103590450 四資四 馬茂源

1. It is known that the HIV test has only 0.1% of false positive and false negative, respectively. However, for a specific group of people, the prevalence of HIV positive rate is 0.01 %. If a person belongs to such a group and is found to be positive in the HIV test, find the probability that the person is really infected.

	true	false
test positive	`0.01% * 99.9%`	`99.99% *  0.1%`
test negative	`0.01% *  0.1%`	`99.99% * 99.9%`
	`0.01%`	`99.99%`

P(really infected | test positive) = precision =

$$\frac{TP}{TP+FP}$$

= `0.01% * 99.9%` / (`0.01% * 99.9%` + `99.99% * 0.1%`) = 0.09083469721767594 = **9.08%**

```
from sklearn.model_selection import train_test_split, cross_val_score
from sklearn.metrics import accuracy_score
from sklearn.neighbors import KNeighborsClassifier
from sklearn.naive_bayes import GaussianNB
from sklearn.datasets import load_iris
import numpy as np
import matplotlib.pyplot as plt
import operator
```

```
iris = load_iris()
print(iris.DESCR)
```

```
Iris Plants Database
====================

Notes
-----
Data Set Characteristics:
```

```
:Number of Instances: 150 (50 in each of three classes)
:Number of Attributes: 4 numeric, predictive attributes and the class
:Attribute Information:
    - sepal length in cm
    - sepal width in cm
    - petal length in cm
    - petal width in cm
    - class:
            - Iris-Setosa
            - Iris-Versicolour
            - Iris-Virginica
:Summary Statistics:

============== ==== ==== ======= ===== ====================
                Min  Max   Mean    SD   Class Correlation
============== ==== ==== ======= ===== ====================
sepal length:   4.3  7.9   5.84   0.83    0.7826
sepal width:    2.0  4.4   3.05   0.43   -0.4194
petal length:   1.0  6.9   3.76   1.76    0.9490  (high!)
petal width:    0.1  2.5   1.20  0.76     0.9565  (high!)
============== ==== ==== ======= ===== ====================

:Missing Attribute Values: None
:Class Distribution: 33.3% for each of 3 classes.
```

2 . UC Irvine has a large repository for various kinds of data. In this problem, you are asked to use the iris dataset (https://archive.ics.uci.edu/ml/datasets/Iris) to perform the experiments. Implement the k-NN classifier for the classification task. To begin one experiment, randomly draw 70 % of the instances for training and the rest for testing. Repeat the drawing and the k-NN classification 10 times and compute the average accuracy. Then, plot the curve of k versus accuracy for k = 1, 3, …, 15. For simplicity, use the Euclidean distance in your computation.

[3]

```
N = 10 #200次的話曲線比較穩定 不過題目說10次就用10次
```

[4]

```
class MyKNeighborsClassifier:

    def __init__(self, n_neighbors=3, **kwargs):
        self._k = n_neighbors
        self._X = self._y = None
        self.set_params(**kwargs)

    def get_params(self, deep=True):
        # suppose this estimator has parameters "alpha" and "recursive"
        return self.__dict__
```

```python
    def set_params(self, **parameters):
        for parameter, value in parameters.items():
            setattr(self, parameter, value)
        return self

    def fit(self, X, y):
        self._X = X
        self._y = y

    def _predict(self, x):
        distances = np.apply_along_axis(lambda x1: np.linalg.norm(x-x1),
                                        1, self._X)
        X_candidates = np.argsort(distances)[:self._k]
        y_candidates = self._y[X_candidates]
        return np.argmax(np.bincount(y_candidates))

    def score(self, X, y_true):
        return accuracy_score(y_true, self.predict(X))

    def predict(self, X):
        return np.apply_along_axis(lambda x: self._predict(x), 1, X)
```

[5]

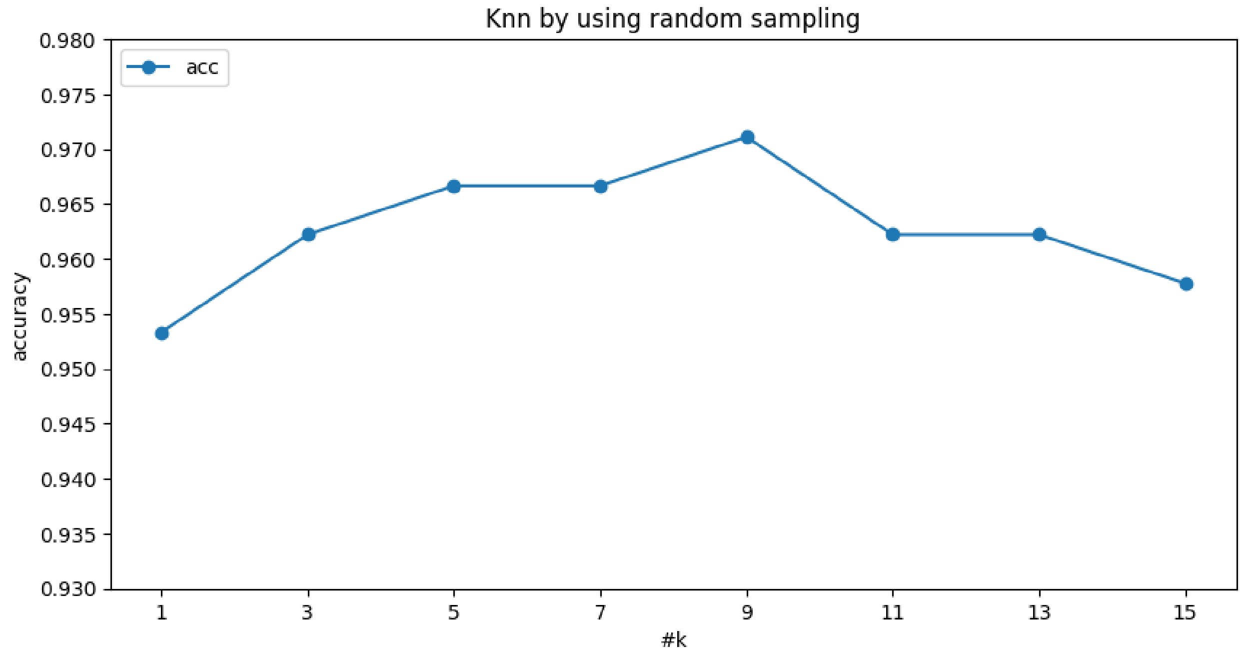
```python
K = list(range(1, 15+1, 2))
Knn_acc = []
for k in K:
    # Repeat the drawing and the k-NN classification 10 times.
    acc = []

    for i in range(N):
        model = MyKNeighborsClassifier(n_neighbors=k)
        X_train, X_test, y_train, y_test = train_test_split(iris.data,
                                                             iris.target,
                                                             test_size=0.3

        model.fit(X_train, y_train)
        acc.append(accuracy_score(y_test, model.predict(X_test)))

    Knn_acc.append(np.mean(acc))


fig = plt.figure(figsize=(10, 5), dpi=100, facecolor='white')
plt.plot(K, Knn_acc, 'o-')
plt.xticks(np.arange(1, 15+1, 2))
plt.yticks(np.arange(0.93, 0.98+0.005, 0.005))
plt.title('Knn by using random sampling')
plt.ylabel('accuracy')
plt.xlabel('#k')
plt.legend(['acc'], loc='upper left')
plt.show()
```



3 . Following problem 2, if you do not have the test dataset (i.e., you have only the 70 % of dataset), how do you determine the optimal value of k? Use your own approach to find such a value and compare the results you have in problem 2 . Comment on your results.

ANS:

I use K-fold cross-validation

Note:

如果以10次來平均的話 根本無法選擇k，Problem2跟3的曲線無法穩定，必須增加到200次以上才會穩定

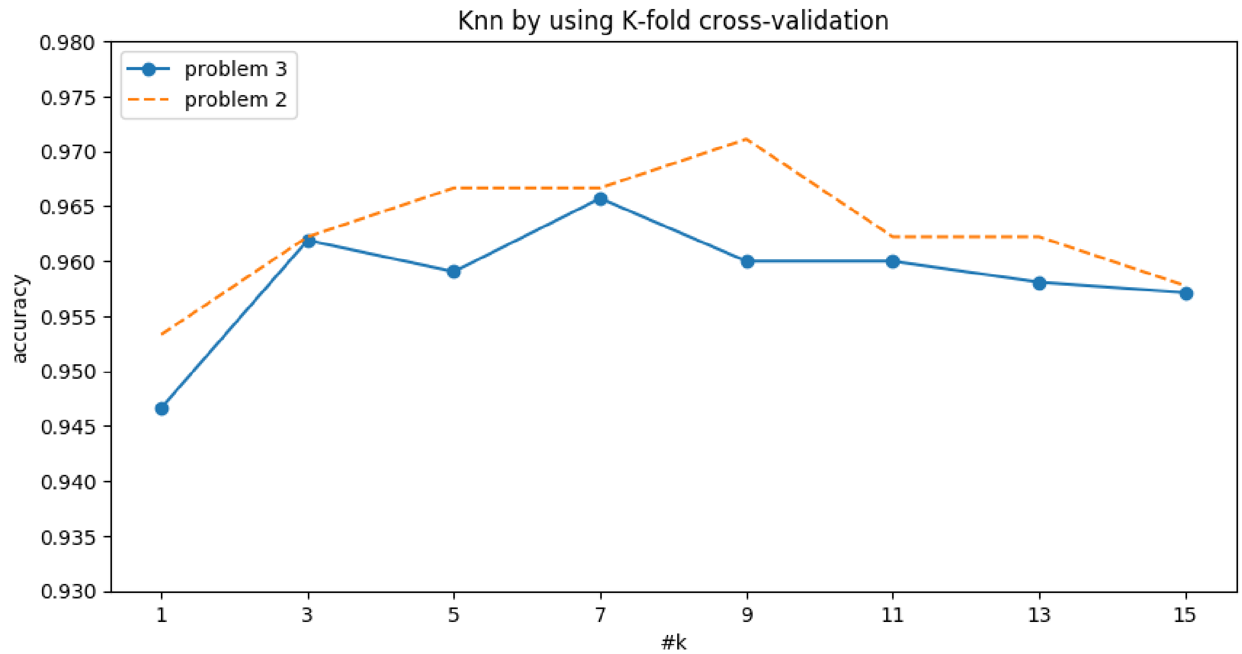
不過就在10次平均下多跑幾次來看，還是可以發現大概在k>9時精準度會開始下降

```
[9]  K = list(range(1, 15+1, 2))
     Knn_acc_vc = []
     for k in K:
         acc = []
         for i in range(N):
             model = MyKNeighborsClassifier(n_neighbors=k)
             # Assuming that I don't have test data
             X_train, _, y_train, _ = train_test_split(iris.data,
                                                       iris.target,
                                                       test_size=0.3)
             acc.append(np.mean(cross_val_score(model, X_train,
```

```
                                        y_train, cv=5)))

    Knn_acc_vc.append(np.mean(acc))

fig = plt.figure(figsize=(10, 5), dpi=100, facecolor='white')
plt.plot(K, Knn_acc_vc, 'o-', label='problem 3')
plt.plot(K, Knn_acc, '--', label='problem 2')
plt.xticks(np.arange(1, 15+1, 2))
plt.yticks(np.arange(0.93, 0.98+0.005, 0.005))
plt.title('Knn by using K-fold cross-validation')
plt.ylabel('accuracy')
plt.xlabel('#k')
plt.legend(loc='upper left')
plt.show()
```



4 . In the class, we covered the naive Bayes classifier, but only with discrete-type features. Consult any paper to learn how to extend this approach to continuous-type features. Explain your finding as an algorithm.

Bayesian probability

$$p(C_k \mid \mathbf{x}) = \frac{p(C_k)\, p(\mathbf{x} \mid C_k)}{p(\mathbf{x})}$$

When dealing with continuous data, a typical assumption is that the continuous values associated with each class are distributed according to a Gaussian distribution.

$$P(x = v|c) = \frac{1}{\sqrt{2\pi\sigma_c^2}}\, e^{-\frac{(v-\mu_c)^2}{2\sigma_c^2}}$$

The joint model can be expressed as

$$\begin{aligned} p(C_k \mid x_1, \dots, x_n) &\propto p(C_k, x_1, \dots, x_n) \\ &\propto p(C_k)\ p(x_1 \mid C_k)\ p(x_2 \mid C_k)\ p(x_3 \mid C_k)\ \cdots \\ &= p(C_k) \prod_{i=1}^{n} p(x_i \mid C_k)\,. \end{aligned}$$

We difine classifier like this

$$\hat{y} = \underset{k \in \{1,\dots,K\}}{\operatorname{argmax}}\ p(C_k) \prod_{i=1}^{n} p(x_i \mid C_k).$$

5 . Repeat problem 2 with your algorithm in problem 4. Compare the accuracy of naive Bayes classifier with the k-NN.

[7]

```
class MyGaussianNB:

    def __init__(self, **kwargs):
        self._class = {}
        self.set_params(**kwargs)

    def get_params(self, deep=True):
        return self.__dict__

    def set_params(self, **parameters):
        for parameter, value in parameters.items():
            setattr(self, parameter, value)
        return self

    def fit(self, X, y):
        for Ck in np.unique(y):
            data = X[np.where(y == Ck)]
            self._class[Ck] = {'data':data}
            self._class[Ck]['mean'] = np.mean(data, axis=0)
            self._class[Ck]['var'] = np.var(data, axis=0)
            self._class[Ck]['prior'] = (data.shape[0]
                                        / X.shape[0])

    def _likelihood(self, v, feature_index, Ck):
        var = self._class[Ck]['var'][feature_index]
        mean = self._class[Ck]['mean'][feature_index]
        exp = np.exp((-1 * (v - mean)**2) / (2 * var**2))
        return (1 / (np.sqrt(2 * np.pi * var**2))) * exp

    def _predict(self, x):
        class_prob = {Ck:None for Ck in self._class.keys()}

        for Ck, v in class_prob.items():
```

```python
                prior = self._class[Ck]['prior']
                class_prob[Ck] = prior
                for i, xn in enumerate(x):
                    class_prob[Ck] *= self._likelihood(xn, i, Ck)

            result = sorted(class_prob.items(),
                            key=operator.itemgetter(1),
                            reverse=True)
            return result[0][0]

    def score(self, X, y_true):
        return accuracy_score(y_true, self.predict(X))

    def predict(self, X):
        return np.apply_along_axis(lambda x: self._predict(x), 1, X)
```

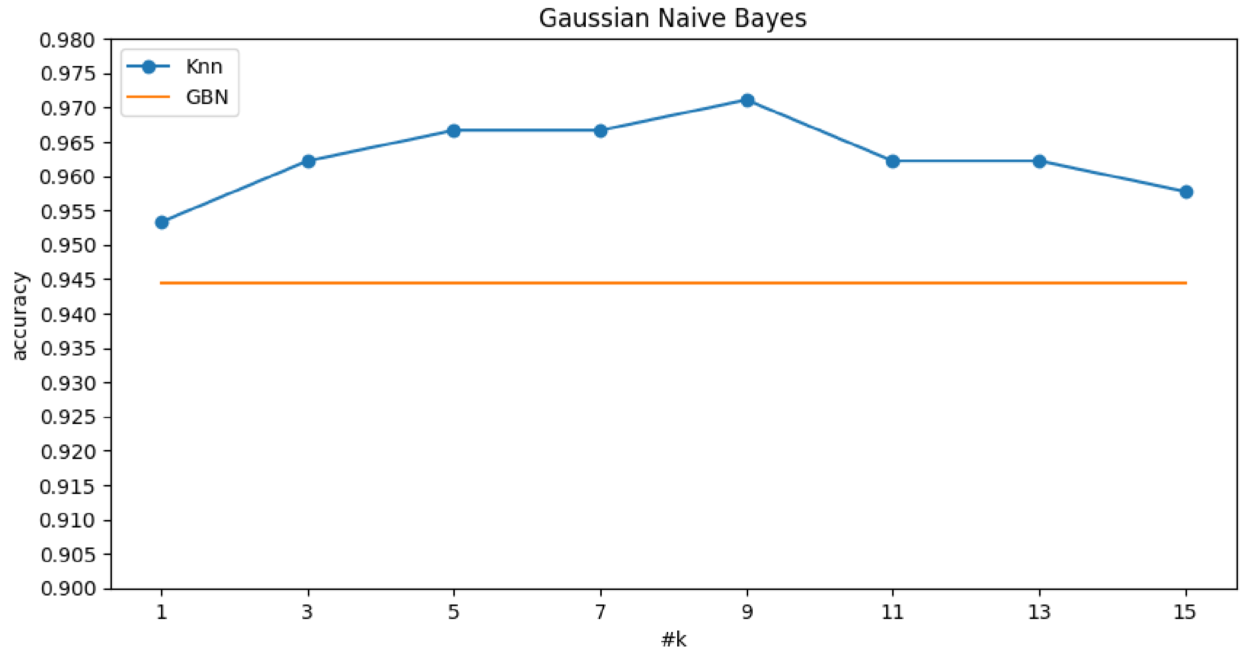
[8]

```python
K = list(range(1, 15+1, 2))
GNB_acc = []

for i in range(N):
    GNB_model = MyGaussianNB()
    X_train, X_test, y_train, y_test = train_test_split(iris.data,
                                                        iris.target,
                                                        test_size=0.3)
    GNB_model.fit(X_train, y_train)
    GNB_acc.append(accuracy_score(y_test, GNB_model.predict(X_test)))

print('acc:%.3f'%(np.mean(GNB_acc)))
fig = plt.figure(figsize=(10, 5), dpi=100, facecolor='white')
plt.plot(K, Knn_acc, 'o-', label='Knn')
plt.plot(K, [np.mean(GNB_acc)]*len(K), label='GBN')
plt.xticks(np.arange(1, 15+1, 2))
plt.yticks(np.arange(0.90, 0.98+0.005, 0.005))
plt.title('Gaussian Naive Bayes')
plt.ylabel('accuracy')
plt.xlabel('#k')
plt.legend(loc='upper left')
plt.show()
```

```
acc:0.944
```



Gaussian Naive Bayes
Knn
GBN
accuracy
0.980
0.975
0.970
0.965
0.960
0.955
0.950
0.945
0.940
0.935
0.930
0.925
0.920
0.915
0.910
0.905
0.900
1
3
5
7
9
11
13
15
#k