CI HW 1

Monday, 6 April 2020 19:04

A1) • $d(a_i \mid p) = \sqrt{(x_i - x)^2 + (y_i - y)^2}$

• Case 1 : $p(\bar{d}_n(a_{i,p}) \mid p) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e$

– Likelihood fct.

• $\prod_{n=1}^{N} P(\bar{d}_n(a_{i,p}) \mid p)$

– Log-likelihood

• $\ln P(\bar{d}_n(a_{i,p}) \mid p) = \sum_{n=1}^{N} \ln P(\bar{d}_n(a_{i,p}) \mid$

$\Rightarrow \ln\left(\frac{1}{\sqrt{2\pi\sigma_i^2}}\right) + \sum_{n=1}^{N} \ln\left(e^{-\frac{(\bar{d}_n(a_{i,p}) - d(a_i}{2\sigma_i^2}}\right.$

$$\frac{\left(\bar{d}_n(a_i, p) - d(a_i, p)\right)^2}{2\sigma_i^2}$$

$$p) = L\left(\bar{d}_n(a_i, p) \mid p\right)$$

$$, p))^2$$

$$)$$

$$\Leftrightarrow \ln\left(\frac{\cdots}{\sqrt{2\pi\sigma_i^2}}\right) + \sum_{n=1} -\frac{(d_n(a_i,p) - d(a_i,p))}{2\sigma_i^2}$$

$$\Leftrightarrow \ln(1) - \frac{1}{2}\cdot\ln(2\pi\sigma_i^2) + \sum_{n=1}^{N} -\frac{[\bar{d}_n(a_i,p}{}$$

$$= -\frac{1}{\cancel{2}}\cdot\left[\frac{1}{\cancel{2}\pi\sigma_i^2}\cdot\overset{2}{\cancel{4}}\pi + \sum_{n=1}^{N} -\frac{\cancel{2}[\bar{d}_n(a_i,p) -}{\sigma_i^4}\right.$$

$$= -\frac{\cancel{\pi}}{\cancel{\pi}\sigma_i^2} + \sum_{n=1}^{N}\frac{[\bar{d}_n(a_i,p) - d(a_i,p)]^2}{\sigma_i^4} \stackrel{!}{=}$$

$$\Leftrightarrow \frac{1}{\sigma_i^2} = \sum_{n=1}^{N}\frac{[\bar{d}_n(a_i,p) - d(a_i,p)]^2}{\sigma_i^4}$$

$$\Leftrightarrow \sigma_i^2 = \sum_{n=1}^{N}[\bar{d}_n(a_i,p) - d(a_i,p)]^2$$

$$- \lambda_i(\bar{d}_n(a_i,n) -$$

$$) - d(a_i, p)\Big]^2$$
$$\overline{\quad 2\sigma_i^2}$$

$$\Big| \quad \frac{d}{d\sigma_i^2}$$

$$d(a_i, p)\Big]^2$$

$$\Big]$$

$$0$$

$$d_p(a_i, p))$$

Case 2 : $p(d_n(a_{i,p}) | p) = \begin{cases} \lambda_i e \\ 0 \end{cases}$

- log - likelihood

$$\mathcal{L}(\bar{d_n}(a_{i,p}) | p) = \sum_{n=1}^{N} \ln P(\bar{d_n}(a_{i,p} | p))$$

$$= \ln(\lambda_i) + \sum_{n=1}^{N} [-\lambda_i (\bar{d_n}(a_{i,}$$

$$= \frac{1}{\lambda_i} - \sum_{n=1}^{N} \bar{d_n}(a_{i,p}) - d_n($$

$$\lambda_i = \begin{cases} \dfrac{1}{\sum_{n=1}^{N} \bar{d_n}(a_{i,p}) - d_n(a_{i,p})} \\ 0 \end{cases}$$

$, \quad d_n(a_i, p) \geq d_n(a_i, p)$

$, \quad else$

$p) - d_n(a_i, p))] \quad \Big| \frac{\partial}{\partial \lambda_i}$

$a_i, p)$

$\overline{d_n}(a_i, p) \geq d_n(a_i, p)$

$, \quad else$