Morning

1. Compute the magnitude of and find the unit vector in the direction of:
   - $[1, 2]$
   - $[-3, 4]$
   - $[2, -4, 1]$
2. Prove that any 3 (distinct) vectors in $\mathbb{R}^2$ are not independent.
3. Compute the angle between $[2, 1, 3]$ and $[2, 1, 0]$
4. Using Gram-Schmidt, create an orthonormal basis for $\mathbb{R}^3$ starting with $[2, 1, 3]$, $[0, 2, 1]$ and $[2, 0, -1]$
5. Solve:

$$x + y + z = 2$$

$$x + 3y + 3z = 0$$

$$x + 3y + 6z = 3$$

   by hand.
6. Compute the dimension of the subspace of $\mathbb{R}^4$ spanned by $[-1, 0, 3, -2]$, $[2, 1, 0, 0]$, $[-2, 0, 6, -4]$, $[-7, -1, 15, -10]$
7. Show that $[1, 1]$ and $[1, 0]$ form a basis for $\mathbb{R}^2$.

Afternoon

6. Multiply

$$\begin{bmatrix} 0 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$$

   and

$$\begin{bmatrix} 1 & 2 \\ -4 & 2 \\ 0 & 6 \end{bmatrix}$$

7. (Challenge) Let V_4 be the vector space of polynomials in x of degree at most 4 (i.e. $3 + x + 5x^2 - 2x^4$). Show that the derivative $\frac{d}{dx} : V_4 \to V_4$ is linear. Also choose a basis for V_4 and compute the matrix representation of $\frac{d}{dx}$.
8. In problem 7, what's $col(A)$? What's $ker(A)$? What are their dimensions?
9. Compute the determinant of

$$\begin{bmatrix} -1 & 4 \\ 1 & 2 \end{bmatrix}$$

10. How many solutions does

$$x + 2y = 3$$

$$x + y = 10$$

   have?

11. Compute the inverse of

$$\begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 1 \\ -2 & 0 & 5 \end{bmatrix}$$

12. Compute the QR decomposition of the matrix in problem 11.
13. Compute the QR decomposition of

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$