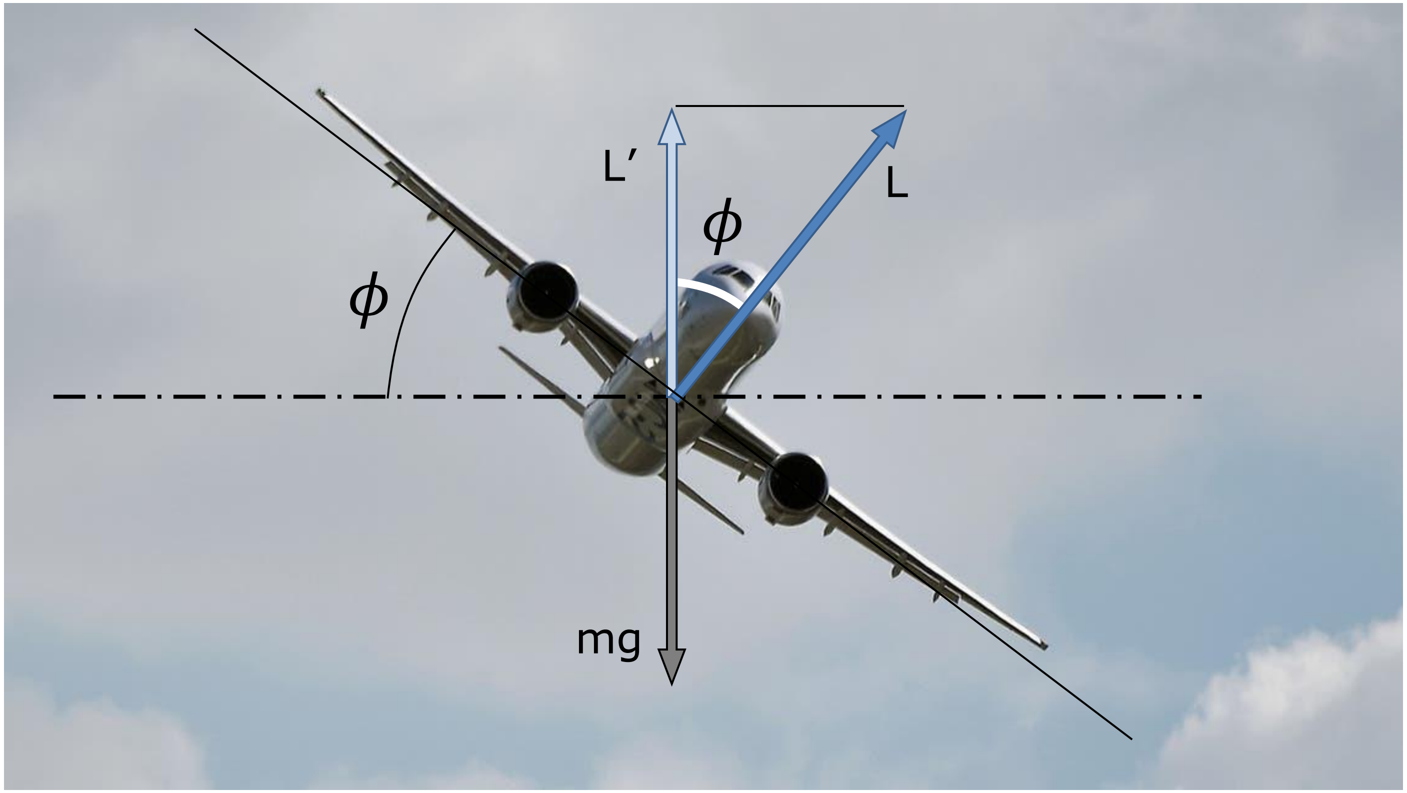
Exercise 3 - Solution



Maneuver Performance

Geometry of a Level Turn

use ailerons

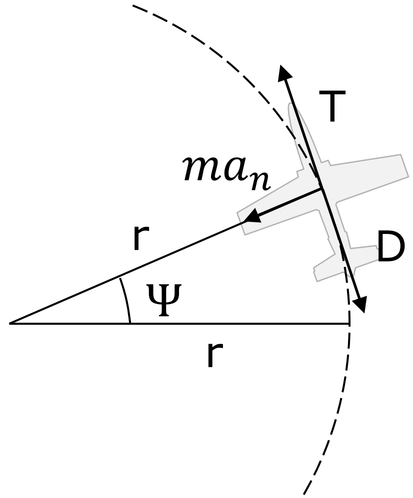


Bank Angle ϕ

Load Factor $n = \dfrac{L}{mg}$ $\qquad cos\phi = \dfrac{L'}{L} = \dfrac{mg}{L} \Rightarrow n = \dfrac{1}{cos\phi}$

Bank angle closely related to load factor

Equations of Motion – Stationary Level Turn



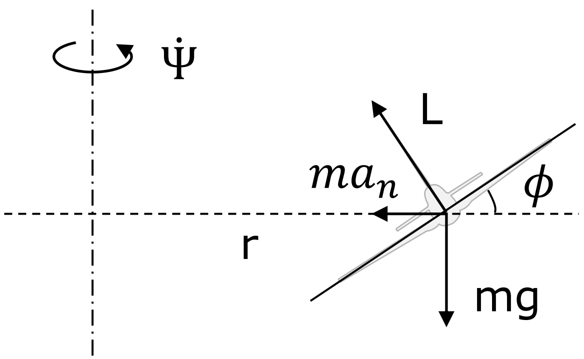
$V = const$ $\gamma = 0$

$r = const$

$T = D$ In flight-path direction

$L\,cos\phi = mg$ Vertical to the flight path

$L\,sin\phi = ma_n$ Normal (radial direction)

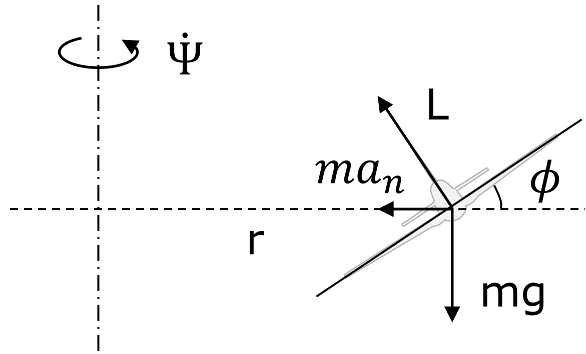


Centripetal acceleration

$$a_n = \frac{V^2}{r} = V\dot{\Psi}$$

$\frac{V}{r}$

Turn-Rate and Turn-Radius



Load-factor

$$n = \frac{L}{mg} = \frac{1}{cos\phi}$$ bank angle

with

$$tan\phi = \frac{ma_n}{mg} = \frac{V\dot{\Psi}}{g}$$

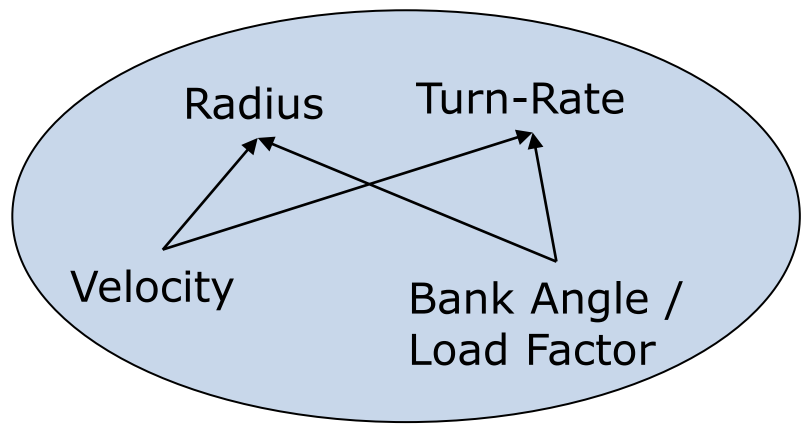
EoM in radial direction

$$L\ sin\phi = ma_n \qquad \text{with} \qquad a_n = \frac{V^2}{r}$$

Turn-rate

$$\dot{\Psi} = \frac{g\ tan\phi}{V} \quad [rad/s]$$

$$= \frac{g\ tan\phi}{V}\frac{180}{\pi} \quad [°/s]$$

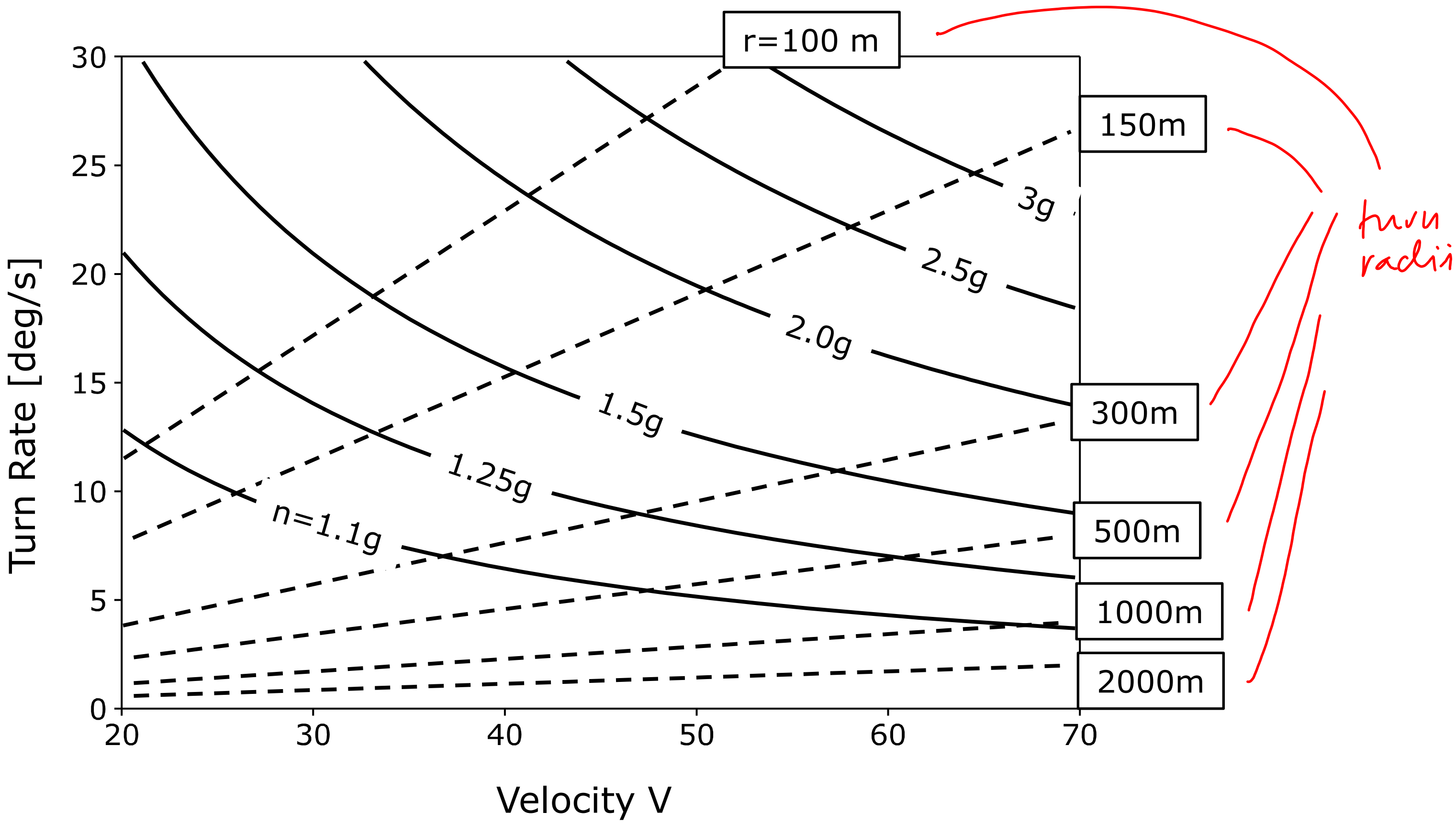


Turn-radius

$$r = \frac{mV^2}{L\ sin\phi} = \frac{mV^2}{\frac{mg}{cos\phi}\ sin\phi} = \frac{V^2}{g\ tan\phi} = \frac{V^2}{gn\ sin\phi} = \frac{V^2}{g\sqrt{n^2-1}}$$

Load-factor / Turn Radius Grid

Only geometry. Therefore does not depend on the aircraft and the atmosphere



Does not show if an aircraft is actually capable of executing a turn

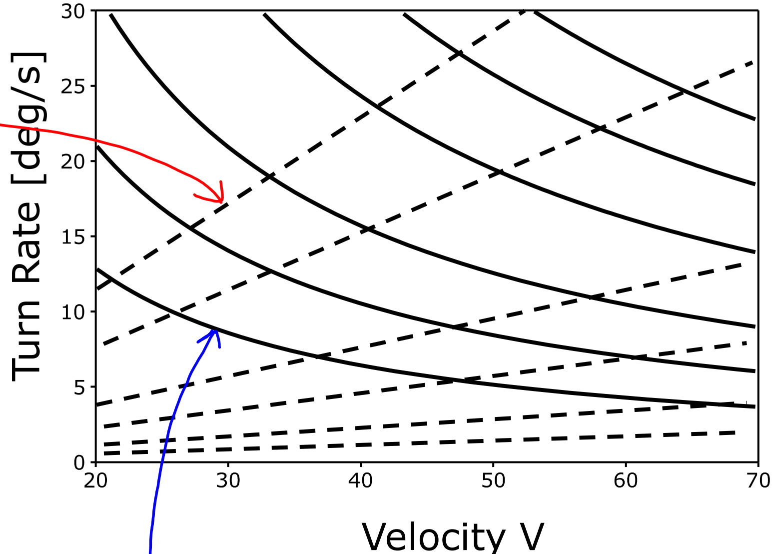
as the eqns. we didn't use aerodynamics ; only geometry

Load-factor / Turn Radius Grid

Turn at constant radius (turn radius grid)

$$\boxed{\dot{\Psi} = \frac{V}{r}} \qquad r = const$$

Note: radius lines originate from (0,0)



Turn at constant load factor
(load factor grid)

$$\dot{\Psi} = \frac{g\, tan\phi}{V} = \frac{ng\, sin\phi}{V} = \frac{ng\sqrt{1-\frac{1}{n^2}}}{V} = \frac{g}{V}\sqrt{n^2-1}$$

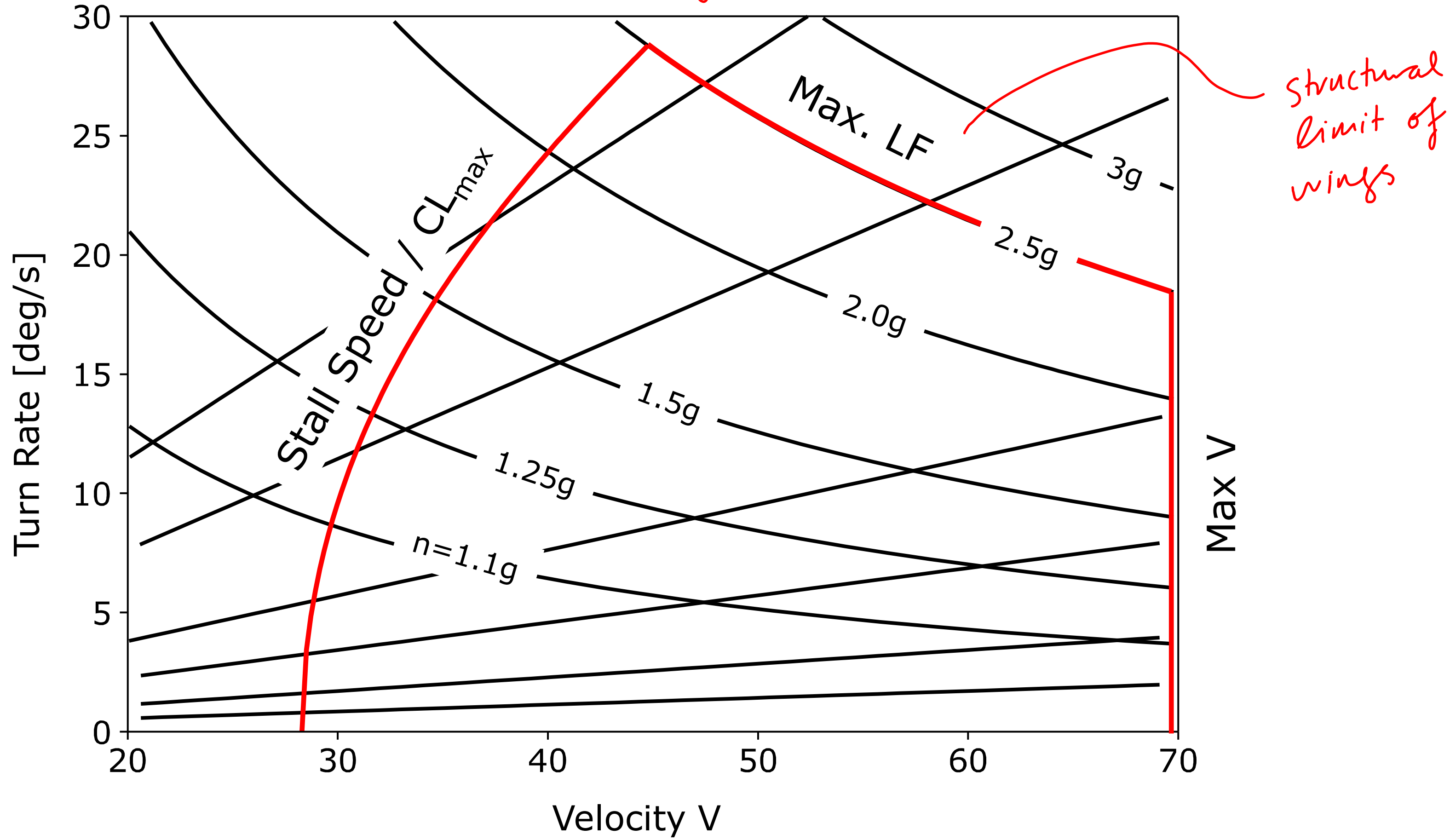
$$n = const \qquad \boxed{\dot{\Psi} = \frac{g}{V}\sqrt{n^2-1}}$$

$$n = \frac{1}{cos\phi}$$

$$sin\phi = \sqrt{1-\cos^2\phi} = \sqrt{1-\frac{1}{n^2}}$$

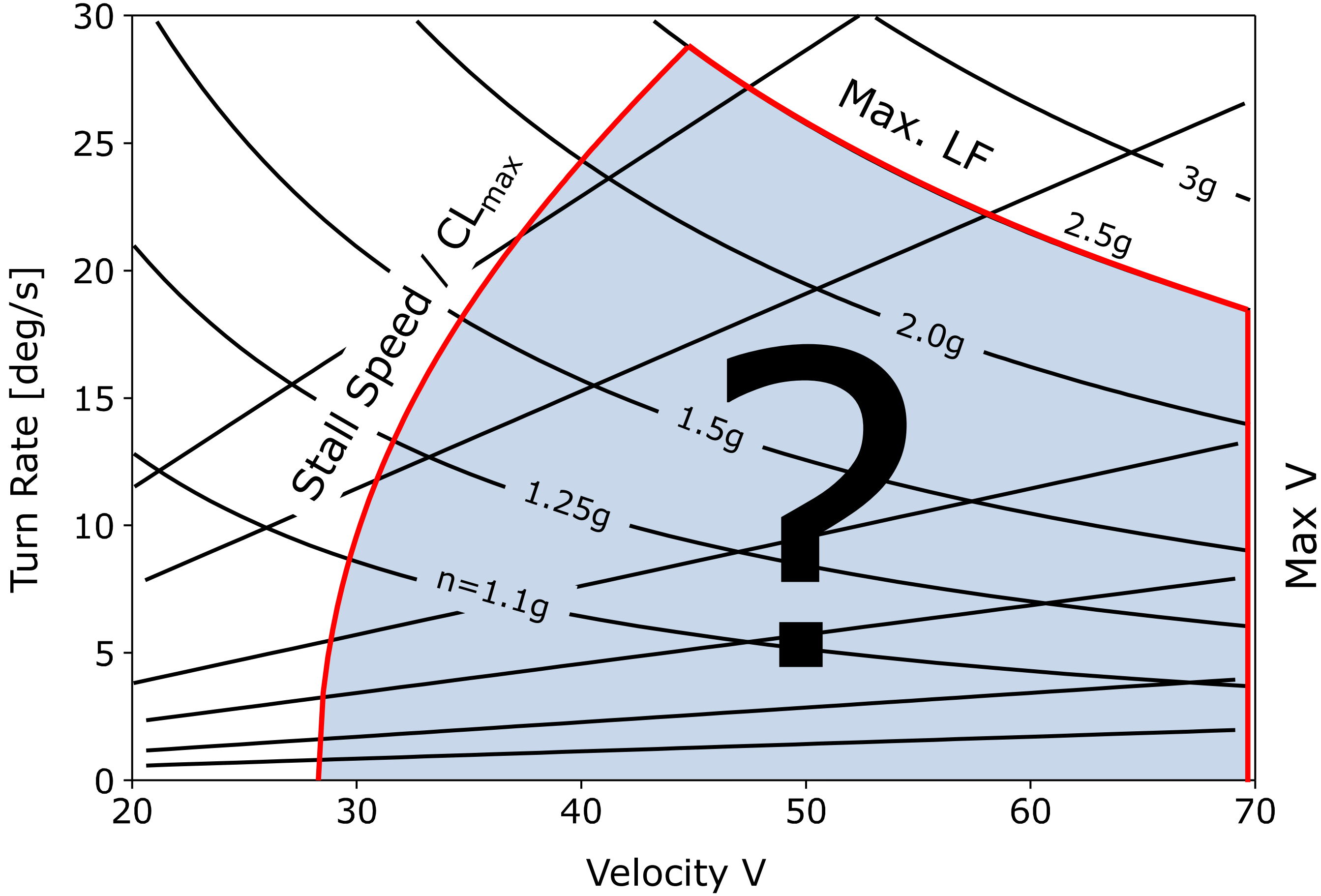
Limits

add limits of airplane



$$L_{max} = \frac{1}{2}\rho V^2 S_{ref} c_{Lmax} = n\, mg \qquad V_{stall} = \sqrt{\frac{2mg\, n}{\rho c_{Lmax} S_{ref}}} \qquad \boxed{\dot{\Psi} = \frac{180}{\pi}\frac{g\sqrt{n^2-1}}{V_{stall}}}$$

Maneuver Performance



Maneuver performance inside the limits can be calculated using the EoM, the thrust and the aerodynamic characteristics

$D = f(V, n)$ Sustained turn: $\frac{dV}{dt} = 0 \quad T_{max} = D$

stay @ same altitude

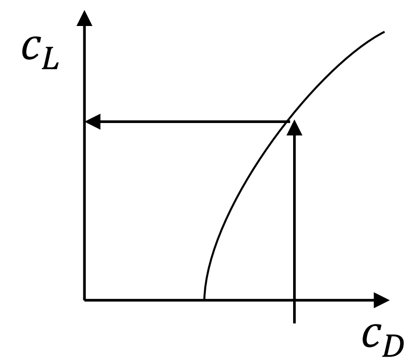
Sustained Level Turn

Calculation procedure (for h= … m)

valid for a specific altitude

- Calculate level V_{max}
- For each $V_i = V_{max} - i * \Delta V$
  - Get the maximum thrust $T_{max} = f(V_i)$
  - Calculate the drag coefficient with the condition $\mathrm{D} = T_{max}$

$$D = \frac{1}{2}\rho V_i^2 S_{ref} c_D \implies c_D = \frac{2T_{max}}{\rho V_i^2 S_{ref}}$$



  - From the drag polar, get the lift coefficient c_L that corresponds to the calculated c_D
  - If $c_L > c_{Lmax}$ stop calculation
  - From c_L calculate the load factor n

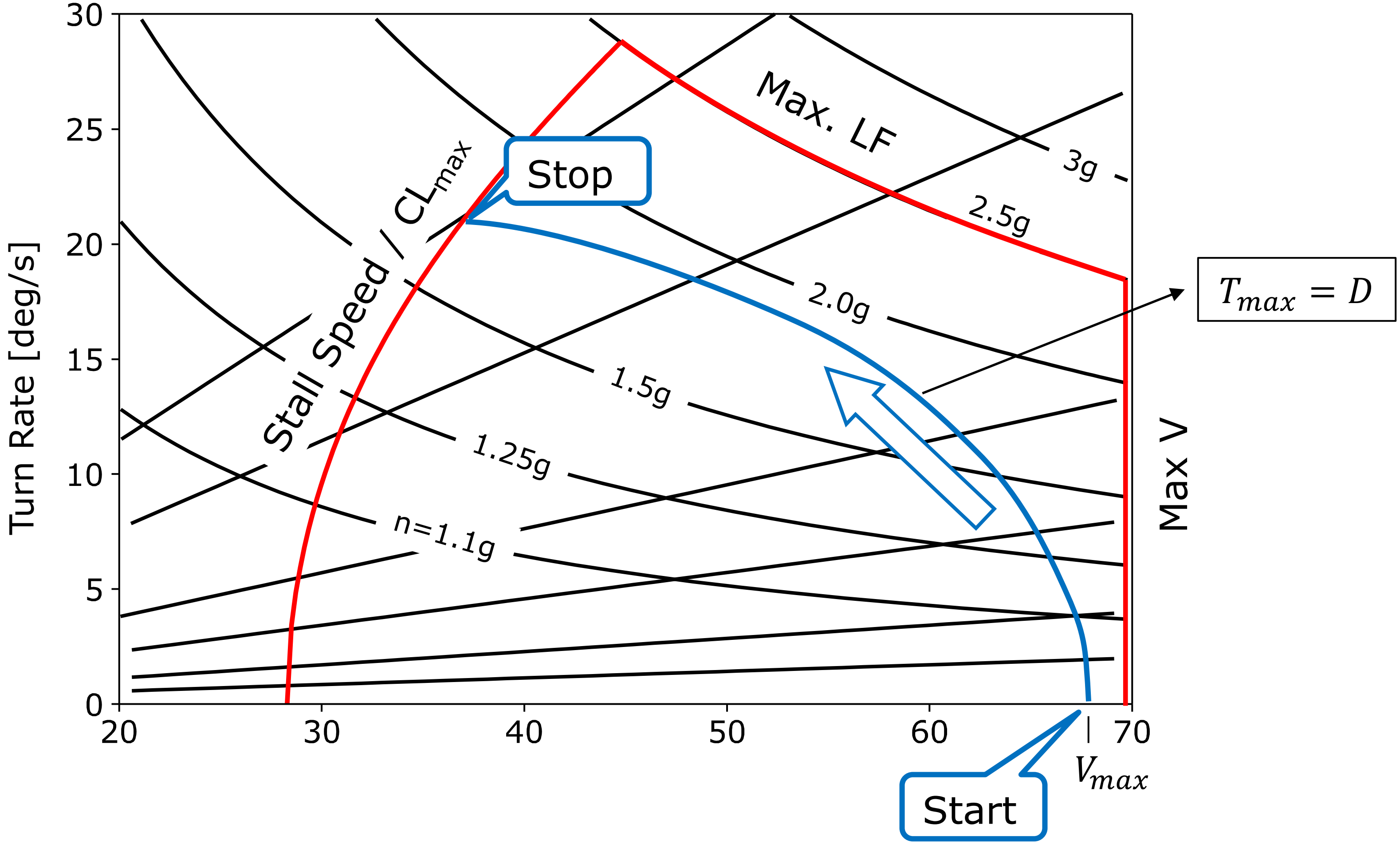
$$L = \frac{1}{2}\rho V^2 S_{ref} c_L \quad n = \frac{L}{mg}$$

  - If $n > 1$ calculate turn-rate $\dot{\Psi}$

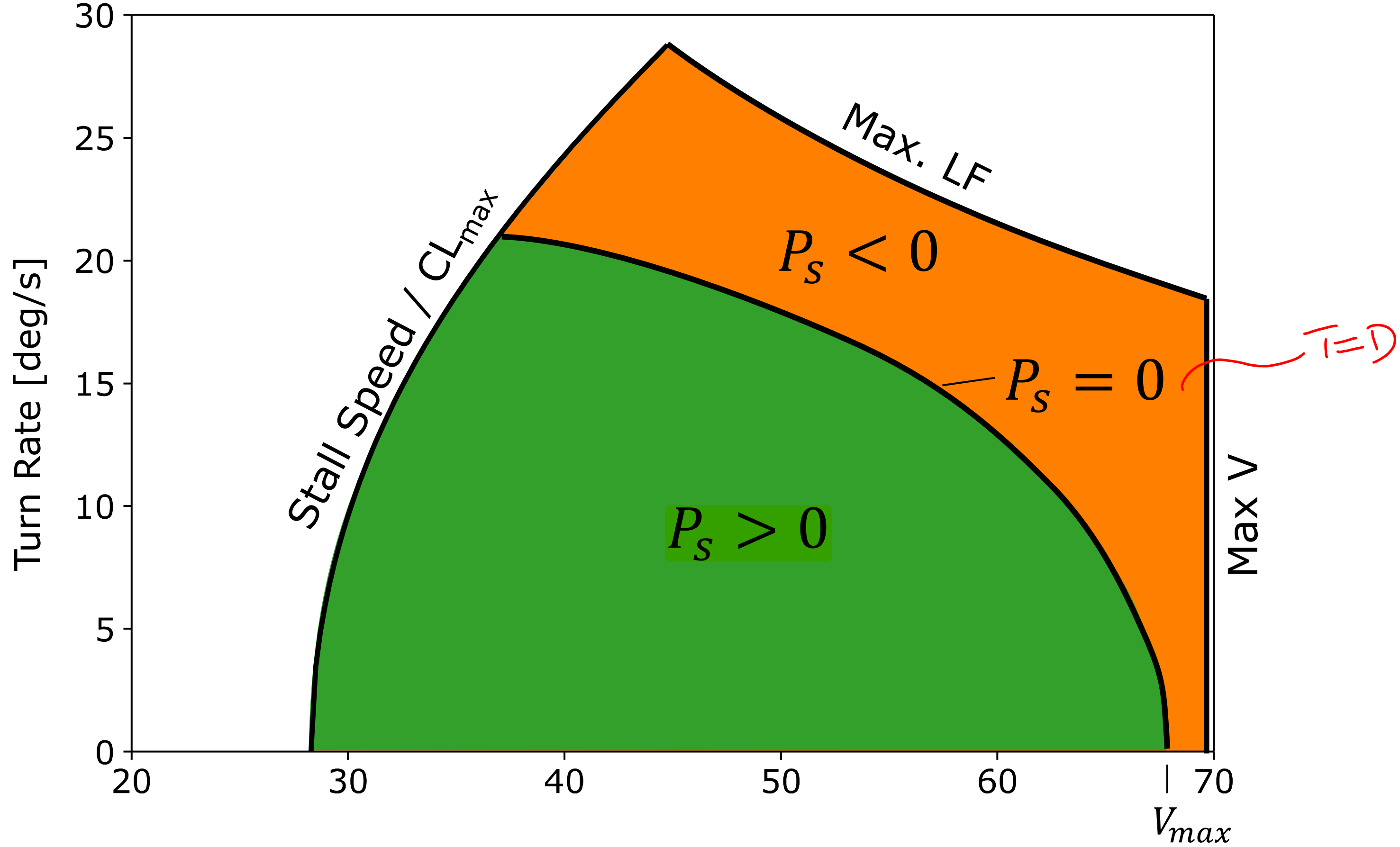
$$\dot{\Psi} = \frac{180}{\pi}\frac{\mathrm{g}\sqrt{n^2 - 1}}{V} \quad [°/s]$$

Plot the result together with the turn-rate and radius grid.
Repeat for different altitudes

Sustained Turn Performance



Turn Performance & SEP



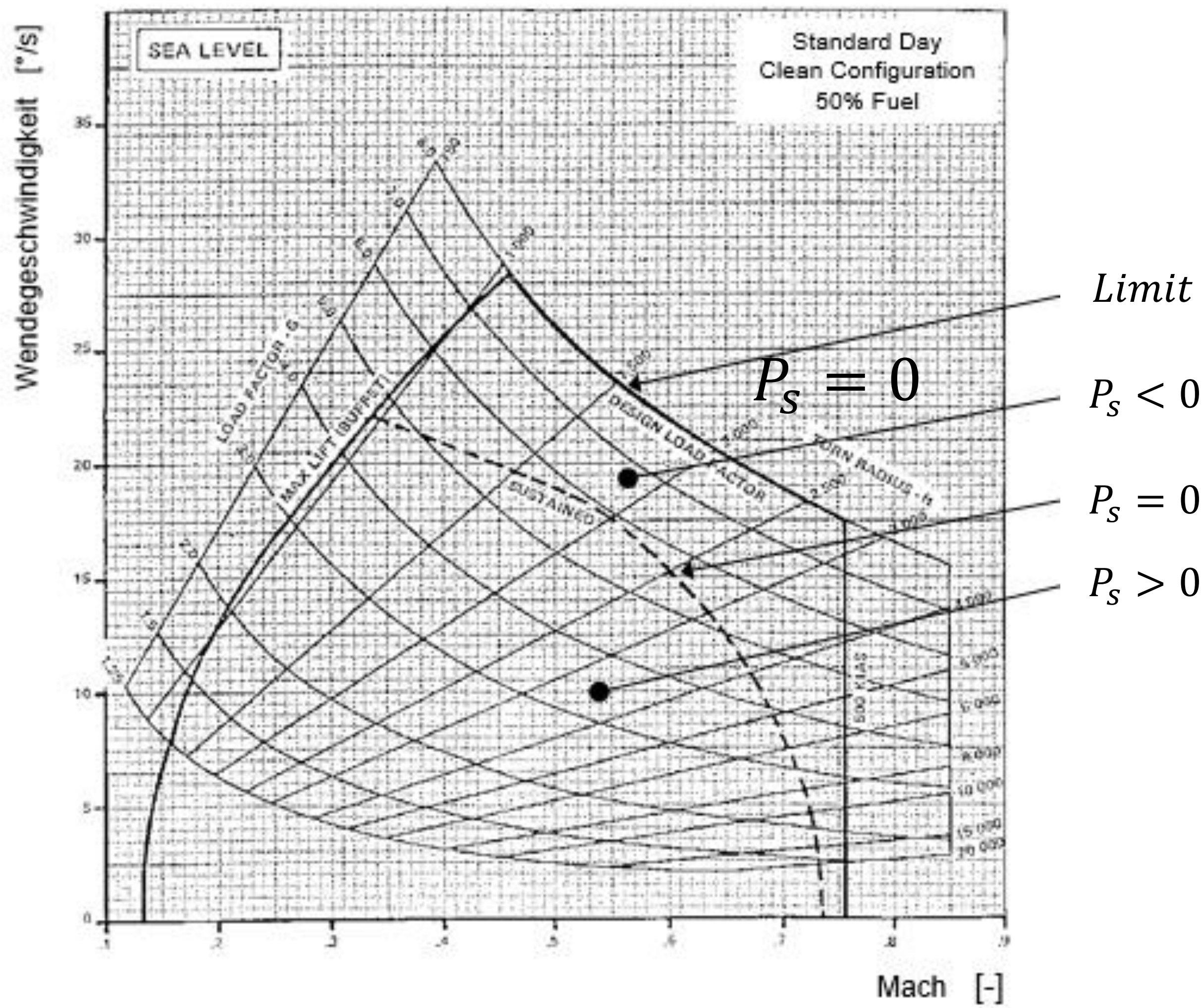
$$P_s = V\frac{(T-D)}{mg}$$

$P_s > 0$ $\quad T_{max} > D$

$P_s < 0$ $\quad D > T_{max}$

[or just calculate SEP & get this grid w/ contour lines!]

Example

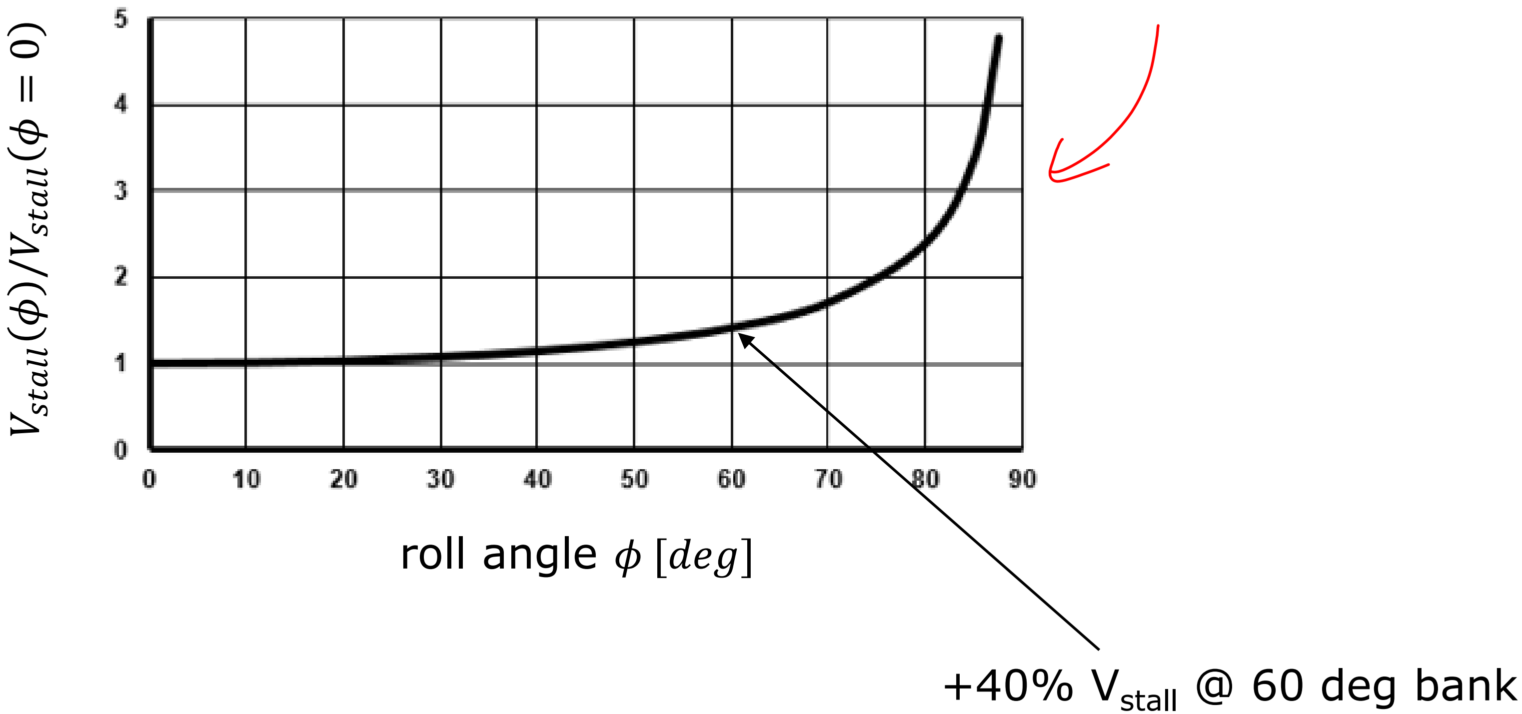

SEA LEVEL
Standard Day
Clean Configuration
50% Fuel
Wendegeschwindigkeit [°/s]
Mach [-]
LOAD FACTOR - G
MAX LIFT (BUFFET)
SUSTAINED
DESIGN LOAD FACTOR
TURN RADIUS - ft
Limit
$P_s = 0$
$P_s < 0$
$P_s = 0$
$P_s > 0$

Roll angle vs. stall speed (level turn)

$$L_{max} = \frac{1}{2}\rho V^2 S_{ref} c_{Lmax} = \frac{mg}{cos\phi}$$

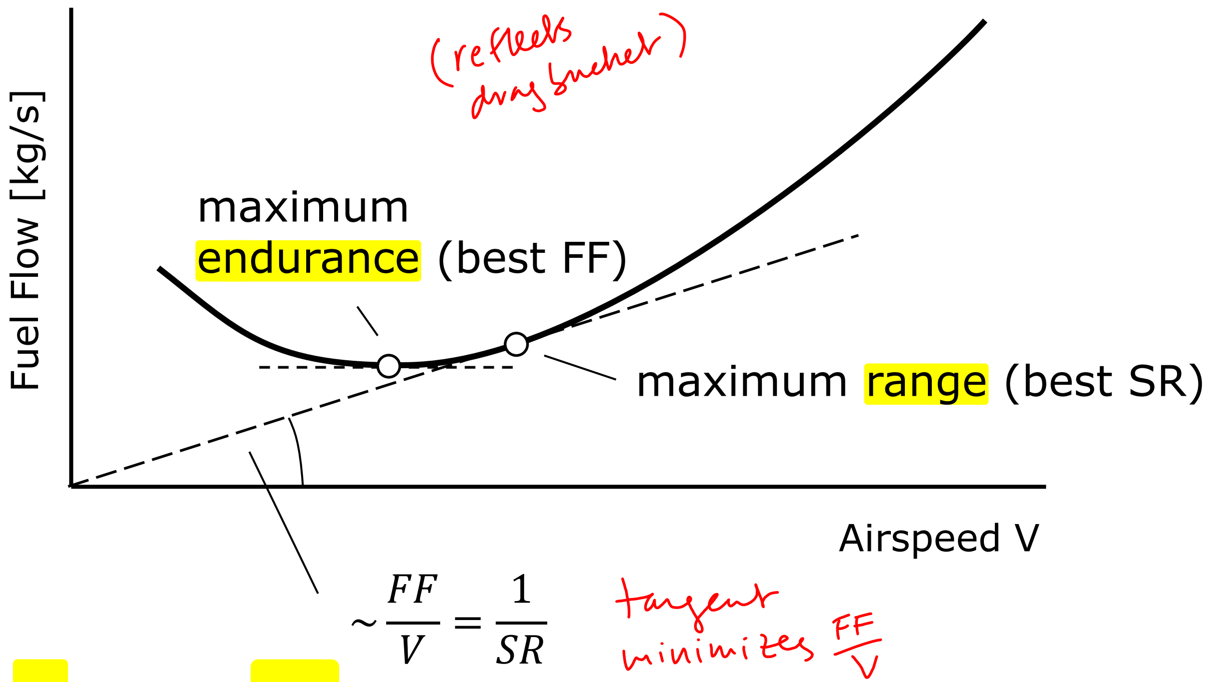
$$V_{stall}(\phi) = \sqrt{\frac{2mg}{\rho c_{Lmax} S_{ref} cos\phi}}$$

$$V_{stall}(\phi) = V_{stall}\sqrt{\frac{1}{\cos(\phi)}}$$



Range and Endurance

Fuel Flow



$$FF = \frac{dm_f}{dt} \quad \dot{m}_f$$

$$V = \frac{dR}{dt}$$

(no wind)

$$SR = \frac{dR}{dm_f} = \frac{dR}{m_f}\frac{dt}{dt} = \frac{V}{FF}$$

SR: Specific Range (or SAR, Specific Air Range)

$TSFC = \dfrac{FF}{T}$ Thrust specific fuel consumption $\Rightarrow$ Jet Propulsion

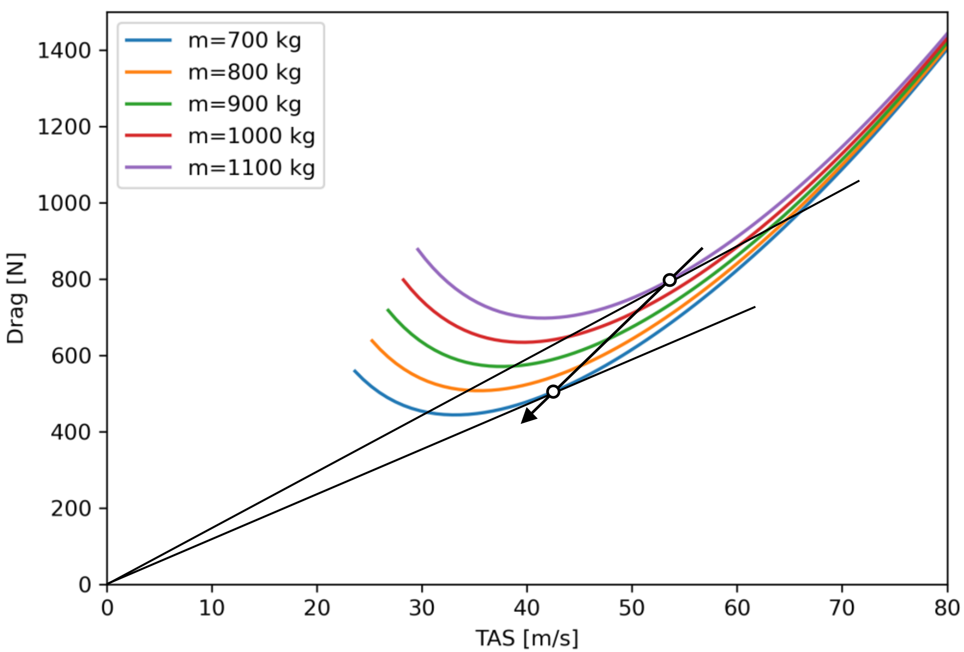
$BSFC = \dfrac{FF}{P}$ Brake-specific fuel consumption $\Rightarrow$ Propeller Propulsion

Cruise Drag

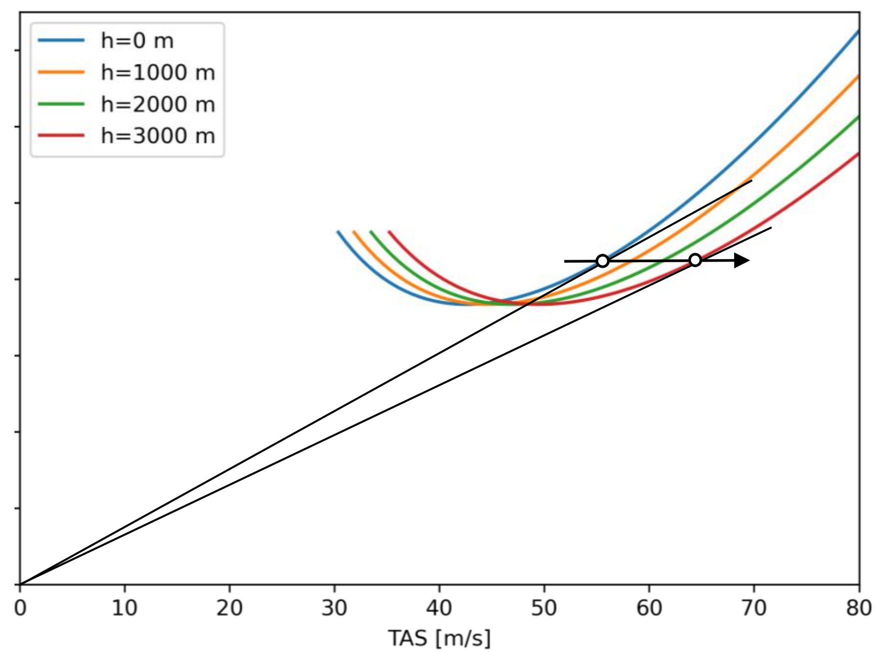
Jet Propulsion

mass ↓ ∴ ↑ altitude
↓ speed to counteract : ↑ altitude

Effect of mass on SR



Effect of altitude on SR



$$TSFC = \frac{FF}{T}$$ If $TSFC$=const., $FF \sim T$

(Note: TSFC ↑ w/ altitude as air gets cooler ⇒ ΔT ↑ in Brighton cycle)

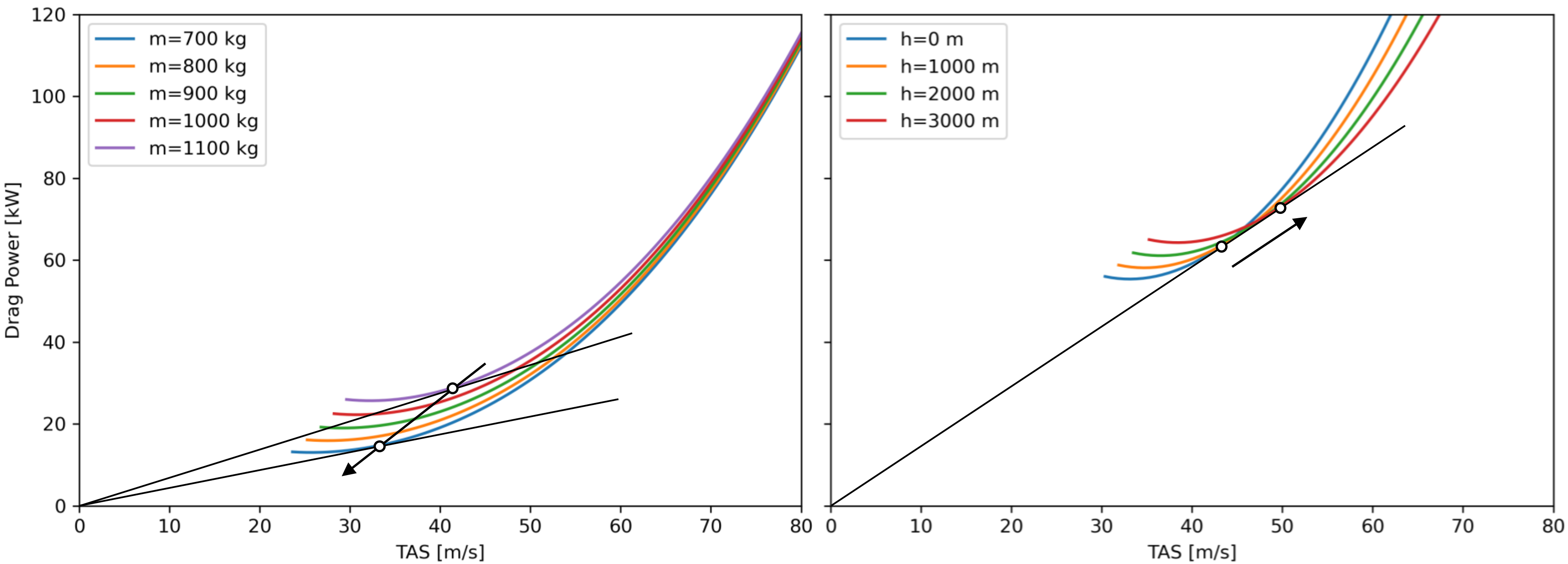
with $T = D$ $\Longrightarrow$ $SR_{max} = \left(\frac{V}{D}\right)_{max}$

- mass ↓ → drag ↓
- decrease V to stay at SR_{max} when getting lighter
- fly longer (SR ↑) by flying higher

Cruise Power

Propeller Propulsion $P = DV$

Effect of mass on SR Effect of altitude on SR



$BSFC = \frac{FF}{P}$ If $BSFC$=const., $FF \sim P$

$\Rightarrow$ $SR_{max} = \left(\frac{V}{P}\right)_{max}$

Endurance - Jet

$$E = \frac{m_f}{FF}$$

m_f — Fuel mass
FF — Fuel flow

$$E = \int_{t1}^{t2} dt = \int_{W1}^{W2} \frac{1}{dW/dt} dW \qquad \frac{dW}{dt} = -\mathrm{FF}\ \mathrm{g}$$

$$TSFC = \frac{FF}{T}$$ Thrust specific fuel consumption

$$\frac{dW}{dt} = -TSFC\ g\ T$$

Unit of TSFC $\left[\frac{kg}{s}\frac{1}{N}\right]$!

$$E = \int_{W1}^{W2} \frac{1}{dW/dt} dW = \int_{W1}^{W2} \frac{1}{-TSFC\ g\ T} dW \quad \Big| \quad \frac{W}{W}$$

$$\int_a^b dx = -\int_b^a dx$$

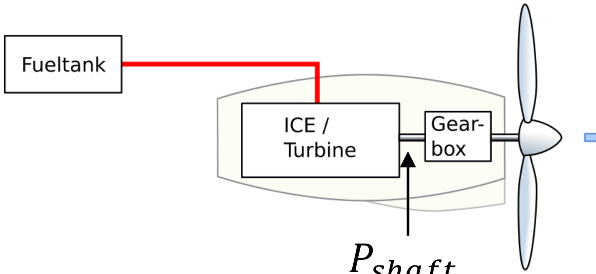
$T = D \qquad L = W$ level cruise

$$E = \int_{W2}^{W1} \frac{1}{TSFC\ g} \frac{L}{D} \frac{1}{W} dW$$ if $\frac{L}{D}$ and $TSFC$ are assumed constant

$$\Rightarrow \quad E = \frac{1}{TSFC\ g} \frac{L}{D} \ln\left[\frac{W_1}{W_2}\right]$$ good starting point

Endurance - Prop

$$BSFC = \frac{FF}{P_{shaft}} \qquad DV = P_{shaft}\,\eta_{prop}$$



$$\frac{BSFC\ DV}{\eta_{prop}} = FF = -\frac{dW}{dt}\frac{1}{g}$$

$$E = \int_{W1}^{W2} \frac{1}{dW/dt}\,dW = \int_{W1}^{W2} \frac{\eta_{prop}}{-BSFC\ g\ DV}\,dW$$

$$E = \int_{W2}^{W1} \frac{\eta_{prop}\,W}{BSFC\ g\ DV\ W}\,dW = \int_{W2}^{W1} \frac{\eta_{prop}}{BSFC\ g\ V}\frac{L}{D}\frac{1}{W}\,dW$$

if $\frac{L}{D}$ and $BSFC$ are assumed constant & V = const.

$$\Rightarrow \quad E = \frac{\eta_{prop}}{BSFC\ g}\frac{1}{V}\frac{L}{D}\ln\left[\frac{W_1}{W_2}\right]$$

$V \downarrow$ for higher endurance!

Range

(distance vs. endurance : time)

$$SR = \frac{dR}{dW_{fuel}}$$

Specific range [km/kg]

$$SR = \frac{dR}{dt}\frac{dt}{dW_{fuel}} = \frac{V}{dW/dt}$$

$$R = \int_{W1}^{W2} \frac{V}{dW/dt} dW$$

$$R = \int_{W1}^{W2} \frac{V}{dW/dt} dW = -\int_{W1}^{W2} \frac{V}{TSFC\ g\ T} dW$$

$$\frac{dW}{dt} = -TSFC\ g\ T$$

$$T = D$$

$$L = W$$

$$R = \int_{W2}^{W1} \frac{V}{TSFC\ g} \frac{L}{D} \frac{1}{W}\ dW$$

$$\boxed{R = \frac{V}{TSFC\ g} \frac{L}{D} \ln\left[\frac{W_1}{W_2}\right]}$$

Breguet Range Equation

For prop:

$$R = \frac{\eta_{prop}}{BSFC\ g} \frac{L}{D} \ln\left[\frac{W_1}{W_2}\right]$$

Assumptions

In order to integrate the endurance or range equation

$$R = \int_{W2}^{W1} \frac{V}{TSFC\ g} \frac{L}{D} \frac{1}{W}\, dW$$

we needed to set some assumptions:

$V = const.$ $\quad L/D = const. \Longrightarrow c_L = const.$ (drag polar: CL/CD depends on CL)

$TSFC = const.$

Cruise Flight Schedule

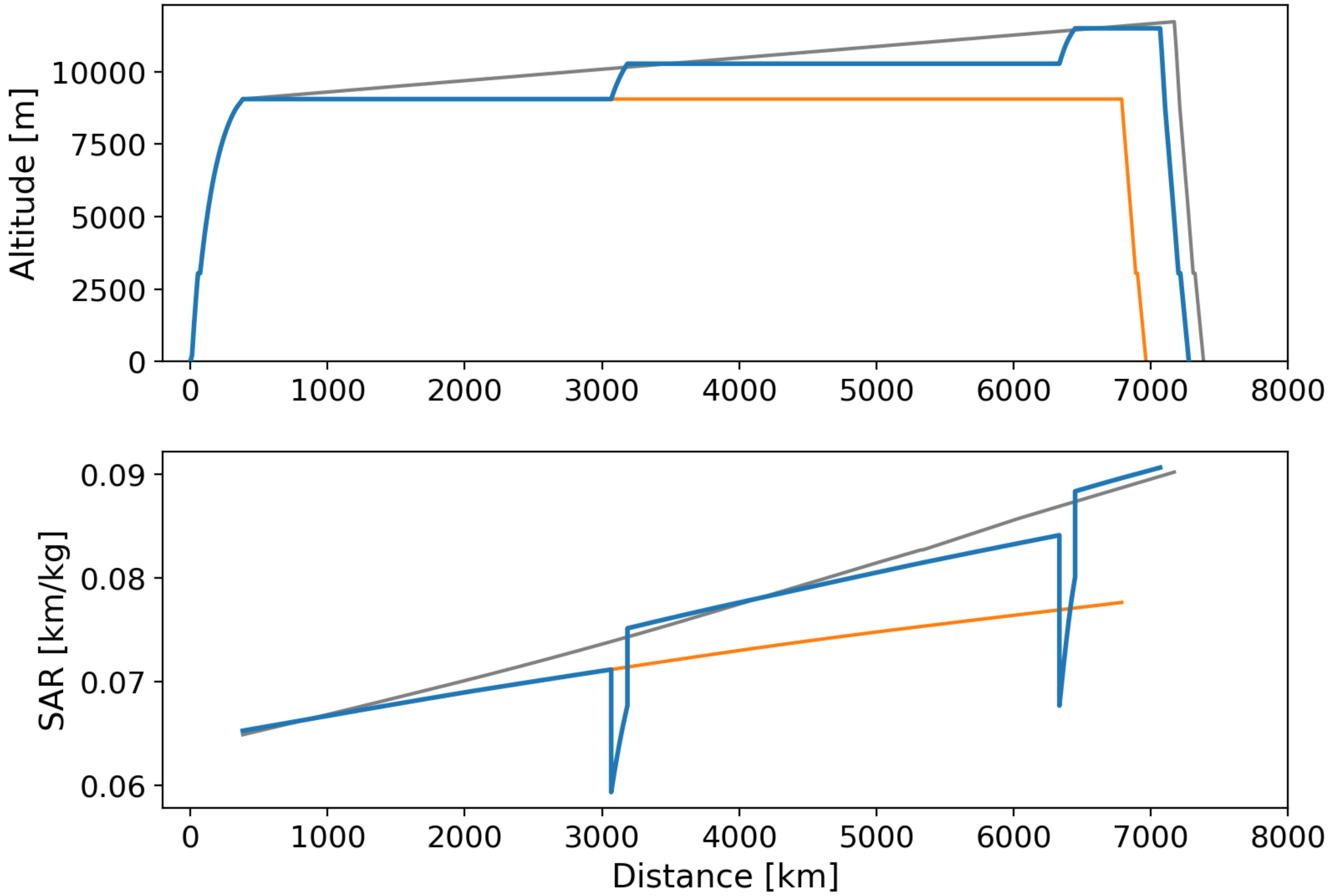
From L=mg $\quad V = \sqrt{\dfrac{mg}{\rho S_{ref} c_L}}$ ← $const.$

As mass decreases, density needs to decrease: required climbing

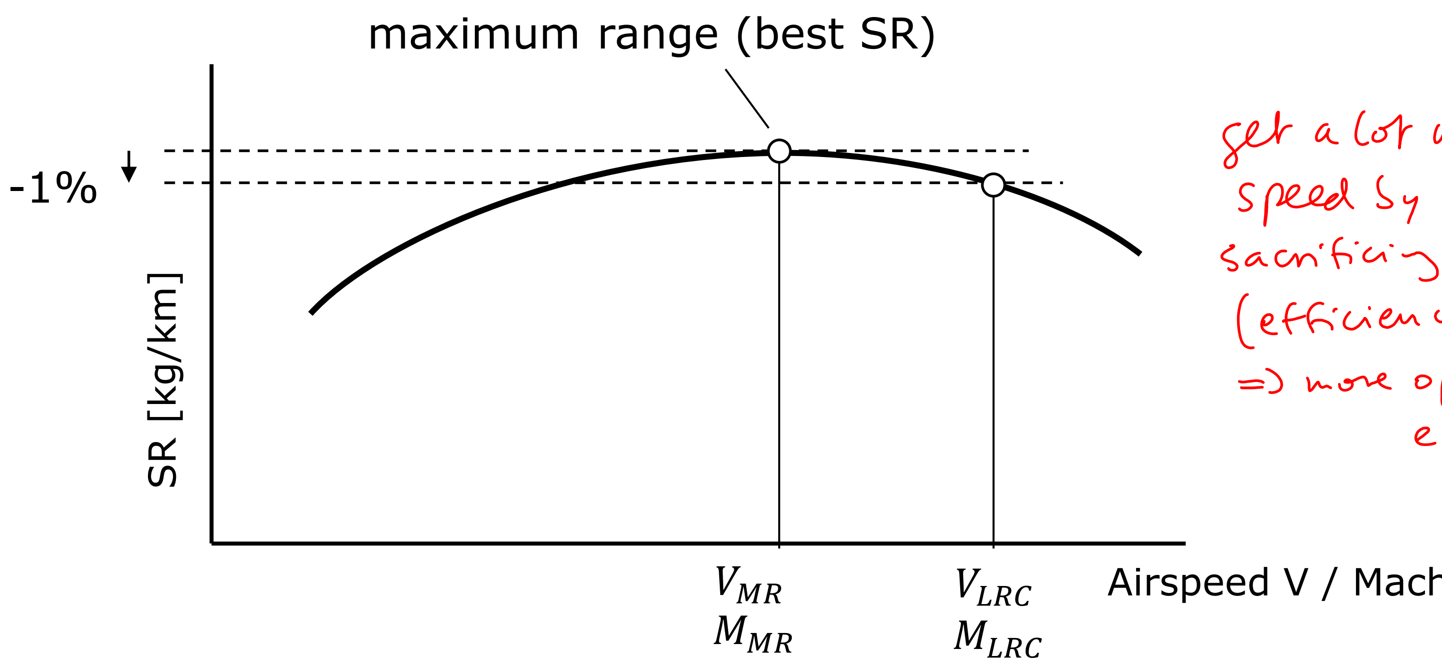
→ the assumptions require climbing !

Cruise Schedule Example

Boeing 747-100 Cruise SR



LRC – Long Range Cruise



get a lot more speed by sacrificing 1% SR (efficiency)
=> more operationally economic

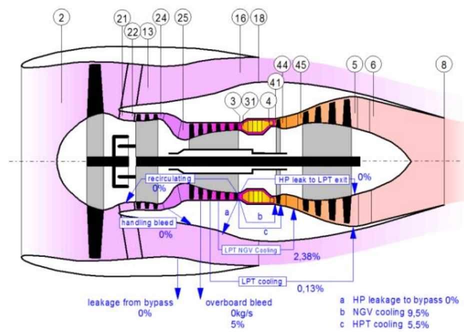
MR – Maximum Range Cruise

LRC – Long Range Cruise 99% of maximum SR

TSFC – Engine Deck

In reality:

$$TSFC = f(h, M, T)$$

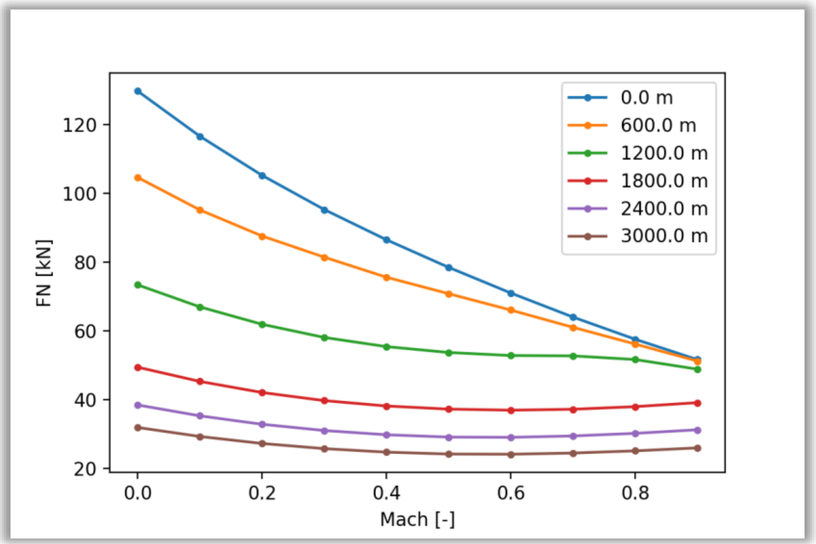


Gas cycle computations can provide full engine decks

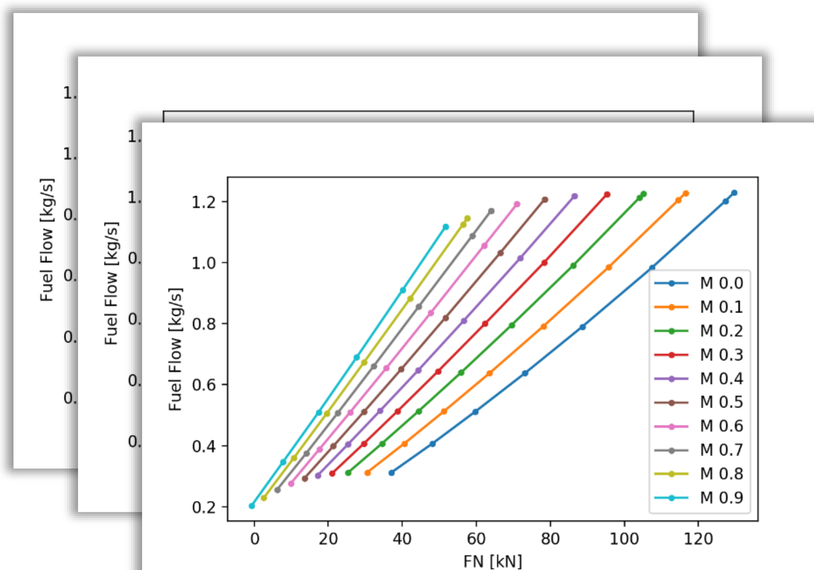
(e.g GasTurb, *NPSS*, etc..)

German American citizens only

Max. thrust table



Fuel Flow Tables for each altitude and thrust



Mission Performance Methods

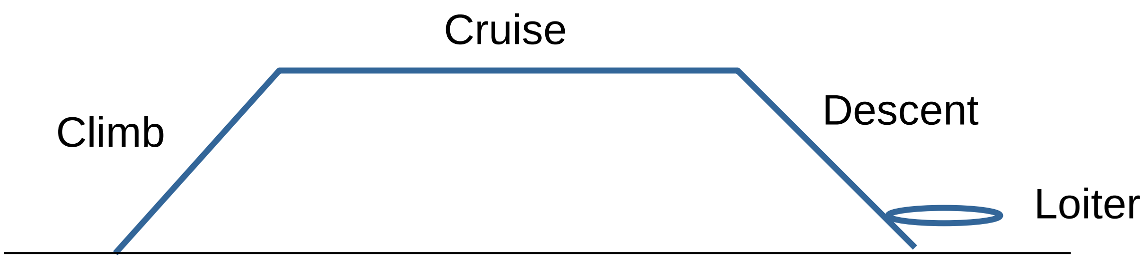
lots of assumptions => know when it breaks down!

fidelity ↑ ↓

Analytic	▪ Integration of eq. of motion: Breguet range equation ▪ Fast and simple ▪ Only works for constant conditions (e.g. constant CL cruise, simplified engine characteristics)
2D Point-Mass needs engine deck	▪ Numerical integration of linear momentum equation ▪ Neglect rotational dynamics ▪ Segmented mission ▪ Dataset of trimmed polars (aerodynamics) and installed thrust / fuel flow
6-DOF needs a lot of data	▪ Numerical integration of momentum and angular momentum equations ▪ Requires moments of inertia ▪ Requires definition of control surfaces and control concept ▪ Three dimensional trajectories ▪ Allows for the simulation of sub-systems

Mission Performance

Elements of a mission profile:
- Climb
- Cruise
- Descent
- Loiter (Holding)



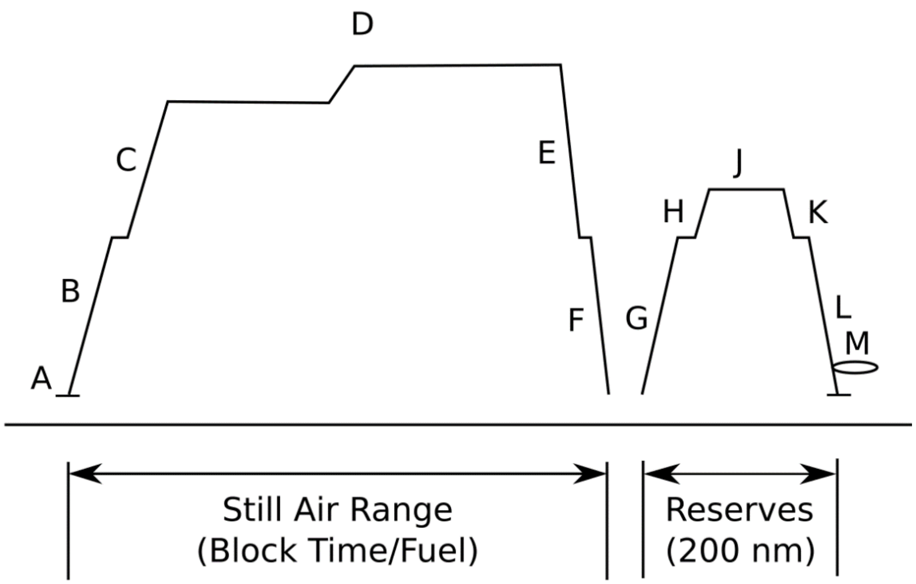
Results of interest:
- Maximum range *or* required fuel

Parameters:
- Climb Speed
- Cruise Altitude
- Cruise Speed
- Descent Speed
- Loiter Speed

Note: the fuel required for take-off or landing is very small compared to the overall mission fuel

Mission Performance

A: Ground Op. & Takeoff
B: Climb (250 KCAS to 10 kft)
C: Climb unrestricted to ICA
D: Cruise
E: Descent unrestricted to 10 kft
F: Descent at 250 KCAS



Same procedure for reserves (G-L)

Final reserve (M): 30 minutes loiter

KCAS: Knots Calibrated Air Speed
Knots: nautical miles per hour (1 nm = 1.852 km)
ICA: Initial Cruise Altitude

$\left(\text{knots} = \frac{\text{nautical miles } (1.852\text{ km})}{\text{h}} \right)$

Long-Haul vs Short-Haul

Boeing 747-100

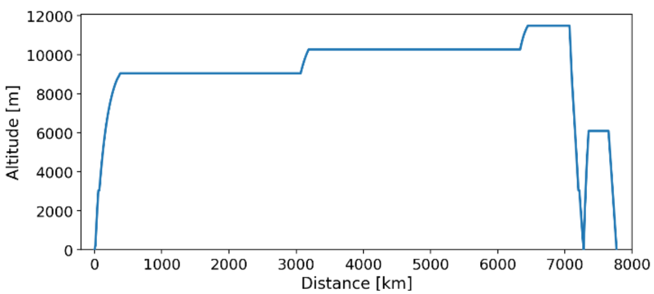
TOM: 330.2 t
Payload: 45.9 t
Fuel: 121.95 t

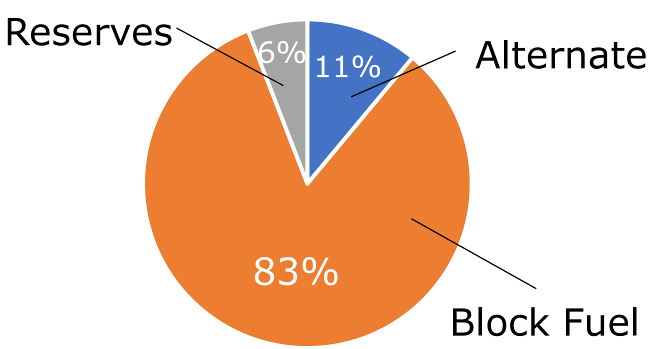
~ excludes loiter reserves

Block Fuel: 101.3 t
Block Range: 7'280 km

Cruise SR
Start 0.0652 km/kg
End 0.0906 km/kg

↑ efficiency





Airbus A320

Note: $\frac{km}{kg\ fuel}$

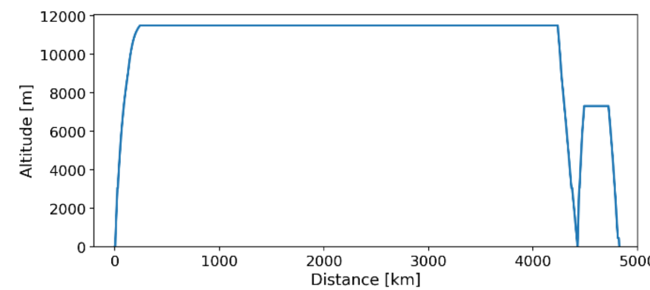
can't be compared directly as it doesn't take the payload carried into account!

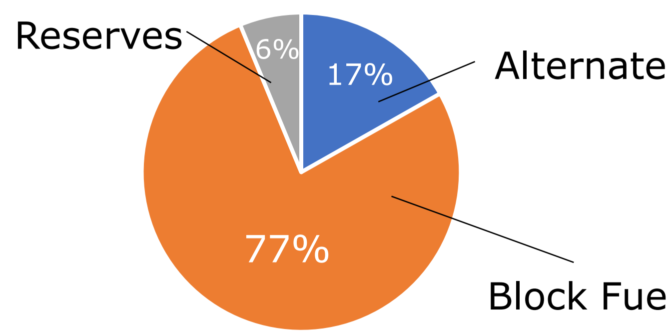
TOM: 71.3 t
Payload: 10 t
Fuel: 17.18 t

Block Fuel: 13.2 t
Block Range: 4'430 km

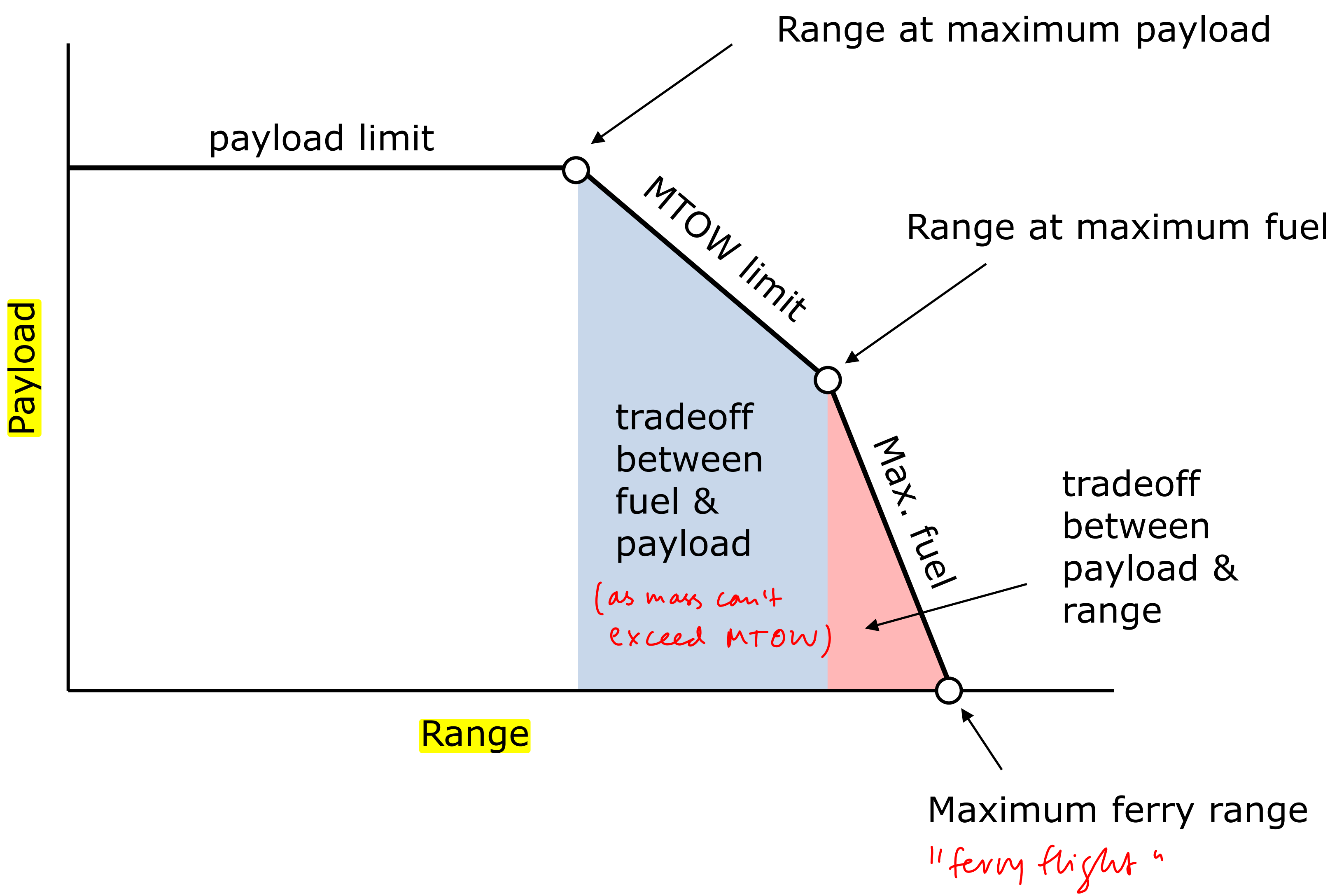
Cruise SR
Start 0.334 km/kg
End 0.381 km/kg

not a big change => no step in mission





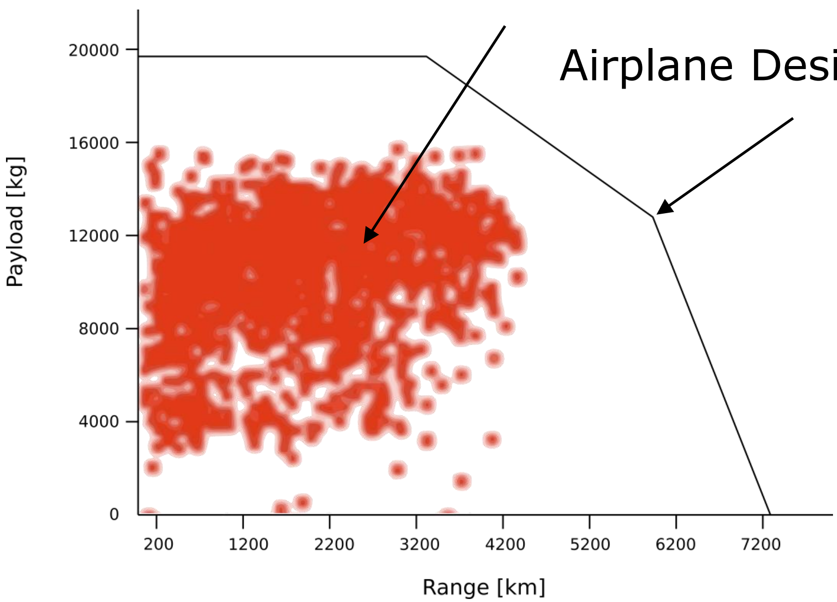
Payload-Range Chart



Example US domestic flights for a representative airplane type (737 / A320)

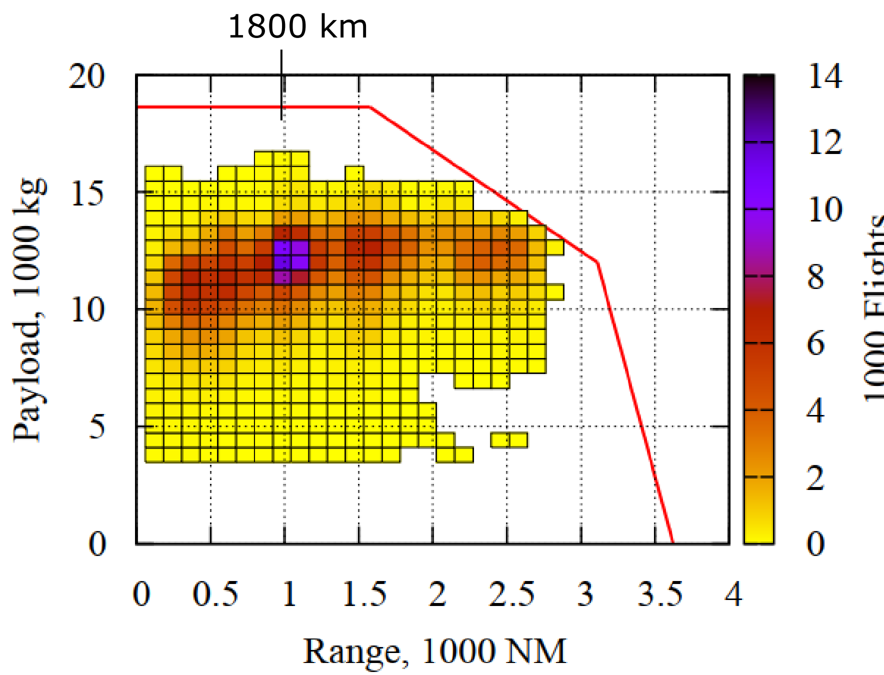
Actual Missions

Airplane Design Mission



Adapted after: Trends in Aircraft Efficiency and Design Parameters - Zeinali, M. and Rutherford, D. Data: US Domestic flights, 2006

A320

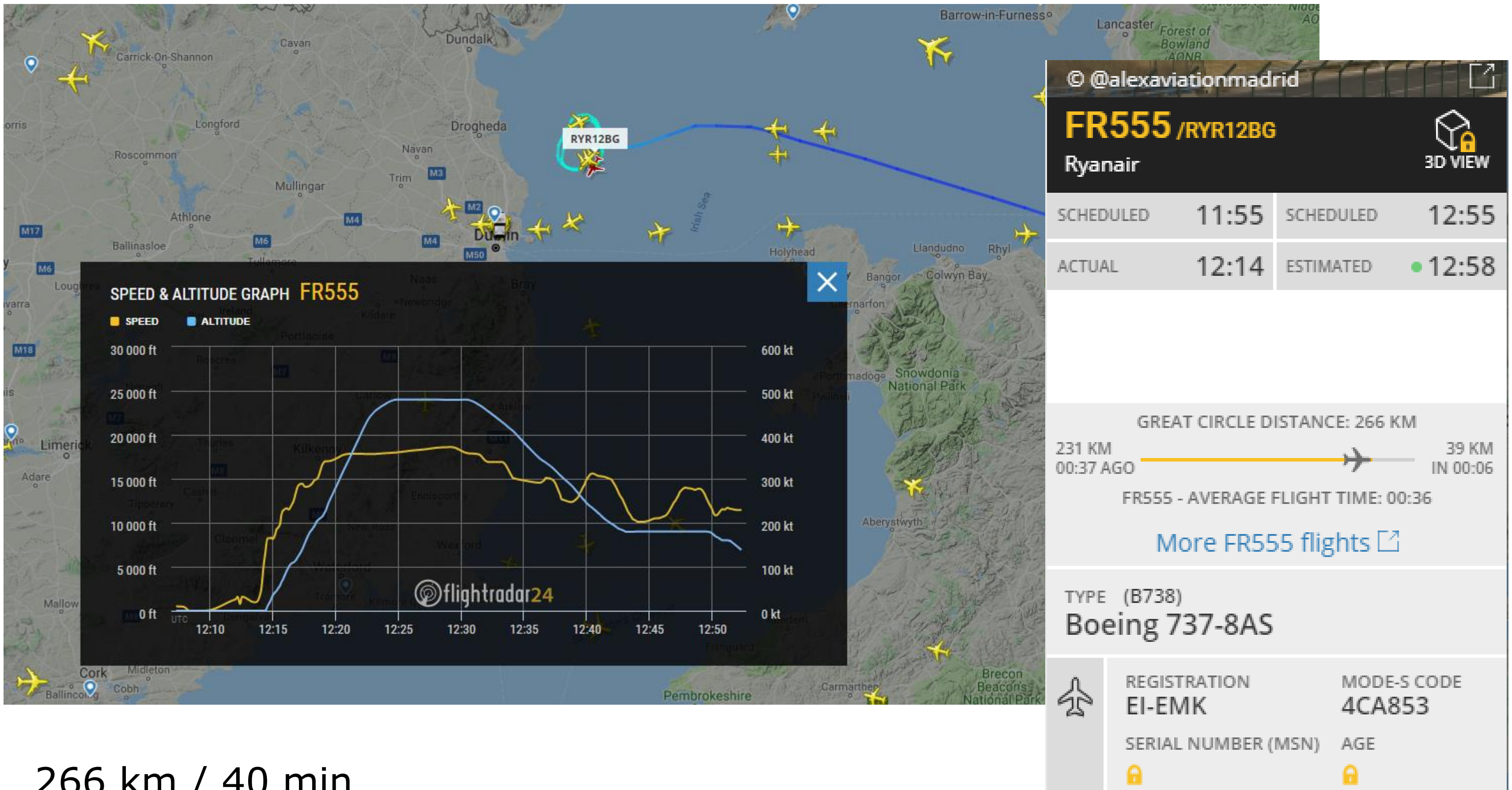


- airplanes not used for mission they were designed for!

Husemann, M., Schäfer, K., & Stumpf, E. (2018). Flexibility within flight operations as an evaluation criterion for preliminary aircraft design. *Journal of Air Transport Management*, *71*, 201-214.

Short-Haul Example

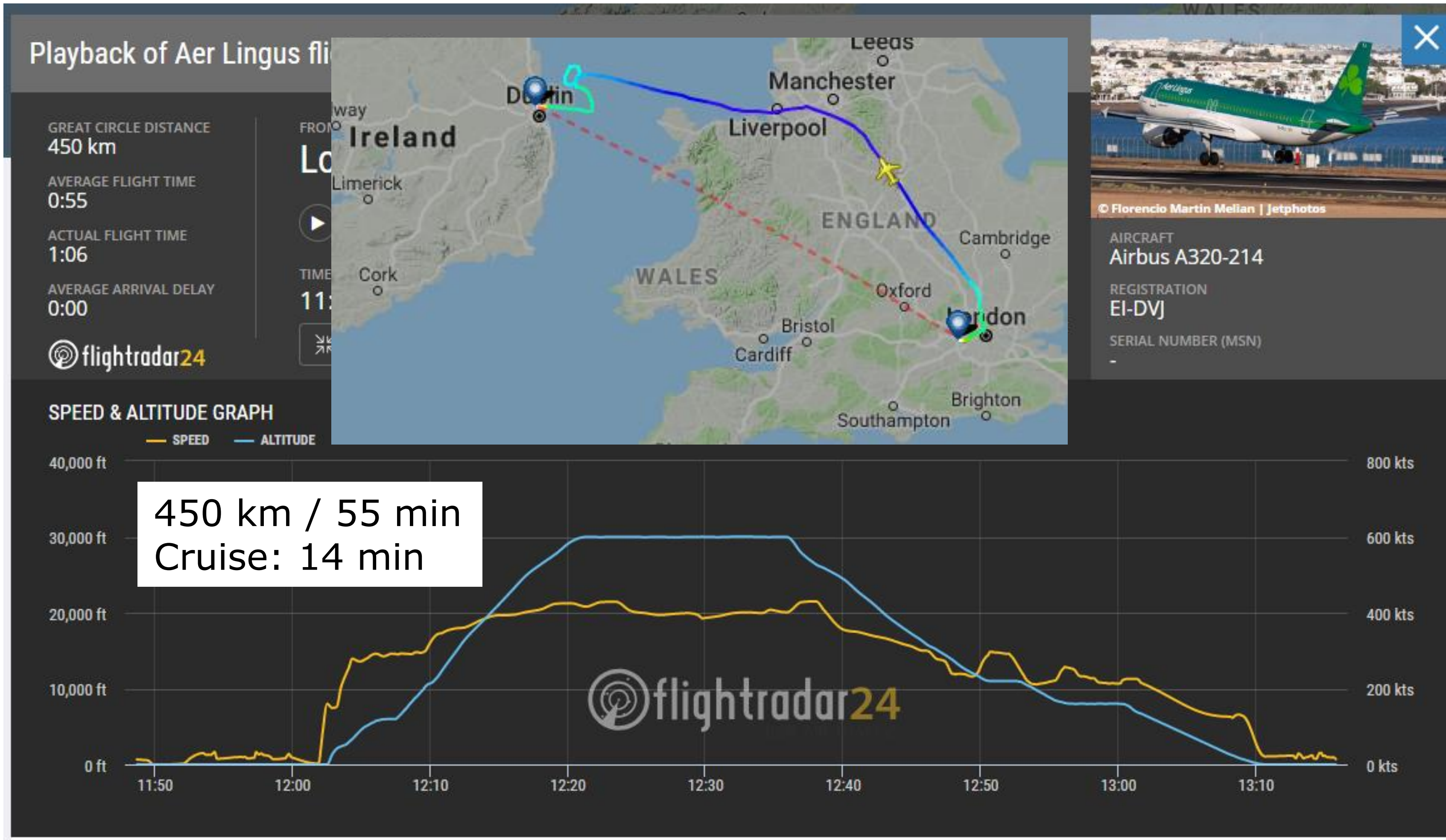
Boeing 737 - Manchester to Dublin



266 km / 40 min
Cruise: 5 min

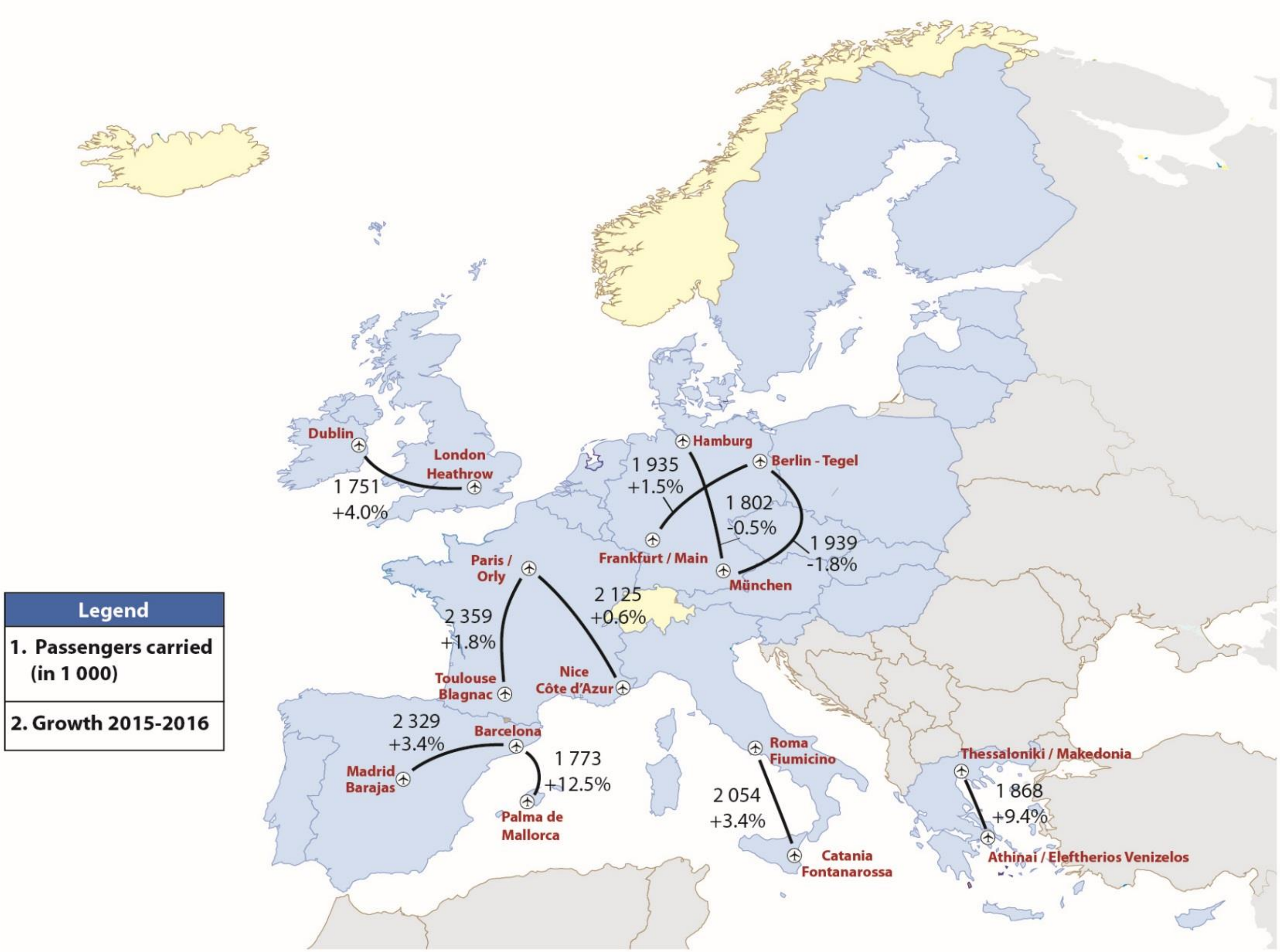
Short-Haul Example

Airbus A320 – London to Dublin



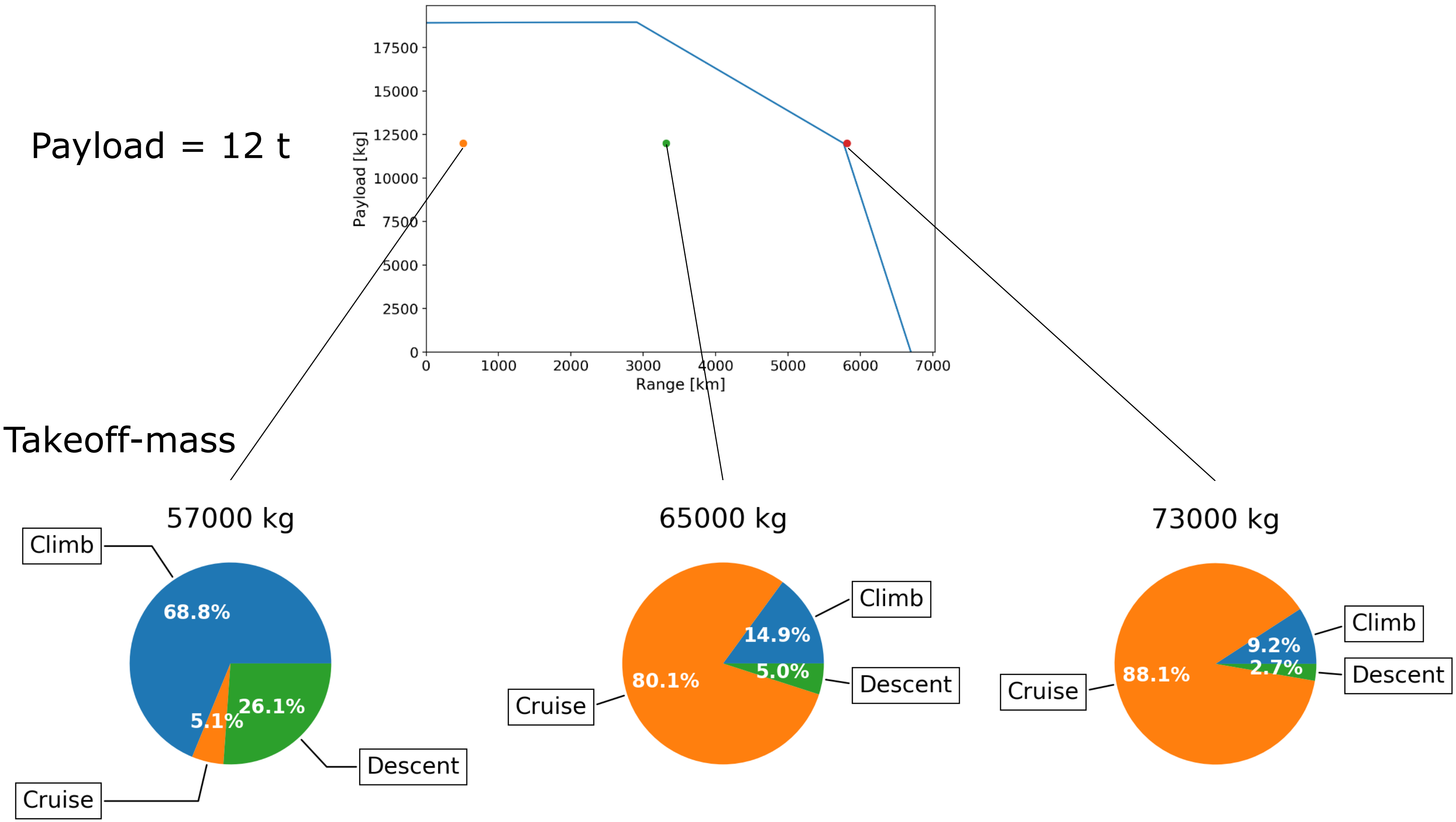
450 km / 55 min
Cruise: 14 min

Top 10 EU Airport Pairs



Source: https://ec.europa.eu/eurostat/statistics-explained/images/8/8e/Top_10_airport_pairs_within_the_EU28_2016.jpg

Example Mission Fuel A320



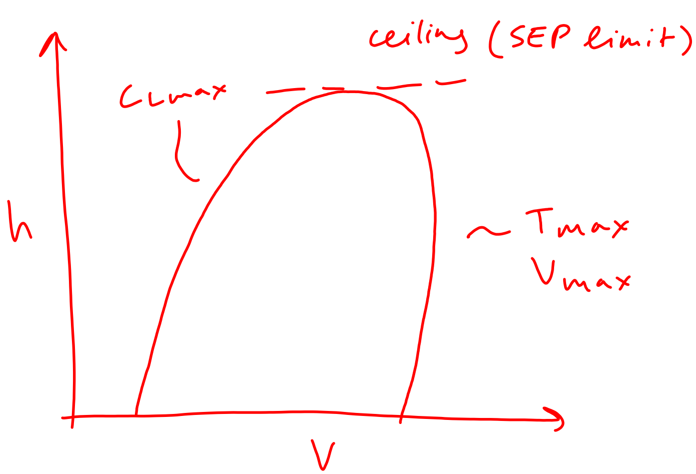
Block Fuel in % per flight phase

39

V-n Diagram

"FLIGHT ENVELOPES"

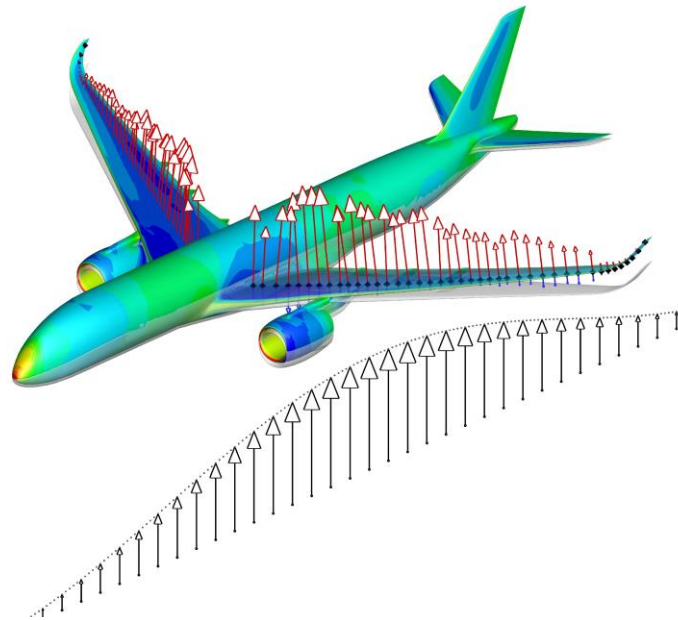
seen so far:

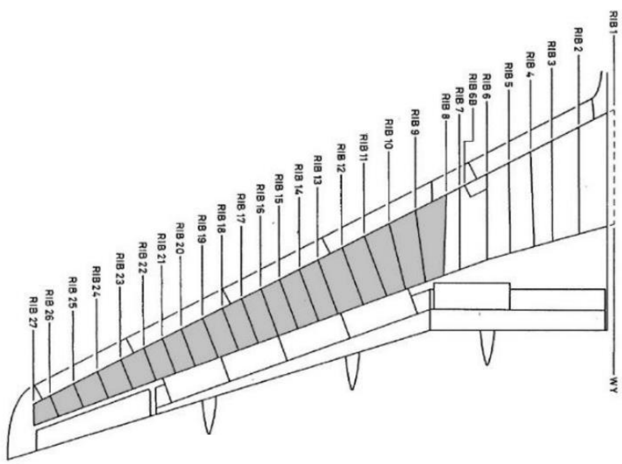


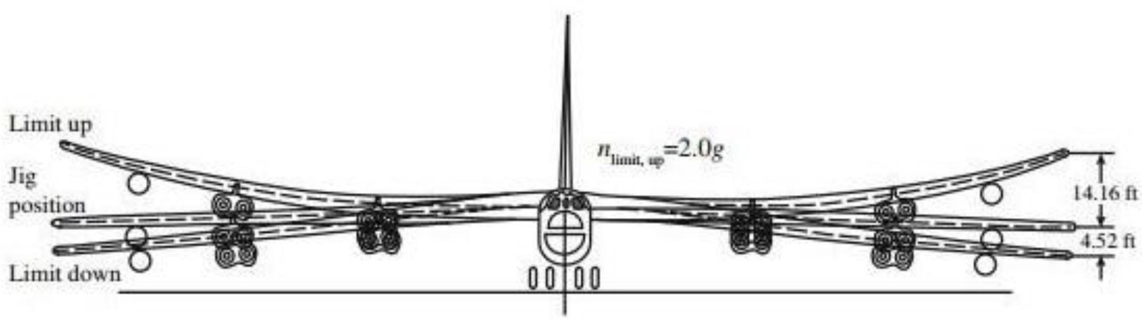
Velocity vs Load Factor Diagram

Load factor due to:
- Maneuvering
- Gusts

Higher load factor -> more mass -> less performance





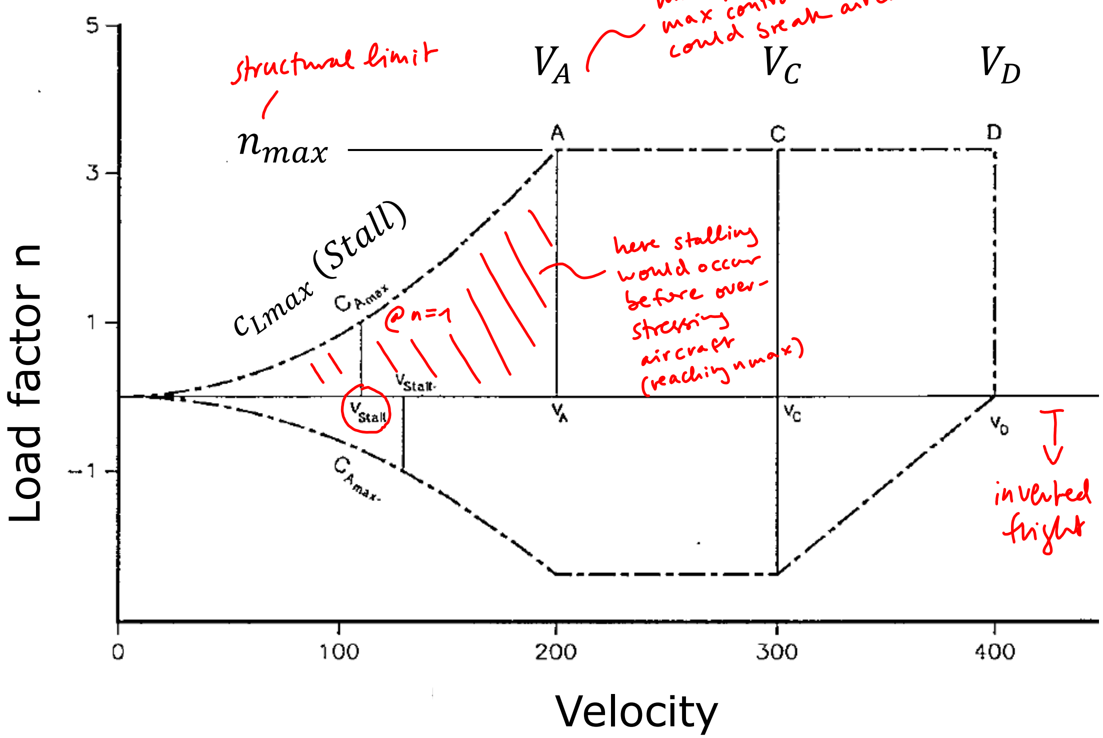


V-n Diagram

Design maneuvering speed V_A

$$V_A = V_{stall}\sqrt{n_{max}}$$

min speed at which max control surface deflection could break aircraft (reach n max)



$$n = \frac{\rho V^2 S_{ref}}{2mg} c_{Lmax} = \left(\frac{V}{V_{stall}}\right)^2$$

Example for CS-23 normal and commuter categories

$$n_{max} = 2.1 + \frac{24000}{W + 10000}$$

W: MTOW in lbs

V_C Design cruising speed

$$V_C \geq 33\sqrt{\frac{W}{S}}\ [KTS]$$

W/S = wing loading at design maximum take-off weight lb/ft²

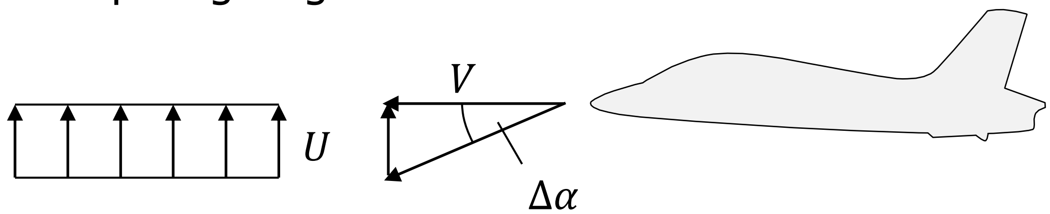
V_D Design dive speed

$$V_D \geq 1.4\ V_{c,min}$$

Gust Envelope

Sharp-edged gust

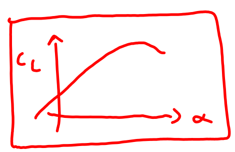
"normalized standard gust"



$$\Delta\alpha = \frac{U}{V} \quad \text{[rad]}$$

AoA ↑ => L ↑ => $n = \frac{L}{mg}$ ↑ !

$$\Delta L = \frac{\rho V^2}{2} S_{ref} \underbrace{\frac{dc_L}{d\alpha}}_{\text{lift slope}} \Delta\alpha$$



$$n = \frac{L}{mg} = \frac{(mg + \Delta L)}{mg} = 1 + \frac{\Delta L}{mg}$$

$$\boxed{n = 1 + \frac{\rho V S_{ref} \frac{dc_L}{d\alpha} U}{2mg}}$$

Certification Specification CS-23

$$n = 1 + k_g \frac{\rho V S_{ref} \frac{dc_L}{d\alpha} U}{2mg}$$

• factors that "soften" the gust:

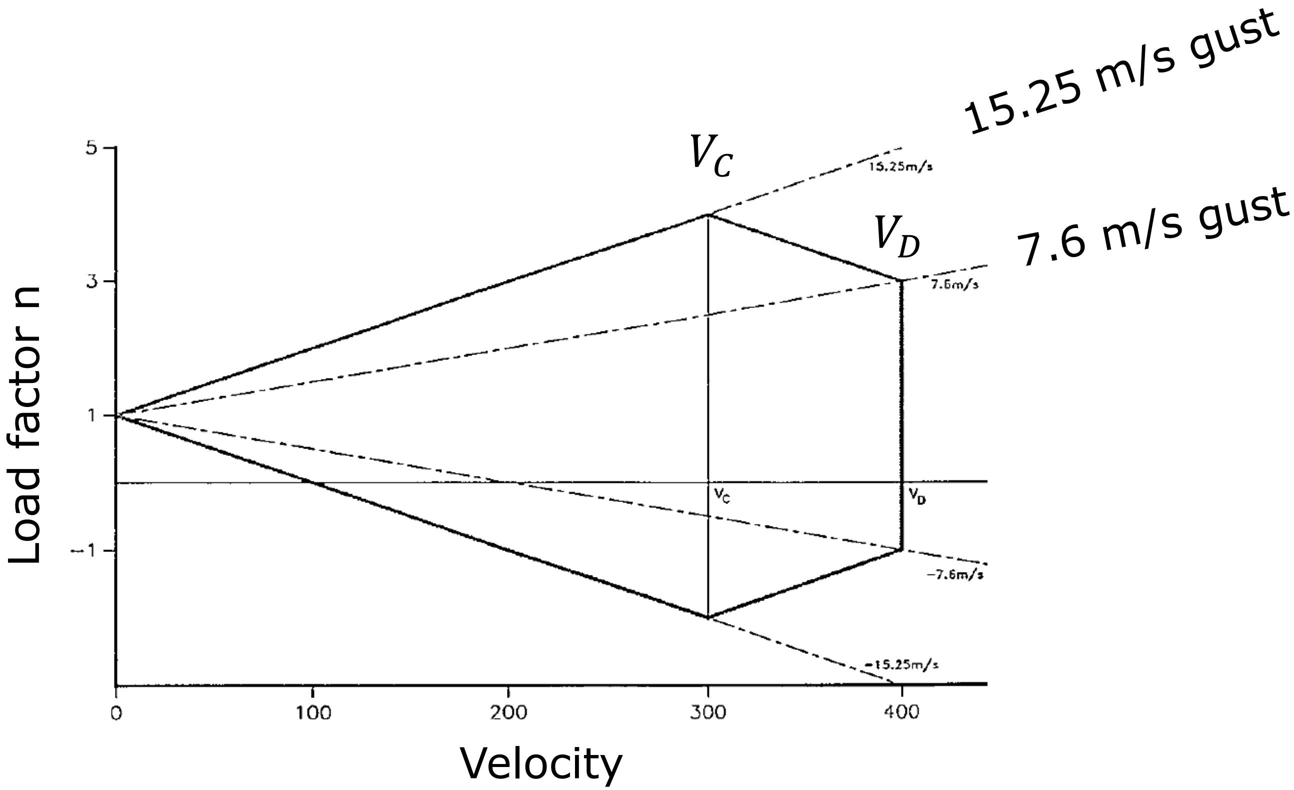
$$k_g = \frac{0.88\mu_g}{5.3 + \mu_g}$$

k_g gust alleviation factor

$$\mu_g = \frac{2m}{\rho S_{ref} MAC \frac{dc_L}{d\alpha}}$$

μ_g airplane mass ratio

Gust Load – CS-23



CS-23: The airplane needs to withstand

a $15.25\frac{m}{s}$ gust at V_C

a $7.6\frac{m}{s}$ gust at V_D

$$n = 1 + k_g \frac{\rho V S_{ref} \frac{dc_L}{d\alpha} \mathrm{U}}{2mg}$$

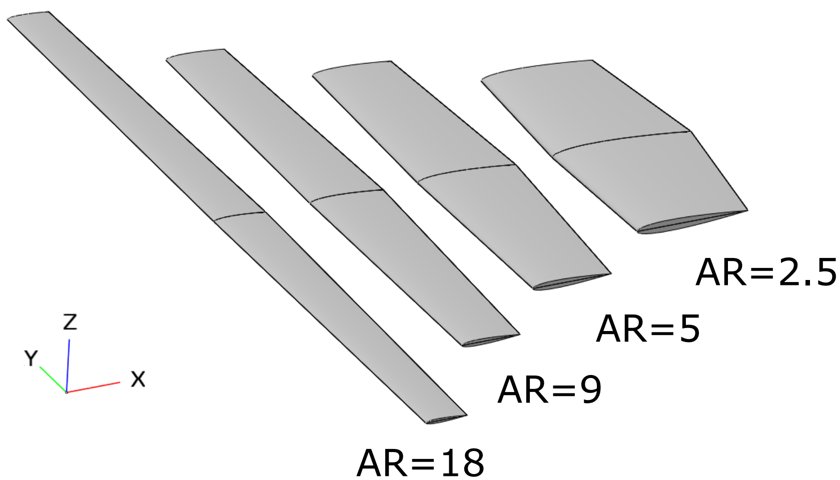
Wing Lift-Slope

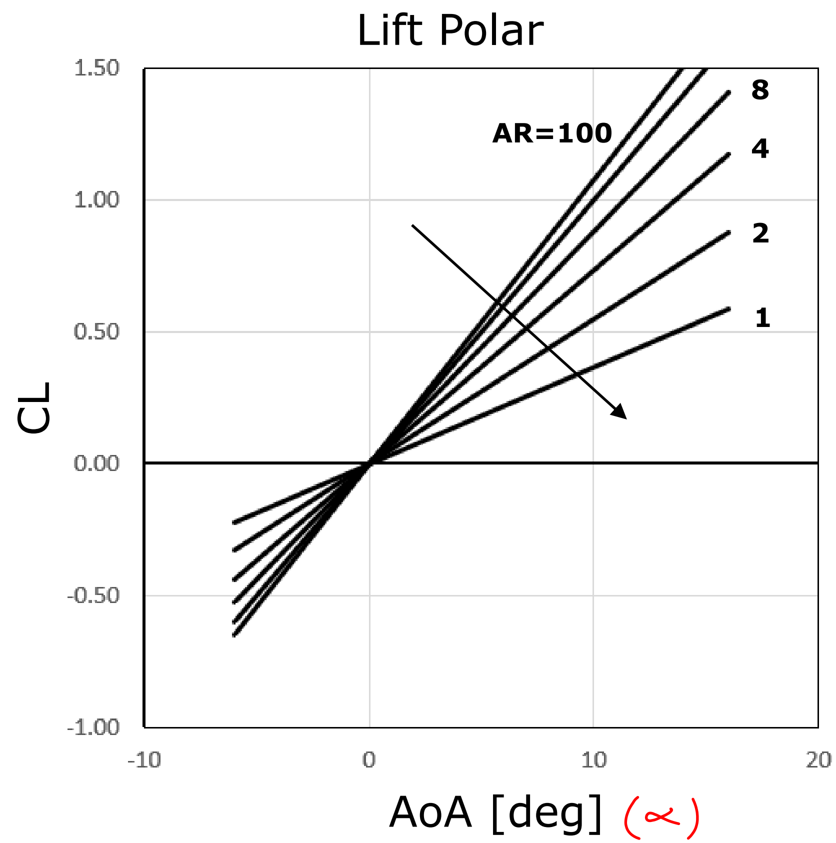
$$n = 1 + k_g \frac{\rho V S_{ref} \frac{dc_L}{d\alpha} \mathrm{U}}{2mg}$$

$$\frac{dc_L}{d\alpha} = c_{l_\alpha} \frac{AR}{AR+2}$$

$$c_{D_i} = \frac{1}{\pi A R e} c_L^2$$

$$AR = \frac{b^2}{S_{ref}}$$



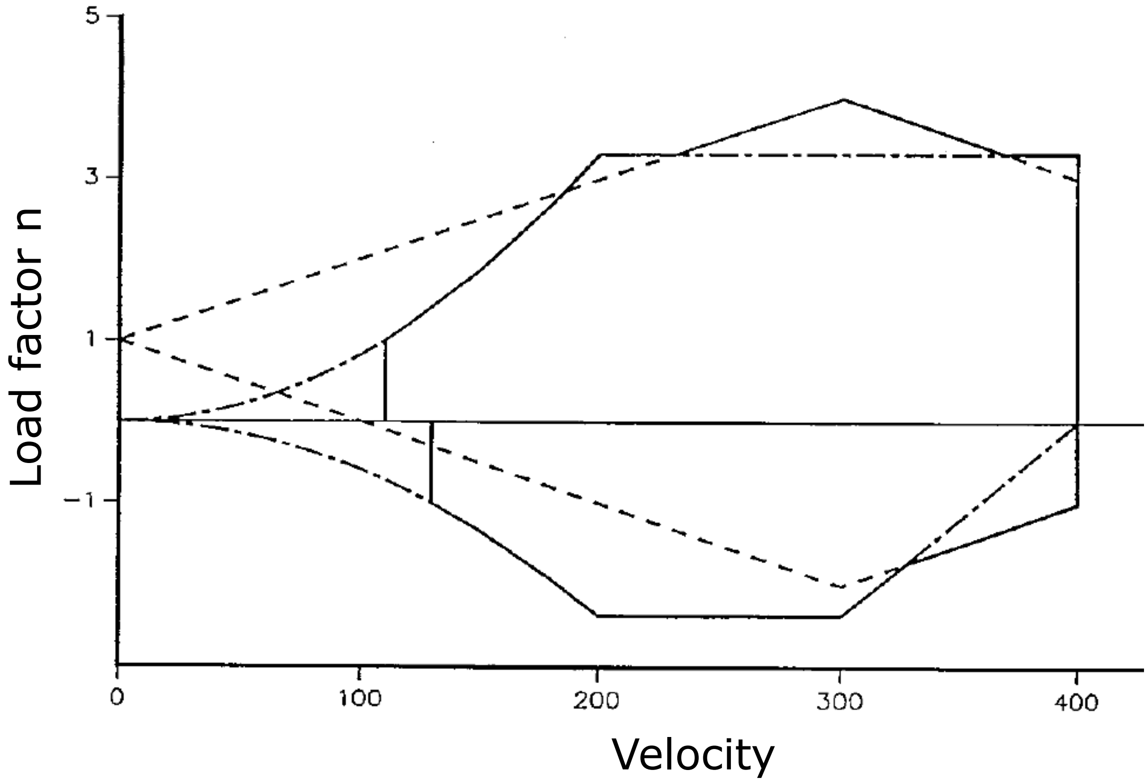


⇒ gust response is "better" ($\Delta\alpha \downarrow$) if wing has ↓ AR (less slender)

∴ tradeoff S/W efficiency (high AR) & gust response (low AR)

V-n combined with gust loads

Safe operating envelope in still and turbulent air



Airspeed Indicator