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Discrete: Problem Set 3

1) $p \to q \equiv (p \wedge \neg q) \to F$

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$p \to q$	$(p \wedge \sim q) \to F$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	F	T	T

yes, they are logically equivalent

2) given: 1) $\neg t \vee \neg u$ 2) $s \to t \wedge u$ 3) $(\neg r \vee \neg f) \to (s \wedge L)$

1	$\sim t \vee \sim u$	1st hypothesis
2	$\sim(t \wedge u)$	DM 1
3	$s \to t \wedge u$	2nd hypothesis
4	$\sim s$	MT 2,3
5	$\sim(r \wedge f) \to \sim(s \wedge L)$	DM hypothesis 3
6	$\sim(\sim(r \wedge f)) \vee \sim(s \wedge \sim L)$	Def of implication
7	$\sim(\sim(r \wedge f))$	Disjunctive Syllogism 6
8	$(r \wedge f)$	double negation
9	r	simplification

3) I believe the first error to be in B because you cannot infer universal instantiation in such a way because $P(c)$ would need to equal $\forall_x (P(x) \vee Q(x))$ not just $\forall_x P(x)$, the next in C because you cannot simplify through an "$\vee$" but only through an "$\wedge$".

4)

$$\left.\begin{array}{l} 1)\ (p \vee \sim t) \rightarrow h \\ 2)\ \sim h \vee (m \wedge k) \\ 3)\ (m \wedge k \wedge q) \rightarrow r \\ 4)\ \sim q \rightarrow (\sim h \wedge \sim s) \\ 5)\ p \vee \sim t \end{array}\right\} \text{given}$$

6) $\therefore r$

7.	h	MT 1, 5
8.	$(m \wedge k)$	hypothesis 3, disjunctive elimination
9.	$(m \wedge k \wedge h)$	conjunction 7, 8
10.	$q \vee (\sim h \wedge \sim s)$	definition of implies
11.	$(q \vee \sim h) \wedge (q \vee \sim s)$	distribution 10
12.	$q \wedge (q \vee \sim s)$	disjunctive syllogism 7, 11
13.	$q \vee \sim s$	absorption 12
14.	q	MT 4, 7
15.	$(m \wedge k \wedge q)$	conjunction
16.	$(m \wedge k \wedge q)$	hypothesis 3
17.	r	MP 16

5) Start: this house is next to a lake — C.

- so treasure is not in kitchen: — a.
- b becomes false cause of — C. A.
- so there is no elm tree in frontyard — b
- So treasure is under flagpole — D

it would have to be behind the flagpole because for (d) to be a true statement atleast one requirement must be met, and since we know the 1st part is false, we can conclude that the 2nd part must be true.

6) a) Assume A is a knight, then b must be lying
   - which contradicts

   Assume B is a knight, the A mus be lying
   - which can be true

   B = knight   A = knave

b) assume C is a knight, which contradicts because knights cannot lie

   assume C is a knave, which makes sense as it allows C to be still lying

   C = knave   D = knight

c) Assume E is a knight, which works

   Assuming the other results in same

   - but since there is only one right amount, we know there exists 1 knave.

6) D. assume 0 knights
↳ U tells truth so not possible
assume 1 knight
↳ Z, W both tell truth
assume 2 knights
↳ only Y and W tell truth, so we can conclude there are 2 knights: Y and W and 4 knaves: U, V, X, Z

7) He errors when making the final existential generalization because he treats $(P(c) \wedge Q(c))$ to have the same $P(c)$ for some $c \in U$ which is not logically equivalent to the premise

8) a. $(p \wedge t) \rightarrow (r \vee s)$
b. $q \rightarrow (u \wedge t)$
c. $u \rightarrow p$
d. $\sim s$
$\therefore q \rightarrow r$
proof:

1. q — assuming true
2. $(u \wedge t)$ — modus ponens b and 1
3. $u \rightarrow p$ — given c
4. p — modus ponens 3,
5. $(p \wedge t)$ — 4, 2
6. $\sim s$ — given
7. $(r \vee s)$ — 5, a
8. r — DS 6, 7
9. $q \rightarrow (p \wedge t)$ — HS 5, b
10. $q \rightarrow r$ — HS 9, a

9) Premises

a. $P \rightarrow Q$

b. $R \rightarrow S$

c. $\neg Q \vee \neg S$

$\therefore \neg P \vee \neg R$

proof:

1) $\neg Q \vee \neg S$ given
2) $Q \rightarrow \neg S$ rule of inference/implication ; C
3) $\neg S \rightarrow \neg R$ MT b and 2
4) $Q \rightarrow \neg R$ HS 2,3
5) $P \rightarrow Q$ given a
6) $P \rightarrow \neg R$ HS 4 and 5
7) $\neg P \vee \neg R$ implication

QED

10) D = logic is difficult

L = students like logic

M = Math is easy

premises:

1) $D \vee \sim L$
2) $M \rightarrow \sim D$

a) $L \rightarrow \sim M$

proof: if L is true then D has to be true by DS, then using MT if $\sim D$ is false the M is $\sim M$, thus $L \rightarrow \sim M$ QED

b) $\sim M \rightarrow \sim L$

proof: we let D be true, which means $\sim M$ is true because of MT but premise 1 can still be true with $\sim L$ being false, so in this situation $\sim M$ is true and $\sim L$ is false which contradicts $\sim M \rightarrow \sim L$

→

10) C) $\sim M \vee D$
proof: this is false. If M is true then $\sim D$ is true (from 2) and if M is true then $\sim M$ is false. Therefore in this situation, you have $\sim M$ = false and D = false which makes conclusion $\sim M \vee D$ false.

D) $\sim D \vee \sim M$
proof: if $\sim D$ is true because statement 2, M can either be T because MT or F which makes the conclusion true and if $\sim D$ is false then $\sim M$ is true because of MT. therefore the conclusion is true.

E) $\sim L \rightarrow (\sim M \vee \sim D)$
proof: this is true, because referencing part D $(\sim M \vee \sim D)$ will always hold true, therefore the conclusion doesn't rely on if $\sim L$ is true/false. Thus even if $\sim L \wedge \sim M \wedge \sim D$ were true, there are no contradictions as $\sim L$ can be true when D is true. (statement 1)

11) a. $\forall x (P(x) \wedge Q(x))$

b. $\exists x (R(x) \wedge \sim Q(x))$

c. $\exists x (R(x) \wedge \sim P(x))$

d. yes because of b: we know some excuses can be unsatifactory, but because of a, we know that these excuses that are unsatisfactory should have non-clear explainations

12) Premises:

a. if superman were able and willing to prevent evil, he would do so
$(W \wedge A) \rightarrow P$

b. if superman were unable to prevent evil, he would be impotent
$A \rightarrow I$

c. if he were unwilling to prevent evil, he would be malevolent
$\sim W \rightarrow M$

d. superman does not prevent evil
$\sim P$

e. if superman exists, he is neither impotent or malevolent
$E \rightarrow (\sim M \wedge \sim I)$

∴ Therefore, Superman does not exist

1. $\sim W \vee \sim A$	from a and d MT
2. $A \rightarrow \sim W \rightarrow M$	HS 3
3. $M \vee I$	HS 2, b
4. $\sim(\sim I \wedge \sim M)$	DM 3
5. $\sim E$	MT 4 ; e