Sample Means of an Exponential Distribution

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The Exponential Distribution

The exponential distribution represents the distribution of waiting times in a Poisson process with rate of change λ.

The probability density function for the exponential distribution is

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The distribution is supported on the interval $[0, \infty)$.

The mean of an exponential distribution is $\frac{1}{\lambda}$ and the standard deviation is also also $\frac{1}{\lambda}$.

In R the exponential distribution is simulated with rexp(n, lambda) where lambda is the rate parameter.

```
n <- 40
lambda <- 0.2

# our expected mean and standard deviation
1/lambda
```

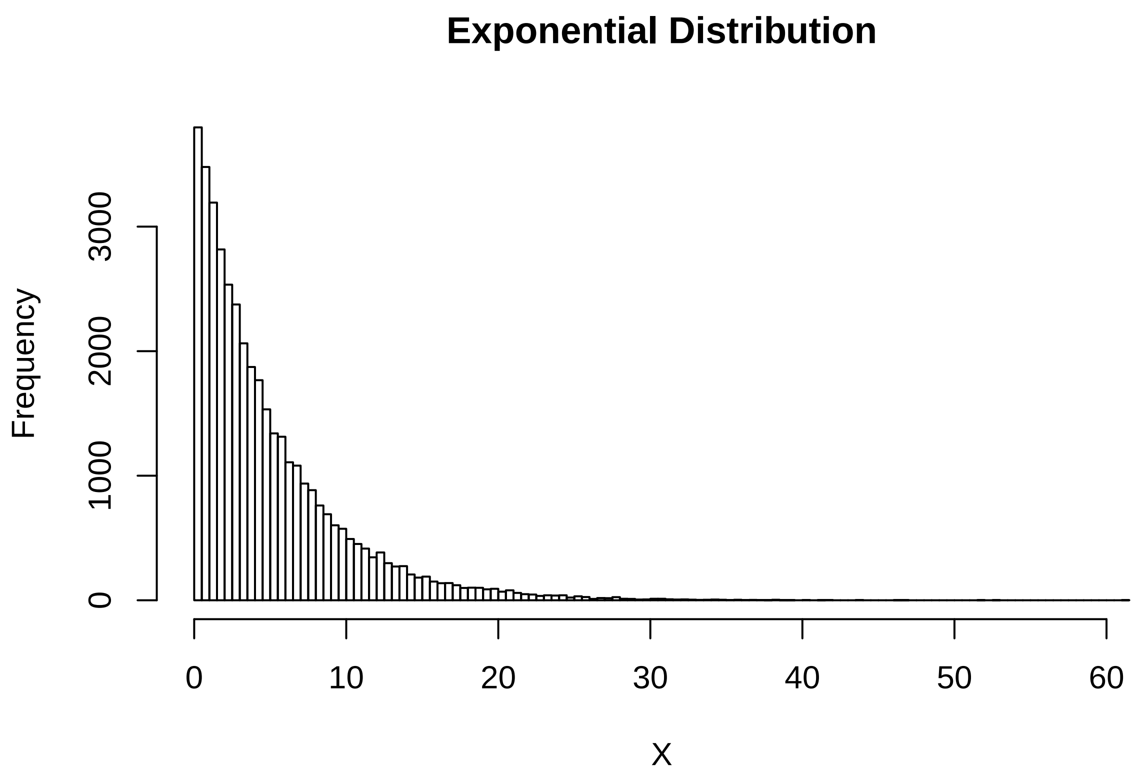
```
## [1] 5
```

We expect to observe a mean and standard deviation of 5 from our simulation.

Simulation

To get a more intuitive understanding of the exponential distribution, let us first see what it looks like. We ask R to give us 40,000 random values from the distribution, and plot them on a histogram.

```
# 1,000 simulations
num.sims <- 1000
X <- rexp(n*num.sims,lambda)
hist(X,breaks=100, main = "Exponential Distribution")
```



Mean and Standard Deviation

We compute the mean and standard deviation of the random values and compare them to our expected value and standard deviation of $\frac{1}{\lambda}$.

```
print(Xbar<-mean(X))
```

```
## [1] 4.962
```

The mean for the 40,000 generated random values is 4.9616.

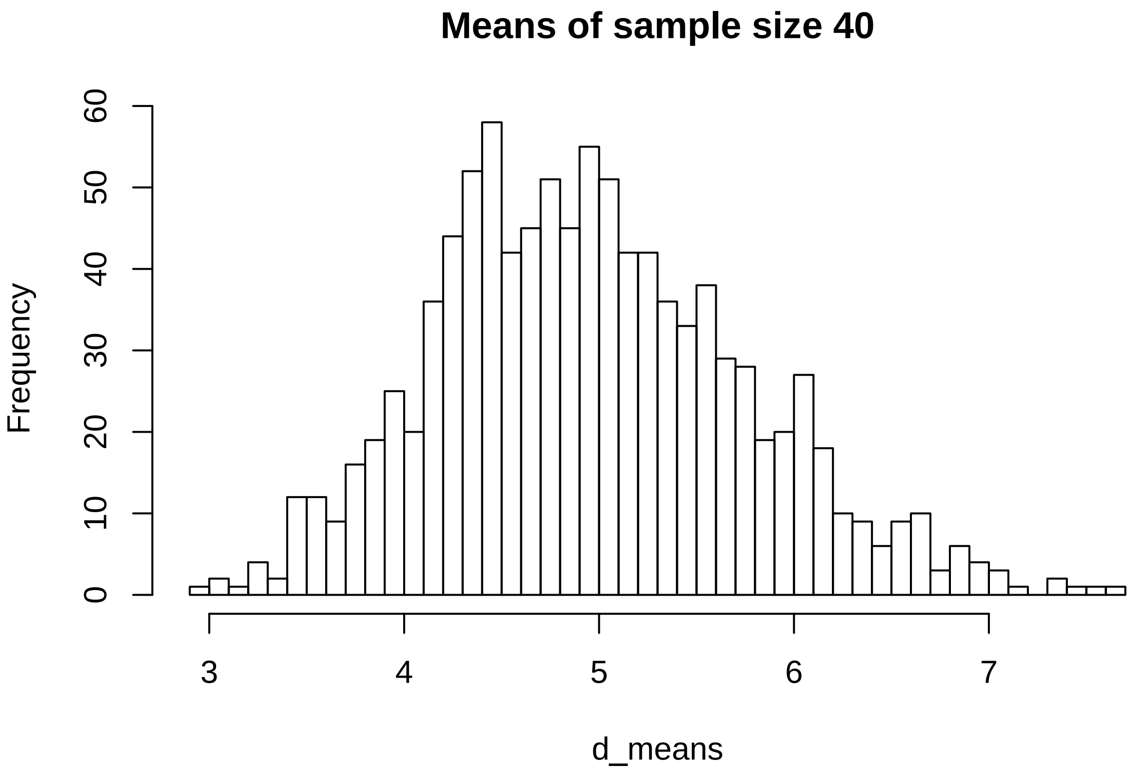
```
sd(X)
```

```
## [1] 4.974
```

The standard deviation for the random values is 4.9738.

Both the observed mean and standard deviation are close to $\frac{1}{\lambda}$ which we know to be 5.

```
X2 <- matrix(X,nrow=num.sims)
d_means <- apply(X2,1,mean)
hist(d_means,breaks=50, main ="Means of sample size 40")
```


Means of sample size 40
Frequency
0
10
20
30
40
50
60
3
4
5
6
7
d_means