Briana Parchment

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Truth Tables

Problem 1

Truth tables? -2

a) NAND
The NAND operator applies AND to its operants and then complements the result by applying NOT. We write p NAND q to denote NOT(p and q).

b) NOR
Takes the OR of its operands and complements the result; p NOR q denotes NOT (p OR q).

c) ≡ "if and only if"
p ≡ q is true when both p and q are true, or when both are false, but not otherwise

Problem 2

a) $\overline{(p \to q)} \equiv$ (NOT p OR q)

p	q	(p->q)	$\overbrace{\text{(NOT p OR q)}}^{a}$	(p->q) ≡ a
0	0	1	1	1
0	1	1	1	1
1	0	0	0	1
1	1	1	1	1

b) p -> (q -> (r OR NOT p))

p	q	r	$\overbrace{\text{(r OR NOT p)}}^{a}$	$\overbrace{\text{(q -> a)}}^{b}$	p->b
1	1	1	1	1	1
1	1	0	0	0	0
1	0	0	0	1	1
1	0	1	1	1	1
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1

c) (p OR q) → (p AND q)

p	q	(p OR q)	(p AND q)	(p OR q) ~~pRo~~ → (p AND q)
1	1	1	1	1
1	0	1	0	0
0	1	1	0	0
0	0	0	0	1

start with this

Problem 3

To what set operator does p AND NOT q correspond?

Answer: NAND exclusion -1

Problem 4

$\underbrace{p \rightarrow q}_{\text{True}}$

p: It is ~~raining~~ ~~pouring~~ pouring

q: It is ~~pouring~~ raining

$\underbrace{q \rightarrow p}_{\text{False}}$ If it is raining, → then it is pouring. → Statement not true

NAND

p	q	r	(p NAND q) NAND r	p NAND (q NAND r)
1	1	1	1	1
1	1	0	1	0
1	0	0	1	0
1	0	1	0	0

NOR? -1

Problem 5

-2

The boolean operator ~~OR~~ does not depend on their first or second argument.

p, q, not p, not q, true, false

Problem 6

P	Q	P AND Q
0	0	0
0	1	0
1	0	0
1	1	1

P	Q	P OR Q
0	0	0
0	1	1
1	0	1
1	1	1

P	NOT P
0	1
1	0

P	Q	$P \to Q$
0	0	1
0	1	1
1	0	0
1	1	1

P	Q	$P \equiv Q$
0	0	1
0	1	0
1	0	0
1	1	1

P	Q	P NAND Q
0	0	1
0	1	1
1	0	1
1	1	0

P	Q	P NOR Q
0	0	1
0	1	0
1	0	0
1	1	0

P	Q	$P \otimes Q$
0	0	0
0	1	1
1	0	1
1	1	0

P	Q	$\neg(p \to q)$
0	0	0
0	1	0
1	0	1
1	1	0

P	Q	P AND $\bar{P}$
0	0	0
0	1	0
1	0	0
1	1	0

P	Q	P OR $\bar{P}$
0	0	1
0	1	1
1	0	1
1	1	1

P	Q	$q \to p$
0	0	1
0	1	0
1	0	1
1	1	1

p	q	$\neg(q \to p)$
0	0	0
0	1	1
1	0	0
1	1	0

p	$\bar{p}$
0	1
1	0

q	$\bar{q}$
1	0
0	1

p	q	NOT$(p \otimes q)$
0	0	1
0	1	0
1	0	0
1	1	1

Problem 7

a)

p	q	$\otimes$
0	0	0
0	1	1
1	0	1
1	1	0

b) $\otimes$ is associative, ~~not~~ commutative

−1

Problem 8

−2

$(\bar p + \bar q + r)\ (\bar p + \bar q + r)\cdot(\bar p + q + \bar r)\ (\bar p + q + r)\ (p + \bar q + \bar r)(p + \bar q + r)(p + q + \bar r)(p+q$

Problem 9

a) $(\bar p \bar q r) + (p \bar q \bar r) + (p \bar q r) + (p q \bar r) + (p q r)$ −1

b) $(\bar p \bar q \bar r) + (\bar p q r) + (\bar p q \bar r)$

c) $(\bar x \bar y c) + (\bar x y \bar c) + (x \bar y \bar c) + (x y c)$

Problem 10

a)

pq \ rs	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	0	1
10	1	1	1	1

✓

b)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	0	0	0
10	1	1	0	1

✓

c)

pq \ rs	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	1	1
10	1	0	1	0

✓

d)

pqr \ s	0	1
0	1	1
1	0	1

−1

e) pqrs <10

pqrs	0	1
0	1	1
1	0	1

Problem 11

a) $q\bar{r}s + qrs + qr\bar{s} + pqs + \bar{q}\bar{r}s + \bar{p}\bar{q}r$

−2

b) $qr\bar{s} + \bar{p}\bar{q}\bar{r}s + \bar{p}\bar{q}r + p\bar{r}s$

c) $pqrs + pqr\bar{s} + \bar{p}qrs + p\bar{q}rs$

d) $\bar{p}\bar{q}\bar{r}s + pqrs$

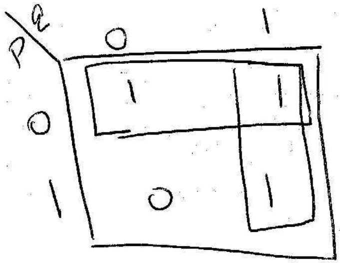
e) $p\bar{q}rs + \bar{p}\bar{q}rs$

10

(pg. 7)

Problem 12

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1



$\bar p \bar q = 1$ $pq = 1$

$\bar p q = 1$

$\bar p \bar q + \bar p q + pq =$ Sum of Products

Problem 13

-2

a) $\bar p \bar q \bar r \bar s, pqrs$

b) $\bar p qrs, pq\bar r s, pqr\bar s, pqr\bar s, p\bar q rs$

c) $\bar p \bar q \bar r \bar s, pq\bar r \bar s, \bar p \bar q rs, p\bar q \bar r s, \bar p qr\bar s, p\bar q r\bar s$

d) $pqr\bar s$

e) $\bar p \bar q \bar r \bar s < 10$

Problem 14

-2

a) $\bar p \bar q rs, \bar p \bar q r \bar s, q\bar r \bar s, qrs, pq\bar r \bar s, p\bar q \bar r \bar s, p\bar q rs$

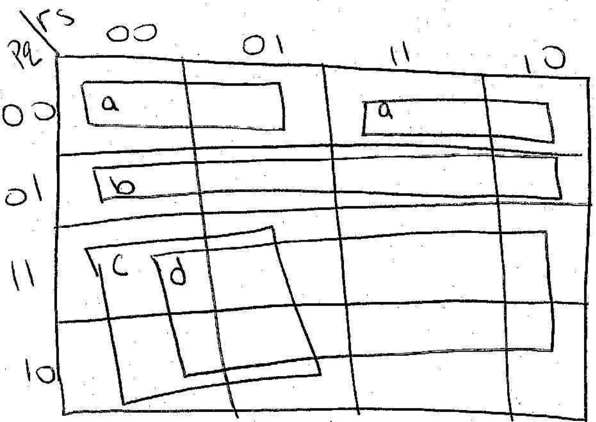
b) $\bar p q \bar r \bar s, \bar p qrs, qr\bar s, p\bar q \bar r \bar s, \bar p q \bar r \bar s$

c) $pqrs, pqr\bar s, p\bar q rs, \bar p qrs$

d) $\bar p \bar q \bar r \bar s, pqrs$

e) $\bar p \bar q \bar r \bar s, pqrs$

Problem 15



a) ~~16~~ 32 b) ~~8~~ 16 c) ~~4~~ 8 d) ~~2~~ 8

-2

Problem 16

a) $pqr \to p+q$

p	q	r	p+q	E
0	0	0	0	1
0	0	1	0	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Tautology

✓

b) $((p \to q)(q \to r)) \to (p \to r)$

p	q	r	$p \to q$	$q \to r$	$p \to r$	E
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	1
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

Tautology

✓

c) $(p \to q) \to p$

p	q	$p \to q$	$(p \to q) \to p$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

Not a Tautology

✓

d) $(p \equiv (q+r)) \to (q \to pr)$

p	q	r	$q+r$	$p \equiv (q+r)$	$q \to pr$	E
0	0	0	0	1	1	1
0	0	1	1	0	0	1
0	1	0	1	1	1	1
0	1	1	1	0	0	0
1	0	0	0	0	0	1
1	0	1	1	1	1	1
1	1	0	1	0	1	1
1	1	1	1	1	1	1

-1

Not a Tautology

(Pg 10)

Problem 17

a) Test whether the expression is a tautology, by making a truth table, and the last row of the table should contain all true values.

b) The satisfiability problem ~~could~~ can be solved using inherent intractability.

Problem 18

1) $p \equiv p$

p	q	$p \equiv q$
0	0	1
0	1	0
1	0	0
1	1	1

2) $(p \equiv q) \equiv (q \equiv p)$

p	q	$p \equiv q$	$q \equiv p$	$a \equiv b$
0	0	1	1	1
0	1	0	0	1
1	0	0	0	1
1	1	1	1	1

Tautology

3) Transitive Law

p	q	r	$\overbrace{p \equiv q}^{a}$	$\overbrace{q \equiv r}^{b}$	$\overbrace{p \equiv r}^{c}$	$\overbrace{(a \text{ AND } b)}^{d}$	$d \to c$
0	0	0	1	1	1	1	1
0	0	1	1	0	0	0	1
0	1	0	0	0	1	1	1
0	1	1	0	1	0	0	1
1	0	0	0	1	0	0	1
1	0	1	0	0	1	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Tautology

4) Equivalence of Negations

p	q	$\overbrace{(p \equiv q)}^{a}$	$\bar{p}$	$\bar{q}$	$\overbrace{\bar{p} \equiv \bar{q}}^{b}$	$a \equiv b$
0	0	1	1	1	1	1
0	1	0	1	0	0	1
1	0	0	0	1	0	1
1	1	1	0	0	1	1

tautology

5) Commutative law for AND: ~~p(qr) ≡ (pq)r~~ $pq \equiv qp$

pq	$pq \equiv qp$	
0 0	1	tautology
0 1	1	
1 0	1	
1 1	1	

6) Associative law for OR: $p(qr) \equiv (pq)r$

pqr	$p(qr) \equiv (pq)r$	
0 0 0	1	tautology
0 0 1	1	
0 1 0	1	
0 1 1	1	
1 0 0	1	
1 0 1	1	
1 1 0	1	
1 1 1	1	

7) Commutative law for OR
~~p(~~ $(p+q) \equiv (q+p)$

$p\ q$	$(p+q) \equiv (q+p)$	
0 0	1	tatology
0 1	1	
1 1	1	
1 0	1	

8) Associative Law for OR: $(p+(q+r)) \equiv ((p+q)+r)$

$p\ q\ r$	$(p+(q+r)) \equiv ((p+q)+r)$	
0 0 0	1	tautology
0 0 1	1	
0 1 0	1	
1 1 1	1	
1 1 0	1	
1 0 1	1	

9) Distributive law of AND & over OR: $p(q+r) \equiv (pq+pr)$

p q r	$p(q+r) \equiv (pq+pr)$
0 0 0	1
0 0 1	1
0 1 0	1
1 1 1	1
1 1 0	1
1 0 1	1

} tautology

10) 1(True) is the identity for AND: $(p \text{ AND } 1) \equiv p$

p	$(p \text{ AND } 1) \equiv p$
0	1
1	1

} tautology

11) 0(False) is the identity for OR: $p \text{ OR } 0 \equiv p$

p	$p \text{ OR } 0 \equiv p$
0	1
1	1

} tautology

12) 0 is the annihilator for AND: $(p \text{ AND } 0) \equiv 0$

p	$(p \text{ AND } 0) \equiv 0$
0	1
1	1

} tautology

13) Elimination of double negations: (NOT NOT p) $\equiv$ p (pg.1

p	NOT p	(NOT NOT p) $\equiv$ p
0	1	1
1	0	1

$\}$ tautology

14) The distributive law for OR over AND: $(p+qr) \equiv ((p+q)(p+$

p q r	$(p+qr) \equiv ((p+q)(p+r))$
0 0 0	1
0 0 1	1
0 1 0	1
0 1 1	1
1 1 1	1
1 1 0	1
1 0 1	1

$\}$ tautology

15) 1 is the annihilator for OR: (1 OR p) $\equiv$ 1

p	(1 OR p) $\equiv$ 1
1	1
0	1

$\}$ tautology

16) Idempotence of AND: pp $\equiv$ p

p	pp $\equiv$ p
0	1
1	1

$\}$ tautology

(problem 18)

(pg. 15)

17) Idempotence of OR: $p+p \equiv p$

p	$p+p \equiv p$
0	1
1	1

} tautology

18) Subsumption

a) $(p+pq) \equiv p$

p	q	$(p+pq) \equiv p$
0	0	1
0	1	1
1	1	1
1	0	1

} tautology

b) $p(p+q) \equiv p$

p	q	$p(p+q) \equiv p$
0	0	1
0	1	1
1	1	1
1	0	1

} tautology

19) Elimination of certain negations

a) $p(\bar{p}+q) \equiv pq$

p	q	$\bar{p}$	$(\bar{p}+q) \equiv pq$
1	1	0	1
1	0	0	1
0	0	1	1
0	1	1	1

} tautology

20) DeMorgan's laws (pg. 16)

a) $\text{NOT}(pq) \equiv \bar{p} + \bar{q}$

p	q	$\bar{p}$	$\bar{q}$	$\text{NOT}(pq) \equiv \bar{p} + \bar{q}$
0	0	1	1	1
0	1	1	0	1
1	1	0	0	1
1	0	0	1	1

tautology

b) $\text{NOT}(p+q) \equiv \overline{pq}$

p	q	$\bar{p}$	$\bar{q}$	$\text{NOT}(p+q) \equiv \overline{pq}$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

tautology

c) $\overbrace{(\text{NOT}(p_1 p_2 \cdots p_k)) \equiv (\bar{p}_1 + \bar{p}_2 + \cdots + \bar{p}_k)}^{a}$

p_1	p_2	p_k	(a)
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	1	1	1
1	1	0	1
1	0	1	1

tautology

d) $\overbrace{(\text{NOT}(p_1 + p_2 + \cdots + p_k)) \equiv (\bar{p}_1 \bar{p}_2 \cdots \bar{p}_k)}^{a}$

p_1	p_2	a
0	0	1
0	1	1
1	1	1
1	0	1

tautology

(problem 18)
(pg. 17)

21) $\overbrace{((p \to q) \text{ AND } (q \to p))}^{a} \equiv (p \equiv q)$

p	q	$p \to q$	$q \to p$	a
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	1	1	1

tautology

22) $(p \equiv q) \to (p \to q)$

p	q	$(p \to q)$	$(p \equiv q) \to (p \to q)$
0	0	1	1
0	1	1	1
1	1	1	1
1	0	0	1

tautology

23) Transitivity of implication: $\overbrace{((p \to q) \text{ AND } (q \to r))}^{a} \to (p \to r)$

p	q	r	$p \to q$	$q \to r$	$p \to r$	a
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	1	1
1	1	1	1	1	1	1
1	1	0	1	0	0	1
1	0	1	0	1	1	1

tautology

(problem 18)
(pg. 18)

24) Implication w/ AND and OR

a) $(p \to q) \equiv (\bar{p} + q)$

p	q	$p \to q$	$(p \to q) \equiv (\bar{p} + q)$
0	0	1	1
0	1	1	1
1	1	1	1
1	0	0	1

} tautology

b) $\overbrace{(p_1 p_2 \cdots p_n \to q) \equiv (\overline{p_1} + \overline{p_2} + \ldots + \overline{p_n} + q)}^{a}$

p_1	p_2	q	a
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	1	1	1
1	1	0	1
1	0	1	1

} tautology

Problem 19

1) $(x+y) \equiv (x+y)$

2) $(((x+y) \equiv (yz)) \equiv ((yz) \equiv (x+y)))$

3) $(((x+y) \equiv yz)$ AND $(yz \equiv x)) \rightarrow ((x+y) \equiv x$

4) $(((x+y) \equiv yz) \equiv ((\bar{x}+\bar{y}) \equiv y\bar{z}))$

5) $((x+y)(yz)) \equiv ((yz)(x+y))$

6) $(x+y)(yz(x)) \equiv (x+y(yz)x)$

7) $((x+y)+yz) \equiv (yz+(x+y))$

8) $((x+y)+(yz+x)) \equiv (((x+y)+x))$

9) $((x+y)(yz+x)) \equiv (x+y(yz)+(x+y(x))$

10) $((x+y)$ AND $1) \equiv (\cancel{x+a}\; x+y)$

11) $\cancel{x+}\;(x+y)$ OR $0 \equiv (x+y)$

12) $((x+y)$ AND $0) \equiv 0$

13) (NOT NOT $(x+y)) \equiv (x+y)$

14) $((x+y)+yz(x)) \equiv (((x+y)+yz)((x+y)+x))$

15) $(1$ OR $(x+y)) \equiv 1$

16) $((x+y)(x+y)) \equiv (x+y)$

17) $((x+y)+(x+y)) \equiv (x+y)$

18) a) $((x+y)+(x+y)(yz)) \equiv (x+y)$

b) $((x+y)((x+y)(yz))) \equiv (x+y)+yz$

19) a) $((x+y)((\bar{x}+\bar{y})+yz)) \equiv ((x+y)(yz))$

b) $((x+y)+(\bar{x}+\bar{y})(yz)) \equiv (x+y)+yz$

20) a) ~~$NOT(pq) \equiv \bar{p}$~~

$NOT((x+y)(yz)) \equiv (\bar{x}+\bar{y})+(\bar{y}\ \bar{z})$

b) $NOT((x+y)+yz) \equiv (\bar{x}+\bar{y})(\bar{y}\bar{z})$

c) $NOT((x+y_1)(x+y_2)(x+y_k)) \equiv ((\bar{x}+\bar{y}_1)+(\bar{x}+\bar{y}_2)+\ldots(\bar{x}+\bar{y}_k))$

d) $(NOT((x+y_1)+(x+y_2)+\ldots+(x+y_k))) \equiv (((\bar{x}+\bar{y}_1)(\bar{x}+\bar{y}_2))\ldots(\bar{x}+\bar{y}_k))$

21) $(((x+y)\to yz)\ AND\ (yz\to(x+y))) \equiv ((x+y)\equiv yz)$

22) $((x+y)\equiv yz)\to((x+y)\to yz)$

23) $(((x+y)\to yz)\ AND\ (yz\to x))\to((x+y)\to x)$

24) a) $((x+y)\to yz) \equiv ((\bar{x}+\bar{y})+yz)$

b) $((x+y_1)(x+y_2)\ldots(x+y_n)\to y_z) \equiv ((\bar{x}+\bar{y}_1)+(\bar{x}+\bar{y}_2)+\ldots(\bar{x}+\bar{y}_n)+yz)$

Problem 20

a) Distributive law for OR over AND

$(pq+rs) \rightarrow (p+r)(p+s)(q+r)(q+s)$ ✓

b) $(pq+p\bar{q}r) \rightarrow p(q+r)$

Elimination of Certain Negations

$pq+pr$

Associative Law for AND

$p(q+r)$

Problem 21

$p+pq$ Distristributive Law for OR over AND

$p+pq \equiv p(p+q)$ Subsumption ✓

$p(p+q) \equiv p$

Problem 22

−2

1) $\text{NOT}(pq+p\bar{r})$

$\text{NOT}(pq)+(p\bar{r})$

$(\bar{p}+\bar{q})+(p\bar{r})$

$\bar{q}pr$

2) $\text{NOT}(\text{NOT}p+q(\text{NOT}(r+\bar{s})))$

$\text{NOT}(pq(\text{NOT}(r+\bar{s})))$

$\text{NOT}(\overline{pq}(\bar{r}+s))$

$pq(r+\bar{s})$

$p(\bar{q}+r+\bar{s})$

Problem 23

1) $w\bar{x} + \cancel{y}\, w\bar{x}y + \bar{z}\bar{x}w$

$y + \bar{z}\bar{x}\, w$ Subsumption $w\bar{x}$

-2

2) $(w+\bar{x})(w+y+z)(\bar{w} + \cancel{x}\,\bar{x}+y)(\bar{x})$

$\bar{x}\,(w+y+\bar{z})(\bar{w}+\bar{x}+\bar{y})$