Problem Set Number 1

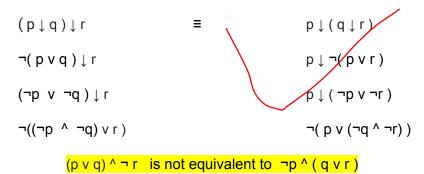
1. Exclusive Or (*): This is associative because the highlighted columns have the same results as each other.

р	q	r	p⊕q	q⊕r	(p ⊕ q) ⊕ r	p ⊕ (q ⊕ r)
F	F	F	F	F	F	F
F	F	Т	F	Т	Т	Т
F	Т	F	Т	Т	Т	Т
F	Т	Т	Т	F	F	F
Т	F	F	Т	F	Т	Т
Т	F	Т	Т	Т	F	F
Т	Т	F	F	Т	F	F
Т	Т	Т	F	F /	Т	Т

NAND : Nand is not associative, we prove this by doing the actual associative property in the following:

(p v q) v ¬r is not equivalent to ¬p v (q ^ r)

NOR: Nor is not associative, we prove this by doing the actual associative property in the following:



2.

3.

(a) Not equivalent because DeMorgan's law would require it to be ¬(p^q) and (¬p v ¬q) for it to be equivalent and the truth table does not line up

р	q	¬р	¬q	qr v pr	pr ^ qr
Т	Т	F	F	F	F
Т	F	F	T/	Т	F
F	Т	A	F	Т	F
F	F	Т	Т	Т	Т

(b) Not equivalent logically equivalent because it does not follow the distributive laws property:

р	q	r	(p v q)	(p ^ r)	(p v q) v (p ^ r)	(p v q) ^ r
Т	Т	Т	Т	Т	Т	Т
Т	Т	F	Т	F	Т	F
Т	F	Т	Т	Т	Т	T
Т	F	F	Т	F	\T	F
F	Т	Т	Т	F	ţ	T
F	Т	F	Т	F	T	F
F	F	Т	F	F	F	F
F	F	F	F	F	F	F

4.

- (a) Sam is not an orange belt or Dave is not a red belt
- (b) The train is not late and my watch is not fast

5. They are not logically equivalent because the truth tables don't align.

р	q	r	p→q	q→r	p→(q→r)	(p→q)→r
F	F	F	Т	Т	Т	F
F	F	Т	Т	Т	Т	Т
F	Т	F	Т	F	Т	F
F	Т	Т	Т	Т	Т	Т
Т	F	F	F	Т	Т	Т
Т	F	Т	F	Т	Т	Т
Т	Т	F	Т	F	F	F
Т	Т	Т	Т	Т	T	Т

- c. If a number is divisible by 9, then it is divisible by 3.
- 6. *d*. If Jim passes the course, then he will have done his homework regularly.
 - (a) If I catch the 8:05 bus, then I will be on time for work.
 - (b) If there are two 45 degree angles, then this triangle is a right triangle.
 - (c) If it is divisible by 3, then this number can be divisible by 9.
 - (d) If he does homework regularly, then Jim can pass the class.

7.

(a)
$$p \rightarrow q v r$$
 can be = $q or r$

p ^
$$\neg q \rightarrow$$
 becomes $(q \vee r) \wedge \neg q \rightarrow r$ which results in $r \rightarrow r$

p ^ ¬r \rightarrow q becomes (q v r) ^ ¬r \rightarrow q which results in q \rightarrow q THUS

$$q \vee r \equiv r \rightarrow r OR q \rightarrow q$$

- (b) "If n is a prime number, then n is odd or n is 2"
 - 1. If n is a prime number and not odd, then n is 2
 - 2. If n is a prime number and n is not 2, then n is odd.

- (a) m = 25 cannot be a factor of n=10 because a number cannot be a factor of a number less than it
- (b) Some places where R(m,n) is false are from the following values:
 - (i) m = 27; n = 9
 - (ii) m = 16; n = 8
 - (iii) m = 12; n = 6
- (c) R(m,n) is true if m = 5 and n = 10 because :

$$((n^2)/m) =$$

$$100/5 = 20$$

And
$$n/m = 10/5$$

= 2

Which results that m = 5 is a factor that goes into n^2 and n evenly

(a) 16, through the formula (2^2)^n, where n is the number of variables:

(b)

T, T, T, T	рv ¬q
T, T, T, F	pvq
T, T, F, F	p
T, F, F, F	p^q
F, F, F, F	p ^ ¬p
F, F, F, T	pr\q
F, F, T, T	р
F, T, T, T	¬p v ¬q
F, T, F, T	¬q
T, F ,T, F	q
T, F, F, T	$p \leftarrow \to q$
F, T, T, F	$p \leftarrow \rightarrow \neg q$
T, T, F, T	$\neg p \to \neg q$
T, F, T, T	$p \rightarrow q$
F, F, T, F	¬p ^ q
F, T, F, F	p ^ ¬q
.,.,.,.	۲ ۹

c) it would be 256 because $2^{(2^n)}$: where n = 3

р	q	p ^ q	pvq	$(p \land q) \rightarrow (p \lor q)$
F	Т	F	Т	T
F	F	F	F	T
Т	Т	Т	Т	Ī
Т	F	F	Т	T

Yes, it equals True for every situation in (p ^ q) \rightarrow (p v q) ; so it is a tautology.

