ETH zürich

Robot Dynamics

Fixed-wing UAVs: Dynamic Modeling and Control

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Aerial Robotics: Lectures and Exercises

22.11 (today)	fixed-wing lecture
23.11	no exercise
29.11	rotary-wing lecture
30.11	no exercise
06.12	fixed-wing case-study
07.12	fixed-wing exercise
09.12	*RSL & ASL open lab (17:00 -20:00)*
13.12	rotary-wing case-study
14.12	rotary-wing exercise

Contents | Fixed-wing UAVs

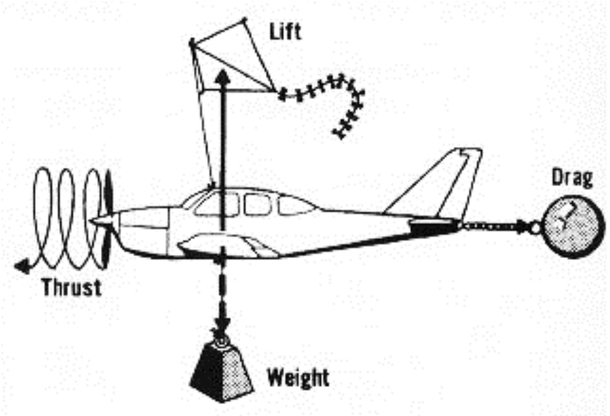
1. Introduction
2. Aerodynamic Basics
3. Aircraft Dynamic Modeling
4. Fixed-wing Control

Introduction

Introduction | Fixed-wing aircraft

https://upload.wikimedia.org/wikipedia/commons/a/a5/Quadrotor_drone_in_blue_ski_01.JPG

- Definition: fixed-wing aircraft are capable of flight using **wings** that generate **lift** caused by the vehicle's **forward airspeed** and the **shape (geometry)** of the wings.
- → **Aerodynamic lift**



https://www.flightsimbooks.com/flightsimhandbook/43-1.jpg

Introduction | Fixed-wing aircraft

https://upload.wikimedia.org/wikipedia/commons/a/a5/Quadrotor_drone_in_blue_ski_01.JPG

"If the wings are traveling faster than the fuselage, it's probably a helicopter...and therefore, unsafe."

Fixed-wing aircraft:

- Efficient → long range / endurance (cover larger areas, faster, and stay airborne longer!)
- Mechanically simple
- Require infrastructure (e.g. catapult for take-off, landing strip/arresting system)
- Unless hybrid – no vertical take-off/landing (VTOL)

Rotary-wing aircraft:

- Highly maneuverable
- VTOL capable, no infrastructure needed
- Mechanically simple (multirotor) or complex (single-rotor / helicopter)
- Inefficient (short range / endurance)

Introduction | "Small" fixed-wing UAVs

Introduction | “Small” fixed-wing UAVs

- Wide variety of configurations for different purposes / applications
- No community standard – requires modeling and specific tuning per platform!

Aerodynamic Basics

Aerodynamic Basics | Basic Principles

viscosity: how much fluid resists motion

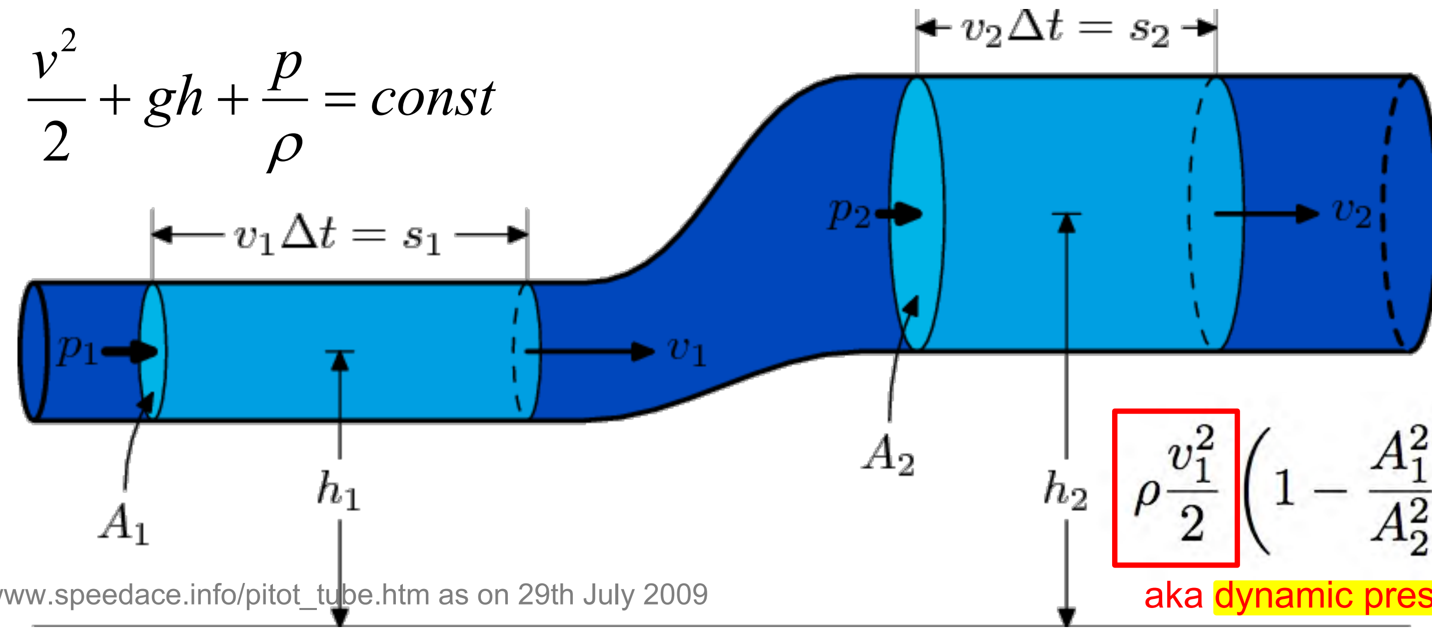
Analysis on differential volumes:

no viscosity

- Navier-Stokes Equation (viscous, compressible flows)
- Without viscosity: Euler Equation air: "inviscid"
- Incompressible along streamline: **Bernoulli Equation** (up to close to Mach 1 valid)

$\rho = const.$

$$\frac{v^2}{2} + gh + \frac{p}{\rho} = const$$

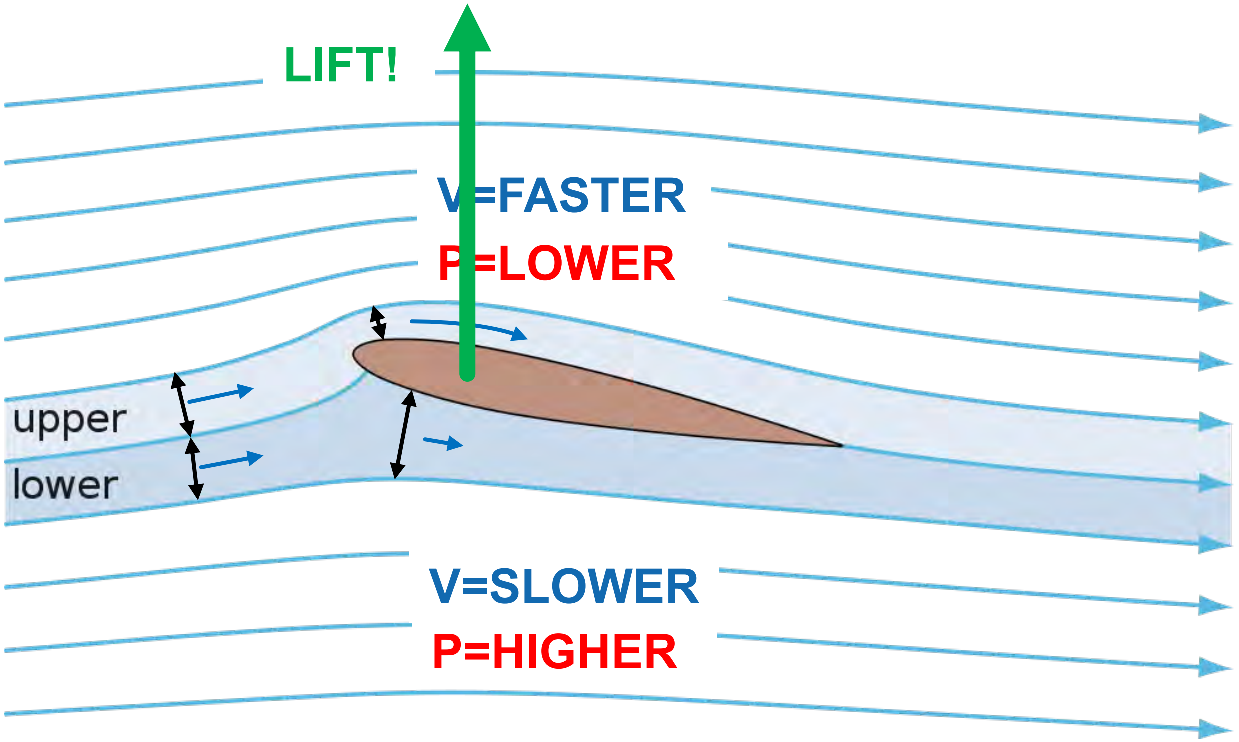


$$\boxed{\rho \frac{v_1^2}{2}} \left(1 - \frac{A_1^2}{A_2^2}\right) = p_2 - p_1$$

aka dynamic pressure

www.speedace.info/pitot_tube.htm as on 29th July 2009

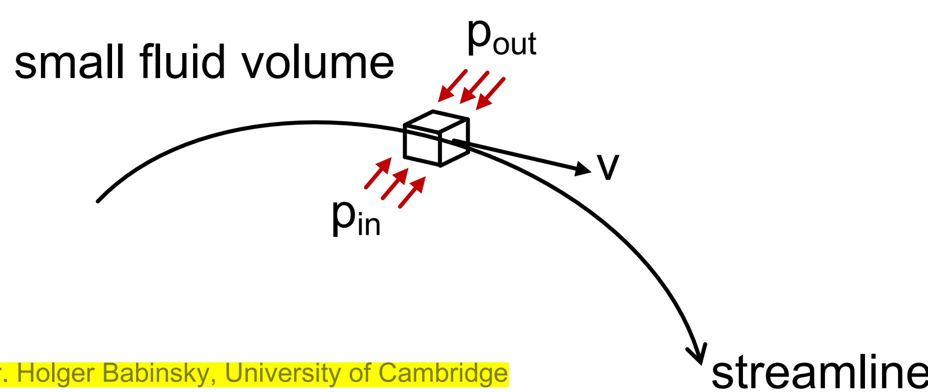
Aerodynamic Basics | Basic Principles



https://commons.wikimedia.org/wiki/File:Streamlines_around_a_NACA_0012.svg

Aerodynamic Basics | Basic Principles

- Watch out for common misconceptions of lift!
  - E.g. the "distance traveled" argument for speed difference — BS
- But…Bernoulli isn't the whole story!
- A second perspective:
  - streamline curvature ↔ pressure gradients



s.t. particle stays on streamline

centripetal force:
$A^*p_{out} > A^*p_{in}$

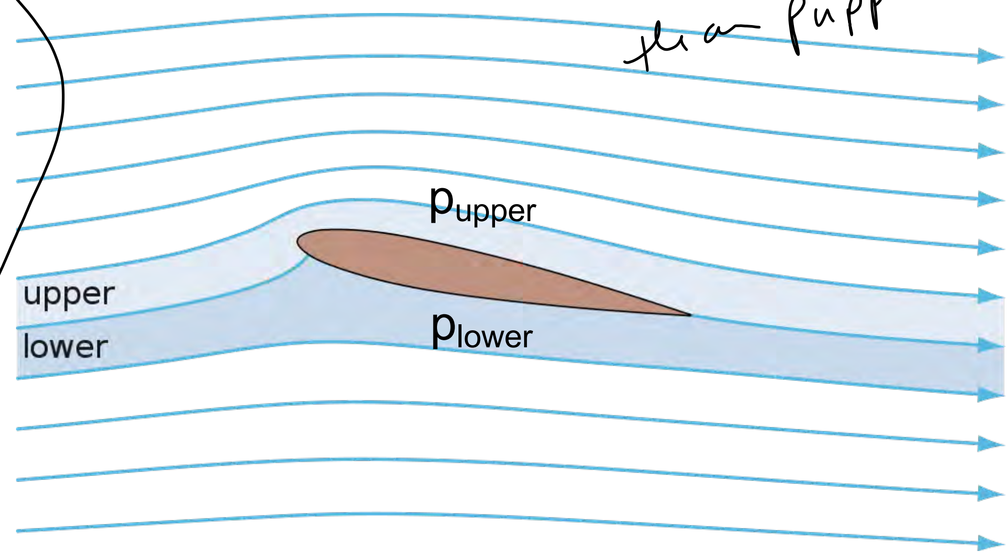
Nice lecture by Dr. Holger Babinsky, University of Cambridge
https://www.youtube.com/watch?v=XWdNEGr53Gw

Aerodynamic Basics | Basic Principles

- $p_{upper} < p_{atm}$
- $p_{lower} > p_{atm}$
- Therefore: $p_{upper} < p_{lower}$

LIFT!

p_{atm} – should be higher than p_{upper}



p_{atm}

should be lower than p_{lower}!

https://commons.wikimedia.org/wiki/File:Streamlines_around_a_NACA_0012.svg

Laminar flow: particles along smooth streamline

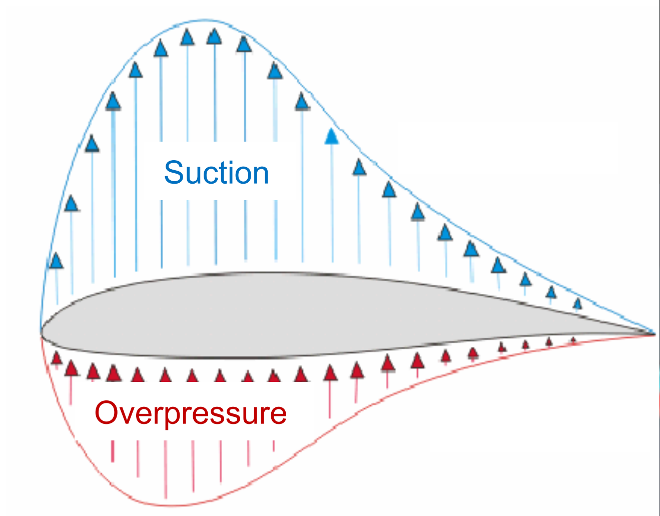
separated flow: flow has reversed direction

Aerodynamic Basics | Airfoils

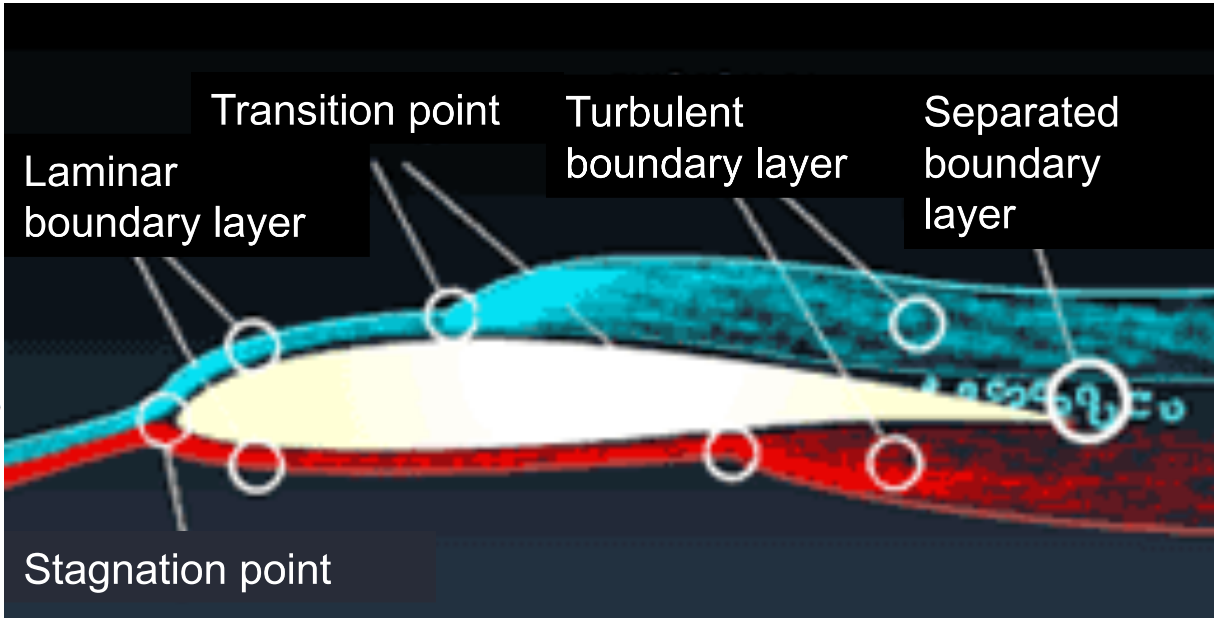
could still be attached i.e. generate lift

2-Dimensional Flow Analysis

- Flow field (pressure distribution, laminar/turbulent) highly dependent on **angle of attack**, Reynolds number and Mach number



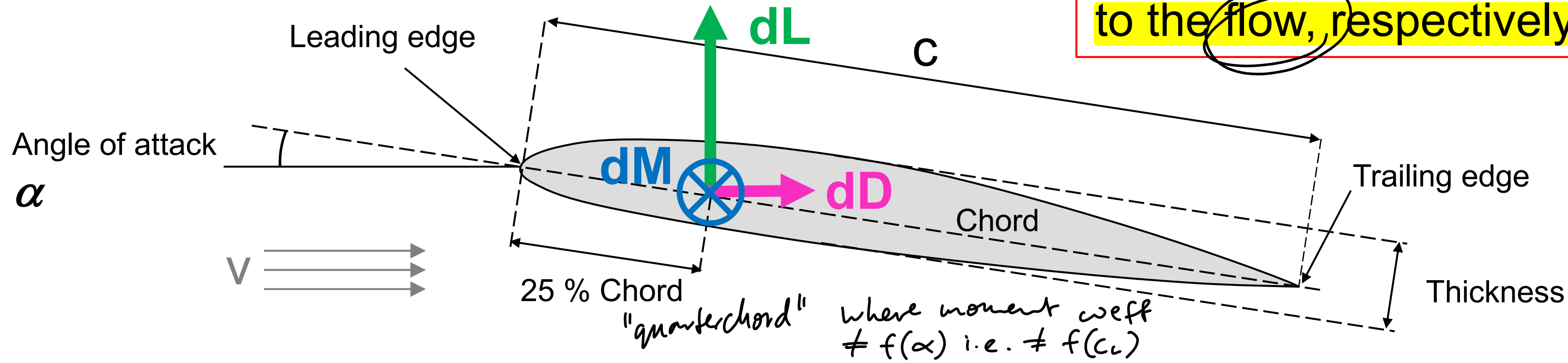
www.thuro.at/aerodynamik2.htm



http://www.thuro.at/anims/abloesung.gif

Aerodynamic Basics | Airfoils

Lift and **drag** are defined **perpendicular** and **parallel** to the flow, respectively.



Pressure distribution can be reduced to two forces and one moment ***per unit length*** *(spanwise/out of slide)*:

Lift force $$dL = C_l \frac{\rho}{2} \overbrace{c \cdot \underset{\text{along span}}{dy}}^{\text{area}} \cdot V^2$$

Drag force $$dD = C_d \frac{\rho}{2} c \cdot dy \cdot V^2$$

Moment $$dM = C_m \frac{\rho}{2} c^2 \cdot dy \cdot V^2$$

ρ : Density of fluid (air) [kg/m³]

c : Chord length [m]

V : Flight speed (w.r.t. air) [m/s]

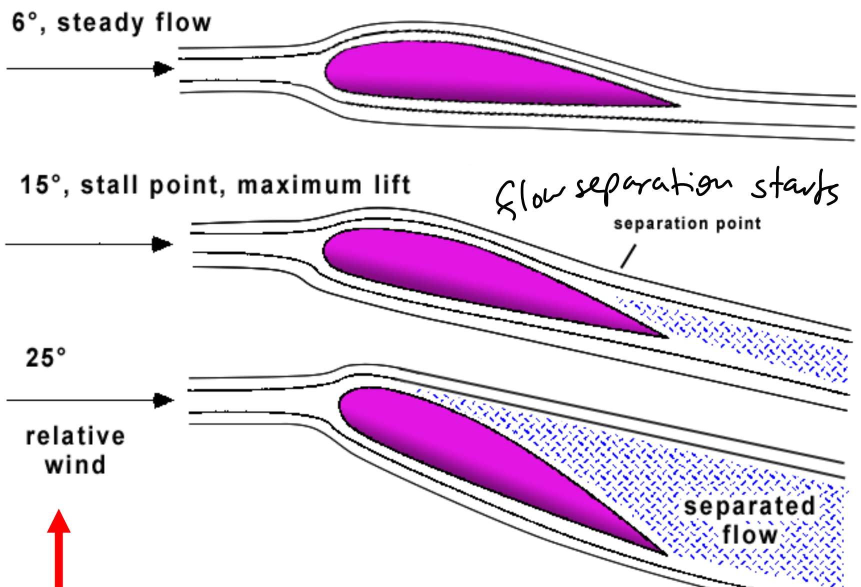
C_l : Airfoil lift coefficient [-]

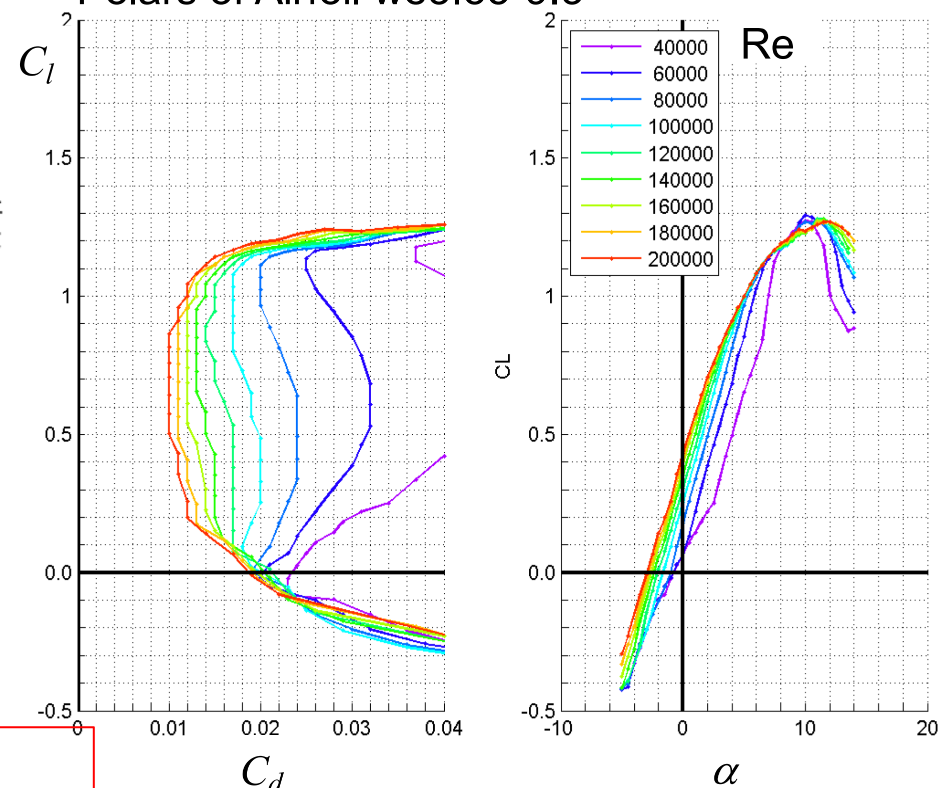
C_d : Airfoil drag coefficient [-]

C_m : Airfoil moment coefficient [-]

Aerodynamic Basics | Angle of attack / stall

Stall when flow starts to separate (lift ↓↓)





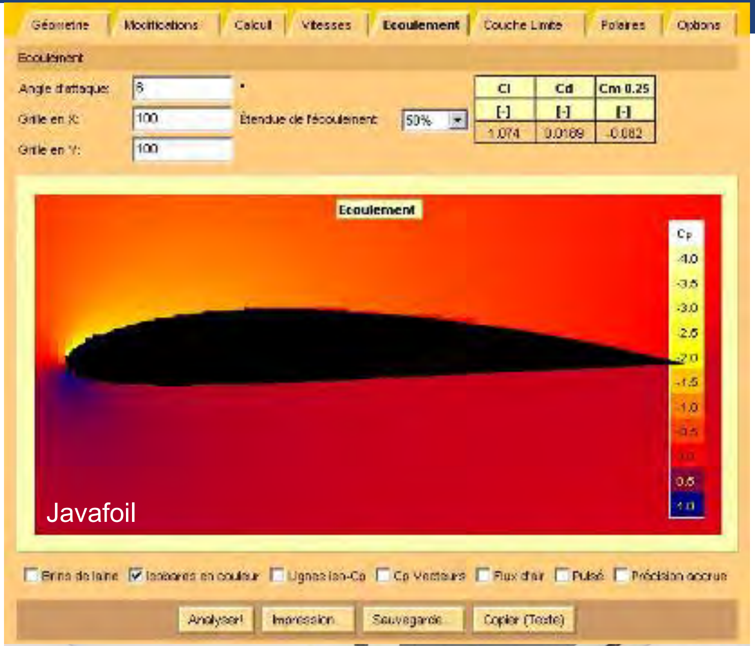
Note: These are just representative numbers. The stall angle will vary per airfoil!

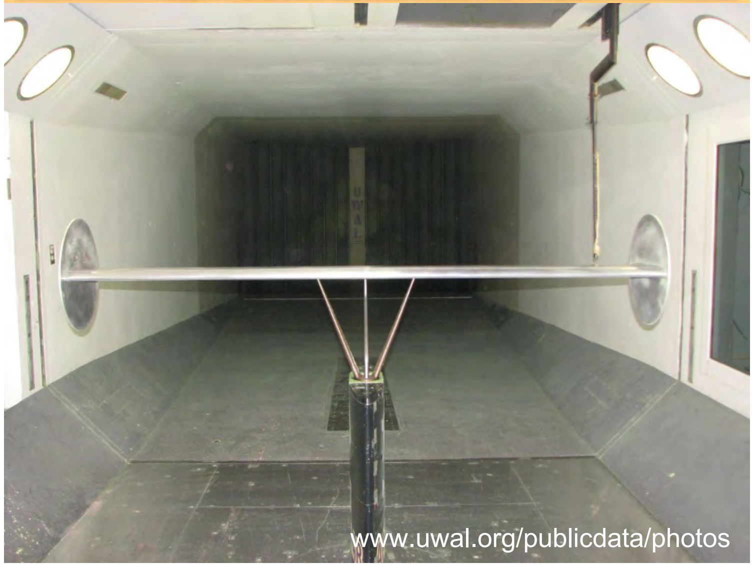
// purposefully introduce turbulent flow on wing s.t. flow stays attached longer

Airfoil Lift, Drag and Moment

Methods to determine airfoil lift, drag and moment coefficients:

- **Theoretically** using 2D-CFD software
  - Javafoil
    http://www.mh-aerotools.de/
  - Xfoil
    http://raphael.mit.edu/xfoil/
- **Experimentally** in a *wind tunnel*
  - Extruded airfoil mounted on a measurement system
  - Laminar flow produced by fans





T. Stastny, R. Siegwart. "Nonlinear Model Predictive Guidance for Fixed-wing UAVs Using Identified Control Augmented Dynamics". International Conference on Unmanned Aerial Systems (ICUAS). 2018.

Airfoil Lift, Drag and Moment

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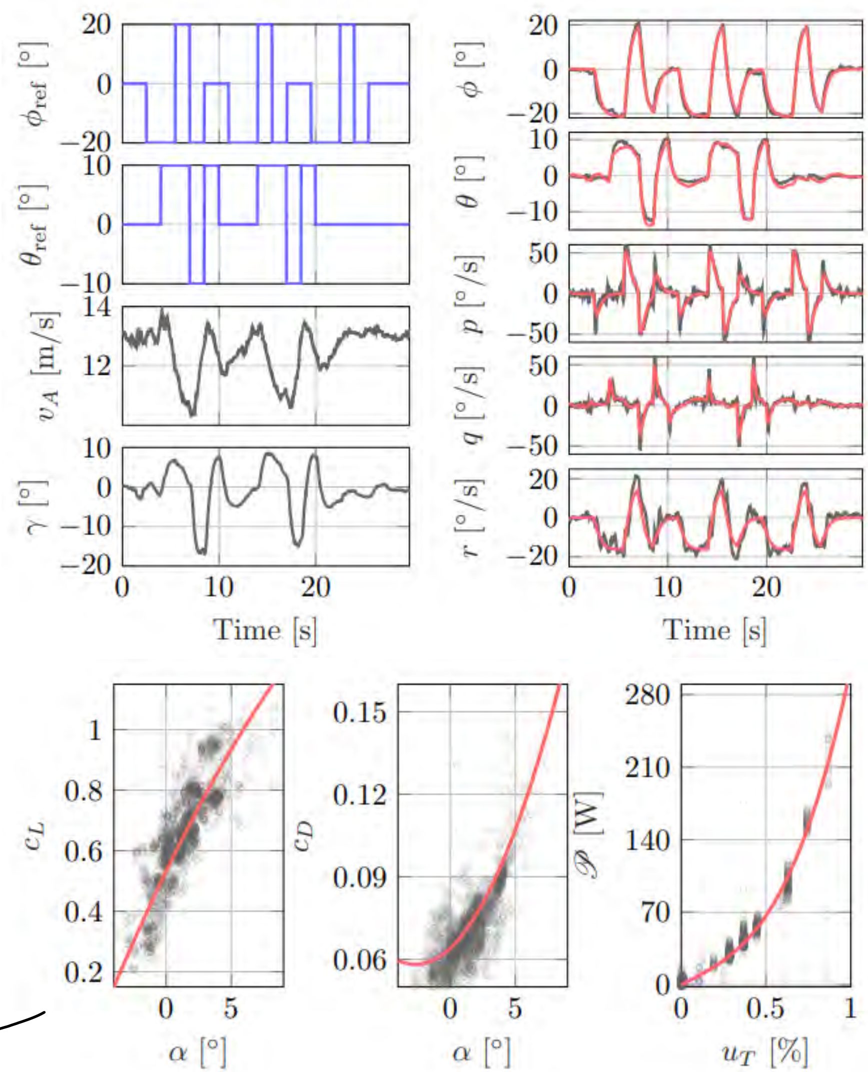
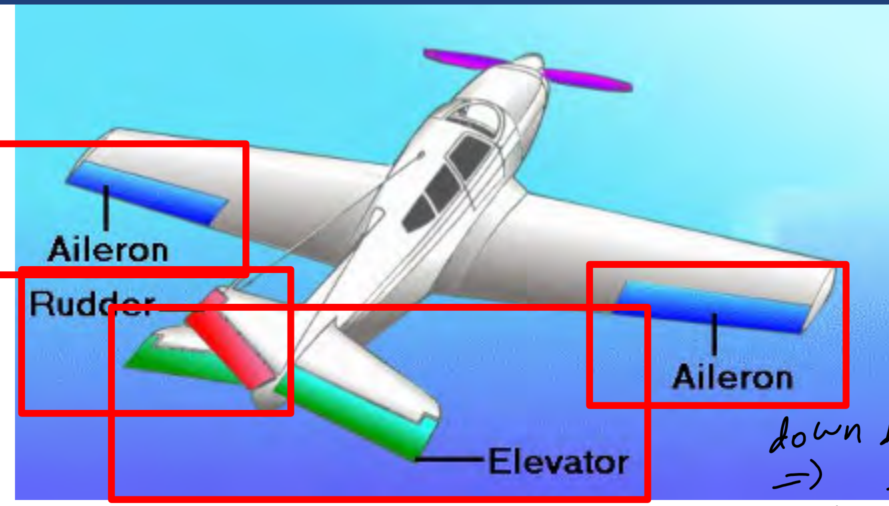
- **Experimentally** in a *wind tunnel*
  - Extruded airfoil mounted on a measurement system
  - Laminar flow produced by fans
- **Experimentally** from *flight data*
  - System identification on logged sensor measurements of static and dynamic maneuvers



C omme

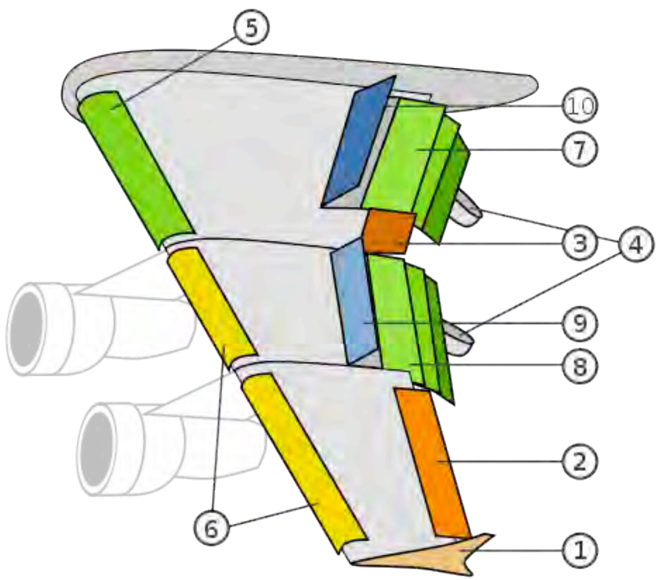
Control surfaces

- For **small** airplanes, the standard control surfaces are:
  - Ailerons (rolling)
  - Elevator (pitching)
  - Rudder (yawing)



down => left roll

- For larger airplanes, they can be more complex…



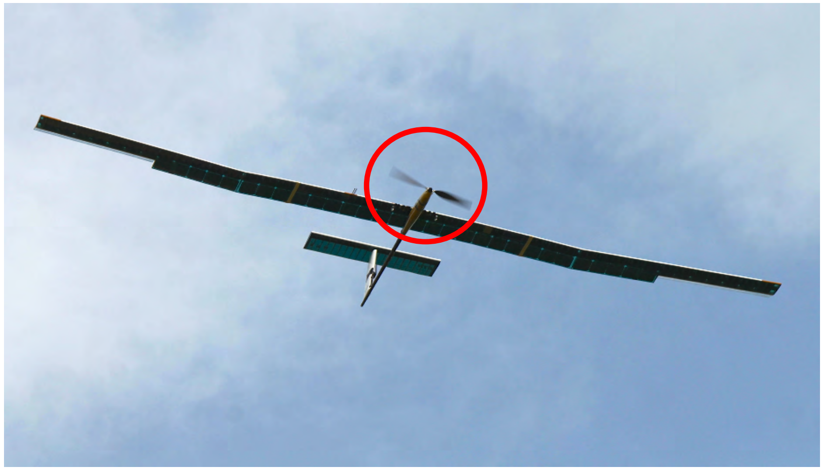
Ailerons:
2. Low-Speed Aileron
3. High-Speed Aileron

Lift increasing flaps and slats:
4. Flap track fairing
5. Krüger flaps
6. Slats
7. Three slotted inner flaps
8. Three slotted outer flaps

Spoilers:
9. Spoilers
10. Spoilers-Air brakes

Propulsion Group | Placement

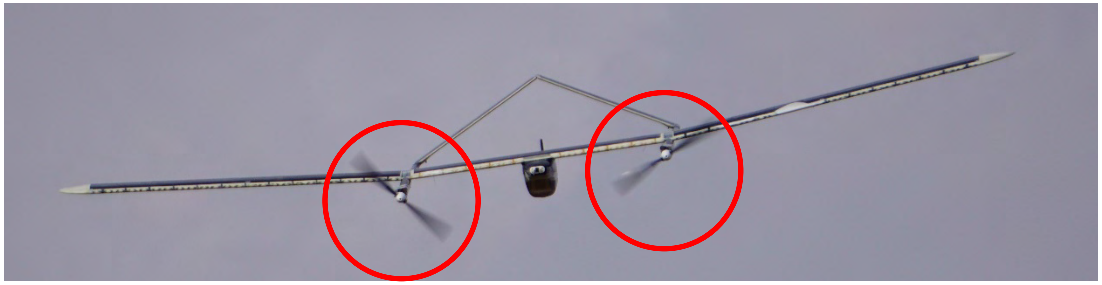
pusher e.g. for camera



In the front…

In the back…

On the wings…



Aircraft Dynamic Modeling

Why Model the Dynamics of an Airplane?

- **System analysis**: model allows evaluating flight characteristics
  - Stability
  - Controllability
  - Power required ➔ fuel needs
  - Controllability in the case of actuator failure
- **Autopilot design and simulation**: model allows comparing different control techniques and autopilot parameter tuning (test in sim)
  - Gain of time and money
  - Closed-loop control analysis, higher performance of the autopilot
  - No risk of damage compared to real tests
- **Pilot training (in Simulator)**
  - Allows simulating and training especially emergency situations

Aircraft Dynamic Modeling | Intro

- Dynamics of an airplane
  ... Are very different from an acrobatic aircraft to a jetliner airplane
  ... but the principles remain the same for all
  - Wings, stabilizers roll yaw pitch incr. lift for take off & landing slow plane down
  - control surfaces (ailerons, rudder, elevator, flaps, spoilers, …)
  - propulsion group (motor-gearbox-propeller, turbine, rocket, …)

- In this lecture, we will model a typical fixed-wing UAV
  1. Assumptions/simplifications
  2. Kinematics (frames of reference + state definitions)
  3. Dynamics (forces + moments)
  4. Equations of motion (Newton)

Aircraft Dynamic Modeling | Intro

~ 1st order time constant

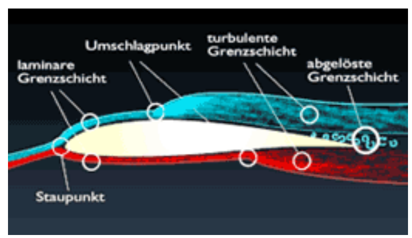
$\mathbf{u}$

Actuator Dynamics

δ

Propulsion

Aerodynamics



$\mathbf{F}, \mathbf{M}$

Body Dynamics

$\dot{\mathbf{x}}$

$\int$

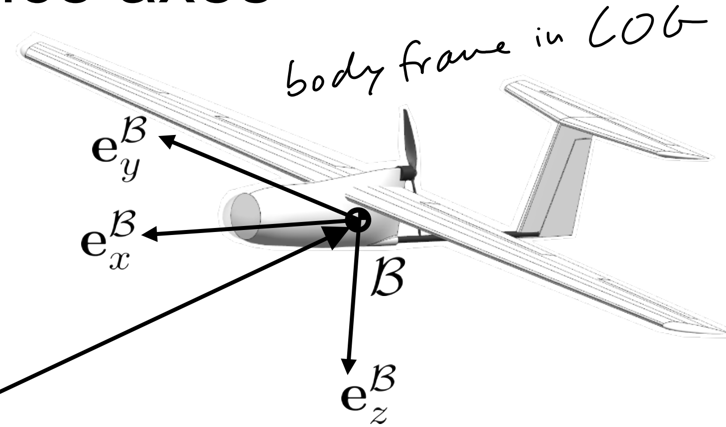
Aircraft Dynamic Modeling | Assumptions and simplifications

- **Definitions**
  - Origin of body-fixed coordinate frame set into center of gravity
- **Assumptions and simplifications**
  - **Rigid** and symmetric structure: constant, (almost) diagonal inertia matrix
  - Constant mass (electric aircraft)
  - Motor without dynamics and without gyroscopic effects (can be adapted)
  - Aerodynamics:
    - We don't enter stall (operation in the linear lift domain)
    - Lump fuselage lift/drag
    - Neglect side-force from sideslip
    - No interference effects (e.g. prop wash)

Aircraft Dynamic Modeling | On rigidity …

Aircraft Kinematics

Aircraft Kinematics | Reference axes



Inertial (local) reference frame $\mathcal{I}$

- North, East, **Down** (or NED)
- *Flat Earth assumption*
- Where to define the origin?
  - E.g. UTM (Universal Transverse Mercator) coordinates
  - "Home" (or start) position ← more typical for MAV

$_{\mathcal{I}}\mathbf{r}$

inertial frame

$\mathcal{I}$

$$\mathbf{e}_x^{\mathcal{I}} \equiv N \ (\text{Northing})$$

$$\mathbf{e}_y^{\mathcal{I}} \equiv E \ (\text{Easting})$$

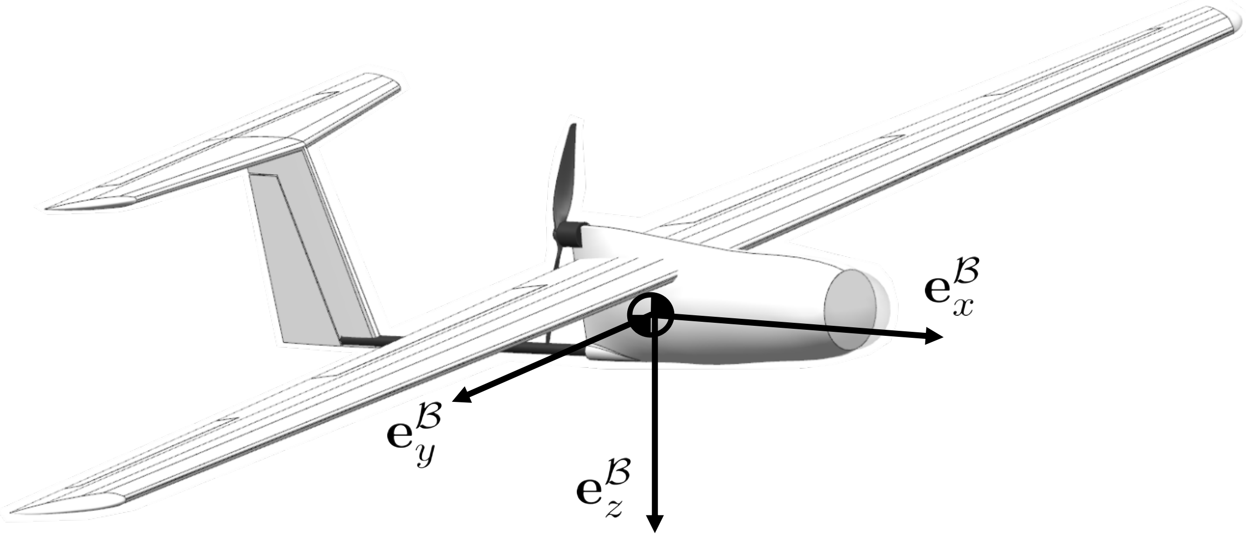
$$\mathbf{e}_z^{\mathcal{I}} \equiv D \ (\text{Down})$$

NED aerospace convention

Body-fixed frame $\mathcal{B}$

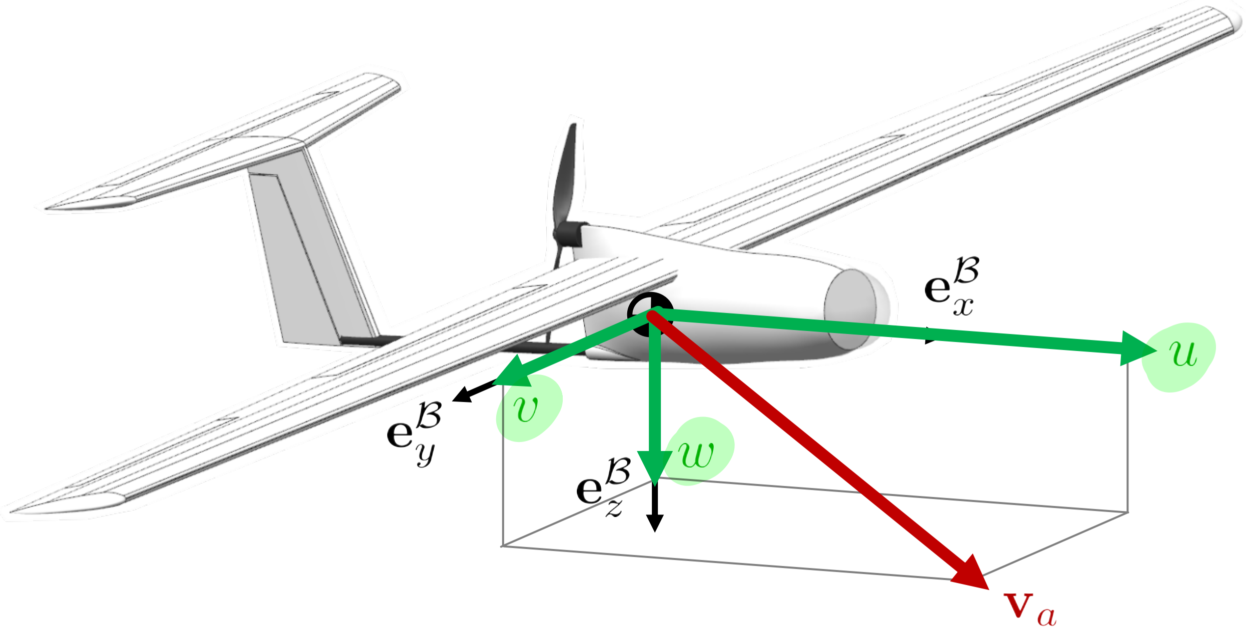
- x out the nose
- y out the **right** wing
- z (with right-hand rule) down
- Origin located at center of gravity

Aircraft Kinematics | Reference axes



Body-frame $\mathcal{B}$

Aircraft Kinematics | Reference axes



Body-frame $\mathcal{B}$

Body velocity:

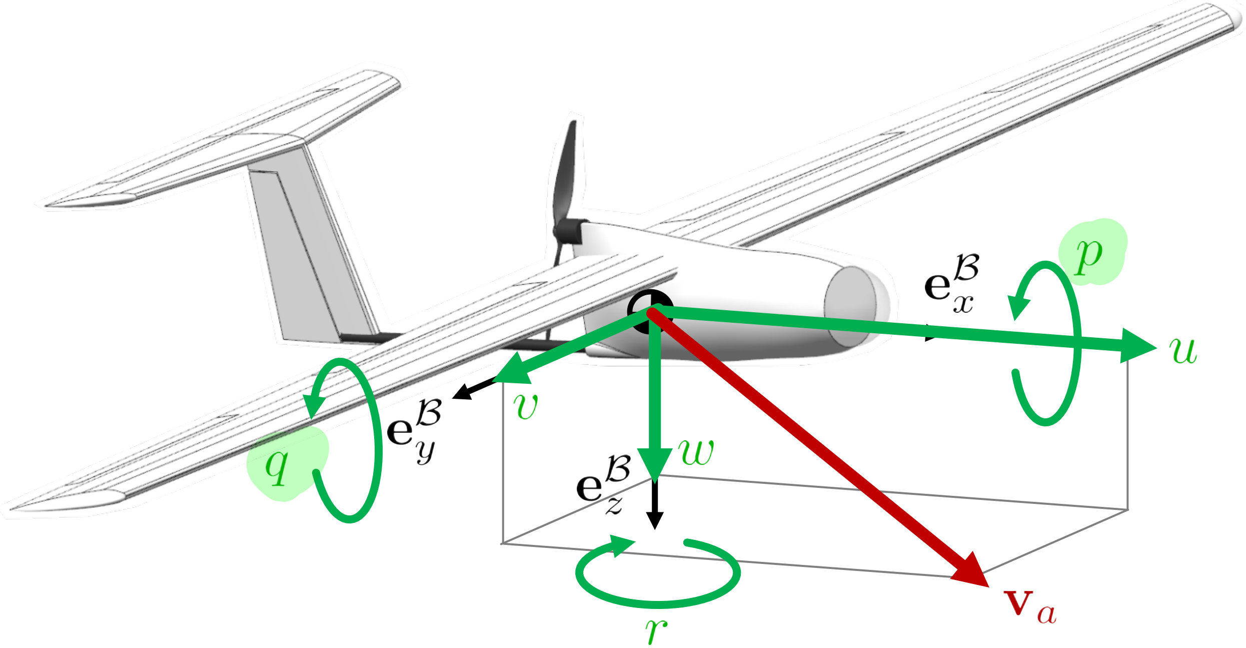
$$_{\mathcal{B}}\mathbf{v}_a = (u, v, w)^T$$

- Air-mass relative speed (airspeed):

$$V = \sqrt{u^2 + v^2 + w^2}$$

- V is always positive

Aircraft Kinematics | Reference axes



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Body rates:

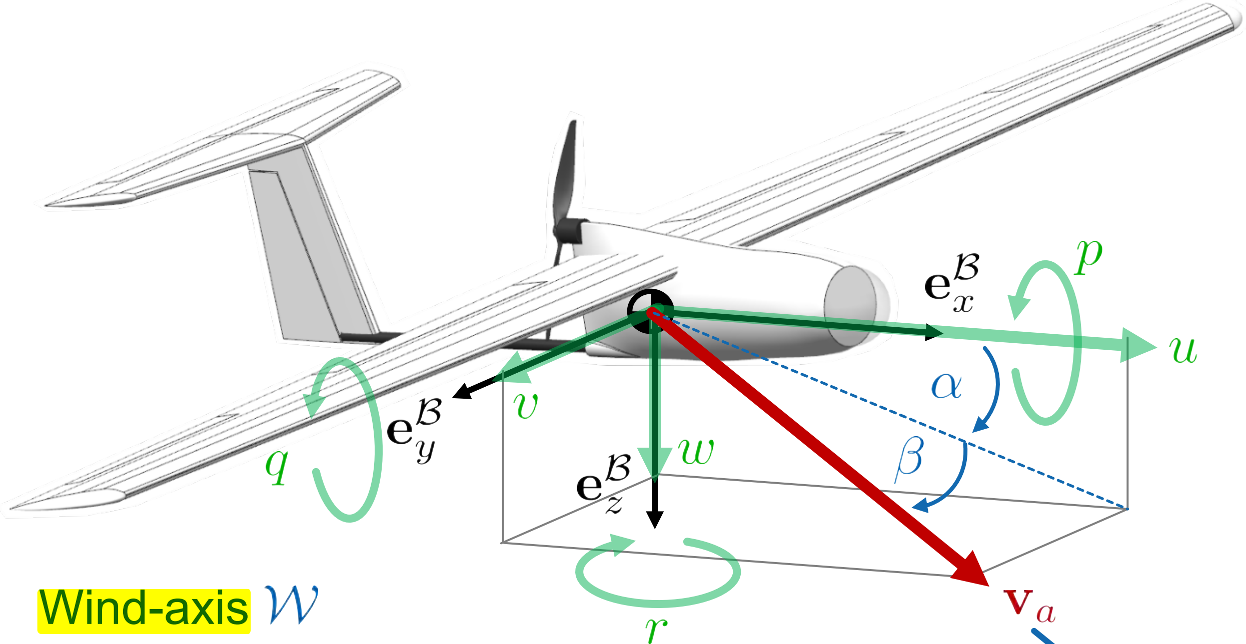
$$_{\mathcal{B}}\omega = (p, q, r)^T$$

roll rate

pitch rate

yaw rate

Aircraft Kinematics | Reference axes



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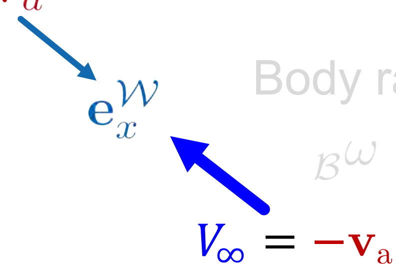
Body rates:

$${}_{\mathcal{B}}\omega = (p, q, r)^T$$

Wind-axis $\mathcal{W}$

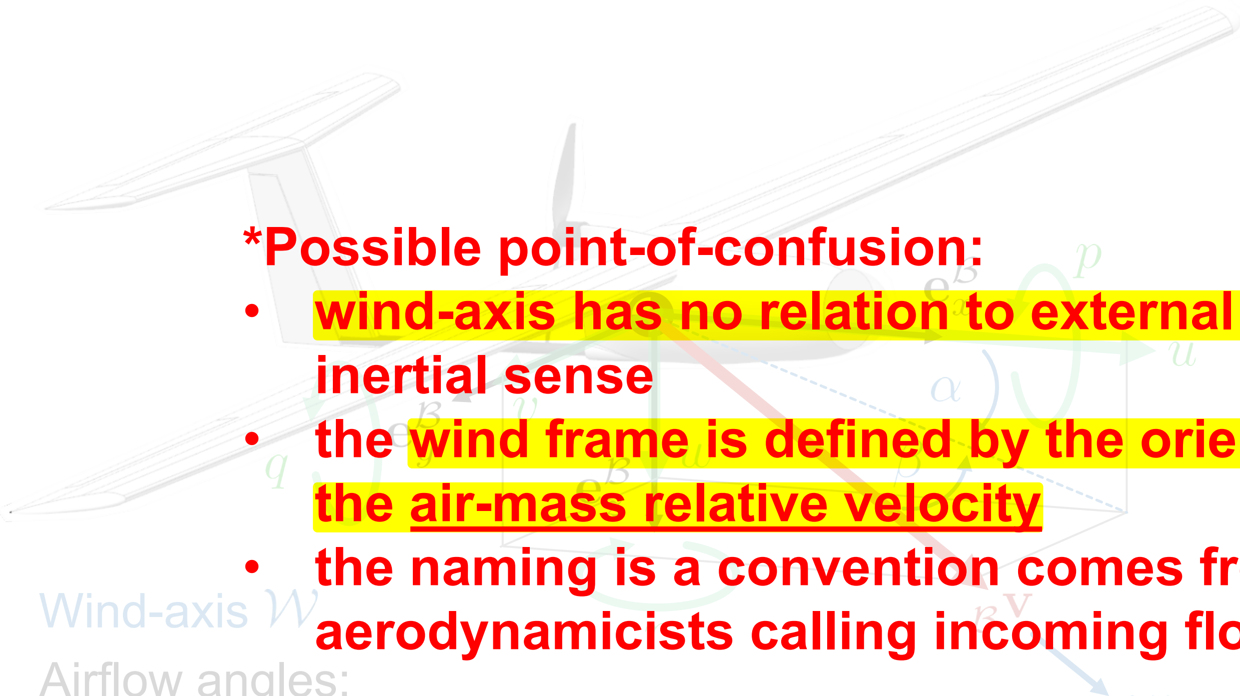
Airflow angles:

- Angle of attack $\alpha = \tan^{-1}(w/u)$
- Sideslip angle $\beta = \sin^{-1}(v/V)$
- Wind frame is opposite to “free-stream” velocity, or “wind” (i.e. the air-mass)



Free-stream velocity

Aircraft Kinematics | Reference axes



***Possible point-of-confusion:**
- **wind-axis has no relation to external wind in the inertial sense**
- **the wind frame is defined by the orientation of the air-mass relative velocity**
- **the naming is a convention comes from aerodynamicists calling incoming flow, "wind"**

Body-frame $\mathcal{B}$

Body velocity:

$$_{\mathcal{B}}\mathbf{v}_a = (u, v, w)^T$$

- Air-mass relative speed (airspeed):

$$V = \sqrt{u^2 + v^2 + w^2}$$

- V is always positive

Body rates:

$$_{\mathcal{B}}\omega = (p, q, r)^T$$

Wind-axis $\mathcal{W}$

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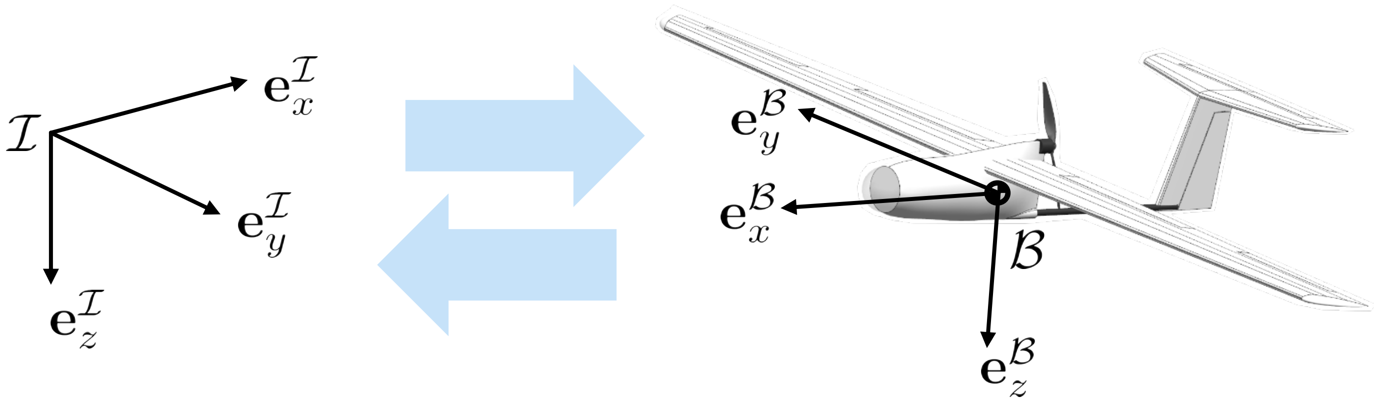
Aircraft Kinematics | Coordinate transformation

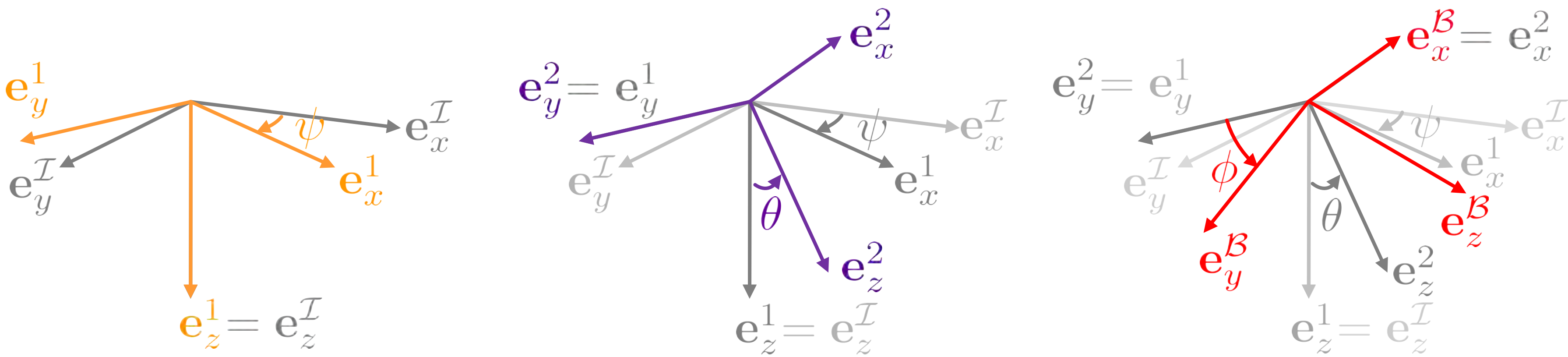
- Euler angles, roll, pitch, and yaw, are used to transform between inertial and body axes.

$$\mathbf{\Theta} = (\phi, \theta, \psi)^T$$



Aircraft Kinematics | Coordinate transformation

Rotation Matrix ($\mathcal{B}$ to $\mathcal{I}$) $\mathbf{C}_{\mathcal{I}\mathcal{B}}$ is parametrized with **3 successive rotations** using the **ZYX Tait-Brian Angles** (specific kind of Euler Angles):



1 **Yaw:**
around - $\mathbf{e}_z^{\mathcal{I}}$: $\mathbf{C}_{\mathcal{I}1}(\psi)$
$\Rightarrow$ Frame 1

2 **Pitch:**
around - $\mathbf{e}_y^{1}$: $\mathbf{C}_{12}(\theta)$
$\Rightarrow$ Frame 2

3 **Roll:**
around - $\mathbf{e}_x^{2}$: $\mathbf{C}_{2\mathcal{B}}(\phi)$
$\Rightarrow$ Frame B

$$\mathbf{C}_{\mathcal{I}\mathcal{B}} = \mathbf{C}_{\mathcal{I}1}(\psi)\,\mathbf{C}_{12}(\theta)\,\mathbf{C}_{2\mathcal{B}}(\phi)$$

Aircraft Kinematics | A note on angular rates

- **Angular Rates:**

Time variation of Tait-Bryan angles $\left(\dot{\phi}, \dot{\theta}, \dot{\psi}\right)$ $\neq$ body angular rates (p, q, r)

$$_{B}\omega = \mathbf{E}_{R,euler,ZYX}(\mathbf{\Theta})\dot{\mathbf{\Theta}}$$

from euler rate to body rate

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$

- **Singularity:** for $\theta = \pm\frac{\pi}{2}$ $\left(det(\mathbf{E}_{R,euler,ZYX}) = -cos(\theta) \right)$
  - "Gimbal Lock"
  - Can be problematic for Euler-angle based controllers …

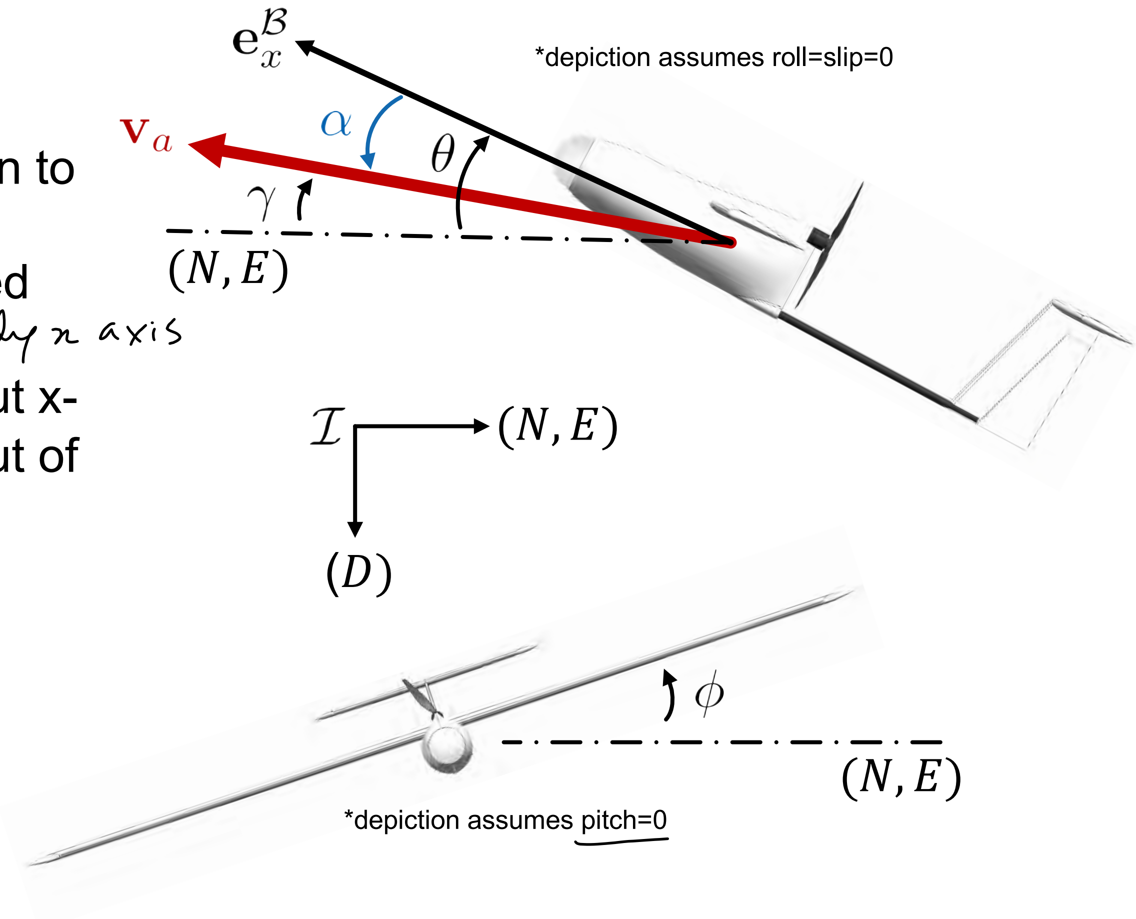
$L_{IB} = L_{BI}^{-1} \neq L_{BI}^{T}$

Aircraft Kinematics | Polar coordinates

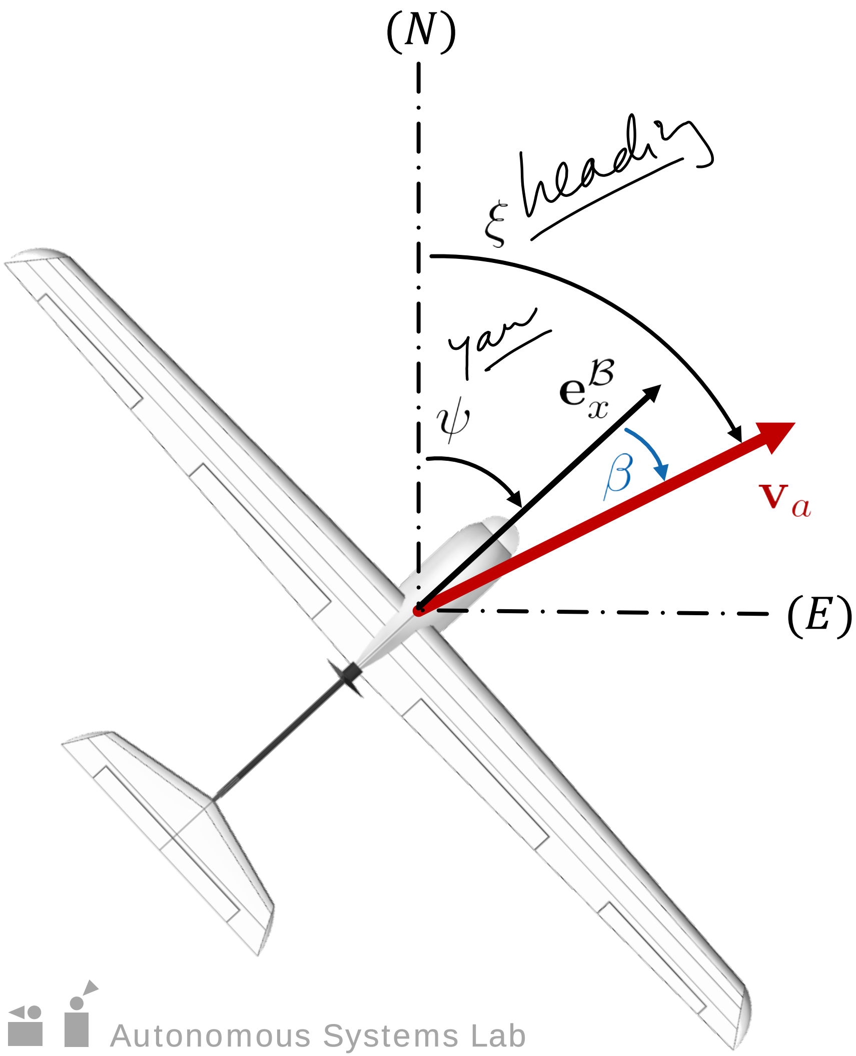
Longitudinal Inertial Frame (*side view)

- θ : Pitch angle, from horizon to x-body axis
- γ : Flight path angle, defined from horizon to $\mathbf{v}_a$ not body x axis
- ϕ : Roll angle, rotation about x-body axis (note: pointing out of slide)

$${}_{\mathcal{I}}\mathbf{v}_a = \mathbf{C}_{\mathcal{I}\mathcal{B}}\, {}_{\mathcal{B}}\mathbf{v}_a$$



Aircraft Kinematics | Polar coordinates

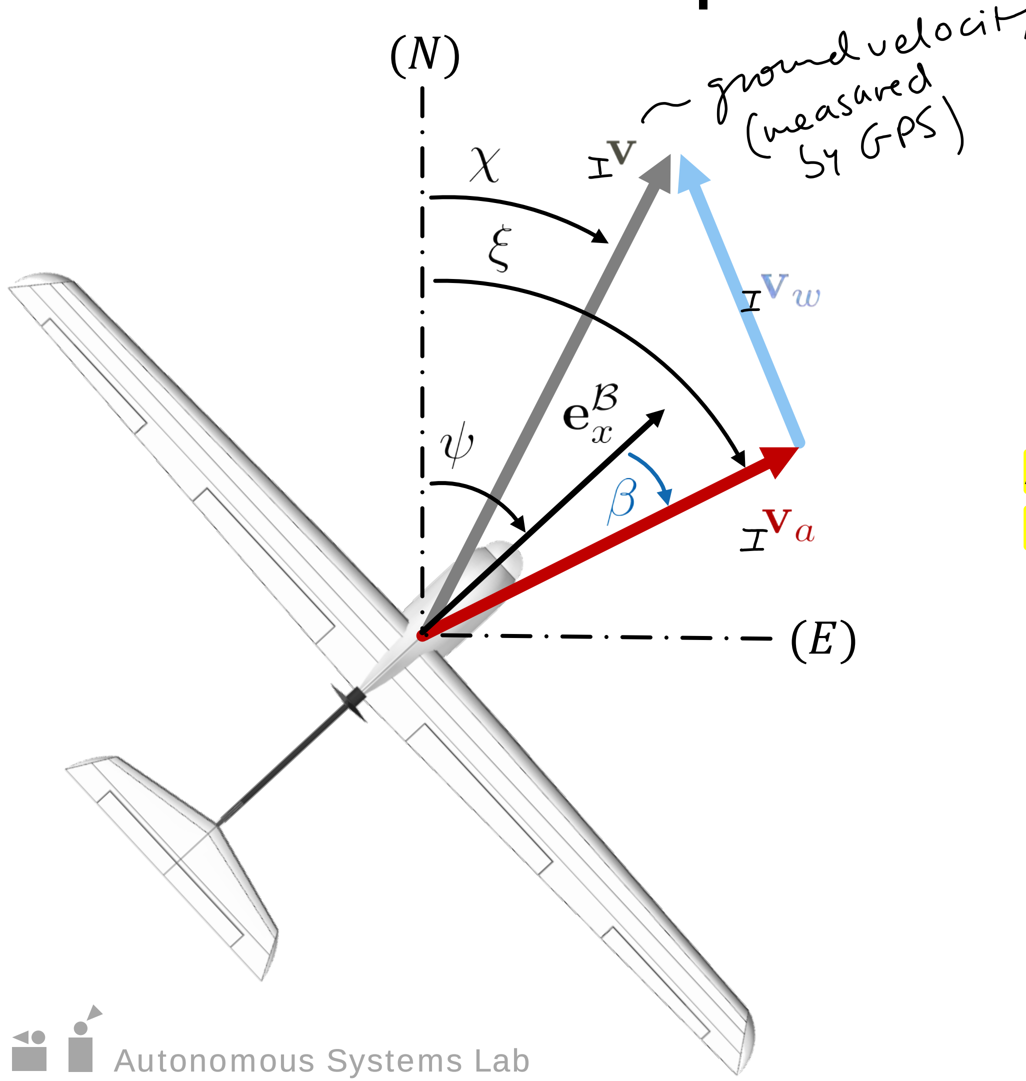


Lateral-directional Inertial Frame (*top view)

- ψ: Yaw angle, from North to x-body axis
- ξ: Heading angle, defined from North to $\mathbf{v}_a$
- Note: this STILL does <u>not</u> consider wind.

heading ≠ yaw

Aircraft Kinematics | Polar coordinates



Lateral-directional Inertial Frame (*top view)

- ψ: Yaw angle, from North to x-body axis
- ξ: Heading angle, defined from North to $\mathbf{v}_a$

Adding a constant, steady horizontal wind:

- $\mathbf{v}_w$: Wind velocity
- $\mathbf{v}$: Ground-based inertial
- velocity (or “ground speed”)
- χ: Course angle, defined from North to $\mathbf{v}$
- Note: the constant, steady wind assumption means only position dynamics are affected.

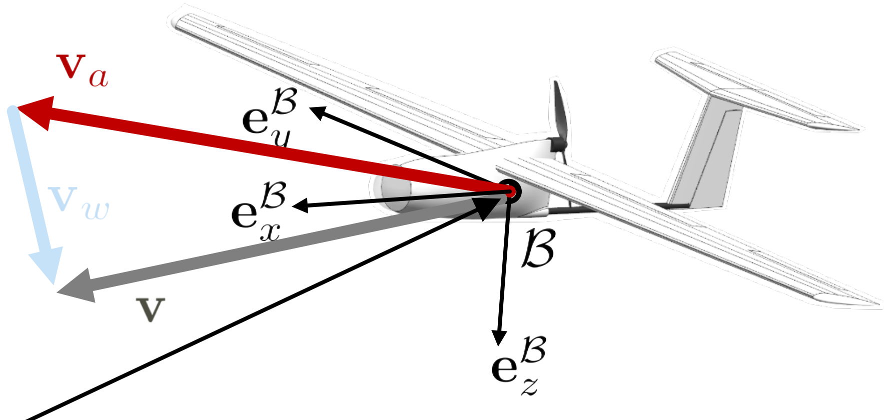
Aircraft Kinematics | Polar coordinates

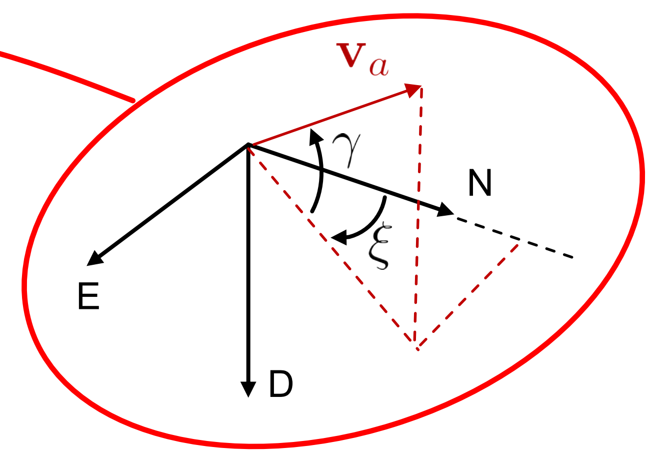
$${}_{\mathcal{I}}\mathbf{v} = {}_{\mathcal{I}}\mathbf{v}_a + {}_{\mathcal{I}}\mathbf{v}_w$$

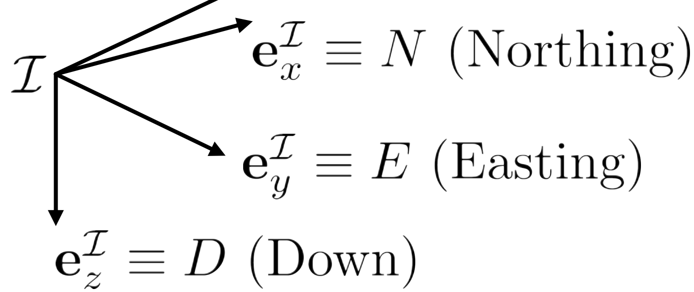
ground velocity

$$= {}_{\mathcal{I}}\dot{\mathbf{r}} = \begin{pmatrix} V\cos\gamma\cos\xi + v_{w,N} \\ V\cos\gamma\sin\xi + v_{w,E} \\ -V\sin\gamma + v_{w,D} \end{pmatrix}$$

(how we are moving)



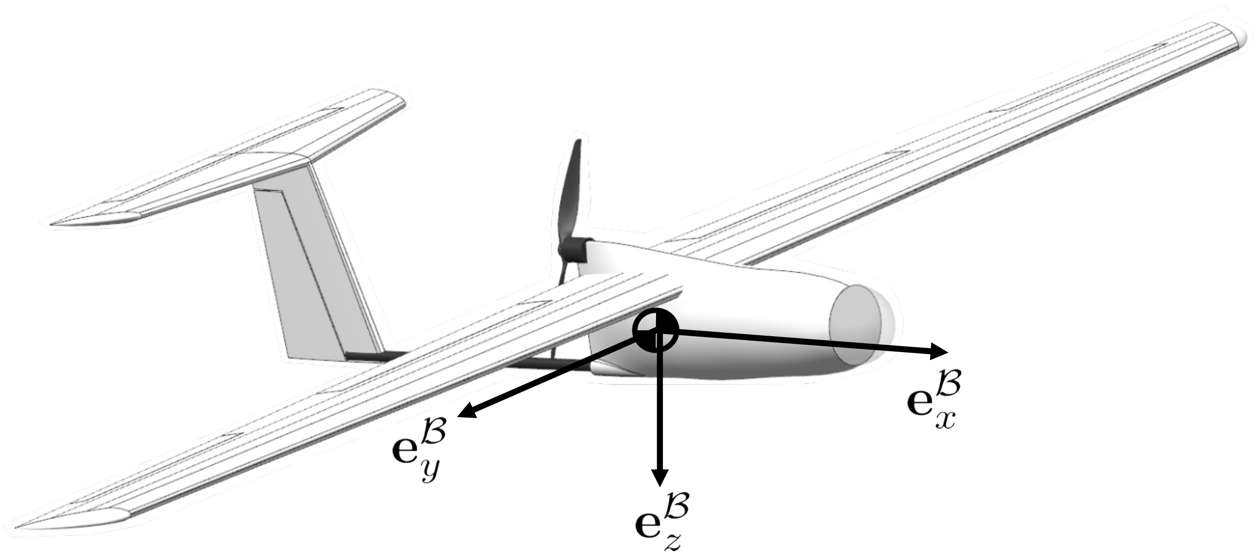




Aircraft Dynamics

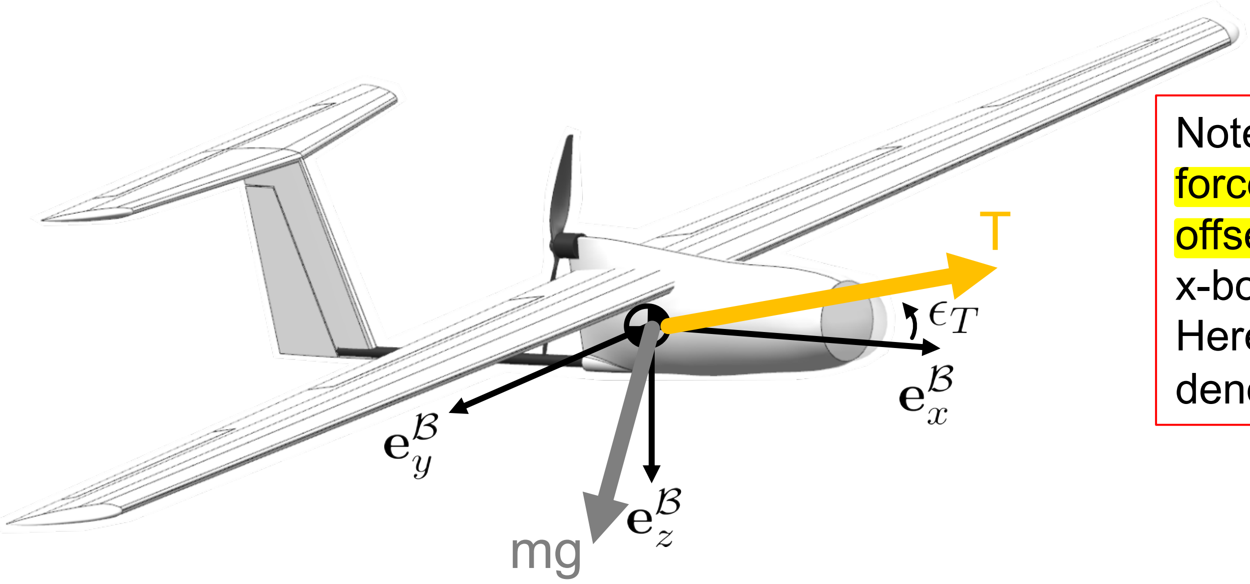
Aircraft Dynamics | Forces & Moments

- Sum forces and moments from all components
- Expressed at center of gravity (CoG) ☞ Newton-Euler equations



Aircraft Dynamics | Forces & Moments

- **Non-aerodynamic forces**
  - Weight at the center of gravity
  - Thrust of propeller

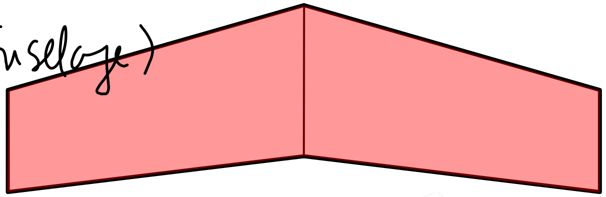


Note: Thrust force can be offset from x-body axis. Here denoted: ϵ_T

Aircraft Dynamics | Forces & Moments

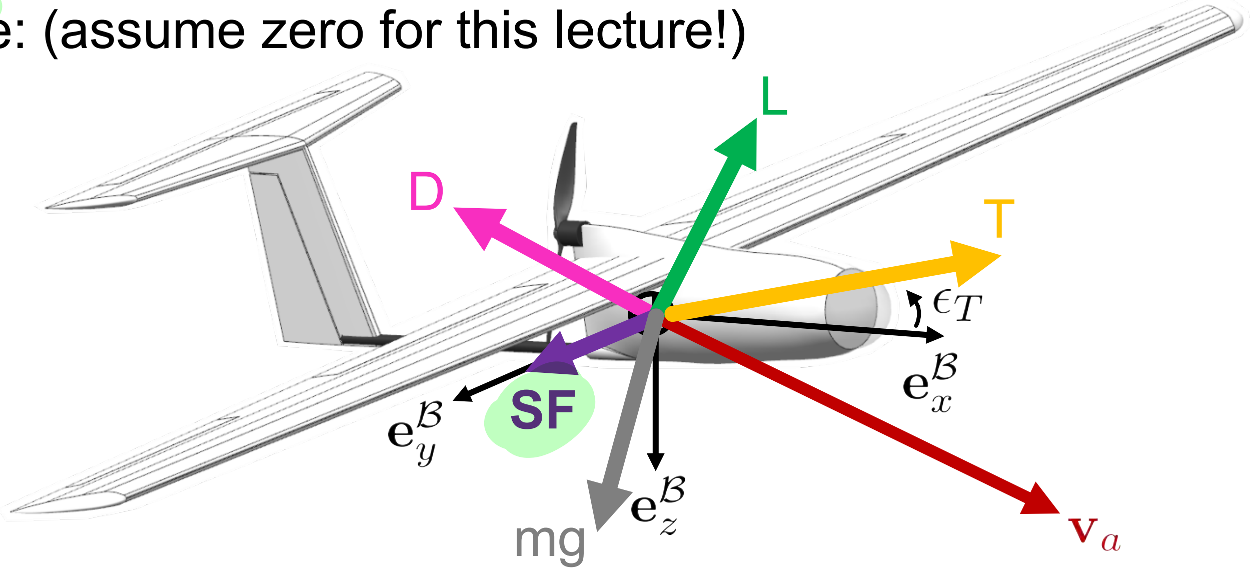
"aircraft lift coeff."

S: Surface (or 'planform') Area



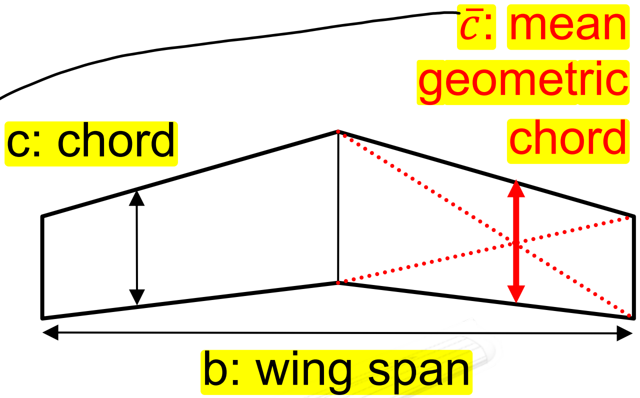
- **Aerodynamic forces**
  - Lift: $L = \frac{1}{2}\rho V^2 \boxed{S} c_L$ lumped lift eqn. (tail, wing, fuselage)
  - Drag: $D = \frac{1}{2}\rho V^2 \boxed{S} c_D$
  - Side-force: (assume zero for this lecture!)

Lift and Drag **always** perpendicular and parallel, respectively, to air velocity $\mathbf{v}_a$

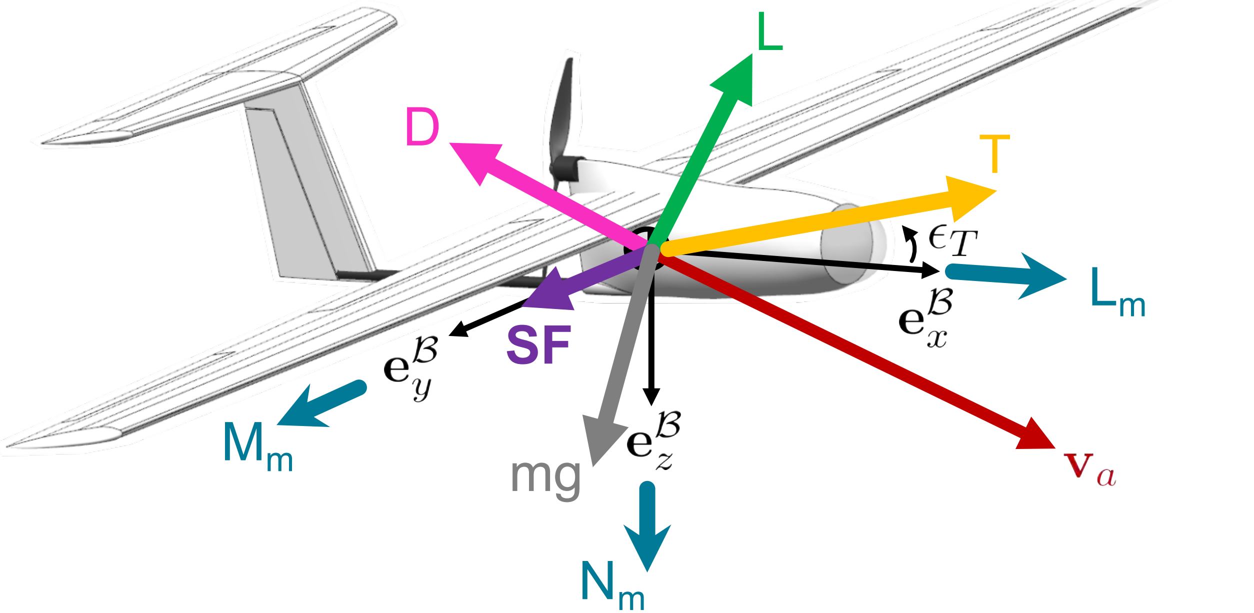


Aircraft Dynamics | Forces & Moments

- **Aerodynamic moments**
  - around x Rolling moment: $L_m = \frac{1}{2}\rho V^2 S \boxed{b} c_l$
  - around y Pitching moment: $M_m = \frac{1}{2}\rho V^2 S \boxed{\bar{c}} c_m$
  - around z Yawing moment: $N_m = \frac{1}{2}\rho V^2 S \boxed{b} c_n$



See more wing geometry definitions in backup slides.



Aircraft Dynamics | Component build-up Approach

- The aerodynamic forces and moments are built up from both static and dynamic components, summed from each part of the aircraft.
  - E.g.

$$
\begin{aligned}
c_L &= f(\alpha, q, \delta_e) \\
&= c_{L_0} + c_{L_\alpha}\alpha + c_{L_q}\hat{q} + c_{L_{\delta_e}}\delta_e \\
&= c_{L_0} + c_{L_\alpha}\alpha + c_{L_{\alpha}^2}\alpha^2 + c_{L_q}\hat{q} + c_{L_{\delta_e}}\delta_e + c_{L_{\alpha\delta_e}}\alpha\delta_e
\end{aligned}
$$

1st order Taylor Expansion (only linear terms)

2nd order Taylor Expansion (some coupled terms)

$$c_{L_\alpha} = \frac{\partial c_L}{\partial \alpha}$$

subscript: wrt what quantity we take derivative

- Other model structures could be used, but the build-up approach **generalizes well in practice**, when in nominal flight regimes

Aircraft Dynamics | Forces and moments

- Represented in body frame, at the CoG:

thrust w/ inclination

$$\sum {}_B\mathbf{F} = \begin{pmatrix} -D\cos\alpha + L\sin\alpha \\ SF \text{ side force} \\ -D\sin\alpha - L\cos\alpha \end{pmatrix} + \begin{pmatrix} T\cos\varepsilon_T \\ 0 \\ -T\sin\varepsilon_T \end{pmatrix} + m\mathbf{C}_{BI}\begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}$$

$$= \begin{pmatrix} T\cos\varepsilon_T - D\cos\alpha + L\sin\alpha - mg\sin\theta \\ SF + mg\sin\phi\cos\theta \\ -T\sin\varepsilon_T - D\sin\alpha - L\cos\alpha + mg\cos\phi\cos\theta \end{pmatrix}$$

*Assuming side-slip = 0

$$\sum {}_B\mathbf{M} = \begin{pmatrix} L_m \\ M_m \\ N_m \end{pmatrix} + \begin{pmatrix} L_{m_T} \\ M_{m_T} \\ N_{m_T} \end{pmatrix}$$

*Moments due to propulsion with subscript T "Thrust"

Aircraft Dynamic Modeling | Equations of motion

$\frac{d}{dt} {}_I v_\omega = 0$

- Translation

constant wind assumption:
$\mathrm{d}/\mathrm{dt}({}_\mathrm{I}\mathbf{v} = {}_\mathrm{I}\mathbf{v_a} + {}_\mathrm{I}\mathbf{v_w}) = \mathrm{d}/\mathrm{dt}({}_\mathrm{I}\mathbf{v_a})$

$$ {}_{\mathcal{B}}\dot{\mathbf{v}}_a = \frac{1}{m}\sum {}_{\mathcal{B}}\mathbf{F} - {}_{\mathcal{B}}\omega \times {}_{\mathcal{B}}\mathbf{v}_a $$

coriolis term

$$ {}_{\mathcal{I}}\dot{\mathbf{r}} = \mathbf{C}_{\mathcal{IB}}\, {}_{\mathcal{B}}\mathbf{v}_a + {}_{\mathcal{I}}\mathbf{v}_w $$

- Rotation

inertia matrix

moments

$$ {}_{\mathcal{B}}\dot{\omega} = {}_{\mathcal{B}}\mathbf{I}^{-1}\left(\sum {}_{\mathcal{B}}\mathbf{M} - {}_{\mathcal{B}}\omega \times ({}_{\mathcal{B}}\mathbf{I}\, {}_{\mathcal{B}}\omega)\right) $$

Symmetric y-axis

$$ {}_{\mathcal{B}}\mathbf{I} = \begin{pmatrix} I_{xx} & \boxed{0} & I_{xz} \\ \boxed{0} & I_{yy} & \boxed{0} \\ I_{xz} & \boxed{0} & I_{zz} \end{pmatrix} $$

Mass moments and products of inertia

Aircraft Dynamic Modeling | Equations of motion

- Translation

constant wind assumption:
$\mathrm{d}/\mathrm{dt}({}_{\mathrm{I}}\mathbf{v} = {}_{\mathrm{I}}\mathbf{v}_{\mathbf{a}} + {}_{\mathrm{I}}\mathbf{v}_{\mathbf{w}}) = \mathrm{d}/\mathrm{dt}({}_{\mathrm{I}}\mathbf{v}_{\mathbf{a}})$

$$_{\mathcal{B}}\dot{\mathbf{v}}_a = \frac{1}{m}\sum {}_{\mathcal{B}}\mathbf{F} - {}_{\mathcal{B}}\omega \times {}_{\mathcal{B}}\mathbf{v}_a$$

$$_{\mathcal{I}}\dot{\mathbf{r}} = \mathbf{C}_{\mathcal{I}\mathcal{B}}\,{}_{\mathcal{B}}\mathbf{v}_a + {}_{\mathcal{I}}\mathbf{v}_w$$

- Rotation

$$_{\mathcal{B}}\dot{\omega} = {}_{\mathcal{B}}\mathbf{I}^{-1}\left(\sum {}_{\mathcal{B}}\mathbf{M} - {}_{\mathcal{B}}\omega \times ({}_{\mathcal{B}}\mathbf{I}\,{}_{\mathcal{B}}\omega)\right)$$

$$\dot{\mathbf{\Theta}} = \mathbf{E}_{R,euler,ZYX}(\mathbf{\Theta})^{-1}\,{}_{\mathcal{B}}\omega$$

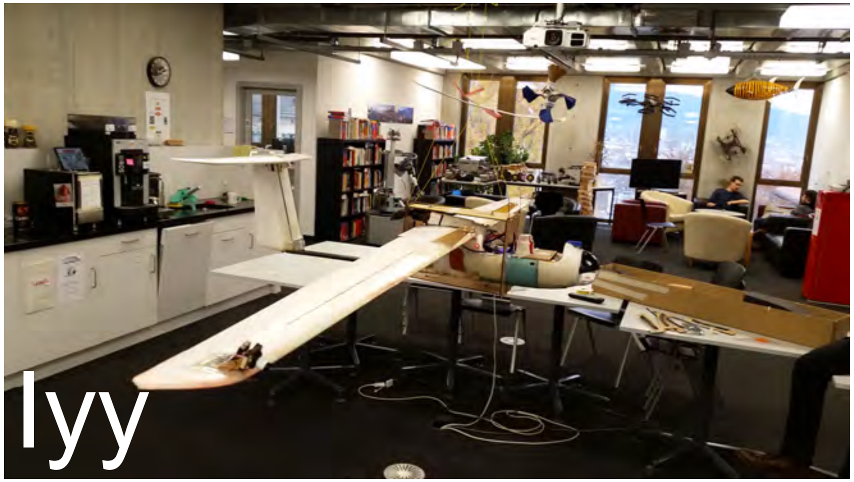
Typically small

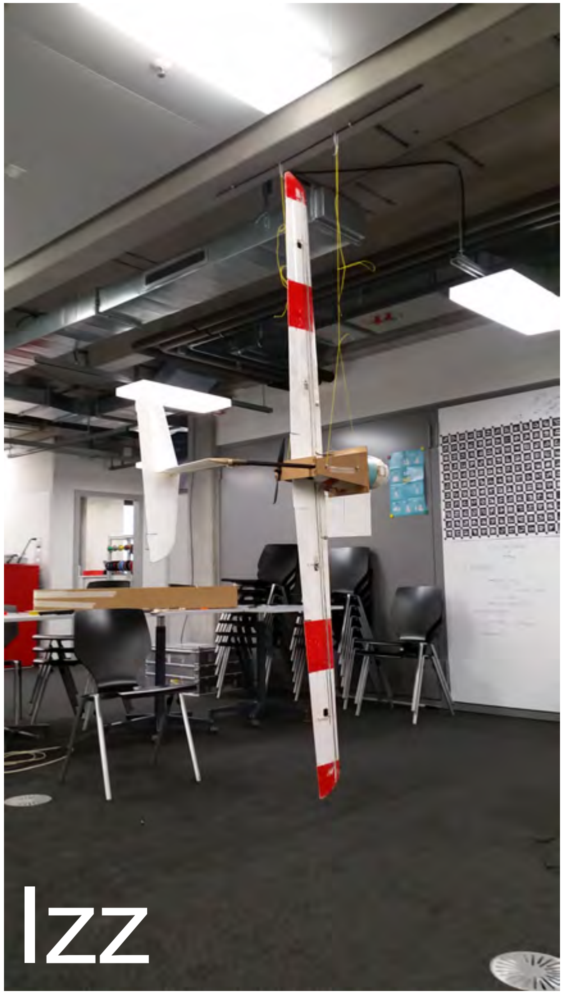
$$_{\mathcal{B}}\mathbf{I} = \begin{pmatrix} I_{xx} & 0 & \boxed{I_{xz}} \\ 0 & I_{yy} & 0 \\ \boxed{I_{xz}} & 0 & I_{zz} \end{pmatrix}$$

Mass moments and products of inertia

Aircraft Dynamic Modeling | Equations of motion

- Side-note on *identification* of mass moments of inertia:





SWING TEST

Aircraft Dynamic Modeling | Equations of motion

- Side-note on *identification* of mass moments of inertia:

SWING TEST

Fixed-wing Control

Fixed-wing Control | Introduction

- Control of airplanes is not so easy!:
  - Inherently non-linear (especially in longitudinal axis)
  - Low control authority
  - Actuator saturation
  - Double integrator characteristics
  - MIMO: 4 inputs, 6 DoF, thus underactuated

Fixed-wing Control | Control Concepts

Many control techniques:

- Cascaded PID loops model-free
- Optimal control
  - LQR
- Robust control
  - H-infinity
  - H-2 loop-shaping
- Adaptive control
- Model Predictive Control
  - Linear/Nonlinear
- (Nonlinear) Dynamic Inversion

Chose according to:

- Computational Power
- Type of flight (e.g. aerobatics vs. level flight)
- Availability / fidelity of model

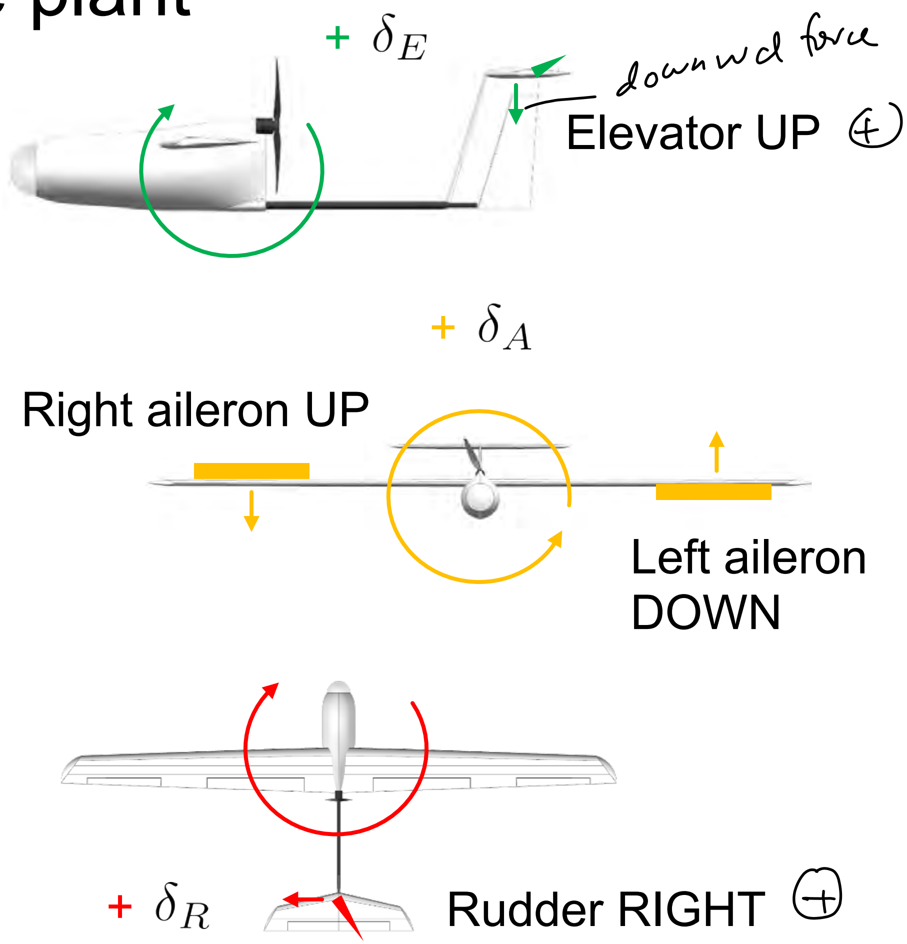
Fixed-wing Control | The plant

- Controls
  - Throttle $\delta_T \in [0, 1]$
  - Elevator $\delta_E \in [-1, 1]$
  - Aileron $\delta_A \in [-1, 1]$
  - Rudder $\delta_R \in [-1, 1]$

*Normalized actuator inputs

Convention: positive deflections cause positive moments

Ailerons may have differential (e.g. more UP for same DOWN can combat adverse yawing)



Fixed-wing Control | The plant

- States
  - Body velocities (u, v, w)
  - Body rates (p, q, r)
  - Euler angles (ϕ, θ, ψ)
  - Inertial position (r_N, r_E, r_D)

Fixed-wing Control | The plant

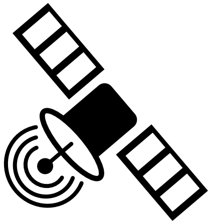
- States
  - Body velocities (u, v, w)
  - Body rates (p, q, r)
  - Euler angles (ϕ, θ, ψ)

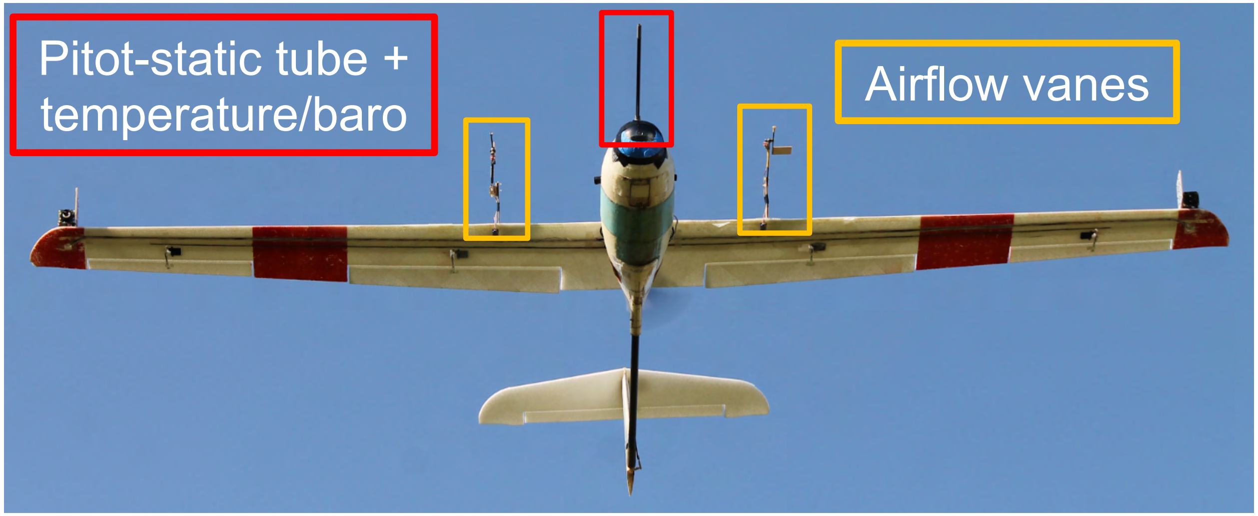
(Common) 'Measurements'

$V, (\beta, \alpha), v_N, v_E, v_D$

quite reliable

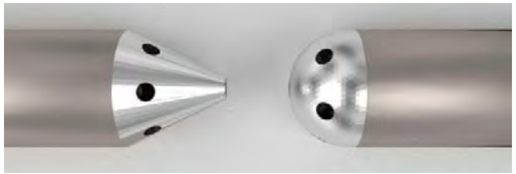
GNSS





more accurate than vane

Multi-hole probe



https://www.vectoflow.de/de/wp-content/uploads/sites/2/2015/11/probe-tips.jpg

Fixed-wing Control | The plant

- States
  - Body velocities (u, v, w)
  - Body rates (p, q, r)
  - Euler angles (ϕ, θ, ψ)
  - Inertial position (r_N, r_E, r_D)

(Common) 'Measurements'

$V, (\beta, \alpha), v_N, v_E, v_D$

p, q, r

'Direct' measurement from IMU gyroscope (still has bias and drift!)

Fixed-wing Control | The plant

- States
  - Body velocities (u, v, w)
  - Body rates (p, q, r)
  - Euler angles (ϕ, θ, ψ)
  - Inertial position (r_N, r_E, r_D)

(Common) 'Measurements'

$V, (\beta, \alpha), v_N, v_E, v_D$

p, q, r

a_x, a_y, a_z, ψ

Body-frame accelerations from IMU accelerometer can give an estimate of pitch and roll (though again susceptible to bias and drift). – much better when measurements are fused with GNSS velocities!

Yaw angle can be estimated from an on-board magnetometer. (again better estimates when more measurements are fused)

Fixed-wing Control | The plant

- States
  - Body velocities (u, v, w)
  - Body rates (p, q, r)
  - Euler angles (ϕ, θ, ψ)
  - Inertial position (r_N, r_E, r_D)

(Common) 'Measurements'

$V, (\beta, \alpha), v_N, v_E, v_D$

p, q, r

a_x, a_y, a_z, ψ

r_N, r_E, r_D

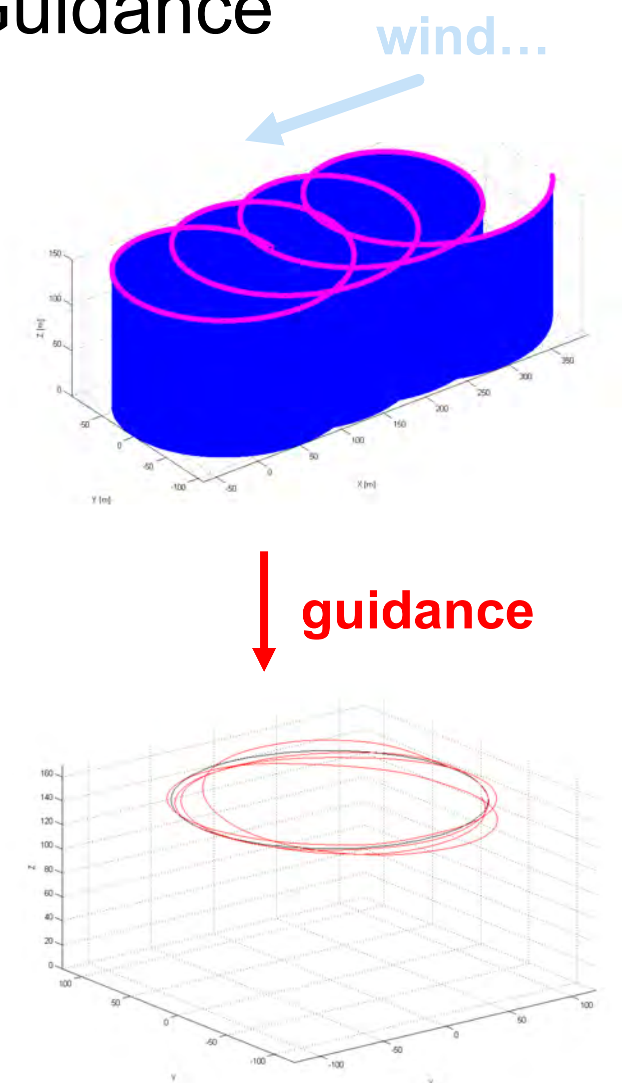
Fixed-wing Control | The plant

- What can/should we feedback and control, and how?

inputs (control surface deflections)

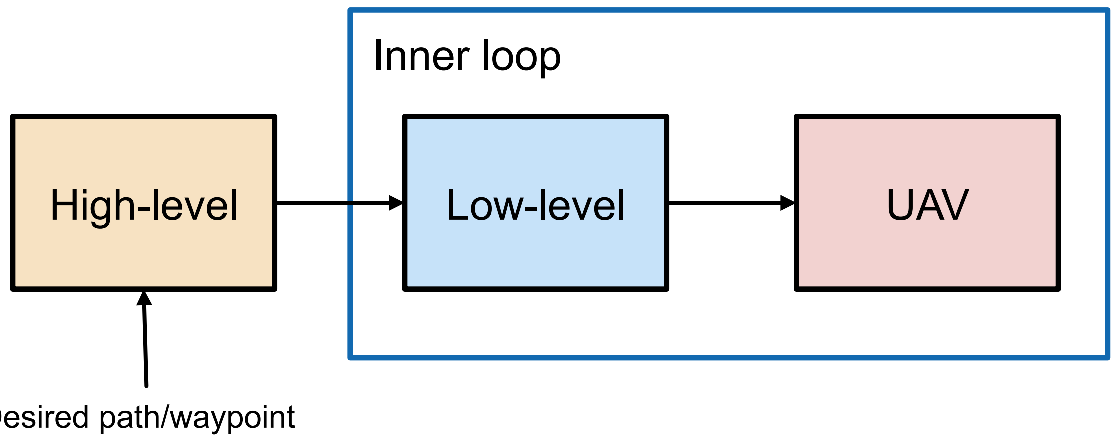
δ_T, δ_E, δ_A, δ_R → UAV PLANT → (u, v, w), (p, q, r), (ϕ, θ, ψ), (r_N, r_E, r_D)

Fixed-wing Control | Control & Guidance

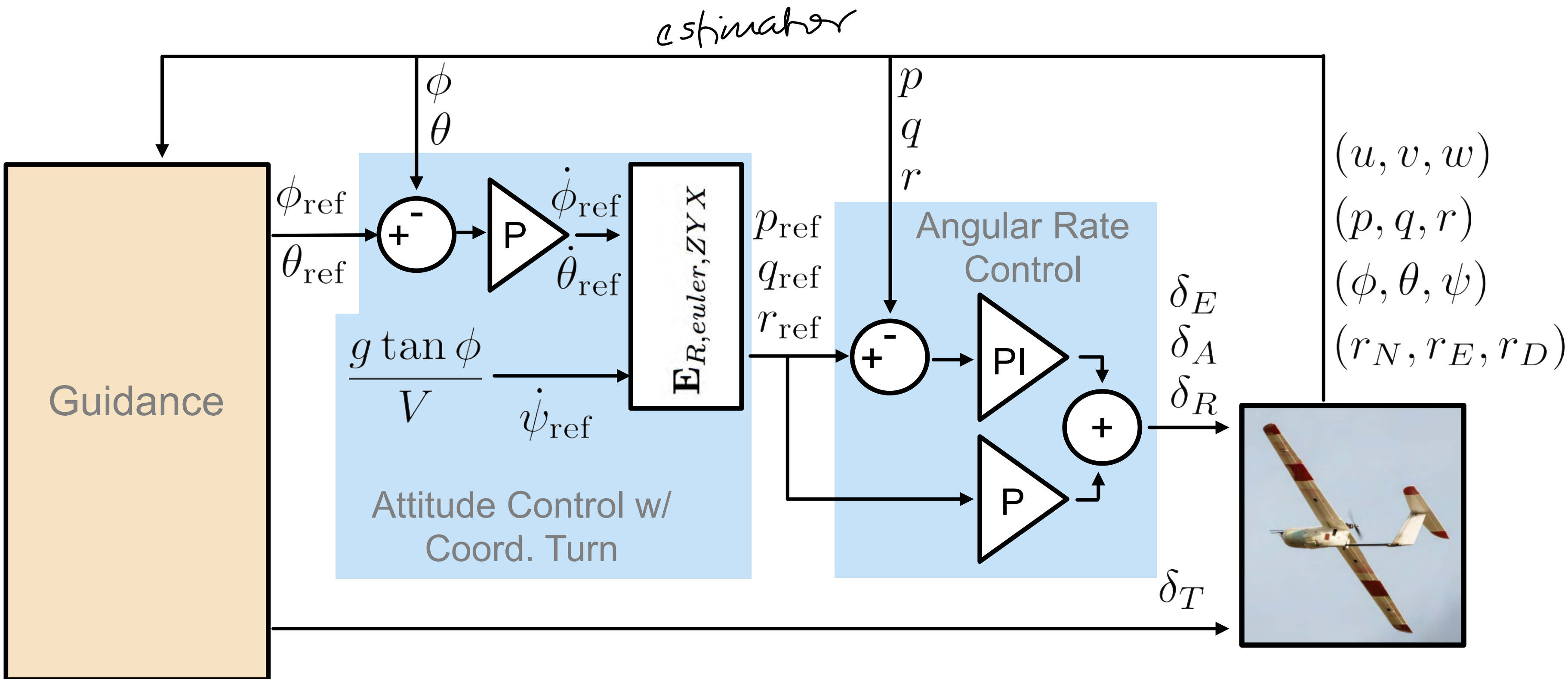


A popular concept: cascaded control loops

- **Control** = low level part
  - Stabilize attitude (and sometimes airspeed)
- **Guidance** = high level part
  - Follow paths or trajectory (position control)
    Effect: Reject constant low frequency perturbation (constant wind)



Fixed-wing Control | Simple cascaded control



Path or trajectory reference

- Need integrator wind-up protection
- * Should scale actuator output with airspeed: $1/V^2$ ~ dynamic pressure scaling
- Bandwidth of inner (low-level) loop should be sufficiently higher than the outer (guidance)

* $L, M \sim v^2$ => if we want error to be a force/moment, do $\frac{1}{V^2}$
lift moment

Fixed-wing Control | Steady level turning flight

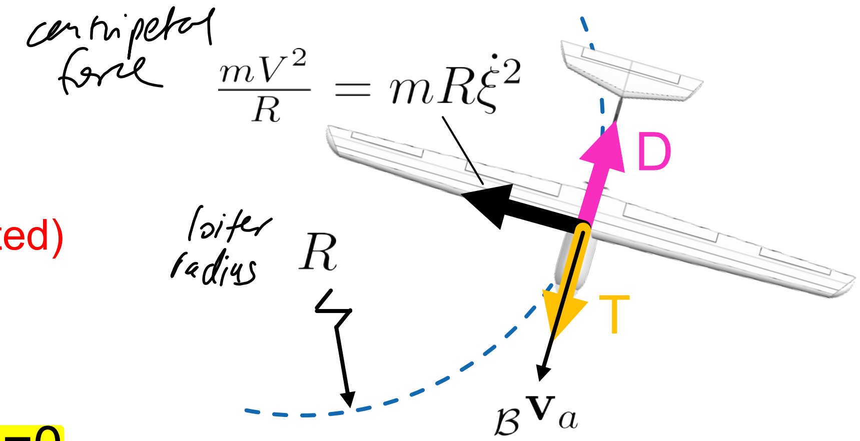
- Turning (not straight)

  $_{\mathcal{B}}\dot{\mathbf{v}}_a = \mathbf{0},\ _{\mathcal{B}}\dot{\omega} = \mathbf{0}$ <- steady (unaccelerated)

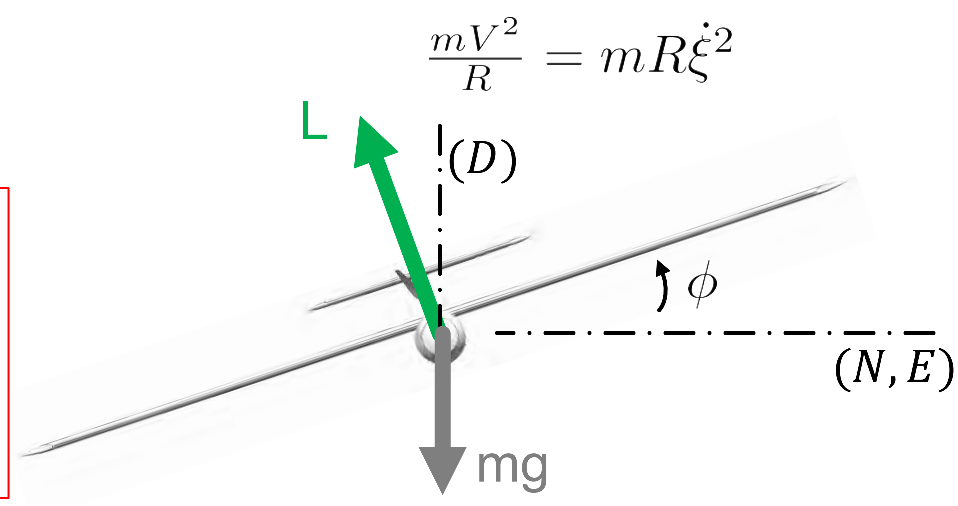
  $\theta = \alpha \rightarrow \gamma = 0$ <- level

  $\phi = \text{const.} \neq 0$ <- turning

  - Demand for **coordinated** turn: SF=0
  - L increases with $\frac{1}{\cos\phi}$
  - V_{min} increases with $\sqrt{\frac{1}{\cos\phi}}$



Recall SF (side-force) is composed of only aerodynamic forces, which must be zero, thus the lateral, centripetal force, here shown as , only comes from N-E lift component



Fixed-wing Control | Steady level turning flight

- Turning (not straight)

$_{\mathcal{B}}\dot{\mathbf{v}}_a = \mathbf{0},\ _{\mathcal{B}}\dot{\omega} = \mathbf{0}$ <- steady (unacceler…

$\theta = \alpha \rightarrow \gamma = 0$ <- level

$\phi = \text{const.} \neq 0$ <- turning

- Demand for **coordinated** turn: Y…

Assuming NO sideslip, …

$$L\cos\phi = mg$$

$$L = \frac{mg}{\cos\phi} \sim \frac{1}{\cos\phi}$$

$$\frac{1}{2}\rho V^2 S c_L(\alpha) \sim \frac{1}{\cos\phi}$$

For const. α

$$V \sim \sqrt{\frac{1}{\cos\phi}}$$

minimum speed

- L increases with $\frac{1}{\cos\phi}$
- V_{min} increases with $\sqrt{\frac{1}{\cos\phi}}$

Recall SF (side-force) is composed of only aerodynamic forces, which must be zero, thus the lateral, centripetal force, here shown as ⟶ , only comes from N-E lift component

D T $\mathbf{v}_a$ $R\dot{\xi}^2$ ϕ (N,E) mg

Fixed-wing Control | Steady level turning flight

- Heading-rate can be found for a given roll angle with a force balance (assuming $\dot{\psi} \approx \dot{\xi}$)

Note this assumes we have thrust force only acting in the same axis as drag. In reality, thrust force likely will add a small vertical component to the lift (e.g. if we fly at any pitch/angle of attack).

Force balance:

$$L\cos\phi = mg$$
$$D = T$$
$$m\frac{V^2}{R} = L\sin\phi$$

$$\boxed{\begin{aligned} &\frac{L\sin\phi}{L\cos\phi} = \frac{m\frac{V^2}{R}}{mg} \\ &\tan\phi = \frac{V\dot{\xi}}{g} \\ &\rightarrow \dot{\xi} = g\tan\phi/V \\ &\rightarrow \dot{\psi} = g\tan\phi/V \end{aligned}}$$

Robot Dynamics

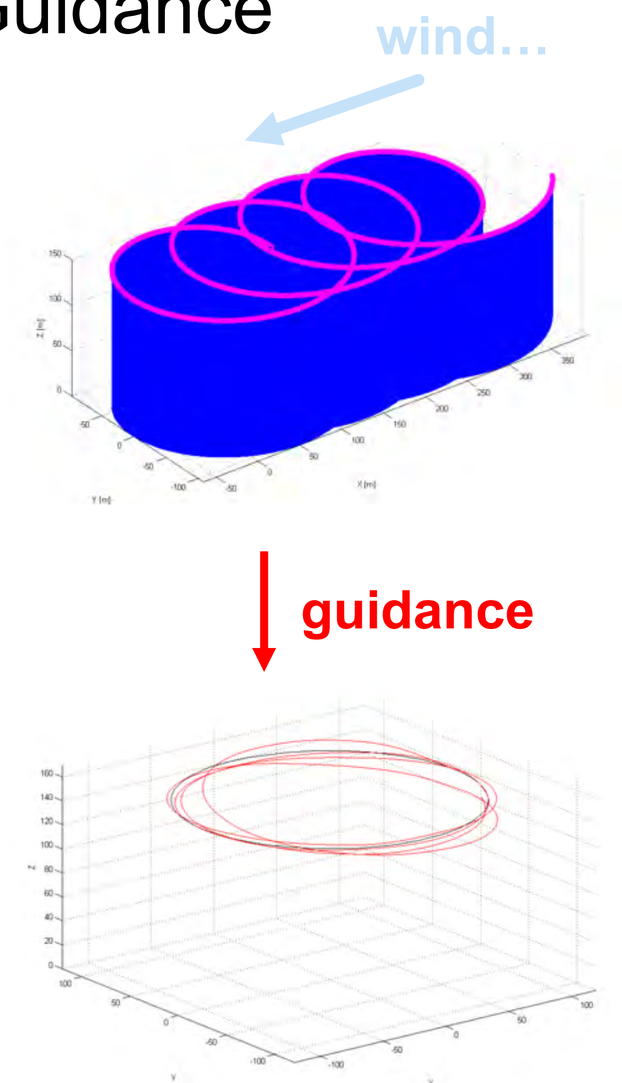
Fixed-wing UAVs: Dynamic Modeling and Control – Part 2

David Rohr

151-0851-00 V

Marco Hutter, Roland Siegwart, and Thomas Stastny

Fixed-wing Control | Control & Guidance



A popular concept: cascaded control loops

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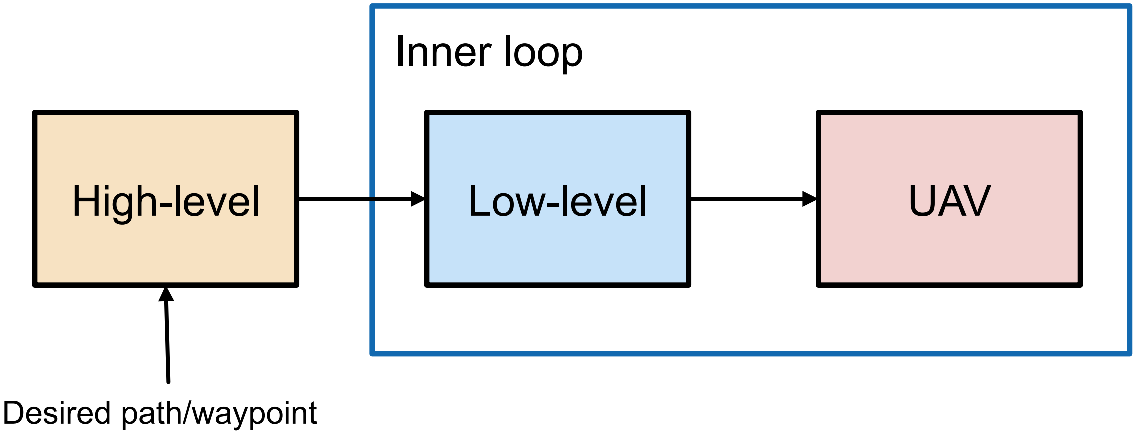
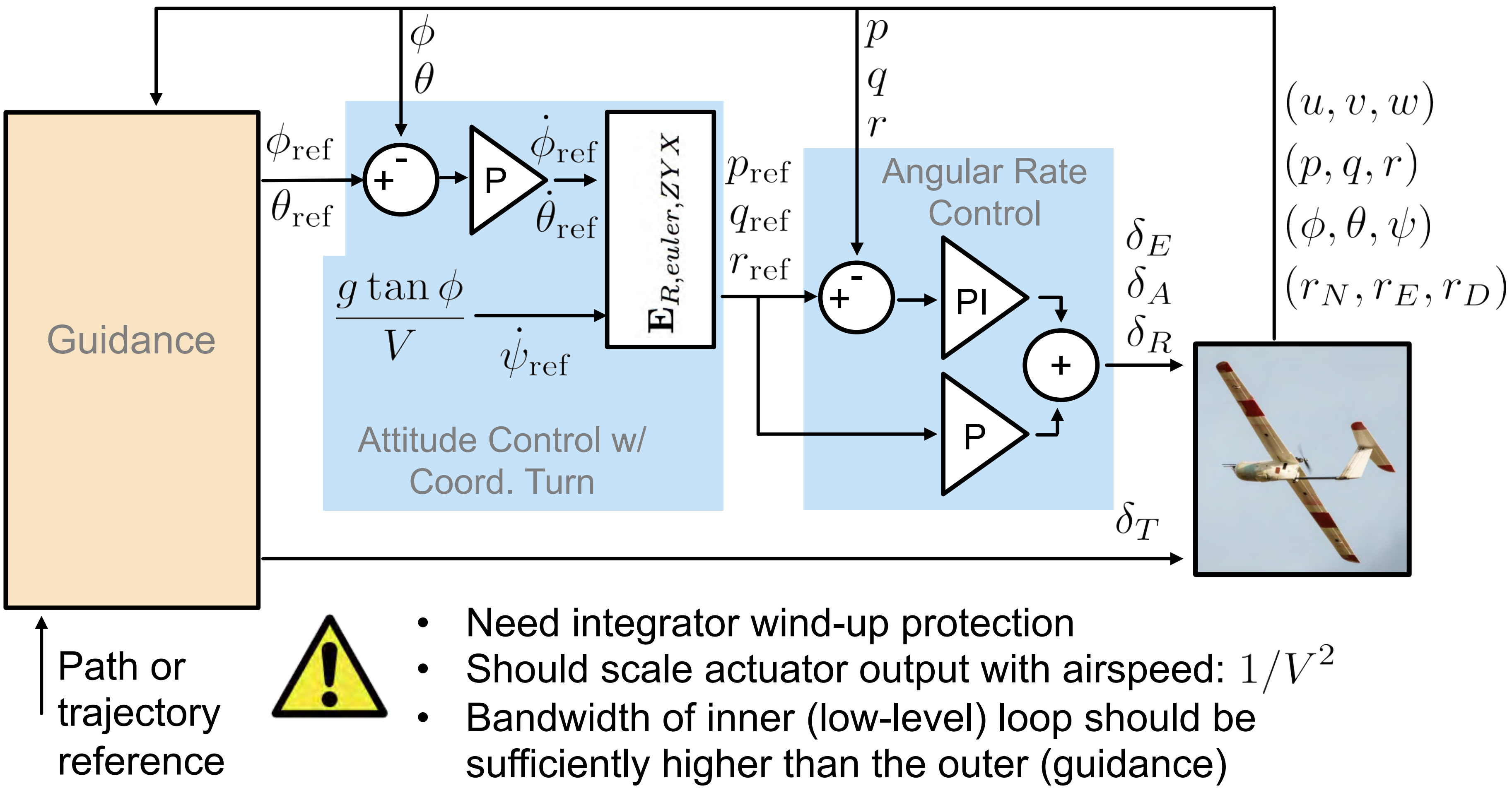


Fixed-wing Control | Simple cascaded control

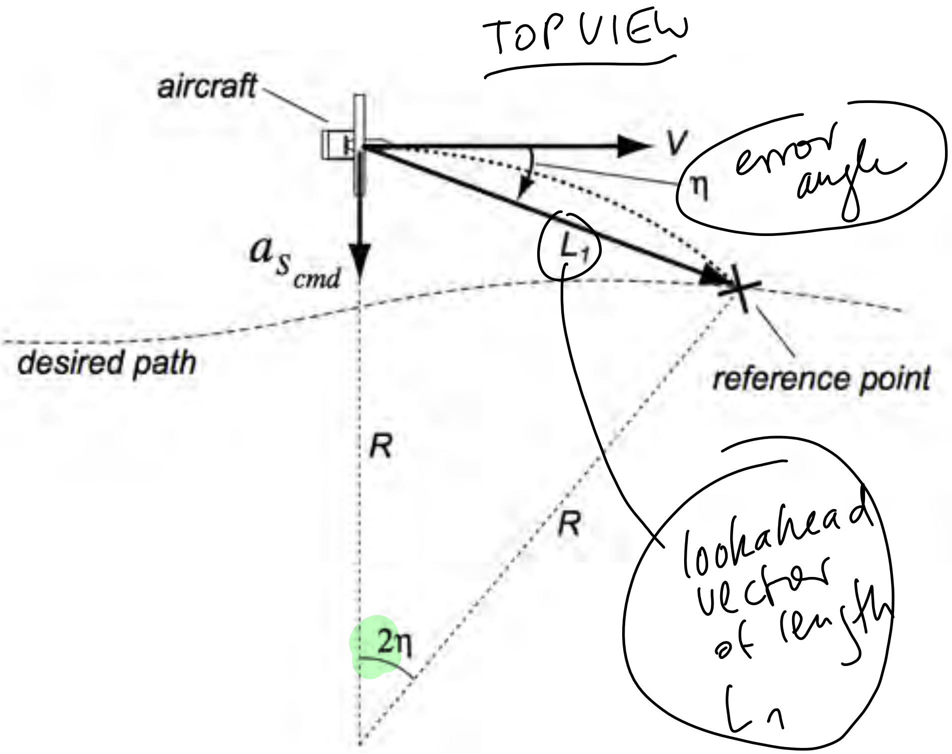


- Need integrator wind-up protection
- Should scale actuator output with airspeed: $1/V^2$
- Bandwidth of inner (low-level) loop should be sufficiently higher than the outer (guidance)

Fixed-wing Control | $\mathcal{L}_1$ Guidance

not required for exam

Lateral-directional path following guidance



$$\sin\eta = \frac{L_1}{2R} \Rightarrow R = \frac{L_1}{2\sin\eta}$$

$$\boxed{a_{s_{cmd}}} = \frac{V^2}{R} = \boxed{2\frac{V^2\sin\eta}{L_1}}$$

$$\Rightarrow \dot{\xi}_d = \frac{a_{s_{cmd}}}{V}$$

$$\dot{\xi}_d = \frac{g\tan\phi_d}{V} \Rightarrow \boxed{\phi_d = \tan^{-1}\left(\frac{a_{s_{cmd}}}{g}\right)}$$

Theory and graphics from:
S. Park, J. Deyst, and J. P. How, “A New Nonlinear Guidance Logic for Trajectory Tracking”, Proceedings of the AIAA Guidance, Navigation and Control Conference, Aug 2004. AIAA-2004-4900

Fixed-wing Control | TECS (Total Energy Control System)

(old, simple approach)

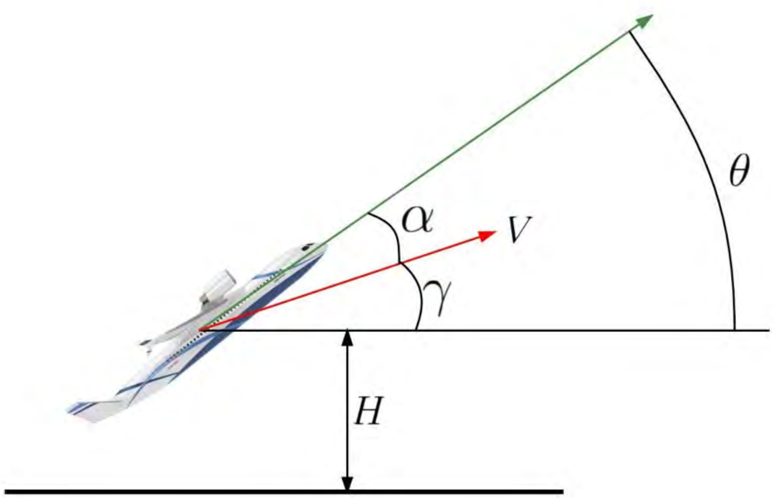
Control Altitude and Airspeed

$$E_{tot} = E_{kin} + E_{pot} = \frac{1}{2}mV^2 + mgH$$

$$\frac{\dot{E}_{tot}}{mg} = \frac{mV\dot{V}}{mg} + \frac{\dot{H}mg}{mg} = \frac{V\dot{V}}{g} + \dot{H}$$

$$\dot{E}_{spec} = \frac{\dot{E}_{tot}}{mgV} = \frac{\dot{V}}{g} + \frac{\dot{H}}{V} = \frac{\dot{V}}{g} + \sin\gamma$$

specific energy rate



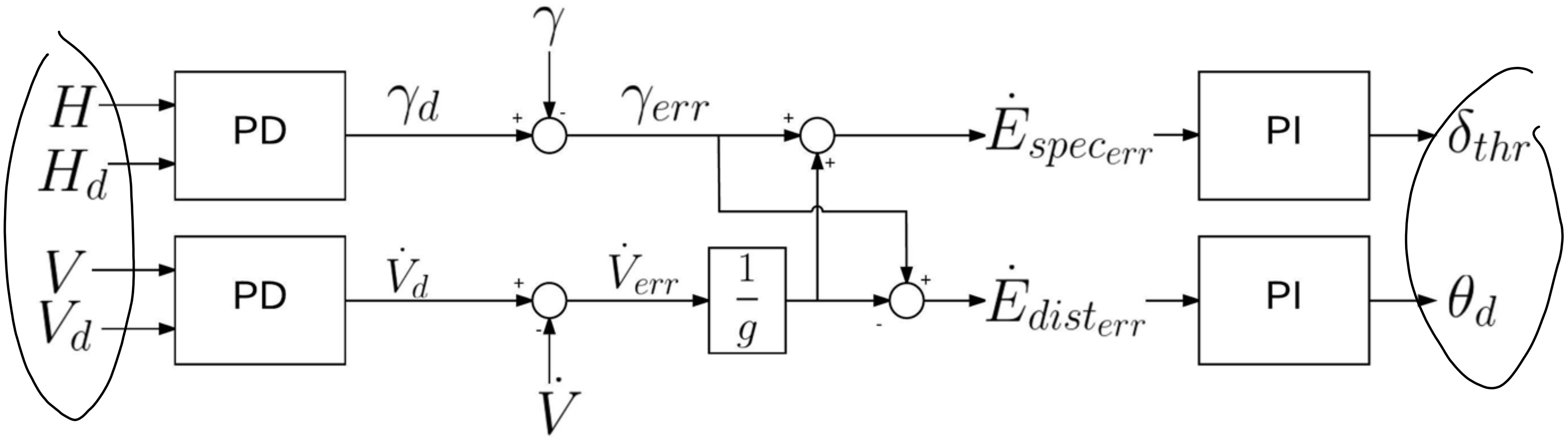
$$\dot{E}_{spec} = \frac{\dot{V}}{g} + sin\gamma \approx \frac{\dot{V}}{g} + \gamma$$

$$\dot{E}_{dist} = \gamma - \frac{\dot{V}}{g}$$

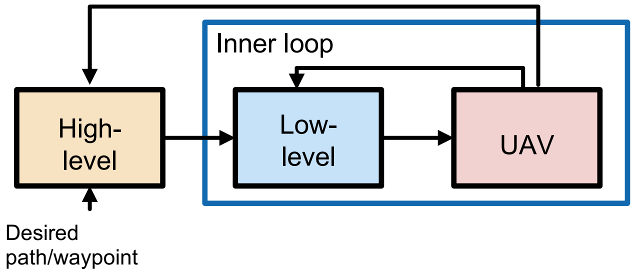
energy distribution rate

Defined as: potential energy rate minus kinetic energy rate

Fixed-wing Control | TECS (Total Energy Control System)



Note the model abstraction



- In lower-level loops, dynamics are modeled from actuators→attitude/airspeed
  - Note that aside from computational costs, these dynamics are challenging to globally identify in a nonlinear, high fidelity form. Thus **linearizations** are often made.
- Higher-level loops often model the aircraft in a three-degrees-of-freedom (3DoF) sense, mapping attitude/airspeed→position
  - Modeling of high-level dynamics does not require identification, as typically only kinematics are used.

Up next

22.11 (today)	fixed-wing lecture
23.11	no exercise
29.11	rotary-wing lecture
30.11	no exercise
06.12	fixed-wing case-study (hybrid = VTOL+ fixed-wing UAVs and path planning)
07.12	fixed-wing exercise (simulation + control)
09.12	*RSL & ASL open lab (17:00 -20:00)*
13.12	rotary-wing case-study
14.12	rotary-wing exercise

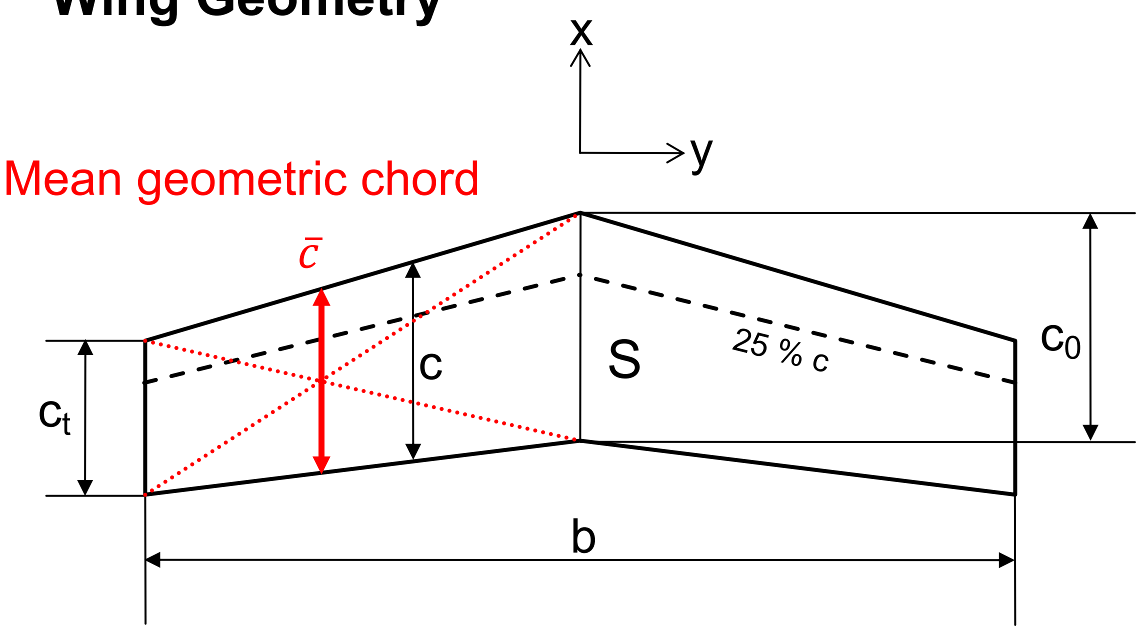
Backup Slides

References

- B.W. McCormick. Aerodynamics, Aeronautics, and Flight Mechanics. Wiley, 1979. ISBN: 9780471030324.
- B. Etkin. Dynamics of Atmospheric Flight . Wiley, 1972. ISBN: 9780471246206.
- G.J.J. Ducard. Fault-Tolerant Flight Control and Guidance Systems: Practical Methods for Small Unmanned Aerial Vehicles . Advances in Industrial Control. Springer, 2009. ISBN: 9781848825611.
- R.W. Beard and T.W. McLain. Small Unmanned Aircraft: Theory and Practice. Princeton University Press, 2012. ISBN: 9780691149219.
- R.F. Stengel. Flight Dynamics. Princeton University Press, 2004. ISBN: 0-691-11407-2

Basics of Aerodynamics | Wing Geometry

Wing Geometry



- b: Wingspan
- c: Chord
- c_0: Root Chord
- c_t: Tip Chord
- S: Reference Area
- AR: Aspect Ratio

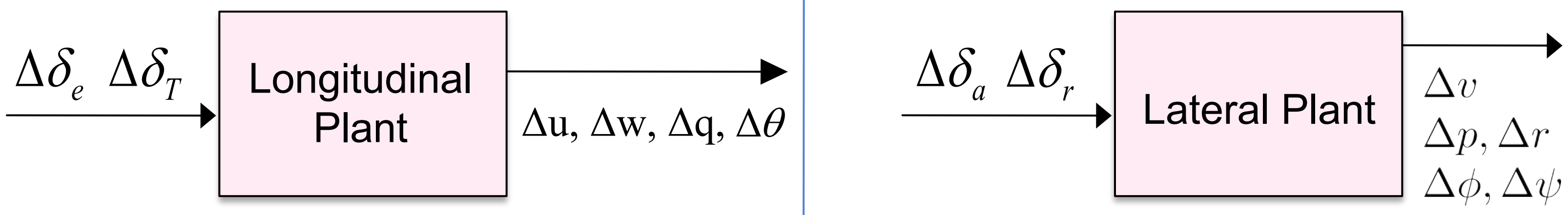
$$AR = \frac{b^2}{S}$$

Modeling for Control | Linearization

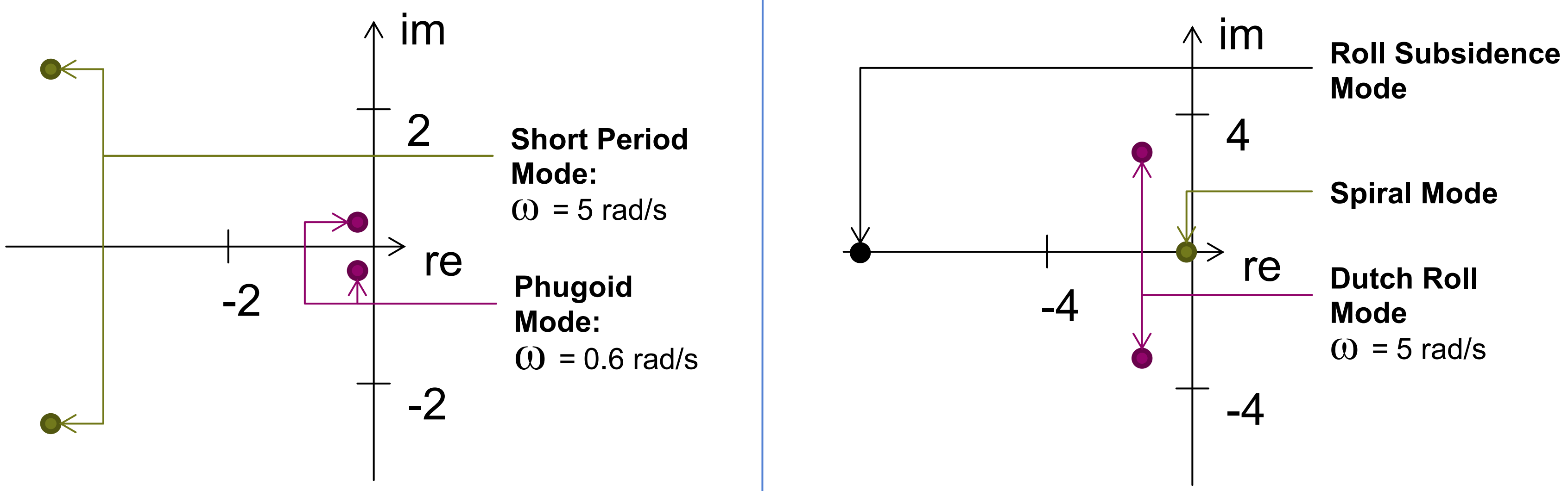
- Decouple plant into lateral-directional and longitudinal states
- Linearize about trim airspeed
  - Typically straight and level steady flight

Modeling for Control | The (linearized) plant

Subsystem



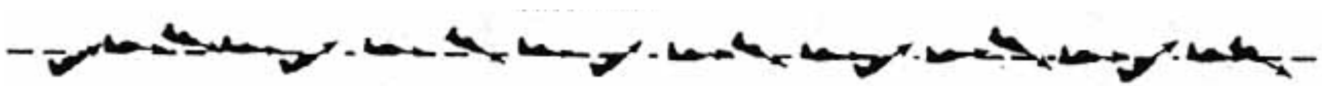
Corresponding Poles (Aerobatic Model Airplane)



Modeling for Control | The (linearized) plant

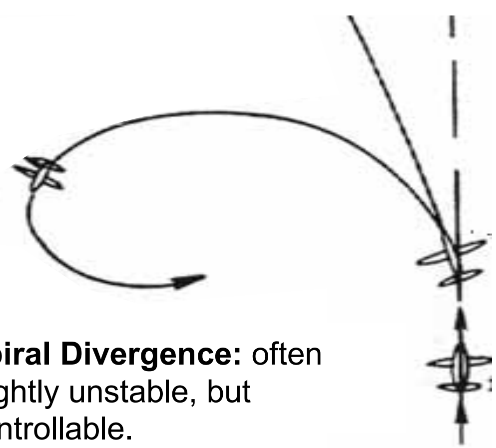
Longitudinal Modes

Phugoid mode: exchange between kinetic and potential energy. **SLOW**

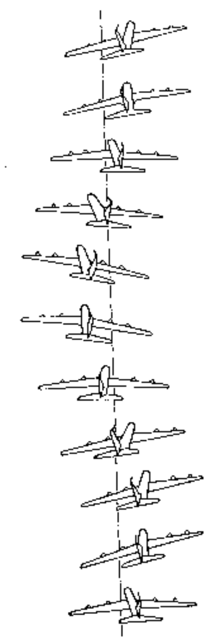


Short Period Mode: oscillation of angle of attack. **FAST**

Lateral-directional Modes



Spiral Divergence: often slightly unstable, but controllable.



Dutch Roll Mode: combined yaw-roll oscillation

Graphics from:
http://history.nasa.gov/SP-367/chapt9.htm and
http://www.fzt.haw-hamburg.de/pers/Scholz/Flugerprobung.html