# Effect of load on the deflection of a cantilever beam fixed at both ends

Word Count: **2696**

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# 1. Introduction

## 1.1 Introduction

Understanding how applied forces affect deflection in cantilever beams is essential for many engineering applications. Some older mountain bikes and cars use an outdated suspension system to absorb impacts called the ‘leaf spring’. This is essentially a collection of thin metal beams which act as cantilever beams fixed at both ends deflecting to absorb impacts of the terrain. As cars and mountain bikes have become heavier and are required to absorb greater forces due to harsher terrain these systems have been antiquated and replaced by newer spring and air suspension systems. Some metal flex pivots and link systems in some downhill bikes still act in a similar fashion however. Therefore by seeing how increasing the loads on a cantilever beam fixed at both ends affects the displacement of the beam at the centre, I could get a better idea why these systems could not be used today. Additionally, this knowledge is crucial for designing structures such as bridges and supports for buildings, where accurate deflection predictions and models are essential.

## 1.2 Research Question

How does the Force applied, in $N$, at the centre of a cantilever beam fixed at both ends affect the displacement produced at the centre, in $m$?

## 1.3 Physics Background Information

A cantilever fixed at both ends will experience a deflection when a net force is applied onto it. When the force is applied near the centre in a downwards direction, the deflection will also occur downwards with maximum deflection at the centre of the beam. I wish to model how this deflection will change on a flat cantilever beam supported at both ends as I change the load placed onto the centre of the beam.

This deflection experienced by a beam supported at both ends can be modelled according to the following formula when the load is applied at the centre[1].

$$x = \frac{FL^3}{48 \times E \times I}$$

Where:

$x$ is the resulting deflection. $(m)$

$F$ is the applied load. $(N)$

$L$ is the length of the beam. $(m)$

$E$ is the beam material's Young's Modulus. ($Pa$ or $Nm^{-2}$)

$I$ is the moment of inertia of the beam. ($kg\ m^2$)

Therefore the equation can be written in the form $y = mx + c$, where

$y = x$,

$m = \frac{L^3}{48 \times E \times I}$,

[1] "Beams - Supported at Both Ends - Continuous and Point Loads." *The Engineering ToolBox*, 2009, https://www.engineeringtoolbox.com/beam-stress-deflection-d_1312.html. Accessed 6 October 2024.

$x = F$,

And, $c = 0$

Thus, the amount of deflection depends on the materials of the beam (Young's Modulus), the geometry (shape & size) of the beam, as well as the applied force.

I aim to investigate how the applied force affects the deflection experienced by the beam. According to Newton’s second law of motion:

$F = ma$

Where $F$ is Force, $m$ is mass, and $a$ is acceleration. And thus, considering the force, $F$, to be $W$, the Weight applied by the masses on earth, we can say that:

$W = mg$

Where $g \approx 9.81$, and $g$ is the acceleration due to gravity observed on the surface of the earth. We can thus alter the applied force by changing the load as the amount of mass placed onto the beam.

Considering the deflection is related to the beam's Young's Modulus, defined as $E$, we could check the accuracy in our results by calculating $E$ for the beam, and comparing this to the expected value for the beam’s material.

Since we know from our initial formula the gradient is equal to

$$m = \frac{L^3}{E \times 48 \times I}$$

We can determine $E$ using the formula

$$E = \frac{L^3}{m \times 48 \times I}$$

Therefore, we initially need to find out the value of the beam’s Area Moment of Inertia, $I$, which we can determine for a rectangular beam using the following formula[2]:

$$I = (\frac{1}{12}) \times L^3 \times w,$$

Where,

$L$ is the length of the beam, and $w$ is the width of the beam

## 1.4 Experimental Variables

| **Variables** | | | |
|---|---|---|---|
| **Variable** | | **Explanation** | **Apparatus** |
| **Independent Variable** | Applied Force onto the beam | Varying the applied force by randomly selecting a range of masses to be used for each trial and weighing them using an electronic balance. | Electronic Balance ( ± 0.0001 Kg )<br><br>(used due to high precision) |
| **Dependant Variable** | Deflection of the beam | Measured by recording the difference between the initial position without weight applied, and the final position once the weight has been applied using a travelling microscope. | Travelling Microscope ( ± 0.00001 m )<br><br>(used due to high precision) |

**Table 1, Independent and Dependant variables**

| **Control Variables** | | |
|---|---|---|
| **Variable** | **Justification** | **Means of Control** |
| Geometry (Length & Shape) of Beam | The geometry of the beam affects its Moment of Inertia, which affects the gradient of the graph | Use the same beam throughout & evenly clamp sides using blocks to control the length over which deflection occurs |
| Young's Modulus of Beam's Material | The Young's Modulus of the beam's material affects the | Use the same metal beam throughout |

[2] “Area Moment of Inertia - Typical Cross Sections I.” *The Engineering ToolBox*, 2008, https://www.engineeringtoolbox.com/area-moment-inertia-d_1328.html. Accessed 6 October 2024.

| | | |
|---|---|---|
| | gradient of the plotted best fit line | |
| Area over which weight is applied | Distributing the weight over a larger or smaller area affects the impact on deflection | Place masses in a stack with the same one at the bottom throughout the entire experiment. This ensures always the same area is in contact with the beam |
| Position where weight is applied | The defined formula assumes the weight to be perfectly applied in the centre of the beam | Measured & marked centre of the beam where the masses will be placed using a ruler. |

**Table 2, Control Variables**

## 1.5 Hypothesis

According to the formula obtained in section 1.3, as I increase the applied force, the deflection should be directly proportional, passing through the origin. The expected direct proportionality can be seen in Figure 1.

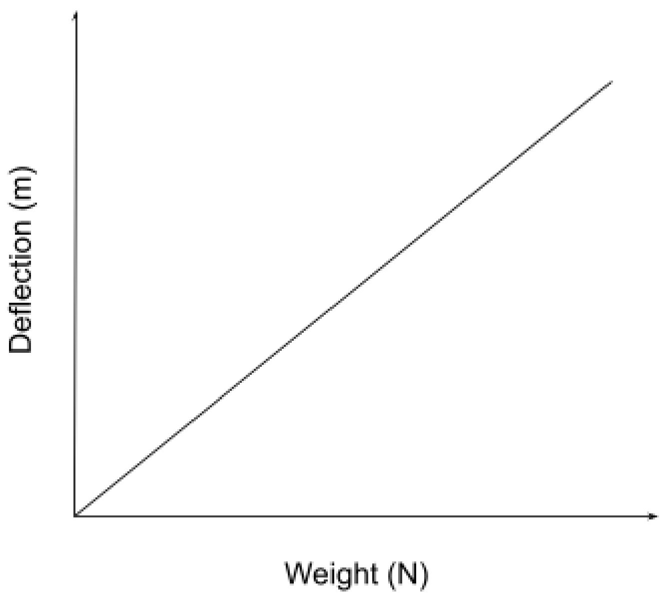



**Figure 1, Sketch showing hypothesised direct proportionality**

The gradient should allow us to determine the Young's Modulus using measurements for the moment of inertia of the beam.

# 2. Experiment

## 2.1 Required Materials

- Aluminium beam, thin and long to enable accurate deflection measurement. (Specifications: Length $0.600\ m \pm 0.001$, Width $0.034\ m \pm 0.001$, Thickness $0.0011\ m \pm 0.0001$, Mass $0.0584\ Kg \pm 0.0001$)
- Slotted masses: which can be stacked to stay applied at the centre of the beam. ( $100\ g \pm 1$ )
- 4 Wooden Blocks of equal dimensions: to hold the beam's edges in a symmetrical manner.
- Pencil: To mark centre of beam
- 2 Clamps: to hold the sides of the beam in place.
- Travelling Microscope: to measure the initial and final position of beam centre for deflection ( $\pm\ 0.00001\ m$ )
- Ruler: to measure beam dimensions and finding the centre for placing the loads ( $\pm\ 0.001\ m$ )
- Electronic Balance: to measure applied mass ( $\pm\ 0.0001\ kg$ )

## 2.2 Method

1. Clamp the metal beam between two wooden blocks, to two level surfaces. (see Figure 2 or Figure 3)
2. Measure the centre of the beam using the ruler and mark that point with an x.
3. Mark an additional dot on the side of the beam facing the travelling microscope.
4. Align the travelling microscope to be currently centred on and viewing the marked dot and record the current value.
5. Select a random sample of mass and measure it using the scale and record the value.
6. Place the weight on the marked centre lightly without applying any additional pressure or dropping it.
7. Move the travelling microscope to the newly positioned dot and record the new value.
8. Remove the weight and repeat the steps 6 and 7 for 2 more times to get a total of 3 values for the given weight.
9. Remove all masses and repeat the steps 5 to 9 for a total of 8 different masses.

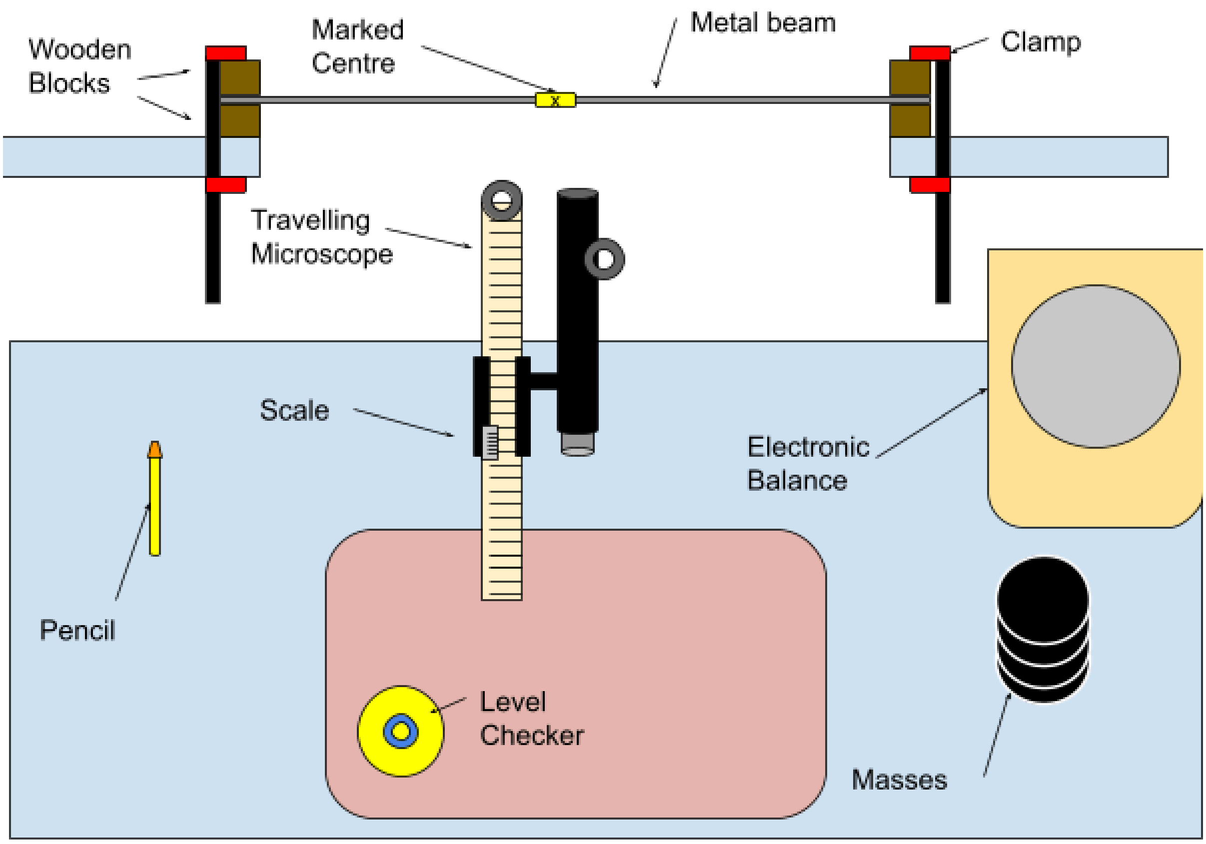



**Figure 2, Diagram displaying Experimental Setup**

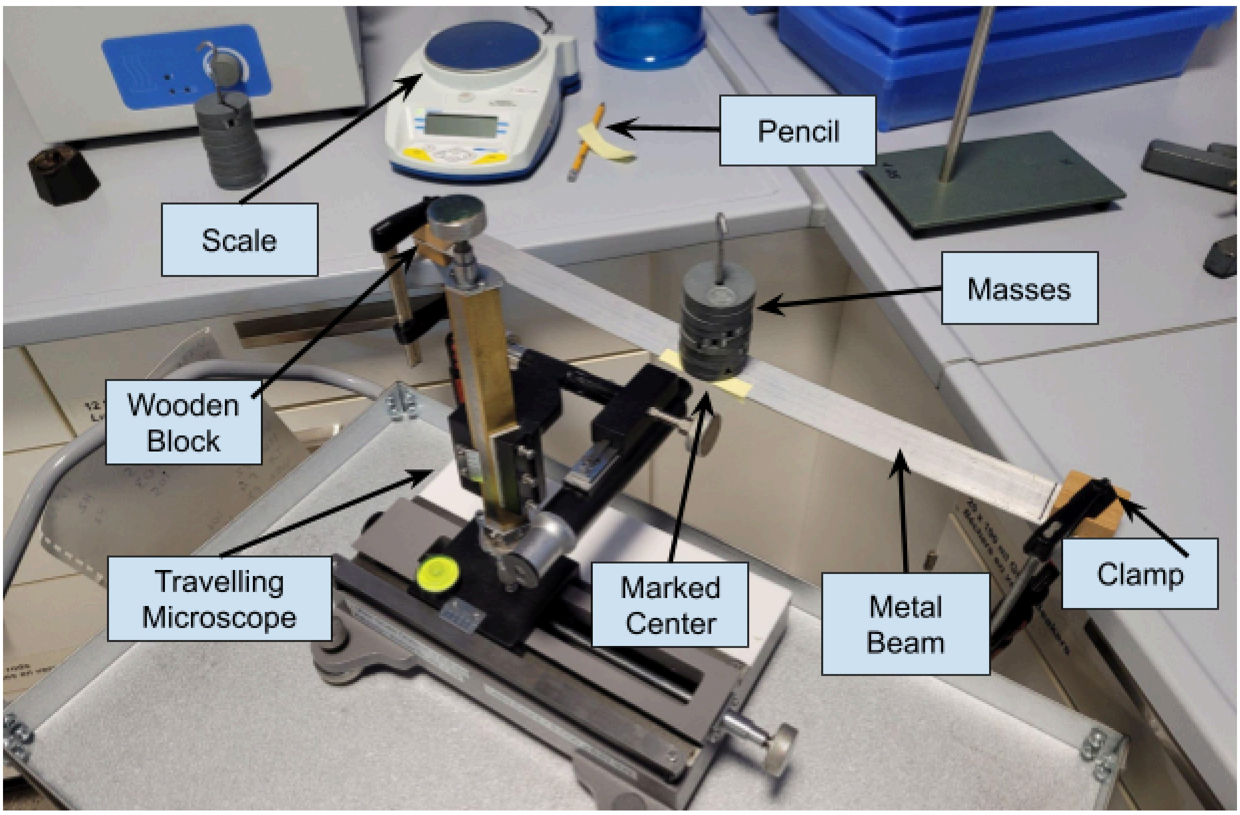



**Figure 3, Labelled Experimental Setup**

## 2.3 Risk Assessment

The beam can break if too much weight and strain is applied, therefore wearing eye protection such as lab glasses can prevent parts from flying into your eyes. Additionally, reducing the applied weight or using a new beam if the current beam seems to be experiencing too much strain.

The masses can slip off the beam and fall, ensure the area under the beam is clear and never step under the beam. Additionally, you can place cushions to catch the masses if they do fall.

# 3. Results

## 3.1 Obtained Data

The beam measurements are displayed in Table 3.

| Beam Measurements (m) | | Uncertainty |
|---|---|---|
| **Length** | **0.600** | **0.001** |
| **Width** | **0.034** | **0.001** |
| **Thickness** | **0.0011** | **0.0001** |
| **Mass** | **0.0584** | **0.0001** |

**Table 3, Beam measurements**

Using the measurements of the beam found in Table 3 we can obtain the beam's moment of inertia, $I$, using the formula obtained in the Physics Background Section 1.3.

$$I = (\frac{1}{12}) \times L^3 \times w$$

$$I = (\frac{1}{12}) \times 0.6^3 \times 0.034 = 0.000612\ Kg\,m^2$$

The uncertainty was obtained using the following formula.

$$\Delta I = I \times (\frac{3 \times \Delta L}{L} + \frac{\Delta w}{w})$$

$$\Delta I = 0.000612 \times (\frac{3 \times (0.001)}{0.6} + \frac{0.001}{0.034}) = 0.0000211\ Kg\,m^2$$

Therefore, the beam's area moment of inertia can be written as below, when accounting for uncertainty.

$$I = (6.12 \pm 0.21) \times 10^{-4}\ Kg\,m^2$$

The resulting data from the experiment is displayed in Table 4, as can be seen below.

| Mass (Kg) | Position (m) | | | | | Deflection, x (m) | Uncertainty in Deflection, Δ x (m) |
|---|---|---|---|---|---|---|---|
| | Weight (N) | Reading 1 | Reading 2 | Reading 3 | Average | | |
| **± 0.01** | **± 0.0001** | **± 0.00001** | **± 0.00001** | **± 0.00001** | | | **± Δ (m)** |
| 0.00000 | 0.0000 | 0.02746 | | | 0.02746 | 0.00000 | **0.00001** |
| 0.50038 | 4.9071 | 0.02644 | 0.02673 | 0.02666 | 0.02661 | 0.00085 | **0.00016** |
| 0.99325 | 9.7405 | 0.02592 | 0.02584 | 0.02613 | 0.02596 | 0.00150 | **0.00016** |
| 1.29173 | 12.6675 | 0.02552 | 0.02578 | 0.02599 | 0.02576 | 0.00170 | **0.00025** |
| 0.19883 | 1.9499 | 0.02708 | 0.02716 | 0.02721 | 0.02715 | 0.00031 | **0.00007** |
| 1.99119 | 19.5269 | 0.02444 | 0.02465 | 0.02471 | 0.02460 | 0.00286 | **0.00015** |
| 0.59635 | 5.8482 | 0.02682 | 0.02677 | 0.02644 | 0.02668 | 0.00078 | **0.00020** |
| 0.79317 | 7.7783 | 0.02638 | 0.02616 | 0.02609 | 0.02621 | 0.00125 | **0.00016** |
| 1.59278 | 15.6198 | 0.02643 | 0.02576 | 0.02596 | 0.02605 | 0.00141 | **0.00034** |

**Table 4, Raw and processed data**

The Weight in Newtons was calculated using the formula defined in the Background Physics Section 1.3.

When the mass was $5.0038 \times 10^{-1}$ Kg, it gave us the following calculation:

$$W = mg$$

$$W = (5.0038 \times 10^{-1}) \times 9.806652 = 4.9071\, N$$

The Average reading for each mass was calculated using the equation:

$$Average = \frac{(reading\ 1 + reading\ 2 + reading\ 3}{3}$$

For instance at $5.0038 \times 10^{-1}$ Kg, we used the following calculation to obtain the average:

$$Average = \frac{(0.02644 + 0.02673 + 0.02666}{3} = 0.02661\, m$$

The average deflection for each reading was calculated using the following formula:

$$x = zero\ weight\ reading - Average\ position\ reading$$

An example calculation for the average deflection at $5.0038 \times 10^{-1}$ Kg is shown below:

$$x = 0.02746 - 0.02661 = 0.00085\, m$$

The Uncertainty in deflection was the calculated individually for each reading using the following equation:

$$\Delta x = \frac{(maximum\ reading - minimum\ reading)}{2} + zero\ reading\ error$$

For $5.0038 \times 10^{-1}$ Kg this gives:

$$\Delta x = \frac{(0.02673 - 0.02644)}{2} + 0.00001 = 0.00016\, m$$

## 3.2 Data Interpretation / Presentation

We plot the Deflection against the Weight, as defined in the Physics Background Section 1.3, whilst including uncertainties in the forms of error bars and defining a line of best fit through the data points, as can be seen in Figure 4.

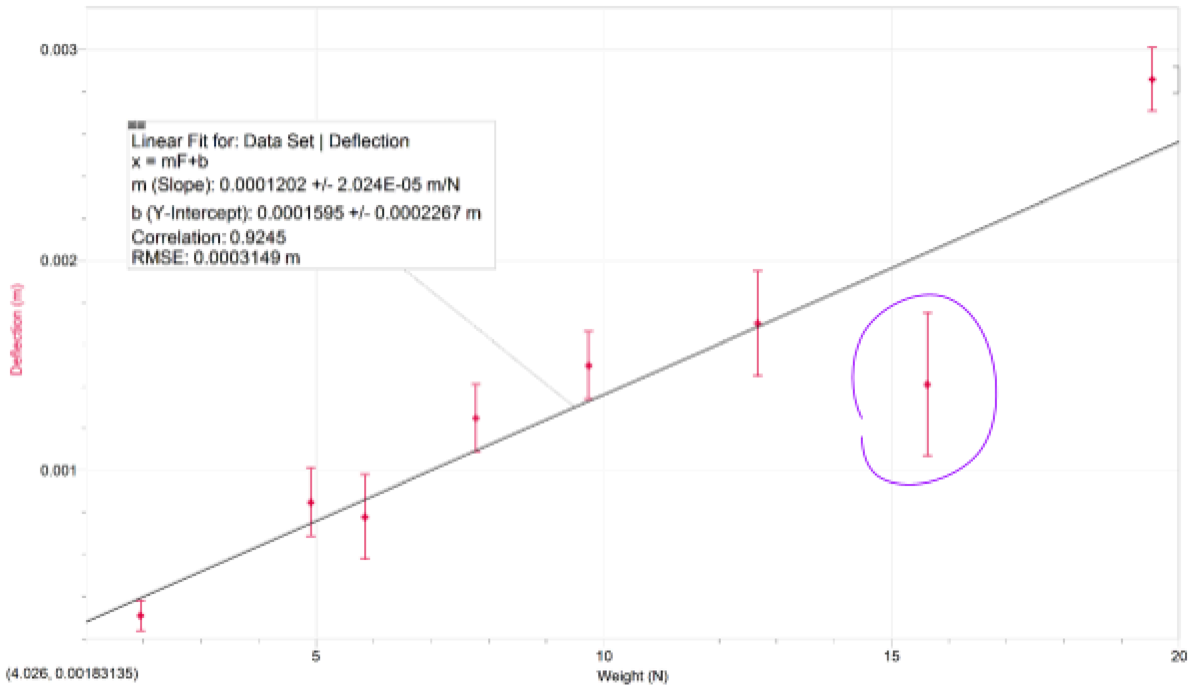



**Figure 4, Logger Pro graph showing all data points. Error bars on the x-axis (Weight) are too small to be visible. See Appendix 1 for a larger version.**

The value at 15.6 N, as highlighted on Figure 4, seems to be furthest from the line of best fit, and also has an extremely large uncertainty, meaning there was probably a human error whilst measuring.

After treating the point at 15.6 N as an outlier and removing it from the dataset we get a much better fitting graph, where a determined line of best fit passes through all the other error bars.

We were then able to establish the boundary cases for the linear fit within the range defined by our error bars, resulting in the maximum and minimum potential values for our gradient. All of these fits, as well as the data, are shown in Figure 5, which allowed us to determine the most probable value for the gradient and y-intercept along with their uncertainty.

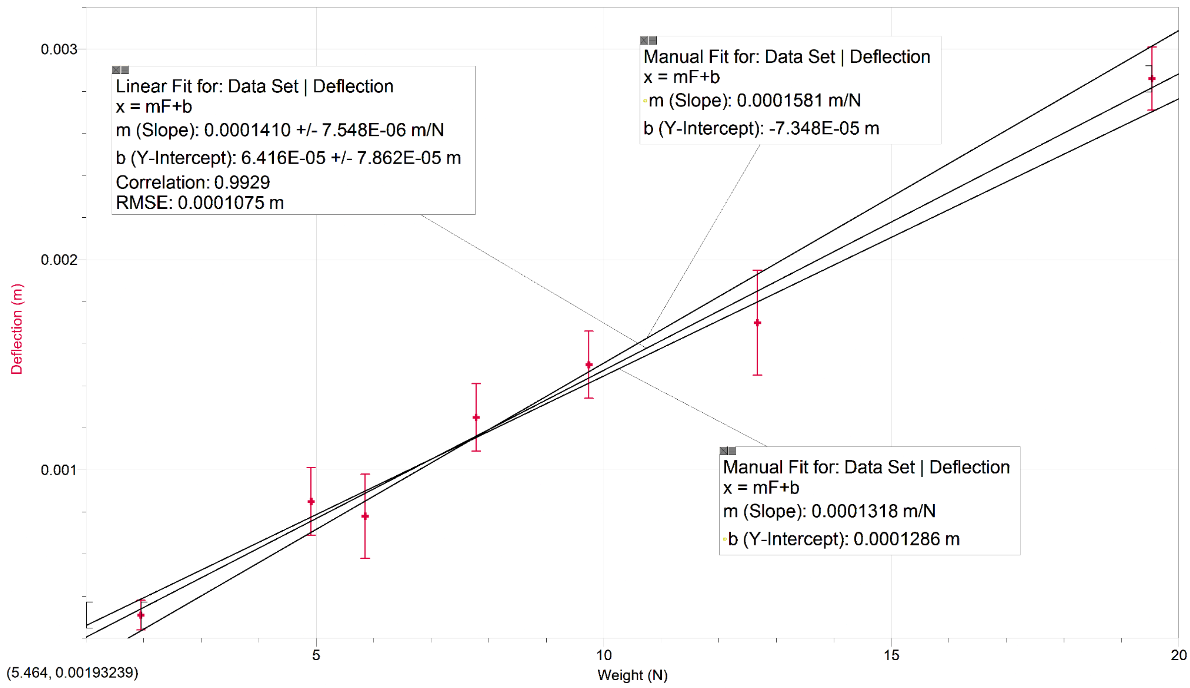



**Figure 5, Logger Pro graph showing data points when the outlier is removed. Error bars on the x-axis (Weight) are too small to be visible. See Appendix 2 for a larger version.**

The values of $m$, the gradient, can be seen in Table 5.

| m | m max | m min | Δ m |
|---|---|---|---|
| **0.0001410** | **0.0001581** | **0.0001318** | **0.00001315** |

**Table 5, Obtained, Maximum, Minimum, and error in gradient**

The values of $c$, the Y-intercept, can be seen in Table 6.

| c | c max | c min | Δ c |
|---|---|---|---|
| **0.0000642** | **0.0001286** | **-0.0000725** | **0.0001005** |

**Table 6, Obtained, Maximum, Minimum, and error in gradient**

The uncertainties in our results for both $m$ and $c$, were determined by doing

$$\Delta x = \frac{(x_{max} - x_{min})}{2}$$

Where $x$ represents $m$ or $c$.

Therefore, rounded to 3 significant figures,

$$m = (1.41 \pm 0.13) \times 10^{-4}\, mN^{-1}$$

and

$$c = (0.64 \pm 1.29) \times 10^{-4}\, m$$

We can thus see the range of uncertainty for $c$, the y-intercept, passes through 0, the origin. According to the linear fit passing through the origin, we can determine the correlation to be directly proportional, as hypothesised in Section 1.5.

We can thus calculate the Young's Modulus of the beam to evaluate the accuracy and precision of our experiment.

The following results were rounded to 4 significant figures as per the highest uncertainty. However, the full obtained values were used for calculations.

According to the formula defined in the Background Physics Section 1.3

$$E = \frac{L^3}{m \times 48 \times I}$$

Which substituting in the known values for Length, Gradient, and moment of Inertia gives

$$E = \frac{(0.600)^3}{0.000141 \times 48 \times 0.000612} = 52150\ MPa$$

We can then obtain the uncertainty in $E$ as follows

$$\Delta E = E \times \left(\frac{\Delta m}{m} + \frac{\Delta I}{I} + \frac{6 \times \Delta L}{L}\right)$$

Which when substituting the values gives

$$\Delta E = 52150 \times \left(\frac{0.00001315}{0.000141} + \frac{0.0000211}{0.000612} + \frac{6 \times (0.001)}{0.600}\right) = 7180\ MPa$$

Therefore we can write the Young's Modulus of the beam's material as follows

$$E = (52.15 \pm 7.180) \times 10^3\ MPa$$

Which can be simplified to $GPa$ and 2 significant figures for readability, to give

$$(52 \pm 7.2)\ GPa$$

Considering the beam's material to be Aluminium, we can compare our result to the broader scientifically accepted value for the Young's Modulus of Aluminium[3] as seen in Table 7.

| **Young's Modulus (GPa)** | | | |
|---|---|---|---|
| **Determined** | **Literature Value** | **Accuracy Error (%)** | **Precision Error (%)** |
| **52150** | **69000** | **24** | **14** |

**Table 7, Our result compared to broader scientifically accepted value**

We can obtain the error in accuracy as a percentage using the following formula

$$Accuracy\ Error = \frac{(E_{literature} - E_{determined})}{E_{literature}} \times 100$$

And the precision as follows

$$Precision\ Error = \frac{\Delta E_{determined}}{E_{determined}} \times 100$$

# 4. Conclusion

## 4.1 Conclusion

The statistical analysis of the data indicates there is a direct proportionality between the applied force, $F\ (N)$, and deflection, $x\ (m)$. A best-fit linear regression in the form $y = mx\ (+\ c)$ fits through all our data points, as well as passing through the origin when accounting for uncertainties represented in the error bars and removing outliers. This supports our initial hypothesis, as well as the broader Physics

[3] "Young's Modulus, Tensile Strength and Yield Strength Values for some Materials." *The Engineering ToolBox*, 2003, https://www.engineeringtoolbox.com/young-modulus-d_417.html. Accessed 6 October 2024.

Background and theoretical equations stating that deflection should directly correlate with the applied load by a factor of $m$, the gradient, which is dependent on the specific beam's dimensions and materials.

We then got a value for the Young's Modulus of the material to be $52.1 \pm 7.18\ GPa$, which gives an error in accuracy of 24% compared to the scientifically accepted value of $69\ GPa$ for Aluminium. A percentage accuracy error of over 24% suggests inaccuracies in our experiment. Our obtained value being smaller than the expected Young's Modulus indicates our beam to be softer than standard aluminium. The precision for our determined Young's Modulus is 14%, which is too high to be considered highly reliable considering levels below 10% can be regarded this way. However my obtained precision is reliable enough at under 20% to reflect moderate confidence, making me think our results are affected by a systematic rather than random error.

Overall, this investigation successfully answered the research question: "How does the Force applied, in N, at the centre of a cantilever beam fixed at both ends affect the displacement produced at the centre, in m?" The results confirmed that as the applied force increases, the deflection at the centre of the beam increases proportionally.

## 4.2 Evaluation

### 4.2.1 Strengths

A notable strength of this experiment was the precision of the Travelling Microscope used to measure the deflection. The use of the Travelling Microscope leads to significantly reduced reading uncertainties in measurements of small deflections which is vital for an experiment such as this one.

### 4.2.2 Limitations and Improvements in order of importance

| **Random Errors** | | |
|---|---|---|
| **Source of Error** | **Impact on the Results** | **Improvement** |
| Beam Irregularities | Could cause uneven deflection, impacting measurements, as parts might bend more or less depending on these defects. | Use a new beam with a highly uniform machined surface such as those from Bunnings[4] |
| Microscope Slightly Displaced | Changes the previously recorded zero reading and would lead to inaccurate measurements | Use Clamps to secure the Travelling Microscope and the platform it's on. |
| Vibrations in the room | Could cause slight movements in the apparatus, affecting measurements | Ensure you are conducting the experiment in a closed room alone with limited disturbances |
| Thermal Expansion of the beam | Changes in beam temperature could change its geometry and thus affect its moment of inertia | Use a surface temperature measuring instrument such as those from Testo[5] to ensure no changes occur |

**Table 8, Random errors.**

[4] "Metal Mate 0.5 x 300 x 900mm Plain Aluminium Sheet - 300mm." *Bunnings*, https://www.bunnings.com.au/metal-mate-0-5-x-300-x-900mm-plain-aluminium-sheet-300mm_p1112749. Accessed 6 October 2024.
[5] "testo 925 – Temperature measuring instrument for TC Type K with App connection." *Testo*, https://www.testo.com/en-UK/testo-925/p/0563-0925. Accessed 6 October 2024.

| Systematic Errors | | | |
|---|---|---|---|
| **Source of Error** | **Impact on the Results** | **Direction on conclusion** | **Improvement** |
| The beam is not made of pure regular aluminium | The Young's Modulus will not correspond with the scientific background | The gradient was greater than expected and thus the beam's material was found to be softer than regular aluminium. | Use a new beam of a pure known material such as those from Bunnings[6] |
| Beam slipping from clamps | The weight will no longer be centred on the beam, potentially inducing torque and twisting forces in the beam, affecting measured deflection. | Torque or Twisting motions in the beam could result in a greater measured gradient than expected.This would result in a smaller Young's Modulus. | Re-measure the zero deflection point and ensure the blocks are still correctly aligned each trial |
| The beam not being level | The weight will affect the beam diagonally, instead of centred, potentially inducing torque and twisting forces in the beam, affecting measured deflection. | Torque or Twisting motions in the beam could result in a greater measured gradient than expected.This would result in a smaller Young's Modulus. | Use a spirit level to ensure the beam is flat, such as the one sold by Acme tools[7] |
| Weight is applied over a small area rather than a point load | The weight will be distributed over an area rather than directly centred, reducing its impact on deflection | Lower Y-Intercept as well as a smaller gradient than expected. This means our obtained Young's Modulus will be greater than the expected value. | Hang the masses from a string attached at the centre, to ensure the weight acts as a point load. |

**Table 9, Systematic errors.**

[6] "Metal Mate 0.5 x 300 x 900mm Plain Aluminium Sheet - 300mm." *Bunnings*, https://www.bunnings.com.au/metal-mate-0-5-x-300-x-900mm-plain-aluminium-sheet-300mm_p1112749. Accessed 6 October 2024.
[7] "Empire Level 48 In. True Blue Box Level." *ACME TOOLS*, https://www.acmetools.com/empire-level-48-in-true-blue-box-level-e7548/045242366798.html. Accessed 6 October 2024.

# 5. References

## 5.1 Works Cited


“Area Moment of Inertia - Typical Cross Sections I.” *The Engineering ToolBox*, 2008, https://www.engineeringtoolbox.com/area-moment-inertia-d_1328.html. Accessed 6 October 2024.

“Beams - Fixed at Both Ends - Continuous and Point Loads.” *The Engineering ToolBox*, 2004, https://www.engineeringtoolbox.com/beams-fixed-both-ends-support-loads-deflection-d_809.html. Accessed 6 October 2024.

“Beams - Fixed at Both Ends - Continuous and Point Loads.” *The Engineering ToolBox*, 2004, https://www.engineeringtoolbox.com/beams-fixed-both-ends-support-loads-deflection-d_809.html. Accessed 6 October 2024.

“Beams - Supported at Both Ends - Continuous and Point Loads.” *The Engineering ToolBox*, 2009, https://www.engineeringtoolbox.com/beam-stress-deflection-d_1312.html. Accessed 6 October 2024.

“Empire Level 48 In. True Blue Box Level.” *ACME TOOLS*, https://www.acmetools.com/empire-level-48-in-true-blue-box-level-e7548/045242366798.html. Accessed 6 October 2024.

Ernst, Laurin. “Moment Of Inertia Formulas For Different Shapes {2024}.” *Structural Basics*, 2024, https://www.structuralbasics.com/moment-of-inertia-formulas/. Accessed 6 October 2024.

“Metal Mate 0.5 x 300 x 900mm Plain Aluminium Sheet - 300mm.” *Bunnings*, https://www.bunnings.com.au/metal-mate-0-5-x-300-x-900mm-plain-aluminium-sheet-300mm_p1112749. Accessed 6 October 2024.

“testo 925 – Temperature measuring instrument for TC Type K with App connection.” *Testo*, https://www.testo.com/en-UK/testo-925/p/0563-0925. Accessed 6 October 2024.

“Young's Modulus, Tensile Strength and Yield Strength Values for some Materials.” *The Engineering ToolBox*, 2003, https://www.engineeringtoolbox.com/young-modulus-d_417.html. Accessed 6 October 2024.

## 5.2 Bibliography

“Area Moment of Inertia - Typical Cross Sections I.” *The Engineering ToolBox*, 2008, https://www.engineeringtoolbox.com/area-moment-inertia-d_1328.html. Accessed 6 October 2024.

“Beams - Fixed at Both Ends - Continuous and Point Loads.” *The Engineering ToolBox*, 2004, https://www.engineeringtoolbox.com/beams-fixed-both-ends-support-loads-deflection-d_809.html. Accessed 6 October 2024.

“Beams - Fixed at Both Ends - Continuous and Point Loads.” *The Engineering ToolBox*, 2004, https://www.engineeringtoolbox.com/beams-fixed-both-ends-support-loads-deflection-d_809.html. Accessed 6 October 2024.

"Beams - Supported at Both Ends - Continuous and Point Loads." *The Engineering ToolBox*, 2009, https://www.engineeringtoolbox.com/beam-stress-deflection-d_1312.html. Accessed 6 October 2024.

"Empire Level 48 In. True Blue Box Level." *ACME TOOLS*, https://www.acmetools.com/empire-level-48-in-true-blue-box-level-e7548/045242366798.html. Accessed 6 October 2024.

Ernst, Laurin. "Moment Of Inertia Formulas For Different Shapes {2024}." *Structural Basics*, 2024, https://www.structuralbasics.com/moment-of-inertia-formulas/. Accessed 6 October 2024.

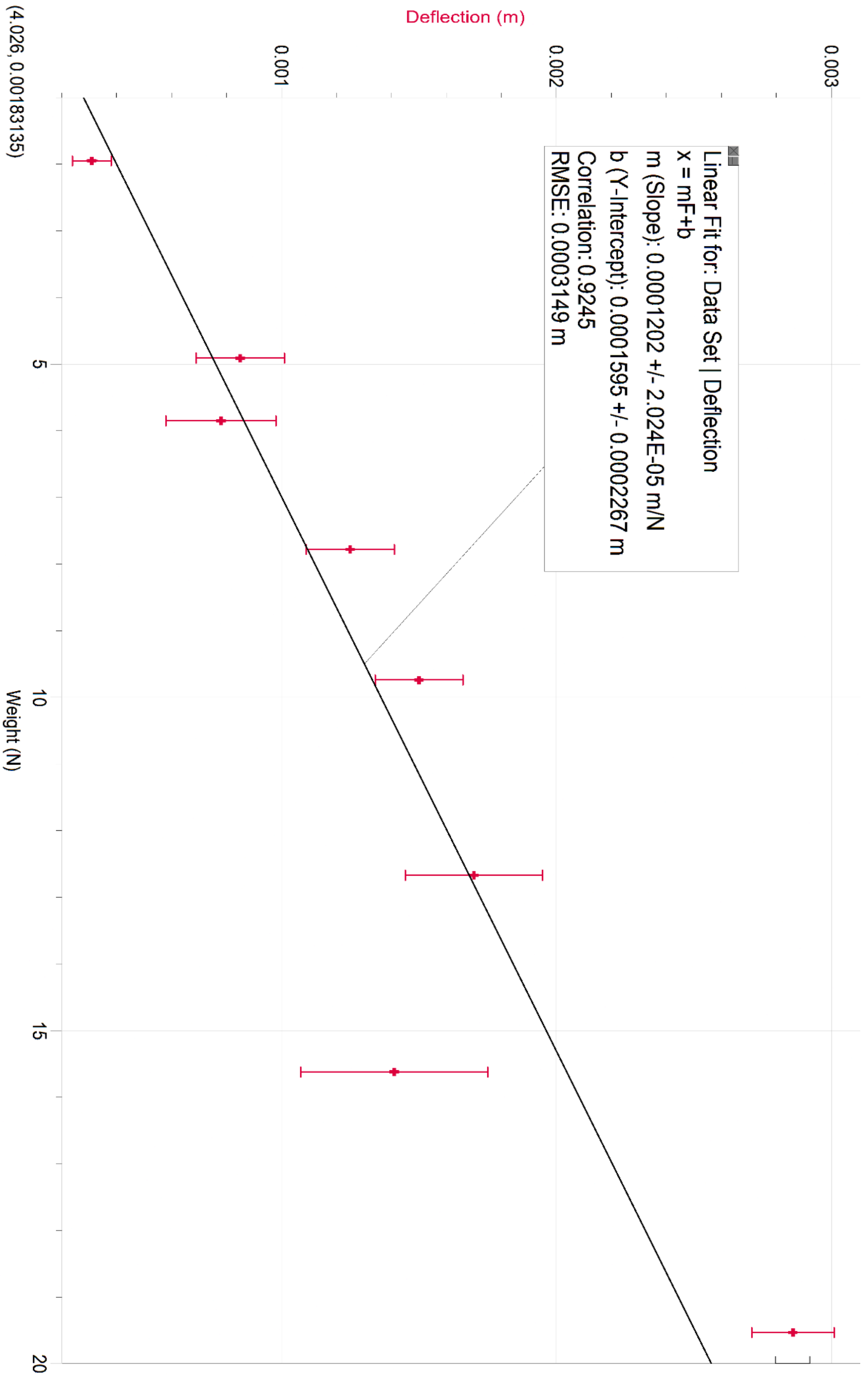
"Metal Mate 0.5 x 300 x 900mm Plain Aluminium Sheet - 300mm." *Bunnings*, https://www.bunnings.com.au/metal-mate-0-5-x-300-x-900mm-plain-aluminium-sheet-300mm_p1112749. Accessed 6 October 2024.

"testo 925 – Temperature measuring instrument for TC Type K with App connection." *Testo*, https://www.testo.com/en-UK/testo-925/p/0563-0925. Accessed 6 October 2024.

"Young's Modulus, Tensile Strength and Yield Strength Values for some Materials." *The Engineering ToolBox*, 2003, https://www.engineeringtoolbox.com/young-modulus-d_417.html. Accessed 6 October 2024.

## 5.3 Appendix

## Appendix 1, Logger Pro graph showing all data



## Appendix 2, Logger Pro graph showing data with the outlier removed