CI HW 1

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Analytical derivation for Case I (Gaussian)

- Likelihood Function:

$$P(\tilde{d}_n(a_i,p) \mid p) = p\left[\prod_{n=0}^{N-1} \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{[\tilde{d}_n(a_i,p) - d(a_i,p)]^2}{2\sigma_i^2}}\right]$$

To convert the product to a sum, we use the logarithm

- Log-likelihood Function:

$$L(\tilde{d}_n(a_i,p) \mid p) = \ln P(\tilde{d}_n(a_i,p) \mid p)$$

$$= \sum_{n=0}^{N-1} \left[\ln\left(\frac{1}{\sqrt{2\pi\sigma_i^2}}\right) + \left[-\frac{[\tilde{d}_n(a_i,p) - d(a_i,p)]^2}{2\sigma_i^2}\right]\right]$$

$$= \sum_{n=0}^{N-1} \left[\ln(1) - \ln\left(\sqrt{2\pi\sigma_i^2}\right) - \frac{[\tilde{d}_n(a_i,p) - d(a_i,p)]^2}{2\sigma_i^2}\right] \quad \Big| \frac{d}{d\sigma_i^2}$$

$$= \sum_{n=0}^{N-1} \left[-\frac{\cancel{4\pi}}{\cancel{4\pi}\sigma_i^2} + \frac{[\tilde{d}_n(a_i,p) - d(a_i,p)]^2}{\sigma_i^4}\right] \overset{!}{=} 0$$

$$\Leftrightarrow \quad \frac{N}{\sigma_i^2} = \sum_{n=0}^{N-1} \frac{[\tilde{d}_n(a_i,p) - d(a_i,p)]^2}{\sigma_i^4}$$

$$\sigma_i^2 = \sum_{n=0}^{N-1} [\tilde{d}_n(a_i,p) - d(a_i,p)]^2 \cdot \frac{1}{N}$$

Analytical derivation for case II:

- $$\ell(\tilde{d}_n(a_i,p) \mid p) = \ln P(\tilde{d}_n(a_i \mid p) \mid p)$$

$$= \sum_{n=0}^{N-1} \ln(\lambda_i) - \lambda_i[\tilde{d}_n(a_i,p) - d(a_i,p)] \quad \Big| \frac{\partial}{\partial\lambda_i}$$

$$= \frac{N}{\lambda_i} - \sum_{n=0}^{N-1}[\tilde{d}_n(a_i,p) - d(a_i,p)] \overset{!}{=} 0$$

$$\Leftrightarrow \quad \lambda_i = \sum_{n=0}^{N-1} \frac{N}{\tilde{d}_n(a_i,p) - d(a_i,p)} \quad \text{if } \hat{d}_n(a_i,p) \geq d_n(a_i,p)$$

Is LSE equal to MLE?

- $$\hat{p}_{ML}(p) = \underset{p}{\operatorname{argmax}} \prod^{N_a-1} p(\tilde{d}(a_i,p) \mid p)$$

- $$\hat{p}_{ML}(n) = \underset{p}{\arg\max} \prod_{i=0}^{N_A-1} p\left(\tilde{d}(a_i,p) \mid p\right)$$

$$= \underset{p}{\arg\max} \prod_{i=0}^{N_A-1} \frac{1}{\sqrt{2\pi\sigma_i^2}} \cdot e^{-\frac{[d_n(a_i,p) - d(a_i,p)]^2}{2\sigma_i^2}} \qquad | \ln$$

$$= \underset{p}{\arg\max} \sum_{i=0}^{N_A-1} \cancel{\ln\left(\frac{1}{\sqrt{2\pi\sigma_i^2}}\right)} \cdot (-) \frac{[\tilde{d}_n(a_i,p) - d_n(a_i,p)]^2}{2\sigma_i^2}$$

→ Die Standardabweichung ist gleich für alle Anker. Da der Faktor nur skalierend ist, fällt dieser weg.

$$= \underset{p}{\arg\min} \sum_{i=0}^{N_A-1} \frac{[\tilde{d}_n(a_i,p) - d_n(a_i,p)]^2}{2\sigma_i^2} = \hat{p}_{LS}(n)$$

Analytical solution for the Jacobian Matrix

- $$[J(p)]_{i,1} = \frac{d}{dx}\left[\tilde{d}_n(a_i,p) - \sqrt{(x_i - x)^2 + (y_i - y)^2}\right]$$

$$= -\frac{1}{2} \frac{[-2(x_i - x)]}{\sqrt{(x_i - x)^2 + (y_i - y)^2}}$$

$$= \frac{x_i - x}{\sqrt{(x_i - x)^2 + (y_i - y)^2}}$$

- $$[J(p)]_{i,2} = \frac{y_i - y}{\sqrt{(x_i - x)^2 + (y_i - y)^2}}$$