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CONSTRAINTS ON DYNAMICAL TRANSPORTS OF ENERGY ON A SPHERICAL PLANET

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ABSTRACT

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The factors which control the total flux of energy across a latitude belt in the atmosphere—ocean system are determined by comparing the flux resulting from various approximations with the observed flux, and by using a one-dimensional heat-balance climate model to calculate the sensitivity of the flux to the efficiency of the dynamical transports. The results show that, as long as a hemisphere is in equilibrium and as long as the structure of the atmosphere—ocean system is dominated by the planetary scale, the total flux is constrained to peak near 35° latitude, the flux per unit area to peak near 45° latitude, and the magnitude of the flux is determined primarily by the solar constant, the size of the earth, the tilt of the earth's axis, and the hemispheric mean albedo. The magnitude of the flux is insensitive to the structure and dynamics of the atmosphere—ocean system, in part because of the high efficiency of the dynamical transport mechanisms and in part because of the negative correlation between local planetary albedo and local thermal emissions to space. These results explain why the total flux and its latitudinal distribution calculated in various experiments with the GFDL general circulation model experienced relatively little modification when the hydrological cycle, mountains, or oceans were removed from the system.

1. INTRODUCTION

Differential heating of the atmosphere—ocean system by solar radiation gives rise to motions which transport energy from low latitudes to high latitudes. In the atmosphere a variety of dynamical mechanisms contribute to this transport (Oort, 1971). Transient eddies, stationary eddies, and mean meridional circulations all make significant contributions, and the energy is transported in different forms, primarily as sensible heat, latent heat, and potential energy. In addition, the transport of sensible heat by ocean currents represents a substantial fraction of the total transport (Oort and Vonder Haar, 1976). The presence of all these different transport mechanisms raises the question, how is the total transport dependent on the particular dynamics and structure of the atmosphere—ocean system?

A preliminary answer to this question can be deduced from a number of experiments that have been carried out at the Geophysical Fluid Dynamics Laboratory (GFDL) with the GFDL numerical general circulation model (GCM). These experiments have examined how the heat transports are affected by the hydrological cycle, i.e. by latent heat (Manabe et al., 1965), by mountains, i.e. by stationary eddies (Manabe and Terpstra, 1974), and by oceans (Manabe et al., 1975). All these experiments indicate that the total meridional energy transport is insensitive to the particular mechanisms which contribute to it. The same insensitivity is also implicit in the results obtained with one-dimensional heat-balance climate models. These models fall into two classes: those which explicitly include transports by all the mechanisms listed above (Sellers, 1969 and 1973); and those which do not (Budyko, 1969; North 1975a, b). In spite of the difference, both classes of models yield very similar results.

These results in turn raise the questions, what does determine the total meridional flux? And to what extent can the insensitivity indicated by the results quoted above be generalized? In order to answer these questions, in this paper we present a number of different calculations of the total meridional transport of heat by the atmosphere—ocean system. All the calculations are based on the one dimensional heat-balance equation, but they employ different approximations and thereby reveal what factors do influence the transport.

2. SIMPLE MODELS

In equilibrium, the total flux of energy across a latitude belt, F, is given by:

$$\frac{dF}{d\phi} = 2\pi R^2 \cos\phi [Q(1-\alpha) - I] \tag{1}$$

where ϕ is latitude, R is the radius of the earth, Q is the mean incident solar radiation per unit area at latitude ϕ, α is the mean albedo at latitude ϕ, and I is the mean emitted thermal radiation per unit area at latitude ϕ. Eq. 1 has been used by Vonder Haar and Oort (1973) and Oort and Vonder Haar (1976) to calculate the annual mean flux $F(\phi)$ in the Northern Hemisphere from satellite observations of $\alpha(\phi)$ and $I(\phi)$. This equation is exact, given only the assumptions of equilibrium and of no heating of the atmosphere—ocean system from below. The structure and, implicitly, the dynamics of the atmosphere—ocean system enter eq. 1 only through the latitudinal variations of α and I.

In view of the apparent lack of sensitivity of F to the structure of the system, in this section we will calculate F for two simple models which completely neglect the structure of the atmosphere—ocean system. In particular, in these models we will assume that α is a constant, equal to its global mean value, α_0, and that I is a constant, I_0, determined by the requirement of global radiative equilibrium. By comparing the resulting solutions for $F(\phi)$ with the actual $F(\phi)$ calculated from satellite observations we can see to what extent $F(\phi)$ is determined by the structure of the atmosphere—ocean system.

In our first model we will neglect the tilt of the earth's axis in calculating F. Therefore:

$$Q = Q_{\perp} = \frac{S_0}{\pi} \cos \phi \tag{2}$$

where S_0 is the solar constant. In all our calculations in this paper we will adopt for S_0 the value of 1.95 cal/cm²/min, or equivalently, 1360 W/m². Upon substituting eq. 2 into eq. 1, integrating, and applying the boundary conditions that $F(\pm\pi/2) = 0$, we obtain:

$$I_0 = \frac{S_0}{4}(1 - \alpha_0) \tag{3}$$

$$F = S_0 R^2 (1 - \alpha_0)\left(\phi + \sin\phi\cos\phi - \frac{\pi}{2}\sin\phi\right) \tag{4}$$

If we let ϕ_0 be the latitude where F peaks, then in this model:

$$\phi_0 = \arccos\frac{\pi}{4} = \pm 38.2^\circ \tag{5}$$

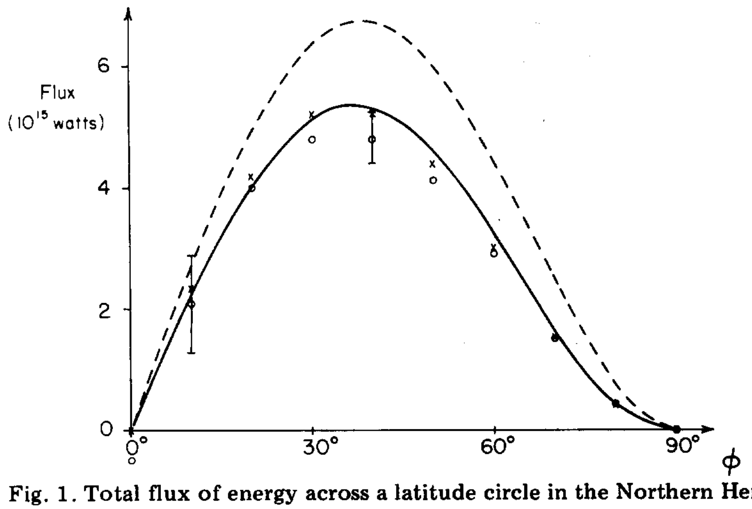

Fig. 1. Total flux of energy across a latitude circle in the Northern Hemisphere vs. latitude: (1) for an earth with no latitudinal structure and no axial tilt (dashed curve), (2) for an earth with no latitudinal structure, but with an axial tilt (solid curve), (3) for the actual earth with axial tilt and latitudinal structure included (circles), and (4) for the approximation given by eq. 14 (x's).

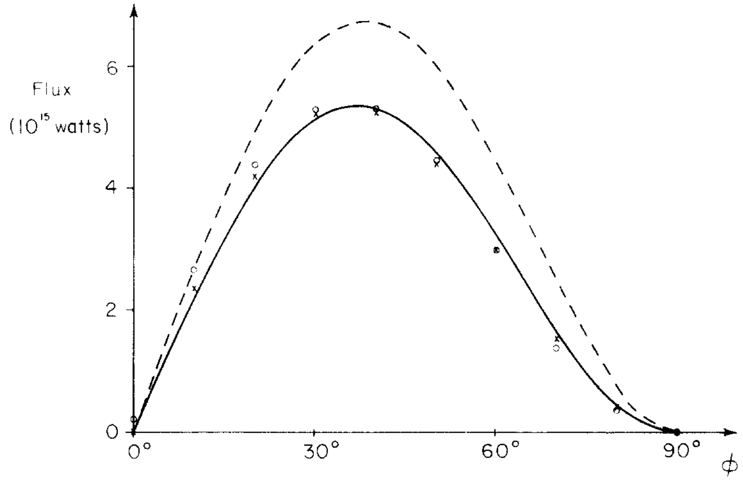

Fig. 2. Same as Fig. 1, but for Southern Hemisphere.

In our second model, we will not neglect the tilt of the earth's axis, but will use the actual distribution of $Q(\phi)$. I_0 is still given by eq. 3, but $Q(\phi)$ can no longer be given a simple analytical representation. Instead $F(\phi)$ for this model was found by numerical integration, using annual mean values of $Q(\phi)$ calculated from the tables given by Ellis and Vonder Haar (1976). $F(\phi)$ in these two models is given in Figs. 1 and 2, for the Northern and Southern Hemispheres, respectively. The dashed curves show $F(\phi)$ for the first model (no axial tilt) and the solid curves show $F(\phi)$ for the second model (axial tilt included). In the calculations the values $R = 6.37 \cdot 10^6$ m and $\alpha_0 = 0.325$ were used. This value of α_0 is the global mean value of α calculated from Ellis and Vonder Haar's (1976) tabulation of satellite observations of albedos. For these two models this value is also the planetary (Bond) albedo. It is slightly greater than the actual planetary albedo since there is no correlation between Q and α in these models.

For comparison, we include in Figs. 1 and 2 the distributions of $F(\phi)$ when the structure of the atmosphere—ocean system is included, i.e., $F(\phi)$ found by integrating eq. 1. These distributions are indicated by the circles in Figs. 1 and 2. The distribution in the Northern Hemisphere is taken from Oort and Vonder Haar (1976), and we have included in Fig. 1 some error bars for this distribution, indicating the probable errors in the individual values due to observational errors. These errors increase equatorward since $F(\phi)$ was found by integrating eq. 1 from the pole with the boundary condition $F(\pi/2) = 0$. The distribution of F in the Southern Hemisphere was calculated by integrating eq. 1 numerically with the boundary condition $F(-\pi/2) = 0$, and with the annual mean values of $\alpha(\phi)$ and $I(\phi)$ for the Southern Hemisphere tabulated by Ellis and Vonder Haar (1976).

Comparing the results of the two simple models with the actual flux distributions shown in Figs. 1 and 2 we see that in fact the structure of the atmosphere—ocean system makes virtually no contribution to the meridional flux. In both hemispheres $F(\phi)$ given by the second model generally agrees with the actual flux within the probable error due to observational uncertainty. The factors that do appear to be important in determining the flux are the solar constant, the axial tilt, the radius of the earth, and the mean albedo. Only the last of these can be considered to be an internal parameter of the atmosphere—ocean system. Since the albedo is due primarily to clouds, ice, and snow, which are strongly constrained by the temperature, even this parameter is strongly constrained by the external parameters, in particular by the solar constant. The insensitivity of the flux is also directly apparent from a comparison of the actual flux in the two hemispheres. In spite of the substantial differences in topography and ocean area of the two hemispheres, which one would expect to lead to substantial differences in the stationary eddy and oceanic components of the meridional heat transport, the fluxes in the two hemispheres are the same within the observational uncertainty.

3. SERIES REPRESENTATION OF THE FLUX

Considerable insight into why the flux is insensitive to the structure and dynamics of the atmosphere—ocean system can be gained by expanding the flux in a Fourier series. For this purpose it is convenient to rewrite eq. 1 in non-dimensional form, and to use the sine of the latitude for the independent variable. We define:

$$x = \sin \phi \tag{6}$$

$$Q = \frac{S_0}{4} s(x) \tag{7}$$

$$I = \frac{S_0}{4} i(x) \tag{8}$$

$$F = \frac{\pi R^2 S_0}{2} f(x) \tag{9}$$

$$a = 1 - \alpha(x) \tag{10}$$

Then eq. 1 can be rewritten as:

$$\frac{\mathrm{d}f}{\mathrm{d}x} = sa - i \tag{11}$$

Since we are interested in a representation of the flux valid in a hemisphere, it is sufficient to expand, s, a, and i in a series of Legendre Polynomials of even order:

$$\left.\begin{aligned} s &= s_0P_0(x) + s_2P_2(x) + s_4P_4(x) + \ldots \\ a &= a_0P_0(x) + a_2P_2(x) + a_4P_4(x) + \ldots \\ i &= i_0P_0(x) + i_2P_2(x) + i_4P_4(x) + \ldots \end{aligned}\right| \quad (12)$$

where:

$$P_0(x) = 1,\ P_2(x) = \tfrac{1}{2}(3x^2 - 1),\ P_4(x) = \tfrac{1}{8}(35x^4 - 30x^2 + 3), \text{ etc.}$$

We have defined s in such a way that its mean value, s_0, is automatically unity. The values of the first few coefficients in these series, calculated from Ellis and Vonder Haar's (1976) tables of the annual mean latitudinal distributions, are given in Table I for both the Northern and Southern Hemispheres; $s(x)$ is the same in both hemispheres.

To obtain the series representation of $f(x)$, we substitute eq. 12 into eq. 11, re-expand the right-hand side of the equation in a series of Legendre Polynomials of even order, integrate, and apply the boundary condition $f(1) = 0$. We obtain:

$$f = \left(s_0a_0 + \frac{s_2a_2}{5} + \ldots - i_0\right)(x - 1) + \left(s_0a_2 + s_2a_0 + \tfrac{2}{7}s_2a_2 + \ldots - i_2\right)\frac{(x^3 - x)}{2} + \ldots \quad (13)$$

Now we consider the approximation in which we neglect all terms in the expansions of eq. 12 which are of fourth order or higher, i.e., in our representation for f we retain only the terms that are explicitly written out in eq. 13. The values of the coefficients shown in Table I indicate that the resulting approximation will have an accuracy of about 10%, which is comparable to the uncertainty introduced by observational errors. All the terms in the expansion of f, except for the first one, which is proportional to

TABLE I

Values of coefficients in the expansions of eq. 12 for the Northern and Southern Hemispheres (N.H. and S.H., respectively)

n	0	2	4
s_n	1	−.473	−.086
a_n (N.H.)	.675	−.192	−.057
i_n (N.H.)	.687	−.165	−.031
a_n (S.H.)	.675	−.217	−.085
i_n (S.H.)	.694	−.179	−.051

$(x-1)$, are automatically zero at the equator, $x = 0$. Therefore, the coefficient of the first term in the expansion must be zero if the hemisphere is to be in equilibrium (no flux across the equator). Within the observational uncertainty, both hemispheres are in fact in equilibrium. For example, we calculate that the ratio of the first coefficient to the second coefficient in the expansion of eq. 13 is less than 0.02 in magnitude for both hemispheres. Therefore we will also neglect the first term in the expansion. We are then left with the simple approximation:

$$f \doteq \tfrac{1}{2}(s_0 a_2 + s_2 a_0 + \tfrac{2}{7} s_2 a_2 - i_2)(x^3 - x) \tag{14}$$

The values of F based on this approximation are shown by the x's in Figs. 1 and 2. We see that they do indeed agree with the more accurately calculated values, within the observational uncertainty.

In deriving the above approximation we have not neglected the structure of the atmosphere—ocean system *per se*. This structure still enters eq. 14 through the coefficients a_2 and i_2. However in this approximation the structure does not affect the *distribution* of the flux with latitude. This distribution is given by a prescribed function $(x^3 - x)$ which has its peak value at a latitude:

$$\phi_0 = \arcsin \frac{1}{\sqrt{3}} = 35.3^\circ \tag{15}$$

This is in excellent agreement with the observations (see Figs. 1 and 2). Only two physical assumptions have been made in deriving this approximation: (1) that the hemisphere is in approximate equilibrium (so the first term in the expansion can be neglected); and (2) that the structure of the atmosphere—ocean system is dominated by the planetary scale (so the third and higher terms in the expansion can be neglected.) The sensitivity of the distribution to the presence of higher-order structure is not great. For example, if in the Northern Hemisphere we retain all the terms in the expansion of f, and arbitrarily assign the third term a magnitude double its observed value, but leave the other terms unchanged, the peak in the flux only shifts to $\phi_0 = 37^\circ$; if the magnitude of the third term is tripled, it only shifts to $\phi_0 = 39^\circ$. Thus we expect that the total meridional flux will peak near 35° latitude as long as the two assumptions stated above continue to hold.

The above approximation, eq. 14, gives formal justification to an approximation sometimes used in climate studies — namely, the approximation that to first order the magnitude of the meridional flux depends on the structure of the atmosphere—ocean system and therefore must be allowed to vary, but that its distribution does not depend on the structure and therefore can be held fixed. For example, in our studies of the effect of baroclinic eddies on atmospheric structure (Stone, 1972, 1973), we implicitly made such an approximation, by specifying the latitudinal distribution of the meridional flux per unit area to be given by:

$$\frac{8}{\pi}\phi\left(1-\frac{2}{\pi}\phi\right)$$

According to eq. 14 the flux per unit area is proportional to:

$$\frac{f}{\cos\phi} \propto \frac{x-x^3}{\sqrt{1-x^2}} \propto \sin 2\phi$$

These two functions both peak at 45° and do not differ by more than 5%. Another example of the use of this kind of approximation is North's two-mode version of the one-dimensional heat-balance models (North, 1975b). In his model the meridional flux is prescribed to have the same latitude distribution as that given by eq. 14.

The existence of the constraint that the flux per unit area must peak near 45° latitude regardless of the structure and dynamics of the atmosphere–ocean system implies that, in climate models, it is of particular importance to represent accurately the dynamical processes and transports that dominate near 45° latitude. Studies of the contributions of different mechanisms to the meridional transports (Oort, 1971; Oort and Vonder Haar, 1976) show that the most important mechanism is transient atmospheric eddies, which contribute 62% of the total transport at 45° N, and that the next most important is ocean transports, which contribute 30% of the total. In fact this constraint offers a simple explanation of why transient eddy activity peaks near 45° N. Fig. 3 shows the mean of the variance of the meridional velocity associated with transient eddies as a function of latitude, taken from Oort and Rasmusson (1971). The variance actually peaks at 50° N. The meridional transport of energy associated with the transient eddies also peaks at 50° N

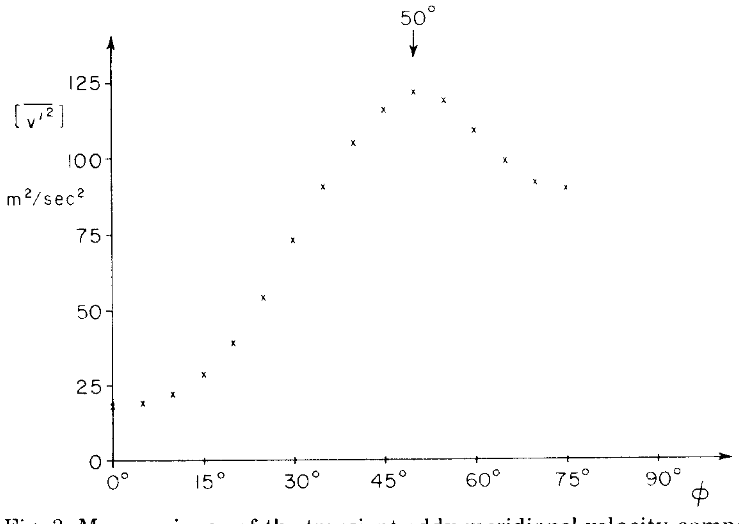

Fig. 3. Mean variance of the transient eddy meridional velocity component vs. latitude from Oort and Rasmusson (1971).

TABLE II

Locations of flux maxima in two sets of experiments with the GFDL GCM

	LATENT HEAT FLUX	ALL OTHER FLUXES	TOTAL FLUX
Moist Model	32° N	7° N, 49° N	40° N
Dry Model	—	39° N	39° N

(a) Effect of hydrological cycle (from Manabe et al., 1965)

	OCEANIC FLUX	ATMOSPHERIC FLUX	TOTAL FLUX
Atmosphere-Ocean Model	23° N	37° N	36° N
Atmosphere Model	—	36° N	36° N

(b) Effect of oceans (from Manabe et al., 1975)

(Oort and Rasmusson, 1971). The displacement of the peak from 45° N to 50° N can be attributed to the presence of the oceans, which contribute a significant transport at 45° N, but not at 60° N (Oort and Vonder Haar, 1976).

This constraint also shows that there must be substantial negative feedback between the latitudinal distributions of flux associated with different kinds of transports. This negative feedback is clearly illustrated in two of the GFDL experiments referred to above. In both the experiment concerned with the effect of the hydrological cycle (Manabe et al., 1965) and the experiment concerned with the effect of the oceans (Manabe et al., 1975) all the components of the meridional flux and their latitudinal distributions were calculated and can be compared. Table II shows the locations of the maxima in the meridional flux in these two experiments. In the first experiment (Table IIa), when the hydrological cycle was included, the meridional flux of latent heat had a single maximum at 32° N, and the sum of all the other meridional fluxes had two peaks, the larger one at 7° N, associated with the Hadley cell, and the smaller one at 49° N, associated with mid-latitude eddies. Nevertheless the total flux had a distribution similar to that given by eq. 14, with a single peak at 40° N. (In this model the series represented by eq. 13 converges more slowly then in the real atmosphere.) However, when the latent heat fluxes were removed, the distribution of the sum of the other fluxes was completely changed, so that it contained a single peak at 39° N. Consequently, the distribution of the total flux was virtually unchanged. Similarly, in the second experiment (Table IIb), when the oceans were removed the distribution of the total flux was unaffected.

4. THE FLUX MAXIMUM

The above approximation, eq. 14, leads to the following expression for the maximum meridional flux at 35° latitude:

$$F_{\max} \doteq \frac{\pi R^2 S_0}{6\sqrt{3}}(-s_2 a_0 - s_0 a_2 - \tfrac{2}{7} s_2 a_2 + i_2) \tag{16}$$

Since this expression does contain a_2 and i_2, a priori we would expect the atmospheric—oceanic structure to affect $F_{\max}$. This apparent dependence can be reconciled with the apparent lack of dependence illustrated in Figs. 1 and 2 by examining the contributions of the individual terms in eq. 16 to the total flux. The break-down for the Northern and Southern Hemispheres (N.H. and S.H. respectively) is shown in Table III. The last three terms, which do depend on the structure of the atmosphere—ocean system, cancel almost exactly. Thus one can represent the flux by an even simpler approximation:

$$f \doteq \tfrac{1}{2} s_2 a_0 (x^3 - x) \tag{17}$$

$$F \doteq \frac{\pi R^2 S_0}{4} s_2 a_0 (x^3 - x) \tag{18}$$

$$F_{\max} \doteq \frac{\pi R^2 S_0}{6\sqrt{3}}(-s_2 a_0) \tag{19}$$

This approximation illustrates mathematically the result stated in section 2, that the flux appears to depend only on the earth's radius (R), the solar constant (S_0), the tilt of the earth's axis (which determines s_2) and the hemispheric mean albedo ($a_0 = 1 - \alpha_0$). If eq. 18 were plotted in Figs. 1 and 2 it would be indistinguishable from the flux calculated from the approximation based on eq. 14.

The cancellation of the structure factors in eq. 16 is primarily due to the correlation between the local absorption of solar radiation and the local emission of thermal radiation, i.e., $a_2 \doteq i_2$ (see Table I). This correlation has a physical basis in the feedback between radiation and temperature: an

TABLE III

Contributions of the different coefficients in eq. 16 to the maximum flux in the Northern Hemisphere (N.H.) and the Southern Hemisphere (S.H.)

	$-s_2 a_0$	$-s_0 a_2$	$-\frac{2}{7} s_2 a_2$	$+i_2$	Sum
N.H.	+.319	+.192	−.026	−.165	+.320
S.H.	+.319	+.217	−.029	−.179	+.328

increased absorption of solar radiation implies higher temperatures, which in turn implies higher thermal emissions to space. However the extent to which these two terms cancel must depend on how far the atmosphere—ocean system is from local radiative equilibrium. Therefore the precise cancellation of the structure terms in eq. 16 may not hold if the structure and dynamics of the atmosphere—ocean system change from those of the current state.

One convenient way of determining the extent to which the flux maximum is dependent on how far the atmosphere—ocean system is from local radiative equilibrium, is to examine the behavior of the system as modeled by a one-dimensional heat-balance climate model. Although such models are greatly simplified versions of the atmosphere—ocean system, they do incorporate the constraints inherent in eq. 1, they do include the feedback between radiation and temperature, and they are sufficiently flexible to allow one to vary how far the system is from radiative equilibrium. For this purpose we will use North's two-mode version of the one-dimensional heat-balance climate models (North, 1975b).

In this model the meridional flux is modeled by a simple diffusion law:

$$F = -2\pi R^2 (1 - x^2)\, D \frac{\mathrm{d}I}{\mathrm{d}x} \tag{20}$$

where D is a dimensionless diffusion coefficient. Therefore the heat balance equation, eq. 1, becomes a differential equation for I, with the non-dimensional form:

$$-D \frac{\mathrm{d}}{\mathrm{d}x}(1 - x^2) \frac{\mathrm{d}i}{\mathrm{d}x} = sa - i \tag{21}$$

The hemisphere is assumed to be in equilibrium so that the boundary conditions for this second order equation are:

$$(1 - x^2)^{1/2} \frac{\mathrm{d}i}{\mathrm{d}x} = 0 \text{ at } x = 0, 1 \tag{22}$$

$s(x)$ is a known function.

The absorption of solar radiation is modeled by:

$$a = \begin{Bmatrix} 0.697 - 0.0779 P_2(x), & x < x_s \\ 0.38, & x > x_s \end{Bmatrix} \tag{23}$$

where x_s, the position of the edge of the ice sheet, is implicitly defined by the relation

$$i(x_s) = 0.585 \tag{24}$$

In eq. 24 the constant has been evaluated for the current value of the solar constant. In the two-mode approximation i is expanded in a series of Legendre polynomials of even order, and truncated after the second term. Thus

F has the same form as in the approximation given by eq. 14. With this truncation the solution of eqs. 21 to 24 reduces to the solution of three coupled algebraic equations for i_0, i_2, and x_s. The details are given by North (1975b).

The value of D still has to be specified in order to solve the equations. North determined D by choosing it so that x_s had a value of 0.95, corresponding to the current climate. This yielded a value $D = 0.382$. However, we are interested in examining the behavior of the solution as a function of D. D is in fact a measure of the efficiency of the dynamical fluxes in transporting energy from low to high latitudes. When $D = 0$, the system is in local radiative equilibrium, and as D increases the system deviates more and more from local radiative equilibrium. Thus by varying D one can gain insight into the circumstances under which F_{max} depends on the structure and dynamics of the atmosphere—ocean system.

Fig. 4 shows F_{max} as a function of D, calculated from eq. 20 and the solutions to eqs. 21 to 24. The cusp in the curve is caused by the disappearance of the polar ice cap for high efficiencies, i.e., a polar ice cap exists for $D < 0.395$, but not for $D > 0.395$. Fig. 4 shows that in fact F_{max} is insensitive to the dynamical efficiency as long as $D > 0.1$. For example, F_{max} does not deviate by more than 10% from a constant value in the range $0.15 < D < 0.85$. In fact there are two other values of D which give the same value of F_{max} as $D = 0.382$, namely 0.194, and 0.502. These values correspond to climates radically different from the current climate, i.e., an ice age with the ice cap extending to 44° latitude, and an ice-free earth, respectively. The ability of one-dimensional heat-balance models to correctly predict the ob-

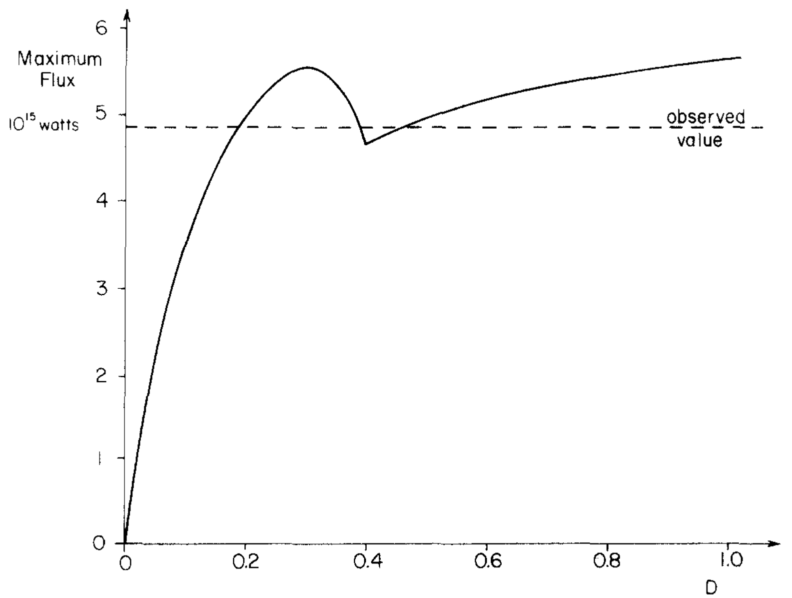

Fig. 4. Maximum total meridional flux as a function of the diffusion coefficient.

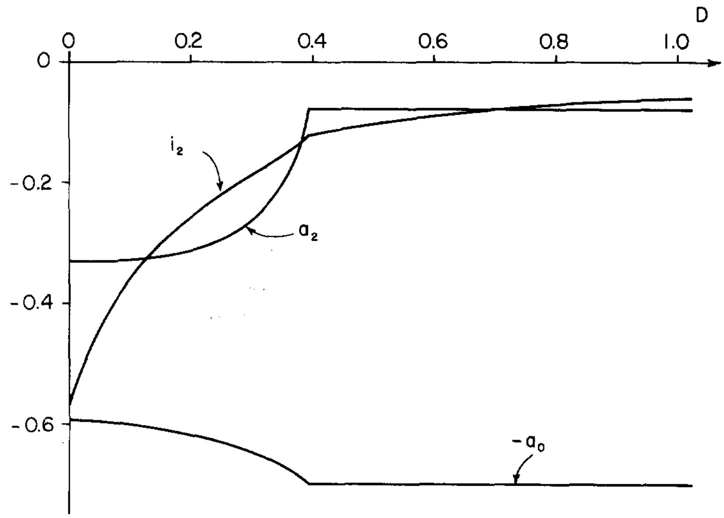

Fig. 5. Values of the expansion coefficients as a function of the diffusion coefficient.

served values of F even though they have not been tuned to do so is sometimes quoted as an important verification of the models' validity (Paltridge, 1975; North, 1975b). Fig. 4 shows that in fact this is not a good check of the models' validity. As long as the external parameters and the current value of the mean albedo have been specified correctly F_{max} is constrained to have about the right value.

The reasons for the insensitivity of F_{max} are shown by the behavior of the three internal factors which are important in determining F_{max}, namely, a_0, a_2, and i_2. These factors are shown in Fig. 5 as functions of the diffusion coefficient. One can distinguish three basically different regimes, depending on whether the dynamical efficiency is low, intermediate, or high. In the low-efficiency regime, F and D approach zero, and the system is near local radiative equilibrium. F can only approach zero if i_2 and/or a_2 are important in determining F_{max} (see eq. 16), and therefore the flux must be sensitive to the structure of the atmosphere—ocean system in this regime. Figs. 4 and 5 show that this regime occurs for $D < 0.1$.

In the high-efficiency regime the atmosphere approaches homogeneity, i.e., $i_2 \to 0$. a_2 also approaches an asymptotic value, although in general this value will not be zero because of the zenith-angle dependence of cloud and surface albedos. This asymptotic value is not dependent on the structure of the system, but rather on the variation of solar zenith angle with latitude, and thus is determined by the geometry of the system. Therefore in the high-efficiency regime, because of homogeneity, F_{max} approaches an asymptotic value which is the maximum flux allowed by the net differential radiative heating and which is necessarily independent of the structure of the atmosphere—ocean system. In North's model the asymptotic value of a_2 is

-0.0779, which is based on observed albedos. The corresponding asymptotic value of F_{max} is $6.5 \cdot 10^{15}$ W. Thus the observed values of the flux in the Northern and Southern Hemispheres are about 75% of the maximum possible, according to North's model. If one allows for the fact that the observations which North's value of a_2 are based on derive from conditions in which cloud cover increases with latitude rather than being homogeneous, then the true asymptotic value of F_{max} is probably less than $6.5 \cdot 10^{15}$ W. Figs. 4 and 5 show that the high-efficiency regime occurs for $D > 0.4$.

Finally, for $0.1 < D < 0.4$, there is a regime of intermediate efficiency, where F_{max} is not sensitive to D (see Fig. 4), but the structure of the atmosphere—ocean system is (see the curve for i_2 in Fig. 5). The insensitivity of F_{max} in this regime is due to the high degree of correlation between a_2 and i_2 shown in Fig. 5. Physically, this is due to the high negative correlation between planetary albedo and thermal emissions to space. This correlation prevents the net radiative heating and the dynamical flux required for balance from changing very much, even though individually a_2 and i_2 may change substantially.

Since the current efficiency of the atmosphere—ocean system ($D \cong 0.38$) is on the borderline between the intermediate- and high-efficiency regimes, one can explain the insensitivity of the meridional flux to the structure and dynamics of the atmosphere—ocean system found in section 2 as being partly due to the correlation between albedo and thermal emissions, and partly due to the high efficiency of the atmosphere—ocean system. If we exclude changes so extreme as to push the atmosphere—ocean system into the low efficiency regime ($D < 0.1$), then the above results indicate that even extreme changes in the structure and dynamics of the atmosphere—ocean system cannot change the flux by more than about 20%. This places a quantitative limit on the conclusion reached in section 2, that the flux is primarily determined by the solar constant, the axial tilt, the radius of the earth, and the mean albedo.

This result implies that there is a strong negative feedback between the *magnitudes* of the fluxes carried by different mechanisms, just as there is between the latitudinal distributions. Since the sources of this feedback are present in GCM's, one would also expect this feedback to be illustrated by the two GFDL GCM experiments described in section 3. The experiments' results for the magnitudes of the different fluxes, at the latitude where the total flux peaks, are given in Table IV. We see that the changes in the total flux are indeed much smaller than the changes in the individual components. This feedback is also apparent in the third GFDL experiment referred to in the introduction, the one concerned with the effect of mountains (Manabe and Terpstra, 1974). In that experiment only winter conditions were simulated, so hemispheric equilibrium did not occur. Nevertheless, the results did show a strong negative feedback between the different components of the eddy flux: when the mountains were removed, the flux carried by stationary eddies was reduced considerably, but this was largely compensated by a

TABLE IV

Magnitude of different fluxes (10^{15} W) at the latitudes of the peak in the total flux in two sets of experiments with the GFDL GCM

	Latent Heat Flux	All Other Fluxes	Total Flux
Moist Model	2.4	2.2	4.6
Dry Model	0	3.8	3.8

(a) Effect of hydrological cycle (from Manabe et al., 1975)

	Oceanic Flux	Atmospheric Flux	Total Flux
Atmosphere - Ocean Model	0.8	4.6	5.4
Atmosphere Model	0	5.4	5.4

(b) Effect of oceans (from Manabe et al., 1975)

simultaneous increase in the flux carried by transient eddies. Since the GFDL model does not include all the important feedback mechanisms in the atmosphere—ocean system — e.g., cloud cover and albedo are fixed — the results of the GCM experiments cannot be used to obtain a quantitative estimate of the strength of the negative feedback between the various fluxes.

5. CONCLUSIONS

Our calculations based on the one-dimensional heat-balance equation show that, as long as a hemisphere is in equilibrium, and as long as the structure of the atmosphere—ocean system is dominated by the planetary scale, the total meridional flux of energy is constrained to peak near 35° latitude, the flux per unit area is constrained to peak near 45° latitude, and the magnitude of the flux is determined primarily by the solar constant, the earth's radius, the tilt of the earth's axis and the mean albedo. These results explain why eddy activity peaks near 45° latitude and why GCM experiments have found the total flux to be insensitive to the presence of oceans, mountains or the hydrological cycle. They also lead one to expect that the total flux will not be sensitive to other factors which would affect the dynamics, e.g., the static stability, the meridional temperature gradient, and the rotation rate of the atmosphere—ocean system. The insensitivity of the magnitude of the flux is apparently caused by two factors: the correlation between albedo and thermal emissions to space, and the relatively high efficiency of the transport mechanisms in the atmosphere—ocean system.

These results are of interest in developing climate models. The constraints on the latitudinal dependence of the flux give a formal justification for fixing the latitudinal dependence of the total flux in simple models while only allowing its magnitude to respond to changes in climate. The constraint that the flux per unit area must peak near 45° latitude implies that it is particularly important in climate models to represent accurately mid-latitude transport mechanisms, i.e., transports by transient atmospheric eddies and by the oceans. The insensitivity of the flux to internal parameters means that the ability of a climate model to reproduce the observed flux is not a good test of the model's reliability. On the other hand, this same lack of sensitivity means that the flux needed for equilibrium is known accurately beforehand, so that a comparison between the flux in a time-dependent model and the expected flux provides an excellent test as to whether the model has reached equilibrium. This test could be particularly useful for non-equilibrium models of systems with long relaxation times, e.g., the earth's oceans, and the atmospheres of Venus and Jupiter.

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