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Problem Set Number 1

1. Exclusive Or (⊕) : This is associative because the highlighted columns have the same results as each other.

p	q	r	p⊕q	q⊕r	(p ⊕ q) ⊕ r	p ⊕ (q ⊕ r)
F	F	F	F	F	F	F
F	F	T	F	T	T	T
F	T	F	T	T	T	T
F	T	T	T	F	F	F
T	F	F	T	F	T	T
T	F	T	T	T	F	F
T	T	F	F	T	F	F
T	T	T	F	F	T	T

NAND : Nand is not associative, we prove this by doing the actual associative property in the following:

(p | q) | r	≡	p | (q | r)
¬(p ^ q) | r		p | ¬(p v r)
(¬p ^ ¬q) | r		p | (¬p v ¬r)
¬ ((¬p ^ ¬q) ^ r)		¬(p ^ (¬p v ¬r)

(p v q) v ¬r   is not equivalent to   ¬p v (q ^ r)

NOR: Nor is not associative, we prove this by doing the actual associative property in the following:

(p ↓ q) ↓ r	≡	p ↓ (q ↓ r)
¬(p v q) ↓ r		p ↓ ¬(p v r)
(¬p v ¬q) ↓ r		p ↓ (¬p v ¬r)
¬((¬p ^ ¬q) v r)		¬(p v (¬q ^ ¬r))

(p v q) ^ ¬ r is not equivalent to ¬p ^ (q v r)

2.

(a) m ^ ¬c

(b) ¬w ^ (h ^ s)

(c) w ^ ¬ (h ^ s)

(d) (n v k) ^ ¬(n ^ k)

3.

(a) Not equivalent because DeMorgan's law would require it to be ¬(p^q) and (¬p v ¬q) for it to be equivalent and the truth table does not line up

p	q	¬p	¬q	¬q v ¬p	¬p ^ ¬q
T	T	F	F	F	F
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	T

(b) Not equivalent logically equivalent because it does not follow the distributive laws property:

p	q	r	(p v q)	(p ^ r)	(p v q) v (p ^ r)	(p v q) ^ r
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

4.

(a) Sam is not an orange belt or Dave is not a red belt

(b) The train is not late and my watch is not fast

5. They are not logically equivalent because the truth tables don't align.

p	q	r	$p \to q$	$q \to r$	$p \to (q \to r)$	$(p \to q) \to r$
F	F	F	T	T	T	F
F	F	T	T	T	T	T
F	T	F	T	F	T	F
F	T	T	T	T	T	T
T	F	F	F	T	T	T
T	F	T	F	T	T	T
T	T	F	T	F	F	F
T	T	T	T	T	T	T

6.

c. If a number is divisible by 9, then it is divisible by 3.
d. If Jim passes the course, then he will have done his homework regularly.

(a) If I catch the 8:05 bus, then I will be on time for work.

(b) If there are two 45 degree angles, then this triangle is a right triangle.

(c) If it is divisible by 3, then this number can be divisible by 9. −4

(d) If he does homework regularly, then Jim can pass the class.

7.

(a) $p \to q \vee r$ can be = q or r

$p \wedge \neg q \to$ becomes $(q \vee r) \wedge \neg q \to r$ which results in $r \to r$

$p \wedge \neg r \to q$ becomes $(q \vee r) \wedge \neg r \to q$ which results in $q \to q$ THUS

$q \vee r \equiv r \to r$ OR $q \to q$

(b) "If n is a prime number, then n is odd or n is 2"

1. If n is a prime number and not odd, then n is 2
2. If n is a prime number and n is not 2, then n is odd.

8.

(a) m = 25 cannot be a factor of n=10 because a number cannot be a factor of a number less than it

(b) Some places where R(m,n) is false are from the following values:

(i) m = 27 ; n = 9

(ii) m =16 ; n = 8

(iii) m = 12 ; n = 6

(c) R(m,n) is true if m = 5 and n = 10 because :

((n^2)/m) =

100/5 = 20

And n/m = 10/5

= 2

Which results that m = 5 is a factor that goes into n^2 and n evenly

9.

(a) 16, through the formula (2^2)^n, where n is the number of variables:

2^4 = 16

(b)

T, T, T, T	p v ¬q
T, T, T, F	p v q
T, T, F, F	p
T, F, F, F	p ^ q
F, F, F, F	p ^ ¬p
F, F, F, T	¬p ^ ¬q
F, F, T, T	¬p
F, T, T, T	¬p v ¬q
F, T, F, T	¬q
T, F ,T, F	q
T, F, F, T	p ←→ q
F, T, T, F	p ←→ ¬q
T, T, F, T	¬p → ¬q
T, F, T, T	p → q
F, F, T, F	¬p ^ q
F, T, F, F	p ^ ¬q

c) it would be 256 because 2^(2^n): where n = 3

Or 2^8 = 256

10.

p	q	p ^ q	p v q	(p ^ q) → (p v q)
F	T	F	T	T
F	F	F	F	T
T	T	T	T	T
T	F	F	T	T

Yes, it equals True for every situation in (p ^ q) → (p v q) ; so it is a tautology.