Mathematical Exploration Title

Global Population Logistical Growth

Research question

At what specific year has the global population growth reached a turning point according to the logistic growth model?

AA HL Maths Internal Assessment

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1. Introduction

In late 2022 and early 2023, the global population recently exceeded 8 billion people. This has once again raised concerns many, including me, have had for a while surrounding the Earth's ability to sustain future growth. Are we growing at such a rate we are heading to a population collapse? Understanding how this population is growing and changing over time is crucial for policymakers so as to know which burdens the population might face in the upcoming future.

Mathematically, population growth can be modelled in several ways. Generally, populations tend to grow exponentially when resources are abundant. However, as factors such as limited resources and environmental constraints limit the rate of increase this growth cannot continue forever. Mathematicians and Biologists have devised the logistic growth model to incorporate this natural limit to growth, assuming a population grows rapidly at first and then slows as it approaches the maximum population size the environment can sustain - which is called the population's carrying capacity. Therefore, although population dynamics can be complex due to varying external factors which will not be taken into account in the paper, the logistic model provides a useful approximation.

In this exploration, I will use the logistic model to determine the year where the global human population reached its turning point - which is where growth shifts from increasing at an exponential rate to slowing down yearly. This turning point is the point of inflection on the graph and is therefore found where the derivative (or gradient) of the graph is maximum. I will therefore be using Calculus to solve for the point of inflection. If successful, knowing when this shift occurs or has occurred can be invaluable to inform important decisions for urban planning or population policies.

Throughout, I will be rounding to 3 significant figures, unless stated otherwise, so as to keep enough of the original value for precision & significance, whilst making it easier to read & follow. All calculations will still be performed with the original fully precise value, meaning some calculations may reach slightly different results if attempting to perform with the rounded data.

2. Fitting the logistic model

2.1 Getting and visualising the data

I used several sources to best have a range of data from 5000 BC to 2100. The data from 1950 to 2023 is most accurate, with the rest of the data losing in accuracy as it relies on estimates & projections as the dates get further from the current year.

I have a table of the world's population every year from 1950 to 2021[1] as well as a projected world population data for every year from 2023 to 2100[2] which are elaborations of data from the United Nations, Department of Economic and Social Affairs, Population Division. I lastly got a table containing a few years from 5000 BC to 2023[3]. I pulled all the relevant data from each and combined it into a single table

[1] Ritchie, Hannah, et al. “Population Growth.” Ourworldindata, 2021, https://ourworldindata.org/population-growth. Accessed 2 June 2024.

[2] Worldometer. “World Population Projections.” worldometers, https://www.worldometers.info/world-population/world-population-projections/. Accessed 2 June 2024.

[3] worldometer. “World Population by Year.” Worldometer, 2024, https://www.worldometers.info/world-population/world-population-by-year/. Accessed 2 June 2024.

which contains all relevant data as well as an additionally calculated growth shown as change in people in billions, and growth rate represented as a percentage change.

$$growth = current\ population - previous\ population$$

$$growth\ rate = \frac{growth}{population} \times 100$$

The different colours in the years & population columns represent different datasets used, whilst the different colours in the growth rates represent the change in growth from the previous year (green suggests a greater growth than the preceding year). This was done using a series of checks using google sheets[4]: The cell colour is green if the growth rate is greater than the previous year, else if it is less than or equal to the previous value the colour is set to dark red. In all other cases the colour is set to a light red to indicate a smaller growth than the previous year.

Here is a section of the data which includes data breaks between years, for the complete table of raw data see appendix 1.

Year	Population (in Billions)	Growth (People in Billions)	Growth (%)
2100	10.3	-0.0109	-0.106
2022	7.98	0.0658	0.825
1400	0.350	-0.0100	-2.86

Figure 1, Complete compilation of 3 different sets of data containing global population data since -5000 BC to 2100 AD

[4] google. “Google Sheets : feuilles de calcul et modèles en ligne.” Google Workspace, https://workspace.google.com/intl/fr/products/sheets/. Accessed 31 August 2024.

I then plot this data as data points using the Desmos[5] tool, as can be seen in Fig. 2, which allows us to visualise how this data changes over time, and will later allow us to graph our models on top of the data points to see how closely it correlates with the data. When plotting with desmos some points are so close they appear to be a line. The y axis gives the population in billions, and the x axis gives the year.

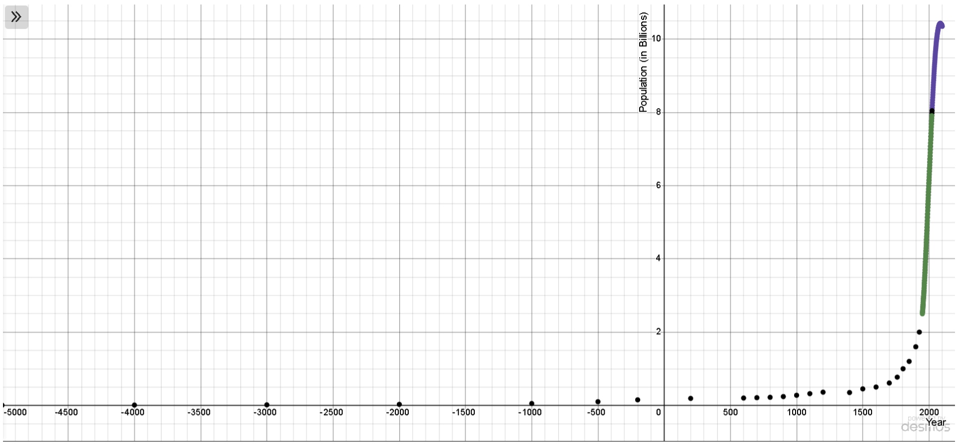

Figure 2, Datapoints of global population in years -5000 BC to 2100 AD

When removing data before 1600 as in Fig. 3, we get a better vision of the part which fits the logistic model.

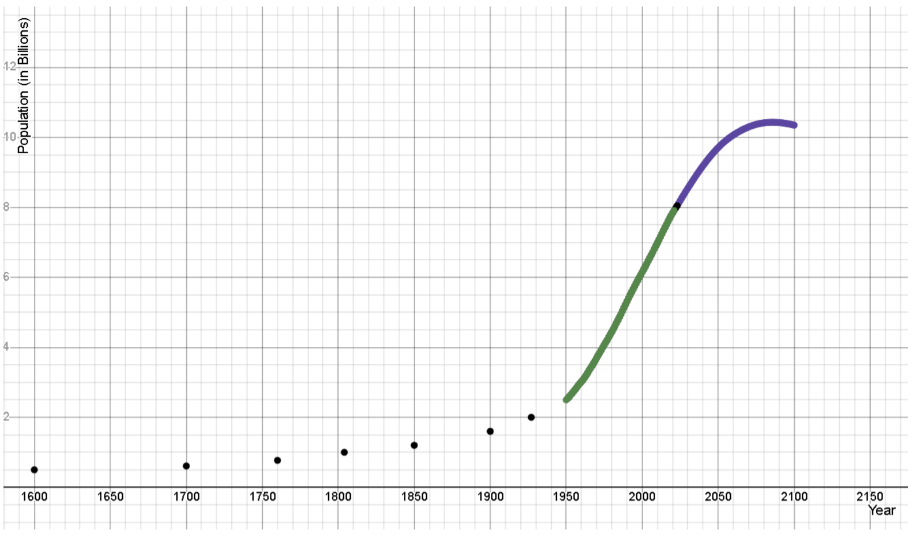

Figure 3, Datapoints of global population in years 1600 to 2100

[5] Desmos. "Desmos online graphing calculator." Desmos, http://desmos.com. Accessed 2 June 2024.

2.2 The logistic growth model

I will be modelling and graphing the global population using different variations of the logistic growth model, where A is a constant, y is the population in billions, K is the carrying capacity, t is the growth rate and x is the year.

The growth model is given as

$$y = \frac{K}{1+Ae^{-tx}}$$

I altered this model to be

$$y = \frac{K}{1+e^{-r(x-m)}}$$

Where r substitutes t, the growth rate.

These models are equivalent as

$$Ae = e^{c}$$

where c is another constant, therefore,

$$Ae^{-tx} = e^{-tx + c}$$

and if we consider another constant, m, where $c = (-\ t)(-\ m)$, then,

$$e^{-tx + c} = e^{-t(x-m)}$$

and therefore

$$Ae^{-tx} = e^{-t(x-m)}$$

Additionally, considering $(x - m)$ is a horizontal translation by m, both constants translate the function along the $x - axis$.

2.3 Fitting data to the logistic growth model

Looking at the previous data points in Fig. 2, we can clearly see that the global population remains extremely low (below 500 Million) up to the year 1600, after which the growth accelerates rapidly. After the 1950s we can then clearly see a shape similar to that of a logistic growth model, especially when complemented with the predicted values. However, what stands out is how the predicted population (in purple) then dips down as the population decreases in around the 2080s.

Since there is such a large uncertainty regarding the earth's projected population, past population, and change in growth I will be using 2 different logistic models to best model the population at different stages throughout time and to best account for earth's uncertainty. I will use an additional exponential model to visualise the population change before the apparent logistic growth and compare how it fits in contrast to the logistic model.

The models range from -5000 (5000 BC) to 2100 AD. The different models used are:

- An exponential model for the population change before the year 1700.
- The logistic model to best fit the total population from 5000 BC to 2100.
- The logistic model to best fit the modern data past the year 1950.

For getting the functions which best fit I use the logger pro[6] tool, as well as manually altering values of variables until getting a function which can no longer be transformed to better fit the data. The values that can be altered to apply

[6] Vernier. "Logger Pro 3." Vernier Science Education, https://www.vernier.com/product/logger-pro-3/. Accessed 2 June 2024.

mathematical transformations in the logistic model include the carrying capacity constant, K, which applies a vertical stretch with the x axis invariant, the growth rate, r, which applies a horizontal stretch with the y axis invariant, and the constant, m, which represents a horizontal translation by m years and will therefore be kept at 4 significant figures to increase accuracy.

The population before the 1700s is not a logistic growth and best fits an exponential growth under this model

$$y = 0.0100 \times 10^{0.000429x} + 0.0190, \{x < 1700\}$$

Which when visualised on desmos gives Figure 4.

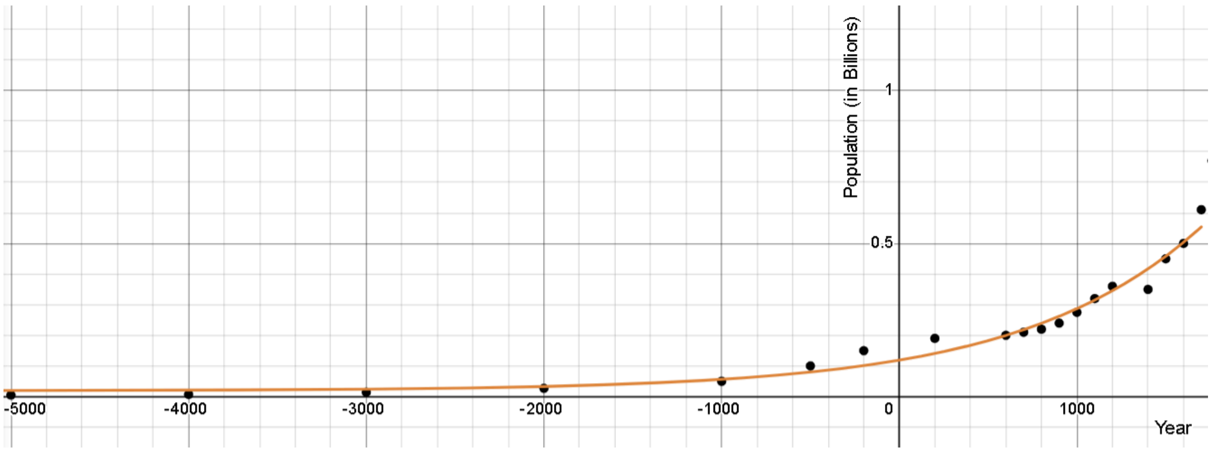

Figure 4, Graph of global population in years -5000 BC to 1700 AD

We can clearly see this section of data fit the exponential model extremely closely, with little doubt of there being a better model for it. However, as we are dealing with population data there is still some random variance causing slight outliers around the year 0 and the model starts losing accuracy after the 1700 AD which is where Figure 4 cuts off.

The best fitting logistical growth model since the 1950's, however, is

$$f(x) = \frac{11.1}{1+(e^{-(0.0311)\times(x-1992)})}$$

Which when graphed on desmos gives Figure 5.

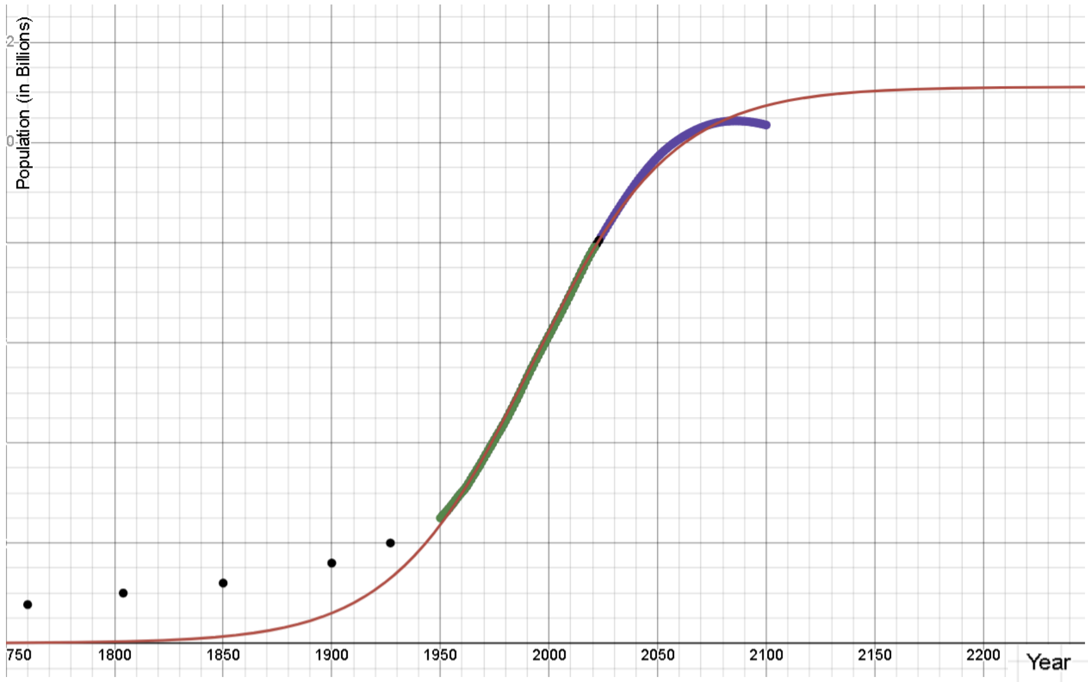

Figure 5, Graph of global population in years 1750 to 2250

This logistic model is clearly distant from the data before 1950, with it predicting a considerably lower population before the rapid increase near 1950. However this model fits the intended segment of data (1950 - 2100) considerably well until around 2024 where the model starts to level out earlier while the predicted data continues to increase until 2080 where it then starts to decrease below the carrying capacity shown on the logistic model. Therefore we can see the logistic model seems to lose accuracy on its extremities where it levels out at a starting and carrying capacity whereas the global population fits our earlier exponential model initially and seems to behave differently around the carrying capacity rather than simply levelling out with no growth.

Finally the best fitting model for the earth's population since 5000 BC to 2100 (our complete dataset) is given as $g(x)$ below to differentiate it from $f(x)$,

$$g(x) = \frac{9.70}{1+(e^{-(0.0372)\times(x-1998)})} + 1.13$$

This is similar to the previous logistic model, however a mathematical transformation has been applied for it to better fit the complete dataset, a vertical translation by 1.13 made by adding 1.13 to the result of the logistic model means the function will initially start at an asymptote of $y = 1.13$ rather than the x axis.

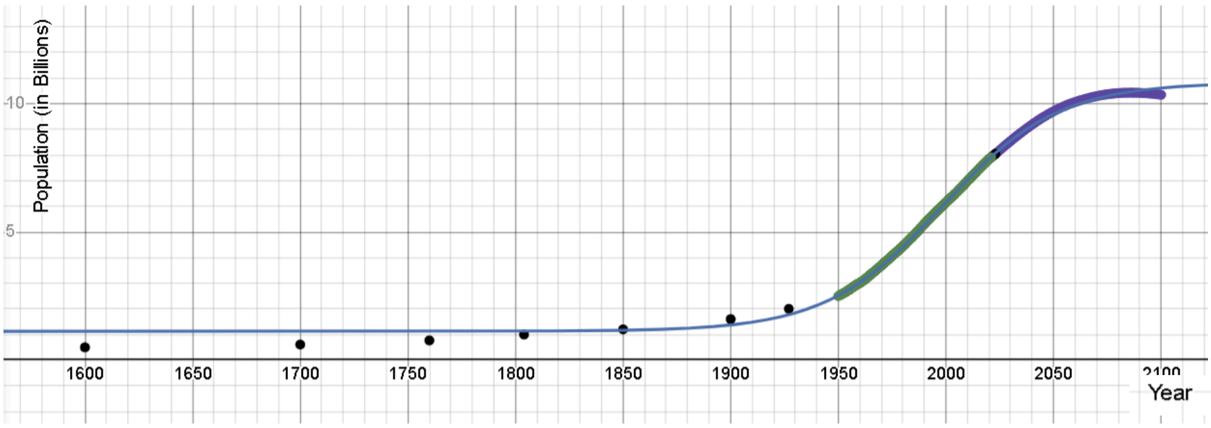

Figure 6, Graph of global population in years 1600 to 2100

This accounts for how we previously saw our model in Figure 5 inaccurately consider the initial human population before the 1950s as it no longer assumes the human population to be non-existent before the logistic growth. However, the data before the year 1800 still gets increasingly inaccurate as the logistic model remains at the asymptote of 1.13 Billion for the human population whilst the real population better fits a smaller exponential model as we saw in Figure 2.

3. Identifying the point of inflection in our best fit model

3.1 Taking the first derivative

Since we want to find the points in our models where the population growth stops increasing, we will start by taking the first derivative of the function $f(x)$, which best fitted the logistical growth since 1950 as this model did not use any additional mathematical transformations outside of the raw logistic model as well as best fitting the modern data which interests us the most.

This will give me the gradient over time of this function, the maximum value of this first derivative will then give me the point of inflection, also known as the turning point ,(where the gradient stops increasing, and the population grows at a decelerating rate).

Using the best fitting logistical growth model since the 1950's

$$f(x) = \frac{11.1}{1+(e^{-(0.0311)\times(x-1992)})}$$

Rewriting this function so that we can differentiate it

$$y = 11.1(1 + (e^{-(0.0311)(x-1992)}))^{-1}$$

The first derivative of this function will be

$$\frac{dy}{dx} = (-1)(11.1)(1 + (e^{-(0.0311)(x-1992)}))^{-2} \times \frac{d(1+(e^{-(0.0311)(x-1992)}))}{dx}$$

Solving the inner derivative we get

$$\frac{dy}{dx} = -\frac{(11.1)(-(0.0311)e^{-(0.0311)(x-1992)})}{(1+(e^{-(0.0311)(x-1992)}))^2}$$

Simplifying fully we end up with

$$\frac{dy}{dx} = \frac{(0.346)e^{-(0.0311)(x-1992)})}{(1+(e^{-(0.0311)(x-1992)}))^2}$$

3.2 Taking the second derivative

To find the maximum of the first derivative I will need the second derivative

Therefore we are simply taking the derivative of the function

$$\frac{dy}{dx} = \frac{(0.346)e^{-(0.0311)(x-1992)})}{(1+(e^{-(0.0311)(x-1992)}))^2}$$

We can simplify this down to a negative exponent

$$\frac{dy}{dx} = (0.346)e^{-(0.0311)(x-1992)})(1 + (e^{-(0.0311)(x-1992)}))^{-2}$$

Which therefore allows us to use the chain rule

$$\frac{d^2y}{dx^2} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Where

$$v\frac{du}{dx} = (0.346)(e^{-(0.0311)(x-1992)}) \times \frac{d(1+(e^{-(0.0311)(x-1992)}))^{-2}}{dx}$$

And

$$\frac{dv}{dx}u = \frac{d(0.346)e^{-(0.0311)(x-1992)}}{dx} \times (1 + (e^{-(0.0311)(x-1992)}))^{-2}$$

Solving for the inner derivatives

$$\frac{du}{dx} = (-\ 2)(1\ +\ (e^{-(0.0311)(x-1992)})^{-3} \times ((e^{-(0.0311)(x-1992)})(-\ 0.0311))$$

$$= \frac{(0.0622)(e^{-(0.0311)(x-1992)})}{(1+(e^{-(0.0311)(x-1992)}))^{3}}$$

And

$$\frac{dv}{dx} = (0.346)\times\ e^{-(0.0311)(x-1992)} \times (-\ 0.0311)$$

$$= (-\ 0.0108)e^{-(0.0311)(x-1992)}$$

Therefore

$$v\frac{du}{dx} = (0.346)(e^{-(0.0311)(x-1992)}) \times \frac{(0.0623)(e^{-(0.0311)(x-1992)})}{(1+(e^{-(0.0311)(x-1992)}))^{3}}$$

$$= \frac{(0.0215)(e^{-(0.0311)(x-1992)})^{2}}{(1+(e^{-(0.0311)(x-1992)}))^{3}}$$

and

$$u\frac{dv}{dx} = (-\ 0.0108)e^{-(0.0311)(x-1992)} \times (1\ +\ (e^{-(0.0311)(x-1992)}))^{-2}$$

$$= \ -\ \frac{(-0.0108)\ e^{-(0.0311)(x-1992)}}{(1+(e^{-(0.0311)(x-1992)}))^{2}}$$

Therefore the full expression gives

$$\frac{d^{2}y}{dx^{2}} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$= \frac{(0.0215)(e^{-(0.0311)(x-1992)})^{2}}{(1+(e^{-(0.0311)(x-1992)}))^{3}} - \frac{(0.0108)\ e^{-(0.0311)(x-1992)}}{(1+(e^{-(0.0311)(x-1992)}))^{2}}$$

3.3 Finding the point of inflection

Since we know our graph (The logistic growth model) only has one point of inflection if we only get one possible answer we can assume this point to be the correct point of inflection without requiring additional checks such as third derivatives.

Therefore, we can get the point of inflection at the point where the second derivative is equal to 0, as this is where the first derivative has a maximum (the gradient of the first derivative is equal to zero).

$$\frac{d^2y}{dx^2} = \frac{(0.0215)(e^{-(0.0311)(x-1992)})^2}{(1+(e^{-(0.0311)(x-1992)}))^3} - \frac{(0.01075)\, e^{-(0.0311)(x-1992)}}{(1+(e^{-(0.0311)(x-1992)}))^2} = 0$$

Therefore

$$\frac{(0.0215)(e^{-(0.0311)(x-1992)})^2}{(1+(e^{-(0.0311)(x-1992)}))^3} = \frac{(0.0108)\, e^{-(0.0311)(x-1992)}}{(1+(e^{-(0.0311)(x-1992)}))^2}$$

and

$$(0.0215)(e^{-(0.0311)(x-1992)})^2 = (0.0108)\, e^{-(0.0311)(x-1992)} \times (1 + (e^{-(0.0311)(x-1992)})$$

$$= (0.0108)(e^{-(0.0311)(x-1992)}) + (0.0108)(e^{-(0.0311)(x-1992)})^2$$

Simplifying this

$$(0.0215)(e^{-(0.0311)(x-1992)}) = (0.0108) + (0.0108)(e^{-(0.0311)(x-1992)})$$

$$(0.0108)(e^{-(0.0311)(x-1992)}) = (0.01075)$$

$$(e^{-(0.0311)(x-1992)}) = 1$$

$$ln(1) = -(0.0311)(x - 1992)$$

$$-\ (0.0311)(x\ -\ 1992)\ =\ 0$$

Which gives us the value for x

$$x\ =\ 1992$$

Therefore we can see that for this function, the point of inflection seems to be equal to the constant we set as m.

4. Generalising for any logistic model

4.1 Generalising our second derivative

When finding the point of inflection for the function f, we found this point of inflection to be at the point m. Considering m applies a horizontal transformation on the function, this made me think if this could always be the function's midpoint, how we could prove that, and if it would be possible to generalise the derivative to then be able to find the point of inflection of each model easier and quicker.

Since we want to generalise the derivatives to get a generalised point of inflection which works for any logistic growth model, we need to use the logistic growth model with the constants not substituted for. We can therefore test if the point of inflection is indeed the x translation and point we set as m, the midpoint.

This time we’ll use the general formula for logistic growth

$$f(x)\ =\ \frac{K}{1+(e^{-r\times(x-m)})}$$

We will use the same techniques as previously to find the first, then second derivatives of

$$y = K(1 + (e^{-(r)(x-m)}))^{-1}$$

Therefore

$$\frac{dy}{dx} = \frac{(K)(r)e^{-(r)(x-m)})}{(1+(e^{-(r)(x-m)}))^2}$$

And therefore, according to the chain rule

$$\frac{d^2y}{dx^2} = ((Kr)(e^{-(r)(x-m)}) \times \frac{d(1+(e^{-(r)(x-m)}))^{-2}}{dx}) + (\frac{d(Kr)e^{-(r)(x-m)}}{dx} \times (1 + (e^{-(r)(x-m)}))^{-2})$$

Solving for the inner derivatives

$$\frac{d^2y}{dx^2} = ((Kr)(e^{-(r)(x-m)}) \times \frac{(2r)(e^{-(r)(x-m)})}{(1+(e^{-(r)(x-m)}))^3}) + ((- Kr^2)e^{-(r)(x-m)} \times (1 + (e^{-(r)(x-m)}))^{-2})$$

Which simplifies the second derivative to

$$\frac{d^2y}{dx^2} = \frac{(2Kr^2)(e^{-(r)(x-m)})^2}{(1+(e^{-(r)(x-m)}))^3} - \frac{(Kr^2)\, e^{-(r)(x-m)}}{(1+(e^{-(r)(x-m)}))^2}$$

4.2 Finding the generalised point of inflection

Now, similar to before, we will find the point of inflection where the second derivative is equal to 0.

Therefore, it will be equal to the x value where the following expression holds true

$$\frac{(2Kr^2)(e^{-(r)(x-m)})^2}{(1+(e^{-(r)(x-m)}))^3} = \frac{(Kr^2)\, e^{-(r)(x-m)}}{(1+(e^{-(r)(x-m)}))^2}$$

Therefore,

$$(2Kr^2)(e^{-(r)(x-m)}) = (Kr^2) + (Kr^2)(e^{-(r)(x-m)})$$

$$(e^{-(r)(x-m)}) = 1$$

$$ln(1) = \; -(r)(x - m)$$

$$-(r)(x - m) \; = \; 0$$

And therefore the expression is true where,

$$x = m$$

Confirming our previous hypothesis holds true. This shows m is indeed the midpoint of the logistic model, and will always be the point of inflection in our altered logistic model.

5. Evaluation

5.1 The range of points of inflection

Since we confirmed that our point of inflection will be the point m, as long as we have positive values of K and r, we will get the points of inflection of the other logistic models to be their value of m.

The best fitting function of the logistic growth model for past the year 1950 is given as

$$f(x) \; = \; \frac{11.1}{1+(e^{-(0.0311)\times(x-1992)})}$$

Therefore,

$$m \; = \; 1992$$

The best fitting logistic growth model for the complete dataset is given as

$$g(x) = \frac{9.70}{1+(e^{-(0.0372)\times(x-1998)})} + 1.13$$

The vertical translation of 1.13 does not affect the inflection point & has no impact on derivatives, therefore meaning this function follows the previous findings and therefore

$$m = 1998$$

5.2 Conclusion

My aim in this investigation has been to predict when the turning point in human population growth occurred through obtaining the second derivatives of the global population modelled by logistic models. Doing this once allowed us to generalise a solution for all models giving us a range of possible points of inflections which we can see across the models appearing consistently between 1992 and 1998.
This shows in the current year (2024) we have passed the point of inflection and the population growth has shifted to a yearly decrease. That means our concerns should perhaps turn away from the effects of a rapidly increasing population and instead focus on more pertinent issues such as eventual population decline. Considering populations experience a "momentum factor", recognizing and addressing these population changes early is key if attempting to control these population shifts.

Overall we got a range of around 6 years across the models, which is much smaller than I would have expected. Since m affects the point of inflection other transformations such as the carrying capacity, K, don't affect the point of inflection meaning uncertainties of the world's carrying capacity (estimated between 8-16 Billion) have no effect on our results. According to the world population data as given by ourworldindata.org[7], in 1988 we can see the population growth shift to a consistent yearly decline with only the years 2007, 2008, 2023 and 2024 standing out as anomalies which strongly aligns with our results as through modelling this data we effectively 'smoothed out' these fluctuations, and thus got a result slightly greater than 1988.

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[7] Ritchie, Hannah, et al. "Population Growth." Ourworldindata, 2021, https://ourworldindata.org/population-growth. Accessed 2 June 2024.

5.4 Future Suggestions

A limitation we encountered during this investigation is that the data used to predict future population could turn out incorrect which could drastically alter our models accuracy. Perhaps testing the effect of removing the future population predictions from the data on the models could help assess the strength of our results.

Additionally, the human population, and even other natural biological populations often do not perfectly follow the logistic model due to not levelling out at the carrying capacity, but rather oscillating above and below it similar to a sinusoidal curve. This seems to be evident in our population data where the population is predicted to start declining in 2087, as the population decreases with a seemingly increasing rate of decrease (exponential negative growth).

Thus we could expand and improve on this research by additionally modelling the population data estimated to oscillate around the carrying capacity with a sine curve. Another interesting aspect would be seeing how the growth has changed in the past following major technological milestones. This could help better predict future trends as we can suspect similar breakthroughs to occur in the future.

Overall these techniques could allow not only a better model for deriving the past point of inflection, but may also permit us to derive points of inflection for the future, where the global population oscillates around the carrying capacity. Where the future concavity of the graph potentially changes back to concave up might be the reference / equilibrium line of the population's sinusoidal oscillation and thus could give a better estimation of the earth’s carrying capacity in a future paper.

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6.3 Appendix

Appendix 1, Complete complied set of data containing global population data since -5000 BC to 2100 AD

Year	Population (in Billions)	Growth (People in Billions)	Growth (%)
2100	10.34932304	-0.010946324	-0.106
2099	10.36026936	-0.010175834	-0.098
2098	10.3704452	-0.009443252	-0.091
2097	10.37988845	-0.008628701	-0.083
2096	10.38851715	-0.007788695	-0.075
2095	10.39630584	-0.006966305	-0.067
2094	10.40327215	-0.00621894	-0.060
2093	10.40949109	-0.00546874	-0.053
2092	10.41495983	-0.004687303	-0.045
2091	10.41964713	-0.003893904	-0.037
2090	10.42354104	-0.003060111	-0.029
2089	10.42660115	-0.002229479	-0.021

2088	10.42883063	-0.001436998	-0.014
2087	10.43026762	-0.000658657	-0.006
2086	10.43092628	0.00024718	0.002
2085	10.4306791	0.001231706	0.012
2084	10.4294474	0.002220995	0.021
2083	10.4272264	0.003203504	0.031
2082	10.4240229	0.004171407	0.040
2081	10.41985149	0.0052143	0.050
2080	10.41463719	0.006370267	0.061
2079	10.40826692	0.007517169	0.072
2078	10.40074975	0.008667753	0.083
2077	10.392082	0.009901603	0.095
2076	10.3821804	0.011186199	0.108
2075	10.3709942	0.012441435	0.120
2074	10.35855276	0.013661271	0.132
2073	10.34489149	0.014831638	0.143
2072	10.33005985	0.015917551	0.154
2071	10.3141423	0.016975592	0.165
2070	10.29716671	0.018091841	0.176
2069	10.27907487	0.019201879	0.187
2068	10.25987299	0.020289211	0.198
2067	10.23958378	0.021310764	0.208
2066	10.21827302	0.022308116	0.218
2065	10.1959649	0.023451125	0.230

2064	10.17251378	0.02456466	0.241
2063	10.14794912	0.025641436	0.253
2062	10.12230768	0.026734615	0.264
2061	10.09557306	0.027839458	0.276
2060	10.06773361	0.029027745	0.288
2059	10.03870586	0.03039575	0.303
2058	10.00831011	0.031869961	0.318
2057	9.97644015	0.033316888	0.334
2056	9.943123262	0.034818393	0.350
2055	9.908304869	0.036404267	0.367
2054	9.871900602	0.038039411	0.385
2053	9.833861191	0.039759148	0.404
2052	9.794102043	0.041445127	0.423
2051	9.752656916	0.043165155	0.443
2050	9.709491761	0.044975615	0.463
2049	9.664516146	0.046741676	0.484
2048	9.61777447	0.048476584	0.504
2047	9.569297886	0.050107082	0.524
2046	9.519190804	0.051647229	0.543
2045	9.467543575	0.053135152	0.561
2044	9.414408423	0.054572003	0.580
2043	9.35983642	0.055939569	0.598
2042	9.303896851	0.057223551	0.615
2041	9.2466733	0.058422808	0.632

2040	9.188250492	0.059589277	0.649
2039	9.128661215	0.060772189	0.666
2038	9.067889026	0.061862656	0.682
2037	9.00602637	0.062819668	0.698
2036	8.943206702	0.063809301	0.713
2035	8.879397401	0.06482223	0.730
2034	8.814575171	0.065776629	0.746
2033	8.748798542	0.066706558	0.762
2032	8.682091984	0.067559239	0.778
2031	8.614532745	0.068391418	0.794
2030	8.546141327	0.069251936	0.810
2029	8.476889391	0.070060599	0.826
2028	8.406828792	0.070851121	0.843
2027	8.335977671	0.071613162	0.859
2026	8.264364509	0.072376056	0.876
2025	8.191988453	0.073152454	0.893
2024	8.118835999	0.073524552	0.906
2023	8.045311447	0.070206291	0.873
2022	7.975105156	0.065810156	0.825
2021	7.909295	0.068342	0.864
2020	7.840953	0.076002	0.969
2019	7.764951	0.081161	1.045
2018	7.68379	0.0839677	1.093
2017	7.5998223	0.0863483	1.136

2016	7.513474	0.0868766	1.156
2015	7.4265974	0.0875838	1.179
2014	7.3390136	0.0884203	1.205
2013	7.2505933	0.0888953	1.226
2012	7.161698	0.0885726	1.237
2011	7.0731254	0.0875224	1.237
2010	6.985603	0.087297	1.250
2009	6.898306	0.0867087	1.257
2008	6.8115973	0.0856489	1.257
2007	6.7259484	0.0845324	1.257
2006	6.641416	0.08324	1.253
2005	6.558176	0.0824246	1.257
2004	6.4757514	0.0818529	1.264
2003	6.3938985	0.0814909	1.275
2002	6.3124076	0.0816606	1.294
2001	6.230747	0.081848	1.314
2000	6.148899	0.0811404	1.320
1999	6.0677586	0.080446	1.326
1998	5.9873126	0.0808316	1.350
1997	5.906481	0.0813357	1.377
1996	5.8251453	0.0819256	1.406
1995	5.7432197	0.0824917	1.436
1994	5.660728	0.0832944	1.471
1993	5.5774336	0.0847473	1.519

1992	5.4926863	0.0864403	1.574
1991	5.406246	0.09007	1.666
1990	5.316176	0.092472	1.739
1989	5.223704	0.09141	1.750
1988	5.132294	0.0913094	1.779
1987	5.0409846	0.0909216	1.804
1986	4.950063	0.088332	1.784
1985	4.861731	0.085895	1.767
1984	4.775836	0.083952	1.758
1983	4.691884	0.0838994	1.788
1982	4.6079846	0.0833571	1.809
1981	4.5246275	0.0806195	1.782
1980	4.444008	0.078425	1.765
1979	4.365583	0.0759254	1.739
1978	4.2896576	0.0738852	1.722
1977	4.2157724	0.0732667	1.738
1976	4.1425057	0.0730685	1.764
1975	4.0694372	0.0739202	1.816
1974	3.995517	0.0752656	1.884
1973	3.9202514	0.0754506	1.925
1972	3.8448008	0.0746376	1.941
1971	3.7701632	0.074773	1.983
1970	3.6953902	0.0747348	2.022
1969	3.6206554	0.0738444	2.040

1968	3.546811	0.071363	2.012
1967	3.475448	0.069031	1.986
1966	3.406417	0.069305	2.035
1965	3.337112	0.0698997	2.095
1964	3.2672123	0.071433	2.186
1963	3.1957793	0.0690926	2.162
1962	3.1266867	0.058316	1.865
1961	3.0683707	0.0491372	1.601
1960	3.0192335	0.0489413	1.621
1959	2.9702922	0.0541842	1.824
1958	2.916108	0.0582412	1.997
1957	2.8578668	0.056864	1.990
1956	2.8010028	0.0549308	1.961
1955	2.746072	0.0540927	1.970
1954	2.6919793	0.0517005	1.921
1953	2.6402788	0.0500078	1.894
1952	2.590271	0.0471406	1.820
1951	2.5431304	0.0438084	1.723
1950	2.499322	0.499322	19.978
1927	2	0.4	20.000
1900	1.6	0.4	25.000
1850	1.2	0.2	16.667
1804	1	0.23	23.000
1760	0.77	0.16	20.779

1700	0.61	0.11	18.033
1600	0.5	0.05	10.000
1500	0.45	0.1	22.222
1400	0.35	-0.01	-2.857
1200	0.36	0.04	11.111
1100	0.32	0.045	14.063
1000	0.275	0.035	12.727
900	0.24	0.02	8.333
800	0.22	0.01	4.545
700	0.21	0.01	4.762
600	0.2	0.01	5.000
200	0.19	0.04	21.053
-200	0.15	0.05	33.333
-500	0.1	0.05	50.000
-1000	0.05	0.023	46.000
-2000	0.027	0.013	48.148
-3000	0.014	0.007	50.000
-4000	0.007	0.002	28.571
-5000	0.005		