Problem 1

The a_V term increases binding energy since binding energy is proportional to the number of nucleons. The a_s term decreases binding energy since the nucleons on the surface of the nucleus has no binding partners. The a_C term decreases binding energy since protons repel each other. The a_a term decreases binding energy since there are usually more neutrons than protons. The effect of a_p term depends on N and Z. When N and Z are both even, the spin of the nucleus is an integer thus obey Bose-Einstein statistics. The wavefunction can then overlap and the nuleons are tighter bound, and binding energy is maximised. Similarly, when N and Z are both odd, binding energy is minimised. When one of them is even and the other is odd, the total effect is 0.

Problem 2

Start by taking the derivative

$$\frac{\mathrm{d}M}{\mathrm{d}Z} = m_p - m_n + 2a_c A^{-\frac{1}{3}} Z + 2a_a \frac{Z - \frac{A}{2}}{A}.$$

By setting the derivative to 0 we get

$$(2a_c A^{-\frac{1}{3}} + 2a_a A^{-1})Z = m_n - m_p + a_a.$$

Thus we have

$$Z = \frac{m_n - m_p + a_a}{2a_c A^{-\frac{1}{3}} + 2a_a A^{-1}} = 60.67.$$

We then compare $M(60, 145) = 134952.43001949298\text{MeV}$ and $M(61, 145) = 134952.16351767798\text{MeV}$ to see that $Z = 61$ is the most stable nucleus.

Problem 3

Fusion is possible up to ^{26}Fe because it has the maximal value of binding energy per nucleon. The product of fusion of ^{26}Fe with any smaller nuclei cannot have a nucleus that has a larger value of binding energy per nucleon.

Problem 4

The ground state is given by

$$\begin{aligned} N :&(1s_{\frac{1}{2}})^2(1p_{\frac{3}{2}})^3 \\ Z :&(1s_{\frac{1}{2}})^2(1p_{\frac{3}{2}})^2 \end{aligned}$$

If only neutrons are excited, the mostly likely configurations are given by

$$\begin{aligned} N :&(1s_{\frac{1}{2}})^2(1p_{\frac{3}{2}})^3(1p_{\frac{1}{2}})^1 \\ Z :&(1s_{\frac{1}{2}})^2(1p_{\frac{3}{2}})^2 \end{aligned}$$

and

$$\begin{aligned} N :&(1s_{\frac{1}{2}})^1(1p_{\frac{3}{2}})^4 \\ Z :&(1s_{\frac{1}{2}})^2(1p_{\frac{3}{2}})^2 \end{aligned}$$

Problem 5

For ${}^{40}_{18}$Ar:

$$\begin{aligned} N:\ &(1s_{\frac{1}{2}})^2(1p_{\frac{3}{2}})^4(1p_{\frac{1}{2}})^2(1d_{\frac{5}{2}})^6(1d_{\frac{3}{2}})^4(1f_{\frac{7}{2}})^2(2s_{\frac{1}{2}})^2 \\ Z:\ &(1s_{\frac{1}{2}})^2(1p_{\frac{3}{2}})^4(1p_{\frac{1}{2}})^2(1d_{\frac{5}{2}})^6(1d_{\frac{3}{2}})^4(2s_{\frac{1}{2}})^2 \end{aligned}$$

${}^{40}_{18}$Ar has even numbers of protons and neutrons, and all of them are paired. As a result, the spin and parity are $J^P = 0^+$.
For ${}^{39}_{19}$K:

$$\begin{aligned} N:\ &(1s_{\frac{1}{2}})^2(1p_{\frac{3}{2}})^4(1p_{\frac{1}{2}})^2(1d_{\frac{5}{2}})^6(1d_{\frac{3}{2}})^4(2s_{\frac{1}{2}})^2 \\ Z:\ &(1s_{\frac{1}{2}})^2(1p_{\frac{3}{2}})^4(1p_{\frac{1}{2}})^2(1d_{\frac{5}{2}})^6(1d_{\frac{3}{2}})^3(2s_{\frac{1}{2}})^2 \end{aligned}$$

In ${}^{40}_{19}$K, all neutrons are paired. There is one unpaired proton in $1d_{\frac{3}{2}}$ level with $l = 2$ and $j = \frac{3}{2}$, so $J^P = \left(\frac{3}{2}\right)^+$.
For ${}^{40}_{20}$Ca:

$$\begin{aligned} N:\ &(1s_{\frac{1}{2}})^2(1p_{\frac{3}{2}})^4(1p_{\frac{1}{2}})^2(1d_{\frac{5}{2}})^6(1d_{\frac{3}{2}})^4(2s_{\frac{1}{2}})^2 \\ Z:\ &(1s_{\frac{1}{2}})^2(1p_{\frac{3}{2}})^4(1p_{\frac{1}{2}})^2(1d_{\frac{5}{2}})^6(1d_{\frac{3}{2}})^4(2s_{\frac{1}{2}})^2 \end{aligned}$$

In ${}^{40}_{20}$Ca, all levels are full-filled. Therefore, $J^P = 0^+$.

${}^{40}_{20}$Ca is stable because it is doubly magic.

Problem 6

We can first calculate j by $16 = 2j + 1$, then $j = \frac{15}{2}$. We then have $l = j \pm \frac{1}{2}$ so that l is either 7 or 8. Given that it has an odd parity, l has to be 7.

Problem 7

The half-lime is given by

$$T_{\frac{1}{2}} = 1600\text{y} = 50\,457\,600\,000\,\text{s}.$$

The lifetime is then given by

$$\tau = \frac{T_{\frac{1}{2}}}{\ln 2} = 72\,794\,929\,295.158\,97\,\text{s}.$$

The total number of $^{226}_{88}\text{Re}$ atoms, N, can then be estimated as

$$N = 0.37\,\text{Bq} * \tau = 26934123839.208817.$$

The amount of substance is then

$$n = \frac{N}{N_A} = 4.472516487510467 \times 10^{14}\text{mol}.$$

The total mass is

$$\begin{aligned} m &= nM_{^{226}_{88}\text{Re}} \\ &= 4.472516487510467 \times 10^{14}\text{mol} \times 226.025\,41\,\text{g/mol} \\ &= 1.0109023728213132 \times 10^{-11}\text{g} \end{aligned}$$

Finally, the percentage is given by

$$\text{percentage} = \frac{m}{1\,\text{g}} \times 100\% = 1.01 \times 10^{-9}\%.$$

Problem 8

$$\text{fission rate} = \frac{10\,\text{kW}}{200\,\text{MeV}} = 6.24 \times 10^{16}\text{s}^{-1}.$$