Results and Analysis

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- The following 10 bonds were selected using the following process: we first pick the bond that is the closest to having time to maturity 6 months, using that maturity date as a basepoint, we further iteratively pick bonds with maturities closest to 6 months after the previous basepoint; in the case of bonds with identical maturity dates, we pick the one with the coupon rate most similar to the other bonds chosen.

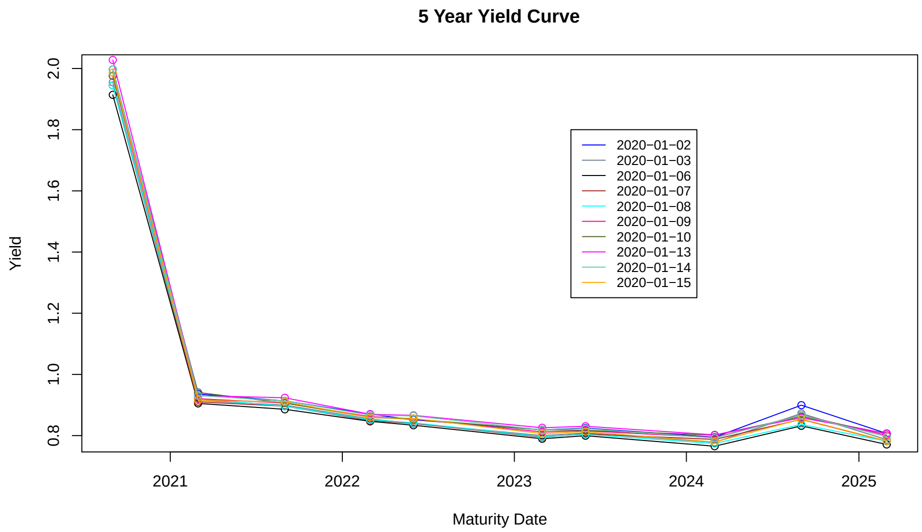
CAN 0.75 Sep 20	CAN 0.75 Mar 21	CAN 0.75 Sep 21	CAN 0.5 Mar 22	CAN 2.75 Jun 22
CAN 1.75 Mar 23	CAN 1.5 Jun 23	CAN 2.25 Mar 24	CAN 1.5 Sep 24	CAN 1.25 Mar 25

- The eigenvalues and eigenvectors tell us the variance structure between the stochastic processes; the eigenvector corresponding to the largest eigenvalue points in the direction in which the data has maximum variance, with variance equal to the eigenvalue, and the eigenvector corresponding to the next largest eigenvalue points in the direction of maximum variance orthogonal to the first eigenvector. The eigenvalues also explain the proportion of the population variance explained by the corresponding principal component given by the eigenvector. From this, we see that the eigenvalues and eigenvectors tell us about the variance structure and the geometry of the data.

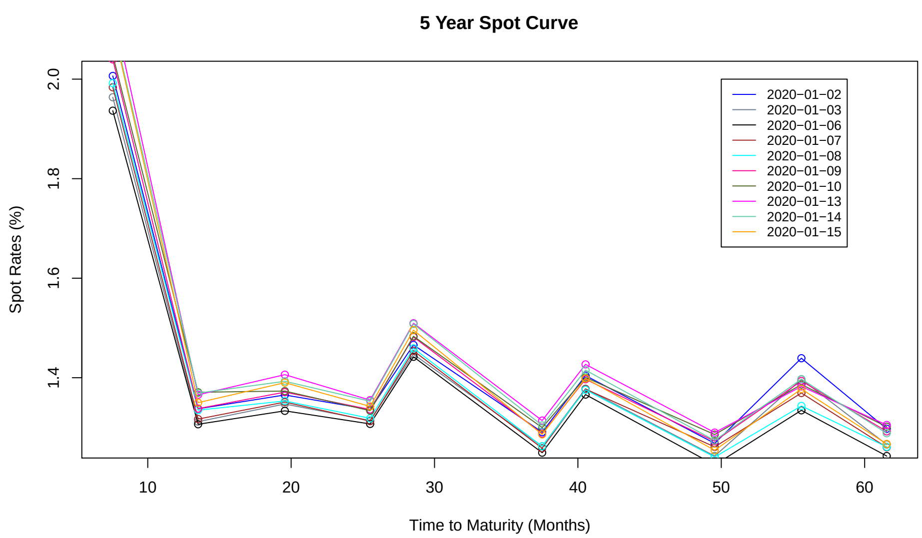
- (a) We first compute the yield to maturity selected bonds. Using the formula $P = \sum_{i=1}^{n-1} \frac{C}{(1+y)^i} + \frac{100+C}{(1+y)^n}$ and by root-finding, we get the following YTM:

CAN 0.75 Sep 20	1.1822%	CAN 1.75 Mar 23	0.8292%
CAN 0.75 Mar 21	0.7527%	CAN 1.5 Jun 23	0.8174%
CAN 0.75 Sep 21	0.7784%	CAN 2.25 Mar 24	0.8331%
CAN 0.5 Mar 22	0.7008%	CAN 1.5 Sep 24	0.8814%
CAN 2.75 Jun 22	0.9393%	CAN 1.25 Mar 25	0.7956%

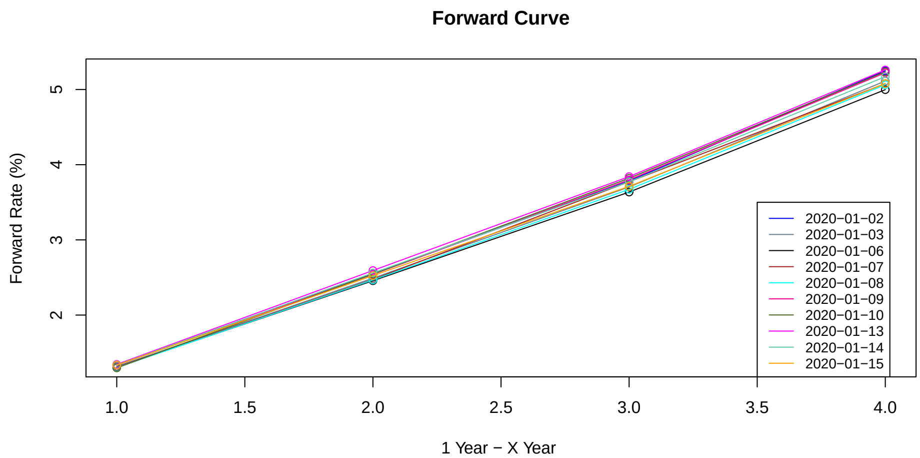
  The 5 year yield curve (under discrete, semi-annual compounding assumption) is shown below; linear interpolation was used in between points that were bootstrapped as we have no a priori information about the yields in between those points, hence linear interpolation is (without further exploration) a good simple choice.



(b) Under the continuous compounding assumption, spot rates can be bootstrapped via the following procedure: take the first bond which is close to TTM 6 months, treat it as a zero-coupon compute its yield, i.e. $s_{\frac{1}{2}} = - \cdot \log(\frac{P}{100+C})$. Then we can iteratively solve for $r_1, r_{\frac{3}{2}}, \ldots, r_5$ using the formula $r_j = \dfrac{-\log\left(\frac{P_j - \sum_{i=1}^{j-1} Ce^{-i \cdot r_i}}{100+C}\right)}{j}$. Then we linearly interpolate between these 10 spot rates to estimate the spot curve. The result is as below for our 10 selected bonds:



(c) We use spot rates r_1, r_2, r_3, r_4, r_5 to compute the forward rates. The 1yr-(j)yr forward rate $f_{1,j}$ can be computed via the formula $f_{1,j} = \dfrac{(1+r_{j+1})^{j+1}}{(1+r_2)} - 1$. This easily gives us the forward rates, from which we again use linear interpolation to estimate the forward curve.



- Picking $X_1, \dots, X_5$ by taking them to be the $2i$-th bonds chronologically from our selected 10, $1 \le i \le 5$ for the yield, Σ_{yield} is:

$$\Sigma_{\text{yield}} = 10^{-4}\begin{pmatrix} 1.9943 & 0.5869 & 1.2783 & 1.3071 & 1.5061 \\ 0.5869 & 0.9993 & 1.2585 & 1.2604 & 1.3986 \\ 1.2783 & 1.2585 & 1.9136 & 2.1778 & 2.2291 \\ 1.3071 & 1.2604 & 2.1778 & 3.6751 & 2.8825 \\ 1.5061 & 1.3986 & 2.2291 & 2.8825 & 2.7845 \end{pmatrix}, \quad \Sigma_{\text{forward}} = 10^{-4}\begin{pmatrix} 3.7573 & 2.3602 & 1.9623 & 1.8725 \\ 2.3602 & 2.9911 & 3.3306 & 3.1200 \\ 1.9623 & 3.3306 & 5.8663 & 4.0821 \\ 1.8725 & 3.1200 & 4.0821 & 3.6884 \end{pmatrix}$$

- The eigenpairs $\{\lambda_i, v_i\}$ in decreasing order of eigenvalues for the matrix Σ_{yield} are:

$$\left\{9.2700 \cdot 10^{-4}, \begin{pmatrix} -0.3160 \\ -0.2698 \\ -0.4383 \\ -0.5844 \\ -0.5420 \end{pmatrix}\right\}, \left\{1.2954 \cdot 10^{-4}, \begin{pmatrix} 0.8877 \\ -0.0853 \\ 0.0586 \\ -0.4467 \\ -0.0408 \end{pmatrix}\right\}, \left\{6.9951 \cdot 10^{-5}, \begin{pmatrix} -0.2541 \\ 0.6415 \\ 0.3971 \\ -0.5882 \\ 0.1420 \end{pmatrix}\right\}$$

$$\left\{7.4058 \cdot 10^{-6}, \begin{pmatrix} 0.1830 \\ 0.5289 \\ 0.0307 \\ 0.3358 \\ -0.7570 \end{pmatrix}\right\}, \left\{2.7788 \cdot 10^{-6}, \begin{pmatrix} 0.1185 \\ 0.4781 \\ -0.8036 \\ 0.0083 \\ 0.3338 \end{pmatrix}\right\}$$

  And the eigenpairs in decreasing order for the matrix Σ_{forward} are

$$\left\{1.2794 \cdot 10^{-3}, \begin{pmatrix} 0.1939 \\ -0.7286 \\ -0.1066 \\ 0.6482 \end{pmatrix}\right\}, \left\{2.7186 \cdot 10^{-4}, \begin{pmatrix} 0.1939 \\ -0.7286 \\ -0.1066 \\ 0.6482 \end{pmatrix}\right\},$$

$$\left\{7.2004 \cdot 10^{-5}, \begin{pmatrix} 0.1939 \\ -0.7286 \\ -0.1066 \\ 0.6482 \end{pmatrix}\right\}, \left\{7.0593 \cdot 10^{-6}, \begin{pmatrix} 0.1939 \\ -0.7286 \\ -0.1066 \\ 0.6482 \end{pmatrix}\right\}$$

  The first (largest) eigenvalue and its associated eigenvector points in the direction of maximum variance, implying that the principal component $v_1^T X$ has maximal variance λ_1 (among linear combinations); this indicates, taking Σ_{yield} for example, that to maximize variance in log return of yield from a portfolio of these bonds, you should sell (short) all the bonds $X_1, \dots, X_5$ as according to the eigenvector v_1.