A Mathematical Fire Spread Model

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Abstract: This paper develops a novel probabilistic model for firebranding spotting which we incorporate into existing models better model the behaviour of forest fires spreading across fuel discontinuities. We combine three models - the Canadian Forest Services Fire Behaviour Prediction (FBP) system, a spotting model that we have modified to account for noise in the spotting process, and a probability of ignition model that we have modified to account for the mass of a firebrand - to obtain a more accurate description of fire behaviour in mature jack pine forest under high to extreme fire weather. We performed sensitivity analysis on the parameters we introduced, examined the robustness of the spotting model, ran an optimization problem to determine the cost of reducing the expected value of area burned below a certain threshold, and looked at the probability that our simulated fire would behave in the same extreme way the fire in Fort McMurray did in 2016. We made revisions to our model to address issues that came up while trying to answer this probabilistic question, and in the end suggest ideas for next steps we could take to further improve the model.

Contents

1 Background

In Canada, forest fire are a landscape-level disturbance that greatly influence forest structure, function, and, in general, the forest environment. On average, it's estimated that $10 - 15$ million hectares of Boreal forest burns globally, with Canadian forest accounting for 2 million hectares of this total on average. It has been estimated that Canadian forest fires release $27\,\mathrm{TgCyr}^{-1}$ and in some years this number can be greater than $100\,\mathrm{TgC}$ - which is the amount of carbon released by motor vehicles in Canada. In addition, to the distributing amount of carbon released, during years with large areas burned the amount of mercury released by Canadian forest approaches the amount of Mercury released by industrial North American **?**. In addition to the environmental concerns, forest fires are incredibly costly. On average Canadian forest fire management agencies spend nearly half a billion dollars in direct fire suppression activities **?**.

Typically fires that burn over 200ha are considered to be large fires. In Canada, these large fires account for only about 3% of the fires that occur in a given year but account for nearly 97% of the area burned **?**. During these large intense forest fires, various sizes of flaming/burning/smouldering wood chips, needles, twigs and other forest material (often called firebrands) are lofted into the atmosphere by the upward draft from the fire. Once lofted these firebrands are carried downwind by the ambient wind where they burn out before landing or make it to the forest floor in either a flaming or smouldering state. Once they land, depending on the recipient fuel conditions, the firebrands can do one of two things - if the recipient fuels have a high moisture content, they can preheat these fuels and dry them out. Or, if the recipient fuels are dry enough, they may be capable of starting a new fire. These new fires are called spot fires and can cause many forest fire management issues. An example of this is Fort McMurray in 2016, where two spotting incidents occurred with the largest of the two being just over 1km **?**. The fire in this area is considered the most costliest insured natural disaster in Canadian history running a bill of an estimated 3.6 billion dollars and a total estimated cost of around 8.9 billion dollars **?**.

Despite the management challenges and dangers posed by fire brand spotting, the main fire behaviour prediction system in Canada - the Fire Behaviour Prediction (FBP) system **?** - does not account for spotting events beyond a standard 20m which it implicitly has built into its fire spread model. Moreover, there are spotting models that do exists **??????**, **???**, **?**, **?**, **?**, and **?**. However, the majority of the models that exists are built around the idea of finding the *maximum* spotting distance and *not* the likelihood that a spot fire will occur.

2 Modelling Question

Throughout the modelling process we had one overarching question:

Under high/extreme fire weather conditions can we create a stochastic fire spread model that accounts for firebrand spotting?

As we worked to answering this question, a couple of secondary questions arose in a natural way:

(i.) What is the probability that a fire will spot/breach a fuel discontinuity

(ii.) How large of a fuel discontinuity do you need to create to bring the expected value of area burned below a certain value? What is the cost of creating a fuel discontinuity large enough to stop firebrand spotting from causing a fire on the other side of a fuel discontinuity?

3 Modelling Approach

We decided to combine three existing models - the Fire Behaviour Prediction (FBP) systems elliptical fire growth model **?**, a fire brand spotting model by **?**, and a probability of ignition model by **?**. The elliptic growth model is a deterministic model that is based on many years of data collection which indicated that with a constant wind speed direction, fire growth is elliptical in nature. The fire brand spotting model is a physical model developed from physical principles. The probability of ignition model is a statistical model based on fitting curves to large samples of data.

As firebrand spotting is particularly random in nature, we decided to incorporate some of this randomness into the model by **?**, thus turning the deterministic spotting model into a stochastic one.

All three of the models used and the addition of the stochastic parts we have included will be discussed in much more detail in section 4.

4 Formulation of Model

4.1 Our Modelling Assumptions

We made the following assumptions in our initial analysis:

1. The landscape the fire is burning in is homogeneous - that is, the FFMC, DMC, DC, FMC, fuel type (Mature Jackpine), and other FBP parameters are constant throughout the landscape. We did this for computational ease and because locally a forest will have this consistency.

2. Wind speed is constant at canopy height. Again, this is for computational ease, but the model we have could easily incorporate this into the model if we really wanted to.

3. Wind blows perpendicularly to the fuel discontinuity.

4. A cell that is in a burning state remains burning for 60 minutes. We could not find any good data on this as it really depends on the moisture content and type of trees that are burning.

5. While a cell is burning, if the lofted height of a firebrand is greater than the height of the canopy, then the firebrand can spot.

6. We chose a rate of firebrand generation to be one firebrand per burning cell per every 5 minutes. We based this on work done by **?**. However, **?** uses a firebrand generation rate of five firebrands per burning cell per second. This is a very large difference and without any actual data we have no way of really justifying either rate. We used 1 firebrand per 5 minute per burning grid for computational ease.

7. If a firebrand lands and causes an ignition, we put a delay time of 30 minutes before we consider the cell to be in a burning state.

8. Firebrand mass is exponentially distributed with mean $0.35g$. This is based on the work of **????**, where after burning different types of pine trees and combining the data, the papers found that the mass distribution of collected firebrands was 0.35g and approximately exponentially distributed. We also used this because firebrand generation should be a memory less process.

9. The length to radius ratio of the cylindrical firebrand is fixed. This is for mathematical convenience and as far as we are aware there are no models existing models that allow this to vary. We used a fixed ratio of $L/r = 6$ which is the same value used in **?**.

10. The lofted height of a particular fire brand is exponentially distributed. Since the fire plume model doesn't account for any turbulence, and we would expect that firebrands don't always loft at their terminal velocity, we would expect that the majority of firebrands are lofted to a much lower height than their maximum loft-able height. Further, it is clear that this should be a process that is memory less. We choose the λ in the exponential distribution such that $Pr(\text{lofted height} = z_{max}) = 0.01$, i.e. a firebrand is lofted to it's maximum height 1% of the the time.

4.2 Elliptical Fire Growth

Our elliptic fire growth model is largely based on the Fire Behaviour Prediction systems elliptic growth model. This model largely relies on two outputs: Fire line intensity (called I from here out) and the forward rate of spread (called ROS from here out). This model is largely based on many decades of data collection and fitting curves to this collection of data and justified by a physical understanding of how these different parts of a forest interact to cause forest fires.

4.2.1 Fire Behaviour Prediction Parameters

FFMC	Fine fuel moisture code	CBH	Crown-base Height (m)
DMC	Duff moisture code	FMC	foliar moisture content
DC	Drought moisture code	CFL	Crown fuel Load (kg/m^3)
a_1, a_2, a_3	Rate of Spread constants	b_1, b_2 b_3	Surface fuels consumed constants

Extreme fire weather values to correspond to a FFMC of 91+, DMC of 51+, and a DC of 340+. High fire weather conditions correspond to FFMC$\in \{87, 90\}$, DMC$\in \{31, 50\}$ and a DC$\in \{241, 340\}$ **?**.

4.2.2 Fire Behaviour Prediction system main outputs

Here we outline the calculations needed to compute the rate of spread and fire line intensity. The initial spread index (ISI) is given by the equations:

$$ISI = 0.208 \cdot f_1(w) \cdot f_2(\text{FFMC}) \tag{1}$$

where FFMC is the fine fuel moisture content and w is the wind speed in km/h and we have two auxilary functions f_1 and f_2 given by:

$$f_1 = \exp(0.05039 \cdot w) \tag{2}$$

$$f_2 = 91.9 \cdot \exp(-0.1386 \cdot mc) \cdot \frac{1 + mc^{5.31}}{4.93 \cdot 10^7} \tag{3}$$

$$mc = 147.2 \cdot (101 - \text{FFMC})/(59.5 + \text{FFMC}) \tag{4}$$

Here equation (4) is a conversion from the fine fuel moisture code (FFMC) into a moisture content. Then the equation for fire intensity is given by

$$I = 300 \cdot TFC \cdot ROS \tag{5}$$

Where TFC is the total fuel consumed by the fire in kg/m^2 , ROS the rate of spread in m/min and I the fireline intensity kW/m. The rate of spread is given by the formula

$$ROS = a_1(1 - \exp(-a_2 \cdot ISI))^{a_3} \tag{6}$$

where a_1, a_2, a_3 are rate of spread constants that depend on the fuel type. For Mature Jackpine (also known as $C - 3$ fuels) we have that $a_1, a_2, a_3 = 110, 0.0444, 3.0$ respectively.

For the total surface fuels consumed (TFC) we need to use calculate the following:

i. Build-up-Index: This is roughly a measure of the amount of fuels that have built up on the forest surface floor.

$$BUI = \begin{cases} \frac{0.8 \cdot DMC \cdot DC}{DMC+0.4 \cdot DC} & \text{if } DMC < 0.4DC \\ DMC - \frac{1-(0.8 \cdot DC)}{DMC+0.4 \cdot DC} \cdot (0.92 + 0.0114 \cdot DMC)^{1.7} & \text{otherwise} \end{cases} \tag{7}$$

Where DMC is the duff moisture code and DC is the drought moisture code.

ii. Surface fuels consumed (SFC):

$$SFC = b_1(1 - \exp(-b_2 \cdot BUI))^{b_3} \tag{8}$$

where SFC is measured in kg/m^2.

iii. Critical surface fire intensity (CSI). This is the intensity needed for a ground fire to turn into a fire that reaches the crown of the tree.

$$CSI = 0.001 \cdot CBH^{1.5} \cdot (460 + 25.9 \cdot FMC)^{1.5} \tag{9}$$

Where CBH is the canopy-base height and FMC is the foliar moisture content (the moisture in the leaves/needles).

iv. Critical rate of spread (CSI):

$$RSO = \frac{CSI}{300 \cdot SFC} \tag{10}$$

v. Crown fraction burned (CFB):

$$CFB = \begin{cases} 1 - \exp(-0.23 \cdot (ROS - RSO) & \text{if } ROS \geq RSO \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

If the rate of spread is greater than the critical rate of spread the fire will transition to burning the crown of the tree. This influences that amount of fuels available to burn and hence the fire intensity.

vi. Total fuel consumed (TFC):

$$TFC = SFC + CFB \cdot CFL \tag{12}$$

Then using equation (6) and (12) we have that the fireline intensity is

$$I = 300 \cdot TFC \cdot ROS \tag{13}$$

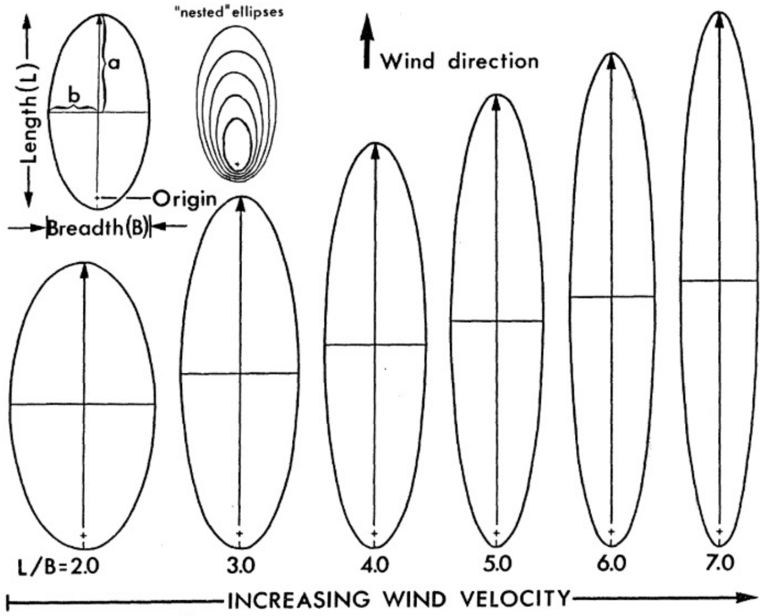
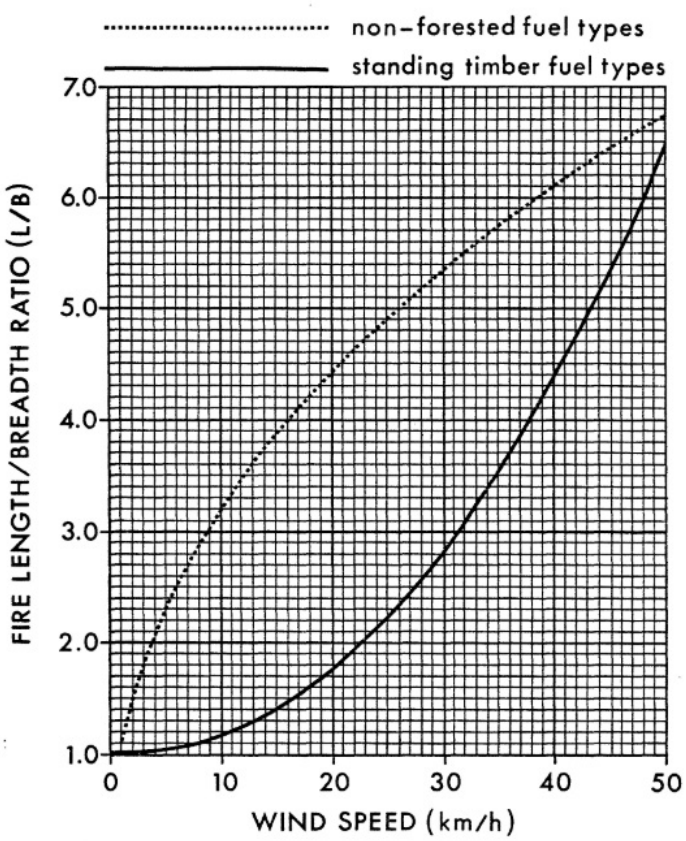


Figure 19. Idealized elliptical shapes of various length-to-breadth (L/B) ratios having the same area for free-burning fires spreading under the influence of increasing wind velocity (L/B = 2.0 to 7.0). Backfire spread assumed to be essentially negligible in all cases.



4.2.3 Elliptic growth model

Now the elliptic fire growth model we employed is a simple model implemented in the FBP system. It describes the growth of a fire as an ellipse, given a wind speed (under the assumption that back burning is negligible). There is a chart they include in the documentation to get the ratio of the length to breadth ratio. See the two figures below which are taken from **?**. To implement this model we needed to be able to define a length to use the length/breadth ratio. To accomplish this, we used the forward rate of spread, *ROS* defined above. Starting from the initial point of the fire(the cell that we ignited) we computed the maximal distance away from this cell at time t as $t \cdot ROS$. Then using the chart below, we were able to use the corresponding length/breadth ratio for the chosen wind speed.

4.3 Firebrand Spotting

4.3.1 Parameters

A_b	cross-sectional area of cylindrical firebrand
a	$(\pi g K)/(2C_d)$
b	$(\pi g \rho_b r_0)/(C_d \rho_a)$
bouy	$g/(\rho_a C_p T)$
C_d	coefficient of drag (1.2)
C_p	specific heat of air ($1.0kJ/kg$)
g	acceleration due to gravity $9.81m/s^2$
C_p	specific heat of air ($1.0kJ/kg$)
g	acceleration due to gravity $9.81m/s^2$
H	canopy height (m)
I	Fireline Intensity (kW/m)
K	burning rate constant (0.0064)
L	length of firebrand (m)
r_0	radius of the firebrand when it leaves the plume (m)
T	ambient temperature ($300K$)
U_H	wind speed at canopy height (m/s)
V_b	volume of cylindrical firebrand
Z	height the firebrand leaves the plume (m)
z_0	0.1313H
α	$\sqrt{b - aZ}$
β	$(3.64^3\pi)/(2.45^5)$
γ	$(9.35^2)/(\pi g H^{2/3})$
η	$\beta\gamma^{5/2}$
ρ_a	density of air ($1.225kg/m^3$)
ρ_b	density of firebrand ($440kg/m^3$)
t^*	time to reach canopy (seconds)

4.3.2 Terminal velocity Assumption

Many authors **????????** use the terminal velocity assumption in their analysis. This is a result of work done by **?** which showed that firebrands reach their terminal velocity relatively quickly (ie, in a couple of seconds). **?** compared simulated results obtained using the terminal velocity assumption and not using it. Their results indicate that their firebrand transport model without the terminal velocity assumption were launched further than brands travelling at terminal velocity. Letting W be the weight of the fire brand, D the drag force on the firebrand and v_b the velocity of the firebrand, we achieve terminal velocity, v_t, when there is no more acceleration, i.e. $W = D$. Writing the mass of the firebrand as $m_b = V_b \rho_b$, where V_b is the volume of the firebrand and ρ_b is the density of the fire brand we solve for the terminal velocity:

$$W = m_b g$$
$$D = \frac{1}{2} C_d \rho_a v_b^2 A_b$$
$$\Rightarrow v_t = \sqrt{\frac{2\rho_b V_b g}{C_d \rho_a A_b}}$$

Based on the work in **????**, we have made the assumption that the firebrands we are modelling are cylindrical in shape. This allows us to write

$$V_b = \pi r^2 L \qquad A_b = 2rL$$

and thus we can simplify the terminal velocity equation to:

$$v_t = \sqrt{\frac{\pi \rho_b g}{\rho_a C_d} r} \tag{14}$$

4.3.3 Maximum Loftable Height

Then using work from **?** and a fire plume model from **?**, **?** finds that the maximum radius of a firebrand that can be lofted as a function of the fire line intensity I is given by

$$r_{max}(I) = \gamma \left(\frac{C_d \rho_a}{\rho_b} \right) I^{2/3} \tag{15}$$

Setting the terminal velocity equal to the maximum rate of up-draught gas flow in the fireplume and using the assumption from **?** that the terminal velocity is always less than or equal to this rate, **?** found the maximum loftable height of a firebrand of radius r_i for a fire intensity I is

$$z_{max}(r_i, I) = \eta \left[\left(\frac{\rho_a}{\rho_b} \right) \left(\frac{C_d}{r_i} \right) \right]^{3/2} I^{5/3} \tag{16}$$

4.3.4 Burning Rate of a Cylindrical Firebrand

The model by **?** ignores any burning of the lofting firebrand, as during this stage the fire is still evaporating whatever moisture is contained in the firebrand. Upon exiting the plume, the firebrand is considered to be burning. Using the burning rate for cylindrical firebrands developed by **?**, we can describe the change in firebrand radius as it burns by

$$\frac{dr}{dt} = \frac{K}{2} \left(\frac{\rho_a}{\rho_b} \right) \frac{dz}{dt} \tag{17}$$

Where $K = 0.0064$ and initial condition $r(0) = r_0$ and $z(0) = Z$ - the height the firebrand leaves the fire plume at. Integrating (17) from 0 to a time t, we have

$$r(t) = r_0 + \frac{K}{2} \left(\frac{\rho_a}{\rho_b} \right) (z(t) - Z) \tag{18}$$

4.3.5 Downwind Launching of Lofted Firebrand

While the firebrand is being lofted it will travel some horizontal distance x_0 (as wind will cause the fire plume to be on a slight angle). Using a simple geometric argument we find that

$$x_0 = Z \tan\theta$$

A factor of 0.7 was introduced by **?** to correct for a possible overestimation of the distance travelled while lofting. In **?**(pg 92), an empirical formula for $\tan\theta$ is used. Combining these we have that x_0 is

$$x_0 = 0.7 \tan\left(\frac{bouy * I}{w^3}\right)^{1/2} \tag{19}$$

Where

$$bouy = \frac{g}{\rho_a C_p T}$$

$g = 9.81 m/s^2$, ρ_a is the density of air, C_p is the specific heat of air at a constant temperature, T is the ambient temperature (298K). This is the initial horizontal position of a lofted firebrand as its about to leave the plume. Now **?** uses the coupled ODES to describe the falling and downwind launching of a firebrand:

$$\frac{dz}{dt} = -v_t \qquad z(0) = Z \tag{20}$$

$$\frac{dx}{dt} = \frac{U_H \log(z(t)/z_0)}{\log(H/z_0)} \qquad x(0) = x_0 \tag{21}$$

where U_H is the wind speed at canopy top height, H is the height of canopy top height, $z_0 = 0.1313H$ and $z(t)$ is the solution to (20). the equation for the burning firebrand $r(t)$ into equation (20), we get the solution to (20) is

$$z(t) = \frac{a}{4}t^2 - \sqrt{b}t + Z \tag{22}$$

From this solution we see that there is a time t^* such that $z(t^*) = H$ where H is the height of the trees where the firebrand lands. This time is given by,

$$t^* = \frac{2}{a}\left(\sqrt{b} - \sqrt{b - a(Z - H)}\right) \tag{23}$$

We use this time because after the firebrand hits the canopy, we have two considerations. Firstly, the canopy serves as a mechanical obstacle that may prevent the firebrand from reaching the ground. Further, even if the firebrand does reach the forest floor, there is no way to know how long it will take for it to fall. Secondly, once the firebrand is at canopy height/below the canopy, the wind speed should decline and so little addition drifting should occur. Now, using the solution to (20) in (21), **?** found an analytic solution to be

$$x(t) = x_0 + \frac{U_H}{\log(H/z_0)}\left\{t(\log\left(\frac{z(t)}{z_0}\right) - 2) + \left(\frac{2\sqrt{b}}{a}\right)\log\left(\frac{Z}{z(t)}\right) + \left(\frac{2\alpha}{a}\right)\log\left(\frac{2Z - (\sqrt{b}+\alpha)t}{2Z - (\sqrt{b}-\alpha)t}\right)\right\}$$

Where $\alpha = \sqrt{b - aZ}$. Evaluating this at t^* to get the distance the firebrand travels before reaching the canopy we get

$$x(t^*) = x_0 + \frac{U_H}{\log(H/z_0)}\left\{t^*(\log\left(\frac{H}{z_0}\right) - 2) + \left(\frac{2\sqrt{b}}{a}\right)\log\left(\frac{Z}{H}\right) + \left(\frac{2\alpha}{a}\right)\log\left(\frac{2Z - (\sqrt{b}+\alpha)t^*}{2Z - (\sqrt{b}-\alpha)t^*}\right)\right\}$$

This final equation, describes the horizontal distance a firebrand travels.

4.4 Probability of Ignition by Firebrand

4.4.1 Parameters

β_0 = -4.8479	β_2 = 0.0258
β_1 = 0	β_3 = 0.6032

For our probability of ignition model we used the equations from **?**. These equations describe the probability of sustained flaming when a flaming match is dropped on different fuel types. These were determined by performing a logistic regression on collected experimental data. The formula we used was for the probability of stained flaming in pine needles. Then we have that the probability of sustained flaming is given by

$$P(sf) = \frac{1}{1 + \exp(-z)} \tag{24}$$

where

$$z = \beta_0 + \beta_1 FFMC + \beta_2 DMC + \beta_3 ISI \tag{25}$$

4.5 Our Contributions

We wanted to add a couple features to the spotting model to account for some of the noise in the firebrand spotting model.

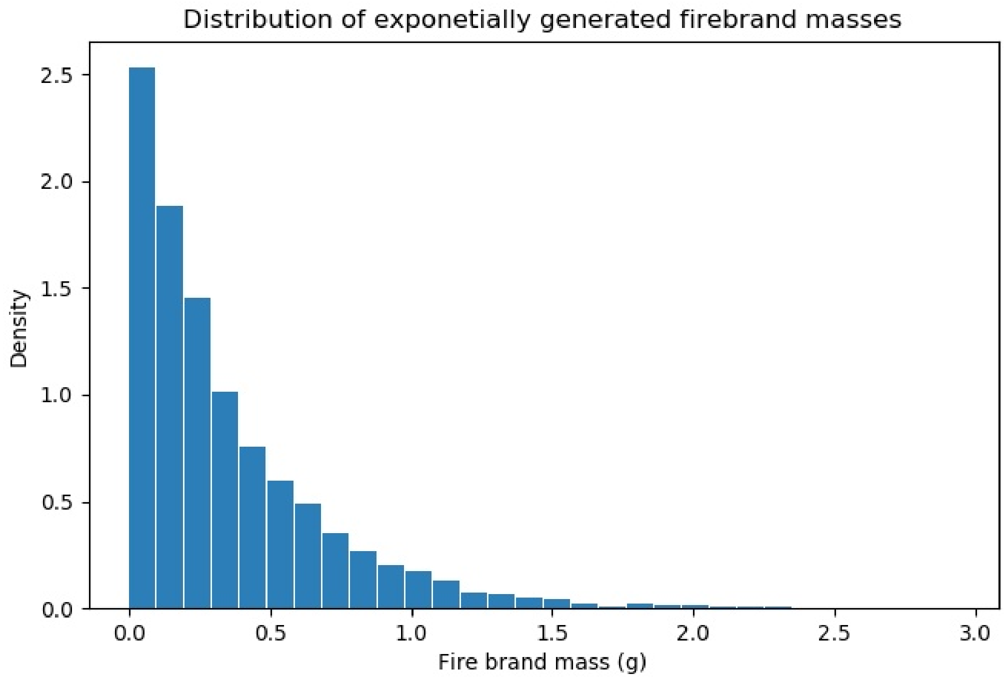
4.5.1 Firebrand Generation

The first feature we added was a mass generation process that generated a random mass based on an exponential distribution with mean 0.35. Once we generated this random mass we wanted to find its radius assuming a cylindrical geometry and fixed length to width ratio of $L/r = 6$. The mass of the firebrand can be written as

$$\frac{m}{1000} = 2\pi r^2 L \rho_b \tag{26}$$

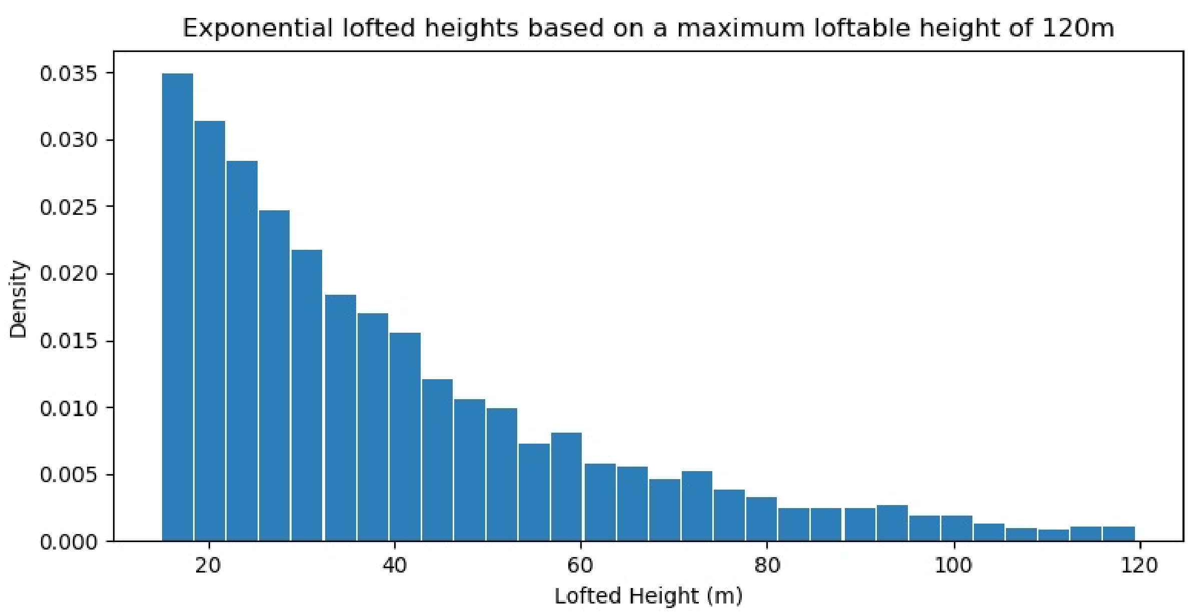
$$\Rightarrow r = \left(\frac{m}{6 \cdot 1000 \pi \rho_b}\right)^{1/3} \tag{27}$$

Where we divide by 1000 to account for the fact the left hand side is in grams and the right hand side in kg. Now this gives us a probabilistic way to generate firebrand radii to use in the lofting process.



4.5.2 Firebrand Lofting

We assumed that firebrand lofting heights are exponentially distributed with mean such that the probability that they reach their maximum loftable height is 1%. We used a truncated exponential distribution to achieve this:



4.5.3 Probability of Ignition

The probability of ignition model we used was based on experimental data where a flaming match was dropped on forest floors and then recorded if the match caused ignition or not. However, it is possible that firebrands are much smaller than a match (which weighs about 0.15g). As such, we wanted to add a factor to weight the probability of ignition to account for this. We used a simple minded approach and weighted the probability of sustained flaming in **?** by a factor of the mass of a firebrand divided by the mass of a match. However, it could be the case that firebrands are also much larger than a match stick. After consulting with Professor Mike Wotton and Professor David Martell of the Faculty of Forestry at the University of Toronto, we decided that there was no good or simple way to incorporate such a factor for larger firebrands. This is for one main reason - larger firebrands will contain more moisture than smaller ones and hence require more energy to reach a flaming state. Hence, these larger firebrands may land in a smouldering state rather than a flaming one. Letting $\underline{m} = 0.15g$, m the mass of a firebrand upon landing, and $P(sf)$ be the probability of a match igniting, the probability of ignition by a fire brand is then,

$$P(ign) = \begin{cases} (m/\underline{m})P(sf) & \text{if } m < \underline{m} \\ P(sf) & \text{otherwise} \end{cases} \tag{28}$$

5 Analysis of Model

5.1 Qualitative Behaviour

We first observe that our model exhibits the qualitative behaviour that we expect it to based on our modelling predictions. The following plots show the evolution of fire under extreme conditions generated by a pre-existing elliptical spread model without firebrand spotting, and our firespread model with spotting.

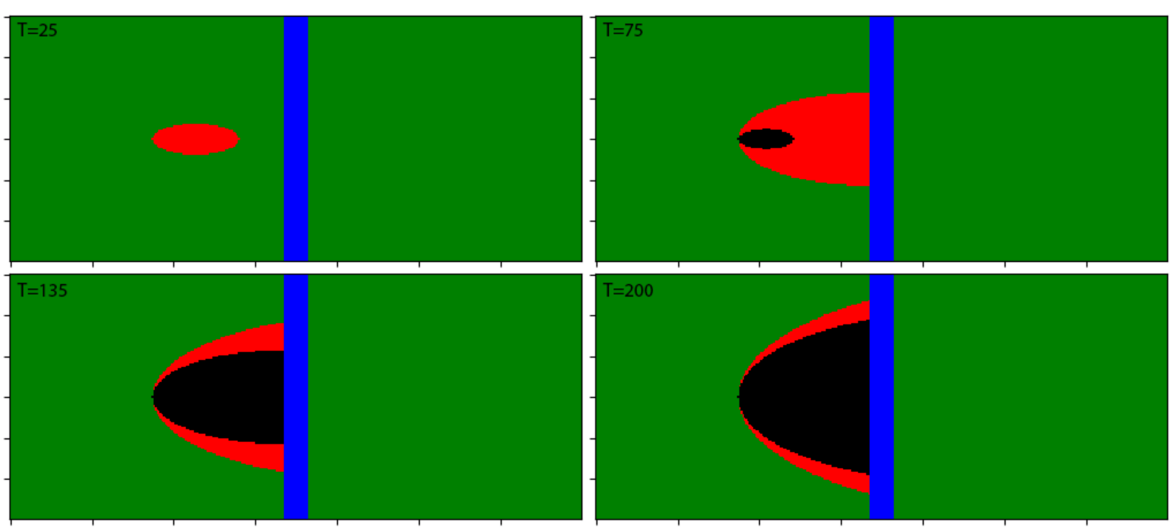


Figure 1: Fire spread under model without spotting

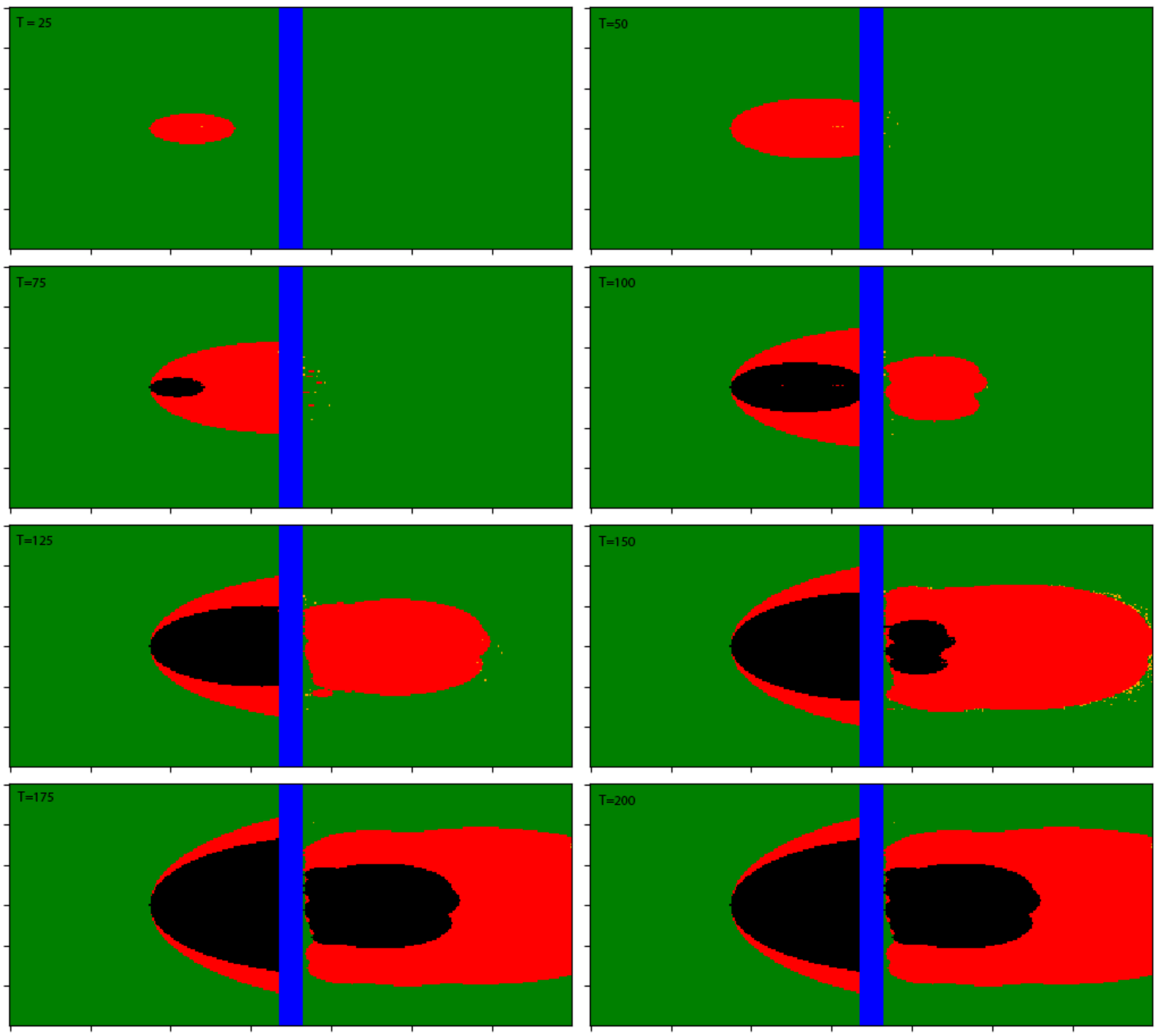


Figure 2: Fire spread under our model with spotting

Note that the green cells represent fuel cells (forest/trees), blue cells represent fuel discontinuties (rivers), red cells represent areas that are currently burning, and black cells represent burn out areas. Animations have been submitted with the report for these plots.

We observe that under sufficiently intense conditions such that firebrand spotting occurs, our model evolves differently from pre-existing models, with the fire being able to cross fuel-discontinuities and continue spreading (we can also observe that it spreads slightly faster downwind, even without fuel discontinuities, than the pre-existing models). Furthermore, the animations for our model reveal qualitative behaviour of fire-spread that is more in line with reality than pre-existing models: fire doesn't actually grow perfectly elliptically as the elliptical fire spread model predicts. Our model has firebrands being launch downwind ahead of the fire, starting their own little blooms of elliptical fire growth. Most firebrands that do not travel too far just get engulfed by the main elliptical fire spread before igniting and creating a local burning ellipse of its own, but some firebrands do travel far enough and cause ignition, creating somewhat irregular shapes of overlapping ellipsoids. However, despite the slight deviation from perfect ellipsoids, our firebrand spotting model still grows roughly in an elliptical shape, which is in line with experimental results.

Furthermore, our model with firebrand spotting incorporated behaves exactly as the pre-existing elliptical fire-spread model does under moderate conditions. This is in line with our expectations; under moderate conditions, we should not expect firebrand spotting to occur, thus becoming a deterministic model.

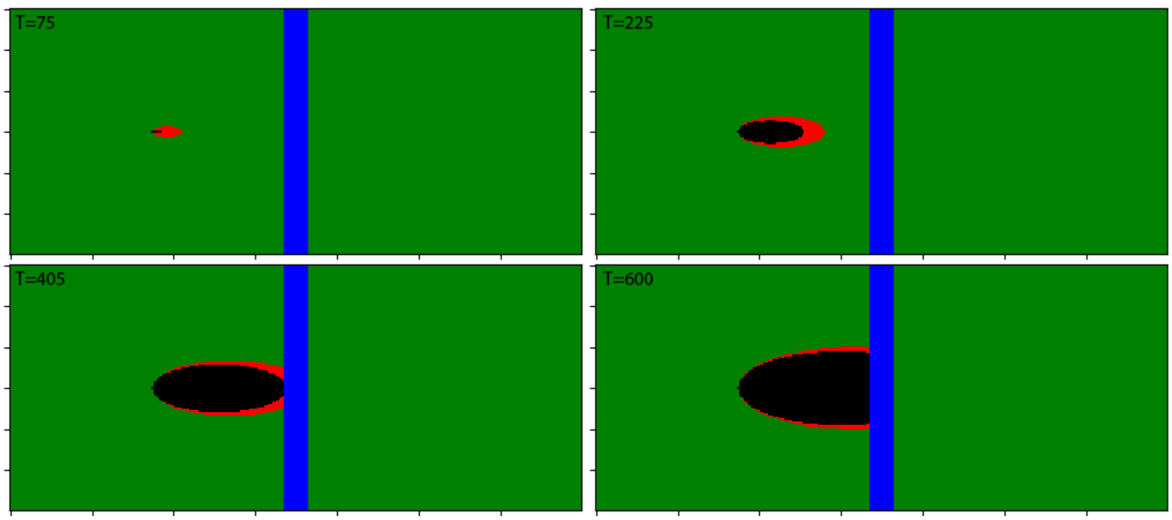


Figure 3: Fire spread model with spotting under moderate conditions

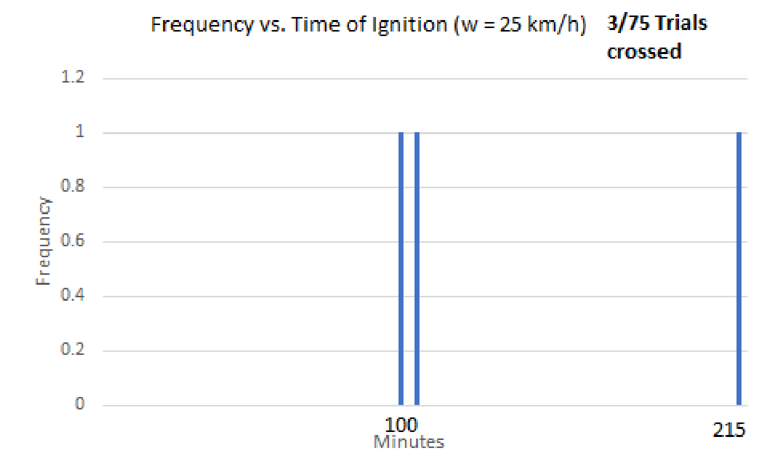
This qualitative behaviour partially answers our primary modelling question: this stochastic model, at the least, predicts accurately the qualitative results expected from firebrand spotting in extreme and moderate weather conditions.

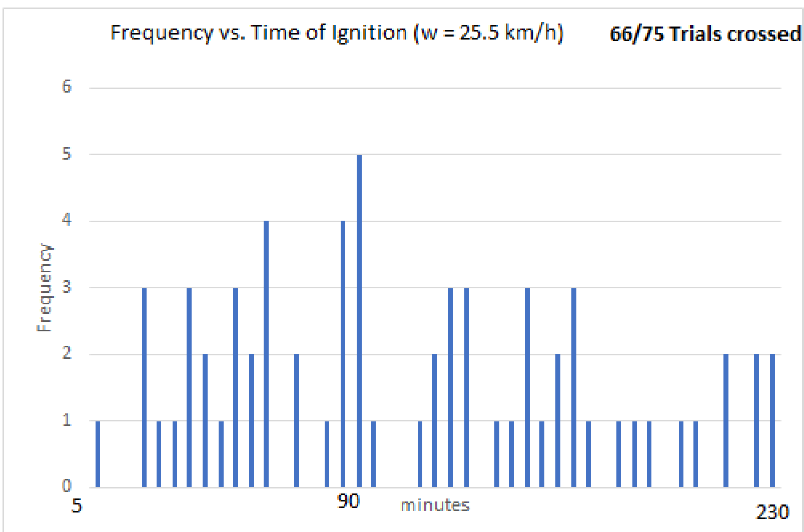
5.2 Fort McMurray Scenario Analysis

The devastating fire that happened at Fort McMurray in 2016 began on the opposite side of the 1.1 km wide Athabasca River, during the night time when wind speeds were between 20-25 km/h, with gusts of wind between 30 - 40 km/h. Ignition on the Fort McMurray side was due to a firebrand, and so this scenario fits perfectly to do an analysis of our model, and see if we can observe fire spotting across the river in this setting. The relevant fire conditions were known prior to the fire **? ?**; in particular, $FFMC = 96$, $DMC = 45$, $DC = 367$, and $FMC = 85$. Since the wind speeds varied, we tested with the 20-25 km/h range, along with wind speeds of 25.5 km/h and 26 km/h, to account for possible gusts of wind. We let the simulations run for 4 hours (simulated time), and for 75 simulations per each wind speed. To reduce simulation time, we started the fires 100 metres from the river.

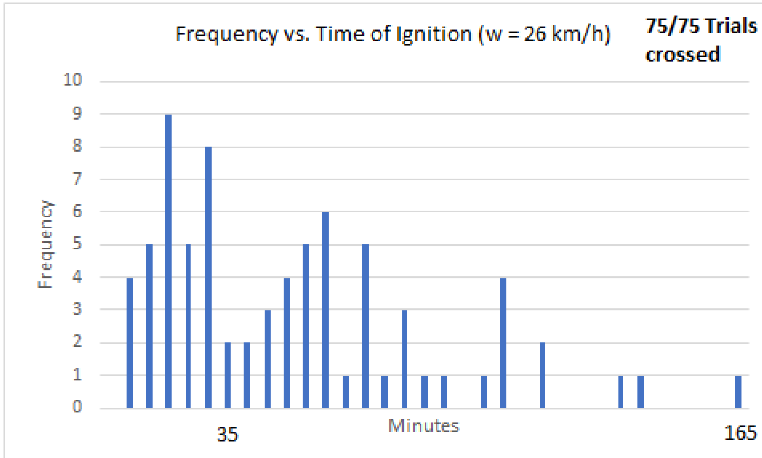
5.2.1 Results

For wind speeds below 25 km/h, during no trials did the fire cross the river. However, for wind speeds at and above 25 km/h, we did see crossing. Below are graphs for each wind speed displaying the time for which crossing occurred, and the frequency of crossing during that time.

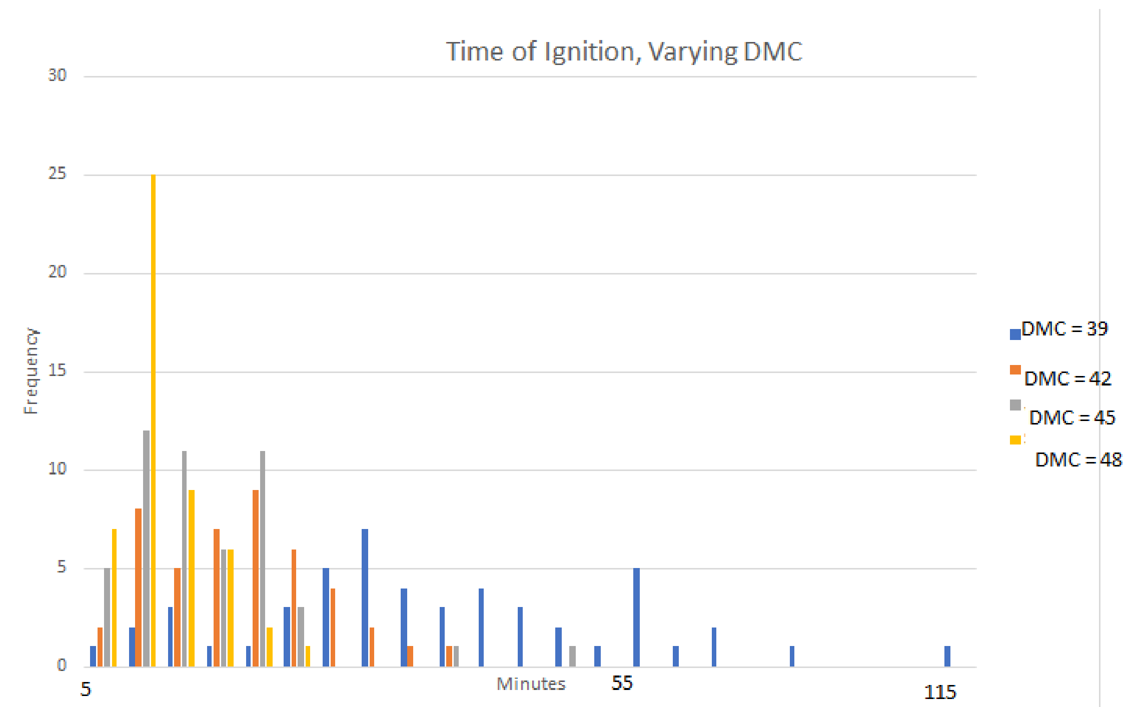




The first observation to make is that, as the wind speed increases, the firebrands are crossing the river more quickly, which is certainly desired behaviour. Second, our model is very sensitive to wind speed. This makes sense, as wind speed is one of the most important parameters in our firebrand model, showing up in each of the three models we are combining; see 5.4 for a further analysis of sensitivity to wind speed. As noted above, high gusts of wind were recorded, which would affect the average wind speed of a firebrand. Therefore, our simulations do predict spotting across this river to be a very real threat, though we would like to quantify this a bit better. We first check whether our simulation results are sensitive to the fuel parameters that were recorded. Below are the

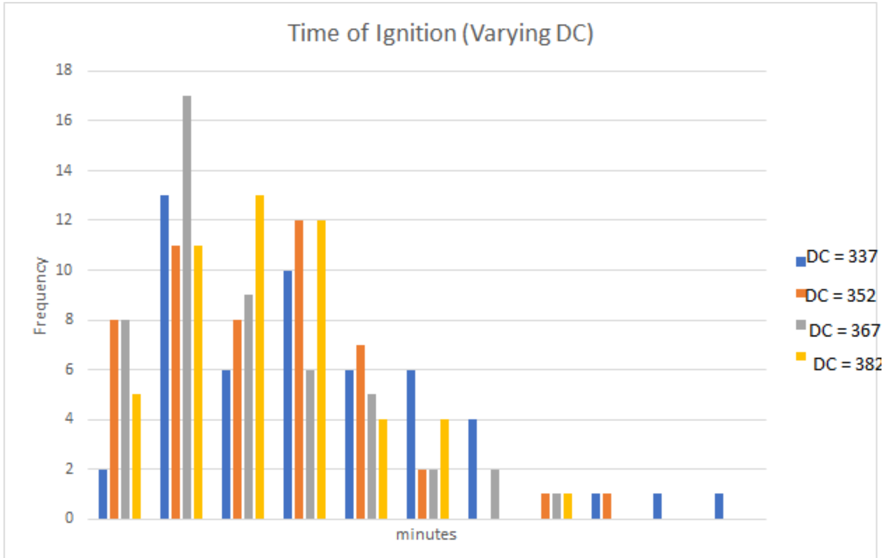


results of varying DMC, DC, and FFMC, and recording the same information as we did with varying wind speed, but now fixing wind speed to be 27 km/h (so that we know the wind speed will be high enough to not be the deciding factor in our trials). We tried four variations per each fuel parameter, with 50 simulations per each specific set of values; see the below figures on pages 17 and 18.



In all trials for DMC and DC, we observed the fire crossing the river. DMC is more sensitive to DC, which makes sense, since DMC measures the moisture content a ground layer above DC. Similarily, FFMC is more sensitive than DMC, as it quantifies surface level moisture:

In the two values of FFMC visible on the graph, the fire crossed the river in every trial. However, for FFMC = 90 and FFMC = 93, the fire did not cross in any of the trials.



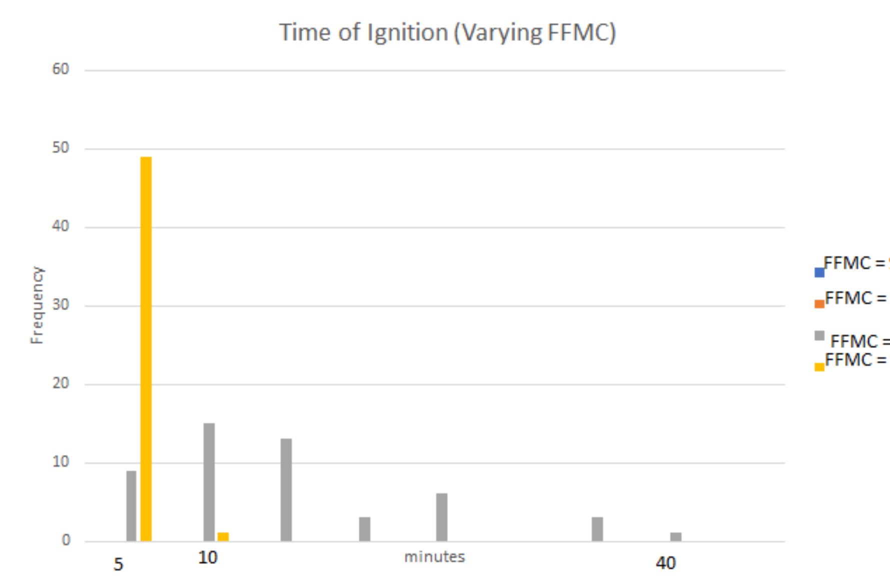
These values are still within the extreme fire hazard range for FFMC, which puts it into perspective how dangerous the fire situation was at Fort McMurray at the time.
To make sure our simulations weren't just predicting spotting across the river for these very extreme cases of Fort McMurray, we ran nominal values for what is considered an extreme fire hazard, as well as a high and moderate fire hazard, with an increased wind at 30 km/h, and same river width (1.1 km). In particular, we chose values in the middle range of each box shown in the chart below:

	FFMC	DMC	DC
moderate	81-86	16-30	141-240
high	87-90	31-50	241-340
extreme	91+	51+	340+

In the moderate and high range, we did not detect the fire crossing, even after 25 trials each. This makes sense, as the river is quite wide, so only the lightest firebrands (and consequently the least likely to start a fire) will make it across. Using the extreme parameters, 20/25 times the fire crossed, at an average time of 131 minutes (starting the fire 100 metres away from the river)

5.2.2 Addressing the Probability Threshold

To see why we don't have spotting below wind speeds of 25 km/h, we have to analyze an assumption we made in our model. Specifically, we said that if a fire brand lands on a fuel cell, and has probability of igniting less than 0.5, we said it did not start a fire, while if it had probability larger than 0.5 we said it did start a fire. We choose this cut off value of 0.5 as the original probability of ignition model by **?** was done by performing a logistic regression and the only meaningful way to interpret the probabilities from this type of regression is to set some threshold to classify events. A natural choice is 0.5. Plotting the average number of firebrands crossing the river above varying thresholds of probability, at wind speeds of 25 km/h and 22.5 km/h, we see why at wind speed 25 km/h there is a



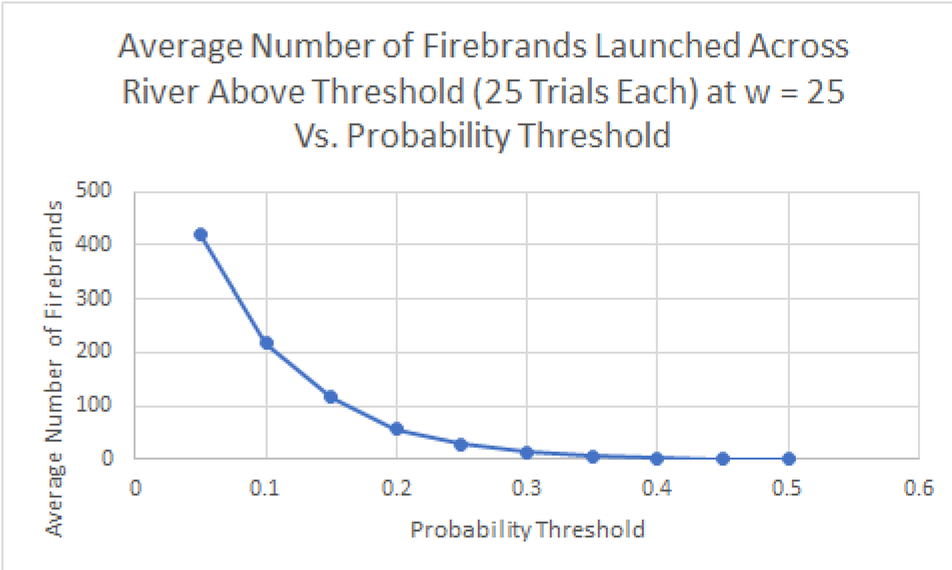
very low chance of spotting (see the two graphs below).

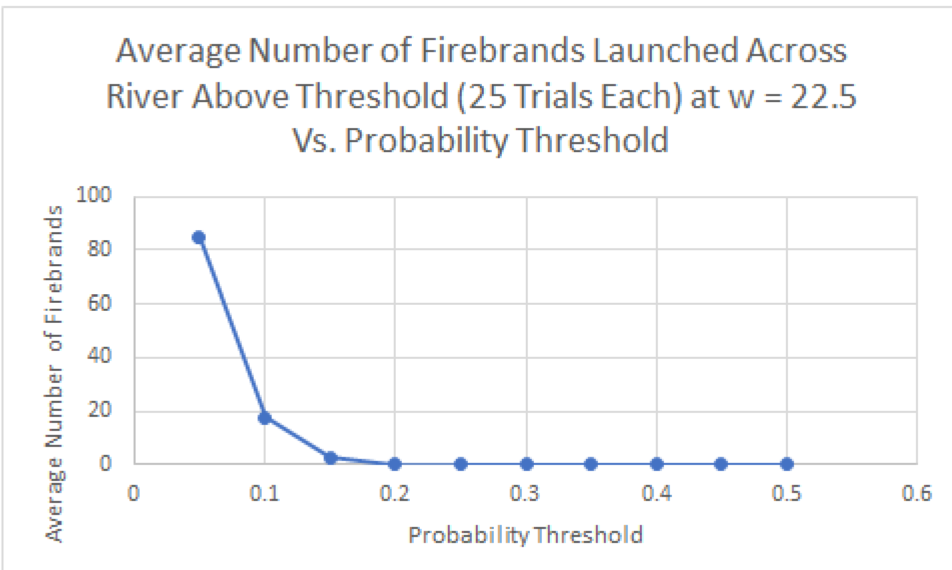
The average number of firebrands crossing the river at a wind speed of 25 km/h and a probability larger than 0.5 is close to zero (about 0.6), and as the threshold decreases, the average number of firebrands crossing increases geometrically. Similarly for the wind speed of 22.5 km/h, though it is very clear that no firebrands above the threshold of 0.5 are crossing. Therefore, our assumption is making our model deterministic at lower wind speeds, but we would like to assign meaningful probabilities to answer the auxiliary question 2i), so some tweaking of our model is required.

5.2.3 Changing our Mass Scaling Assumption

As noted in 4.5.3, the assumption of scaling masses lower than the weight of a match was slightly controversial within the forestry department, and since this makes the outputs in our probability of ignition model not actual probabilities, this assumption goes against our current goal of making this existing model a probabilistic model. We replace this assumption by only assigning nonzero probabilities to firebrands closely resembling matches igniting, for which the distribution in the probability of ignition model comes from. We remark that this assumption is less of an issue for wider rivers, since the firebrands we will discount (burning needles and leaves) will not be capable of travelling great distances. The above table is from a firebrand data collection **?**.

Since the average weight of a twig firebrand without pan screening is 0.03 grams, this seems like a natural mass cut off value for considering firebrands with a non-zero probability of ignition. Implementing this into our model, we are then able to assign a probability, for each simulation, for a single firebrand to cross the river and ignite during the simulation time. Using the Fort McMurray parameters, we immediately observed that our simulations gave us either a probability of 0 or a probability of close to one for

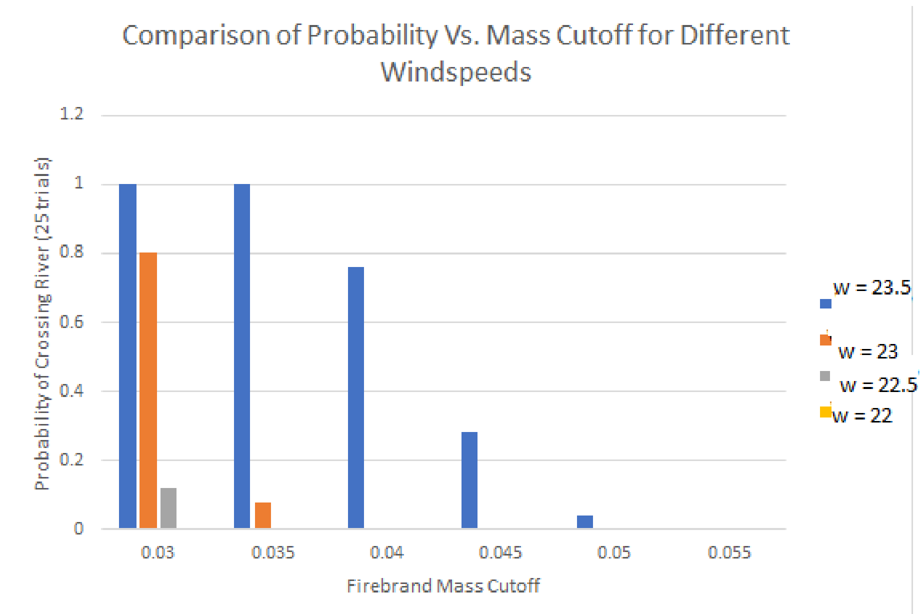




	Quantity		Percent Composition (%)		Total Weight (g)		Percent of Weight (%)		Avg Weight per Firebrand (g)	
Firebrand	Pan w/o Screen	Pan with Screen	Pan w/o Screen	Pan with Screen	Pan w/o Screen	Pan with Screen	Pan w/o Screen	Pan with Screen	Pan w/o Screen	Pan with Screen
Needles	1748	1166	93.9	93.5	10.6	8.7	78.5	59.6	0.01	0.01
Leaves	43	23	2.3	1.8	0.1	0.7	0.9	4.9	0.003	0.03
Twigs	16	7	0.9	0.6	0.5	1.2	4.0	8.0	0.03	0.17
Bark	55	51	3.0	4.1	2.3	4.0	16.6	27.5	0.04	0.08
Total	1862	1247	100.0	100.0	13.5	14.6	100.0	100.0	0.01	0.01

Note: “w/o” denotes “without”

the ignition of a firebrand during 4 hours of simulation time. This is because the forest fire conditions are so extreme, that any firebrand above this weight threshold will start a fire; moreover, spotting distance is a random process in our model, which explains why in some trials we did not get firebrands crossing the river that were able to start a fire. We will use this fact that the probability is either zero or close to one to our advantage, as it allows us to assign a probability of a fire crossing the river to be the average over our outcomes. For less extreme parameters, we do not expect this dichotomy to occur, so to obtain more meaningful simulations, it may be necessary to increase fire brand rate, or the length of the simulation. Below is a graph of our probabilities obtained, varying wind speed and the mass cut off value:



It seems, to be on the safe side of over-predicting, that 0.03g is the better of the choices for the mass cutoff. This probabilistic model gives fairly high probabilities for a fire starting across the river using Fort McMurray parameters and the associated wind range, which is more in line with the "perfect storm" fire conditions present at the time. Moreover, this model does not over-predict, due to the probability for spotting to occur dropping drastically once wind speed is reduced below 22 km/h. This tweaked model is still very sensitive to wind speed, which may be unavoidable, due to its primary role in many components of this model.

5.3 Robustness of Wind Speed Models

The spotting model by **?** uses a logarithmic wind speed profile to describe how the wind speed above the canopy changes as a function of height. This function of lofted height is what is used to describe the horizontal position of the falling firebrand. Another commonly used wind profile is the exponential wind profile **?**,

$$w(z) = w_H \left(\frac{z(t)}{H}\right)^{\beta} \tag{29}$$

Where w_H is the wind speed at canopy height, $z(t)$ the height above canopy at time t, and β is a parameter. The parameter beta is recommended by **?** to be $\beta = 1/7$. Now using numerical integration to solve

$$\frac{dz}{dt} = w_H \left(\frac{z(t)}{H} \right)^{\beta} \tag{30}$$

with the same initial conditions as in (4) we ran the spotting model by **?** in isolation with both wind speed profiles for different values of w_H. For $w_H = 20km/h$ we get the following histograms:

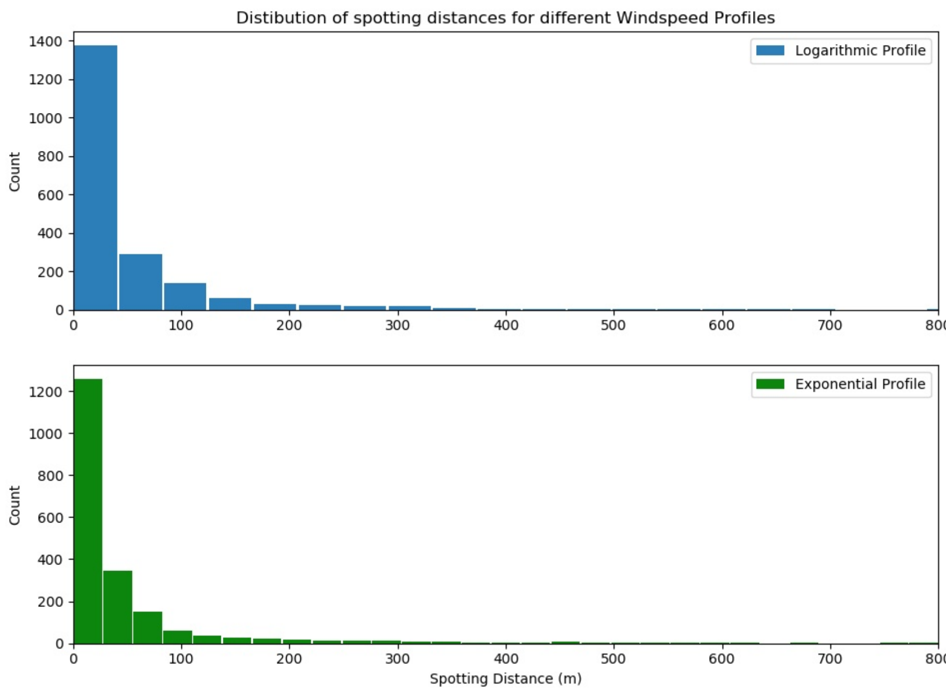


Figure 4: Spotting distance distribution for 2000 iterations at 20km/h and extreme fuel conditions

We see that for spotting distances under $300m$ that both wind speed profiles have for all intents and purposes the same number of firebrands spotted between 0 and 300 meters. Further we see that up to 700 metres both histograms are fairly consistent with one another. Now if we change the wind speed to $30km/h$ and run the same script we get the following histogram:

Overall we see that the histograms are very similar for the two different wind speed profiles and wind speeds. From this, we can conclude that the spotting model is fairly robust with respect to wind speed profiles.

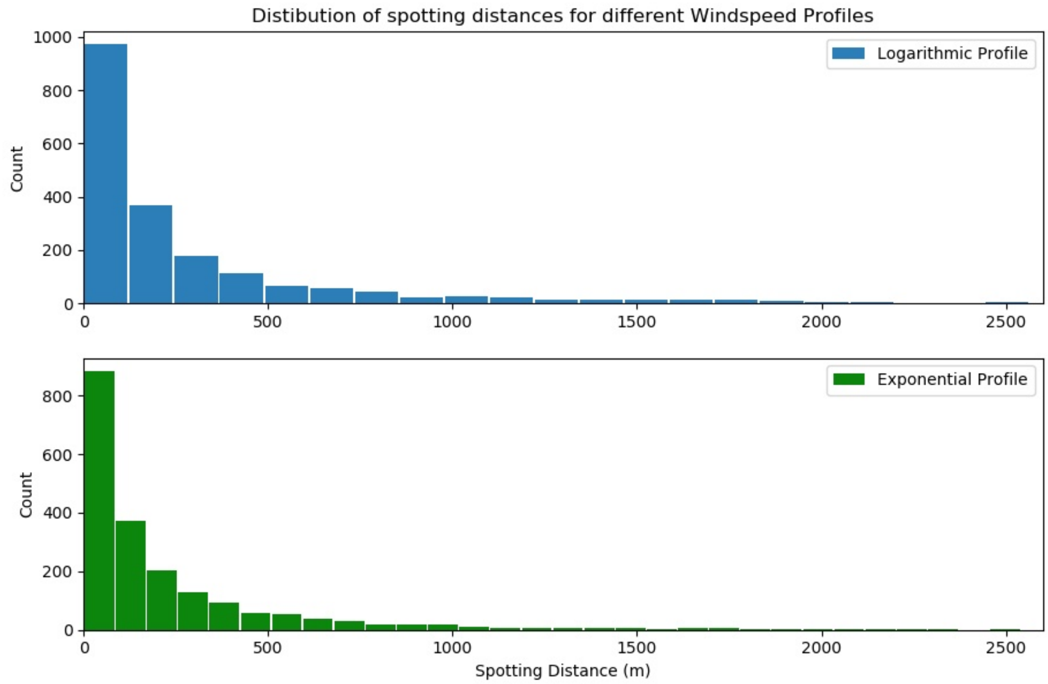


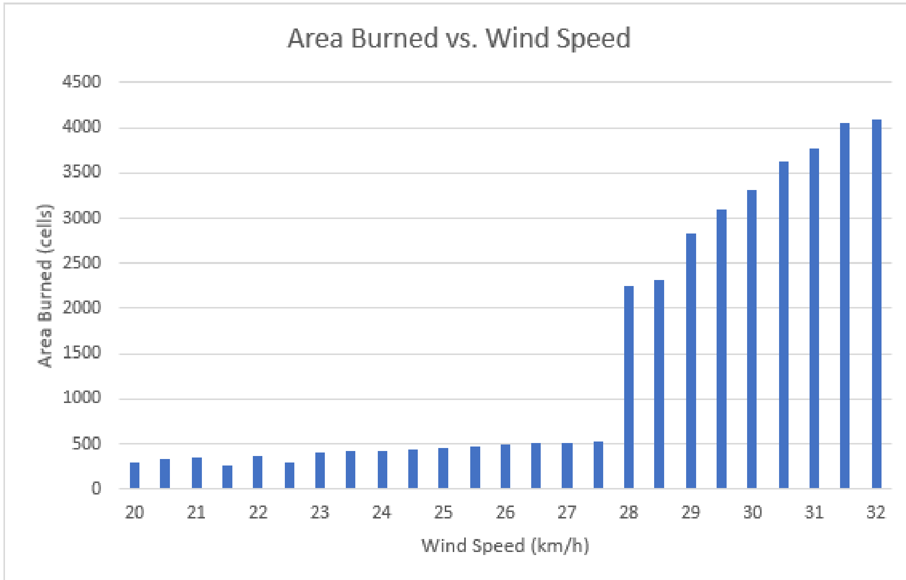
Figure 5: Spotting distance distribution for 2000 iterations at 30km/h and extreme fuel conditions

5.4 Sensitivity Analysis of Parameters

There were some obstacles in conducting sensitivity analysis of parameters, due to do the random aspects of our model and computational limitations. Since any output that we wish to test change in by changing the parameters is a random variable, there is a natural variance in the results that must be taken into account when testing for sensitivity. Therefore, we needed to run multiple simulations and then run analysis on the means/approximate expected values; however, since there's is quite a bit of variance in data given by the simulations, a fairly large number of simulations were required for meaningful analysis.

5.4.1 Wind Speed

As previously mentioned, due the way our model is designed, it should be very sensitive to changes in wind speed, as each individual 'components' of the model that we've combined are highly dependent wind speed. We ran simulations in a fairly high fire weather conditions (enough for spotting), running approximately 50 simulations for each varied parameter value to take an average. Indeed, as we expect, area burned is proportional to wind speed



The above plot also indicates a certain behaviour from our model: for wind speeds below 28km/h, we see that the area burned is significantly lower than that of directly higher values. This is because at these wind speeds, firebrand spotting did not occur; this may be due to the probability threshold as mentioned in section 5.2.2 forcing deterministic behaviour at low wind speeds (although the the behaviour occurs at different numerical values due to different weather conditions). We can actually see that there's a 'discontinuity' around wind speed of 27.856 km/h

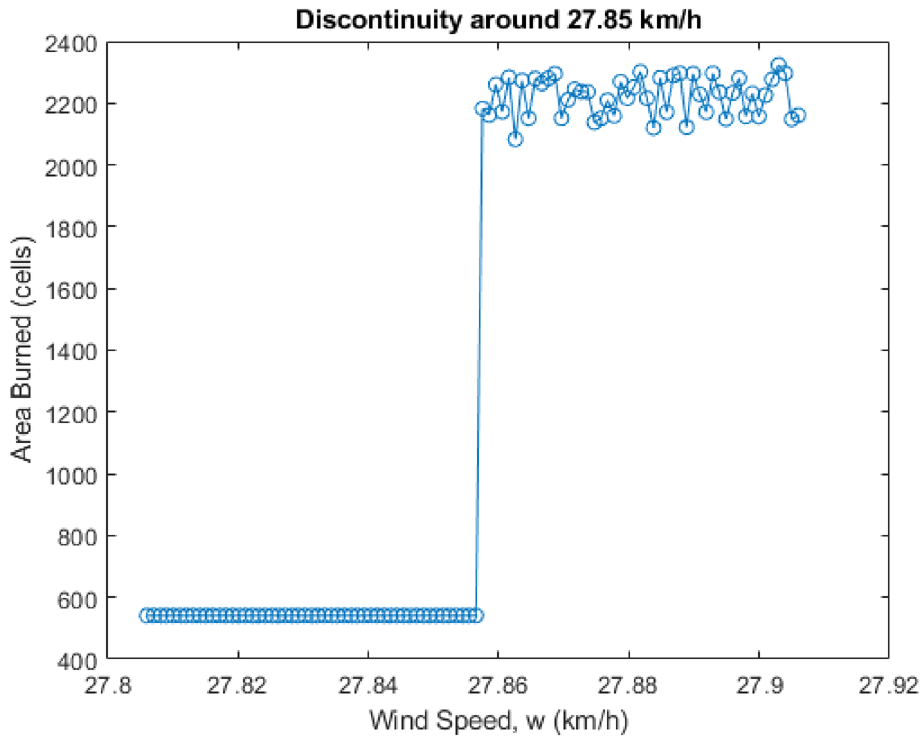
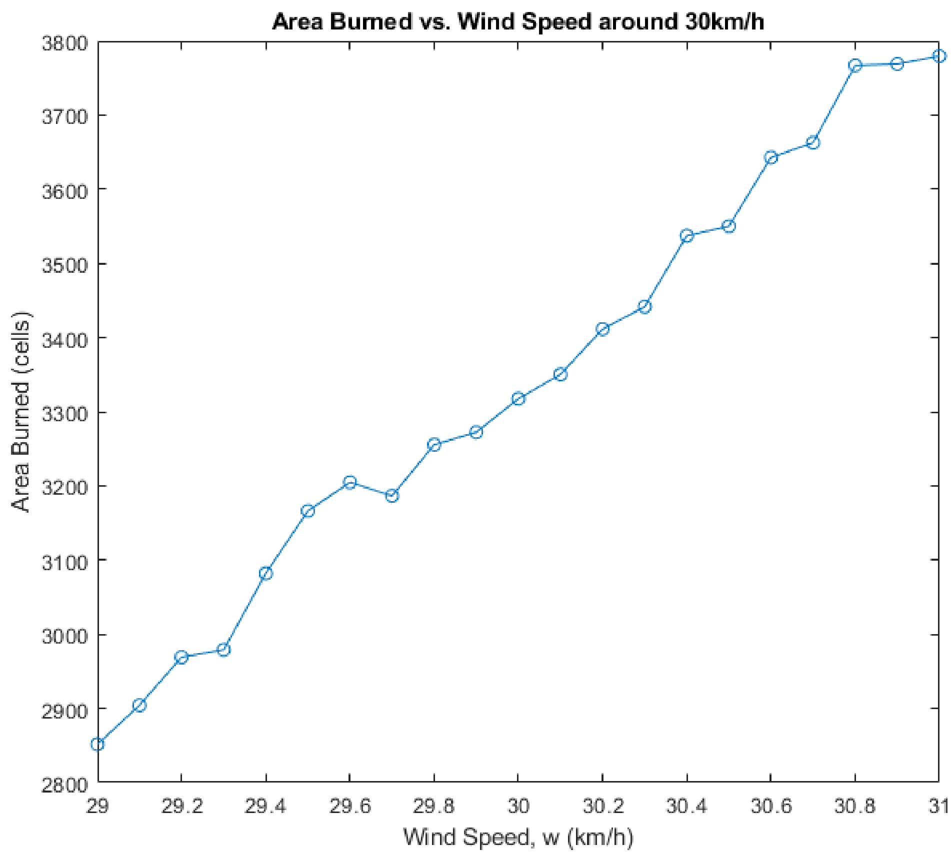


Figure 6: Simulation 100 Wind Speed values around 27.856

This implies that area burned is extremely sensitive with respect to wind speed around these values - the sensitivity at these values is unnaturally high. This kind of discontinuity is unnatural, and as previously stated, perhaps a fully probabilistic model of firebrand ignition instead of the threshold implementation as given in our model here may be more appropriate. Some of this behaviour can be explained by the firebrand spotting model; wind speeds must be high enough for the lofted height of the firebrand to rise above the crown of the forest. However, even so, we should expect a steep decline, not a discontinuity. Also, large amount of the simulations done in this report were conducted using a constant wind speed of 30 km/h (where it's better behaved), so we consider the sensitivity around 30 (plot on the next page).



With wind speeds ranging from 29 to 31 with $\triangle = 0.1$, running 50 simulations each, we compute the sensitivity area burned with respect to wind speed to be 389.5, which is very high. This is in line with our expectations though - our model, as it is, should be very sensitive with respect to wind speed. Going forward, a variable wind speed model may be able to address these issues.

5.5 Application - solving an optimization problem using our model

Due to the costly nature of forest fires, we are interested in whether we can stop the spread of the fire to a certain extent. In practice, it's possible to create fuel discontinuities in forests (e.g. by clearing the land using various means, etc.) and theoretically, when we add another layer/line of discontinuities beside a pre-existing river, these should slow down or stop the spread of fire. A distinct advantage of this firespread model versus pre-existing models is that the stochastic nature of the firebrand spotting model gives us a probabilistic answer to whether fire will cross a fuel discontinuity. Hence, we can run simulations to compute an expected value for fire spread under certain conditions.

Suppose we're working in a 1000m by 1500m grid, with a river (fuel discontinuity) that is 520m wide, with a forest starting on the west side of the river; using parameters ffmc = 90, dmc = 75, and dc = 340 with a fire brand rate of $\frac{1}{15}$ and timsteps of 15 minutes for a 60 minute simulation, the conditions are severe enough that the fire is likely to jump the 520m discontinuity and burn a significant area to the east of the river. Since our grid has cells that are 10m by 10m, each time we create another layer of fuel discontinuities we expand the fuel discontinuity by 10m; furthermore, due to the fact that our grid is homogeneous with equally flammable fuel cells where there isn't a discontinuity, it's obvious that just creating one cell of fuel discontinuity is rather ineffective at preventing firebrand spotting across the river. Instead, we assume that each time we create fuel discontinuities, we create them in layers, i.e. we turn the entire vertical strip of cells into fuel discontinuities. Suppose it costs \$50 to turn a single fuel cell into a fuel discontinuity, and that each cell has a value of \$600. Let L be the number of layers of fuel discontinuities we add, and suppose that the cost of adding another layer $C(L)$ grows quadratically

$$C(L) = 10000L^2 + 65000L$$

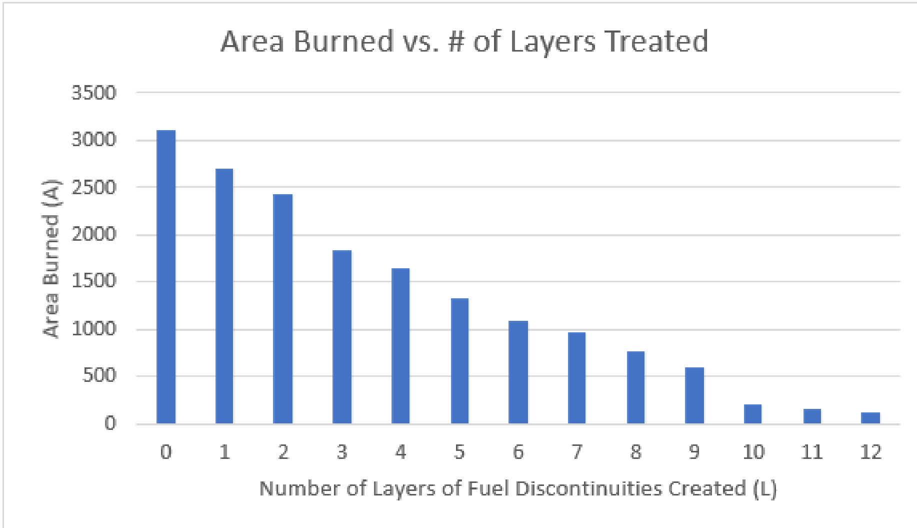
The assumption of quadratic growth seems appropriate. The cost of adding additional fuel layers cannot possibly grow linearly, as each additional layer we need to add implies additional manpower and machinery, which is limited; at a certain point, due to limitations, the cost of obtaining additional resources required to create fuel discontinuities quickly enough will increase, perhaps approximately quadratically. We note that the coefficient of the qudratic term has sensitivity L^2, which is not terribly sensitive as our values of L are relatively small, but it should be noted that this coefficient was somewhat 'cooked up' as a reasonable estimate, and not based on any data (as we were unable to find relevant data). We consider the robustness of this assumption below.

Furthermore, we have a random variable $A(L)$, that is the number of cells burned to the east of the river. Let $T(L)$ be the expected loss in dollars, then

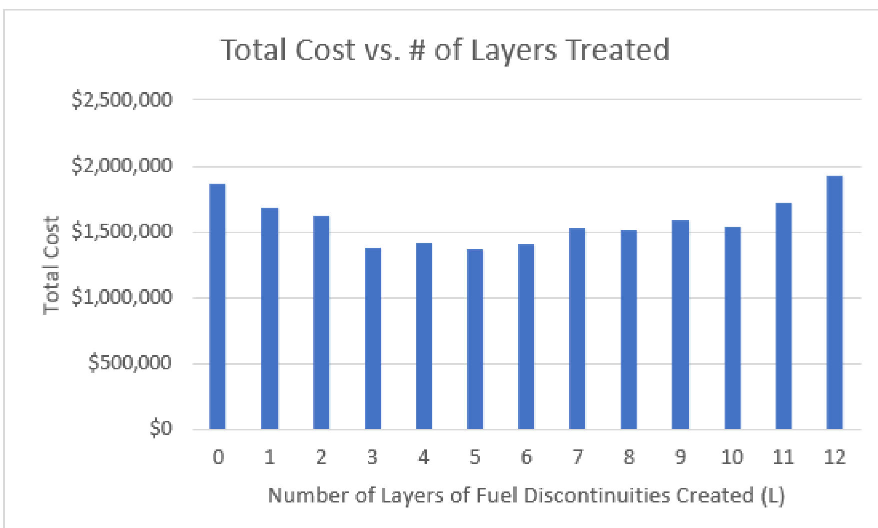
$$T(L) = 600A(L) + C(L) = 600A(L) + 10000L^2 + 65000L$$

Running approximately $n = 300$ simulations and using the above model. The below chart summarizes the results

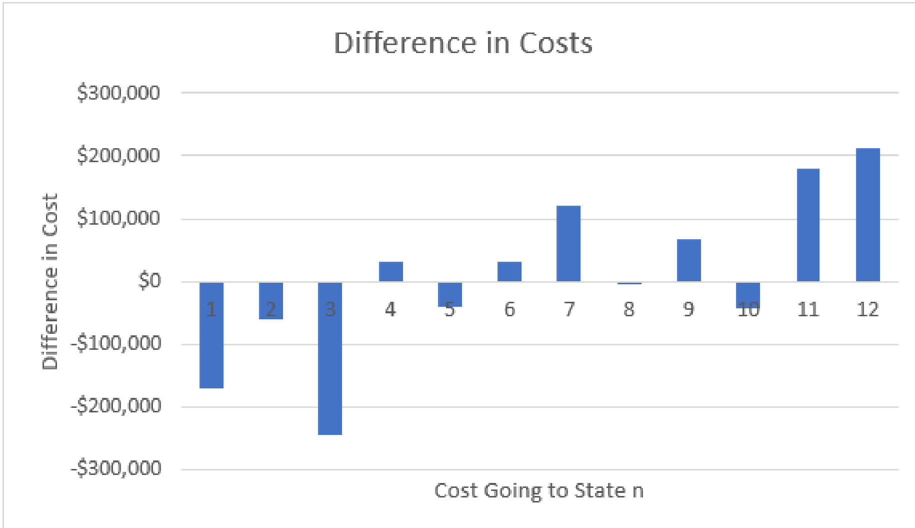
Length of Discontinuity (m)	L	Area Burned, A(L) (a)	Total Loss, T	Difference
520	0	3102.28	\$1,861,368	
530	1	2692.86	\$1,690,714	-\$170,654
540	2	2432.80	\$1,629,680	-\$61,034
550	3	1829.79	\$1,382,874	-\$246,806
560	4	1654.20	\$1,412,520	\$29,646
570	5	1328.39	\$1,372,034	-\$40,486
580	6	1089.20	\$1,403,520	\$31,486
590	7	965.40	\$1,524,240	\$120,720
600	8	765.33	\$1,519,198	-\$5,042
600	9	594.25	\$1,586,550	\$67,352
620	10	205.51	\$1,543,306	-\$43,244
630	11	154.13	\$1,722,478	\$179,172
640	12	123.33	\$1,933,998	\$211,520



As expected, we see that adding additional layers of fuel discontinuities decreases the expected value of the area burned. Furthermore, using this approach, it's fairly easy to answer the question: for a threshold value τ, how large does L need to be such that $E(A(L)) < \tau$. For example, if $\tau = 1000$, our simulations tell us that we would need to create to create 7 additional layers of fuel discontinuities.

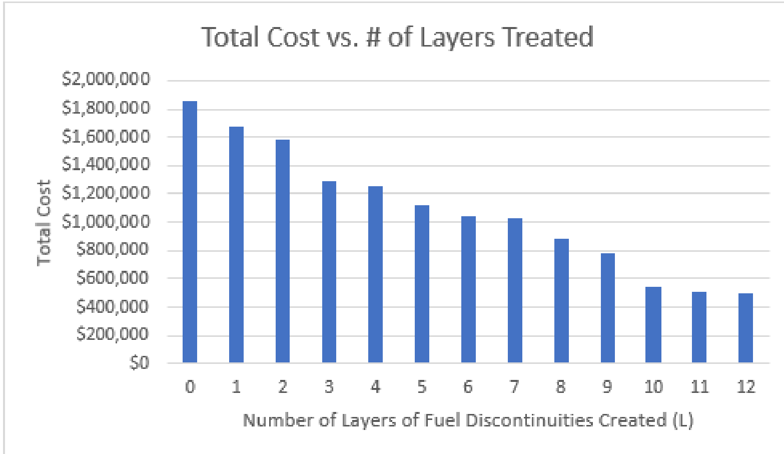


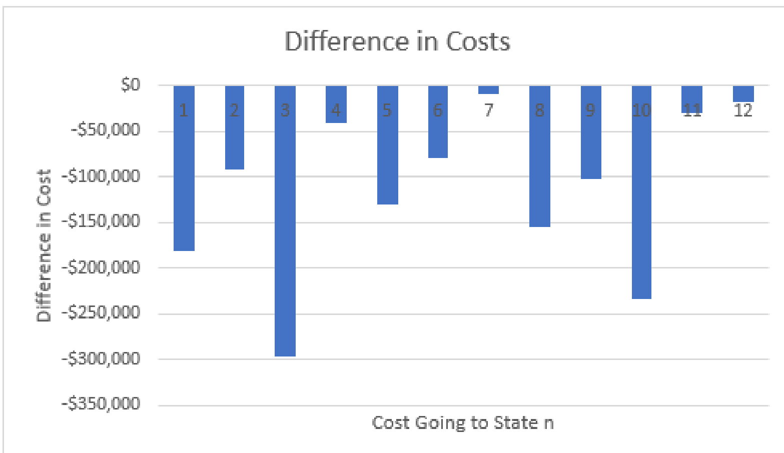
If our objective is to minimize loss, then our simulations indicate that $L = 5$ is the optimal solution, i.e. by creating 5 additional layers of fuel discontinuities, we minimize our expected monetary loss. In general, creating additional layers after 5 increases our project losses, as the cost of creating the layers outweighs the decrease in area burned.



In practical terms, if one's objective is forest fire management, a useful figure to look at may be the change in projected loss from additional layers. Due to resource limitations, it may not be viable to create 5 additional layers of fuel discontinuities, especially if there are time limitations. In this case, instead of searching for a global minimum, finding local minimums within viable ranges is useful. For example, if there's enough time and resources to create 4 additional layers of fuel discontinuities, but not enough to create 5 layers, then our model indicates that it's actually not optimal to create the 4th layer of fuel discontinuities - we're better off if we just create 3, even though we're still decreasing the total area burned. This is a desired attribute caused by our assumption of quadratic growth of cost; there's diminishing returns on creating more fuel discontinuities after a certain threshold, as we would expect. This behaviour does not exist in a linear cost model, as we can see in the results below:

L	Area Burned, A(L) (a)	Total Loss, T	Difference
0	3102.28	$1,861,368	
1	2692.86	$1,680,714	-$180,654
2	2432.80	$1,589,680	-$91,034
3	1829.79	$1,292,874	-$296,806
4	1654.20	$1,252,520	-$40,354
5	1328.39	$1,122,034	-$130,486
6	1089.20	$1,043,520	-$78,514
7	965.40	$1,034,240	-$9,280
8	765.33	$879,198	-$155,042
9	594.25	$776,550	-$102,648
10	205.51	$543,306	-$233,244
11	154.13	$512,478	-$30,828
12	123.33	$493,998	-$18,480





As described above, the assumption of linear growth in the cost of treating land leads to undesired behaviour (we should always create more fuel discontinuities), hence the assumption of quadratic growth seems appropriate.

We should note that the conclusions that can be made from this optimization model is rather weak, due to various assumptions we've made about our model; most importantly, variable wind directions and wind speeds would greatly influence the results of our model, as indicated by the sensitivity of our model to wind speeds. Furthermore, the variance in data and limited computing time for larger samples of simulations decrease the reliability of this result. The intent of this example is to illustrate some of the predictive capabilities of this probabilistic model of firebrand spotting.

6 Response to Questions From Presentation

The main comment we got about or presentation was that we didn't show enough of our model. We hope that section 1-4 show why we decided it wasn't imperative to have this in our presentation. So let's address some of the other questions we got.

1. *Question:* Is it possible to include time varying wind speed?
   Response: Yes it is possible to include this in the model as all the models used have

the ability to incorporate this fairly readily. It would require some set-up of how to actually vary the wind speeds (either randomly, using hourly airport wind speeds or some other method). We have submitted code along with this report, where we have implemented variable wind speeds in our model.

2. *Question:* How does topology affect your model?
Response: We did not include changes in topology in our model. However, the elliptic growth model has an extension that can account for changes in topology. This involves knowing much more information about the landscape as the rate of spread up a slope is different than the rate of spread on flat land. This is something that with a bit of work could likely be implemented. The spotting model does not account for changes in topology downwind from where a firebrand originates and so is not fit to address this question. To this end, there is a special report by Frank Albini from the late 60's early 70's that only exists as a physical paper as far as I am aware that does include the possibility of change in topology. It is the only spotting model we are aware of that has this feature. As such, this could be a subject for further work.

3. *Question:* Is there a most effective type of natural or non-natural fuel discontinuity?
Response: This is a very hard question to answer. Not all provinces in Canada report the case of different fuel treatments (i.e. clear cutting or prescribed burns). This makes it hard to really give a general type of answer. Rivers and lakes provide the best natural option for fuel discontinuities. Instead of making a fuel discontinuity, it may be possible that raising the moisture in an area by dropping water from air tankers would be more effective on influencing fire behaviour. This is difficult to test as there is no known data on the effectiveness of airtanker water deposits on changing fuel conditions (but is currently an active area of research in Ontario).

4. *Question:* What are the computational limitations of the model?
Response: Our implementation of our model in Python was done in such a way that if a firebrand started a fire, we would assign this cell to be another point source for elliptical growth. As one can imagine, if many firebrands are landing on our fuel cells, this will be creating many new point sources for which the code has to run through and update the elliptical growth for. We have taken steps to reduce this computational time by over-riding a firebrand's ability to be a point source of a fire if the fire engulfs an igniting firebrand before it can be classified as a full-blown fire.

7 Conclusion

7.1 Summary of Results

In this project, we combined three models to see if we could get simulate the fire behaviour associated with intense crown fires. Our principle results are as follows:

- On the whole our model was able to produce qualitative behaviour consistent with what we would expect under extreme fire weather conditions. Furthermore, we observe that under moderate fire weather conditions and in low wind speed conditions, firebrand spotting does not occur in our model and is consistent with behaviour expected from pre-existing models

- We tested the robustness of the spotting model used by running many simulations of the model in isolation with different wind speed profiles. Overall, the two wind-speed profiles qualitatively agreed for short spotting distances but differed slightly at long ranges. The logarithmic wind profile employed typically produced more long range spotting distances than the exponential model.

- We attempted to answer the question "what is the probability of a fire crossing a river?" in the context of Fort McMurray, in an attempt to apply our model, and to validate it to some extent. While the results were okay, determining with high probability a fire occurring when wind speeds are past 25 km/h, they left something to be desired, as the reported wind speeds at the time before crossing close to Fort McMurray were between 20-25 km/h. We noticed that our existing models use of a probability threshold restricted our model's usefulness in answering this question below 25 km/h wind speed, so we attempted to revise our model by making it more probabilistic. While there still needs to be more revision on this tweaked model, our initial results are promising, obtaining probabilities of 0.99, 0.8, and 0.12 for wind speeds 23.5, 23, and 22.5, thus giving us a high probability of crossing within the actual wind speed range.

- We also found that our model, under very ideal conditions, is able to generate useful predictions for forest fire management. Applying our model to a optimization problem, we were able to answer the question "how many more layers of fuel discontinuities must be created to bring the expected value of the area burned below a certain threshold?", and "what is the optimal number of additional layers of fuel discontinuities to minimize projected losses?"

- A brief sensitivity analysis showed that the DMC and DC parameters in our model were not sensitive at the Fort McMurray range, while FMC was a little sensitive, partly due to its role in fire ignition (as it quantifies surface level moisture). Wind speed was the most sensitive of all our parameters, which makes sense, due to its central role in the three models we combined.

- A sensitivity analysis of wind speed revealed that as expected, our model is highly dependent and sensitive to wind speed.

7.2 Next Steps

There are a couple obvious ways we could improve/extend our model:

1. Allow for time dependent wind direction and speeds. In our model we used that these were both constant. Changing this would allow for a more accurate representation of how wind behaves.

2. Remove the global homogeneity of the landscape we ran our simulations in. Locally a forest will be fairly homogeneous by classifying it into one of the Canadian Forest Services fuel types (we used C-3 fuels but there are many others). The model will extend readily to this as we would just need to adjust/change the parameters in the three constituent models.

3. Account for changes in topography. This would likely be challenging to implement without much more knowledge or assumptions about our landscape.

4. To address the last question, we could turn this into more of a geographical information systems problem. For example, we could use a digital elevation mapping (DEM) and a land classification mapping of the same area to assign to each pixel in the map an elevation and an initial state (such as forest, roads, river, etc). This addition information may allow to include for more realism in the model. It would also allow for possibly more case study type work to test the predictive capabilities of the model.

5. Many of the other improvements that can be made would really on the collection or creation of more data. For example, the time it takes the different fuel types to burn out under certain conditions, or how long it takes a firebrand to cause an ignition then allow that fire to get up to a fire that the FBP system can model.

6. Based on anecdotal evidence, mountain pine beetle outbreaks have greatly increased the the number, size, and severity of spot fires. The trees that have been killed by these beetles seem to produce (much) larger firebrands than what the experiments by **????** found which in turn is leading to more spot fires. This is a phenomena that is currently unexplainable by any spotting model or any other model as far as we know. In effect, this is something that could be explored.

7. In 5.2, we considered a revised model that we hoped would give us a more satisfactory answer to the question: "What's the probability of fire spotting occurring over a river?". The following are issues with the revised model that still need to be addressed:

   (a) It would be valuable to know how sensitive this probabilistic model is to the other fuel parameters. Simulations can easily be done to determine this, but since this tweaked model does not differ drastically from the original version, it seems unlikely that it will be too sensitive to these parameters, except perhaps FFMC. We simulated FFMC values of 95, 95.5, 96, 96.5, and 97, with 25 simulations each under our normal set-up, along with a wind speed of 23 km/h. We got the probability of the fire crossing for FFMC values below 96.5 to be zero, however at 96.5, and 97 we had 0.96, and 1.00 probability of the fire crossing. Therefore, this tweaked model seems to be more sensitive to FFMC, but it is hard to compare, since we are running a different type of test.

   (b) More simulations need to be run on more normal fire conditions to see how this tweaked model performs, but we believe the trials done using Fort McMurray parameters are a good proof-of-concept.

(c) An issue that may come up is this revised model's performance for smaller rivers, since the light firebrands may be able to travel across short rivers, and, due to our mass cut off, we are neglecting these, which may change results. Again, simulations could be set up to test different mass cutoffs and determine if there is a better one in the case of shorter rivers, or if the cutoff chosen (0.03g) has any effect on the probability of crossing a shorter river.

1 Background

In Canada, forest fire are a landscape-level disturbance that greatly influence forest structure, function, and, in general, the forest environment. On average, it's estimated that $10 - 15$ million hectares of Boreal forest burns globally, with Canadian forest accounting for 2 million hectares of this total on average. It has been estimated that Canadian forest fires release 27 TgCyr^{-1} and in some years this number can be greater than 100 TgC - which is the amount of carbon released by motor vehicles in Canada. In addition, to the distributing amount of carbon released, during years with large areas burned the amount of mercury released by Canadian forest approaches the amount of Mercury released by industrial North American Flannigan et al. (2009). In addition to the environmental concerns, forest fires are incredibly costly. On average Canadian forest fire management agencies spend nearly half a billion dollars in direct fire suppression activities Flannigan et al. (2009).

Typically fires that burn over 200ha are considered to be large fires. In Canada, these large fires account for only about 3% of the fires that occur in a given year but account for nearly 97% of the area burned Stocks et al. (2002). During these large intense forest fires, various sizes of flaming/burning/smouldering wood chips, needles, twigs and other forest material (often called firebrands) are lofted into the atmosphere by the upward draft from the fire. Once lofted these firebrands are carried downwind by the ambient wind where they burn out before landing or make it to the forest floor in either a flaming or smouldering state. Once they land, depending on the recipient fuel conditions, the firebrands can do one of two things - if the recipient fuels have a high moisture content, they can preheat these fuels and dry them out. Or, if the recipient fuels are dry enough, they may be capable of starting a new fire. These new fires are called spot fires and can cause many forest fire management issues. An example of this is Fort McMurray in 2016, where two spotting incidents occurred with the largest of the two being just over 1km KPMG (2017). The fire in this area is considered the most costliest insured natural disaster in Canadian history running a bill of an estimated 3.6 billion dollars and a total estimated cost of around 8.9 billion dollars KPMG (2017).

Despite the management challenges and dangers posed by fire brand spotting, the main fire behaviour prediction system in Canada - the Fire Behaviour Prediction (FBP) system Hirsch (1996) - does not account for spotting events beyond a standard 20m which it implicitly has built into its fire spread model. Moreover, there are spotting models that do exists Albini (1979, 1981b,a, 1983b,a); Albini et al. (2012), Woycheese et al. (1997, 1999); Woycheese (2000), Koo et al. (2012), Wang (2011), Perryman (2009), and Martin and Hillen (2016). However, the majority of the models that exists are built around the idea of finding the *maximum* spotting distance and *not* the likelihood that a spot fire will occur.

2 Modelling Question

Throughout the modelling process we had one overarching question:

Under high/extreme fire weather conditions can we create a stochastic fire spread model that accounts for firebrand spotting?

As we worked to answering this question, a couple of secondary questions arose in a natural way:

(i.) What is the probability that a fire will spot/breach a fuel discontinuity

(ii.) How large of a fuel discontinuity do you need to create to bring the expected value of area burned below a certain value? What is the cost of creating a fuel discontinuity large enough to stop firebrand spotting from causing a fire on the other side of a fuel discontinuity?

3 Modelling Approach

We decided to combine three existing models - the Fire Behaviour Prediction (FBP) systems elliptical fire growth model Hirsch (1996), a fire brand spotting model by Fellini (2018), and a probability of ignition model by Beverly and Wotton (2007). The elliptic growth model is a deterministic model that is based on many years of data collection which indicated that with a constant wind speed direction, fire growth is elliptical in nature. The fire brand spotting model is a physical model developed from physical principles. The probability of ignition model is a statistical model based on fitting curves to large samples of data.

As firebrand spotting is particularly random in nature, we decided to incorporate some of this randomness into the model by Fellini (2018), thus turning the deterministic spotting model into a stochastic one.

All three of the models used and the addition of the stochastic parts we have included will be discussed in much more detail in section 4.

4 Formulation of Model

4.1 Our Modelling Assumptions

We made the following assumptions in our initial analysis:

1. The landscape the fire is burning in is homogeneous - that is, the FFMC, DMC, DC, FMC, fuel type (Mature Jackpine), and other FBP parameters are constant throughout the landscape. We did this for computational ease and because locally a forest will have this consistency.

2. Wind speed is constant at canopy height. Again, this is for computational ease, but the model we have could easily incorporate this into the model if we really wanted to.

3. Wind blows perpendicularly to the fuel discontinuity.

4. A cell that is in a burning state remains burning for 60 minutes. We could not find any good data on this as it really depends on the moisture content and type of trees that are burning.

5. While a cell is burning, if the lofted height of a firebrand is greater than the height of the canopy, then the firebrand can spot.

6. We chose a rate of firebrand generation to be one firebrand per burning cell per every 5 minutes. We based this on work done by Perryman (2009). However, Koo et al. (2012) uses a firebrand generation rate of five firebrands per burning cell per second. This is a very large difference and without any actual data we have no way of really justifying either rate. We used 1 firebrand per 5 minute per burning grid for computational ease.

7. If a firebrand lands and causes an ignition, we put a delay time of 30 minutes before we consider the cell to be in a burning state.

8. Firebrand mass is exponentially distributed with mean $0.35g$. This is based on the work of Manzello et al. (2006, 2007, 2008a,b), where after burning different types of pine trees and combining the data, the papers found that the mass distribution of collected firebrands was 0.35g and approximately exponentially distributed. We also used this because firebrand generation should be a memory less process.

9. The length to radius ratio of the cylindrical firebrand is fixed. This is for mathematical convenience and as far as we are aware there are no models existing models that allow this to vary. We used a fixed ratio of $L/r = 6$ which is the same value used in Koo et al. (2012).

10. The lofted height of a particular fire brand is exponentially distributed. Since the fire plume model doesn't account for any turbulence, and we would expect that firebrands don't always loft at their terminal velocity, we would expect that the majority of firebrands are lofted to a much lower height than their maximum loft-able height. Further, it is clear that this should be a process that is memory less. We choose the λ in the exponential distribution such that $Pr(\text{lofted height} = z_{max}) = 0.01$, i.e. a firebrand is lofted to it's maximum height 1% of the the time.

4.2 Elliptical Fire Growth

Our elliptic fire growth model is largely based on the Fire Behaviour Prediction systems elliptic growth model. This model largely relies on two outputs: Fire line intensity (called

I from here out) and the forward rate of spread (called ROS from here out). This model is largely based on many decades of data collection and fitting curves to this collection of data and justified by a physical understanding of how these different parts of a forest interact to cause forest fires.

4.2.1 Fire Behaviour Prediction Parameters

FFMC	Fine fuel moisture code	CBH	Crown-base Height (m)
DMC	Duff moisture code	FMC	foliar moisture content
DC	Drought moisture code	CFL	Crown fuel Load (kg/m^3)
a_1, a_2, a_3	Rate of Spread constants	b_1, b_2 b_3	Surface fuels consumed constants

Extreme fire weather values to correspond to a FFMC of 91+, DMC of 51+, and a DC of 340+. High fire weather conditions correspond to FFMC$\in \{87, 90\}$, DMC$\in \{31, 50\}$ and a DC$\in \{241, 340\}$ Van Wagner (1987).

4.2.2 Fire Behaviour Prediction system main outputs

Here we outline the calculations needed to compute the rate of spread and fire line intensity. The initial spread index (ISI) is given by the equations:

$$ISI = 0.208 \cdot f_1(w) \cdot f_2(\text{FFMC}) \tag{1}$$

where FFMC is the fine fuel moisture content and w is the wind speed in km/h and we have two auxilary functions f_1 and f_2 given by:

$$f_1 = \exp(0.05039 \cdot w) \tag{2}$$

$$f_2 = 91.9 \cdot \exp(-0.1386 \cdot mc) \cdot \frac{1 + mc^{5.31}}{4.93 \cdot 10^7} \tag{3}$$

$$mc = 147.2 \cdot (101 - \text{FFMC})/(59.5 + \text{FFMC}) \tag{4}$$

Here equation (4) is a conversion from the fine fuel moisture code (FFMC) into a moisture content. Then the equation for fire intensity is given by

$$I = 300 \cdot TFC \cdot ROS \tag{5}$$

Where TFC is the total fuel consumed by the fire in kg/m^2 , ROS the rate of spread in m/min and I the fireline intensity kW/m. The rate of spread is given by the formula

$$ROS = a_1(1 - \exp(-a_2 \cdot ISI))^{a_3} \tag{6}$$

where a_1, a_2, a_3 are rate of spread constants that depend on the fuel type. For Mature Jack-pine (also known as $C - 3$ fuels) we have that $a_1, a_2, a_3 = 110, 0.0444, 3.0$ respectively.

For the total surface fuels consumed (TFC) we need to use calculate the following:

i. Build-up-Index: This is roughly a measure of the amount of fuels that have built up on the forest surface floor.

$$BUI = \begin{cases} \frac{0.8 \cdot DMC \cdot DC}{DMC + 0.4 \cdot DC} & \text{if } DMC < 0.4DC \\ DMC - \frac{1-(0.8 \cdot DC)}{DMC+0.4 \cdot DC} \cdot (0.92 + 0.0114 \cdot DMC)^{1.7} & \text{otherwise} \end{cases} \tag{7}$$

Where DMC is the duff moisture code and DC is the drought moisture code.

ii. Surface fuels consumed (SFC):

$$SFC = b_1(1 - \exp(-b_2 \cdot BUI))^{b_3} \tag{8}$$

where SFC is measured in kg/m^2.

iii. Critical surface fire intensity (CSI). This is the intensity needed for a ground fire to turn into a fire that reaches the crown of the tree.

$$CSI = 0.001 \cdot CBH^{1.5} \cdot (460 + 25.9 \cdot FMC)^{1.5} \tag{9}$$

Where CBH is the canopy-base height and FMC is the foliar moisture content (the moisture in the leaves/needles).

iv. Critical rate of spread (CSI):

$$RSO = \frac{CSI}{300 \cdot SFC} \tag{10}$$

v. Crown fraction burned (CFB):

$$CFB = \begin{cases} 1 - \exp(-0.23 \cdot (ROS - RSO) & \text{if } ROS \geq RSO \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

If the rate of spread is greater than the critical rate of spread the fire will transition to burning the crown of the tree. This influences that amount of fuels available to burn and hence the fire intensity.

vi. Total fuel consumed (TFC):

$$TFC = SFC + CFB \cdot CFL \tag{12}$$

Then using equation (6) and (12) we have that the fireline intensity is

$$I = 300 \cdot TFC \cdot ROS \tag{13}$$

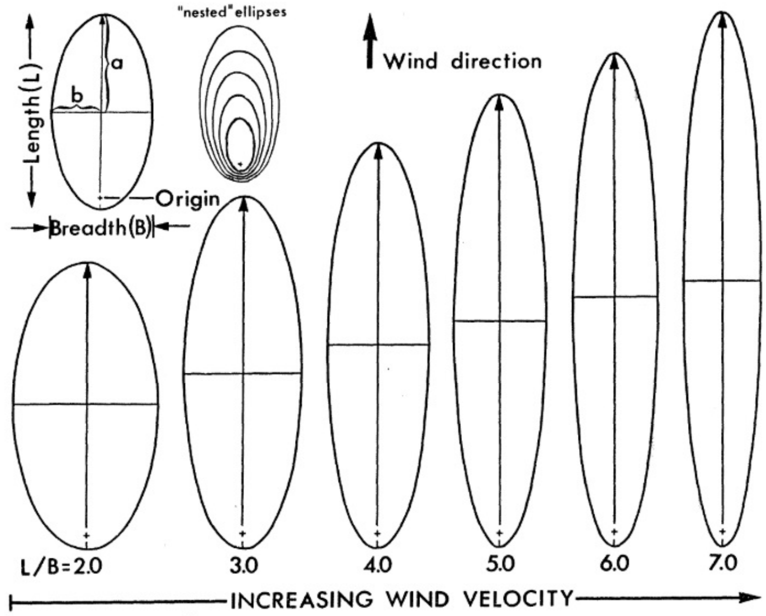
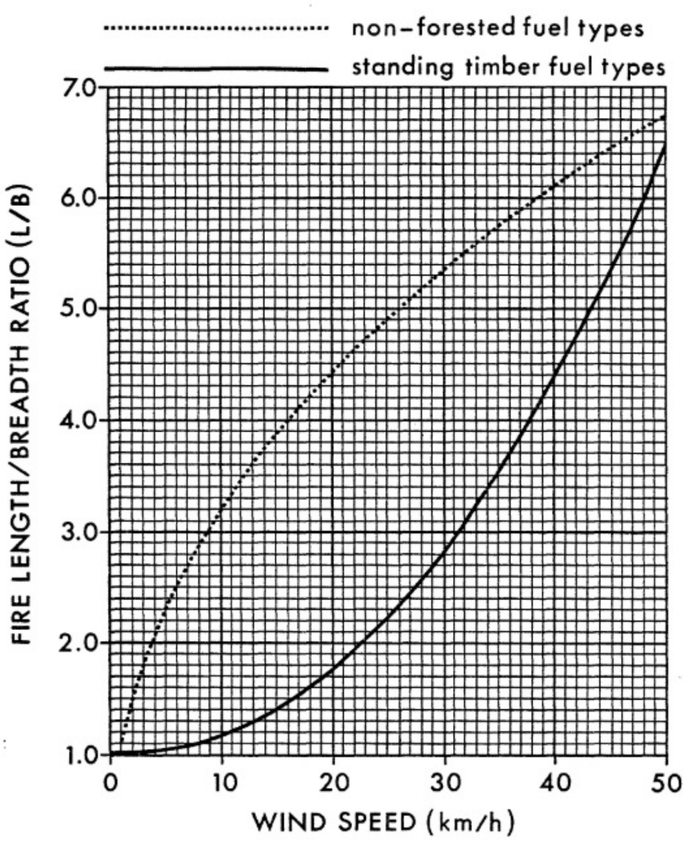


Figure 19. Idealized elliptical shapes of various length-to-breadth (L/B) ratios having the same area for free-burning fires spreading under the influence of increasing wind velocity (L/B = 2.0 to 7.0). Backfire spread assumed to be essentially negligible in all cases.



4.2.3 Elliptic growth model

Now the elliptic fire growth model we employed is a simple model implemented in the FBP system. It describes the growth of a fire as an ellipse, given a wind speed (under the assumption that back burning is negligible). There is a chart they include in the documentation to get the ratio of the length to breadth ratio. See the two figures below which are taken from Hirsch (1996). To implement this model we needed to be able to define a length to use the length/breadth ratio. To accomplish this, we used the forward rate of spread, ROS defined above. Starting from the initial point of the fire(the cell that we ignited) we computed the maximal distance away from this cell at time t as $t \cdot ROS$. Then using the chart below, we were able to use the corresponding length/breadth ratio for the chosen wind speed.

4.3 Firebrand Spotting

4.3.1 Parameters

A_b	cross-sectional area of cylindrical firebrand	T	ambient temperature ($300K$)
a	$(\pi g K)/(2C_d)$	U_H	wind speed at canopy height (m/s)
b	$(\pi g \rho_b r_0)/(C_d \rho_a)$	V_b	volume of cylindrical firebrand
bouy	$g/(\rho_a C_p T)$	Z	height the firebrand leaves the plume (m)
C_d	coefficient of drag (1.2)	z_0	0.1313H
C_p	specific heat of air ($1.0 kJ/kg$)	α	$\sqrt{b - aZ}$
g	acceleration due to gravity $9.81 m/s^2$	β	$(3.64^3 \pi)/(2.45^5)$
C_p	specific heat of air ($1.0 kJ/kg$)	γ	$(9.35^2)/(\pi g H^{2/3})$
g	acceleration due to gravity $9.81 m/s^2$	η	$\beta \gamma^{5/2}$
H	canopy height (m)	ρ_a	density of air ($1.225 kg/m^3$)
I	Fireline Intensity (kW/m)	ρ_b	density of firebrand ($440 kg/m^3$)
K	burning rate constant (0.0064)	t^*	time to reach canopy (seconds)
L	length of firebrand (m)		
r_0	radius of the firebrand when it leaves the plume (m)		

4.3.2 Terminal velocity Assumption

Many authors Albini (1979, 1981a, 1983a,b); Martin and Hillen (2016); Wang (2011); Woycheese et al. (1997, 1999) use the terminal velocity assumption in their analysis. This is a result of work done by Sánchez Tarifa et al. (1965) which showed that firebrands reach their terminal velocity relatively quickly (ie, in a couple of seconds). Koo et al. (2012) compared simulated results obtained using the terminal velocity assumption and not using it. Their results indicate that their firebrand transport model without the terminal velocity assumption were launched further than brands travelling at terminal velocity. Letting W be the weight of the fire brand, D the drag force on the firebrand and v_b the velocity of the firebrand, we achieve terminal velocity, v_t, when there is no more acceleration, i.e.

$W = D$. Writing the mass of the firebrand as $m_b = V_b \rho_b$, where V_b is the volume of the firebrand and ρ_b is the density of the fire brand we solve for the terminal velocity:

$$
\begin{aligned}
W &= m_b g \\
D &= \frac{1}{2} C_d \rho_a v_b^2 A_b \\
\Rightarrow v_t &= \sqrt{\frac{2\rho_b V_b g}{C_d \rho_a A_b}}
\end{aligned}
$$

Based on the work in Manzello et al. (2006, 2007, 2008a,b), we have made the assumption that the firebrands we are modelling are cylindrical in shape. This allows us to write

$$V_b = \pi r^2 L \qquad A_b = 2rL$$

and thus we can simplify the terminal velocity equation to:

$$v_t = \sqrt{\frac{\pi \rho_b g}{\rho_a C_d} r} \tag{14}$$

4.3.3 Maximum Loftable Height

Then using work from Wang (2011) and a fire plume model from Baum and McCaffrey (1989), Fellini (2018) finds that the maximum radius of a firebrand that can be lofted as a function of the fire line intensity I is given by

$$r_{max}(I) = \gamma \left(\frac{C_d \rho_a}{\rho_b} \right) I^{2/3} \tag{15}$$

Setting the terminal velocity equal to the maximum rate of up-draught gas flow in the fireplume and using the assumption from Wang (2011) that the terminal velocity is always less than or equal to this rate, Fellini (2018) found the maximum loftable height of a firebrand of radius r_i for a fire intensity I is

$$z_{max}(r_i, I) = \eta \left[\left(\frac{\rho_a}{\rho_b} \right) \left(\frac{C_d}{r_i} \right) \right]^{3/2} I^{5/3} \tag{16}$$

4.3.4 Burning Rate of a Cylindrical Firebrand

The model by Fellini (2018) ignores any burning of the lofting firebrand, as during this stage the fire is still evaporating whatever moisture is contained in the firebrand. Upon exiting the plume, the firebrand is considered to be burning. Using the burning rate for cylindrical firebrands developed by Albini (1979), we can describe the change in firebrand radius as it burns by

$$\frac{dr}{dt} = \frac{K}{2} \left(\frac{\rho_a}{\rho_b} \right) \frac{dz}{dt} \tag{17}$$

Where $K = 0.0064$ and initial condition $r(0) = r_0$ and $z(0) = Z$ - the height the firebrand leaves the fire plume at. Integrating (17) from 0 to a time t, we have

$$r(t) = r_0 + \frac{K}{2}\left(\frac{\rho_a}{\rho_b}\right)(z(t) - Z) \tag{18}$$

4.3.5 Downwind Launching of Lofted Firebrand

While the firebrand is being lofted it will travel some horizontal distance x_0 (as wind will cause the fire plume to be on a slight angle). Using a simple geometric argument we find that

$$x_0 = Z \tan\theta$$

A factor of 0.7 was introduced by Wang (2011) to correct for a possible overestimation of the distance travelled while lofting. In Alexander (1998)(pg 92), an empirical formula for $\tan\theta$ is used. Combining these we have that x_0 is

$$x_0 = 0.7 \tan\left(\frac{bouy * I}{w^3}\right)^{1/2} \tag{19}$$

Where

$$bouy = \frac{g}{\rho_a C_p T}$$

$g = 9.81 m/s^2$, ρ_a is the density of air, C_p is the specific heat of air at a constant temperature, T is the ambient temperature (298K). This is the initial horizontal position of a lofted firebrand as its about to leave the plume. Now Fellini (2018) uses the coupled ODES to describe the falling and downwind launching of a firebrand:

$$\frac{dz}{dt} = -v_t \qquad z(0) = Z \tag{20}$$

$$\frac{dx}{dt} = \frac{U_H \log(z(t)/z_0)}{\log(H/z_0)} \qquad x(0) = x_0 \tag{21}$$

where U_H is the wind speed at canopy top height, H is the height of canopy top height, $z_0 = 0.1313H$ and $z(t)$ is the solution to (20). the equation for the burning firebrand $r(t)$ into equation (20), we get the solution to (20) is

$$z(t) = \frac{a}{4}t^2 - \sqrt{b}t + Z \tag{22}$$

From this solution we see that there is a time t^* such that $z(t^*) = H$ where H is the height of the trees where the firebrand lands. This time is given by,

$$t^* = \frac{2}{a}\left(\sqrt{b} - \sqrt{b - a(Z - H)}\right) \tag{23}$$

We use this time because after the firebrand hits the canopy, we have two considerations. Firstly, the canopy serves as a mechanical obstacle that may prevent the firebrand from reaching the ground. Further, even if the firebrand does reach the forest floor, there is no way to know how long it will take for it to fall. Secondly, once the firebrand is at canopy height/below the canopy, the wind speed should decline and so little addition drifting should occur. Now, using the solution to (20) in (21), Fellini (2018) found an analytic solution to be

$$x(t) = x_0 + \frac{U_H}{\log(H/z_0)}\left\{ t(\log\left(\frac{z(t)}{z_0}\right) - 2) + \left(\frac{2\sqrt{b}}{a}\right)\log\left(\frac{Z}{z(t)}\right) + \left(\frac{2\alpha}{a}\right)\log\left(\frac{2Z - (\sqrt{b}+\alpha)t}{2Z - (\sqrt{b}-\alpha)t}\right)\right\}$$

Where $\alpha = \sqrt{b - aZ}$. Evaluating this at t^* to get the distance the firebrand travels before reaching the canopy we get

$$x(t^*) = x_0 + \frac{U_H}{\log(H/z_0)}\left\{ t^*(\log\left(\frac{H}{z_0}\right) - 2) + \left(\frac{2\sqrt{b}}{a}\right)\log\left(\frac{Z}{H}\right) + \left(\frac{2\alpha}{a}\right)\log\left(\frac{2Z - (\sqrt{b}+\alpha)t^*}{2Z - (\sqrt{b}-\alpha)t^*}\right)\right\}$$

This final equation, describes the horizontal distance a firebrand travels.

4.4 Probability of Ignition by Firebrand

4.4.1 Parameters

β_0 = -4.8479	β_2 = 0.0258
β_1 = 0	β_3 = 0.6032

For our probability of ignition model we used the equations from Beverly and Wotton (2007). These equations describe the probability of sustained flaming when a flaming match is dropped on different fuel types. These were determined by performing a logistic regression on collected experimental data. The formula we used was for the probability of stained flaming in pine needles. Then we have that the probability of sustained flaming is given by

$$P(sf) = \frac{1}{1 + \exp(-z)} \tag{24}$$

where

$$z = \beta_0 + \beta_1 FFMC + \beta_2 DMC + \beta_3 ISI \tag{25}$$

4.5 Our Contributions

We wanted to add a couple features to the spotting model to account for some of the noise in the firebrand spotting model.

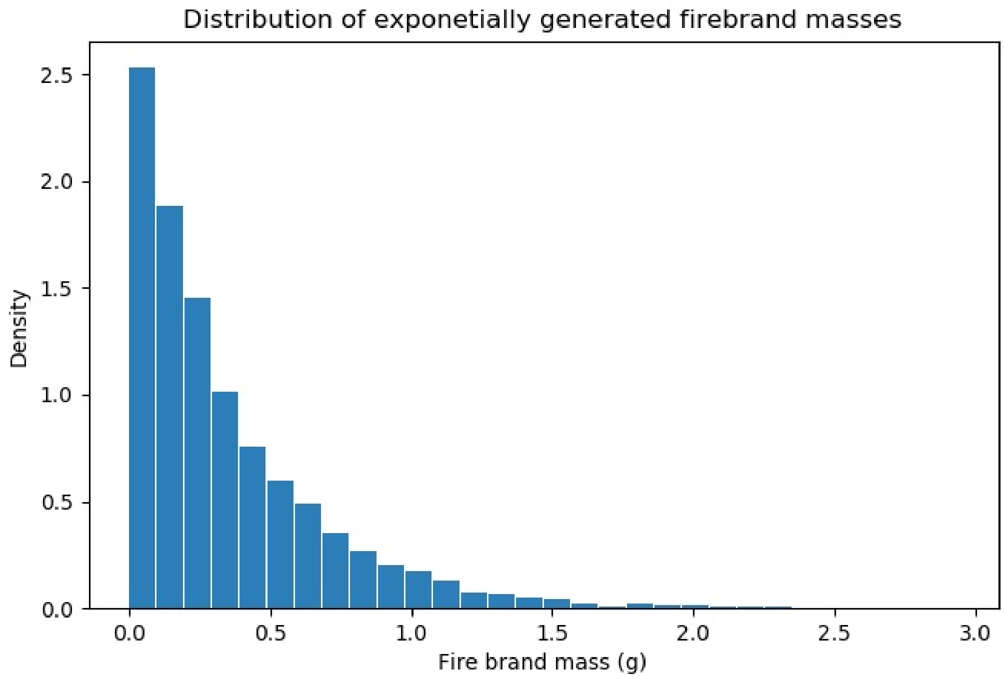
4.5.1 Firebrand Generation

The first feature we added was a mass generation process that generated a random mass based on an exponential distribution with mean 0.35. Once we generated this random mass we wanted to find its radius assuming a cylindrical geometry and fixed length to width ratio of $L/r = 6$. The mass of the firebrand can be written as

$$\frac{m}{1000} = 2\pi r^2 L \rho_b \tag{26}$$

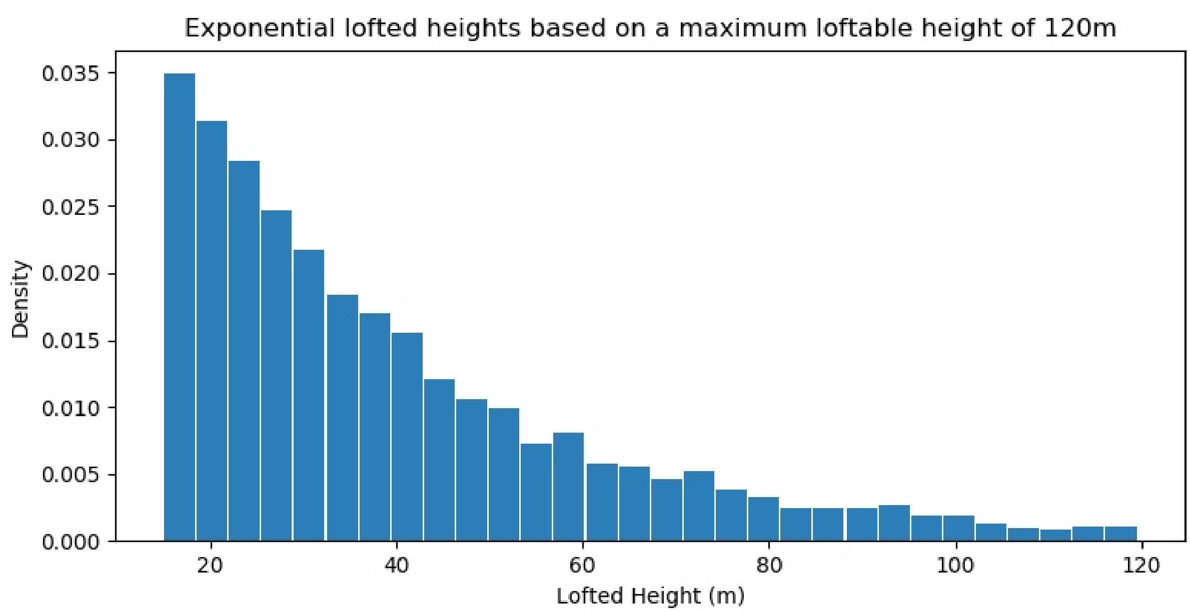
$$\Rightarrow r = \left(\frac{m}{6 \cdot 1000 \pi \rho_b} \right)^{1/3} \tag{27}$$

Where we divide by 1000 to account for the fact the left hand side is in grams and the right hand side in kg. Now this gives us a probabilistic way to generate firebrand radii to use in the lofting process.



4.5.2 Firebrand Lofting

We assumed that firebrand lofting heights are exponentially distributed with mean such that the probability that they reach their maximum loftable height is 1%. We used a truncated exponential distribution to achieve this:



4.5.3 Probability of Ignition

The probability of ignition model we used was based on experimental data where a flaming match was dropped on forest floors and then recorded if the match caused ignition or not. However, it is possible that firebrands are much smaller than a match (which weighs about 0.15g). As such, we wanted to add a factor to weight the probability of ignition to account for this. We used a simple minded approach and weighted the probability of sustained flaming in Beverly and Wotton (2007) by a factor of the mass of a firebrand divided by the mass of a match. However, it could be the case that firebrands are also much larger than a match stick. After consulting with Professor Mike Wotton and Professor David Martell of the Faculty of Forestry at the University of Toronto, we decided that there was no good or simple way to incorporate such a factor for larger firebrands. This is for one main reason - larger firebrands will contain more moisture than smaller ones and hence require more energy to reach a flaming state. Hence, these larger firebrands may land in a smouldering state rather than a flaming one. Letting $\underline{m} = 0.15g$, m the mass of a firebrand upon landing, and $P(sf)$ be the probability of a match igniting, the probability of ignition by a fire brand is then,

$$P(ign) = \begin{cases} (m/\underline{m})P(sf) & \text{if } m < \underline{m} \\ P(sf) & \text{otherwise} \end{cases} \tag{28}$$

5 Analysis of Model

5.1 Qualitative Behaviour

We first observe that our model exhibits the qualitative behaviour that we expect it to based on our modelling predictions. The following plots show the evolution of fire under extreme conditions generated by a pre-existing elliptical spread model without firebrand spotting, and our firespread model with spotting.

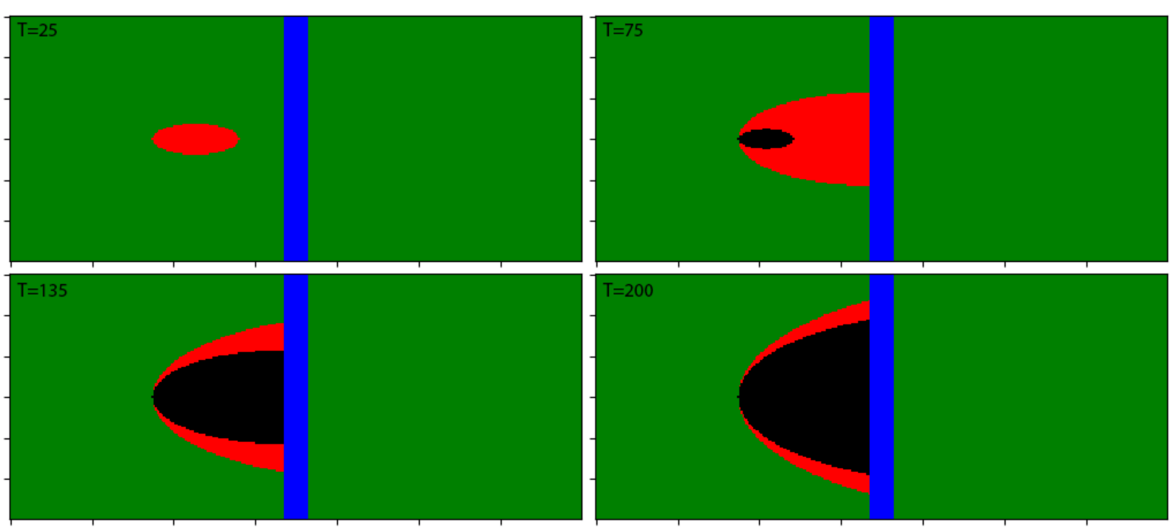


Figure 1: Fire spread under model without spotting

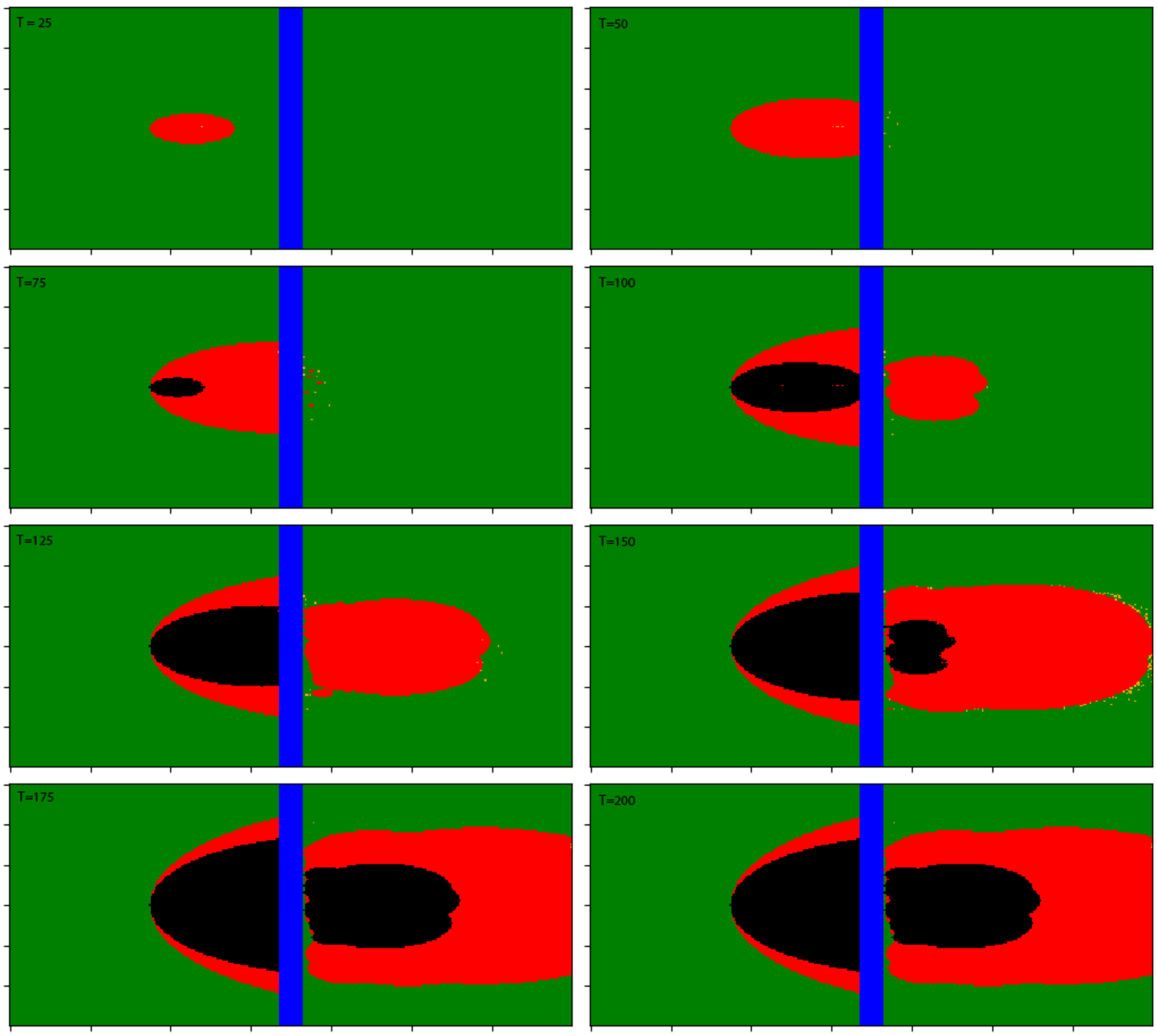


Figure 2: Fire spread under our model with spotting

Note that the green cells represent fuel cells (forest/trees), blue cells represent fuel discontinuties (rivers), red cells represent areas that are currently burning, and black cells represent burn out areas. Animations have been submitted with the report for these plots.

We observe that under sufficiently intense conditions such that firebrand spotting occurs, our model evolves differently from pre-existing models, with the fire being able to cross fuel-discontinuities and continue spreading (we can also observe that it spreads slightly faster downwind, even without fuel discontinuities, than the pre-existing models). Furthermore, the animations for our model reveal qualitative behaviour of fire-spread that is more in line with reality than pre-existing models: fire doesn't actually grow perfectly elliptically as the elliptical fire spread model predicts. Our model has firebrands being launch downwind ahead of the fire, starting their own little blooms of elliptical fire growth. Most firebrands that do not travel too far just get engulfed by the main elliptical fire spread before igniting and creating a local burning ellipse of its own, but some firebrands do travel far enough and cause ignition, creating somewhat irregular shapes of overlapping ellipsoids. However, despite the slight deviation from perfect ellipsoids, our firebrand spotting model still grows roughly in an elliptical shape, which is in line with experimental results.

Furthermore, our model with firebrand spotting incorporated behaves exactly as the pre-existing elliptical fire-spread model does under moderate conditions. This is in line with our expectations; under moderate conditions, we should not expect firebrand spotting to occur, thus becoming a deterministic model.

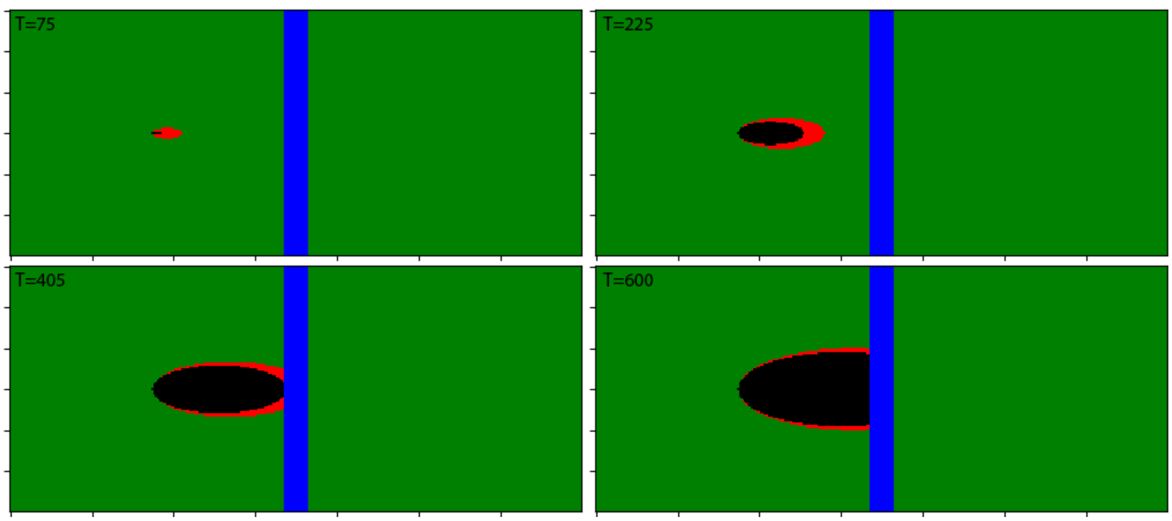


Figure 3: Fire spread model with spotting under moderate conditions

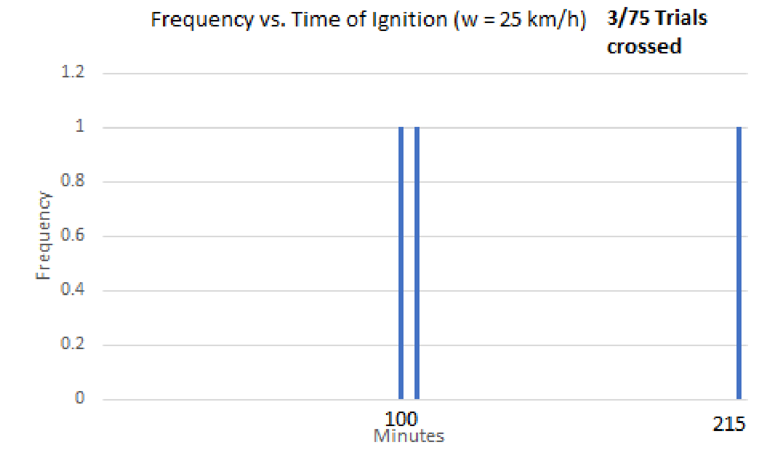
This qualitative behaviour partially answers our primary modelling question: this stochastic model, at the least, predicts accurately the qualitative results expected from firebrand spotting in extreme and moderate weather conditions.

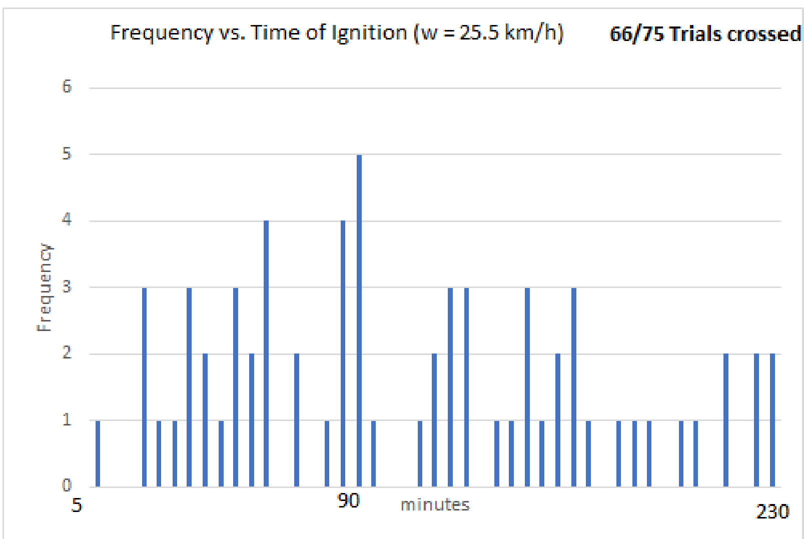
5.2 Fort McMurray Scenario Analysis

The devastating fire that happened at Fort McMurray in 2016 began on the opposite side of the 1.1 km wide Athabasca River, during the night time when wind speeds were between 20-25 km/h, with gusts of wind between 30 - 40 km/h. Ignition on the Fort McMurray side was due to a firebrand, and so this scenario fits perfectly to do an analysis of our model, and see if we can observe fire spotting across the river in this setting. The relevant fire conditions were known prior to the fire KPMG (2017) MNP (2017); in particular, $FFMC = 96$, $DMC = 45$, $DC = 367$, and $FMC = 85$. Since the wind speeds varied, we tested with the 20-25 km/h range, along with wind speeds of 25.5 km/h and 26 km/h, to account for possible gusts of wind. We let the simulations run for 4 hours (simulated time), and for 75 simulations per each wind speed. To reduce simulation time, we started the fires 100 metres from the river.

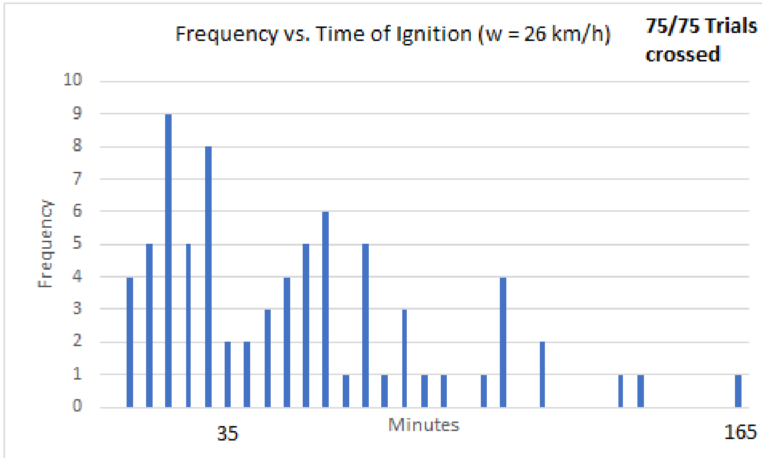
5.2.1 Results

For wind speeds below 25 km/h, during no trials did the fire cross the river. However, for wind speeds at and above 25 km/h, we did see crossing. Below are graphs for each wind speed displaying the time for which crossing occurred, and the frequency of crossing during that time.

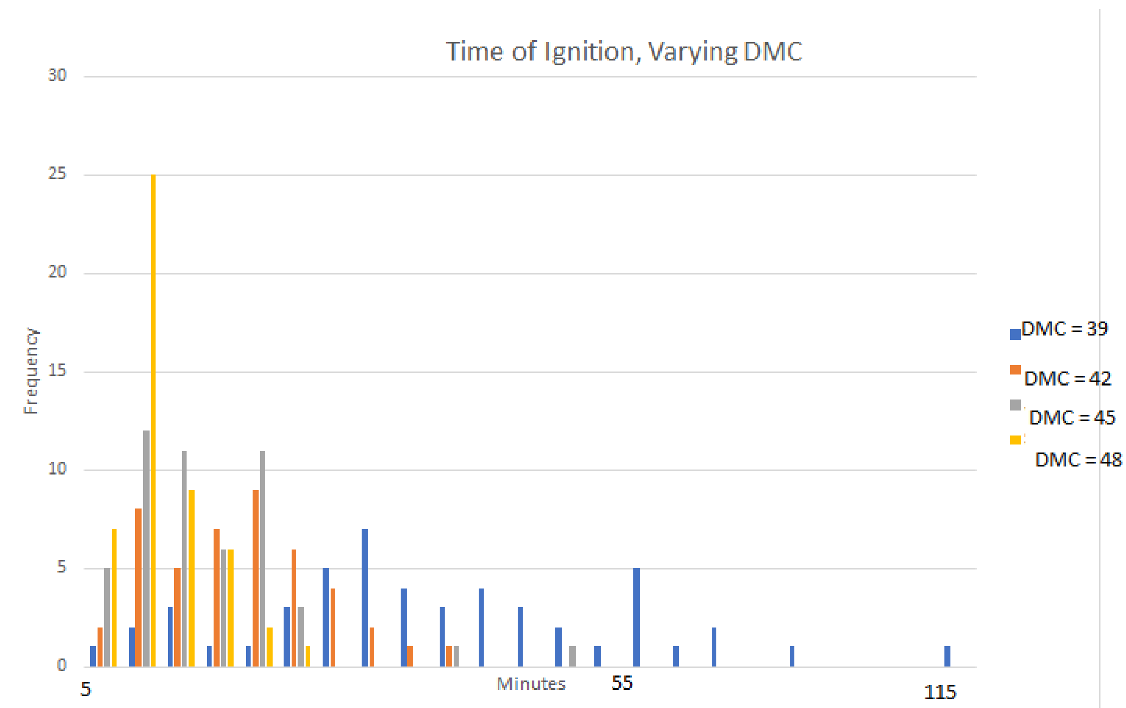




The first observation to make is that, as the wind speed increases, the firebrands are crossing the river more quickly, which is certainly desired behaviour. Second, our model is very sensitive to wind speed. This makes sense, as wind speed is one of the most important parameters in our firebrand model, showing up in each of the three models we are combining; see 5.4 for a further analysis of sensitivity to wind speed. As noted above, high gusts of wind were recorded, which would affect the average wind speed of a firebrand. Therefore, our simulations do predict spotting across this river to be a very real threat, though we would like to quantify this a bit better. We first check whether our simulation results are sensitive to the fuel parameters that were recorded. Below are the

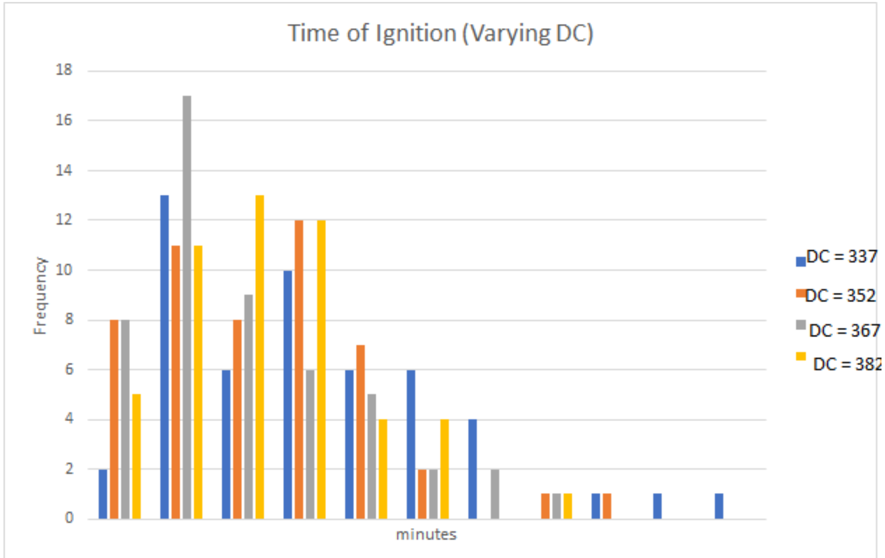


results of varying DMC, DC, and FFMC, and recording the same information as we did with varying wind speed, but now fixing wind speed to be 27 km/h (so that we know the wind speed will be high enough to not be the deciding factor in our trials). We tried four variations per each fuel parameter, with 50 simulations per each specific set of values; see the below figures on pages 17 and 18.



In all trials for DMC and DC, we observed the fire crossing the river. DMC is more sensitive to DC, which makes sense, since DMC measures the moisture content a ground layer above DC. Similarily, FFMC is more sensitive than DMC, as it quantifies surface level moisture:

In the two values of FFMC visible on the graph, the fire crossed the river in every trial. However, for FFMC = 90 and FFMC = 93, the fire did not cross in any of the trials.



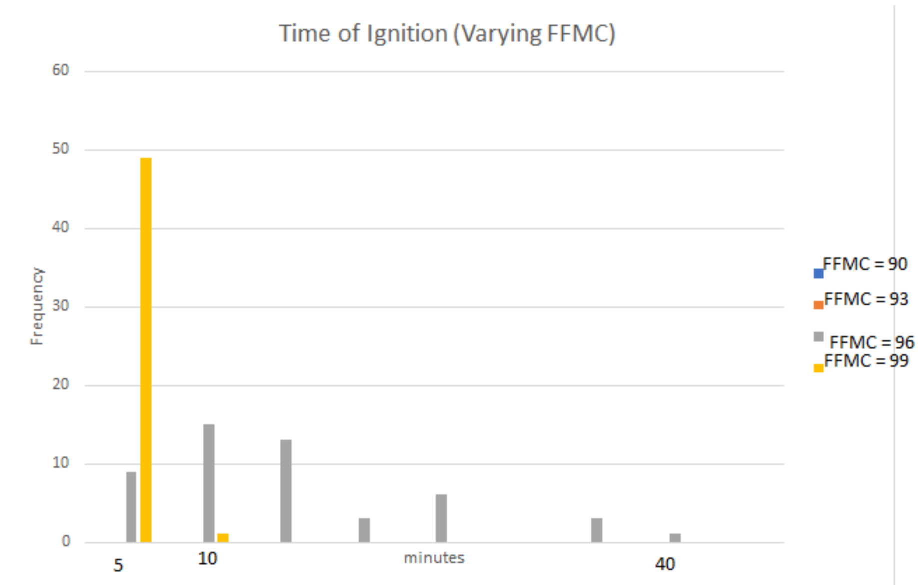
These values are still within the extreme fire hazard range for FFMC, which puts it into perspective how dangerous the fire situation was at Fort McMurray at the time.
To make sure our simulations weren't just predicting spotting across the river for these very extreme cases of Fort McMurray, we ran nominal values for what is considered an extreme fire hazard, as well as a high and moderate fire hazard, with an increased wind at 30 km/h, and same river width (1.1 km). In particular, we chose values in the middle range of each box shown in the chart below:

	FFMC	DMC	DC
moderate	81-86	16-30	141-240
high	87-90	31-50	241-340
extreme	91+	51+	340+

In the moderate and high range, we did not detect the fire crossing, even after 25 trials each. This makes sense, as the river is quite wide, so only the lightest firebrands (and consequently the least likely to start a fire) will make it across. Using the extreme parameters, 20/25 times the fire crossed, at an average time of 131 minutes (starting the fire 100 metres away from the river)

5.2.2 Addressing the Probability Threshold

To see why we don't have spotting below wind speeds of 25 km/h, we have to analyze an assumption we made in our model. Specifically, we said that if a fire brand lands on a fuel cell, and has probability of igniting less than 0.5, we said it did not start a fire, while if it had probability larger than 0.5 we said it did start a fire. We choose this cut off value of 0.5 as the original probability of ignition model by Beverly and Wotton (2007) was done by performing a logistic regression and the only meaningful way to interpret the probabilities from this type of regression is to set some threshold to classify events. A natural choice is 0.5. Plotting the average number of firebrands crossing the river above varying thresholds of probability, at wind speeds of 25 km/h and 22.5 km/h, we see why



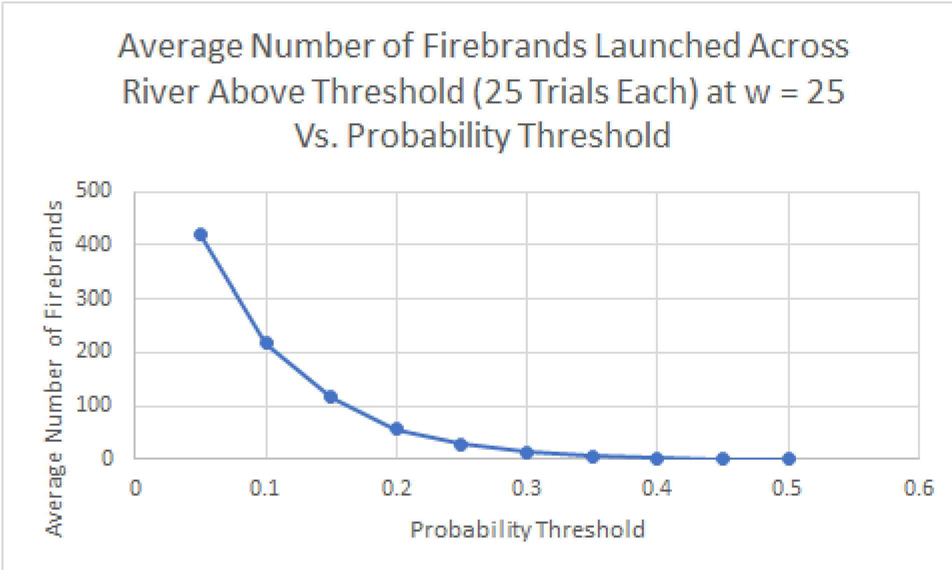
at wind speed 25 km/h there is a very low chance of spotting (see the two graphs below).

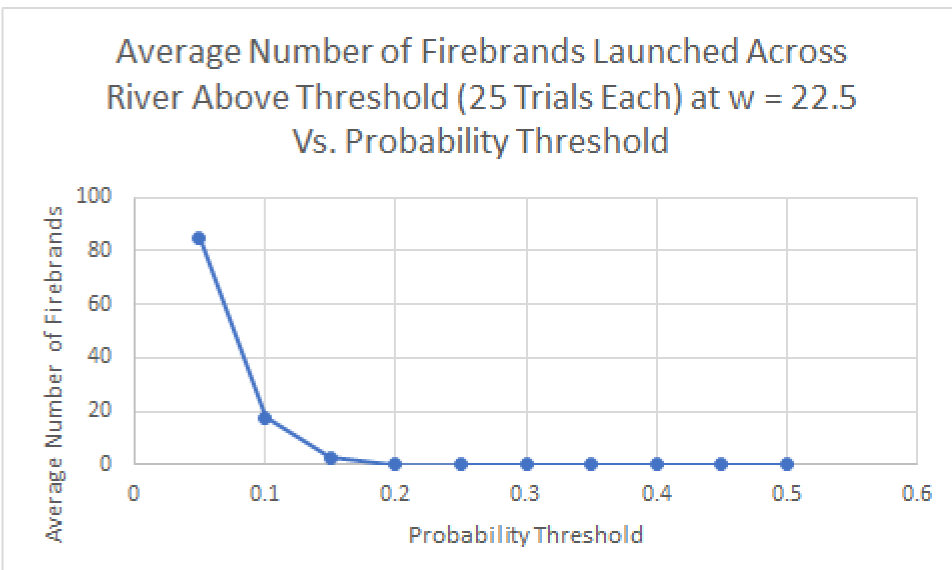
The average number of firebrands crossing the river at a wind speed of 25 km/h and a probability larger than 0.5 is close to zero (about 0.6), and as the threshold decreases, the average number of firebrands crossing increases geometrically. Similarly for the wind speed of 22.5 km/h, though it is very clear that no firebrands above the threshold of 0.5 are crossing. Therefore, our assumption is making our model deterministic at lower wind speeds, but we would like to assign meaningful probabilities to answer the auxiliary question 2i), so some tweaking of our model is required.

5.2.3 Changing our Mass Scaling Assumption

As noted in 4.5.3, the assumption of scaling masses lower than the weight of a match was slightly controversial within the forestry department, and since this makes the outputs in our probability of ignition model not actual probabilities, this assumption goes against our current goal of making this existing model a probabilistic model. We replace this assumption by only assigning nonzero probabilities to firebrands closely resembling matches igniting, for which the distribution in the probability of ignition model comes from. We remark that this assumption is less of an issue for wider rivers, since the firebrands we will discount (burning needles and leaves) will not be capable of travelling great distances. The above table is from a firebrand data collection Kapcak (2015).

Since the average weight of a twig firebrand without pan screening is 0.03 grams, this seems like a natural mass cut off value for considering firebrands with a non-zero probability of ignition. Implementing this into our model, we are then able to assign a probability, for each simulation, for a single firebrand to cross the river and ignite during the simulation time. Using the Fort McMurray parameters, we immediately observed that our simulations gave us either a probability of 0 or a probability of close to one for

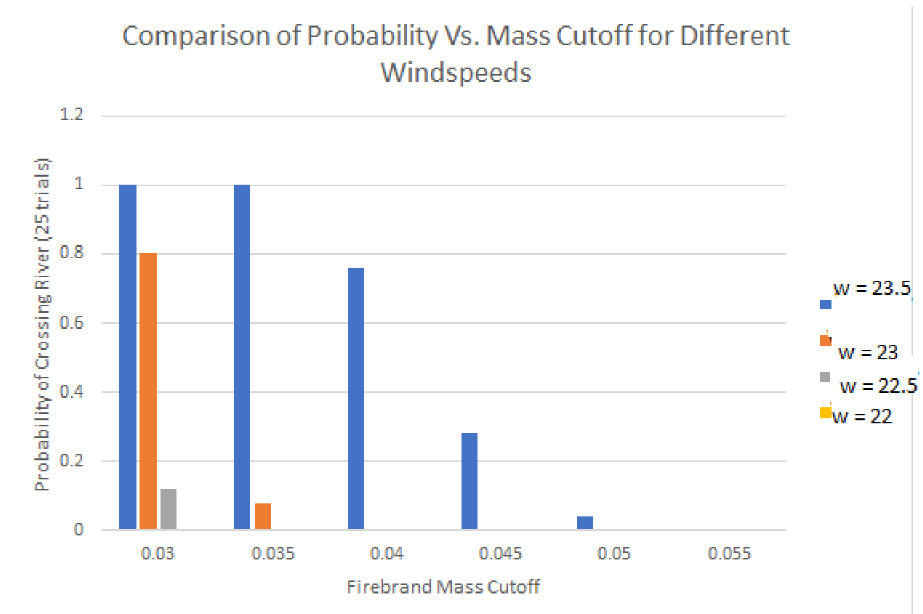




	Quantity		**Percent Composition (%)**		**Total Weight (g)**		**Percent of Weight (%)**		**Avg Weight per Firebrand (g)**	
Firebrand	**Pan w/o Screen**	**Pan with Screen**	**Pan w/o Screen**	**Pan with Screen**	**Pan w/o Screen**	**Pan with Screen**	**Pan w/o Screen**	**Pan with Screen**	**Pan w/o Screen**	**Pan with Screen**
Needles	1748	1166	93.9	93.5	10.6	8.7	78.5	59.6	0.01	0.01
Leaves	43	23	2.3	1.8	0.1	0.7	0.9	4.9	0.003	0.03
Twigs	16	7	0.9	0.6	0.5	1.2	4.0	8.0	0.03	0.17
Bark	55	51	3.0	4.1	2.3	4.0	16.6	27.5	0.04	0.08
Total	1862	1247	100.0	100.0	13.5	14.6	100.0	100.0	0.01	0.01

Note: “w/o” denotes “without”

the ignition of a firebrand during 4 hours of simulation time. This is because the forest fire conditions are so extreme, that any firebrand above this weight threshold will start a fire; moreover, spotting distance is a random process in our model, which explains why in some trials we did not get firebrands crossing the river that were able to start a fire. We will use this fact that the probability is either zero or close to one to our advantage, as it allows us to assign a probability of a fire crossing the river to be the average over our outcomes. For less extreme parameters, we do not expect this dichotomy to occur, so to obtain more meaningful simulations, it may be necessary to increase fire brand rate, or the length of the simulation. Below is a graph of our probabilities obtained, varying wind speed and the mass cut off value:



It seems, to be on the safe side of over-predicting, that 0.03g is the better of the choices for the mass cutoff. This probabilistic model gives fairly high probabilities for a fire starting across the river using Fort McMurray parameters and the associated wind range, which is more in line with the ”perfect storm” fire conditions present at the time. Moreover, this model does not over-predict, due to the probability for spotting to occur dropping drastically once wind speed is reduced below 22 km/h. This tweaked model is still very sensitive to wind speed, which may be unavoidable, due to its primary role in many components of this model.

5.3 Robustness of Wind Speed Models

The spotting model by Fellini (2018) uses a logarithmic wind speed profile to describe how the wind speed above the canopy changes as a function of height. This function of lofted height is what is used to describe the horizontal position of the falling firebrand. Another commonly used wind profile is the exponential wind profile Martin and Hillen

(2016),

$$w(z) = w_H \left(\frac{z(t)}{H} \right)^{\beta} \tag{29}$$

Where w_H is the wind speed at canopy height, $z(t)$ the height above canopy at time t, and β is a parameter. The parameter beta is recommended by Albini (1983b) to be $\beta = 1/7$. Now using numerical integration to solve

$$\frac{dz}{dt} = w_H \left(\frac{z(t)}{H} \right)^{\beta} \tag{30}$$

with the same initial conditions as in (4) we ran the spotting model by Fellini (2018) in isolation with both wind speed profiles for different values of w_H. For $w_H = 20km/h$ we get the following histograms:

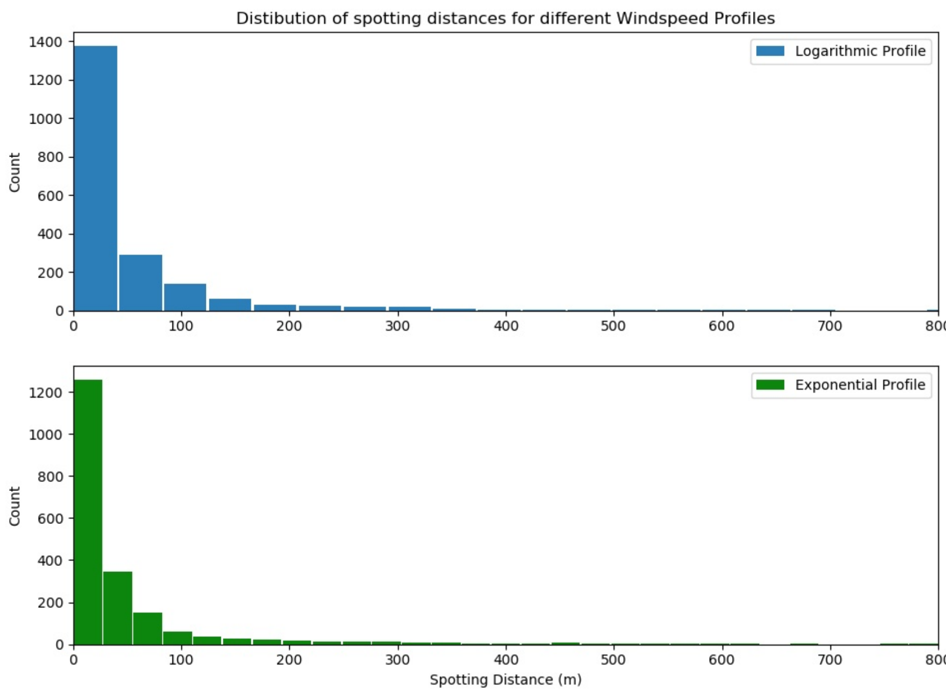


Figure 4: Spotting distance distribution for 2000 iterations at 20km/h and extreme fuel conditions

We see that for spotting distances under $300m$ that both wind speed profiles have for all intents and purposes the same number of firebrands spotted between 0 and 300 meters. Further we see that up to 700 metres both histograms are fairly consistent with one another. Now if we change the wind speed to $30km/h$ and run the same script we get the following histogram:

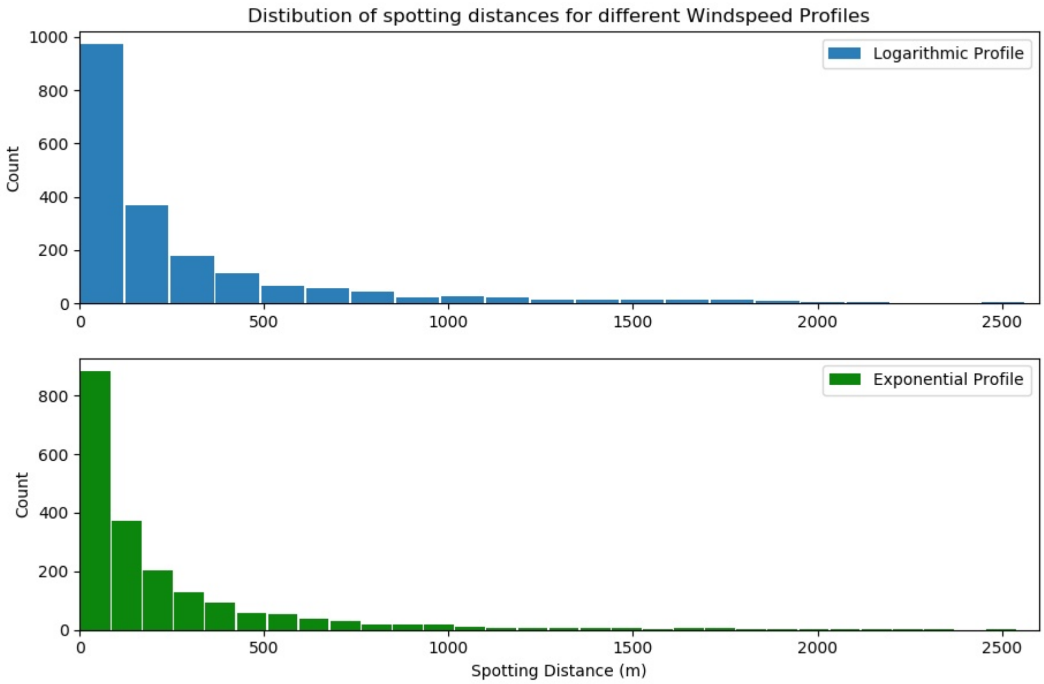


Figure 5: Spotting distance distribution for 2000 iterations at 30km/h and extreme fuel conditions

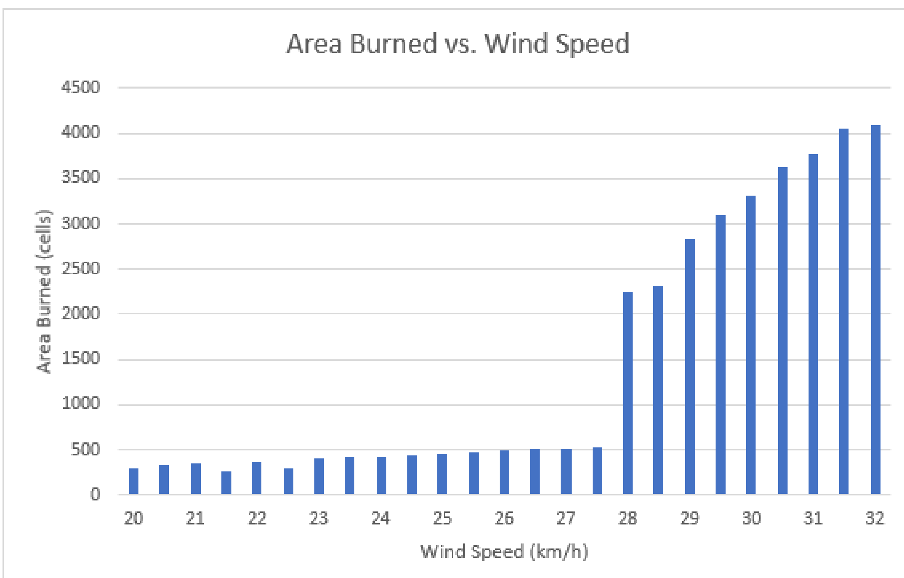
Overall we see that the histograms are very similar for the two different wind speed profiles and wind speeds. From this, we can conclude that the spotting model is fairly robust with respect to wind speed profiles.

5.4 Sensitivity Analysis of Parameters

There were some obstacles in conducting sensitivity analysis of parameters, due to do the random aspects of our model and computational limitations. Since any output that we wish to test change in by changing the parameters is a random variable, there is a natural variance in the results that must be taken into account when testing for sensitivity. Therefore, we needed to run multiple simulations and then run analysis on the means/approximate expected values; however, since there's is quite a bit of variance in data given by the simulations, a fairly large number of simulations were required for meaningful analysis.

5.4.1 Wind Speed

As previously mentioned, due the way our model is designed, it should be very sensitive to changes in wind speed, as each individual 'components' of the model that we've combined are highly dependent wind speed. We ran simulations in a fairly high fire weather conditions (enough for spotting), running approximately 50 simulations for each varied parameter value to take an average. Indeed, as we expect, area burned is proportional to wind speed



The above plot also indicates a certain behaviour from our model: for wind speeds below 28km/h, we see that the area burned is significantly lower than that of directly higher values. This is because at these wind speeds, firebrand spotting did not occur; this may be due to the probability threshold as mentioned in section 5.2.2 forcing deterministic behaviour at low wind speeds (although the the behaviour occurs at different numerical values due to different weather conditions). We can actually see that there's a 'discontinuity' around wind speed of 27.856 km/h

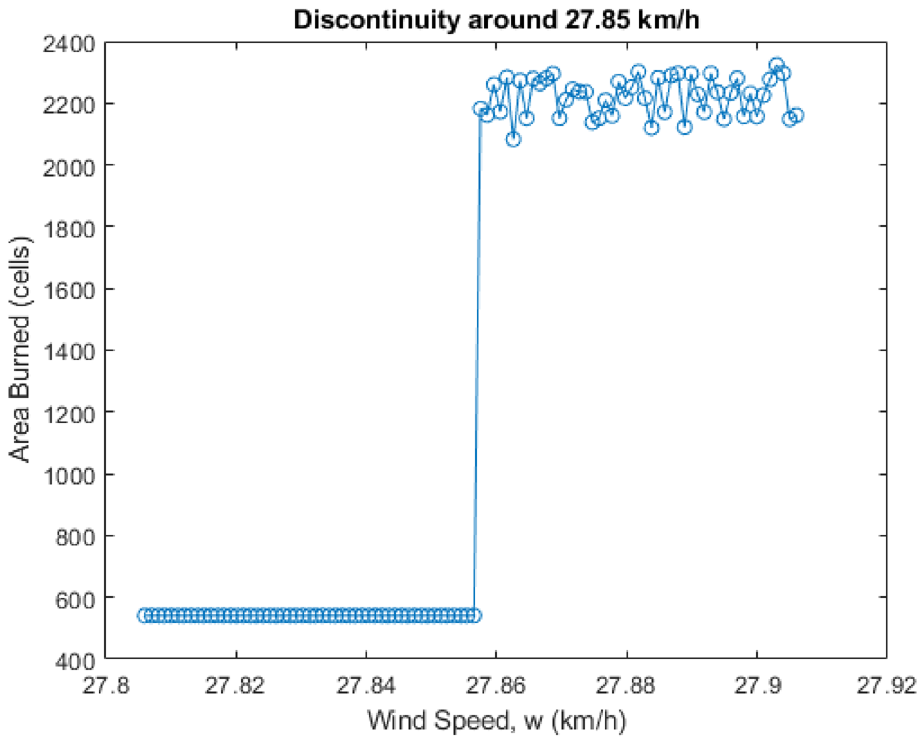
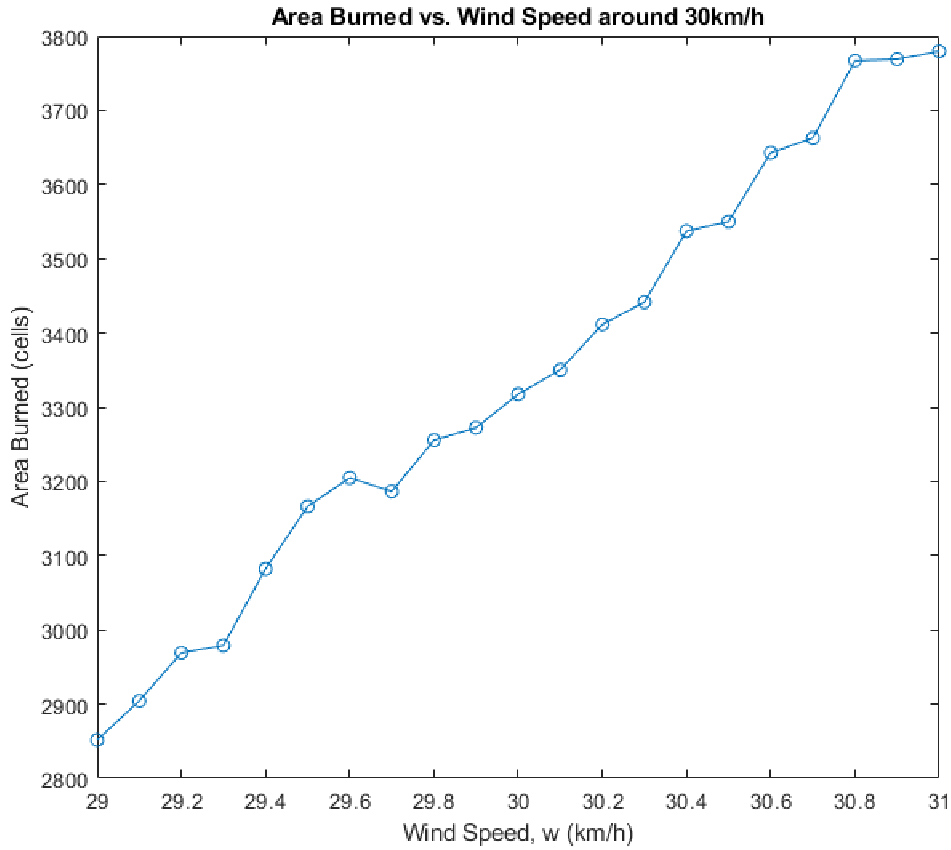


Figure 6: Simulation 100 Wind Speed values around 27.856

This implies that area burned is extremely sensitive with respect to wind speed around these values - the sensitivity at these values is unnaturally high. This kind of discontinuity is unnatural, and as previously stated, perhaps a fully probabilistic model of firebrand ignition instead of the threshold implementation as given in our model here may be more appropriate. Some of this behaviour can be explained by the firebrand spotting model; wind speeds must be high enough for the lofted height of the firebrand to rise above the crown of the forest. However, even so, we should expect a steep decline, not a discontinuity. Also, large amount of the simulations done in this report were conducted using a constant wind speed of 30 km/h (where it's better behaved), so we consider the sensitivity around 30 (plot on the next page).



With wind speeds ranging from 29 to 31 with $\triangle$ = 0.1, running 50 simulations each, we compute the sensitivity area burned with respect to wind speed to be 389.5, which is very high. This is in line with our expectations though - our model, as it is, should be very sensitive with respect to wind speed. Going forward, a variable wind speed model may be able to address these issues.

5.5 Application - solving an optimization problem using our model

Due to the costly nature of forest fires, we are interested in whether we can stop the spread of the fire to a certain extent. In practice, it's possible to create fuel discontinuities in forests (e.g. by clearing the land using various means, etc.) and theoretically, when we add another layer/line of discontinuities beside a pre-existing river, these should slow down or stop the spread of fire. A distinct advantage of this firespread model versus pre-existing models is that the stochastic nature of the firebrand spotting model gives us a probabilistic answer to whether fire will cross a fuel discontinuity. Hence, we can run simulations to compute an expected value for fire spread under certain conditions.

Suppose we're working in a 1000m by 1500m grid, with a river (fuel discontinuity) that is 520m wide, with a forest starting on the west side of the river; using parameters ffmc = 90, dmc = 75, and dc = 340 with a fire brand rate of $\frac{1}{15}$ and timsteps of 15 minutes for a 60 minute simulation, the conditions are severe enough that the fire is likely to jump the 520m discontinuity and burn a significant area to the east of the river. Since our grid has cells that are 10m by 10m, each time we create another layer of fuel discontinuities we expand the fuel discontinuity by 10m; furthermore, due to the fact that our grid is homogeneous with equally flammable fuel cells where there isn't a discontinuity, it's obvious that just creating one cell of fuel discontinuity is rather ineffective at preventing firebrand spotting across the river. Instead, we assume that each time we create fuel discontinuities, we create them in layers, i.e. we turn the entire vertical strip of cells into fuel discontinuities. Suppose it costs $50 to turn a single fuel cell into a fuel discontinuity, and that each cell has a value of $600. Let L be the number of layers of fuel discontinuities we add, and suppose that the cost of adding another layer $C(L)$ grows quadratically

$$C(L) = 10000L^2 + 65000L$$

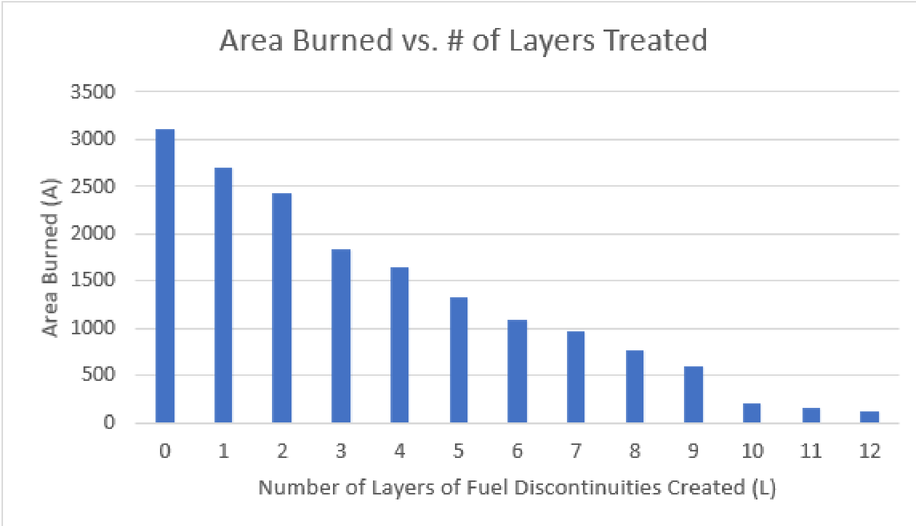
The assumption of quadratic growth seems appropriate. The cost of adding additional fuel layers cannot possibly grow linearly, as each additional layer we need to add implies additional manpower and machinery, which is limited; at a certain point, due to limitations, the cost of obtaining additional resources required to create fuel discontinuities quickly enough will increase, perhaps approximately quadratically. We note that the coefficient of the qudratic term has sensitivity L^2, which is not terribly sensitive as our values of L are relatively small, but it should be noted that this coefficient was somewhat 'cooked up' as a reasonable estimate, and not based on any data (as we were unable to find relevant data). We consider the robustness of this assumption below.

Furthermore, we have a random variable $A(L)$, that is the number of cells burned to the east of the river. Let $T(L)$ be the expected loss in dollars, then

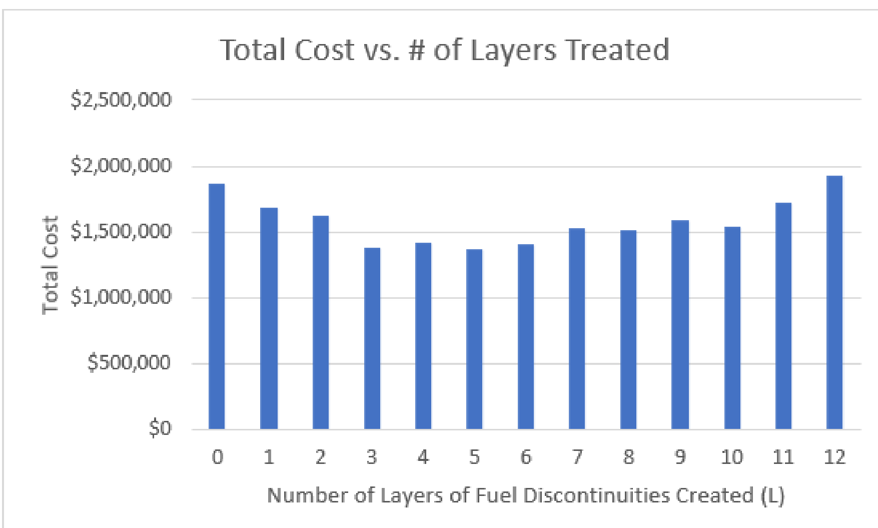
$$T(L) = 600A(L) + C(L) = 600A(L) + 10000L^2 + 65000L$$

Running approximately $n = 300$ simulations and using the above model. The below chart summarizes the results

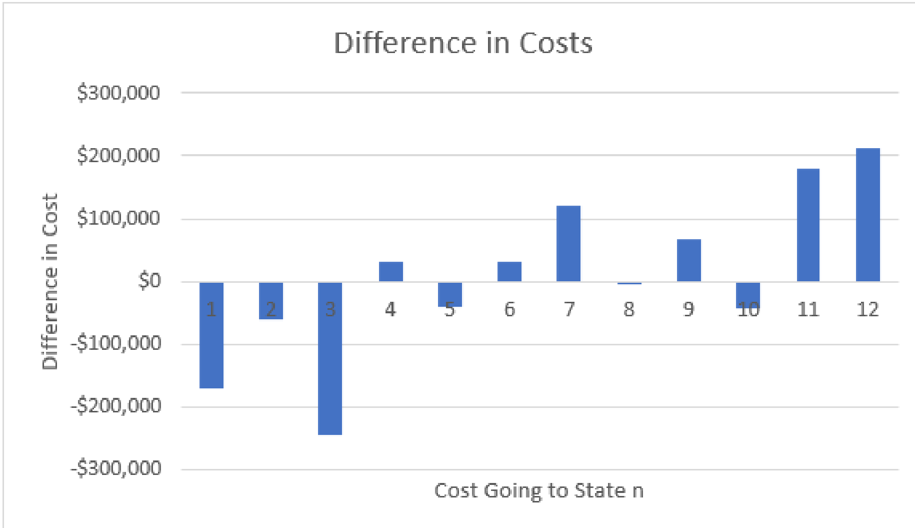
Length of Discontinuity (m)	L	Area Burned, A(L) (a)	Total Loss, T	Difference
520	0	3102.28	$1,861,368	
530	1	2692.86	$1,690,714	-$170,654
540	2	2432.80	$1,629,680	-$61,034
550	3	1829.79	$1,382,874	-$246,806
560	4	1654.20	$1,412,520	$29,646
570	5	1328.39	$1,372,034	-$40,486
580	6	1089.20	$1,403,520	$31,486
590	7	965.40	$1,524,240	$120,720
600	8	765.33	$1,519,198	-$5,042
600	9	594.25	$1,586,550	$67,352
620	10	205.51	$1,543,306	-$43,244
630	11	154.13	$1,722,478	$179,172
640	12	123.33	$1,933,998	$211,520



As expected, we see that adding additional layers of fuel discontinuities decreases the expected value of the area burned. Furthermore, using this approach, it's fairly easy to answer the question: for a threshold value τ, how large does L need to be such that $E(A(L)) < \tau$. For example, if $\tau = 1000$, our simulations tell us that we would need to create to create 7 additional layers of fuel discontinuities.



If our objective is to minimize loss, then our simulations indicate that $L = 5$ is the optimal solution, i.e. by creating 5 additional layers of fuel discontinuities, we minimize our expected monetary loss. In general, creating additional layers after 5 increases our project losses, as the cost of creating the layers outweighs the decrease in area burned.



In practical terms, if one's objective is forest fire management, a useful figure to look at may be the change in projected loss from additional layers. Due to resource limitations, it may not be viable to create 5 additional layers of fuel discontinuities, especially if there are time limitations. In this case, instead of searching for a global minimum, finding local minimums within viable ranges is useful. For example, if there's enough time and resources to create 4 additional layers of fuel discontinuities, but not enough to create 5 layers, then our model indicates that it's actually not optimal to create the 4th layer of fuel discontinuities - we're better off if we just create 3, even though we're still decreasing the total area burned. This is a desired attribute caused by our assumption of quadratic growth of cost; there's diminishing returns on creating more fuel discontinuities after a certain threshold, as we would expect. This behaviour does not exist in a linear cost model, as we can see in the results below:

L	Area Burned, A(L) (a)	Total Loss, T	Difference
0	3102.28	$1,861,368	
1	2692.86	$1,680,714	-$180,654
2	2432.80	$1,589,680	-$91,034
3	1829.79	$1,292,874	-$296,806
4	1654.20	$1,252,520	-$40,354
5	1328.39	$1,122,034	-$130,486
6	1089.20	$1,043,520	-$78,514
7	965.40	$1,034,240	-$9,280
8	765.33	$879,198	-$155,042
9	594.25	$776,550	-$102,648
10	205.51	$543,306	-$233,244
11	154.13	$512,478	-$30,828
12	123.33	$493,998	-$18,480

7 Response to Questions From Presentation

The main comment we got about or presentation was that we didn't show enough of our model. We hope that section 1-4 show why we decided it wasn't imperative to have this in our presentation. So let's address some of the other questions we got.

1. *Question:* Is it possible to include time varying wind speed?
   Response: Yes it is possible to include this in the model as all the models used have the ability to incorporate this fairly readily. It would require some set-up of how to actually vary the wind speeds (either randomly, using hourly airport wind speeds or some other method). We have submitted code along with this report, where we have implemented variable wind speeds in our model.

2. *Question:* How does topology affect your model?
   Response: We did not include changes in topology in our model. However, the elliptic growth model has an extension that can account for changes in topology. This involves knowing much more information about the landscape as the rate of spread up a slope is different than the rate of spread on flat land. This is something that with a bit of work could likely be implemented. The spotting model does not account for changes in topology downwind from where a firebrand originates and so is not fit to address this question. To this end, there is a special report by Frank Albini from the late 60's early 70's that only exists as a physical paper as far as I am aware that does include the possibility of change in topology. It is the only spotting model we are aware of that has this feature. As such, this could be a subject for further work.

3. *Question:* Is there a most effective type of natural or non-natural fuel discontinuity?
   Response: This is a very hard question to answer. Not all provinces in Canada report the case of different fuel treatments (i.e. clear cutting or prescribed burns). This makes it hard to really give a general type of answer. Rivers and lakes provide the best natural option for fuel discontinuities. Instead of making a fuel discontinuity, it may be possible that raising the moisture in an area by dropping water from air tankers would be more effective on influencing fire behaviour. This is difficult to test as there is no known data on the effectiveness of airtanker water deposits on changing fuel conditions (but is currently an active area of research in Ontario).

4. *Question:* What are the computational limitations of the model?
   Response: Our implementation of our model in Python was done in such a way that if a firebrand started a fire, we would assign this cell to be another point source for elliptical growth. As one can imagine, if many firebrands are landing on our fuel cells, this will be creating many new point sources for which the code has to run through and update the elliptical growth for. We have taken steps to reduce this computational time by over-riding a firebrand's ability to be a point source of a fire if the fire engulfs an igniting firebrand before it can be classified as a full-blown fire.

8 Conclusion

8.1 Summary of Results

In this project, we combined three models to see if we could get simulate the fire behaviour associated with intense crown fires. Our principle results are as follows:

- On the whole our model was able to produce qualitative behaviour consistent with what we would expect under extreme fire weather conditions. Furthermore, we observe that under moderate fire weather conditions and in low wind speed conditions, firebrand spotting does not occur in our model and is consistent with behaviour expected from pre-existing models
- We tested the robustness of the spotting model used by running many simulations of the model in isolation with different wind speed profiles. Overall, the two wind-speed profiles qualitatively agreed for short spotting distances but differed slightly at long ranges. The logarithmic wind profile employed typically produced more long range spotting distances than the exponential model.
- We attempted to answer the question ”what is the probability of a fire crossing a river?” in the context of Fort McMurray, in an attempt to apply our model, and to validate it to some extent. While the results were okay, determining with high probability a fire occurring when wind speeds are past 25 km/h, they left something to be desired, as the reported wind speeds at the time before crossing close to Fort McMurray were between 20-25 km/h. We noticed that our existing models use of a probability threshold restricted our model's usefulness in answering this question below 25 km/h wind speed, so we attempted to revise our model by making it more probabilistic. While there still needs to be more revision on this tweaked model, our initial results are promising, obtaining probabilities of 0.99, 0.8, and 0.12 for wind speeds 23.5, 23, and 22.5, thus giving us a high probability of crossing within the actual wind speed range.
- We also found that our model, under very ideal conditions, is able to generate useful predictions for forest fire management. Applying our model to a optimization problem, we were able to answer the question ”how many more layers of fuel discontinuities must be created to bring the expected value of the area burned below a certain threshold?”, and ”what is the optimal number of additional layers of fuel discontinuities to minimize projected losses?”
- A brief sensitivity analysis showed that the DMC and DC parameters in our model were not sensitive at the Fort McMurray range, while FMC was a little sensitive, partly due to its role in fire ignition (as it quantifies surface level moisture). Wind speed was the most sensitive of all our parameters, which makes sense, due to its central role in the three models we combined.
- A sensitivity analysis of wind speed revealed that as expected, our model is highly dependent and sensitive to wind speed.

8.2 Next Steps

There are a couple obvious ways we could improve/extend our model:

1. Allow for time dependent wind direction and speeds. In our model we used that these were both constant. Changing this would allow for a more accurate representation of how wind behaves.

2. Remove the global homogeneity of the landscape we ran our simulations in. Locally a forest will be fairly homogeneous by classifying it into one of the Canadian Forest Services fuel types (we used C-3 fuels but there are many others). The model will extend readily to this as we would just need to adjust/change the parameters in the three constituent models.

3. Account for changes in topography. This would likely be challenging to implement without much more knowledge or assumptions about our landscape.

4. To address the last question, we could turn this into more of a geographical information systems problem. For example, we could use a digital elevation mapping (DEM) and a land classification mapping of the same area to assign to each pixel in the map an elevation and an initial state (such as forest, roads, river, etc). This addition information may allow to include for more realism in the model. It would also allow for possibly more case study type work to test the predictive capabilities of the model.

5. Many of the other improvements that can be made would really on the collection or creation of more data. For example, the time it takes the different fuel types to burn out under certain conditions, or how long it takes a firebrand to cause an ignition then allow that fire to get up to a fire that the FBP system can model.

6. Based on anecdotal evidence, mountain pine beetle outbreaks have greatly increased the the number, size, and severity of spot fires. The trees that have been killed by these beetles seem to produce (much) larger firebrands than what the experiments by Manzello et al. (2006, 2007, 2008b,a) found which in turn is leading to more spot fires. This is a phenomena that is currently unexplainable by any spotting model or any other model as far as we know. In effect, this is something that could be explored.

7. In 5.2, we considered a revised model that we hoped would give us a more satisfactory answer to the question: "What's the probability of fire spotting occurring over a river?". The following are issues with the revised model that still need to be addressed:

   (a) It would be valuable to know how sensitive this probabilistic model is to the other fuel parameters. Simulations can easily be done to determine this, but since this tweaked model does not differ drastically from the original version, it seems unlikely that it will be too sensitive to these parameters, except perhaps

FFMC. We simulated FFMC values of 95, 95.5, 96, 96.5, and 97, with 25 simulations each under our normal set-up, along with a wind speed of 23 km/h. We got the probability of the fire crossing for FFMC values below 96.5 to be zero, however at 96.5, and 97 we had 0.96, and 1.00 probability of the fire crossing. Therefore, this tweaked model seems to be more sensitive to FFMC, but it is hard to compare, since we are running a different type of test.

(b) More simulations need to be run on more normal fire conditions to see how this tweaked model performs, but we believe the trials done using Fort McMurray parameters are a good proof-of-concept.

(c) An issue that may come up is this revised model's performance for smaller rivers, since the light firebrands may be able to travel across short rivers, and, due to our mass cut off, we are neglecting these, which may change results. Again, simulations could be set up to test different mass cutoffs and determine if there is a better one in the case of shorter rivers, or if the cutoff chosen (0.03g) has any effect on the probability of crossing a shorter river.

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