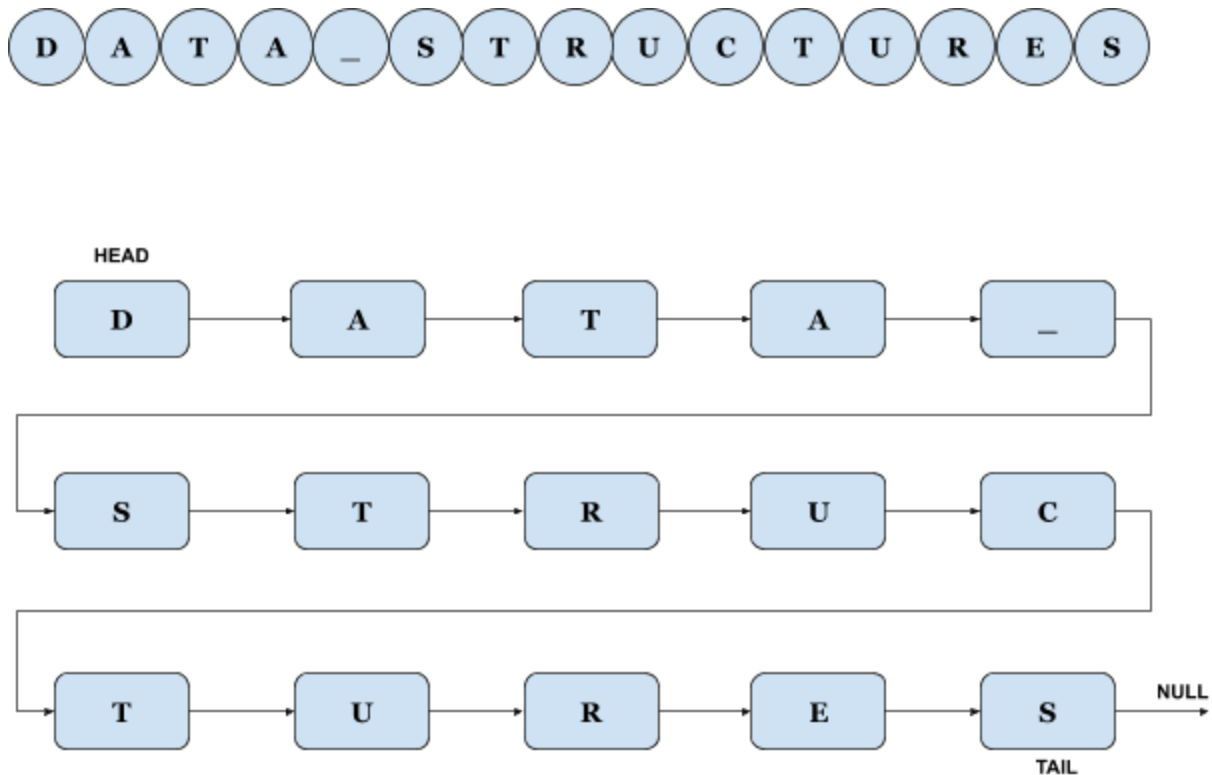

CS261 Data Structures

Assignment 3

v 1.06 (revised 1/22/2021)

Your Very Own Linked List



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General Instructions

1. Programs in this assignment must be written in Python v3 and submitted to Gradescope before the due date specified in the syllabus. You may resubmit your code as many times as necessary. Gradescope allows you to choose which submission will be graded.
2. In Gradescope, your code will run through several tests. Any failed tests will provide a brief explanation of testing conditions to help you with troubleshooting. Your goal is to pass all tests.
3. We encourage you to create your own test programs and cases even though this work won't have to be submitted and won't be graded. Gradescope tests are limited in scope and may not cover all edge cases. Your submission must work on all valid inputs. We reserve the right to test your submission with more tests than Gradescope.
4. Your code must have an appropriate level of comments. At a minimum, each method should have a descriptive docstring. Additionally, put comments throughout the code to make it easy to follow and understand.
5. You will be provided with a starter "skeleton" code on which you will build your implementation. Methods defined in the skeleton code must retain their names and input / output parameters. Variables defined in the skeleton code must also retain their names. We will only test your solution by making calls to methods defined in the skeleton code and by checking values of variables defined in the skeleton code. You can add more helper methods and variables as needed.

However, certain classes and methods cannot be changed in any way. Please see the comments in the skeleton code for guidance. In particular, the content of any methods pre-written for you as part of the skeleton code must not be changed.

6. Both the skeleton code and code examples provided in this document are part of assignment requirements. They have been carefully selected to demonstrate requirements for each method. Refer to them for a detailed description of expected method behavior, input / output parameters, and handling of edge cases. Code examples may include assignment requirements not explicitly stated elsewhere.
7. For each method, you can choose to implement a recursive or iterative solution **(except in Part I of the assignment, where a recursive solution is required)**. When using a recursive solution, be aware of maximum recursion depths on large inputs. We will specify the maximum input size that your solution must handle.

Part 1 - Summary and Specific Instructions

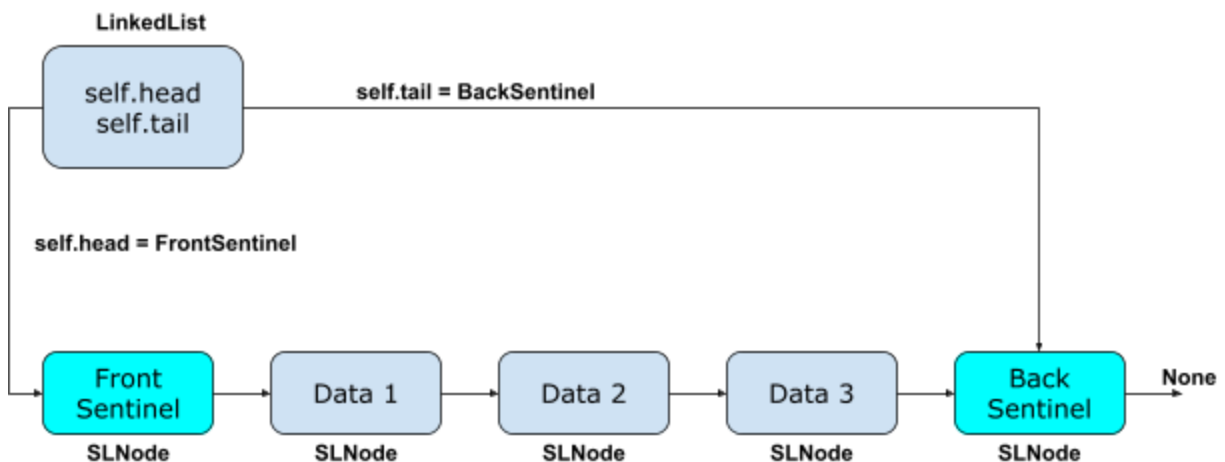
1. Implement Deque and Bag ADT interfaces with a Singly Linked List data structure by completing the skeleton code provided in the file `sll.py`. Once completed, your implementation will include the following methods:

```

add_front(), add_back()
insert_at_index()
remove_front(), remove_back()
remove_at_index()
get_front(), get_back()
remove()
count()
slice()

```

2. We will test your implementation with different types of objects, not just integers. We guarantee that all such objects will have correct implementations of methods `__eq__`, `__lt__`, `__gt__`, `__ge__`, `__le__`, and `__str__`.
3. The number of objects stored in the list at any given time will be between 0 and 900 inclusive.
4. Variables in the `SLNode` and `LinkedList` classes are not marked as private. For this portion of the assignment, you are allowed to access and change their values directly. You are not required to write getter or setter methods for them.
5. **RESTRICTIONS:** You are not allowed to use ANY built-in Python data structures or their methods. **All implementations in this part must be done recursively. You are not allowed to use while- or for- loops in this part of the assignment.**
6. This implementation must be done with the use of front and back sentinel nodes.



add_front(self, value: object) -> None:

This method adds a new node at the beginning of the list (right after the front sentinel).

Example #1:

```
lst = LinkedList()
print(lst)
lst.add_front('A')
lst.add_front('B')
lst.add_front('C')
print(lst)
```

Output:

```
SLL[]
SLL[C -> B -> A]
```

add_back(self, value: object) -> None:

This method adds a new node at the end of the list (right before the back sentinel).

Example #1:

```
lst = LinkedList()
print(lst)
lst.add_back('C')
lst.add_back('B')
lst.add_back('A')
print(lst)
```

Output:

```
SLL[]
SLL[C -> B -> A]
```

insert_at_index(self, index: int, value: object) -> None:

This method adds a new value at the specified index position in the linked list. Index 0 refers to the beginning of the list (right after the front sentinel).

If the provided index is invalid, the method raises a custom "SLLException". Code for the exception is provided in the skeleton file. If the linked list contains N nodes (not including sentinel nodes in this count), valid indices for this method are [0, N] inclusive.

Example #1:

```
lst = LinkedList()
test_cases = [(0, 'A'), (0, 'B'), (1, 'C'), (3, 'D'), (-1, 'E'), (5, 'F')]
for index, value in test_cases:
    print('Insert of', value, 'at', index, ': ', end='')
    try:
        lst.insert_at_index(index, value)
        print(lst)
    except Exception as e:
        print(type(e))
```

Output:

```
Insert of A at 0 : SLL [A]
Insert of B at 0 : SLL [B -> A]
Insert of C at 1 : SLL [B -> C -> A]
Insert of D at 3 : SLL [B -> C -> A -> D]
Insert of E at -1 : <class '__main__.SLLException'>
Insert of F at 5 : <class '__main__.SLLException'>
```

remove_front(self) -> None:

This method removes the first node from the list. If the list is empty, the method raises a custom "SLLException". Code for the exception is provided in the skeleton file.

Example #1:

```
lst = LinkedList([1, 2])
print(lst)
for i in range(3):
    try:
        lst.remove_front()
        print('Successful removal', lst)
    except Exception as e:
        print(type(e))
```

Output:

```
SLL[1 -> 2]
Successful removal SLL[2]
Successful removal SLL[]
<class '__main__.SLLException'>
```

remove_back(self) -> None:

This method removes the last node from the list. If the list is empty, the method raises a custom "SLLException". Code for the exception is provided in the skeleton file.

Example #1:

```
lst = LinkedList()
try:
    lst.remove_back()
except Exception as e:
    print(type(e))
lst.add_front('Z')
lst.remove_back()
print(lst)
lst.add_front('Y')
lst.add_back('Z')
lst.add_front('X')
print(lst)
lst.remove_back()
print(lst)
```

Output:

```
<class '__main__.SLLException'>
SLL []
SLL [X -> Y -> Z]
SLL [X -> Y]
```


remove_at_index(self, index: int) -> None:

This method removes a node from the list given its index. Index 0 refers to the beginning of the list.

If the provided index is invalid, the method raises a custom "SLLException". Code for the exception is provided in the skeleton file. If the list contains N elements (not including sentinel nodes in this count), valid indices for this method are [0, N - 1] inclusive.

Example #1:

```
lst = LinkedList([1, 2, 3, 4, 5, 6])
print(lst)
for index in [0, 0, 0, 2, 2, -2]:
    print('Removed at index:', index, ': ', end='')
    try:
        lst.remove_at_index(index)
        print(lst)
    except Exception as e:
        print(type(e))
print(lst)
```

Output:

```
SLL [1 -> 2 -> 3 -> 4 -> 5 -> 6]
Removed at index: 0 : SLL [2 -> 3 -> 4 -> 5 -> 6]
Removed at index: 0 : SLL [3 -> 4 -> 5 -> 6]
Removed at index: 0 : SLL [4 -> 5 -> 6]
Removed at index: 2 : SLL [4 -> 5]
Removed at index: 2 : <class '__main__.SLLException'>
Removed at index: -2 : <class '__main__.SLLException'>
SLL [4 -> 5]
```

get_front(self) -> object:

This method returns the value from the first node in the list without removing it. If the list is empty, the method raises a custom "SLLError". Code for the exception is provided in the skeleton file.

Example #1:

```
lst = LinkedList(['A', 'B'])
print(lst.get_front())
print(lst.get_front())
lst.remove_front()
print(lst.get_front())
lst.remove_back()
try:
    print(lst.get_front())
except Exception as e:
    print(type(e))
```

Output:

```
A
A
B
<class '__main__.SLLError'>
```

get_back(self) -> object:

This method returns the value from the last node in the list without removing it. If the list is empty, the method raises a custom "SLLError". Code for the exception is provided in the skeleton file.

Example #1:

```
lst = LinkedList([1, 2, 3])
lst.add_back(4)
print(lst.get_back())
lst.remove_back()
print(lst)
print(lst.get_back())
```

Output:

```
4
SLL [1 -> 2 -> 3]
3
```

remove(self, value: object) -> bool:

This method traverses the list from the beginning to the end and removes the first node in the list that matches the provided "value" object. The method returns True if some node was actually removed from the list. Otherwise, it returns False.

Example #1:

```
lst = LinkedList([1, 2, 3, 1, 2, 3, 1, 2, 3])
print(lst)
for value in [7, 3, 3, 3, 3]:
    print(lst.remove(value), lst.length(), lst)
```

Output:

```
SLL [1 -> 2 -> 3 -> 1 -> 2 -> 3 -> 1 -> 2 -> 3]
False 9 SLL [1 -> 2 -> 3 -> 1 -> 2 -> 3 -> 1 -> 2 -> 3]
True 8 SLL [1 -> 2 -> 1 -> 2 -> 3 -> 1 -> 2 -> 3]
True 7 SLL [1 -> 2 -> 1 -> 2 -> 1 -> 2 -> 3]
True 6 SLL [1 -> 2 -> 1 -> 2 -> 1 -> 2]
False 6 SLL [1 -> 2 -> 1 -> 2 -> 1 -> 2]
```

count(self, value: object) -> int:

This method counts the number of elements in the list that match the provided "value" object.

Example #1:

```
lst = LinkedList([1, 2, 3, 1, 2, 2])
print(lst, lst.count(1), lst.count(2), lst.count(3), lst.count(4))
```

Output:

```
SLL [1 -> 2 -> 3 -> 1 -> 2 -> 2] 2 3 1 0
```

slice(self, start_index: int, size: int) -> object:

This method returns a new LinkedList object that contains the requested number of nodes from the original list starting with the node located at the requested start index. If the original list contains N nodes, a valid `start_index` is in range `[0, N - 1]` inclusive. Runtime complexity of your implementation must be $O(N)$.

If the provided start index is invalid, or if there are not enough nodes between the start index and the end of the list to make a slice of the requested size, this method raises a custom "SLLException". Code for the exception is provided in the skeleton file.

Example #1:

```
lst = LinkedList([1, 2, 3, 4, 5, 6, 7, 8, 9])
ll_slice = lst.slice(1, 3)
print(lst, ll_slice, sep="\n")
ll_slice.remove_at_index(0)
print(lst, ll_slice, sep="\n")
```

Output:

```
SLL [1 -> 2 -> 3 -> 4 -> 5 -> 6 -> 7 -> 8 -> 9]
SLL [2 -> 3 -> 4]
SLL [1 -> 2 -> 3 -> 4 -> 5 -> 6 -> 7 -> 8 -> 9]
SLL [3 -> 4]
```

Example #2:

```
lst = LinkedList([10, 11, 12, 13, 14, 15, 16])
print("SOURCE:", lst)
slices = [(0, 7), (-1, 7), (0, 8), (2, 3), (5, 0), (5, 3), (6, 1)]
for index, size in slices:
    print("Slice", index, "/", size, end="")
    try:
        print(" --- OK: ", lst.slice(index, size))
    except:
        print(" --- exception occurred.")
```

Output:

```
SOURCE: SLL [10 -> 11 -> 12 -> 13 -> 14 -> 15 -> 16]
Slice 0 / 7 --- OK: SLL [10 -> 11 -> 12 -> 13 -> 14 -> 15 -> 16]
Slice -1 / 7 --- exception occurred.
Slice 0 / 8 --- exception occurred.
Slice 2 / 3 --- OK: SLL [12 -> 13 -> 14]
Slice 5 / 0 --- OK: SLL []
Slice 5 / 3 --- exception occurred.
Slice 6 / 1 --- OK: SLL [16]
```

Part 2 - Summary and Specific Instructions

1. Implement a Stack ADT class by completing the skeleton code provided in the file `max_stack_sll.py`. You will use the `LinkedList` data structure that you implemented in part 1 of this assignment as the underlying data storage for your Stack ADT.
2. Your Stack ADT implementation will include standard Stack methods and also one new method - `get_max()`:

```
push()  
pop()  
top()  
get_max()
```

3. We will test your implementation with different types of objects, not just integers. We guarantee that all such objects will have correct implementations of methods `__eq__`, `__lt__`, `__gt__`, `__ge__`, `__le__`, and `__str__`.
4. The number of objects stored in the Stack at any given time will be between 0 and 900 inclusive. The Stack must allow for storage of duplicate objects.
5. RESTRICTIONS: You are not allowed to use ANY built-in Python data structures or their methods. You must solve this portion of the assignment by importing the `LinkedList` class that you wrote in part 1 and using class methods to write your solution.

You are also not allowed to directly access any variables of the `LinkedList` or `SLNode` classes. All work must be done by only using class methods.

push(self, value: object) -> None:

This method adds a new element to the top of the stack. It must be implemented with $O(1)$ runtime complexity.

Example #1:

```
s = MaxStack()
print(s)
for value in [1, 2, 3, 4, 5]:
    s.push(value)
print(s)
```

Output:

```
MAX STACK: 0 elements. SLL []
MAX STACK: 5 elements. SLL [5 -> 4 -> 3 -> 2 -> 1]
```

pop(self) -> object:

This method removes the top element from the stack and returns its value. It must be implemented with $O(1)$ runtime complexity. If the stack is empty, the method raises a custom "StackException". Code for the exception is provided in the skeleton file.

Example #1:

```
s = MaxStack()
try:
    print(s.pop())
except Exception as e:
    print("Exception:", type(e))
for value in [1, 2, 3, 4, 5]:
    s.push(value)
for i in range(6):
    try:
        print(s.pop())
    except Exception as e:
        print("Exception:", type(e))
```

Output:

```
Exception: <class '__main__.StackException'>
5
4
3
2
1
Exception: <class '__main__.StackException'>
```

top(self) -> object:

This method returns the value of the top element of the stack without removing it. It must be implemented with $O(1)$ runtime complexity. If the stack is empty, the method raises a custom "StackException". Code for the exception is provided in the skeleton file.

Example #1:

```
s = MaxStack()
try:
    s.top()
except Exception as e:
    print("No elements in stack", type(e))
s.push(10)
s.push(20)
print(s)
print(s.top())
print(s.top())
print(s)
```

Output:

```
No elements in stack <class '__main__.StackException'>
MAX STACK: 2 elements. SLL [20 -> 10]
20
20
MAX STACK: 2 elements. SLL [20 -> 10]
```

get_max(self) -> object:

This method returns the maximum value currently stored in the stack. It must be implemented with $O(1)$ runtime complexity. If the stack is empty, the method raises a custom "StackException". Code for the exception is provided in the skeleton file.

Example #1:

```
s = MaxStack()
for value in [1, -20, 15, 21, 21, 40, 50]:
    print(s, ' ', end='')
    try:
        print(s.get_max())
    except Exception as e:
        print(type(e))
    s.push(value)
while not s.is_empty():
    print(s.size(), end='')
    print(' Pop value:', s.pop(), ' get_max after:', end='')
    try:
        print(s.get_max())
    except Exception as e:
        print(type(e))
```

Output:

```
MAX STACK: 0 elements. SLL [] <class '__main__.StackException'>
MAX STACK: 1 elements. SLL [1] 1
MAX STACK: 2 elements. SLL [-20 -> 1] 1
MAX STACK: 3 elements. SLL [15 -> -20 -> 1] 15
MAX STACK: 4 elements. SLL [21 -> 15 -> -20 -> 1] 21
MAX STACK: 5 elements. SLL [21 -> 21 -> 15 -> -20 -> 1] 21
MAX STACK: 6 elements. SLL [40 -> 21 -> 21 -> 15 -> -20 -> 1] 40
7 Pop value: 50 get_max after: 40
6 Pop value: 40 get_max after: 21
5 Pop value: 21 get_max after: 21
4 Pop value: 21 get_max after: 15
3 Pop value: 15 get_max after: 1
2 Pop value: -20 get_max after: 1
1 Pop value: 1 get_max after: <class '__main__.StackException'>
```


Part 3 - Summary and Specific Instructions

1. Implement Deque and Bag ADT interfaces with a circular doubly linked list data structure by completing the skeleton code provided in the file `cdll.py`. Once completed, your implementation will include the following methods:

Methods identical to Singly Linked List in Part 1:

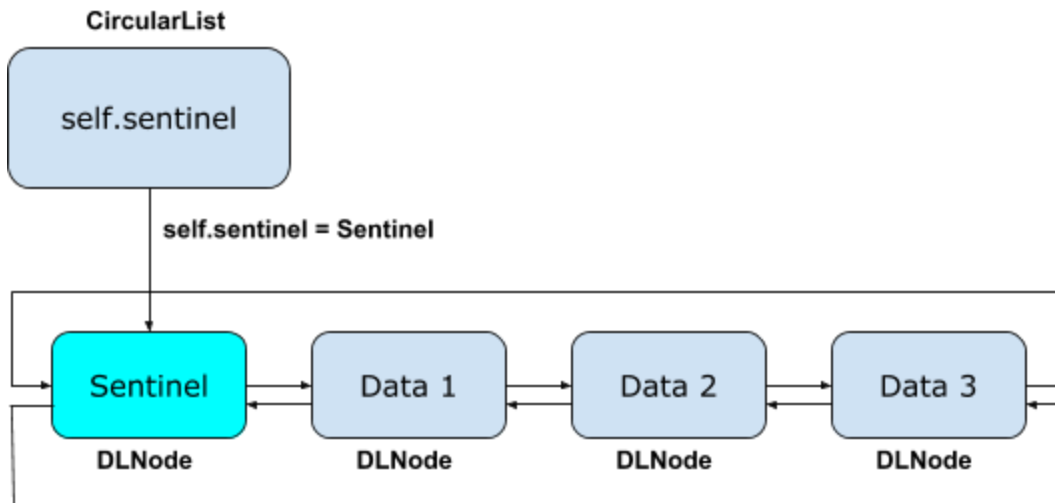
```
add_front(), add_back()
insert_at_index()
remove_front(), remove_back()
remove_at_index()
get_front(), get_back()
remove()
count()
```

**** Note that you do not need to implement the `slice()` method.**

Additional methods:

```
swap_pairs(), reverse(), sort()
rotate(), remove_duplicates()
odd_even(), add_integer()
```

2. We will test your implementation with different types of objects, not just integers. We guarantee that all such objects will have correct implementations of methods `__eq__`, `__lt__`, `__gt__`, `__ge__`, `__le__`, and `__str__`.
3. The number of objects stored in the list at any given time will be between 0 and 900 inclusive.
4. This implementation must be done with the use of a single sentinel node.



5. Variables in the `DLNode` and `CircularList` classes are not marked as private. For this portion of the assignment, you are allowed to access and change their values directly. You are not required to write getter or setter methods for them.
6. **RESTRICTIONS:** You are not allowed to use ANY built-in Python data structures or their methods.
7. Methods `get_front()`, `get_back()`, `add_front()`, `add_back()`, `remove_front()`, and `remove_back()` must be implemented such that they have $O(1)$ runtime complexity.
8. All methods that have identical names to methods in Part 1 must work the same way as the methods you implemented for a Singly Linked List in Part 1 of this assignment. Please refer to part 1 of this document for a detailed description of each method and its expected input and output values.

Note that, in the case where an exception needs to be raised, the name of the exception should be `CDLLError`, not `SLLException`. Code for the exception is provided in the skeleton file.

9. When using code examples provided in part 1 of this document, your list must be created from the class `CircularList`, not from the class `LinkedList`. Also, note that the `__str__` method uses different characters for separating nodes (to reflect the fact that this is a Doubly Linked List).

For example, if a code sample from part 1 says:

```
lst = LinkedList([1, 2, 3])
lst.add_back(4)
print(lst)
```

Output:

```
SLL [1 -> 2 -> 3 -> 4]
```

Then, for the purposes of part 3, you would change it as follows (also note a slightly different output format):

```
lst = CircularList([1, 2, 3])
lst.add_back(4)
print(lst)
```

Output:

```
CDLL [1 <-> 2 <-> 3 <-> 4]
```

swap_pairs(self, index1: int, index2: int) -> None:

This method swaps two nodes given their indices. Swapping must be done by changing node pointers. You are not allowed to just swap the values of the two nodes.

If any of the provided indices is invalid, the method raises a custom "CDLLEnception". Code for the exception is provided in the skeleton file. If the linked list contains N nodes (not including sentinel nodes in this count), valid indices for this method are [0, N - 1] inclusive.

Example #1:

```
lst = CircularList([0, 1, 2, 3, 4, 5, 6])
test_cases = ((0, 6), (0, 7), (-1, 6), (1, 5),
              (4, 2), (3, 3), (1, 2), (2, 1))

for i, j in test_cases:
    print('Swap nodes ', i, j, ' ', end='')
    try:
        lst.swap_pairs(i, j)
        print(lst)
    except Exception as e:
        print(type(e))
```

Output:

```
Swap nodes 0 6 CDLL [6 <-> 1 <-> 2 <-> 3 <-> 4 <-> 5 <-> 0]
Swap nodes 0 7 <class '__main__.CDLLEnception'>
Swap nodes -1 6 <class '__main__.CDLLEnception'>
Swap nodes 1 5 CDLL [6 <-> 5 <-> 2 <-> 3 <-> 4 <-> 1 <-> 0]
Swap nodes 4 2 CDLL [6 <-> 5 <-> 4 <-> 3 <-> 2 <-> 1 <-> 0]
Swap nodes 3 3 CDLL [6 <-> 5 <-> 4 <-> 3 <-> 2 <-> 1 <-> 0]
Swap nodes 1 2 CDLL [6 <-> 4 <-> 5 <-> 3 <-> 2 <-> 1 <-> 0]
Swap nodes 2 1 CDLL [6 <-> 5 <-> 4 <-> 3 <-> 2 <-> 1 <-> 0]
```

reverse(self) -> None:

This method reverses the order of the nodes in the list. The reversal must be done “in place” without creating any copies of existing nodes or an entire existing list. You are not allowed to swap values of the nodes; the solution must change node pointers. Your solution must have $O(N)$ runtime complexity.

Example #1:

```
test_cases = (  
    [1, 2, 3, 3, 4, 5],  
    [1, 2, 3, 4, 5],  
    ['A', 'B', 'C', 'D']  
)  
for case in test_cases:  
    lst = CircularList(case)  
    lst.reverse()  
    print(lst)
```

Output:

```
CDLL [5 <-> 4 <-> 3 <-> 3 <-> 2 <-> 1]  
CDLL [5 <-> 4 <-> 3 <-> 2 <-> 1]  
CDLL [D <-> C <-> B <-> A]
```

Example #2:

```
lst = CircularList()  
print(lst)  
lst.reverse()  
print(lst)  
lst.add_back(2)  
lst.add_back(3)  
lst.add_front(1)  
lst.reverse()  
print(lst)
```

Output:

```
CDLL []  
CDLL []  
CDLL [3 <-> 2 <-> 1]
```

Example #3:

```
class Student:
    def __init__(self, name, age):
        self.name, self.age = name, age

    def __eq__(self, other):
        return self.age == other.age

    def __str__(self):
        return str(self.name) + ' ' + str(self.age)

s1, s2 = Student('John', 20), Student('Andy', 20)
lst = CircularList([s1, s2])
print(lst)
lst.reverse()
print(lst)
print(s1 == s2)
```

Output:

```
CDLL [John 20 <-> Andy 20]
CDLL [Andy 20 <-> John 20]
True
```

Example #4:

```
lst = CircularList([1, 'A'])
lst.reverse()
print(lst)
```

Output:

```
CDLL [A <-> 1]
```

sort(self) -> None:

This method sorts the content of the list in non-descending order. The sorting must be done “in place” without creating any copies of existing nodes or an entire existing list. You are not allowed to swap values of the nodes; the solution must change node pointers.

You can implement any sort method of your choice. Sorting does not have to be very efficient or fast; a simple insertion or bubble sort will suffice. However, runtime complexity of your implementation cannot be worse than $O(N^2)$. Duplicates in the list can be placed in any relative order in the sorted list (in other words, your sort does not have to be ‘stable’).

For this method, you may assume that elements stored in the linked list are all of the same type (either all numbers, or strings, or custom objects, but never a mix of those). You do not need to write checks for this condition.

Example #1:

```
test_cases = (
    [1, 10, 2, 20, 3, 30, 4, 40, 5],
    ['zebra2', 'apple', 'tomato', 'apple', 'zebra1'],
    [(1, 1), (20, 1), (1, 20), (2, 20)]
)
for case in test_cases:
    lst = CircularList(case)
    print(lst)
    lst.sort()
    print(lst)
```

Output:

```
CDLL [1 <-> 10 <-> 2 <-> 20 <-> 3 <-> 30 <-> 4 <-> 40 <-> 5]
CDLL [1 <-> 2 <-> 3 <-> 4 <-> 5 <-> 10 <-> 20 <-> 30 <-> 40]
CDLL [zebra2 <-> apple <-> tomato <-> apple <-> zebra1]
CDLL [apple <-> apple <-> tomato <-> zebra1 <-> zebra2]
CDLL [(1, 1) <-> (20, 1) <-> (1, 20) <-> (2, 20)]
CDLL [(1, 1) <-> (1, 20) <-> (2, 20) <-> (20, 1)]
```

rotate(self, steps: int) -> None:

This method 'rotates' the linked list by shifting positions of its elements right or left `steps` number of times. If `steps` is a positive integer, elements should be rotated right. Otherwise, rotation is to the left. All work must be done "in place" without creating any copies of existing nodes or an entire existing list. You are not allowed to swap values of the nodes; the solution must change node pointers. Please note that the value of `steps` parameter can be very large (from -10^9 to 10^9). The solution's runtime complexity must be $O(N)$, where N is the length of the list.

Example #1:

```
source = [_ for _ in range(-20, 20, 7)]
for steps in [1, 2, 0, -1, -2, 28, -100]:
    lst = CircularList(source)
    lst.rotate(steps)
    print(lst, steps)
```

Output:

```
CDLL [15 <-> -20 <-> -13 <-> -6 <-> 1 <-> 8] 1
CDLL [8 <-> 15 <-> -20 <-> -13 <-> -6 <-> 1] 2
CDLL [-20 <-> -13 <-> -6 <-> 1 <-> 8 <-> 15] 0
CDLL [-13 <-> -6 <-> 1 <-> 8 <-> 15 <-> -20] -1
CDLL [-6 <-> 1 <-> 8 <-> 15 <-> -20 <-> -13] -2
CDLL [-6 <-> 1 <-> 8 <-> 15 <-> -20 <-> -13] 28
CDLL [8 <-> 15 <-> -20 <-> -13 <-> -6 <-> 1] -100
```

Example #2:

```
lst = CircularList([10, 20, 30, 40])
for j in range(-1, 2, 2):
    for _ in range(3):
        lst.rotate(j)
    print(lst)
```

Output:

```
CDLL [20 <-> 30 <-> 40 <-> 10]
CDLL [30 <-> 40 <-> 10 <-> 20]
CDLL [40 <-> 10 <-> 20 <-> 30]
CDLL [30 <-> 40 <-> 10 <-> 20]
CDLL [20 <-> 30 <-> 40 <-> 10]
CDLL [10 <-> 20 <-> 30 <-> 40]
```

Example #3:

```
lst = CircularList()
lst.rotate(10)
print(lst)
```

Output:

```
CDLL []
```

remove_duplicates(self) -> None:

This method deletes all nodes that have duplicate values from a sorted linked list, leaving only nodes with distinct values. All work must be done “in place” without creating any copies of existing nodes or an entire existing list. You are not allowed to change values of the nodes; the solution must change node pointers. The solution’s time complexity must be $O(N)$.

You may assume that the list is sorted. You do not need to write checks for this. You may also assume that all elements in the list are of the same type and can be compared with each other using a `==` operator.

Example #1:

```
test_cases = (
    [1, 2, 3, 4, 5], [1, 1, 1, 1, 1],
    [], [1], [1, 1], [1, 1, 1, 2, 2, 2],
    [0, 1, 1, 2, 3, 3, 4, 5, 5, 6],
    list("abccd"),
    list("005BCDDEEFI")
)
for case in test_cases:
    lst = CircularList(case)
    print('INPUT :', lst)
    lst.remove_duplicates()
    print('OUTPUT:', lst)
```

Output:

```
INPUT : CDLL [1 <-> 2 <-> 3 <-> 4 <-> 5]
OUTPUT: CDLL [1 <-> 2 <-> 3 <-> 4 <-> 5]
INPUT : CDLL [1 <-> 1 <-> 1 <-> 1 <-> 1]
OUTPUT: CDLL []
INPUT : CDLL []
OUTPUT: CDLL []
INPUT : CDLL [1]
OUTPUT: CDLL [1]
INPUT : CDLL [1 <-> 1]
OUTPUT: CDLL [1]
INPUT : CDLL [1 <-> 1 <-> 1 <-> 2 <-> 2 <-> 2]
OUTPUT: CDLL [1]
INPUT : CDLL [0 <-> 1 <-> 1 <-> 2 <-> 3 <-> 3 <-> 4 <-> 5 <-> 5 <-> 6]
OUTPUT: CDLL [0 <-> 2 <-> 4 <-> 6]
INPUT : CDLL [a <-> b <-> c <-> c <-> d]
OUTPUT: CDLL [a <-> b <-> d]
INPUT : CDLL [0 <-> 0 <-> 5 <-> B <-> C <-> D <-> D <-> E <-> E <-> F <-> I]
OUTPUT: CDLL [5 <-> B <-> C <-> F <-> I]
```


odd_even(self) -> None:

This method regroups list nodes by first grouping all ODD nodes together followed by all EVEN nodes (references are to the node numbers in the list (starting from 1), not their values). Please see the code examples below for additional details.

All work must be done “in place” without creating any copies of existing nodes or an entire existing list. You are not allowed to change values of the nodes; the solution must change node pointers. The solution’s time complexity must be $O(N)$.

Example #1:

```
test_cases = (
    [1, 2, 3, 4, 5], list('ABCDE'),
    [], [100], [100, 200], [100, 200, 300],
    [100, 200, 300, 400],
    [10, 'A', 20, 'B', 30, 'C', 40, 'D', 50, 'E']
)
for case in test_cases:
    lst = CircularList(case)
    print('INPUT :', lst)
    lst.odd_even()
    print('OUTPUT:', lst)
```

Output:

```
INPUT : CDLL [1 <-> 2 <-> 3 <-> 4 <-> 5]
OUTPUT: CDLL [1 <-> 3 <-> 5 <-> 2 <-> 4]
INPUT : CDLL [A <-> B <-> C <-> D <-> E]
OUTPUT: CDLL [A <-> C <-> E <-> B <-> D]
INPUT : CDLL []
OUTPUT: CDLL []
INPUT : CDLL [100]
OUTPUT: CDLL [100]
INPUT : CDLL [100 <-> 200]
OUTPUT: CDLL [100 <-> 200]
INPUT : CDLL [100 <-> 200 <-> 300]
OUTPUT: CDLL [100 <-> 300 <-> 200]
INPUT : CDLL [100 <-> 200 <-> 300 <-> 400]
OUTPUT: CDLL [100 <-> 300 <-> 200 <-> 400]
INPUT : CDLL [10 <-> A <-> 20 <-> B <-> 30 <-> C <-> 40 <-> D <-> 50 <-> E]
OUTPUT: CDLL [10 <-> 20 <-> 30 <-> 40 <-> 50 <-> A <-> B <-> C <-> D <-> E]
```

add_integer(self, num: int) -> None:

Assume the content of the linked list represents an integer such that each digit is stored in a separate node (you do not need to write checks for this).

This method will receive an integer `num`, add it to the number already stored in the linked list and store the result of the addition back in the list nodes, one digit per node. Please see examples below for more details.

This addition must be done 'in place', by changing the values of the existing nodes to the extent possible. However, since the result of the addition may have more digits than nodes in the original linked list, you may need to add some new nodes. However, you should only add the minimum number of nodes necessary and not recreate the entire linked list.

For example, if the original list stored the number 123 as [1 <-> 2 <-> 3], and the second integer had the value of 5, no new nodes must be created since the result of the addition can be stored in the same three nodes as [1 <-> 2 <-> 8]. If the original list was [1 <-> 2 <-> 3], and the second integer had the value of 999, you are allowed to create one new node so that the entire result can fit [1 <-> 1 <-> 2 <-> 2].

Example #1:

```
test_cases = (
    ([1, 2, 3], 10456),
    ([], 25),
    ([2, 0, 9, 0, 7], 108),
    ([9, 9, 9], 9_999_999),
)

for list_content, integer in test_cases:
    lst = CircularList(list_content)
    print('INPUT :', lst, 'INTEGER', integer)
    lst.add_integer(integer)
    print('OUTPUT:', lst)
```

Output:

```
INPUT : CDLL [1 <-> 2 <-> 3] INTEGER 10456
OUTPUT: CDLL [1 <-> 0 <-> 5 <-> 7 <-> 9]
INPUT : CDLL [] INTEGER 25
OUTPUT: CDLL [2 <-> 5]
INPUT : CDLL [2 <-> 0 <-> 9 <-> 0 <-> 7] INTEGER 108
OUTPUT: CDLL [2 <-> 1 <-> 0 <-> 1 <-> 5]
INPUT : CDLL [9 <-> 9 <-> 9] INTEGER 9999999
OUTPUT: CDLL [1 <-> 0 <-> 0 <-> 0 <-> 0 <-> 9 <-> 9 <-> 8]
```

Part 4 - Summary and Specific Instructions

1. Implement a Queue ADT class by completing the skeleton code provided in the file `queue_from_stacks.py`. You will use the MaxStack ADT that you implemented in part 2 of this assignment as the underlying data storage for your Queue ADT.
2. Your Queue ADT implementation will include the following methods:

```
enqueue()  
dequeue()
```

3. We will test your implementation with different types of objects, not just integers. We guarantee that all such objects will have correct implementations of methods `__eq__`, `__lt__`, `__gt__`, `__ge__`, `__le__`, and `__str__`.
4. The number of objects stored in the Queue at any given time will be between 0 and 900 inclusive. The Queue must allow for storage of duplicate elements.
5. RESTRICTIONS: You are not allowed to use ANY built-in Python data structures or their methods. You must solve this portion of the assignment by importing the MaxStack class that you wrote in part 2 and using class methods to write your solution.

You are also not allowed to directly access any variables of the MaxStack class. All work must be done by only using class methods (`push()`, `pop()`, `top()`, `size()`, and `is_empty()`). You don't need to use the method `get_max()` in your implementation.

enqueue(self, value: object) -> None:

This method adds a new value to the end of the queue. It must be implemented with $O(1)$ runtime complexity.

Example #1:

```
q = Queue()
print(q)
for value in [1, 2, 3, 4, 5]:
    q.enqueue(value)
print(q)
```

Output:

```
QUEUE: 0 elements. MAX STACK: 0 elements. SLL []
QUEUE: 5 elements. MAX STACK: 5 elements. SLL [5 -> 4 -> 3 -> 2 -> 1]
```

dequeue(self) -> object:

This method removes and returns the value at the beginning of the queue. It must be implemented with a runtime complexity of not worse than $O(N)$.

If the queue is empty, the method raises a custom "QueueException". Code for the exception is provided in the skeleton file.

Example #1:

```
q = Queue()
for value in [1, 2, 3, 4, 5]:
    q.enqueue(value)
print(q)
for i in range(6):
    try:
        print(q.dequeue(), q)
    except Exception as e:
        print("No elements in queue", type(e))
```

Output:

```
QUEUE: 5 elements. MAX STACK: 5 elements. SLL [5 -> 4 -> 3 -> 2 -> 1]
1 QUEUE: 4 elements. MAX STACK: 4 elements. SLL [5 -> 4 -> 3 -> 2]
2 QUEUE: 3 elements. MAX STACK: 3 elements. SLL [5 -> 4 -> 3]
3 QUEUE: 2 elements. MAX STACK: 2 elements. SLL [5 -> 4]
4 QUEUE: 1 elements. MAX STACK: 1 elements. SLL [5]
5 QUEUE: 0 elements. MAX STACK: 0 elements. SLL []
No elements in queue <class '__main__.QueueException'>
```