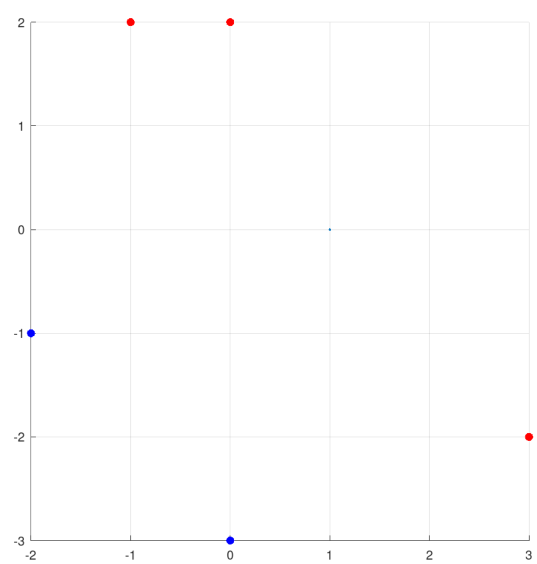
Name : Febrian Nugroho
NIM :2301930551

Answer of Assignment II
Adaptive Linear Neuron

Dataset:

1. $\left\{ x_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, t_1 = 1 \right\}$
2. $\left\{ x_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, t_2 = 1 \right\}$
3. $\left\{ x_3 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, t_3 = 1 \right\}$
4. $\left\{ x_4 = \begin{bmatrix} -2 \\ -1 \end{bmatrix}, t_4 = -1 \right\}$
5. $\left\{ x_5 = \begin{bmatrix} 0 \\ -3 \end{bmatrix}, t_5 = -1 \right\}$

Initial Weight and Bias:
w $= \begin{bmatrix} 3.0 & 1.0 \end{bmatrix}$
b $= 1.0$
$\alpha = 0.05$



Algorithm

for each iteration,
1. calculate output:

$$\begin{aligned} a_j &= \sum w_{ij}p + b, \text{ where} \\ a_j &= \text{Output}, w = \text{Weight}, p = \text{Input}, \text{ and } b = \text{Bias} \end{aligned}$$

2. compute loss function / error:

$$\begin{aligned} E(k) &= (t_k - a_k)^2, \text{ where} \\ E(k) &= \text{Least Square Error }, \\ t_k &= \text{Desired target input}, \\ a_k &= \text{Output} \end{aligned}$$

3. Update weights:

$$\begin{aligned} w_{new} &= w_{old} + \Delta w, \text{ where} \\ w_{new} &= \text{updated weight} \\ w_{old} &= \text{initial weight} \\ \Delta w &= 2\alpha(t-a)p \end{aligned}$$

4. Update bias:

$$\begin{aligned} b_{new} &= b_{old} + \Delta b, \text{ where} \\ b_{new} &= \text{updated bias} \\ b_{old} &= \text{initial bias} \\ \Delta b &= 2\alpha(t-a) \end{aligned}$$

5. Repeat until error is sufficiently low / zero

Training

Epoch 1
1-th Iteration:
Calculate output:

$$\begin{aligned} a_1 &= w.x + b \\ &= \begin{bmatrix} 3.0 & 1.0 \end{bmatrix} . \begin{bmatrix} -1 \\ 2 \end{bmatrix} + 1 \qquad (1) \\ a_1 &= 0 \end{aligned}$$

Calculate error:

$$\begin{aligned}
E &= (t-a)^2 \\
E &= (1-0)^2 \\
E &= 1.0
\end{aligned} \tag{2}$$

Update weights:

$$\begin{aligned}
w_{new} &= w_{old} + 2\alpha e p^T \\
w_{new} &= \begin{bmatrix} 3 & 1 \end{bmatrix} + (2)(0.05)(1.0)(\begin{bmatrix} -1 & 2 \end{bmatrix}) \\
w_{new} &= \begin{bmatrix} 3 & 1 \end{bmatrix} + \begin{bmatrix} -0.1 & 0.2 \end{bmatrix} \\
w_{new} &= \begin{bmatrix} 2.9 & 1.2 \end{bmatrix}
\end{aligned} \tag{3}$$

Update bias:

$$\begin{aligned}
b_{new} &= b_{old} + 2\alpha e \\
b_{new} &= 1.0 + (2)(0.05)(1) \\
b_{new} &= 1.0 + 0.1 \\
b_{new} &= 1.1
\end{aligned} \tag{4}$$

2-nd Iteration:
Calculate output:

$$\begin{aligned}
a_1 &= w.x + b \\
&= \begin{bmatrix} 2.9 & 1.2 \end{bmatrix} . \begin{bmatrix} 0 \\ 2 \end{bmatrix} + 1.1 \\
a_1 &= 3.5
\end{aligned} \tag{5}$$

Calculate error:

$$\begin{aligned}
E &= (t-a)^2 \\
E &= (1-3.5)^2 \\
E &= 6.25
\end{aligned} \tag{6}$$

Update weights:

$$\begin{aligned}
w_{new} &= w_{old} + 2\alpha e p^T \\
w_{new} &= \begin{bmatrix} 2.9 & 1.2 \end{bmatrix} + (2)(0.05)(-2.5)(\begin{bmatrix} 0 & 2 \end{bmatrix}) \\
w_{new} &= \begin{bmatrix} 2.9 & 1.2 \end{bmatrix} + \begin{bmatrix} 0 & -0.5 \end{bmatrix} \\
w_{new} &= \begin{bmatrix} 2.9 & 0.7 \end{bmatrix}
\end{aligned} \tag{7}$$

Update bias:

$$\begin{aligned}
b_{new} &= b_{old} + 2\alpha e \\
b_{new} &= 1.1 + (2)(0.05)(-2.5) \\
b_{new} &= 1.1 + -0.25 \\
b_{new} &= 0.85
\end{aligned} \tag{8}$$

3-rd Iteration:
Calculate output:

$$\begin{aligned} a_1 &= w.x + b \\ &= \begin{bmatrix} 2.9 & 0.7 \end{bmatrix} . \begin{bmatrix} 3 \\ -2 \end{bmatrix} + 0.85 \\ a_1 &= 3.5 \end{aligned} \tag{9}$$

Calculate error:

$$\begin{aligned} E &= (t - a)^2 \\ E &= (1 - 3.5)^2 \\ E &= 6.25 \end{aligned} \tag{10}$$

Update weights:

$$\begin{aligned} w_{new} &= w_{old} + 2\alpha e p^T \\ w_{new} &= \begin{bmatrix} 2.9 & 1.2 \end{bmatrix} + (2)(0.05)(-2.5)(\begin{bmatrix} 0 & 2 \end{bmatrix}) \\ w_{new} &= \begin{bmatrix} 2.9 & 1.2 \end{bmatrix} + \begin{bmatrix} 0 & -0.5 \end{bmatrix} \\ w_{new} &= \begin{bmatrix} 2.9 & 0.7 \end{bmatrix} \end{aligned} \tag{11}$$

Update bias:

$$\begin{aligned} b_{new} &= b_{old} + 2\alpha e \\ b_{new} &= 1.1 + (2)(0.05)(-2.5) \\ b_{new} &= 1.1 + -0.25 \\ b_{new} &= 0.85 \end{aligned} \tag{12}$$