

Now we calculate the derivative of output with respect to input for the output layer

$$\frac{\partial \text{out}_1}{\partial \text{in}_1} = \frac{\partial (e^{\text{in}_1} / (e^{\text{in}_1} + e^{\text{in}_2} + e^{\text{in}_3}))}{\partial \text{in}_1}$$

We know that $\frac{\partial \text{out}_1}{\partial \text{in}_1} = (e^{\text{in}_1} (e^{\text{in}_2} + e^{\text{in}_3})) / (e^{\text{in}_1} + e^{\text{in}_2} + e^{\text{in}_3})^2$

$$\therefore \begin{bmatrix} \frac{\partial \text{out}_1}{\partial \text{in}_1} \\ \frac{\partial \text{out}_2}{\partial \text{in}_2} \\ \frac{\partial \text{out}_3}{\partial \text{in}_3} \end{bmatrix} = \begin{bmatrix} (e^{\text{in}_1} (e^{\text{in}_2} + e^{\text{in}_3})) / (e^{\text{in}_1} + e^{\text{in}_2} + e^{\text{in}_3})^2 \\ (e^{\text{in}_2} (e^{\text{in}_1} + e^{\text{in}_3})) / (e^{\text{in}_1} + e^{\text{in}_2} + e^{\text{in}_3})^2 \\ (e^{\text{in}_3} (e^{\text{in}_1} + e^{\text{in}_2})) / (e^{\text{in}_1} + e^{\text{in}_2} + e^{\text{in}_3})^2 \end{bmatrix} = \begin{bmatrix} e^{1.8658} (e^{2.2292} + e^{2.8204}) / (e^{1.8658} + e^{2.2292} + e^{2.8204})^2 \\ e^{2.2292} (e^{1.8658} + e^{2.8204}) / (e^{1.8658} + e^{2.2292} + e^{2.8204})^2 \\ e^{2.8204} (e^{1.8658} + e^{2.2292}) / (e^{1.8658} + e^{2.2292} + e^{2.8204})^2 \end{bmatrix}$$

$$= \begin{bmatrix} 6.4611 & 9.2492 & 16.7836 \\ 16.7836 & 6.4611 & 9.2492 \\ 9.2492 & 16.7836 & 6.4611 \end{bmatrix} \begin{bmatrix} 0.1591 \\ 0.2025 \\ 0.2497 \end{bmatrix} = \begin{bmatrix} 1.0584789 \\ 2.144691 \\ 2.644004 \end{bmatrix}$$

with respect to each weight.

Now we proceed to calculate derivative for each input neuron with respect to each weight.

$$\frac{\partial \text{in}_1}{\partial W_{k11}} = \frac{\partial (h_{2\text{out}_1} * W_{k11}) + (h_{2\text{out}_2} * W_{k11}) + (h_{2\text{out}_3} * W_{k11})}{\partial W_{k11}}$$

$$\frac{\partial \text{in}_1}{\partial W_{k11}} = \frac{\partial (h_{2\text{out}_1} * W_{k11})}{\partial W_{k11}} = h_{2\text{out}_1}$$

Similarly in similar fashion, we can get the others as:

$$\begin{bmatrix} \frac{\partial \text{in}_1}{\partial W_{k11}} \\ \frac{\partial \text{in}_2}{\partial W_{k12}} \\ \frac{\partial \text{in}_3}{\partial W_{k13}} \end{bmatrix} = \begin{bmatrix} h_{2\text{out}_1} \\ h_{2\text{out}_2} \\ h_{2\text{out}_3} \end{bmatrix} = \begin{bmatrix} 0.1591 \\ 0.2025 \\ 0.2497 \end{bmatrix}$$