

CS-2102

Discrete Math

Problem Set 1, Due Friday February 3, 2017 by 5 pm.

Remember that you can work one other person. Be sure to give adequate explanation for your solutions, lack of clarity and completeness may result in points being deducted. Answers given with no work or justification may have points taken off.

(9 pts, 3 pts each) 1. Prove or disprove that the logical operators exclusive or, NAND and NOR are associative. You may use truth tables or simplification laws. (Hint: look in the lecture notes if you are not sure what you need to prove in order for a logical operator to be associative.)

(8 pts, 2 pts each) 2. Write the following statements in symbolic form using the symbols \neg , \wedge , \vee and the indicated letters to represent component statements.

- (a) Juan is a math major but not a computer science major. (m = "Juan is a math major", c = "Juan is a computer science major")
- (b) Let h = "John is healthy," w = "John is wealthy," and s = "John is wise."
John is not wealthy but he is healthy and wise.
- (c) John is wealthy, but he is not both healthy and wise.
- (d) Either Olga will go out for tennis or she will go out for track but not both. (n = "Olga will go out for tennis," k = "Olga will go out for track")

(4 pts, 2 pts each) 3. Are the following two statements logically equivalent? Justify your answers using truth tables and include a few words of explanation:

- (a) $\neg(p \wedge q)$ and $\neg p \wedge \neg q$
- (b) $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$

(4 pts, 2 pts each) 4. Use De Morgan's laws to write negations for the following statements:

- (a) Sam is an orange belt and Dave is a red belt.
- (b) The train is late or my watch is fast.

(4 pts) 5. Determine if the following statement forms are logically equivalent: (Use any method that you would like)

$$p \rightarrow (q \rightarrow r) \text{ and } (p \rightarrow q) \rightarrow r$$

(8 pts, 2 pts each) 6. Rewrite the following statements in if-then form.

- (a) Catching the 8:05 bus is a sufficient condition for my being on time for work.
- (b) Having two 45° angles is a sufficient condition for this triangle to be a right triangle.

- (c) Being divisible by 3 is a necessary condition for this number to be divisible by 9.
- (d) Doing homework regularly is a necessary condition for Jim to pass the class.

(6 pts, 3 pts each) 7.

- (a) Show that the following statement forms are all logically equivalent (Use any method you would like).
 $p \rightarrow q \vee r$, $p \wedge \neg q \rightarrow r$, and $p \wedge \neg r \rightarrow q$.
- (b) Use the logical equivalences established in part a to rewrite the following sentence in two different ways: "If n is a prime number, then n is odd or n is 2."

(3 pts, 1 pt each) 8. Let $R(m, n)$ be the predicate "If m is a factor of n^2 then m is a factor of n ," with domain for both m and n being the set of integers.

- (a) Explain why $R(m, n)$ is false if $m = 25$ and $n = 10$.
- (b) Give values different from those in the previous part for which $R(m, n)$ is false.
- (c) Explain why $R(m, n)$ is true if $m = 5$ and $n = 10$.

(9 pts, 3 pts each) 9.

- (a) Determine the number m of nonequivalent propositions involving the two propositional variables p and q . Hint one way to answer this question is to think about the number of unique columns in a truth table that can be made using only 2 propositional variables.
- (b) For each of these unique columns come up with an expression, in terms of p and q , that have that truth table.
- (c) What would m be if we allow for 3 variables?

(2 pts) 10. Verify that $p \wedge q \rightarrow p \vee q$ is a tautology.