*Spring Semester 2023*

# AIRCRAFT AERODYNAMICS & FLIGHT MECHANICS

**23.02.2023**

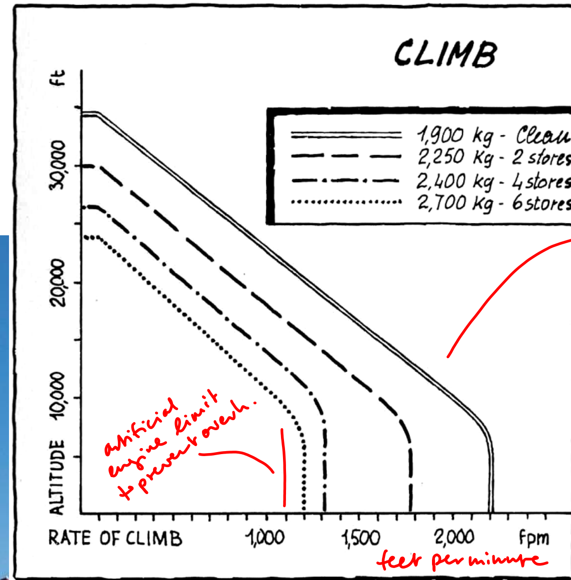
Dr. Marc Immer ALR Aerospace

This lecture is adapted with permission from the lecture “Ausgewählte Kapitel der Flugtechnik” by Dr. Jürg Wildi

# Aircraft Performance

# Typical Performance Characteristics

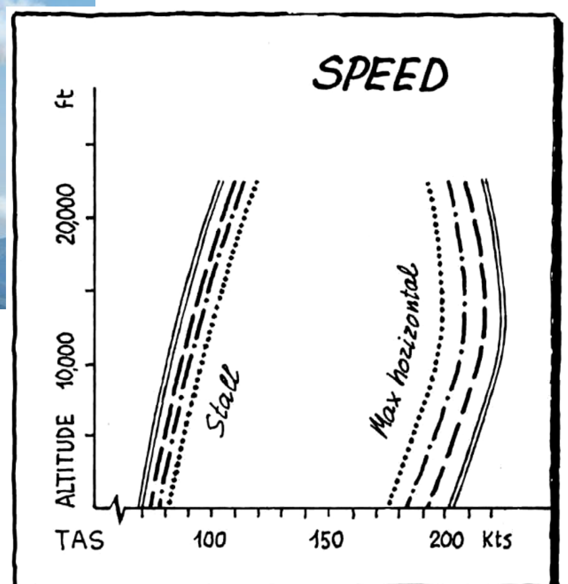
## Point Performance

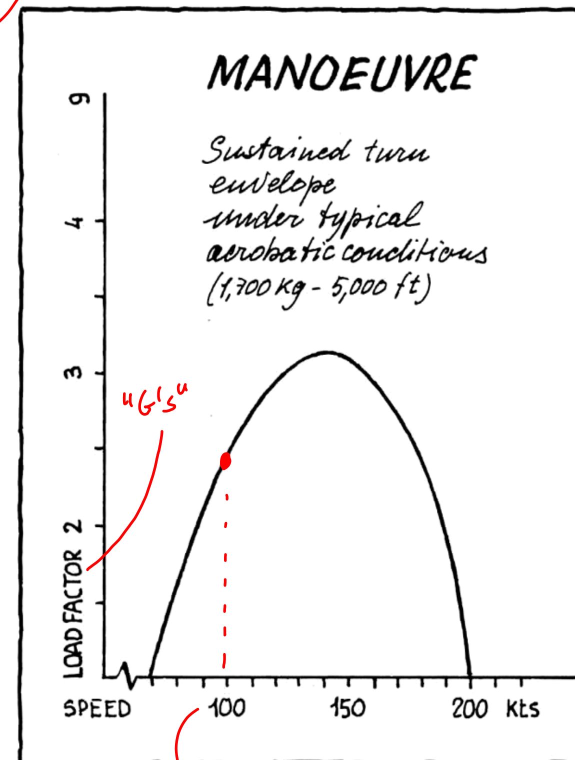



artificial engine limit to prevent overh.

feet per minute

engine has less air
=> power limit
(air less dense w/ alt. ↑)



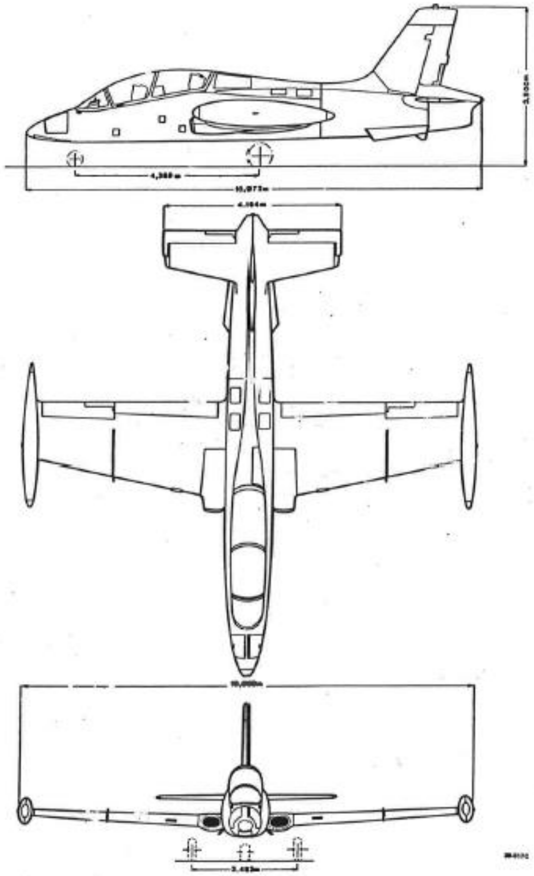

4 G's

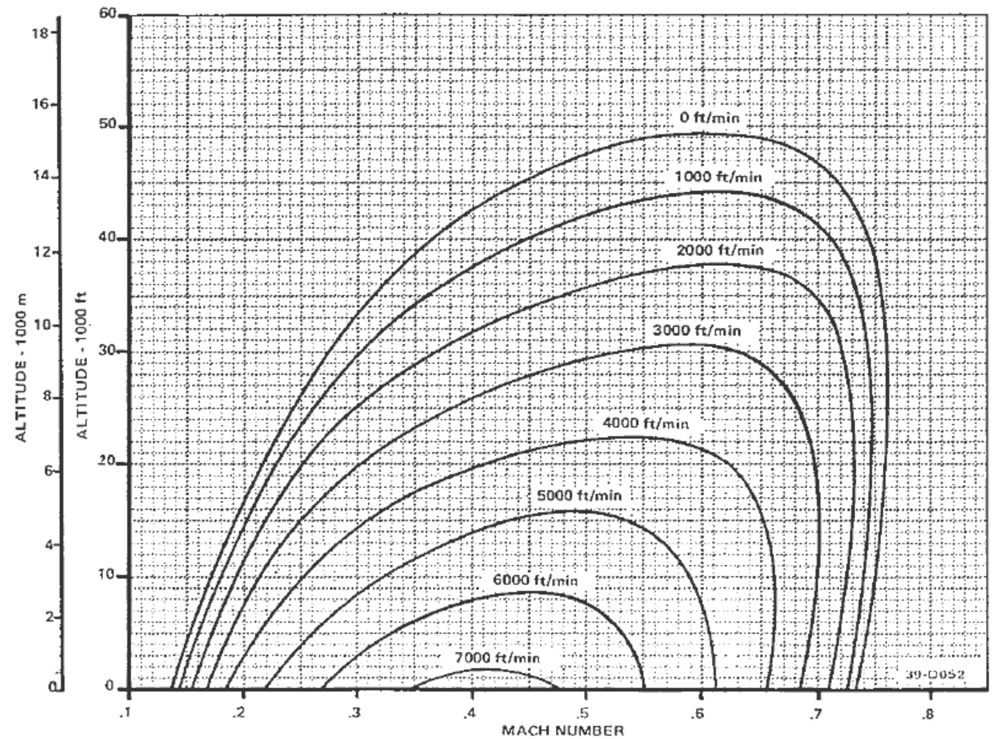
(simplified)

e.g. pull 2.5 g
& you can turn
w/o losing speed

# Point Performance – Specific Excess Power

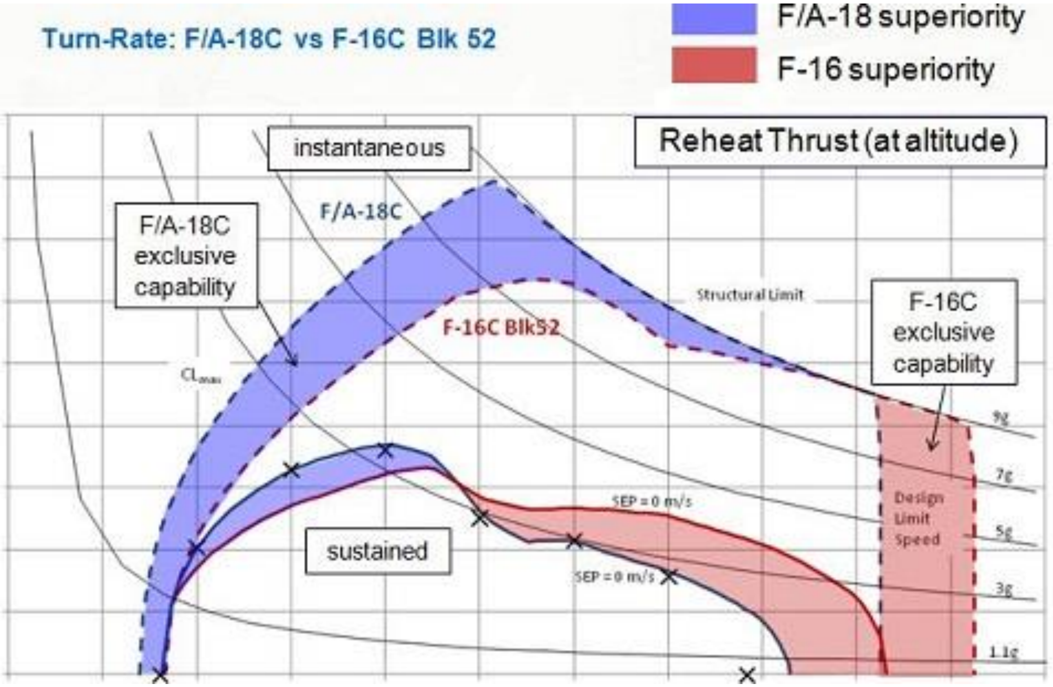
SEP or $P_s$





# Energy Maneuverability - SEP

## Maneuver Performance



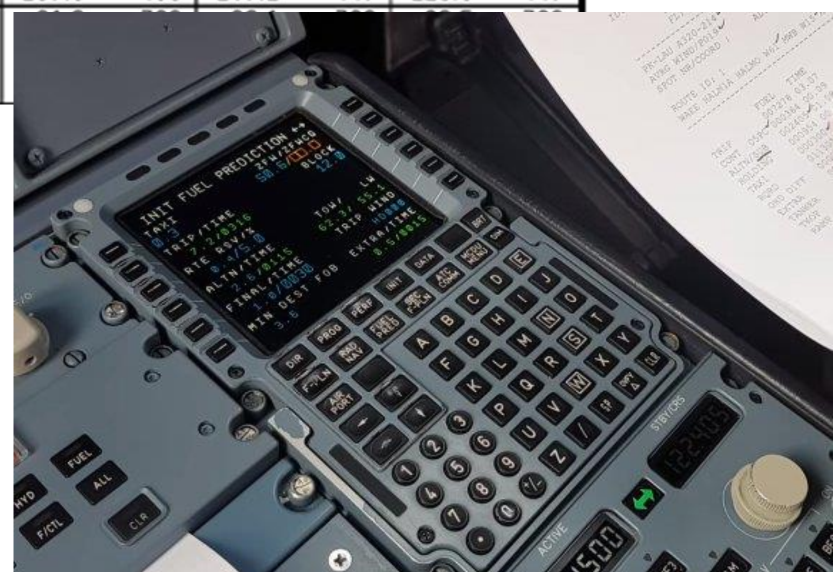
at one state

# Point Performance - Cruise

**CRUISE - M.78**

| MAX. CRUISE THRUST LIMITS<br>NORMAL AIR CONDITIONING<br>ANTI-ICING OFF | | | | | | | | ISA<br>CG=33.0% | | N1 (%)<br>KG/H/ENG<br>NM/1000KG | | MACH<br>IAS (KT)<br>TAS (KT) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| WEIGHT (1000KG) | FL290 | | FL310 | | FL330 | | FL350 | | FL370 | | FL390 | |
| 50 | 80.8 | .780 | 80.6 | .780 | 80.4 | .780 | 80.4 | .780 | 80.9 | .780 | 82.0 | .780 |
| | 1305 | 302 | 1211 | 289 | 1127 | 277 | 1053 | 264 | 994 | 252 | 954 | 241 |
| | 176.9 | 462 | 188.9 | 458 | 201.3 | 454 | 213.5 | 450 | 225.0 | 447 | 234.6 | 447 |
| 52 | 80.9 | .780 | 80.7 | .780 | 80.6 | .780 | 80.7 | .780 | 81.3 | .780 | 82.5 | .780 |
| | 1315 | 302 | 1222 | 289 | 1139 | 277 | 1066 | 264 | 1011 | 252 | 975 | 241 |
| | 175.6 | 462 | 187.3 | 458 | 199.2 | 454 | 210.8 | 450 | 221.2 | 447 | 229.3 | 447 |
| 54 | 81.1 | .780 | 80.9 | .780 | 80.9 | .780 | 81.0 | .780 | 81.7 | .780 | 82.9 | .780 |
| | 1324 | 302 | 1232 | 289 | 1152 | 277 | 1082 | 264 | 1030 | 252 | 1000 | 241 |
| | 174.3 | 462 | 185.7 | 458 | 196.8 | 454 | 207.8 | 450 | 217.2 | 447 | 223.6 | 447 |
| 56 | 81.2 | .780 | 81.1 | .780 | 81.1 | .780 | | | | | | |
| | 1335 | 302 | 1244 | 289 | 1166 | 277 | | | | | | |
| | 173.0 | 462 | 184.0 | 458 | 194.6 | 454 | | | | | | |

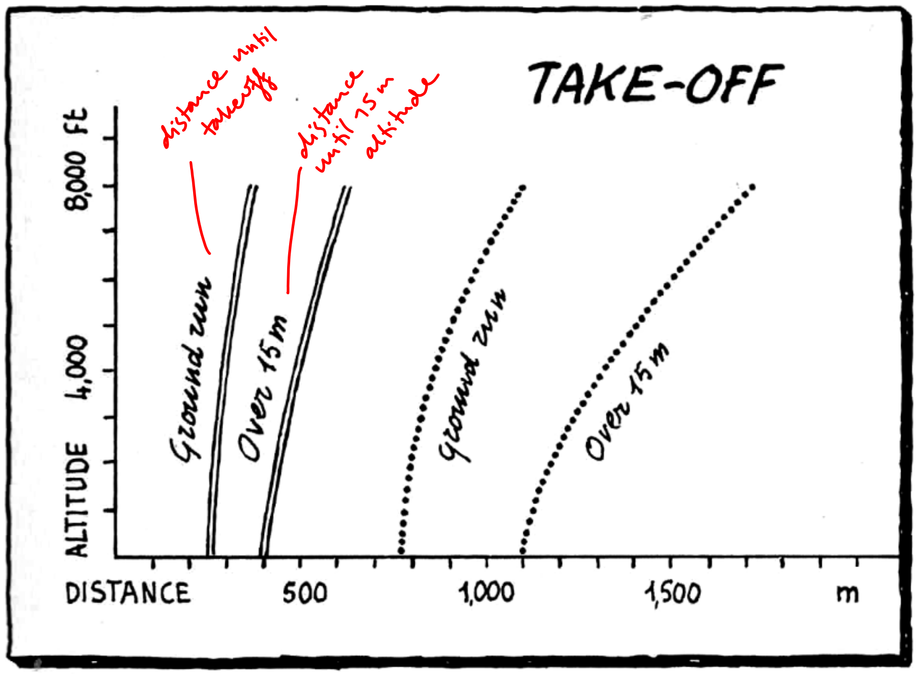
Specific Range and Fuel Flow

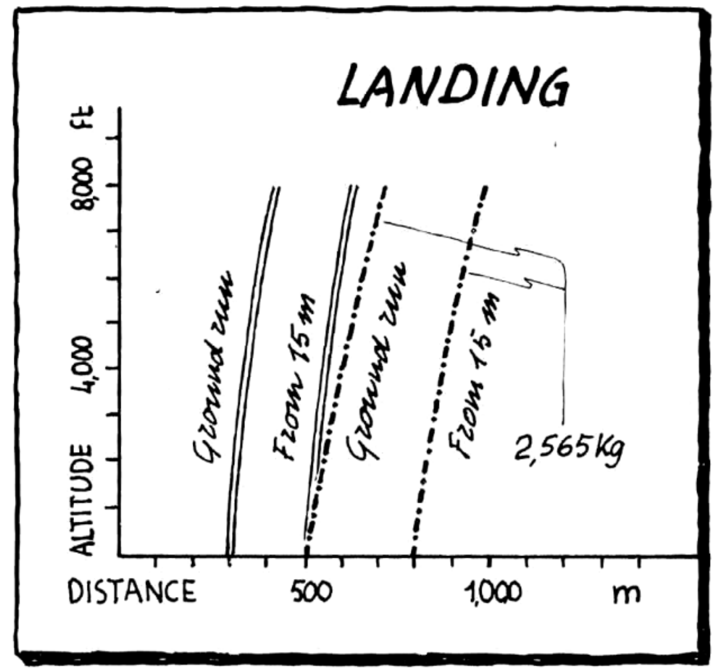


## ***Field Performance*** ***(TOL)***

2) Take-Off and Landing Performance

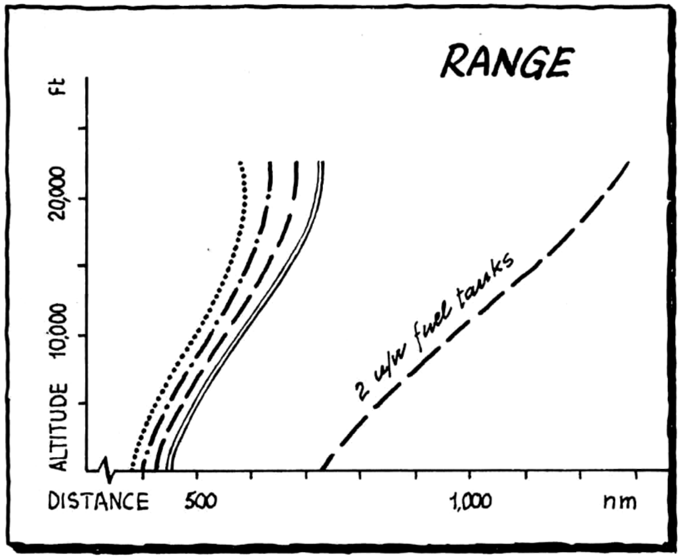
less air to burn fuel => less thrust/power



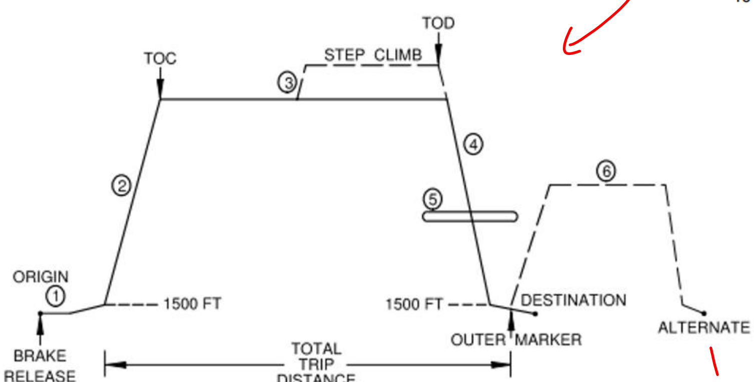

And also rejected take-off (RTO) and balanced field length (BFL)

# Mission Performance




Range and endurance for a specified mission

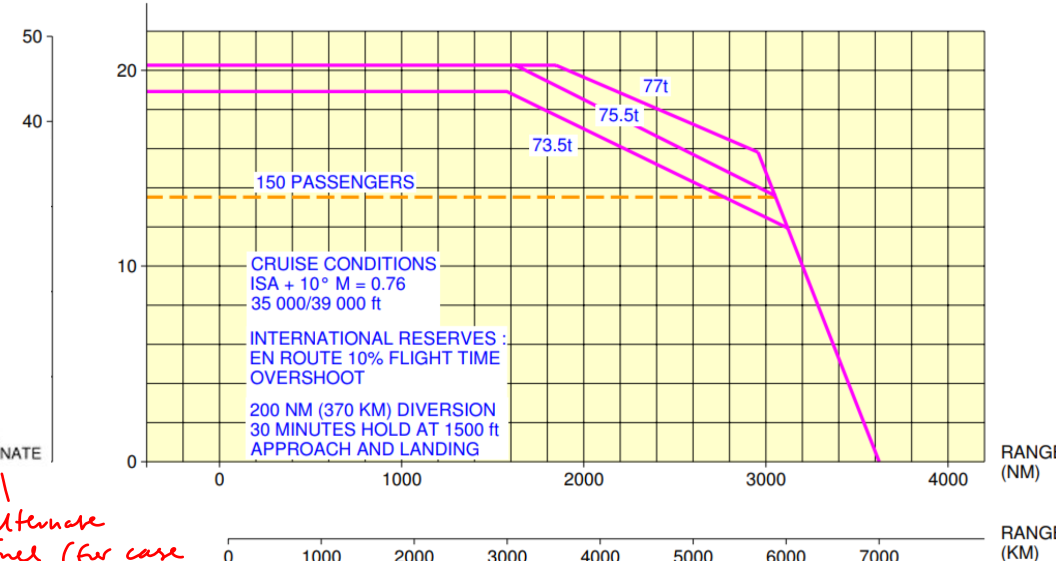

for this mission profile:




alternate fuel (for case when dest. runway closed)

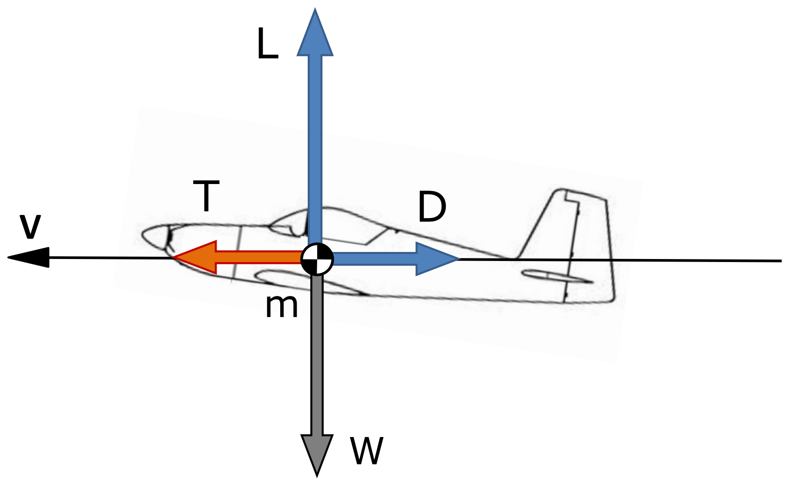
Mission profile with alternate & fuel reserves

Airliner Payload-Range Chart



# Performance

# Solving Performance Questions




$W = f(FF, t)$ — hydrocarbon (fuel) based planes i.e. not electric aircraft

$L = f(V, h, n, t)$

$D = f(V, h, L)$ — more lift => more drag

$T = f(V, h, power\ setting)$

V: velocity

h: altitude

FF: fuel flow

n: load factor, "g"

t: time

Note: assuming standard conditions (ISA atmosphere)

## Performance Model

Input Data: $m, L, D, T, FF,$ Limits

EoM

**Numerical Procedures**

(flight simulation)

(point performance not sufficient; need data for every state)

**Analytical Procedures**

Output: climb speed, range, turn-rate, take-off distance, …

# Simplifications and Assumptions

for analytical procedures

Example model simplifications

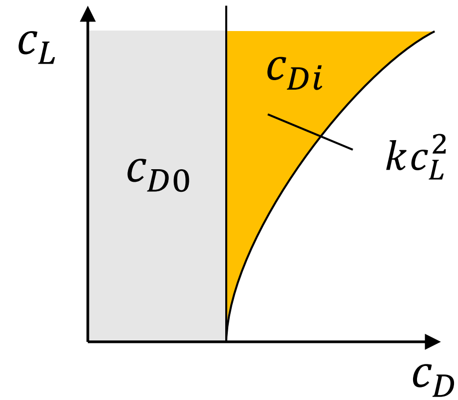
$$m = const$$

$$c_D = c_{D0} + kc_L^2 \qquad k = \frac{1}{\pi ARe}$$

$T = const$ (jet) or $P = const$ (prop)

dramatic simplification

thrust constant for all airspeed




works well for subsonic, incompressible flow

Note: no altitude (Reynolds) dependency

Non-linear terms: small angle approximations

$\sin\theta = \theta$ or $\sin\theta = 0$ for small $\theta$

$\cos\theta = 1$

$\tan\theta = \theta$

# Energy and Momentum Approach

Two ways to approach aircraft performance questions

Rate of change of total energy

$$\frac{dE_{tot}}{dt} = \frac{dE_{pot}}{dt} + \frac{dE_{kin}}{dt}$$

where

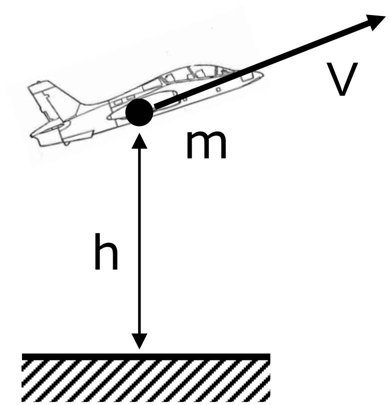
$$E_{pot} = mgh$$

$$E_{kin} = \frac{1}{2}mV^2$$

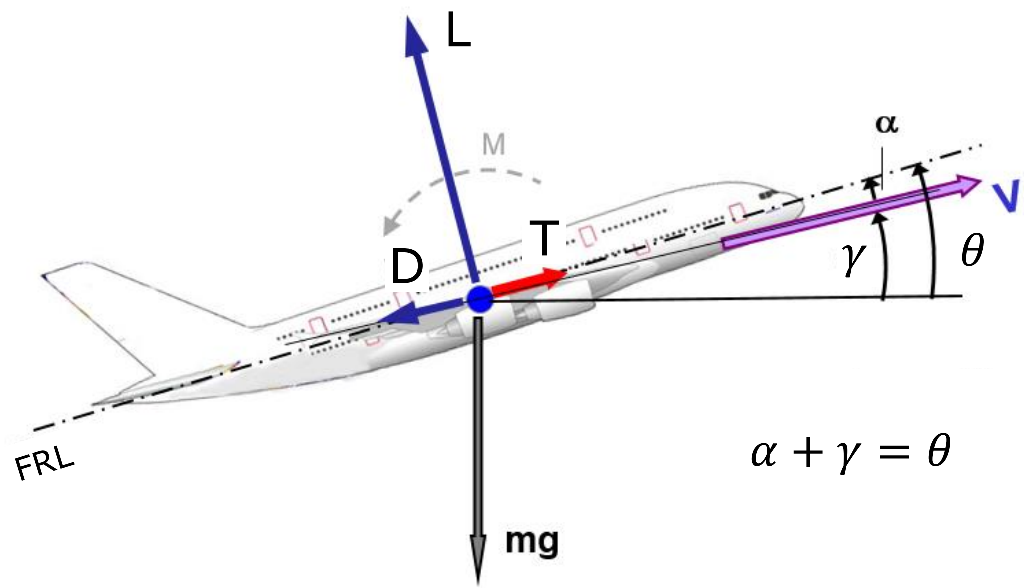
Rate of change of linear momentum

$$m\frac{d\boldsymbol{V}}{dt} = \sum \boldsymbol{F}_{ext}$$

Note: V and F are vectors



# Definitions




$$\alpha + \gamma = \theta$$

FRL: Fuselage Reference Line

**Aerodynamic Forces and Moments**

| | | |
|---|---|---|
| L, $F_L$ | [N] | Lift |
| D, $F_D$ | [N] | Drag |
| M | [Nm] | Pitching Moment |

**Mass**

| | | |
|---|---|---|
| mg | [N] | Weight |

**Flight Path**

| | | |
|---|---|---|
| V | [m/s] | Velocity |
| γ | [°] | Flight path angle<br>Climb angle<br>(direction of velocity) |

**Relative Wind**

| | | |
|---|---|---|
| α | [°] | Angle of Attack (AoA) |

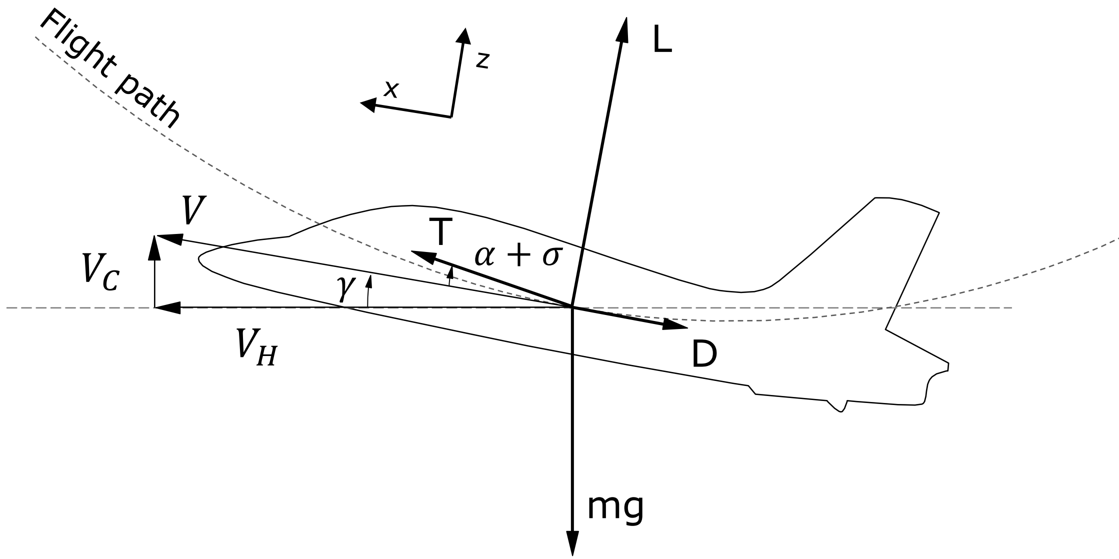
**Attitude**

| | | |
|---|---|---|
| Θ | [°] | Pitch Angle<br>(what you see when looking out the window) |

# Equations of Motion

## Velocity vector coordinate frame

x is along velocity (flight path direction)




In flight path direction

$$T\cos(\alpha+\sigma) - D - mg\,\sin(\gamma) = m\frac{dV}{dt}$$

Normal to flight path

$$L + T\sin(\alpha+\sigma) - mg\,\cos(\gamma) = \boxed{mV\frac{d\gamma}{dt}}$$

pull up rate

apparent force
due to rotating ref. frame

Kinematic relationships

climb rate

$$\frac{dh}{dt} = V_C = V\sin(\gamma)$$

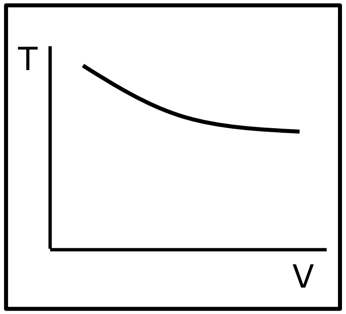
$$\frac{dx}{dt} = V_H = V\cos(\gamma)$$

$\alpha$: angle of attack
$\sigma$: thrust incidence angle

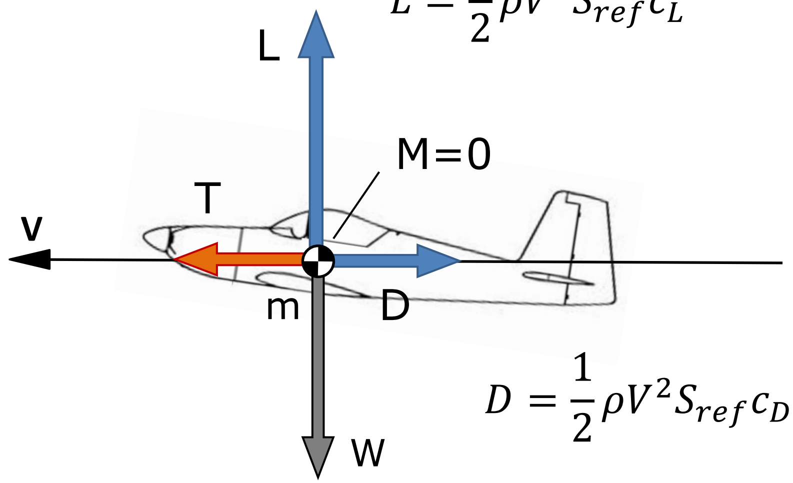
how engine is mounted

Note: often small angle approximations are used

# Aircraft Performance - Forces

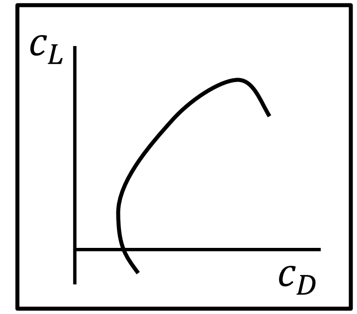



Thrust Characteristics




$$L = \frac{1}{2}\rho V^2 S_{ref} c_L$$

$$D = \frac{1}{2}\rho V^2 S_{ref} c_D$$




Aerodynamics

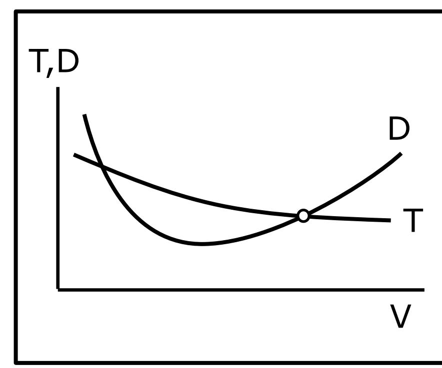
Example: stationary horizontal cruise

$\boxed{L = mg} \rightarrow c_L \rightarrow c_D \rightarrow D \rightarrow \boxed{T = D}$

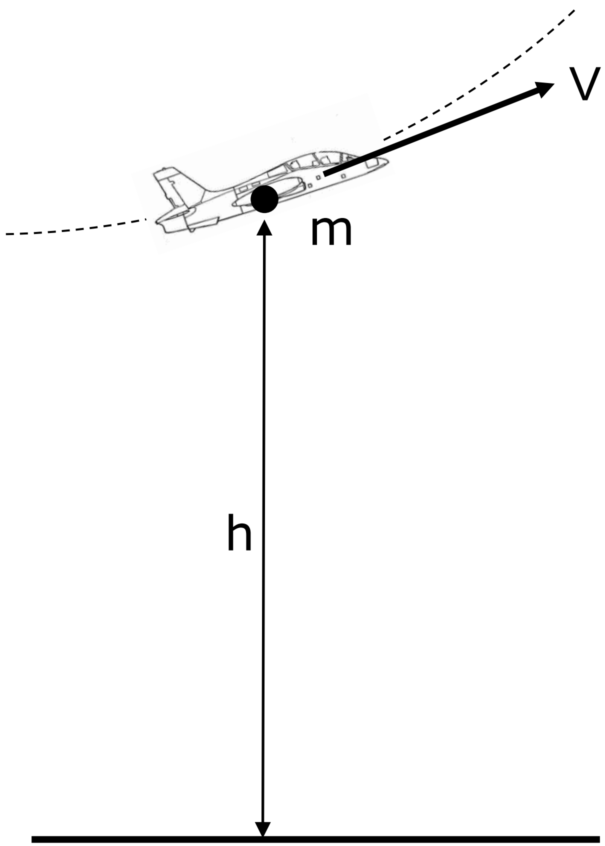
$L = \frac{1}{2}\rho V^2 S_{ref} c_L$

polar

$D = \frac{1}{2}\rho V^2 S_{ref} c_D$



# Aircraft Performance - Energy




## Total Energy

$$E = E_{pot} + E_{kin}$$

$$E = mgh + \frac{1}{2}mV^2 \qquad [Nm = J]$$

## Specific Energy

$$E = mgh + \frac{1}{2}mV^2 \qquad \Big| \; \frac{1}{mg}$$

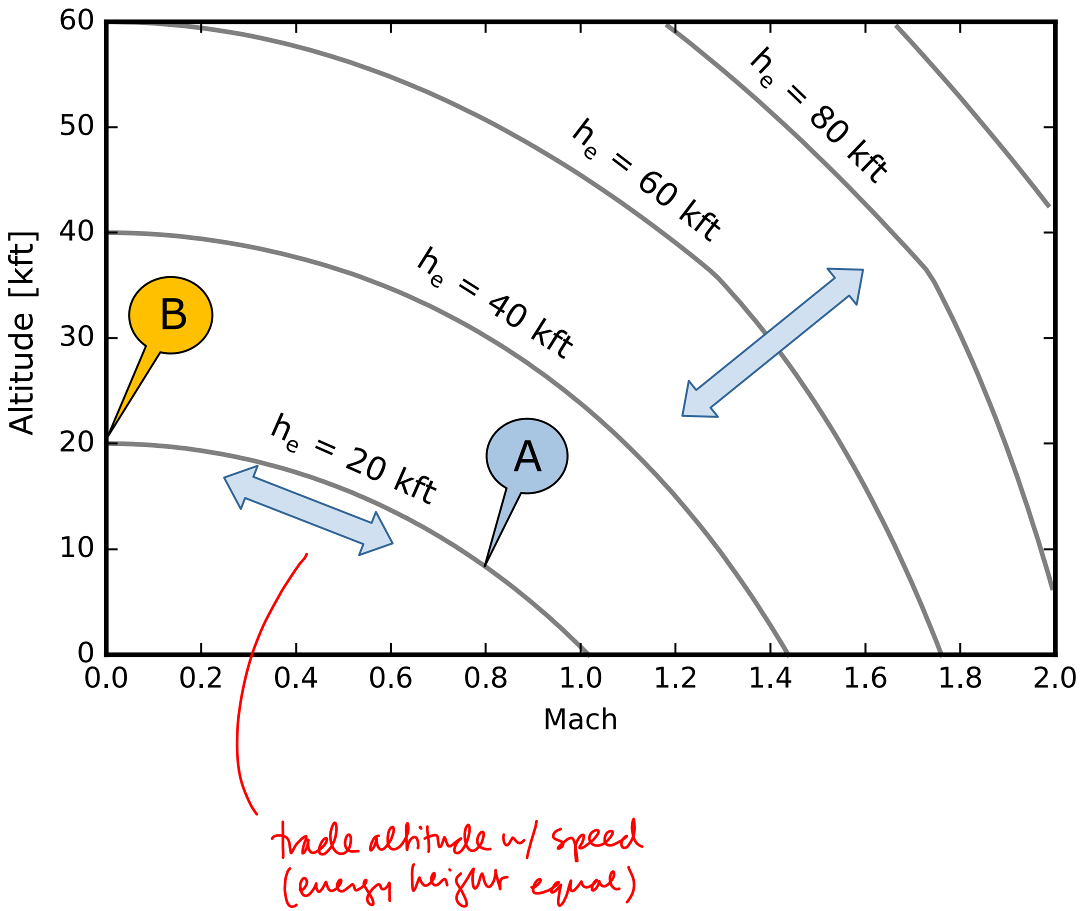
$$\boxed{h_e = h + \frac{V^2}{2g}} \qquad [m]$$

⇨ *Energy height*

# Energy Height

$$h_e = h + \frac{V^2}{2g}$$

Theoretical altitude if all kinetic energy is converted to altitude without losses




Point A and point B have the same energy height

**Energy Management**

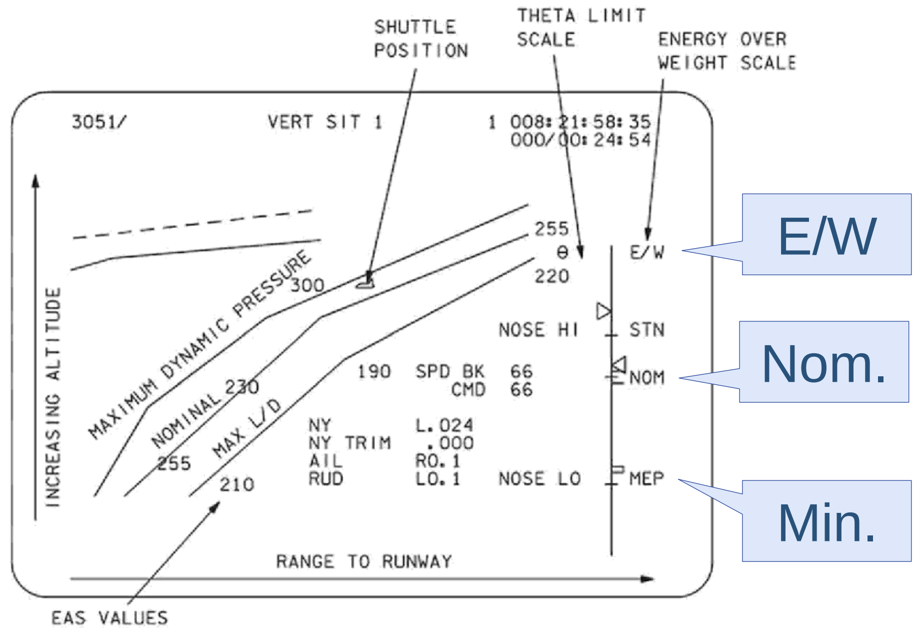
Changing the airspeed by pitching the aircraft
→ fast

Adding or removing energy to/from the aircraft: changing the energy height
→ slow

## *Energy Management Example*

### Space Shuttle TAEM:
### Terminal Area Energy Management

$$E/W \Rightarrow \frac{E}{mg} = h_s$$




Example of navigation display for TAEM flight.

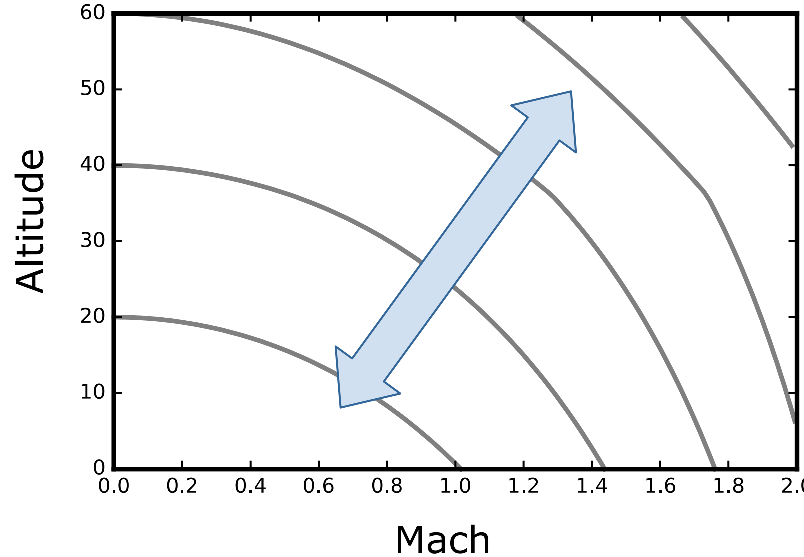
Sources:
- http://www.aviation-art.net/Gallery2013/shuttlecomeshome.jpg
- Sivolella, Davide (2014): To orbit and back again : how the Space Shuttle flew in space. Springer.

Dissipate energy by:
- Turning (increases drag)
- Speed-brake

# *Specific Excess Power (SEP)*

The rate of change of the energy height is called Specific Excess Power (SEP)

SEP: $P_s = \frac{dh_e}{dt}$




$$h_e = h + \frac{V^2}{2g} \quad \Big| \quad \frac{d}{dt}$$

$$\frac{dh_e}{dt} = \frac{dh}{dt} + \frac{d}{dt}\left(\frac{V^2}{2g}\right) = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt}$$

climb rate

with $\frac{dh}{dt} = V_C$

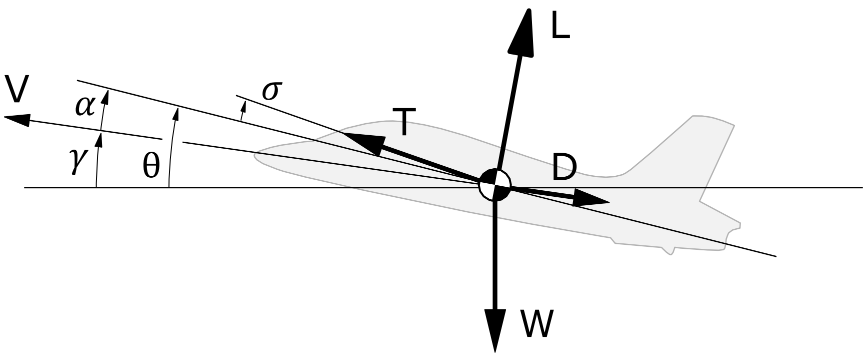
$|P_s| \uparrow$ means energy is bled faster

$$\boxed{P_S = \underbrace{V_C}_{\text{climb rate}} + \underbrace{\frac{V}{g}\frac{dV}{dt}}_{\text{acceleration}}} \quad \left[\frac{m}{s}\right]$$

With available SEP an aircraft can
- Climb
- Accelerate

e.g. given SEP & observed climb rate, you could find out how fast you can accel.

# Calculate Specific Excess Power (SEP)




$$P_S = V_C + \frac{V}{g}\frac{dV}{dt}$$

EoM in velocity direction:

$$m\frac{dV}{dt} = T\cos(\alpha+\sigma) - D - mg\,\sin(\gamma) \quad \Big| \; \frac{V}{mg}$$

$$\frac{V}{g}\frac{dV}{dt} = \frac{V}{mg}(T\cos(\alpha+\sigma) - D) - V\sin(\gamma)$$

rearrange

$$V\sin(\gamma) + \frac{V}{g}\frac{dV}{dt} = \frac{V}{mg}(T\cos(\alpha+\sigma) - D)$$

with $V_C = V\sin(\gamma)$

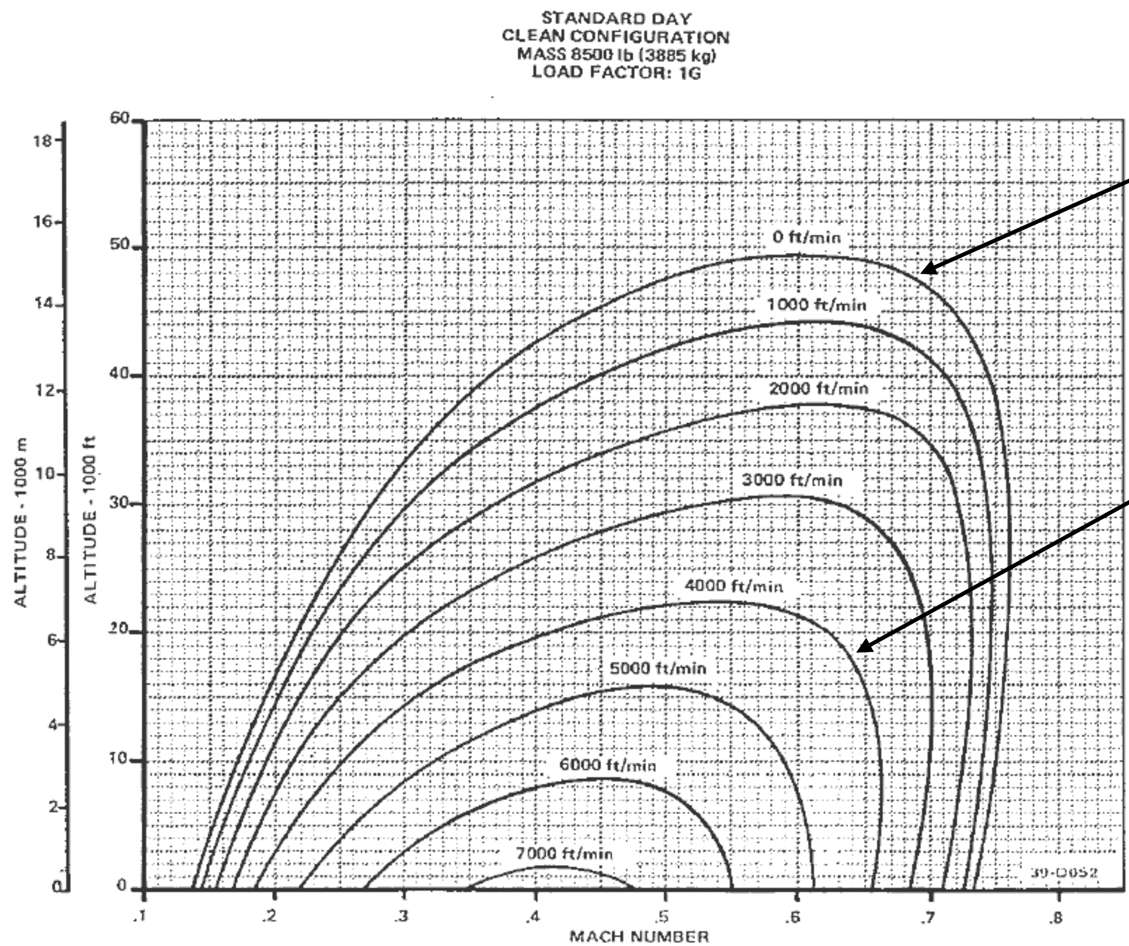
$$\Rightarrow \quad P_S = \frac{V}{mg}(T\cos(\alpha+\sigma) - D)$$

$\Rightarrow$ Small angle assumption

$$P_S = V\frac{(T-D)}{mg}$$

# *Specific Excess Power (SEP)*

## SEP Envelope (H-M Diagram)




$$P_S = V_C + \frac{V}{g}\frac{dV}{dt}$$

$P_S = 0$ $\quad n = 1$

stationary horizontal flight

$P_S > 0$

$n = 1$ and $\frac{dV}{dt} = 0$

sustained climb rate

Note: $T = T_{max}$

Load factor $n = \dfrac{L}{mg}$

# Literature

## Aerodynamics and Performance

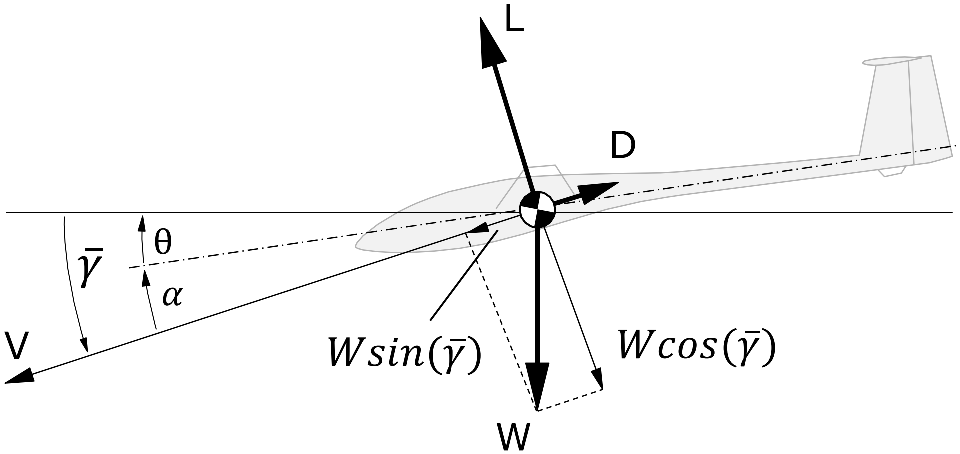

- McCormick, B. W. (1995). Aerodynamics Aeronautics and Flight Mechanics
- Saarlas, M. (2006). Aircraft performance. John Wiley & Sons
- John D. Anderson Jr. (1998). Aircraft Performance & Design. McGraw-Hill Education.
- Airbus (2002). Getting to grips with aircraft performance


## Aircraft Design


- Raymer, D. (2012). Aircraft design: a conceptual approach. American Institute of Aeronautics and Astronautics AIAA
- Nicolai, L. M., & Carichner, G. E. (2010). Fundamentals of aircraft and airship design, Volume 1–Aircraft Design. American Institute of Aeronautics and Astronautics AIAA

# Gliding Flight


# Gliding Flight




Stationary glide

$$\frac{dV}{dt} = 0 \qquad \frac{d\gamma}{dt} = 0$$

$$T = 0$$

$$\cancel{T cos(\alpha + \sigma)} - D - mg\, sin(\gamma) = m\frac{dV}{dt}$$

$$L + \cancel{T sin(\alpha + \sigma)} - mg\, cos(\gamma) = mV\frac{d\gamma}{dt}$$

$\Rightarrow$

$$D = mg\, sin(\bar{\gamma})$$

$$L = mg\, cos(\bar{\gamma})$$

## Gliding Flight

$$D = mg\,sin(\bar{\gamma})$$

$$L = mg\,cos(\bar{\gamma})$$

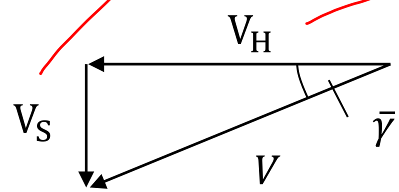
$$\frac{D}{L} = \frac{sin(\bar{\gamma})}{cos(\bar{\gamma})} = \tan(\bar{\gamma})$$

$$\tan(\bar{\gamma}) = \bar{\gamma} = \frac{V_S}{V_H}$$

$$\frac{c_D}{c_L} = \frac{V_S}{V_H} = \frac{\Delta h}{\Delta s}$$

sink speed

horiz. velocity

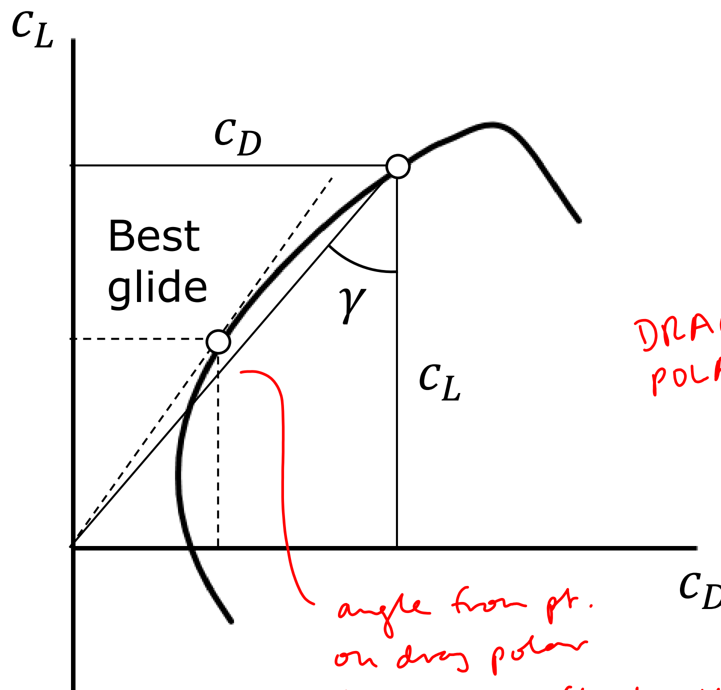



VELOCITY TRIANGLE

**Best glide**
Smallest flight path angle
Largest glide distance

Definition *glide ratio*

$$\frac{c_L}{c_D} = \frac{1}{\tan(\bar{\gamma})}$$




DRAG POLAR

angle from pt. on drag polar to origin = flight path angle (smallest <=> longest glide!)

## Gliding Flight

Form the equations of motion follows:

$L = mg\cos\bar{\gamma}$ $\qquad \cos\bar{\gamma} = \frac{c_L}{\sqrt{c_L^2 + c_D^2}}$

$$V = \sqrt{\frac{2mg}{\rho S_{ref} c_L} cos\bar{\gamma}} = \sqrt{\frac{2mg}{\rho S_{ref}\sqrt{c_L^2 + c_D^2}}}$$

$$V_S = V sin\bar{\gamma} = V\frac{c_D}{c_L} cos\bar{\gamma} = \sqrt{\frac{2mg}{\rho S_{ref}} \frac{c_D^2}{c_L^3} \cos^3\bar{\gamma}} = \sqrt{\frac{2mg}{\rho S_{ref}\sqrt{c_L^2 + c_D^2}} \frac{c_D^2}{c_L^2} \cos^2\bar{\gamma}}$$

$\frac{c_D}{c_L} = \tan\bar{\gamma} = \frac{\sin\bar{\gamma}}{\cos\bar{\gamma}}$

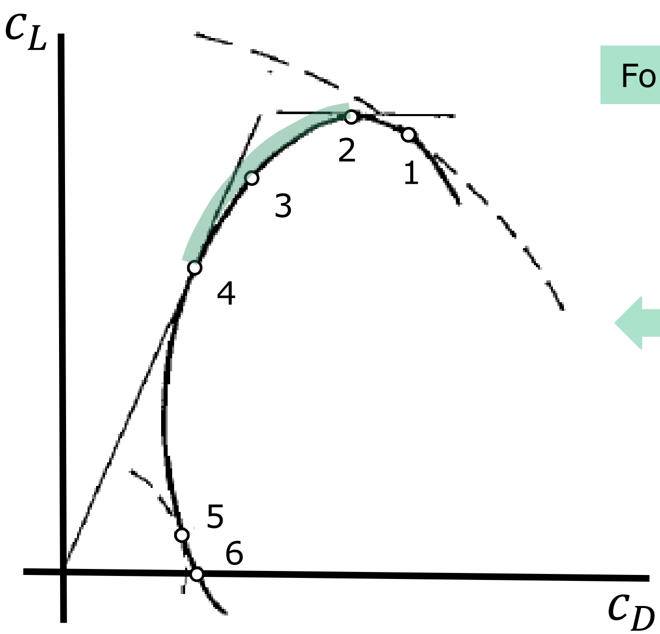
$$V_H = V cos\bar{\gamma} = \sqrt{V^2 - V_S^2} = \sqrt{\frac{2mg}{\rho S_{ref}\sqrt{c_L^2 + c_D^2}}\left(1 - \frac{c_D^2}{c_L^2}\cos^2\bar{\gamma}\right)}$$

For simplifications, use:

If $c_D \ll c_L$ $\qquad \sqrt{c_L^2 + c_D^2} = c_L$ $\qquad$ Small glide angles: $cos\bar{\gamma} = 1$

## *Gliding Flight – Speed Polar*

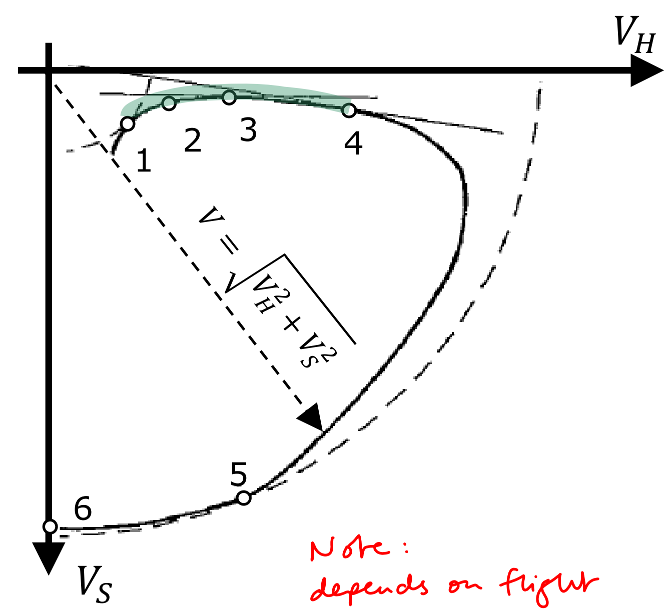
SPEED POLAR




For small flight path angles $\gamma$

$$V = \sqrt{\frac{2mg}{\rho S_{ref} c_L}}$$

$$V_S = \sqrt{\frac{2mg}{\rho S_{ref}} \frac{c_D^2}{c_L^3}}$$




only aerodynamics

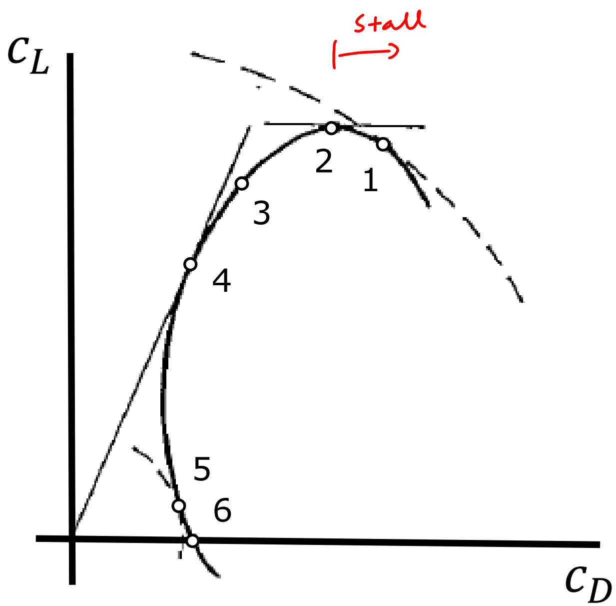
(indep. on flight condition)

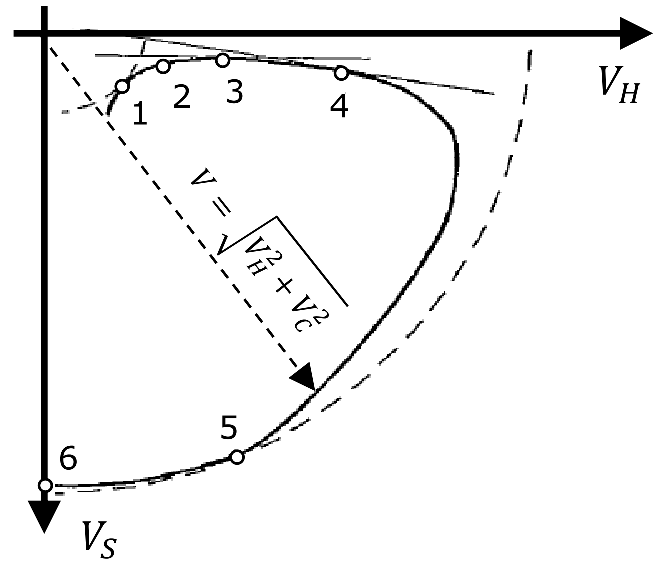
aerodynamics
+ aircraft data (mass, wing area)
+ atmospheric conditions (altitude, temperature, density)

Note:
depends on flight condition
(mass, altitude, etc.)

mass does not change glide angle,
but speed at which you glide optimally

## Gliding Flight – Velocities




3 Min. sink speed
4 Best glide angle

1 Min. velocity
2 Min. horizontal velocity
5 Max. velocity
6 Vertical dive

→ to stay in the air for longest

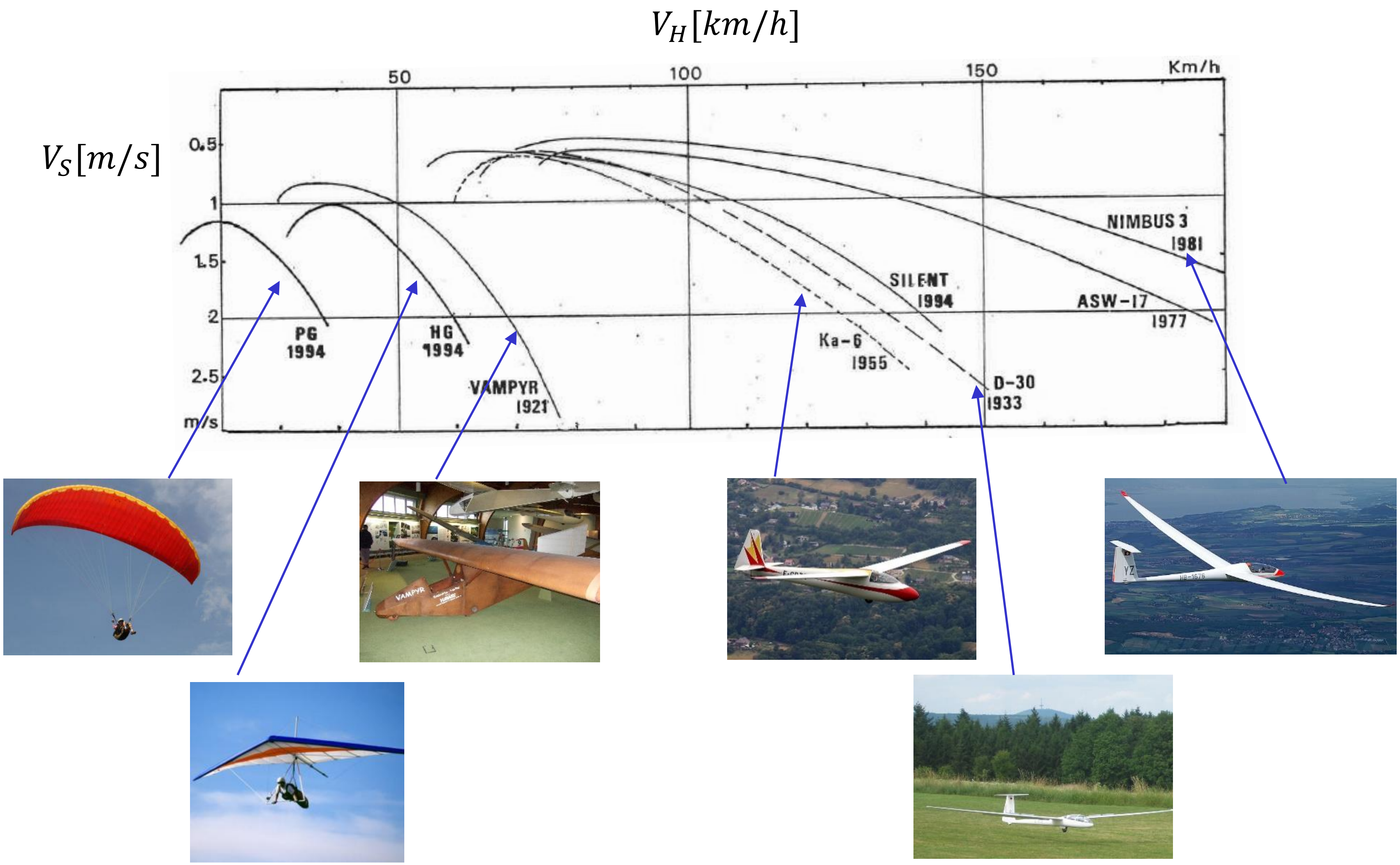
→ to cover most distance

$$V_{H,min} = \sqrt{\frac{2mg}{\rho S_{ref} c_{Lmax}}}$$

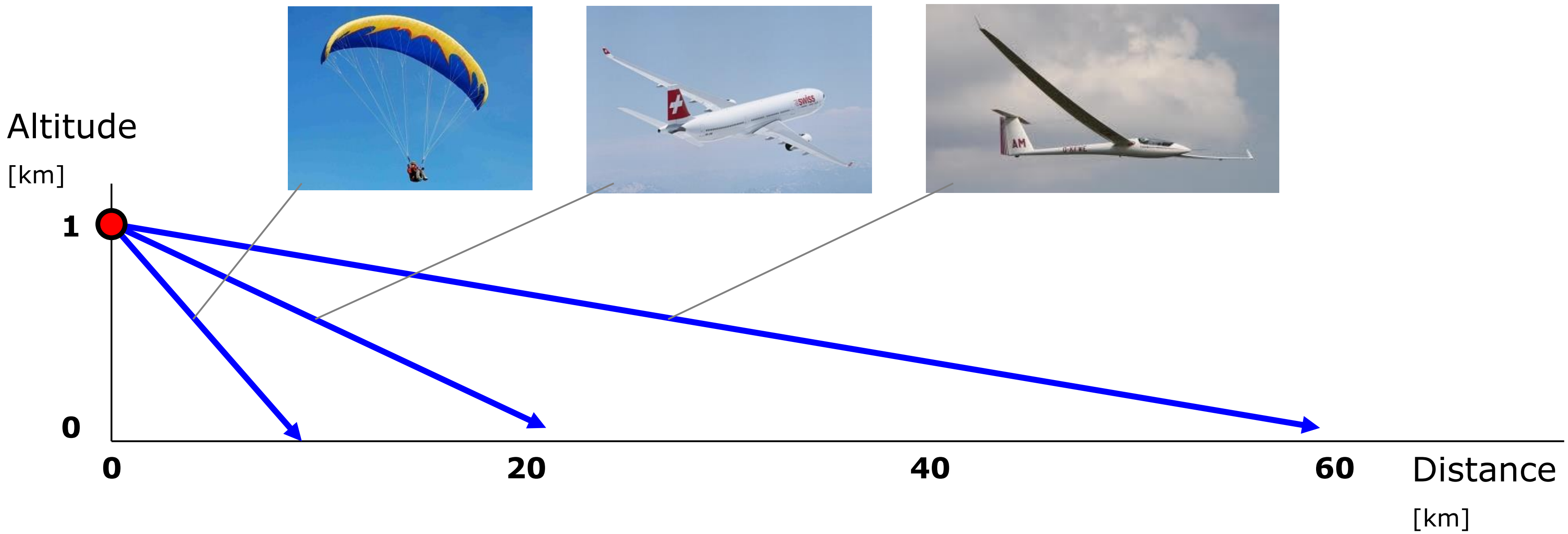
$$\tan(\bar{\gamma}_{min}) = \frac{1}{\left(\frac{c_L}{c_D}\right)_{max}}$$

$$V_{S,min} = \sqrt{\frac{2mg}{\rho S_{ref}} \frac{1}{\left(\frac{c_L^3}{c_D^2}\right)_{max}}}$$

## Gliding Flight



# *Gliding Flight*

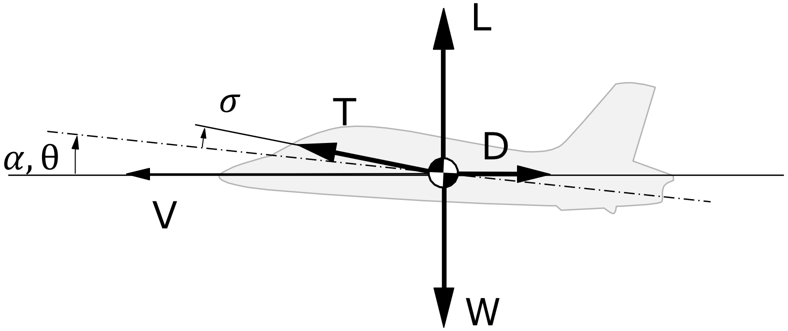





Source:
https://en.wikipedia.org/wiki/List_of_airline_flights_that_required_gliding

# Cruise Flight

# Horizontal Cruise Flight


$$\gamma = 0 \qquad \frac{d\gamma}{dt} = 0$$

$$T\cos(\alpha + \sigma) - D = m\frac{dV}{dt}$$

$$L + T\sin(\alpha + \sigma) - mg = 0$$

$$V = const. \quad \Rightarrow \quad m\frac{dV}{dt} = 0$$

# Horizontal Cruise Flight

$$\underbrace{T cos(\alpha + \sigma)}_{1} - D = 0$$

$$L + \underbrace{T sin(\alpha + \sigma)}_{0} - mg = 0$$

$\Longrightarrow$ Stationary, level cruise:

$$\boxed{T = D}$$

$$\boxed{L = mg}$$

w/ small angle approx.

Drag force characteristics (function of airspeed)

$$D = \frac{1}{2}\rho V^2 S_{ref} c_D$$

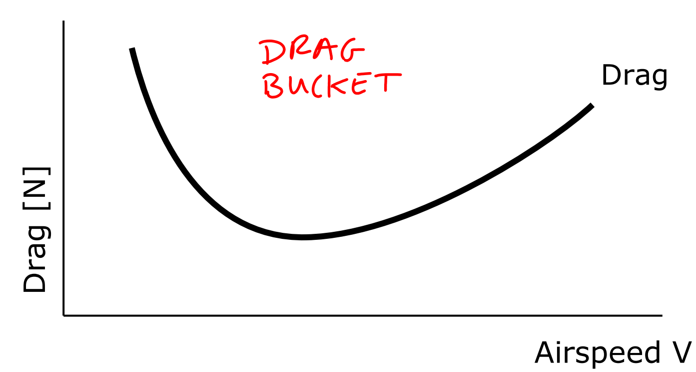
$$c_D = c_{D0} + k c_L^2$$

(simplified model) ⚠ parabolic drag model

$$D = \frac{1}{2}\rho V^2 S_{ref} c_{D0} + \frac{1}{2}\rho V^2 S_{ref} k c_L^2 \quad (1)$$

$$L = \frac{1}{2}\rho V^2 S_{ref} c_L = mg \quad \Longrightarrow \quad \mathrm{c_L} = \frac{2mg}{\rho V^2 S_{ref}} \quad (2)$$

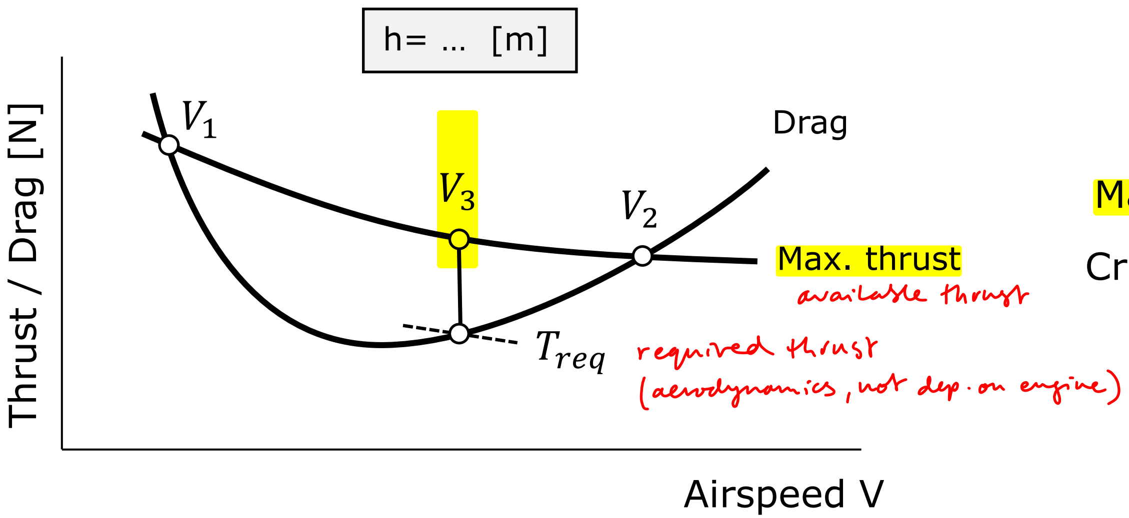
$$(2) \rightarrow (1) \quad D = \frac{1}{2}\rho V^2 S_{ref} c_{D0} + k \frac{(mg)^2}{\frac{1}{2}\rho V^2 S_{ref}}$$



# Horizontal Cruise Flight

Stationary horizontal cruise at maximum thrust is only possible at $V_1$ or $V_2$

$V_1 < V < V_2$ : velocity range    Note: $V_{min} = V_1$ or $V_{Stall}$



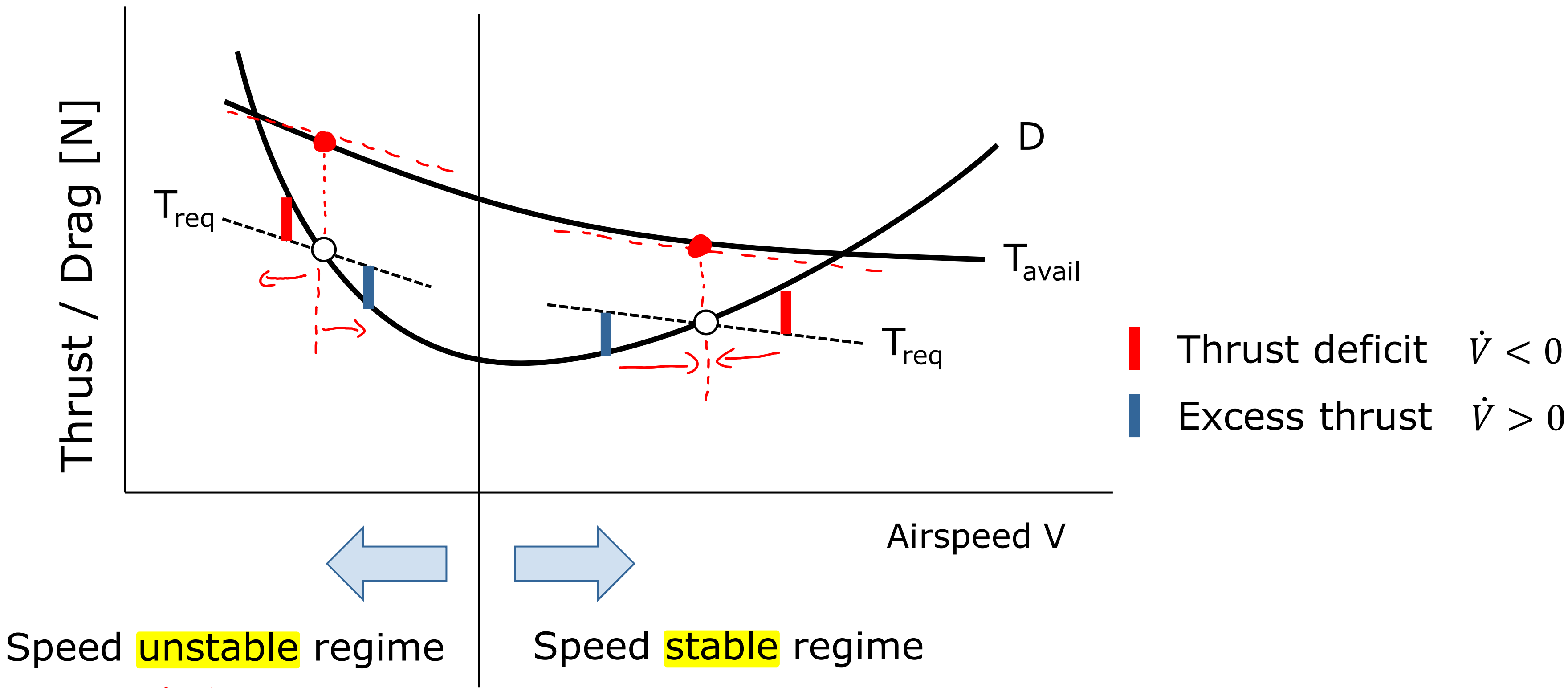

Max thrust = thrust available

Cruise drag = thrust required

At $V_3$ there is an excess thrust $(T_{max} - D) > 0$ ⇨ SEP>0

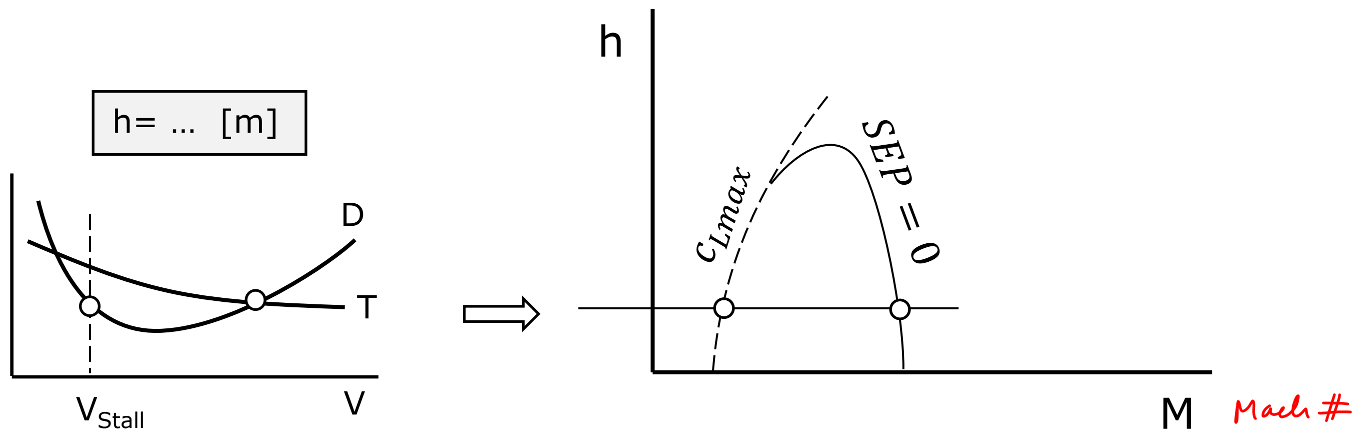
For stationary flight at $V_3$: $T_{req} = D$

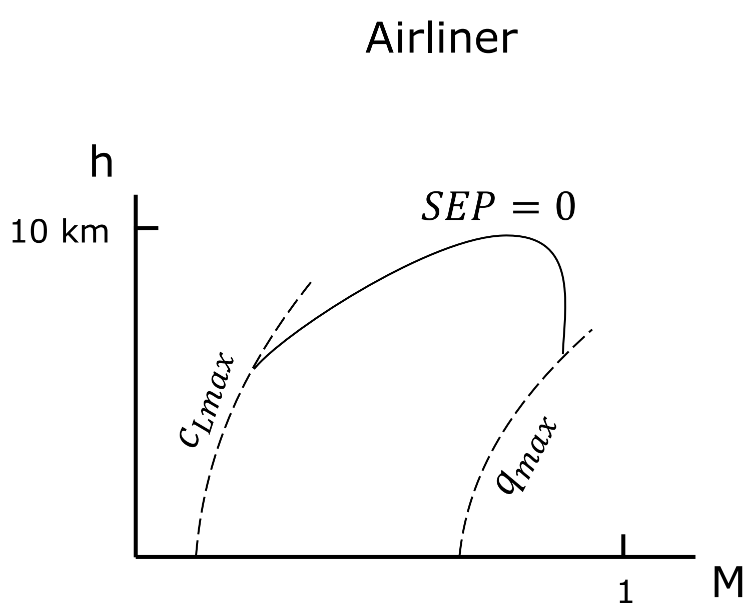
## *Horizontal Cruise Flight – Speed Stability*

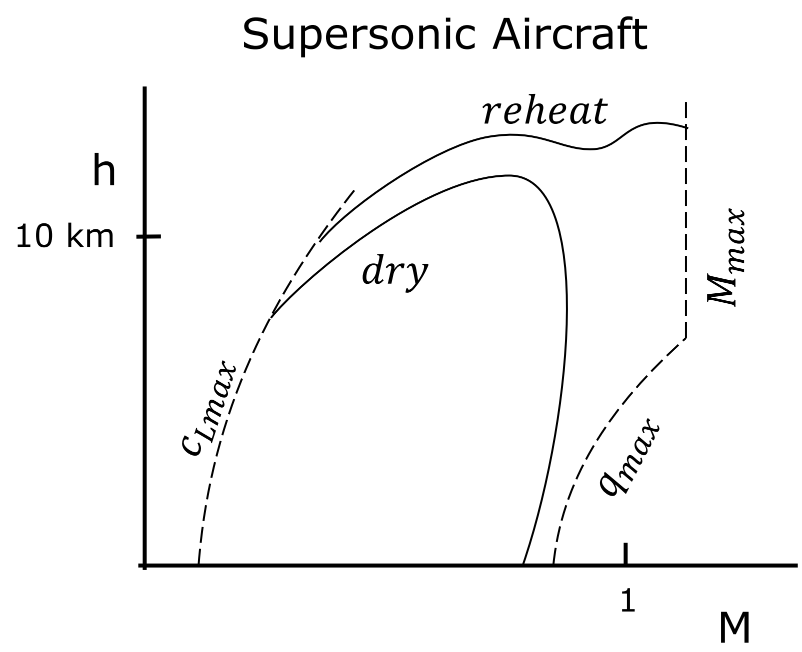



need to change thrust setting to stabilize!

## *Horizontal Cruise Flight – Envelope*



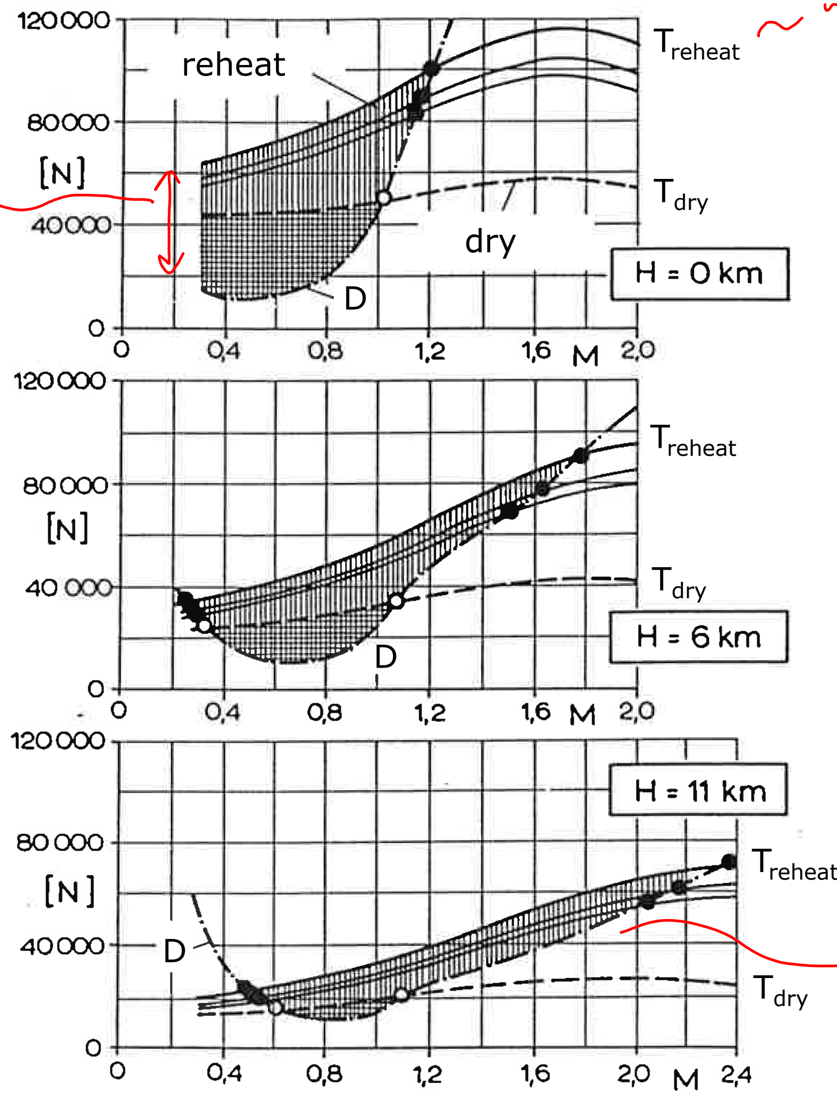



# Horizontal Cruise Flight – Supersonic

T–D high ⟺ SEP high ⟹ high accel.

DRAG

~ w/ afterburner




Reheat / afterburner thrust

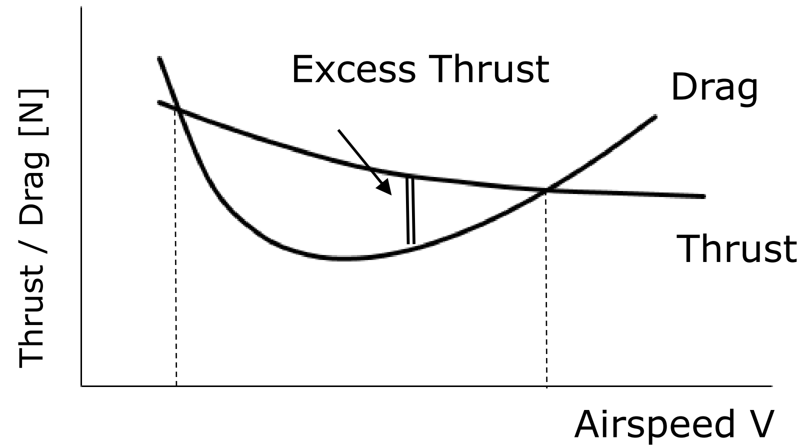
Note: $D_{min}$ does not change for different H

low SEP (T – D) ⟹ slow accel.

## *Analytical solutions*

This requires analytical models for the drag (thrust required) **AND** the thrust! ⚠

↳ depends on altitude (less dense air => less fuel can be injected)




Examples:

**Propulsion (Thrust)**

Turbojet engine: T = const. (only a function of altitude, not airspeed)
Turboprop engine: P = const. (P=TV)

plus some altitude dependency modeled with the ISA atmosphere (density)

**Aerodynamics (Drag)**

$c_D = c_{D0} + k c_L^2$ (simplified model)

$$D = \frac{1}{2}\rho V^2 S_{ref} c_{D0} + k \frac{(mg)^2}{\frac{1}{2}\rho V^2 S_{ref}} \quad \text{with} \quad k = \frac{1}{\pi A R e}$$

## Horizontal Cruise Flight

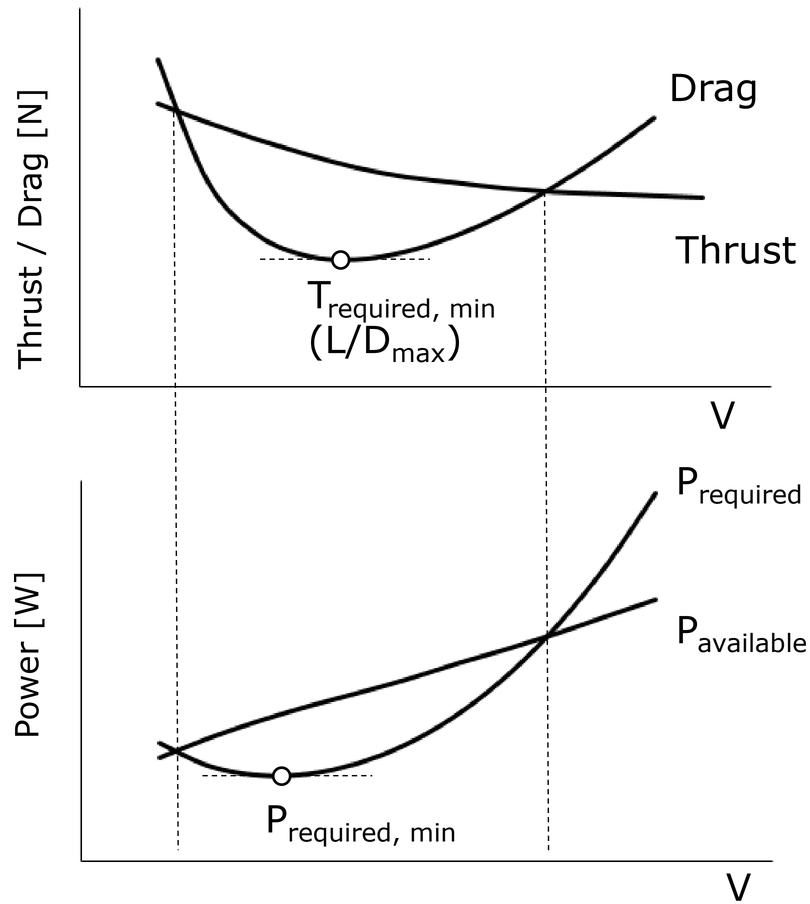
**Required thrust** (Drag)

$$T_{req} = \frac{1}{2}\rho S_{ref} c_{D0}\ V^2 + \frac{2}{\pi A R e}\frac{(mg)^2}{\rho S_{ref}}\frac{1}{V^2}$$

**Required power** multiply drag by V

$$P_{req} = DV = T_{req}V$$

$$P_{req} = \frac{1}{2}\rho S_{ref} c_{D0}\ V^3 + \frac{2}{\pi A R e}\frac{(mg)^2}{\rho S_{ref}}\frac{1}{V}$$




Procedure: set $\frac{dP_{req}}{dV} = 0$ or $\frac{dT_{req}}{dV} = 0$ to find speeds and corresponding $c_L$

# Horizontal Cruise Flight

**Available thrust**

Jet $T_{avail} = const.$

Prop $T_{avail} = \dfrac{P_{avail}}{V}$ $P_{avail} = const.$

**Available power**

Jet $P_{avail} = T_{avail}V$ $T_{avail} = const.$

Prop $P_{avail} = const.$

**Specific Excess Power (SEP)**

$$P_s = (P_{available} - P_{requried})/mg$$

$$P_s = V\frac{(T-D)}{mg}$$