Finite series

Instructor: Pei-Chiang Shao (邵培強)

Department of Mathematics, National Central University
Jhongli District, Taoyuan City 32001, Taiwan (R.O.C.)

E-mail: shaopj823@gmail.com

The colon operator : and sum() function

Example 1. Calculate the following finite series:

1. $a = 1 + 2 + 3 + \cdots + 100$
2. $b = 2 + 4 + 6 + \cdots + 100$
3. $c = 1^2 + 2^2 + 3^2 + \cdots + 100^2$
4. $d = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{100}$

```
% matlab code
a = sum(1:1:100)
b = sum(2:2:100)
c = sum((1:100).^2)
d = sum(1./(1:100))
```

The colon operator : and sum() function

Example 2. Calculate the following finite series:

$$S = 1 + \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots + \frac{99}{100}$$

```
% matlab code
a = 1:99 ;
b = 2:100 ;
S = 1 + sum(a./b)
```

The colon operator : and sum() function

Example 3. Calculate the following finite series:

$$R = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots + \frac{1}{99} - \frac{1}{100}$$

```
% matlab code
% first way :
% R = (1 + 1/3 + ... + 1/99) - (1/2 + 1/4 + ... + 1/100)
S = sum(1./(1:2:100))
T = sum(1./(2:2:100))
R = S - T
% second way :
x = 1./(1:100) ;  % x = [1, 1/2, 1/3, 1/4, ..., 1/100]
R = sum(x(1:2:100)) - sum(x(2:2:100)) ;
% third way :
x = 1./(1:100) ;
p = -(-1).^(1:100) ;  % alternating {1,-1} sequence
R = sum(x.*p) ;
```

Using for loop

Example 4. Execute the following matlab code and see what happened.

```
% matlab code
s = 0
s = s + 1
s = s + 2
s = s + 3
s = s + 4
s = s + 5
s = s + 6
s = s + 7
s = s + 8
s = s + 9
s = s + 10
```

Using for loop

Example 5. Calculate the following finite series using for loop:

$s = 1 + 2 + 3 + \cdots + 10$

```
% matlab code
s = 0
for k = 1 : 10
    s = s + k
end
```

Using for loop

Example 6. Calculate the following finite series using for loop:

$s = 2 + 4 + 6 + \cdots + 100$

```
% matlab code
x = 2 : 2 : 100 ;

s1 = sum(x) ;

s2 = 0 ;
for k = x
    s2 = s2 + k
end

Check = (s1 == s2)
```

Using for loop

Example 7. Use for loop to compute

$$S = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots + \frac{1}{99^2} + \frac{1}{100^2}$$

```
% matlab code
S = 0 ;
for k = 1 : 100
    S = S + (1/(k^2)) ;
end
S
```

Using for loop

Example 8. Use for loop to compute

$$S = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots + \frac{1}{9} - \frac{1}{10}$$

```
% matlab code
S = 0 ;
for k = 1 : 2 : 9
    S = S + (1/k) - (1/(k+1)) ;
end
S
```

Using for loop

Quiz 1. Compute

$$S = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} - \frac{1}{128} + \frac{1}{256} - \frac{1}{512} + \frac{1}{1024}.$$

The prod() function

Example 9. Calculate 12!.

```
% matlab code
x = 1 : 12 ;
P1 = prod(x) ;

P2 = 1 ;
for k = x
    P2 = P2 * k ;
end

Check = (P1 == P2)
```

The factorial() and nchoosek() functions

Example 10. Calculate C_2^5.

```
% matlab code
n = 5 ;
k = 2 ;

C1 = (n*(n-1))/(k*(k-1))
C2 = prod(1:n)/(prod(1:k)*prod(1:n-k))
C3 = factorial(n)/(factorial(k)*factorial(n-k))
C4 = nchoosek(n,k)
```

Using for loop

Quiz 2. Compute

$$\frac{1}{2!}+\frac{3}{4!}+\frac{5}{6!}+\frac{7}{8!}+\cdots+\frac{51}{52!}$$

Using for loop

Quiz 3. Compute

$$\frac{1}{2}+\frac{3}{4}+\frac{7}{6}+\frac{13}{8}+\frac{21}{10}+\cdots+\frac{2451}{100}$$