

# Computation with polynomials



Instructor: Pei-Chiang Shao (邵培強)

Department of Mathematics, National Central University  
Jhongli District, Taoyuan City 32001, Taiwan (R.O.C.)

E-mail: [shaopj823@gmail.com](mailto:shaopj823@gmail.com)

## Computation with polynomials

---

**Example 1.** Construct polynomials  $p(x) = x^3 - 3x^2 + 5x - 1$  and  $q(x) = x^4 - 12x^3 + 31x^2 + 15$  in matlab.

```
% matlab code  
p = [1 -3 5 -1]  
q = [1 -12 31 0 15]
```

## Computation with polynomials

---

**Example 2.** Given the roots  $r_1 = -1, r_2 = 0, r_3 = 1$  of a polynomial  $p$ , construct this polynomial in matlab using `poly()` function.

Hint:  $p(x) = (x - r_1)(x - r_2)(x - r_3) = (x + 1)x(x - 1) = x^3 - x$

```
% matlab code
r = [-1 0 1]           % root vector
p = poly(r)             % coefficient vector
ps = poly2str(p, 'x')   % polynomial string
```

## Computation with polynomials

---

**Example 3.** Given the root vector  $[-0.5, -0.3 + 0.4i, -0.3 - 0.4i]$  of a polynomial. Construct this polynomial.

```
% matlab code  
p = poly([-0.5 -0.3+0.4i -0.3-0.4i])  
ps = poly2str(p, 'y')
```

## Computation with polynomials

---

**Example 4.** Find the roots of the polynomial

$p(x) = x^3 - 10x^2 + 31x - 30$ . That is, find  $[r_1, r_2, r_3]$  such that  $p(x) = (x - r_1)(x - r_2)(x - r_3)$ .

```
% matlab code  
p = [1 -10 31 -30]  
r = roots(p)  
  
q = poly(r)
```

## Computation with polynomials

---

**Example 5.** Evaluate the values of  $p(x) = x^3 + 2x^2 - 5x + 6$  at  $x = -1.5, 0, 1.5$ .

```
% matlab code
p = [1 2 -5 6]

x = 1
y = polyval(p, x)

x = [-1.5 0 1.5]
y = polyval(p, x)
```

## Computation with polynomials

---

**Example 6.** Plot the graph of the polynomial

$$p(x) = x^3 + 2x^2 - 5x + 6.$$

```
% matlab code
p = [1 2 -5 6]
x = -2:0.1:2
y = polyval(p, x)

figure
plot(x,y)
```

## Computation with polynomials

---

**Example 7.** Given two polynomials  $a(x) = x^3 + 2x^2 + 3x + 4$  and  $b(x) = 4x^2 + 9$ . Plot the graph of  $s(x) := a(x) + b(x)$  on  $[-1, 1]$ .

```
% matlab code
a = [1 2 3 4]
b = [0 4 0 9]    % a and b should have the same length
s = a + b

x = -1:0.1:1
y = polyval(s, x)
figure
plot(x,y)
```



## Computation with polynomials

---

**Example 8.** Given two polynomials  $a(x) = x^3 + 2x^2 + 3x + 4$  and  $b(x) = 4x^2 + 9$ . Plot the graph of  $s(x) := a(x)b(x)$  on  $[-1, 1]$ .

```
% matlab code
a = [1 2 3 4]
b = [4 0 9]      % a and b can have different length
s = conv(a,b)     % convolution of a and b

x = -1:0.1:1
y = polyval(s, x)
figure
plot(x,y)
```

## Computation with polynomials

---

**Example 9.** Given two polynomials  $a(x) = x^3 + 2x^2 + 3x + 4$  and  $b(x) = 4x^2 + 9$ . Find  $q(x)$  and  $r(x)$  such that  $a(x) - r(x) = b(x)q(x)$ .

```
% matlab code
a = [1 2 3 4]
b = [4 0 9] % notice: the first element cannot be zero
[q, r] = deconv(a,b) % deconvolution
qs = poly2str(q, 'x')
rs = poly2str(r, 'x')

LHS = a - r
RHS = conv(b, q)
```

## Computation with polynomials

---

Quiz 1. Plot the graph of polynomial

$$p(x) = (x^2 + 2x + 2)(5x + 4)(x^3 - 1)(3x^2 + 6x - 2) \text{ on } [-1, 1].$$

## Computation with polynomials

---

**Example 10.** Given a polynomial  $p(x) = 2x^3 - 5x^2 + 6x - 1$ . Plot  $p(x)$  and  $p'(x)$  on  $[-1, 1]$ .

```
% matlab code
p = [2 -5 6 -1]
Dp = polyder(p)
Dps = poly2str(Dp, 'x')

x = -1:0.01:1 ;
y = polyval(p, x) ;
Dy = polyval(Dp, x) ;

figure
plot(x,y)
hold on
plot(x, Dy, 'r')
```

## Computation with polynomials

---

**Quiz 2.** Given two polynomials  $a(x) = x^2 + 3x + 5$  and  $b(x) = 2x^2 + 4x + 6$ . Compute the derivative of  $a(x)b(x)$  and  $a(x)/b(x)$ .

Hint: 
$$\left[ \frac{a(x)}{b(x)} \right]' = \frac{a'(x)b(x) - a(x)b'(x)}{b(x)^2}.$$

## Computation with polynomials

---

**Example 11.** Evaluate the definite integral

$$I = \int_{-1}^3 (3x^4 - 4x^2 + 10x - 25) dx.$$

```
% matlab code
a = -1 ;
b = 3 ;
p = [3 0 -4 10 -25]
Intp = polyint(p)      % the indefinite integral of p
I = polyval(Intp, b) - polyval(Intp, a)
```

## Computation with polynomials

---

Quiz 3. Evaluate the definite integral

$$I = \int_0^2 (4x^5 - 2x^3 + 6)(7x^2 + 3x - 2) dx.$$