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82-2=80

problem set 2

1) a. There exists an integer that is ~~both~~ prime and not odd.
↳ True, the number 2

b. If x is prime then x is not a perfect square.
↳ True, one is not a prime number

c. There exists an integer that is odd and a perfect square.
↳ True, the number 25, the perfect square of 5 that is odd.

2) a. negation: $\exists x \in \mathbb{Z}, \neg(x < 0)$
English: There exists an integer that is not less than zero

b. negation: $\forall x \in \mathbb{Z}, \neg(x = x+1)$
English: For all integers x does not equal x + 1.

c. negation: $\forall x \in \mathbb{N}, \neg(x > 10)$
English: For all natural numbers, x is not greater than 10

d. negation: $\exists x \in \mathbb{N}, \neg(x > 10)$
English: There is a natural number that is not greater than 10.

3) a) $\forall$ fish x, x has gils
negation: $\exists x \in$ fish, $\neg$(has gils)

b) $\forall$ computers c, c has a CPU
negation: $\exists c \in$ computer, $\neg$(has CPU)

c) $\exists$ a movie m, such that m is over 6 hours long
negation: $\forall m \in$ movie, $\neg$(m > 6 hours long)

d) $\exists$ a band b, such that b has won at least 10 grammy awar
negation: $\forall b \in$ band, $\neg$(b $\geq$ 10 grammy awards)

4) a) There is a student s that has seen the movie Casablanca

b) If there is a student, then the student S has seen starwars

c) For all students s, there exists a movie m such that all student have seen movie m

d) There is a movie M, such that for all students s, Student s has seen movie m.

e) There is a student s, there is a student t, and there is a movie m such that s is not the same as t and both student s has seen movie m and student t has seen movie m.

f) There is a student s, there is a student t, and for all movies m, such that s is not the same as t and if student s has seen movie m, then student t has seen movie m.

5) The absence of error messages during translation of a computer program is a required element but not the only requirement and therefore is not enough for reasonable program correctness.

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6) a) For all positive integers x, there exists a positive integer y such that $x = y + 1$.
↳ False, goes wrong when $x = 1$, y would have to equal 0, which is not a positive number

b) For all integers x, there exists an integer y such that $x = y + 1$
↳ True, if you consider 0 an integer, so when $x = 1$, y can equal 0

c) There exists a real number x, such that for all real numbers y, $x = y + 1$
↳ False, for a single x value not all y values will be able to make the statement true (only one y-value)

d) For all positive real numbers x, there exists a positive real number y, such that $xy = 1$
↳ True, whatever x is, y will equal $1/x$.

e) For all integers x, there exists a real number y such that $xy = 1$.
↳ True, because there has to be some sort of real number that will multiply x down to 1 (y will equal $1/x$ just like in D)

6) f. for all positive integers x and for all positive positive integers y, their exists a positive integer z such that $z = x - y$.

↳ False, if X is greater than or equal to Y, then Z will be 0 or negative.

g. For all integers x and for all integers y, their exists an integer Z such that $z = x - y$

↳ True, since Z can be all integers; as the subtraction of two integers will result as an integer

h. There exists a positive real number U such that all positive real numbers V, $UV < V$

True. $U = .1$ $V = 10$ $UV < V$ because $1 < 10$ (as long as U is between 0 and 1)

7) a) $\forall x (P(x) \rightarrow Q(x))$ and $\forall x P(x) \rightarrow \forall x Q(x)$

Explaination: through an equivalence method, we know that $\forall x (P(x) \rightarrow Q(x)) \equiv \forall x (\sim P \vee q)$, based off the rules of quantifiers, we know that $\forall x$ cannot be distributed into an "or" statement which makes it impossible for the two to be logically equivalent. In $\forall x (P(x) \rightarrow Q(x))$ $P(x)$ is sufficient for $Q(x)$ to be true, while in $\forall x P(x) \rightarrow \forall x Q(x)$, $P(x)$ is a necessary condition

b) $\exists x (P(x) \rightarrow Q(x))$ and $\forall x P(x) \rightarrow \exists x Q(x)$

1. $\exists x (\sim P(x) \vee Q(x))$ equality | 1. $\neg(\forall x P(x)) \rightarrow \exists x Q(x)$ Distribute

2. $\exists x \sim P(x) \vee \exists Q(x) \equiv$ 2. $\exists x \sim P(x) \vee \exists x Q(x)$ equality

So...

yes they are logically equivalent.

c. No they are not logically equivalent.
$\forall x (P(x) \leftrightarrow Q(x))$ is asking for all "sets" of where $P(x) \leftrightarrow Q(x)$ is valid, where as $\forall x P(x) \leftrightarrow \forall x Q(x)$ is saying all values of $P(x) \leftrightarrow$ all values of $Q(x)$; so one is asking for all within a "set" in x while the other says for all $P(x)$ to be true if and only if all $Q(x)$. So basically $\forall x (P(x) \leftrightarrow Q(x))$ is asking for all values where $(P(x) \leftrightarrow Q(x))$, where as the other says All $P(x)$ if and only if all $Q(x)$.

$\longrightarrow$

d) $\forall x(A \to P(x))$ and $A \to \forall x P(x)$

since A does not rely on X, the $\forall x$ in $\forall x(A \to P(x))$ skips over the "$A \to$" and goes to the $P(x)$

$A \to \forall x(P(x))$ which is $A \to \forall x P(x)$ which is indeed equivalent to $A \to \forall x P(x)$ by null quantification

e) $\forall x(P(x) \to A)$ and $\exists x P(x) \to A$

in $\forall x(P(x) \to A)$, they are asking for all the values of X that meet the condition $(P(x) \to A)$; whereas $\exists x P(x) \to A$ says there exists an X where $(P(x) \to A)$, which includes all values of X that meet that condition, which makes the two equations logically equivalent.

f) Not equivalent because you cannot distribute Universal quantifiers to "or" statements

8) a) True; any number would work because squaring it will make it a real number

b) False; if X is a negative number then you cannot squareroot y and get a negative

c) True; there will always be a recipricol y that will make $xy = 1$ because $X \neq 0$.

d) False; there's no one X that can make all y in $xy = 1$.

e) True; for all real numbers $\frac{x+y}{2}$ will result in a real number Z

9) a) $A = \{x \in \mathbb{R} \mid x \leq b\}$

b) 4, 5, 69

c) no, because the values of x reach ∞ on the upper range

d) 1.5, 2, 2.5

e) $A = \{x \in \mathbb{R} \mid x > b\}$