CS-2102
Discrete Math
Problem Set 3, Due Friday February 17, 2016 by 5 pm.

Remember that you can work one other person. Be sure to give adequate explanation for your solutions, lack of clarity and completeness may result in points being deducted. <u>Answers given with no work or justification may have points taken off.</u>

(4 pts) 1. Prove the following statement. $p \to q \equiv (p \wedge \neg q) \to F$.

(3 pts) 2. Given the following:

(1) $\neg t \vee \neg u$
(2) $s \to t \wedge u$
(3) $(\neg r \vee \neg f) \to (s \wedge l)$

Can we conclude *r*? (Hint you will need to use De Morgan's Law)

(3 pts) 3. Identify all the errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true then $\forall x P(x) \vee \forall x Q(x)$.

(a)	$\forall x(P(x) \vee Q(x))$	Premise
(b)	$P(c) \vee Q(\mathrm{c})$	Universal instantiation from (a)
(c)	$P(c)$	Simplification from (b)
(d)	$\forall x P(x)$	Universal generalization from (c)
(e)	$Q(c)$	Simplification from (b)
(f)	$\forall x Q(x)$	Universal generalization from (e)
(g)	$\forall x P(x) \vee \forall x Q(x)$	Conjunction from (d) and (f)

(4 pts) 4. Prove the following using the rules of inference discussed in class

(1) $(p \vee \neg t) \to h$
(2) $\neg h \vee (m \wedge k)$
(3) $(m \wedge k \wedge q) \to r$
(4) $\neg q \to (\neg h \wedge \neg s)$
(5) $p \vee \neg t$
(6) $\therefore r$

(2 pts) 5. In the back of an old cupboard you discover a note signed by the famous pirate Edward Teach AKA Blackbeard (a favorite of my alma mater ECU. His hometown was in Bath NC). It seems that he had a bizarre sense of humor and loved logic puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements and challenged the reader to use them to figure out the location of the treasure.

(a) If this house is next to a lake, then the treasure is not in the kitchen.
(b) If the tree in the front yard is an elm, then the treasure is in the kitchen.
(c) This house is next to a lake.
(d) The tree in the front yard is an elm or the treasure is buried under the flagpole.
(e) If the tree in the back yard is an oak, then the treasure is in the garage.

Where is the treasure hidden? Be sure to justify your answer.

(15 pts, 3pts for a-c each and 6 points for d) 6. You are visiting an island containing two types of people: knights who always tell the truth and knaves who always lie. (Hint make some assumptions, such as "assume A is a knight, then" or "suppose B is telling the truth" and try to show that these assumptions can't be true. This is called proof by contradiction.)

(a) Two natives A and B address you as follows:
A says: Both of us are knights.
B says: A is a knave.
What are A and B?
(b) Two different natives C and D approach you but only C speaks.
C says: Both of us are knaves.
What are C and D?
(c) You then encounter natives E and F.
E says: F is a knave.
F says: E is a knave.
How many knaves are there?
(d) Finally, you meet a group of six natives, U, V, W, X, Y, and Z, who speak to you as follows:
U says: None of us is a knight.
V says: At least three of us are knights.
W says: At most three of us are knights.
X says: Exactly five of us are knights.
Y says: Exactly two of us are knights.
Z says: Exactly one of us is a knight.
Which of these six are knights and which are knaves?

(2 pts)7. What is wrong with this argument? Given the premise $\exists x P(x) \land \exists x Q(x)$, use simplification to obtain $\exists x P(x)$; use existential instantiation to obtain $P(c)$ for some element c; use simplification again to obtain $\exists x Q(x)$; use existential instantiation to obtain $Q(c)$ for some element c; use conjunction to conclude that $P(c) \land Q(c)$; and finally, use existential generalization to conclude that $\exists x(P(x) \land Q(x))$. Point out the flaw(s) with this argument.

(5 pts) 8. Show that the argument form with premises $(p \land t) \to (r \lor s)$, $q \to (u \land t)$, $u \to p$ and $\neg s$ and conclusion $q \to r$ is valid.

(3 pts) 9. Come up with your own (different from the 8 given in class) rule of inference and prove that it is a valid rule of inference. It doesn't have to be complicated, in fact many of you may have come across one or two in the work you did for class. Remember that a rule of inference has some statements as premises (3 at the max for this problem) and one conclusion that you can draw from these premises.

(10 pts, 2 each) 10. Use the following two assumptions:

(1) "Logic is difficult or not many students like logic."
(2) "If mathematics is easy, then logic is not difficult."

By translating these assumptions into statements involving propositional variables and logical connectives, determine whether each of the following are valid conclusions of these assumptions: (Hint: if they are **not** valid conclusions give an example of when the assumptions are true and the conclusion is false, if they are valid show that if the assumptions are true the conclusion is also true.)

(a) If many students like logic, then mathematics is not easy.
(b) That not many students like logic, if mathematics is not easy.
(c) That mathematics is not easy or logic is difficult.
(d) That logic is not difficult or mathematics is not easy.
(e) That if not many students like logic, then mathematics is not easy or logic is not difficult.

(8 pts, 2 pts each) 11. Let $P(x)$, $Q(x)$, and $R(x)$ be the statements "x is a clear explanation," "x is satisfactory," and "x is an excuse," respectively. Suppose that the domain for x consists of all English text. Express each of the following statements using quantifiers, logical connectives, and $P(x)$, $Q(x)$, and $R(x)$.

(a) All clear explanations are satisfactory
(b) Some excuses are unsatisfactory
(c) Some excuses are not clear explanations
(d) Does (c) follow from (a) and (b)?

(5 pts) 12. Prove that the following argument, taken from *Logic: Techniques of Formal Reasoning* by D. Kalish and R. Montague, is valid.

> If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent. If he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exists. .

Hint 1: There are a couple of ways to solve this. One way requires using the rule of inference resolution. Hint 2: When solving this problem remember our discussion of the honor pledge and what we really mean when we say “I have neither given nor received aid on this exam”. What we mean by this is “I have not given and I have not received”. Use this same meaning in the above argument.