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Total 69

Problem Set 4

1) First use universal instantiation so: $P(c)$ if $c \in U$

$(((P(c) \vee Q(c)) \wedge ((\neg P(c) \wedge Q(c)) \rightarrow R(c))) \rightarrow (\neg R(c) \rightarrow P(c))$

4

1. $Q(c) \wedge ((\neg P(c) \wedge Q(c)) \rightarrow R(c)) \rightarrow (\neg R(c) \rightarrow P(c))$
   ↳ Resolution
2. $(\neg P(c) \wedge Q(c)) \rightarrow R(c)$ simplification
3. $\neg R(c) \rightarrow P(c)$

These steps are incomplete, should have shown more steps.

2) a. prove that for all odd integers, n, $[\frac{n}{2}] = \frac{n-1}{2}$

let $n = 2k+1$ then solve for k

What does it prove anyway?

0

$[n/2] = [k + .5]$

$\frac{n-1}{2} = k$ ... $[\frac{n}{2}] = \frac{n-1}{2}$

b. prove that for all even integers, n, $[\frac{n}{2}] = \frac{n}{2}$

let $n = 2k$ then solve for k

0

$[\frac{n}{2}] = [k] = k = \frac{n}{2}$ ... $[\frac{n}{2}] = \frac{n}{2}$

This does not prove anything. What does the [n] mean anyway?

→

3) a) if r and s are rational, then set $r = p/q$ and $s = a/b$ where p, q, a, b are integers, therefore:

$$\frac{r+s}{2} = \frac{\frac{pb+qa}{qb}}{2}$$

since p, a, q, b are ints and ints are closed under addition, subtraction, multiplication and division. Thus $\frac{pb+qa}{qb}$ is rational since

4 they are integers and when that is divided by the integer 2, the same rules follow and the number must still be rational therefore $\frac{r+s}{2}$ is rational.

b) $a < b$ so $c = b - a$ and $b = c + a$

then plugging into formula: $\frac{a+b}{2}$

4 we get $a < \frac{2a+c}{2} < c+a$ ... so ...

$2a < 2a + c < 2c + 2a$ ... therefore ..

$\boxed{0 < c < 2c}$ Only if c>0

c) given r and s we can prove they are rational from corollary a) and from there we can prove there

4 is a number between r and s due to corollary b) therefore given any 2 rational numbers r and s with $r < s$, there is another rational number between r and s.

4) let $x = m^2 - n^2$ and $y = 2mn$ so...

$$(m^2-n^2)^2 + (2mn)^2 = z^2$$
$$m^4 - 2m^2n^2 + n^4 + 4m^2n^2 = z^2$$
$$m^4 + 2m^2n^2 + n^4 = z^2$$
$$(m^2+n^2)^2 = z^2$$

4

$z = m^2 + n^2$ $\quad x = m^2 - n^2$ $\quad y = 2mn$ so...

$$(m^2-n^2)^2 + (2mn)^2 = (m^2+n^2)^2$$

so...

$$m^4 - 2m^2n^2 + n^4 + 4m^2n^2 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2$$

5) a) when $n = -1$ $\quad -3(-1)^2 - 14(-1) - 8 = \boxed{1}$

which is a prime number which shows

2

there exists an integer n that makes

$-3(n)^2 - 14n - 8$ result as a prime number

b) let $m = 2$ and $n = 18$ which are both $\mathbb{Z}^+$

so $mn = 36$ which is a perfect square of 6

2

but m and n are not perfect squares

because $\sqrt{m}$ and $\sqrt{n}$ do not result as integers

that are positive

c) let $a = 2k+1$ and $b = 2k$ so that a and

b are 2 consecutive integers, so prove

$a^2 - b^2$ is odd...

2

$$(2k+1)^2 - (2k)^2$$

$$4k^2 + 4k + 1 - 4k^2 \rightarrow 4k+1$$

with $4k+1$ we know that the difference of 2 squares of any two consecutive integers is odd because $4k+1$ will always result as an odd integer →

6) a) let m and n be perfect squares. Which implies $m = a^2$ and $n = b^2$ where a and b are integers..

$$m + n + 2\sqrt{mn} = a^2 + b^2 + 2\sqrt{a^2 b^2}$$

3

$$a^2 + b^2 + 2ab$$

$$a^2 + 2ab + b^2$$

$$= (a+b)^2$$

which results in $(a+b)^2$ which is a perfect square

b) let $p(x) = 2^x - 1$ therefore we can test prime numbe

$p(3) = 7$ $p(5) = 31$

$p(7) = 127$ $p(11) = 2047$

3

23 89

so when $p = 11$ in $2^p - 1$, p is prime and the result is not prime, therefore this diproves the statement

→

6) c) a, b, c, d are four consecutive integers
so $a=k$ $b=k+1$ $c=k+2$ $d=k+3$ therefore
$abcd=(k)(k+1)(k+2)(k+3)$

$(k^2+k)(k+2)(k+3)$
$(K^3+2k^2+k^2+2k)(k+3)$
3 $K^4+2k^3+k^3+2k^2+3k^3+6k^2+3k^2+6k$
$=K^4+6k^3+11k^2+6k$

this product +1 is $K^4+6k^3+11k^2+6k+1$ which equals $(k^2+3k+1)^2$ therefore the sum of 4 consecutive integers is one less than a perfect square

d. nonnegative real numbers a and b, $\sqrt{ab}=\sqrt{a}\cdot\sqrt{b}$
set $a=x^2$, $b=y^2$
3 so $\sqrt{x^2y^2}=\sqrt{x^2}\cdot\sqrt{y^2}$
$\rightarrow$ $xy=x\cdot y$
$xy=xy$ Therefore, the statement is true

e. nonnegative real numbers a and b, $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$
Let $a=1$ and $b=4$
$\sqrt{a+b}=\sqrt{a}+\sqrt{b}$
$\sqrt{1+4}=\sqrt{1}+\sqrt{4}$
3 $\sqrt{5}=1+2$
$\sqrt{5}=3$
$\sim 2.236\neq 3$, therefore, this is not a true statement

7) a. when odd: let r and s be represented by $(2k+1)$ where $(2k+1)$ represents any odd integer. r = s = 2k+1 ?

Then $(x-r)(x-s)$ becomes $(x-(2k+1))(x-(2k+1))$

Then factor out:

$$x^2 - ((2k+1)x - (2k+1)x) + (2k+1)(2k+1)$$

Difference of 2 odds = even   factor of 2 odds = odd

$$x^2 \pm (2k)x \pm (2k+1)$$

↳ which is the equation we get when r and s are both odd values

when even: let r and s be represented by $(2k)$ where $(2k)$ represents any even integer

Then $(x-r)(x-s)$ becomes $(x-(2k))(x-(2k))$

Then factor out:

$$x^2 - ((2k)x - (2k)x) + (2k)(2k)$$

Difference of 2 evens = even   factor of 2 evens = even

$$x^2 \pm (2k)x \pm (2k)$$

↳ which is the equation we get when r and s are even

when even/odd: let r represent an odd integer as $(2k+1)$ and s represent an even integer $(2k)$

$(x-(2k+1))(x-(2k))$ → factor out

$$x^2 - ((2k)x - (2k+1)x) + (2k)(2k+1)$$

difference of even and odd results odd   factor of even and odd results even

3

so when factoring when r is an odd value and s is an even value we should get:

$$x^2 \pm (2k+1)x \pm (2k)$$

7) b. we cannot break down $x^2 - 1253x + 255$
because it follows the format:

1 $x^2 - (2k+1)x + (2k+1)$

which is not one of the formulas we proved in a).

C: $(x-r)(x-s)(x-t) = x^3 - (r+s+t)x^2 + (rs+rt+st)x - rst$

combinations possible:
odd · odd · odd
odd · odd · even
odd · even · even
even · even · even

we must test each combo by making an odd as $(2k+1)$ and an even as $(2k)$

odd · odd · odd : let $r = 2k+1$ $s = 2k+1$ $t = 2k+1$ and plug into
$x^3 - (r+s+t)x^2 + (rs+rt+st)x - rst$
where sum of 3 odd values $(r+s+t)$ results odd.
The factor of 2 odd values is odd and, the sum of 3 odd values is odd which makes $(rs+rt+st)$ odd.
The finally the factor of 3 odd values rst results
3 odd, therefore we get:

$$x^3 - (2k+1)x^2 + (2k)x - (2k+1)$$

when all values are odd

d. $x^3 + 7x^2 - 8x - 27$ written as product of three polynomials with integer coefficients
This can be denoted as $x^3 + (2k+1)x^2 - (2k)x - (2k+1)$
3 Therefore it can be written that way as long as
$(x-r)(x-s)(x-t)$ where r, s, and t are odd numbers

8.) a. if $a|b$, that means that there is an integer k that factors in so $b=ak$. Then $a|bc$ must also work because b times any integer
2 c will always have b as a factor of that product. Since b divides evenly into bc and a divides evenly into b, than a must also divide evenly into bc, so if $a|b$, then $a|bc$

b. a, b are integers. $a=2k$, $b=2s$ so they are both even.
2 $ab \rightarrow (2k)(2s) = 4ks$, 4ks is the product of two even integers and will always be divisible by 4 so $\frac{ab}{4} = ks$

c. Yes, this is a sufficient condition. 16 is always divisible by 8 with a factor of 2. Therefore any
2 number that is divisible so $n=16k$ where k is an integer, will also be divisible so $n=8\cdot 2k$. This can be rewritten with integer $d=2k$ so $n=8d$ to further demonstrate the concept.

d. if $a|b$ and $a|c$, then $a|(2b-3c)$
so $b=ak$ and $c=as$ where k and s are integers
$a|(2b-3c)$ because a is already a proven
2 factor of b and c. $\frac{b}{a}$ and $\frac{2b}{a}$ still result in an integer, but the latter is multiplied by a product of two. Similarly, $\frac{c}{a}$ and $\frac{-3c}{a}$ is an integer but the latter is multiplied by -3. Therefore a will divide in $(2b-3c)$ evenly and result in an integer if $a|b$ and $a|c$

8.) e. if $ab|c$, then $a|c$ and $b|c$

$c = abk$ so k is also an integer. An integer times an integer is another integer so ak can $= d$ and

2 $bk = e$. Therefore, $c = ae$ or $c = bd$ which satisfies the condition as e and d are integers

$\therefore$ if $ab|c$, then $a|c$ and $b|c$

f. if $a|(b+c)$, then $a|b$ or $a|c$, where a, b, and c are integers

$b+c = ak$

if $b=4$, $c=5$, and $a=3$, $a|(b+c)$ because

$4+5 = 3k \rightarrow 3 = k$ which is an (integer) and

satisfies the conditions. However $a \nmid b$ and $a \nmid c$ because $3|4$ and $3|5$ does not make

2 a true statement since the k in $b=ak$ or $c=ak$ is not an integer. Therefore, this statement is false.

3

9.) a. If $\frac{\sqrt[3]{x}+5}{x^2+6} = \frac{1}{x}$, then $x \neq 8$

plug in 8: $\frac{\sqrt[3]{8}+5}{8^2+6} = \frac{1}{8}$

$\rightarrow \frac{2+5}{64+6} = \frac{1}{8}$

$\frac{7}{70} = \frac{1}{8}$

$\frac{1}{7} \neq \frac{1}{8}$ so $x \neq 8$

3

b. $3x+2y \leq 5$, if $x > 1$, then $y < 1$

$\frac{3x+2y}{3} \leq \frac{5}{3} \rightarrow x + \frac{2}{3}y \leq \frac{5}{3} \rightarrow x \leq \frac{5-2y}{3}$

$x \leq \frac{5-2y}{3}$ and $x > 1$

so $1 < \frac{5-2y}{3} \rightarrow 3 < 5-2y$

$\frac{-2}{-2} < \frac{-2y}{-2} \rightarrow 1 > y$ ✓