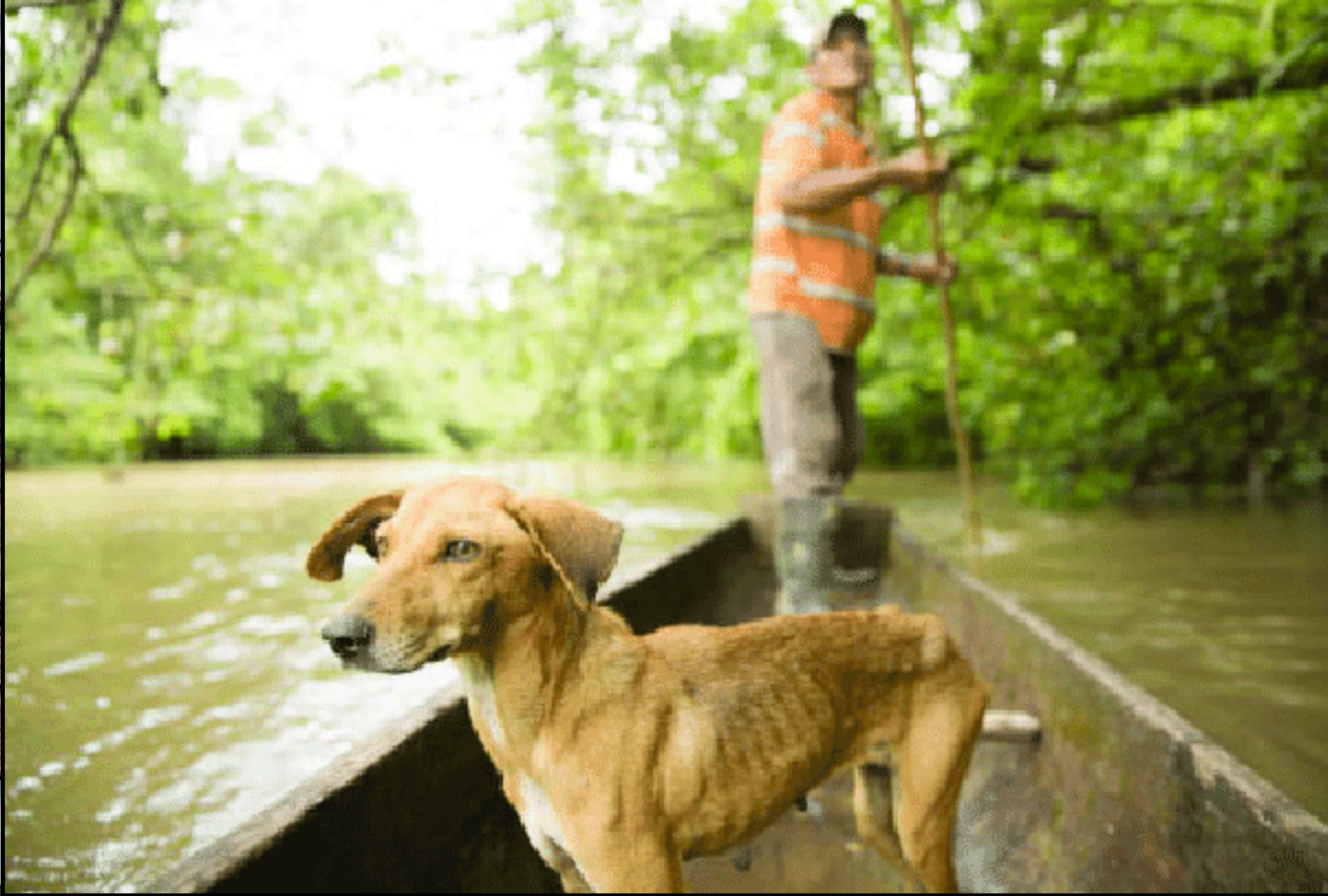
Statistical Rethinking

2022

15: Social Networks




Belize
Mayangna Miskito
Serranilla Ba
Guatemala
Honduras
Bajo Nuevo Bank
El Salvador
Nicaragua
Costa Rica

What Motivates Sharing?

"Up in our country we are human! And since we are human we help each other. We don't like to hear anybody say thanks for that. What I get today you may get tomorrow. Up here we say that by gifts one makes slaves and by whips one makes dogs."

Quoted in Peter Freuchen's 1961 book about the Inuit

What Motivates Sharing?

data(KosterLeckie)

Year of food transfers among 25 households in Arang Dak

25!/(2!(25-2)!) = 300 dyads

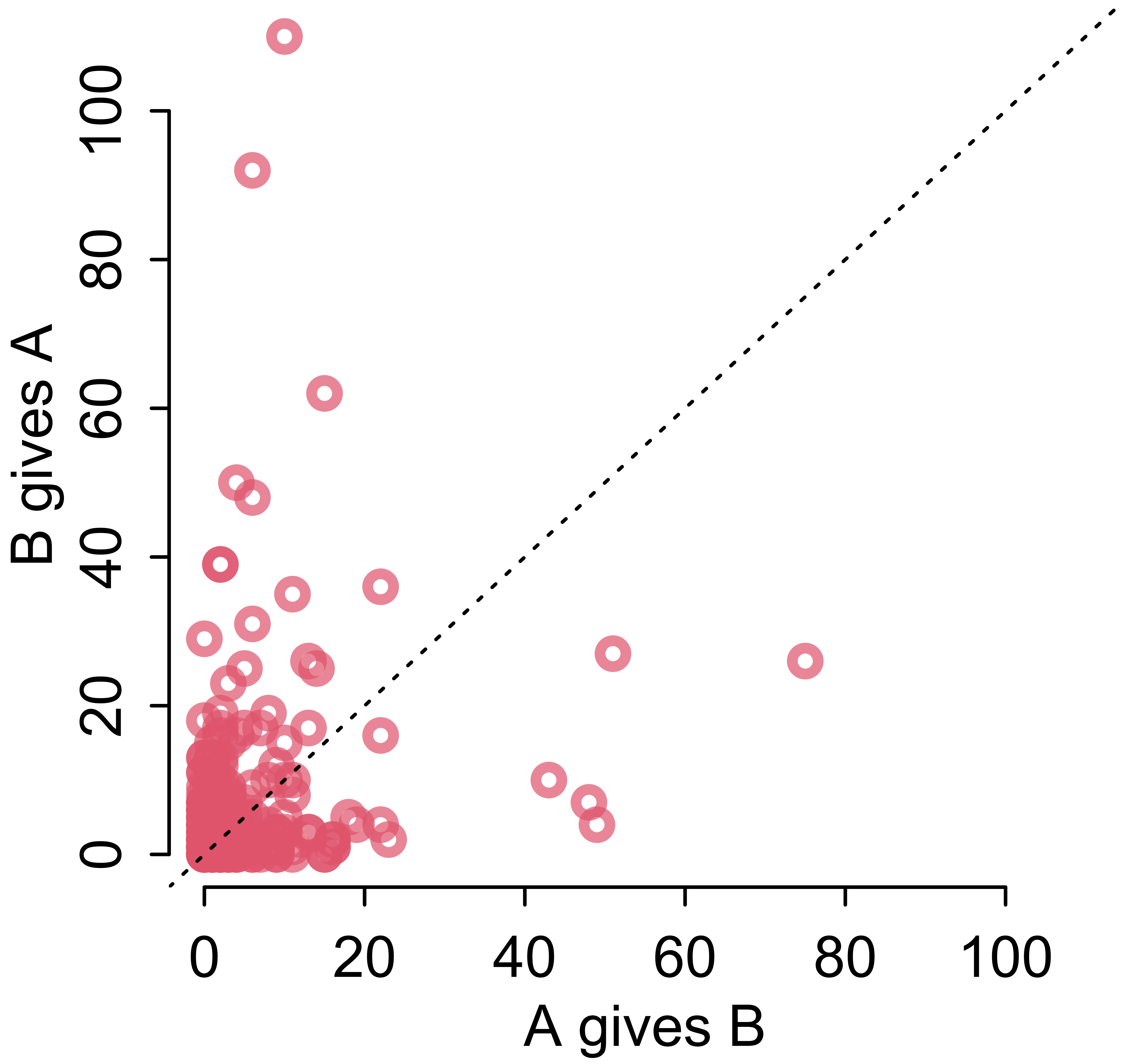
How much sharing explained by reciprocity?
How much by generalized giving?

Which dyads? Which households?

Koster & Leckie 2014

Photo: Dr Karl Frost

What Motivates Sharing?



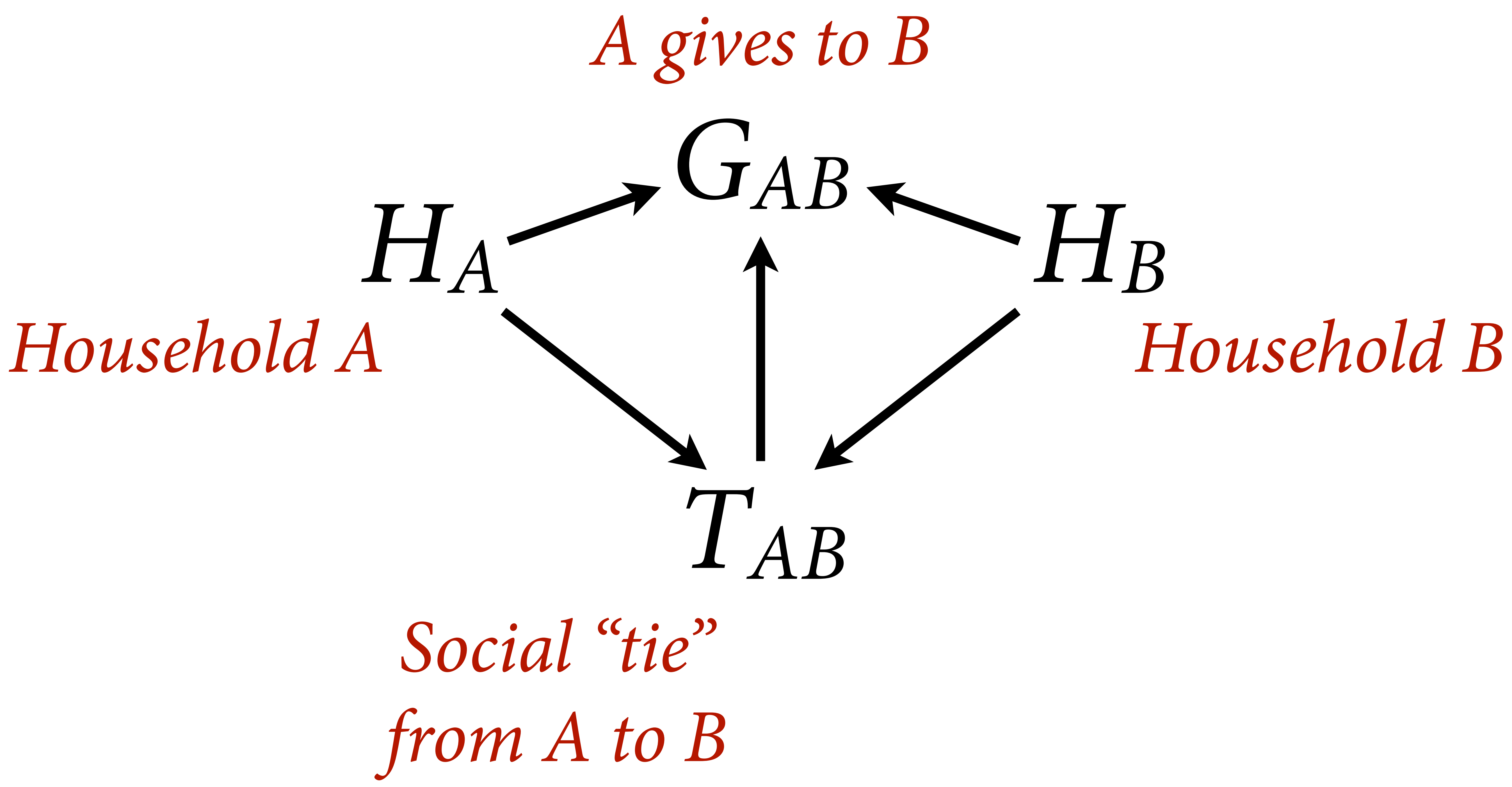
Koster & Leckie 2014

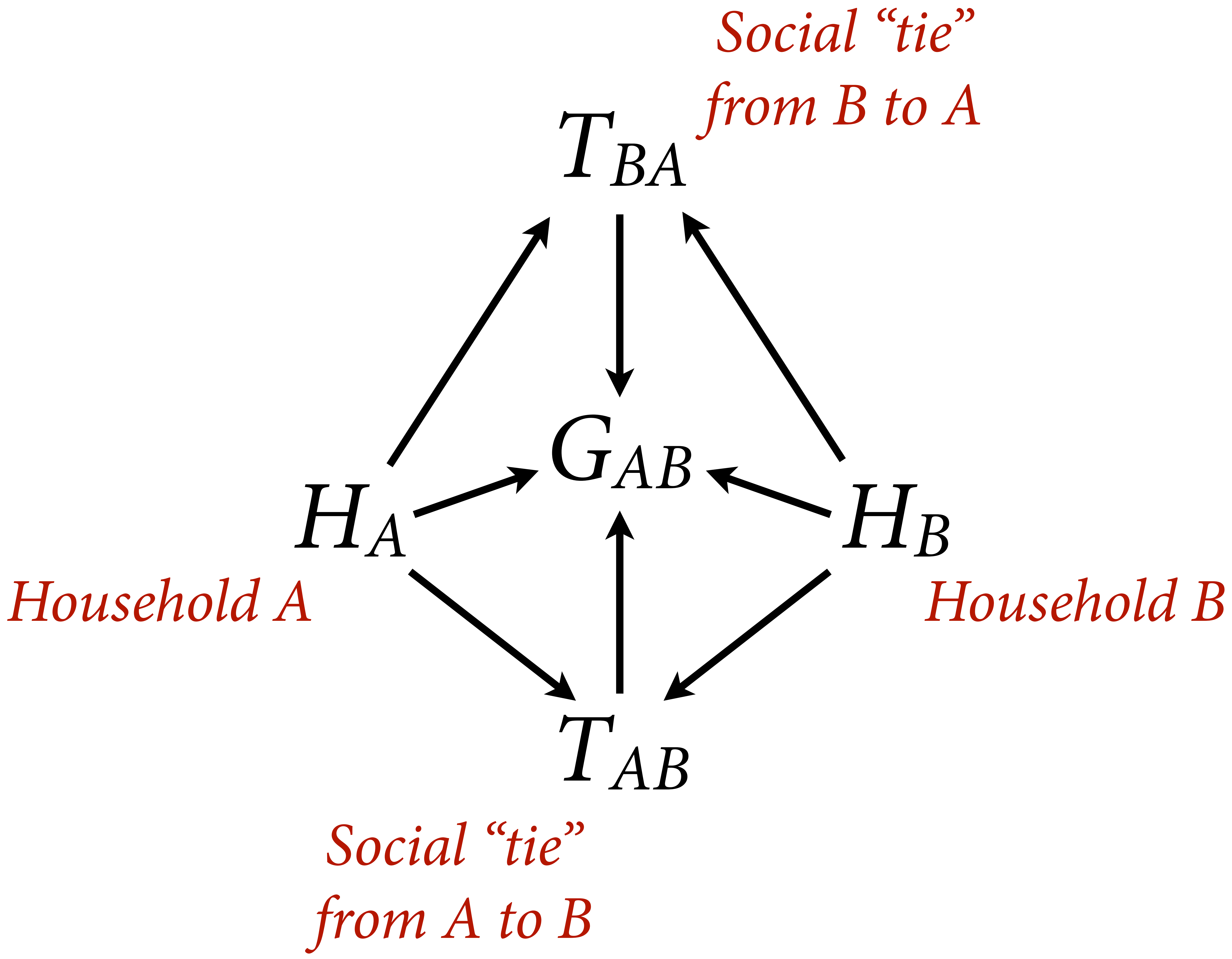
A gives to B

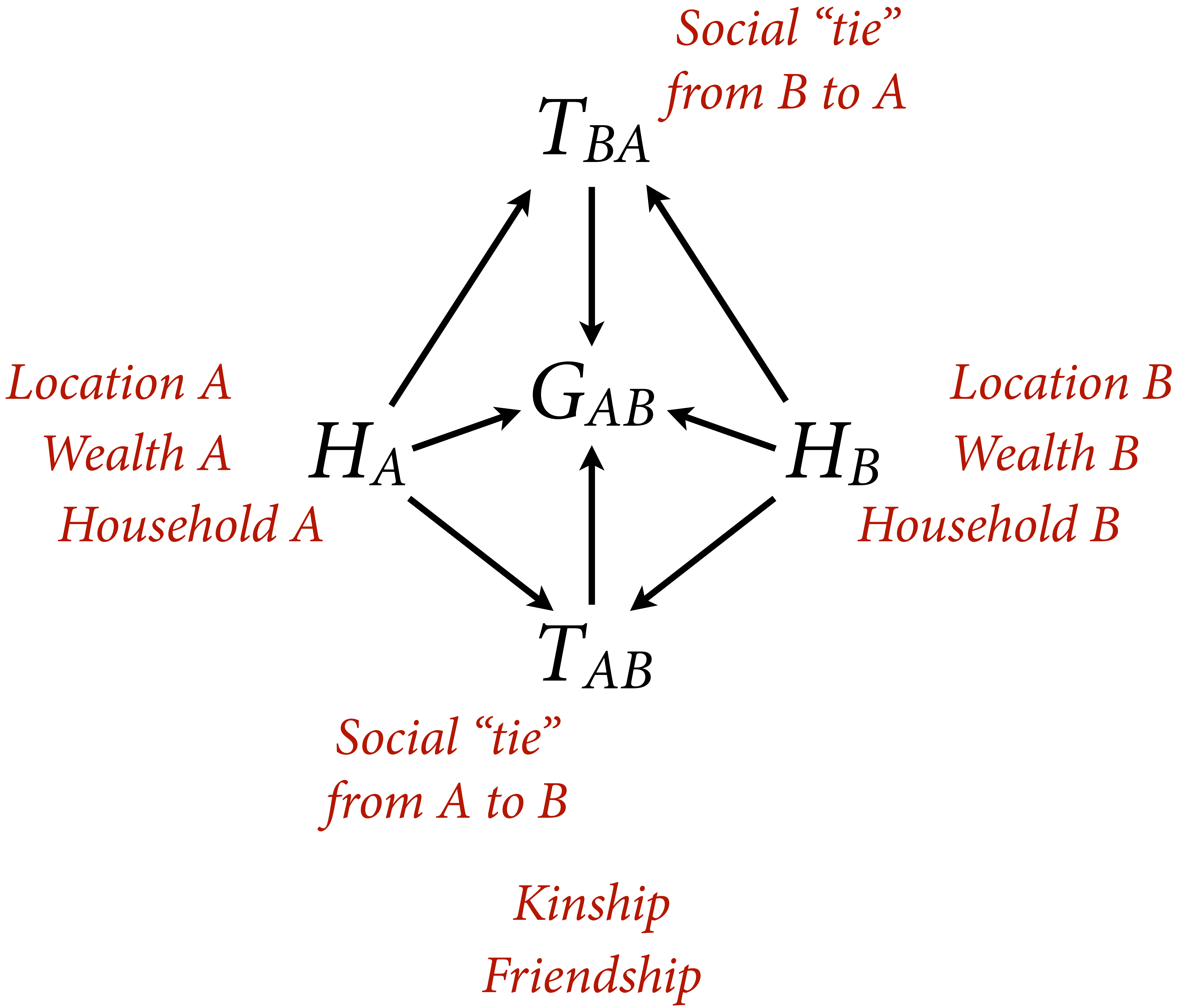
$$H_A \longrightarrow G_{AB} \longleftarrow H_B$$

Household A

Household B


A gives to B
G_{AB}
H_A
H_B
Household A
Household B
T_{AB}
Social "tie"
from A to B


Social "tie"
from B to A
T_{BA}
G_{AB}
H_A
H_B
Household A
Household B
T_{AB}
Social "tie"
from A to B



Social "tie"
from B to A
T_{BA}
Location A
Wealth A
Household A
H_A
G_{AB}
H_B
Location B
Wealth B
Household B
T_{AB}
Social "tie"
from A to B
Kinship
Friendship

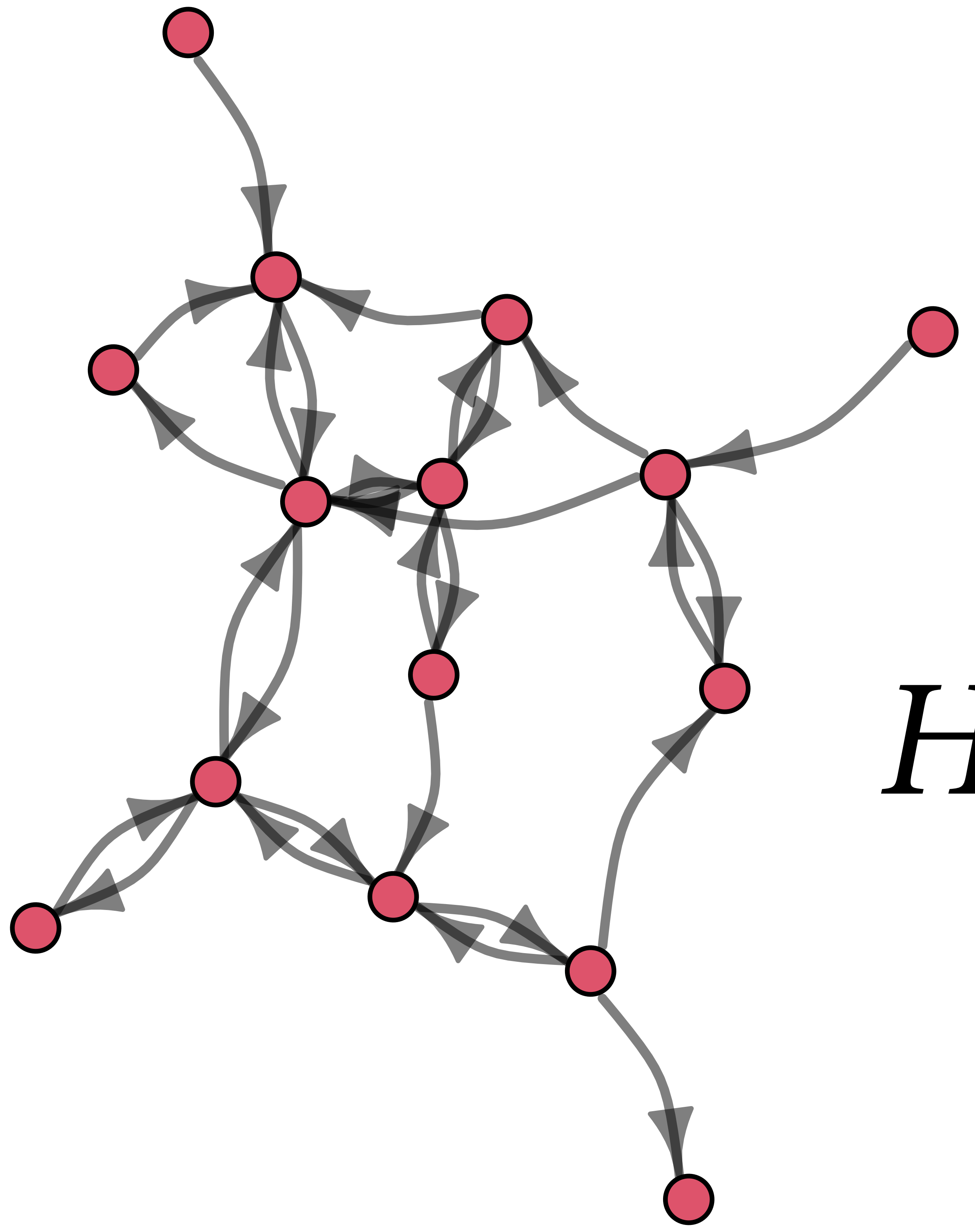
What Motivates Sharing?

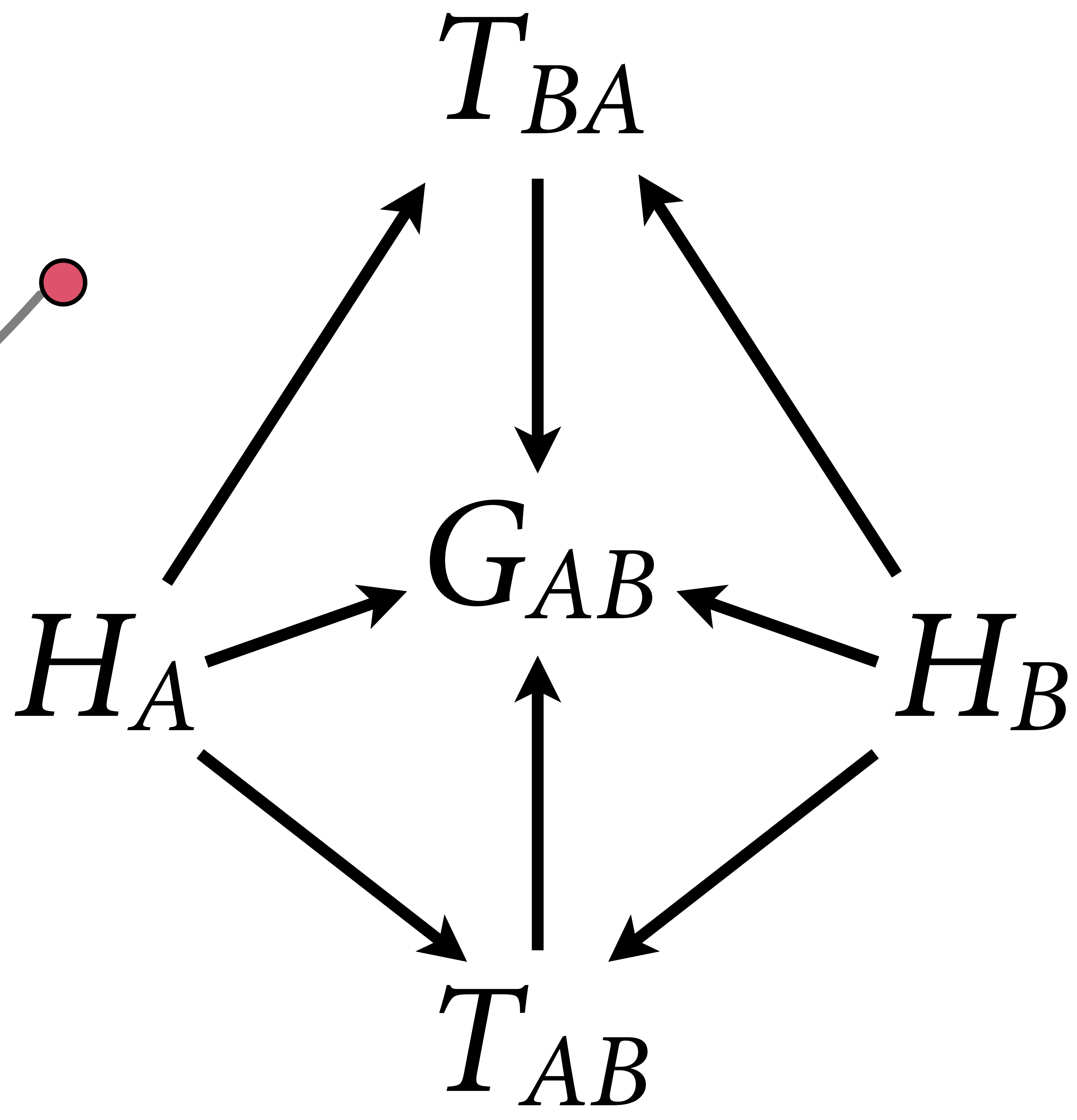
T_{AB} and T_{BA} are not observable

Social network: Pattern of directed exchange

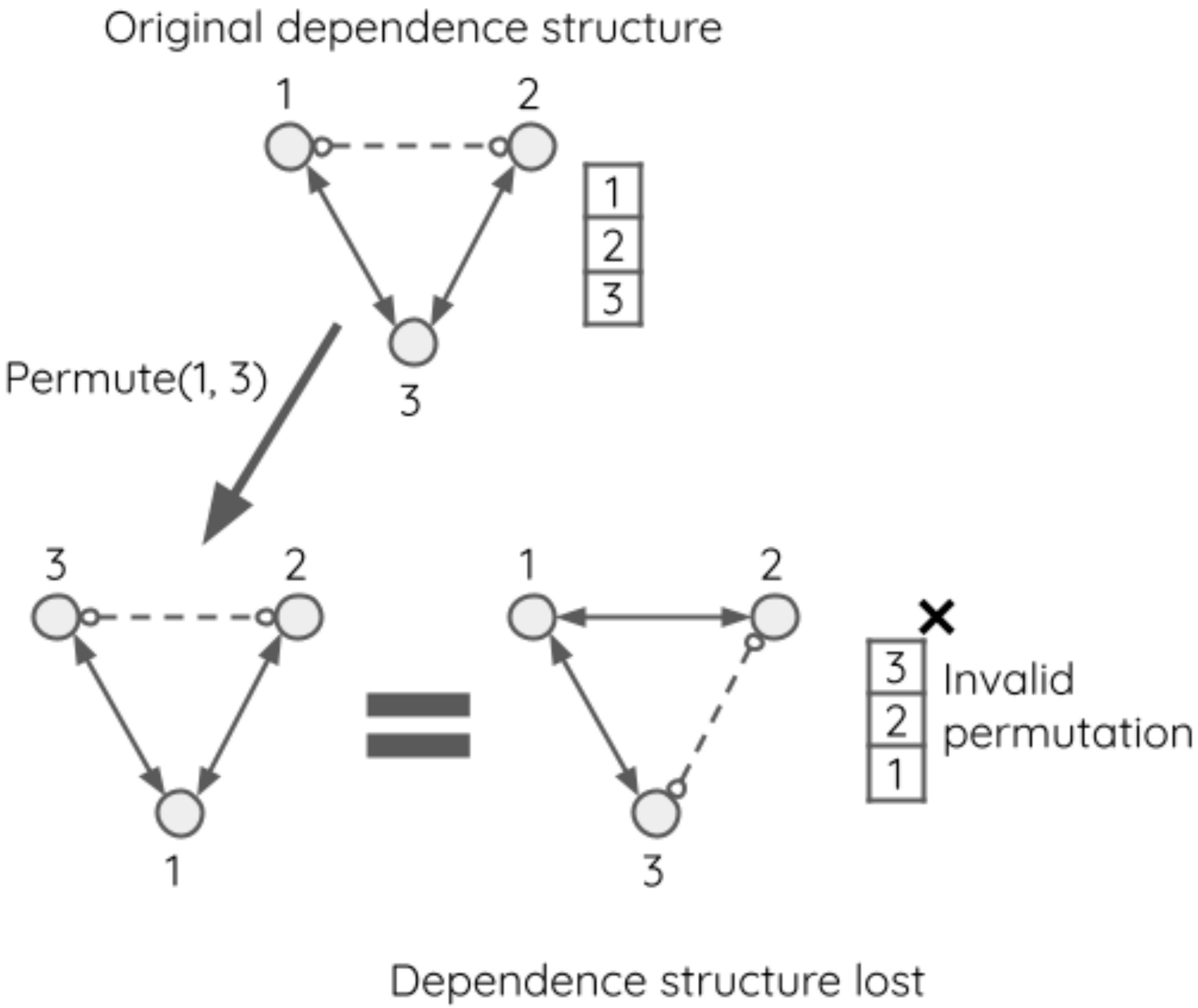
Social networks are abstractions, are not data

What is a principled approach?





Resist Adhockery



Hart et al 2021 Common Permutation Methods in Animal Social Network Analysis Do Not Control for Non-independence

Drawing the Social Owl

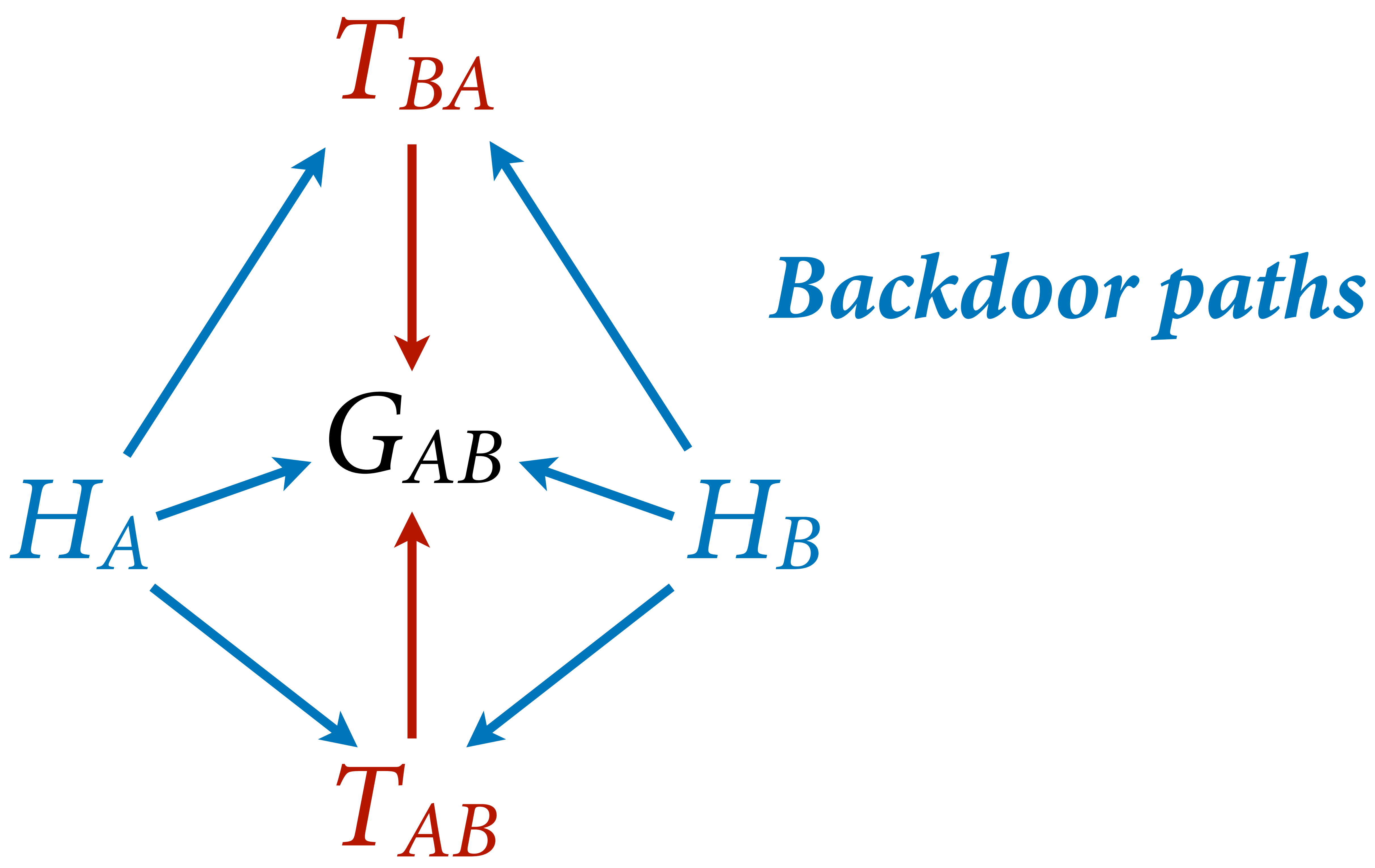
(1) Estimand: Reciprocity & what explains it

(2) Generative model

(3) Statistical model

(4) Analyze sample

$$
\begin{tikzcd}
 & T_{BA} \arrow[d] & \\
H_A \arrow[ur] \arrow[r] \arrow[dr] & G_{AB} & H_B \arrow[ul] \arrow[l] \arrow[dl] \\
 & T_{AB} \arrow[u] &
\end{tikzcd}
$$


T_{BA}
Backdoor paths
G_{AB}
H_A
H_B
T_{AB}

Drawing the Social Owl

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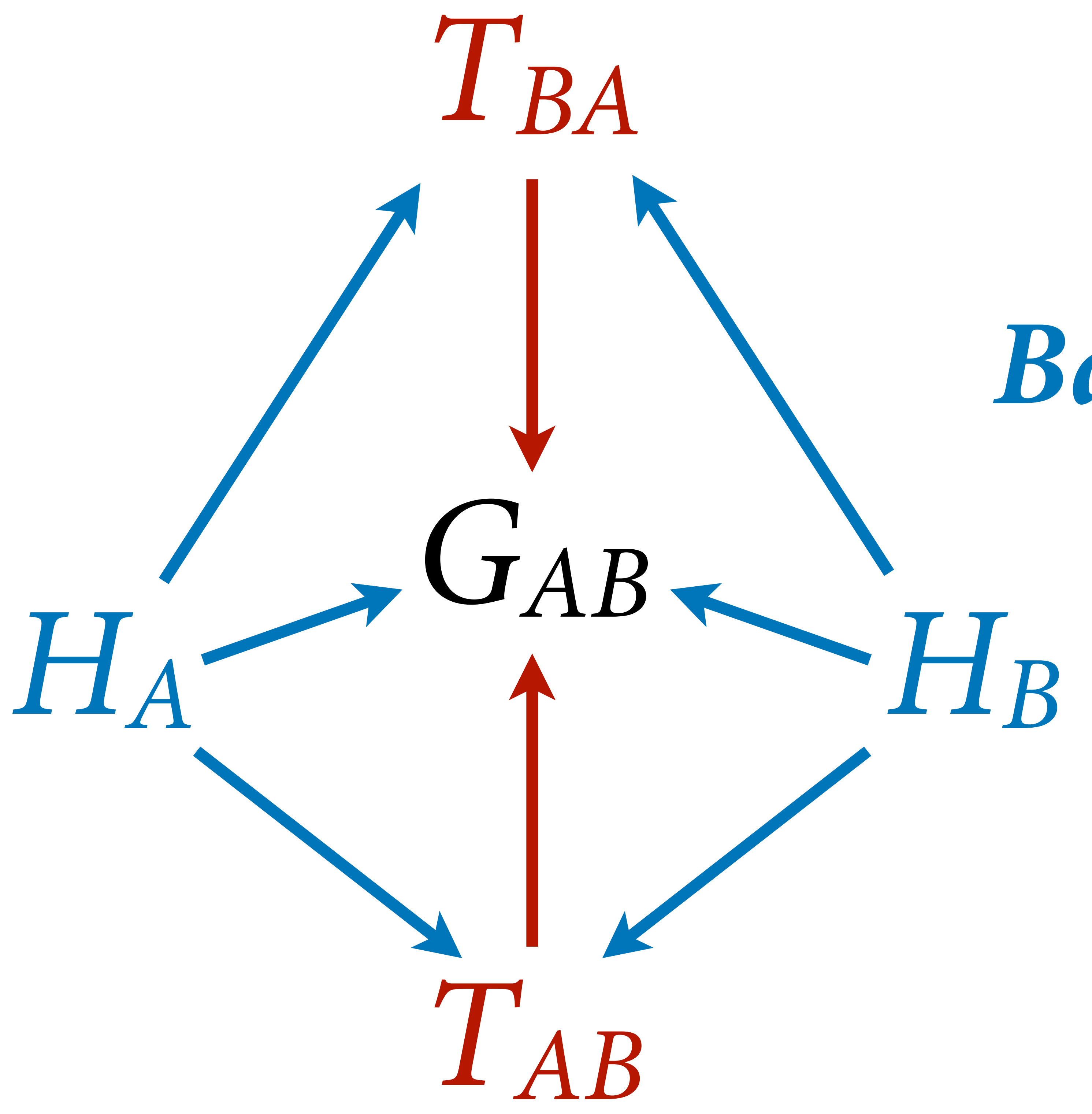

Backdoor paths

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H_A \arrow[ur] \arrow[dr] & G_{AB} & H_B \arrow[ul] \arrow[dl] \\
 & T_{AB} \arrow[u] &
\end{tikzcd}
$$

```r
# N households
N <- 25
dyads <- t(combn(N,2))
N_dyads <- nrow(dyads)
```

```r
# N households
N <- 25
dyads <- t(combn(N,2))
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```

```
> dyads
      [,1] [,2]
 [1,]    1    2
 [2,]    1    3
 [3,]    1    4
 [4,]    1    5
 [5,]    1    6
 [6,]    1    7
 [7,]    1    8
 [8,]    1    9
 [9,]    1   10
[10,]    1   11
[11,]    1   12
[12,]    1   13
[13,]    1   14
[14,]    1   15
[15,]    1   16
[16,]    1   17
[17,]    1   18
[18,]    1   19
[19,]    1   20
[20,]    1   21
[21,]    1   22
[22,]    1   23
[23,]    1   24
[24,]    1   25
[25,]    2    3
[26,]    2    4
[27,]    2    5
[28,]    2    6
[29,]    2    7
[30,]    2    8
[31,]    2    9
[32,]    2   10
[33,]    2   11
[34,]    2   12
[35,]    2   13
[36,]    2   14
[37,]    2   15
[38,]    2   16
[39,]    2   17
[40,]    2   18
[41,]    2   19
[42,]    2   20
[43,]    2   21
[44,]    2   22
[45,]    2   23
[46,]    2   24
[47,]    2   25
[48,]    3    4
[49,]    3    5
[50,]    3    6
[51,]    3    7
[52,]    3    8
[53,]    3    9
[54,]    3   10
[55,]    3   11
[56,]    3   12
[57,]    3   13
[58,]    3   14
[59,]    3   15
[60,]    3   16
[61,]    3   17
[62,]    3   18
[63,]    3   19
[64,]    3   20
[65,]    3   21
[66,]    3   22
[67,]    3   23
[68,]    3   24
[69,]    3   25
[70,]    4    5
[71,]    4    6
[72,]    4    7
[73,]    4    8
[74,]    4    9
[75,]    4   10
[76,]    4   11
[77,]    4   12
[78,]    4   13
[79,]    4   14
[80,]    4   15
[81,]    4   16
[82,]    4   17
[83,]    4   18
[84,]    4   19
[85,]    4   20
[86,]    4   21
[87,]    4   22
[88,]    4   23
[89,]    4   24
[90,]    4   25
[91,]    5    6
```

```r
# N households
N <- 25
dyads <- t(combn(N,2))
N_dyads <- nrow(dyads)

# simulate "friendships" in which ties are shared
f <- rbern(N_dyads,0.1) # 10% of dyads are friends
```

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# now simulate directed ties for all individuals
# there can be ties that are not reciprocal
alpha <- (-3) # base rate of ties; -3 ~= 0.05
y <- matrix(NA,N,N) # matrix of ties
```

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y <- matrix(NA,N,N) # matrix of ties
for ( i in 1:N ) for ( j in 1:N ) {
    if ( i != j ) {
        # directed tie from i to j
        ids <- sort( c(i,j) )
        the_dyad <- which( dyads[,1]==ids[1] & dyads[,2]==ids[2] )
        p_tie <- f[the_dyad] + (1-f[the_dyad])*inv_logit( alpha )
        y[i,j] <- rbern( 1 , p_tie )
    }
}#ij
```

```
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    }
}#ij
```

friends share ties

```r
# now simulate gifts
giftsAB <- rep(NA,N_dyads)
giftsBA <- rep(NA,N_dyads)
lambda <- log(c(0.5,2)) # rates of giving for y=0,y=1
for ( i in 1:N_dyads ) {
    A <- dyads[i,1]
    B <- dyads[i,2]
    giftsAB[i] <- rpois( 1 , exp( lambda[1+y[A,B]] ) )
    giftsBA[i] <- rpois( 1 , exp( lambda[1+y[B,A]] ) )
}
```

```r
# now simulate gifts
giftsAB <- rep(NA,N_dyads)
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    giftsBA[i] <- rpois( 1 , exp( lambda[1+y[B,A]] ) )
}
```
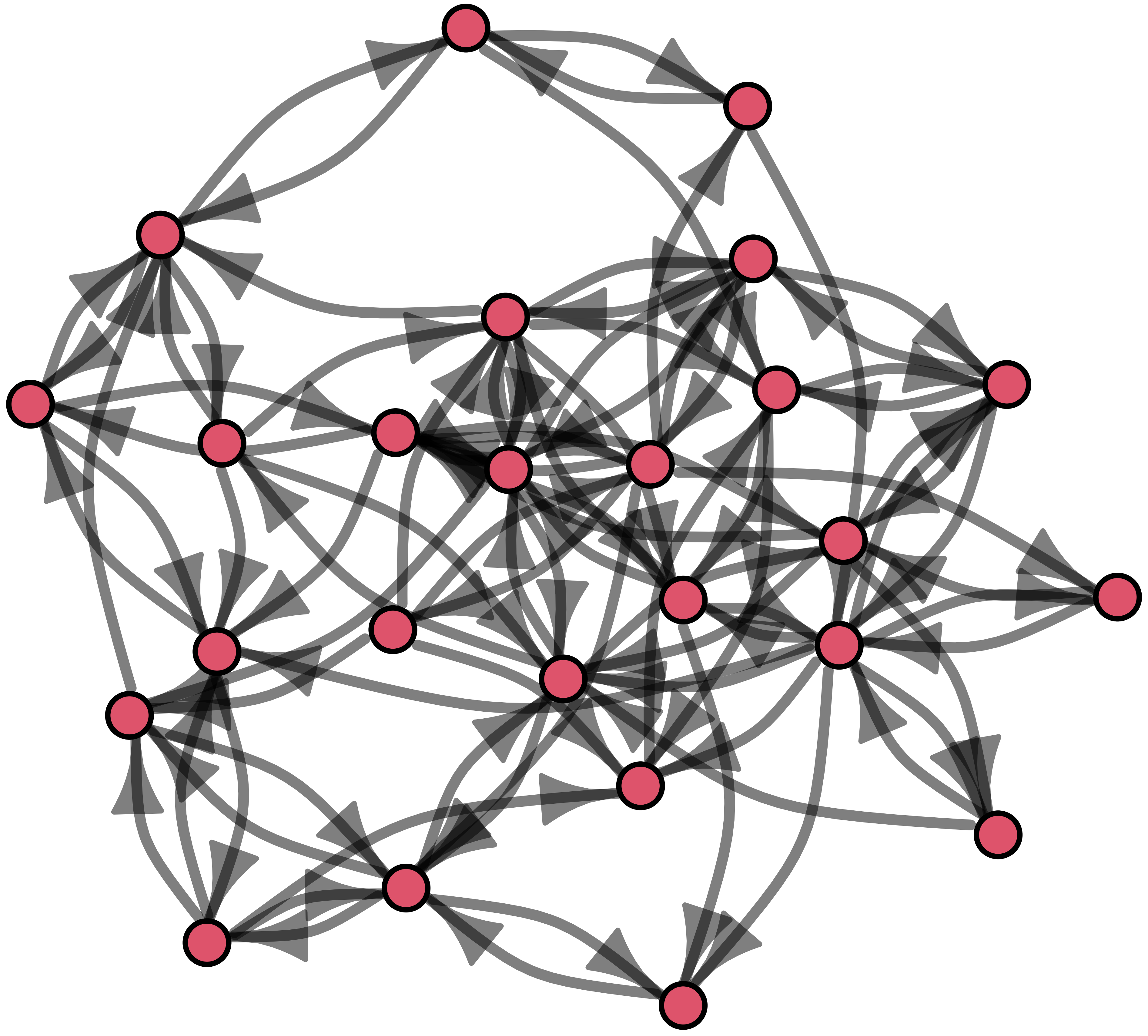

```r
# draw network
library(igraph)
sng <- graph_from_adjacency_matrix(y)
lx <- layout_nicely(sng)
vcol <- "#DE536B"
plot(sng , layout=lx , vertex.size=8 ,
edge.arrow.size=0.75 , edge.width=2 ,
edge.curved=0.35 , vertex.color=vcol ,
edge.color=grau() , asp=0.9 , margin = -0.05 ,
vertex.label=NA )
```



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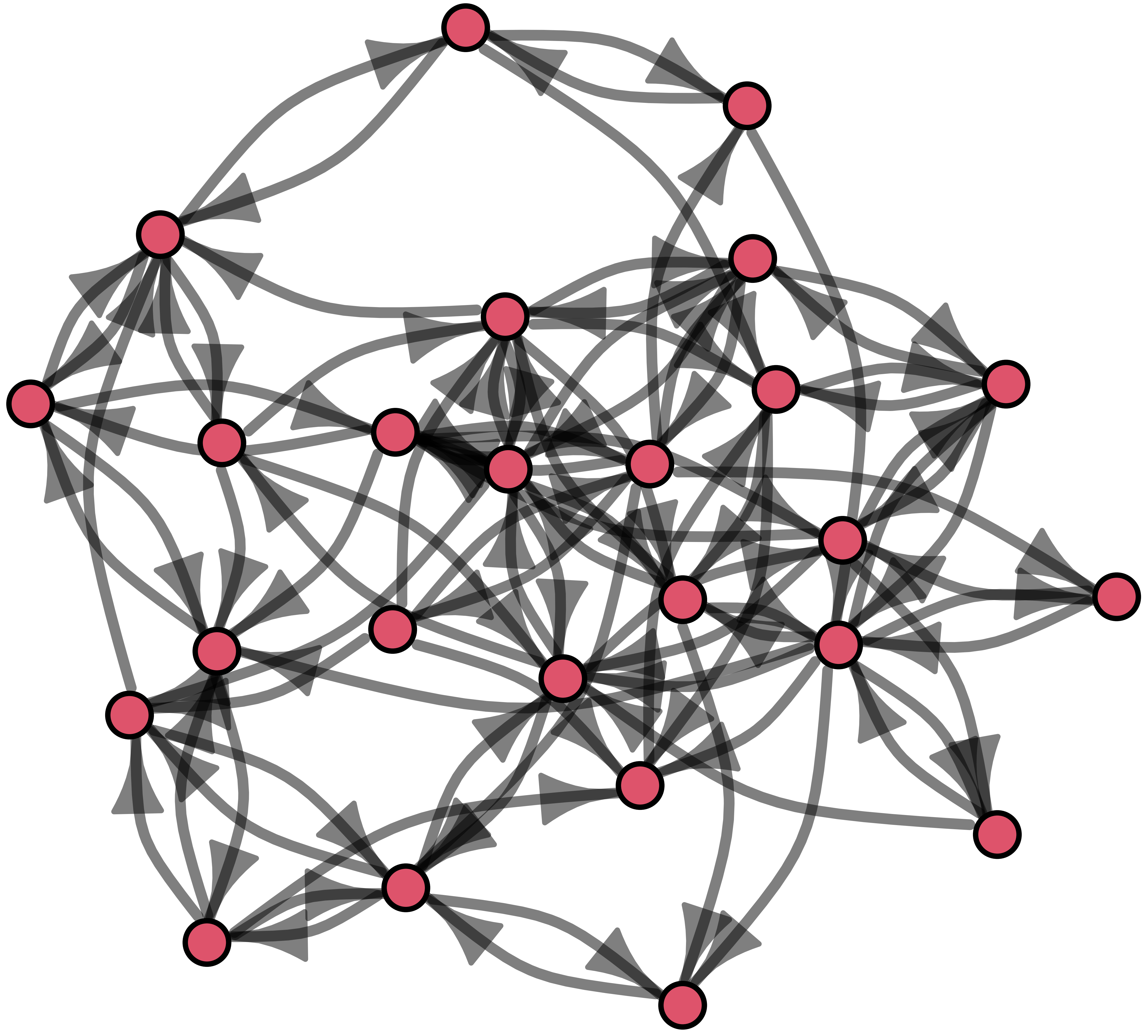

Gifts A to B

$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB}$$

average

Tie A to B

Gifts A to B

$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB}$$

average

Tie A to B

Gifts B to A

$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{BA}) = \alpha + T_{BA}$$

Tie B to A

$$G_{AB} \sim \text{Poisson}(\lambda_{AB}) \qquad G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB} \qquad \log(\lambda_{BA}) = \alpha + T_{BA}$$

The AB dyad $\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \underbrace{\rho\sigma^2}_{\text{covariance within dyads}} & \underbrace{\sigma^2}_{\text{variance among ties}} \end{bmatrix} \right)$

$$G_{AB} \sim \text{Poisson}(\lambda_{AB}) \qquad G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB} \qquad \log(\lambda_{BA}) = \alpha + T_{BA}$$

partial pooling for network ties

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

$$\rho \sim \text{LKJCorr}(2)$$

$$\sigma \sim \text{Exponential}(1)$$

$$\alpha \sim \text{Normal}(0,1)$$

```r
# dyad model
f_dyad <- alist(
    GAB ~ poisson( lambdaAB ),
    GBA ~ poisson( lambdaBA ),
    log(lambdaAB) <- a + T[D,1] ,
    log(lambdaBA) <- a + T[D,2] ,
    a ~ normal(0,1),

    ## dyad effects
    transpars> matrix[N_dyads,2]:T <-
            compose_noncentered( rep_vector(sigma_T,2) , L_Rho_T , Z ),
    matrix[2,N_dyads]:Z ~ normal( 0 , 1 ),
    cholesky_factor_corr[2]:L_Rho_T ~ lkj_corr_cholesky( 2 ),
    sigma_T ~ exponential(1),

    ## compute correlation matrix for dyads
    gq> matrix[2,2]:Rho_T <<- Chol_to_Corr( L_Rho_T )
)

mGD <- ulam( f_dyad , data=sim_data , chains=4 , cores=4 , iter=2000 )
```

```
# dyad model
f_dyad <- alist(
    GAB ~ poisson( lambdaAB ),
    GBA ~ poisson( lambdaBA ),
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```
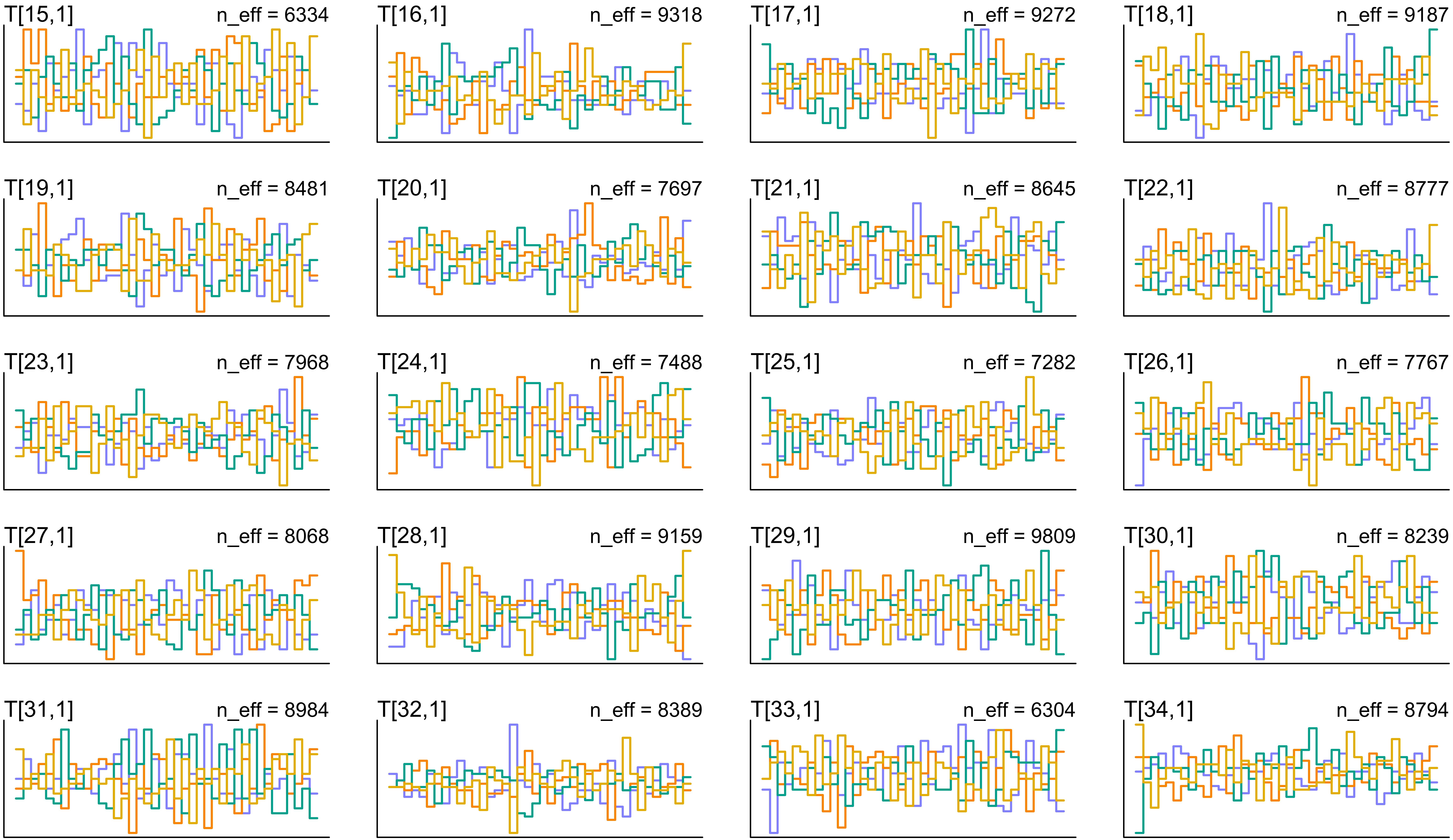
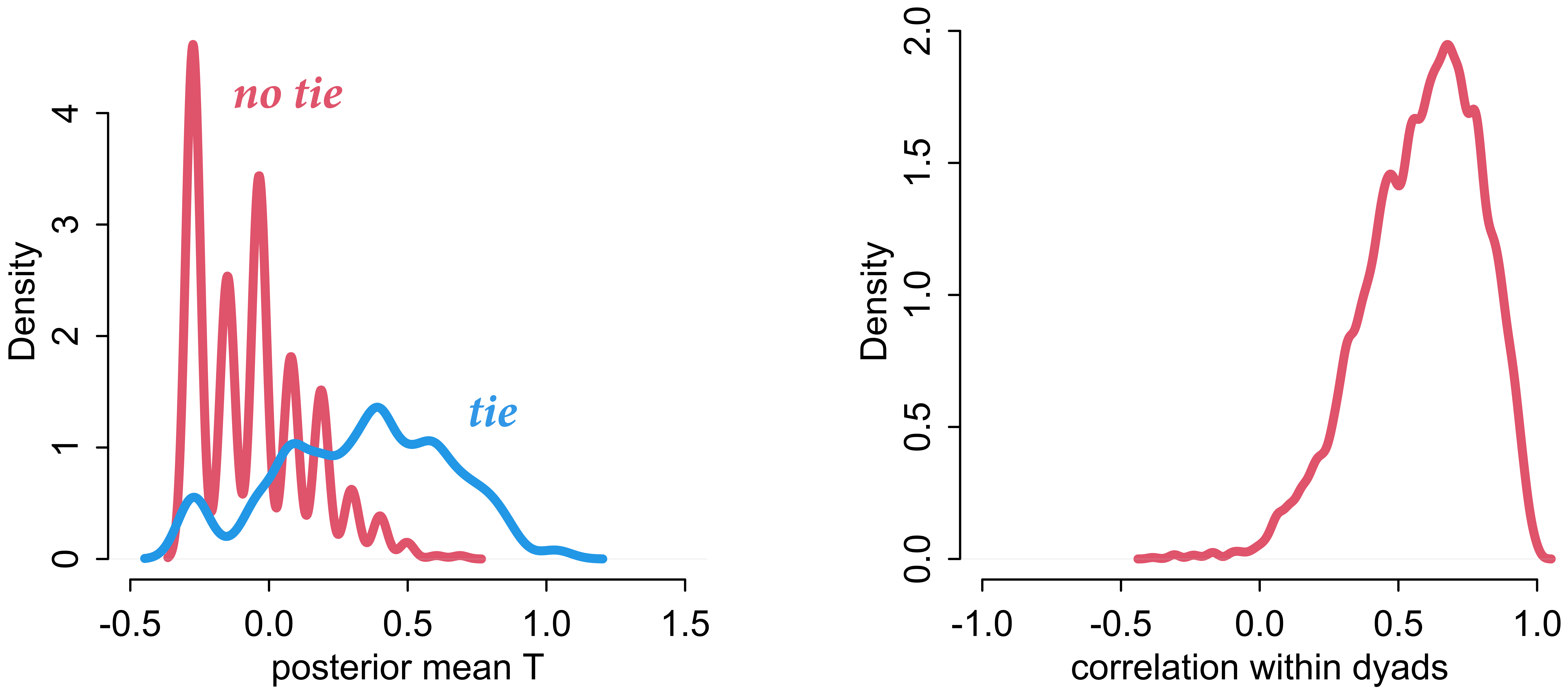
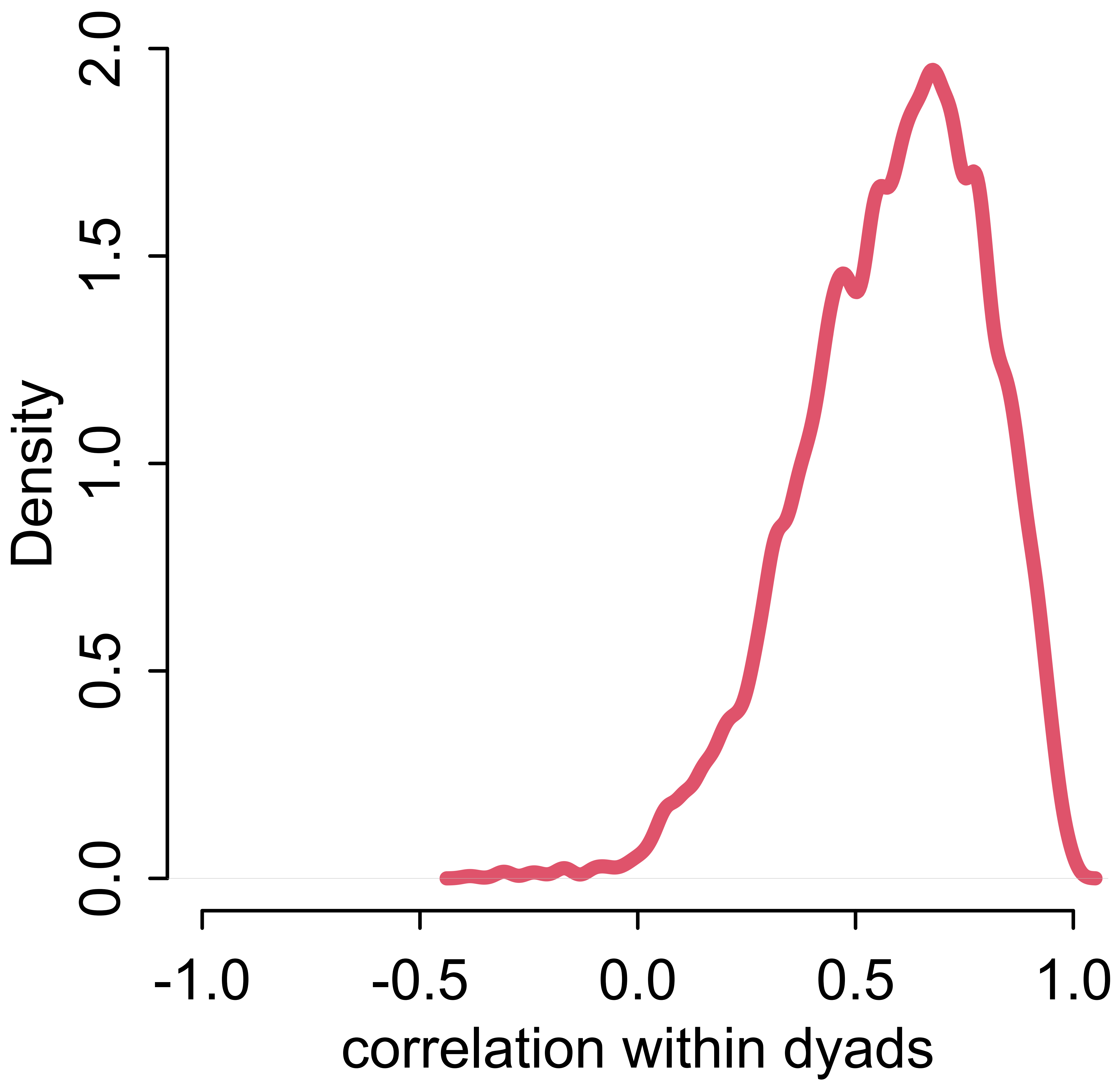

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

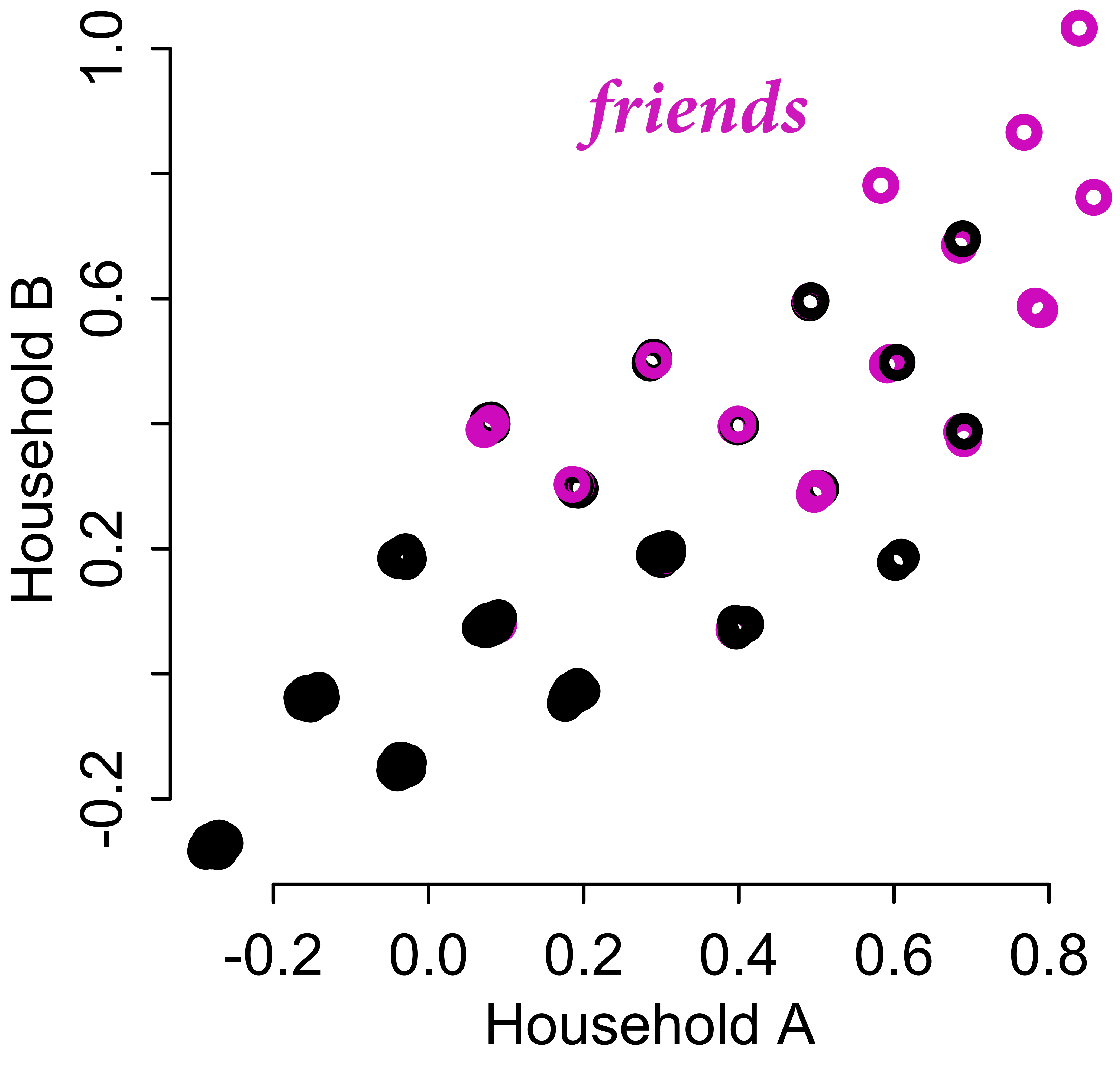

T[15,1]
n_eff = 6334
T[16,1]
n_eff = 9318
T[17,1]
n_eff = 9272
T[18,1]
n_eff = 9187
T[19,1]
n_eff = 8481
T[20,1]
n_eff = 7697
T[21,1]
n_eff = 8645
T[22,1]
n_eff = 8777
T[23,1]
n_eff = 7968
T[24,1]
n_eff = 7488
T[25,1]
n_eff = 7282
T[26,1]
n_eff = 7767
T[27,1]
n_eff = 8068
T[28,1]
n_eff = 9159
T[29,1]
n_eff = 9809
T[30,1]
n_eff = 8239
T[31,1]
n_eff = 8984
T[32,1]
n_eff = 8389
T[33,1]
n_eff = 6304
T[34,1]
n_eff = 8794

Posterior ties



Posterior ties





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(3) Statistical model

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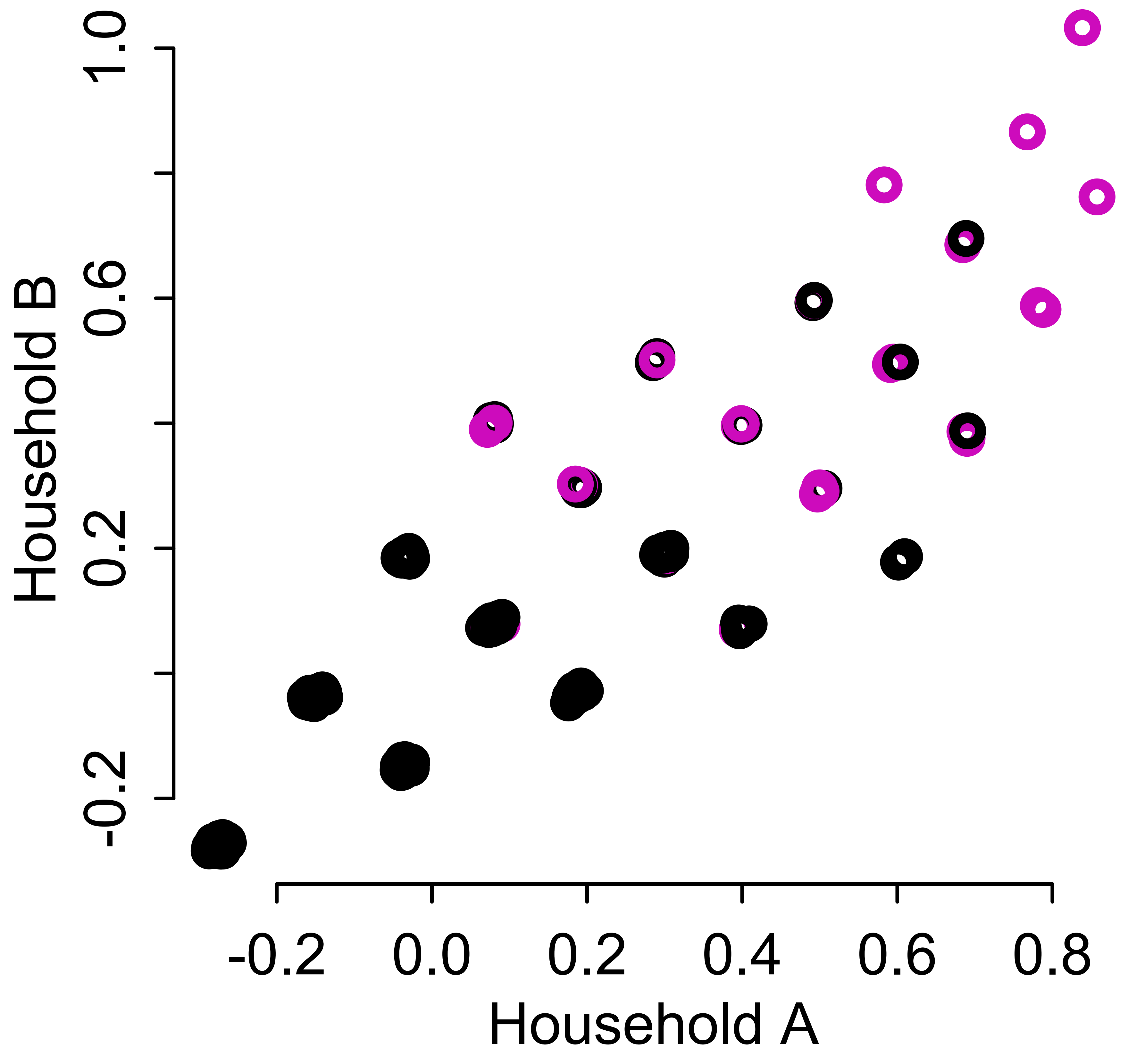

```r
# analyze sample
kl_data <- list(
    N_dyads = nrow(kl_dyads),
    N_households = max(kl_dyads$hidB),
    D = 1:nrow(kl_dyads),
    HA = kl_dyads$hidA,
    HB = kl_dyads$hidB,
    GAB = kl_dyads$giftsAB,
    GBA = kl_dyads$giftsBA )

mGDkl <- ulam( f_dyad , data=kl_data , chains=4 , cores=4 , iter=2000 )

precis( mGDkl , depth=3 , pars=c("a","Rho_T","sigma_T") )
```

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# analyze sample
kl_data <- list(
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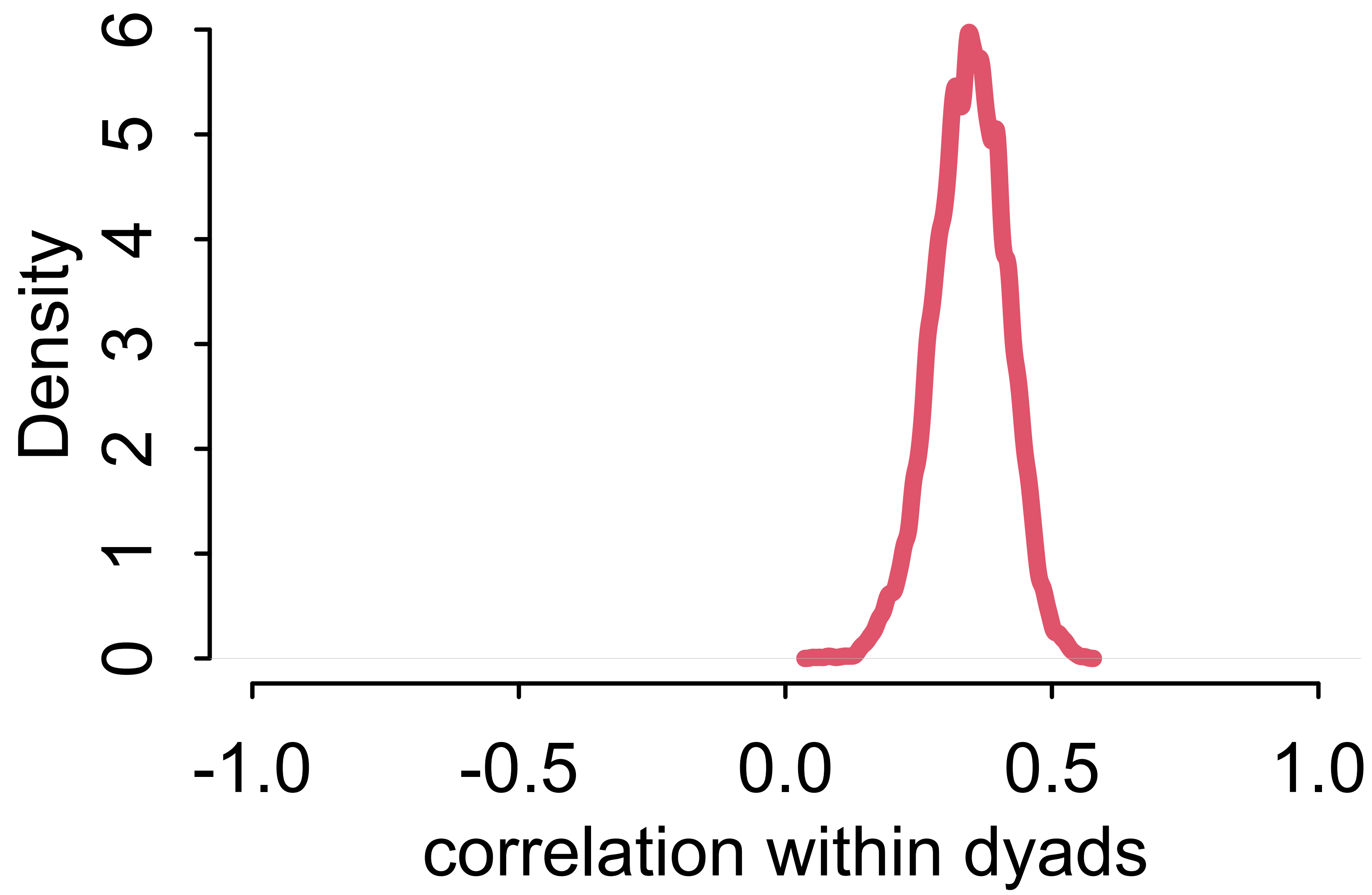
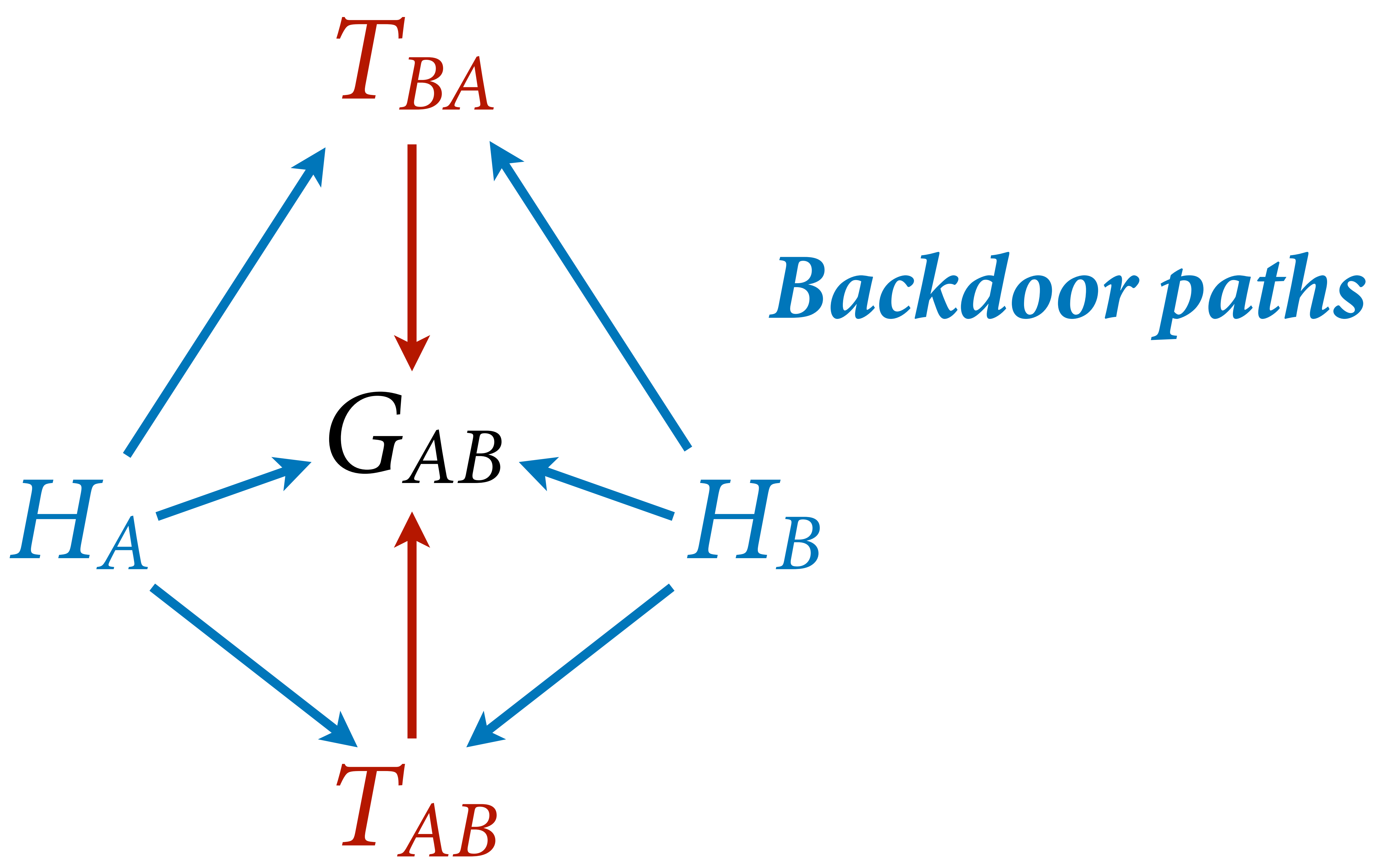
```

	mean	sd	5.5%	94.5%	n_eff	Rhat4
a	0.55	0.08	0.42	0.68	2246	1.00
Rho_T[1,1]	1.00	0.00	1.00	1.00	NaN	NaN
Rho_T[1,2]	0.35	0.07	0.24	0.45	1351	1.00
Rho_T[2,1]	0.35	0.07	0.24	0.45	1351	1.00
Rho_T[2,2]	1.00	0.00	1.00	1.00	NaN	NaN
sigma_T	1.44	0.06	1.35	1.55	1249	1.01




T_{BA}
G_{AB}
H_A
H_B
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Backdoor paths

PAUSE

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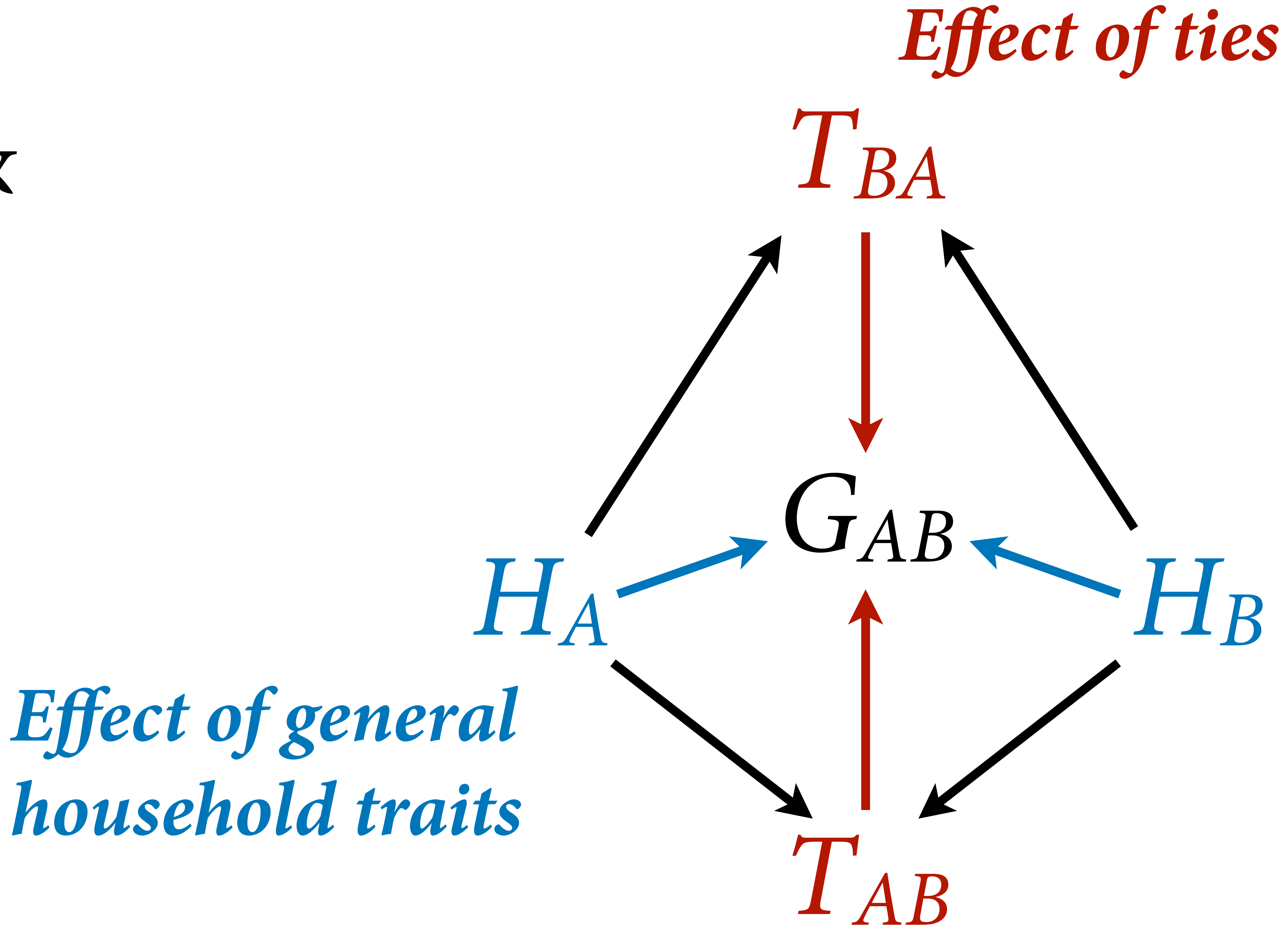

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dyads <- t(combn(N,2))
N_dyads <- nrow(dyads)

# simulate "friendships" in which ties are shared
f <- rbern(N_dyads,0.1) # 10% of dyads are friends

# now simulate directed ties for all individuals
# there can be ties that are not reciprocal
alpha <- (-3) # base rate of ties; -3 ~= 0.05
y <- matrix(NA,N,N) # edge list
for ( i in 1:N ) for ( j in 1:N ) {
    if ( i != j ) {
        # directed tie from i to j
        ids <- sort( c(i,j) )
        the_dyad <- which( dyads[,1]==ids[1] & dyads[,2]==ids[2] )
        p_tie <- f[the_dyad] + (1-f[the_dyad])*inv_logit( alpha )
        y[i,j] <- rbern( 1 , p_tie )
    }
}#ij
```

```
# simulate wealth
W <- rnorm(N) # standardized relative wealth in community
bWG <- 0.5 # effect of wealth on giving - rich give more
bWR <- (-1) # effect of wealth on receiving - rich get less / poor get more

# now simulate gifts
giftsAB <- rep(NA,N_dyads)
giftsBA <- rep(NA,N_dyads)
lambda <- log(c(0.5,2)) # rates of giving for y=0,y=1
for ( i in 1:N_dyads ) {
    A <- dyads[i,1]
    B <- dyads[i,2]
    giftsAB[i] <- rpois( 1 , exp( lambda[1+y[A,B]] + bWG*W[A] + bWR*W[B] ) )
    giftsBA[i] <- rpois( 1 , exp( lambda[1+y[B,A]] + bWG*W[B] + bWR*W[A] ) )
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bWR <- (-1) # effect of wealth on receiving - rich get less / poor get more

# now simulate gifts
giftsAB <- rep(NA,N_dyads)
giftsBA <- rep(NA,N_dyads)
lambda <- log(c(0.5,2)) # rates of giving for y=0,y=1
for ( i in 1:N_dyads ) {
    A <- dyads[i,1]
    B <- dyads[i,2]
    giftsAB[i] <- rpois( 1 , exp( lambda[1+y[A,B]] + bWG*W[A] + bWR*W[B] ) )
    giftsBA[i] <- rpois( 1 , exp( lambda[1+y[B,A]] + bWG*W[B] + bWR*W[A] ) )
}
```

$$G_{AB} \sim \text{Poisson}(\lambda_{AB}) \qquad G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB} \qquad \log(\lambda_{BA}) = \alpha + T_{BA}$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

$$\rho \sim \text{LKJCorr}(2)$$

$$\sigma \sim \text{Exponential}(1)$$

$$\alpha \sim \text{Normal}(0,1)$$

$$
\begin{aligned}
G_{AB} &\sim \text{Poisson}(\lambda_{AB}) \\
\log(\lambda_{AB}) &= \alpha + T_{AB}
\end{aligned}
$$

$$
\begin{aligned}
G_{BA} &\sim \text{Poisson}(\lambda_{BA}) \\
\log(\lambda_{BA}) &= \alpha + T_{BA}
\end{aligned}
$$

$$
\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)
$$

$$
\rho \sim \text{LKJCorr}(2)
$$

$$
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$$
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$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB} + G_A + R_B$$

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$$\log(\lambda_{BA}) = \alpha + T_{BA}$$

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$$\begin{pmatrix} G_A \\ R_A \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_G^2 & r\sigma_G\sigma_R \\ r\sigma_G\sigma_R & \sigma_R^2 \end{bmatrix} \right)$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

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A's giving & receiving

Covariance matrix of household giving & receiving

$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

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$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{BA}) = \alpha + T_{BA} + G_B + R_A$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

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$$\begin{pmatrix} G_A \\ R_A \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{R}_{GR}, \mathbf{S}_{GR} \right)$$

A's giving & receiving

Correlation matrix

Standard deviations

$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB} + G_A + R_B$$

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$$\log(\lambda_{BA}) = \alpha + T_{BA} + G_B + R_A$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

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25 households

300 dyads

600 gift observations

602 social network parameters

53 household parameters

```r
# general model
f_general <- alist(
    GAB ~ poisson( lambdaAB ),
    GBA ~ poisson( lambdaBA ),
    log(lambdaAB) <- a + T[D,1] + gr[HA,1] + gr[HB,2],
    log(lambdaBA) <- a + T[D,2] + gr[HB,1] + gr[HA,2],
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    ## dyad effects - non-centered
    transpars> matrix[N_dyads,2]:T <-
            compose_noncentered(rep_vector(sigma_T,2),L_Rho_T,Z),
    matrix[2,N_dyads]:Z ~ normal( 0 , 1 ),
    cholesky_factor_corr[2]:L_Rho_T ~ lkj_corr_cholesky( 2 ),
    sigma_T ~ exponential(1),

   ## gr matrix of varying effects
    transpars> matrix[N_households,2]:gr <-
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    matrix[2,N_households]:Zgr ~ normal( 0 , 1 ),
    cholesky_factor_corr[2]:L_Rho_gr ~ lkj_corr_cholesky( 2 ),
    vector[2]:sigma_gr ~ exponential(1),

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    gq> matrix[2,2]:Rho_T <<- Chol_to_Corr( L_Rho_T ),
    gq> matrix[2,2]:Rho_gr <<- Chol_to_Corr( L_Rho_gr )
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mGDGR <- ulam(f_general,data=sim_data,chains=4,cores=4,iter=2000)
```

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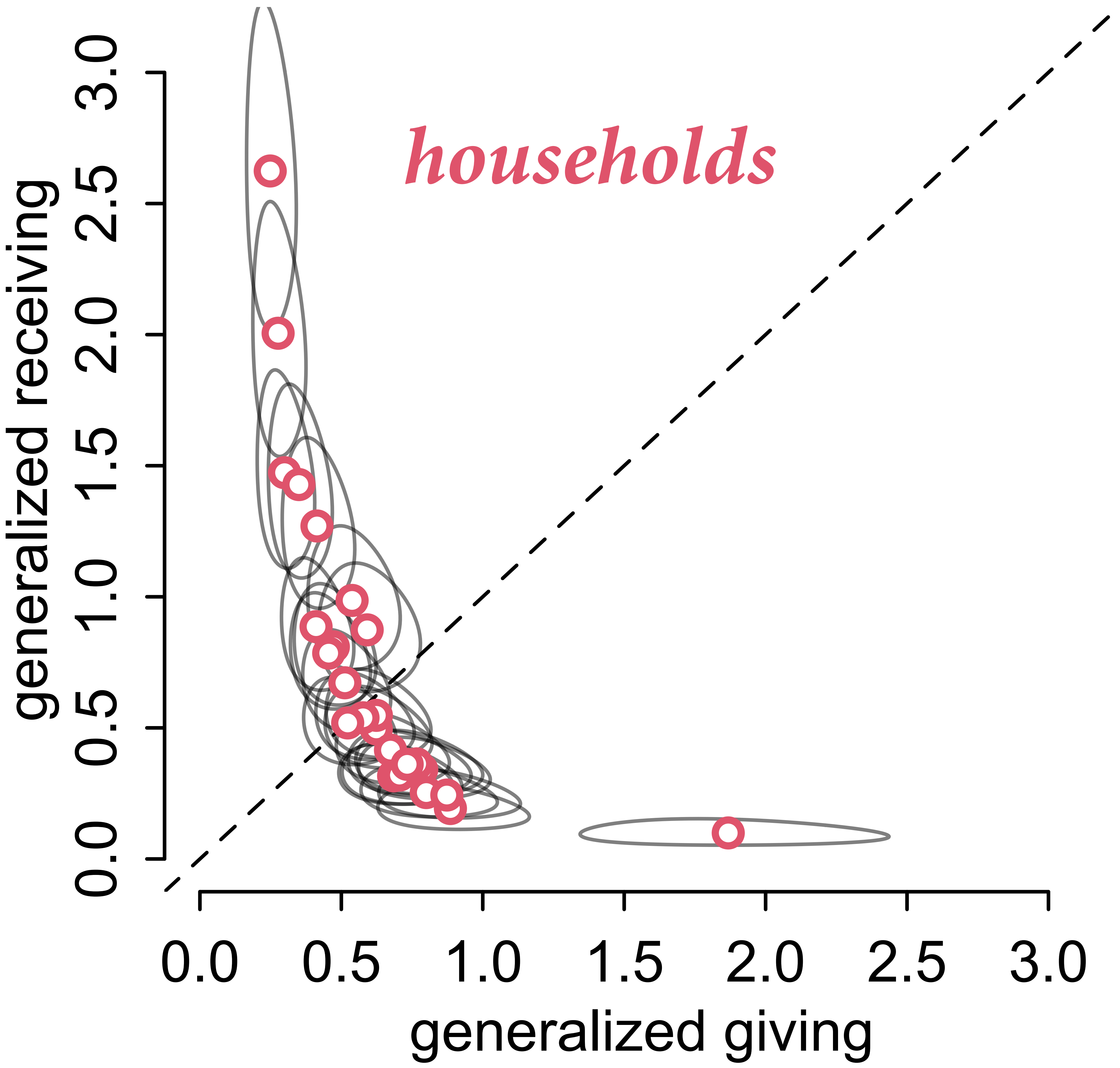
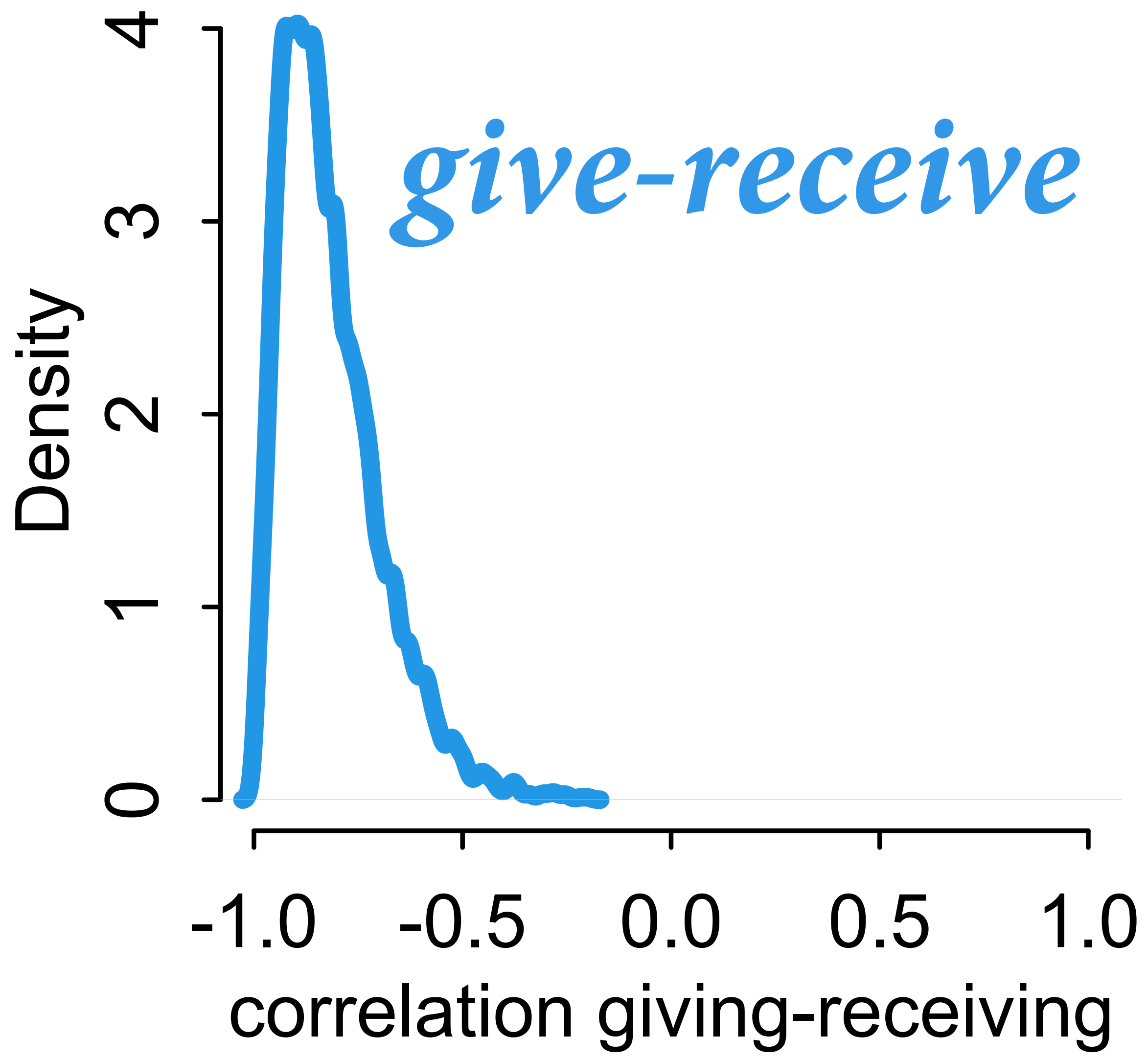
$$\mathbf{R}_{GR} \sim \text{LKJCorr}(2)$$

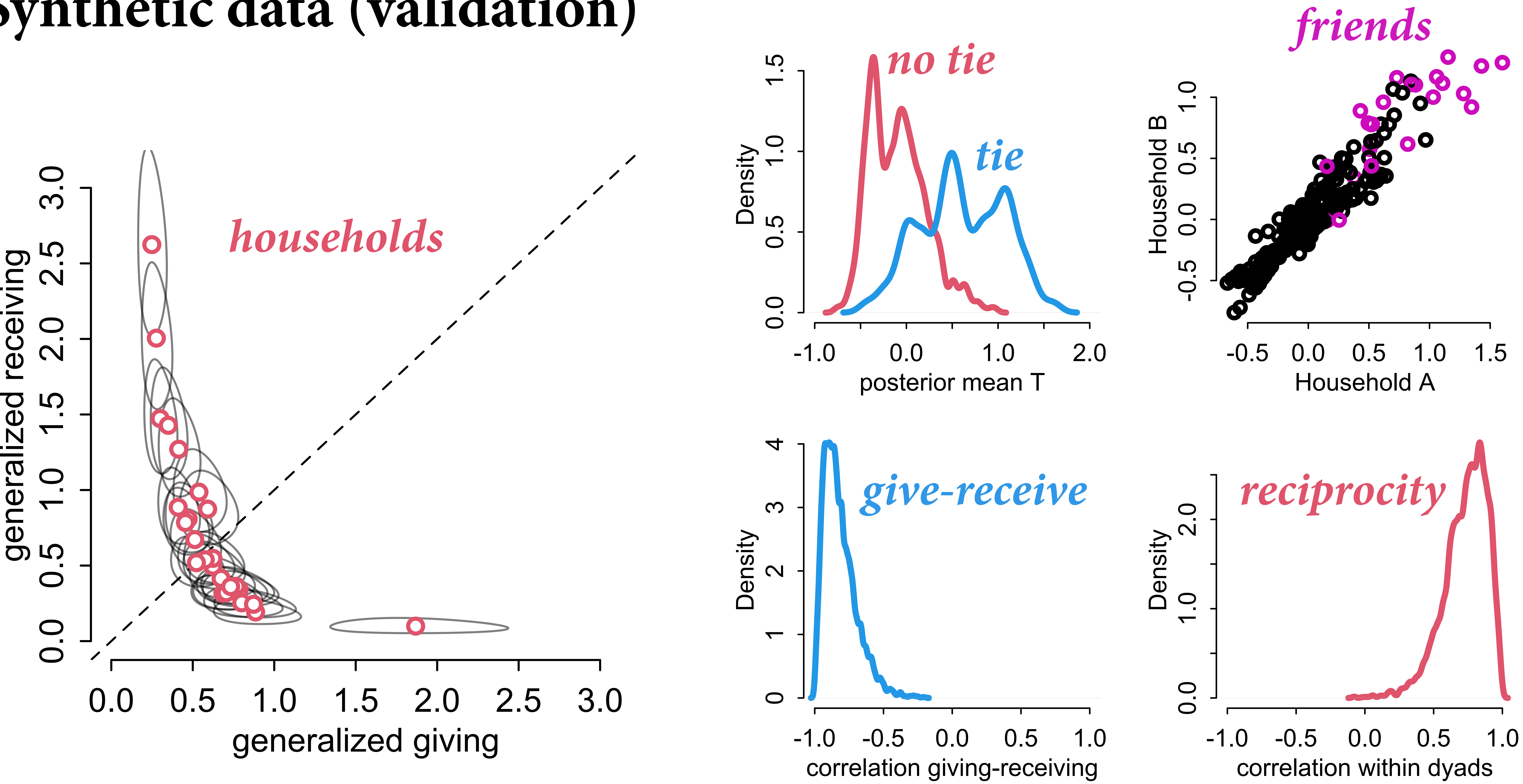
$$\mathbf{S}_{GR} \sim \text{Exponential}(1)$$

Synthetic data (validation)

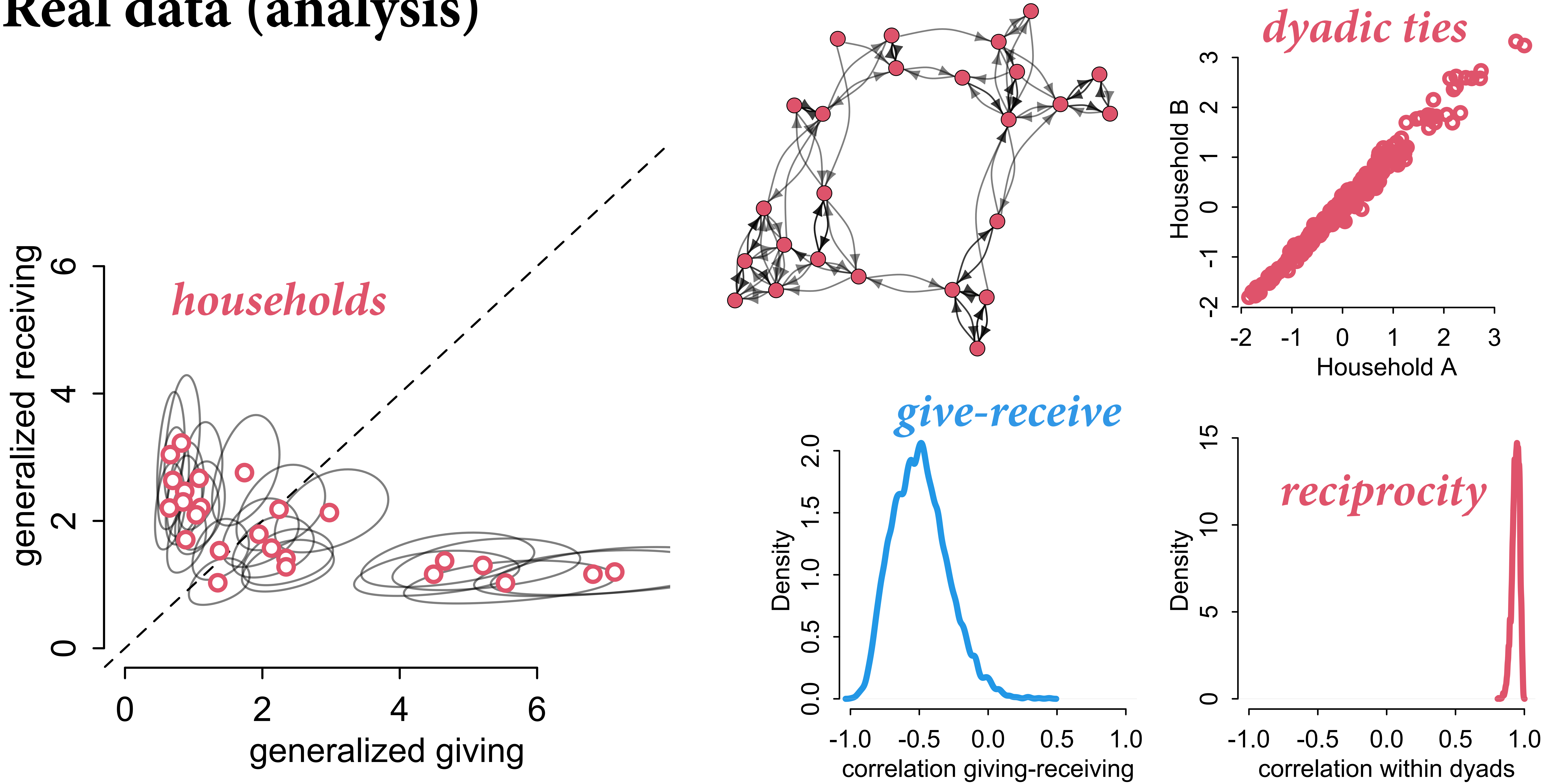




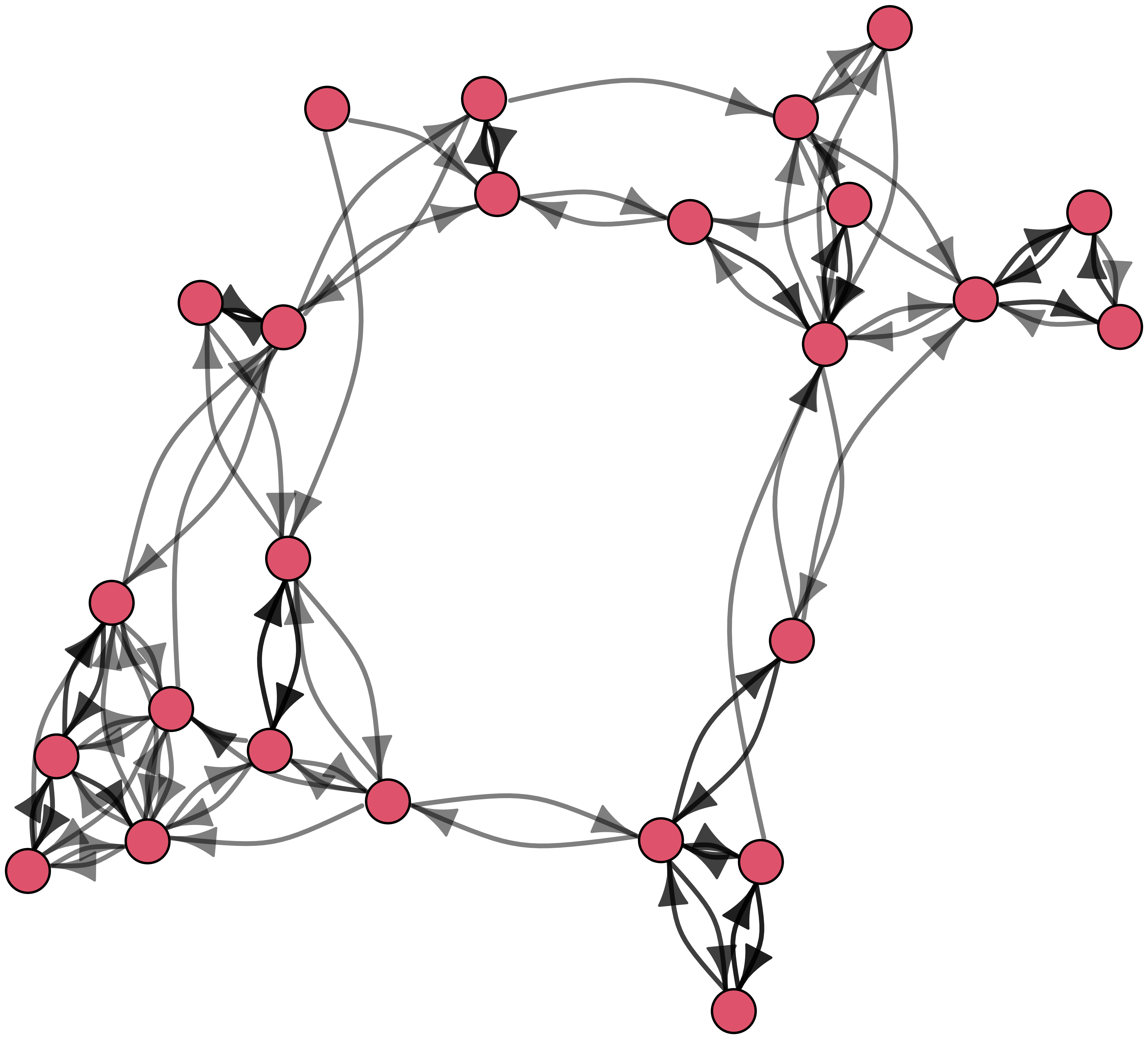
Synthetic data (validation)



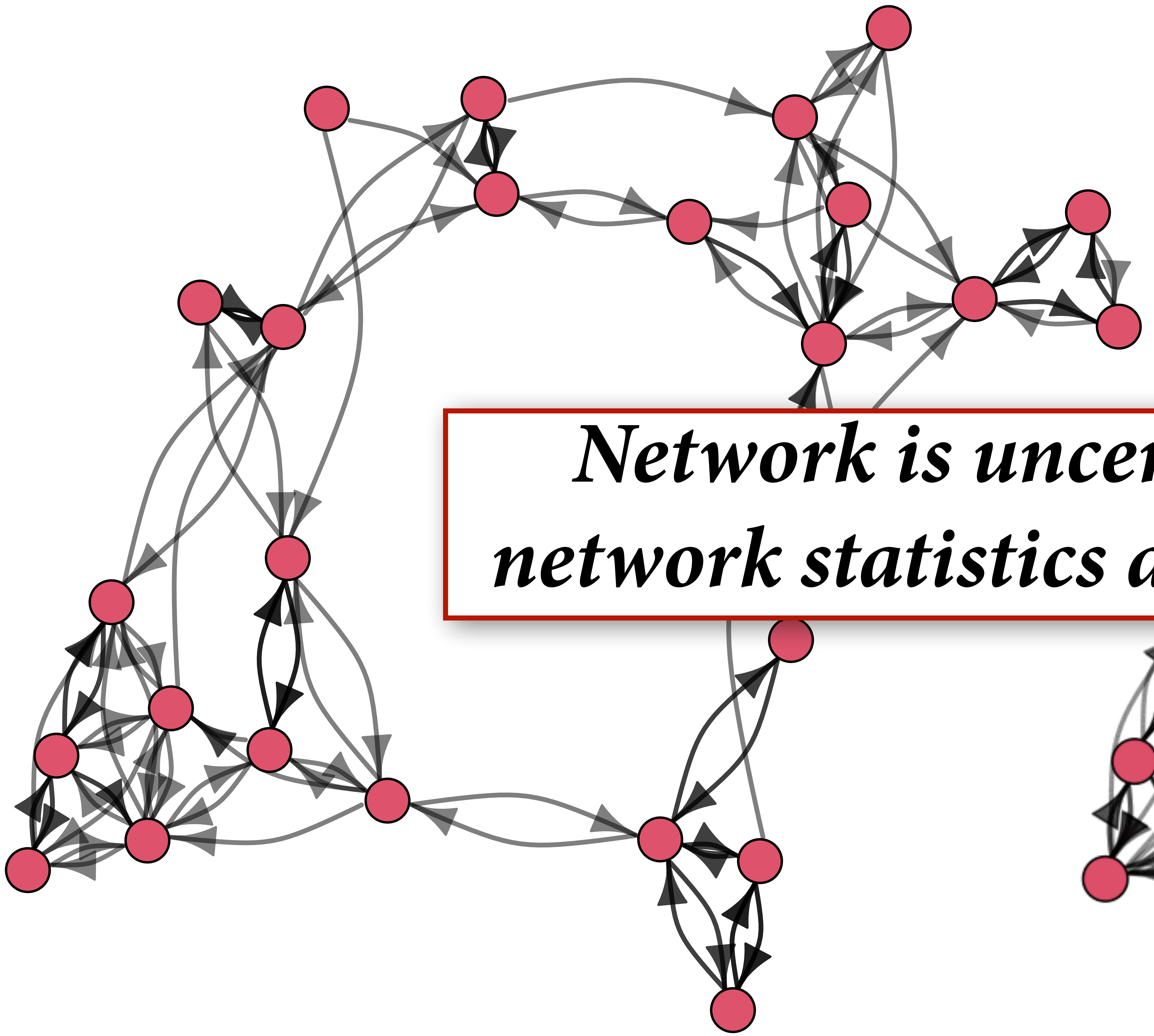
Real data (analysis)

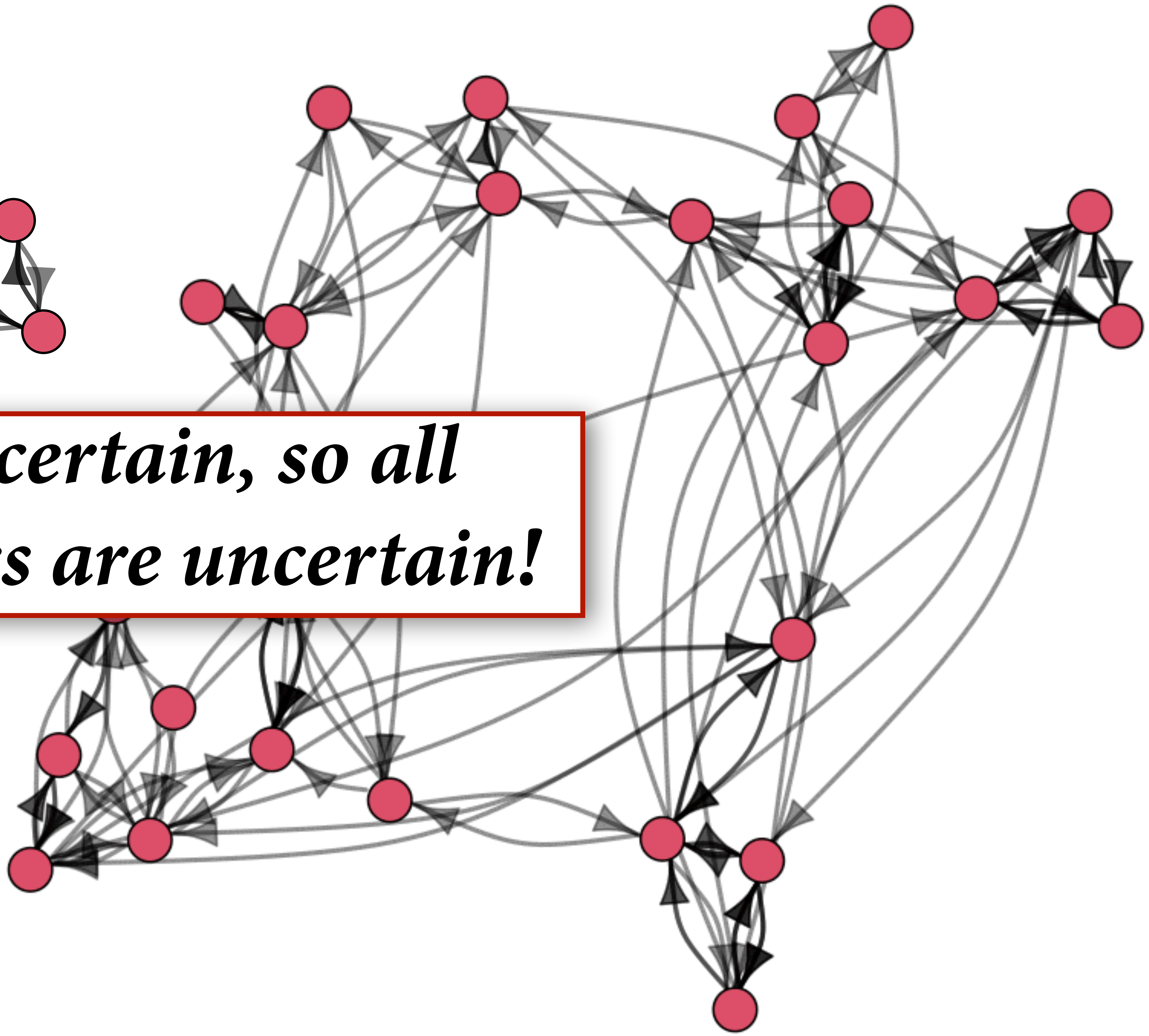


Posterior mean network



Posterior mean network **Samples from posterior**





Network is uncertain, so all network statistics are uncertain!

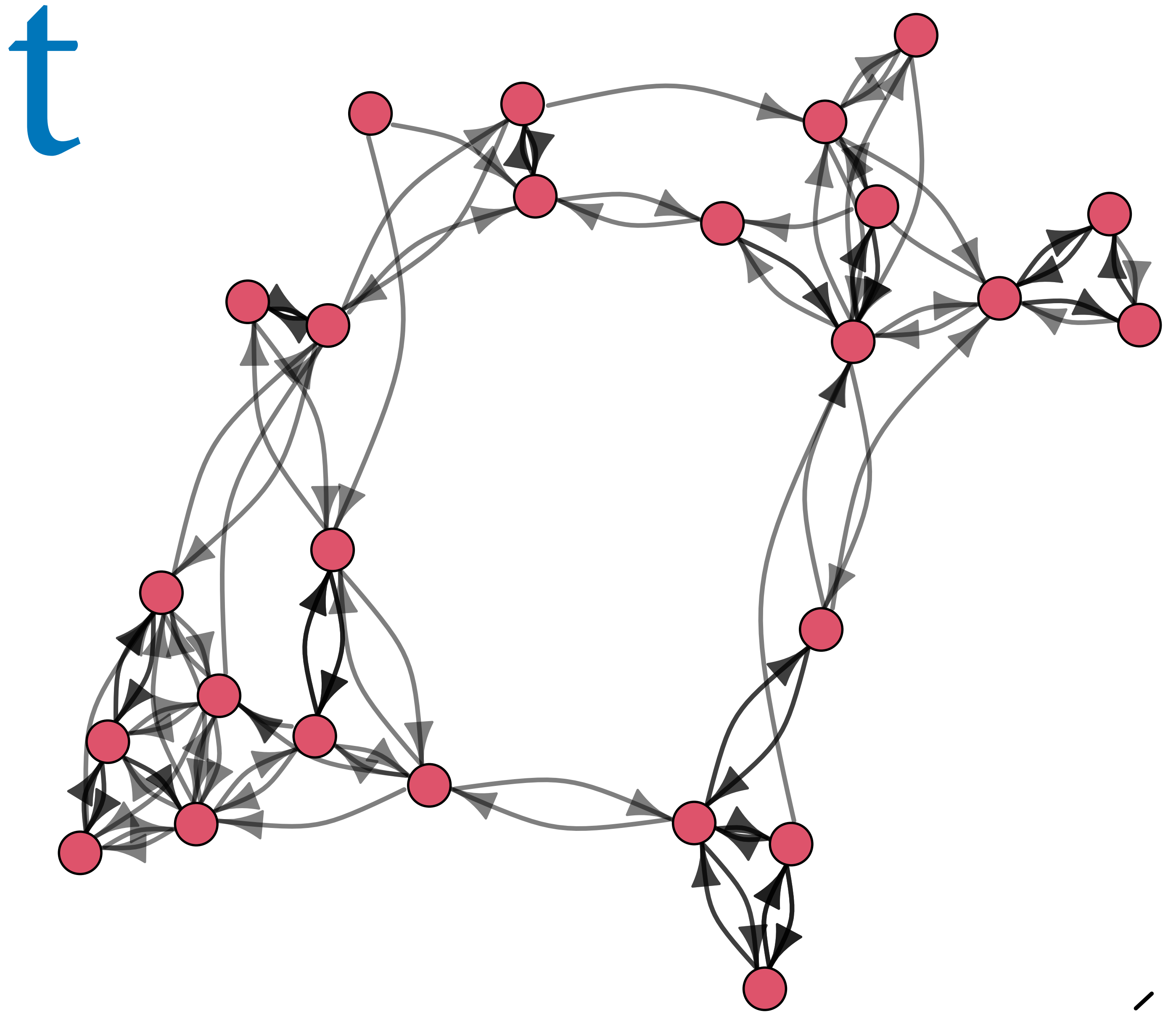
Social Networks Don't Exist

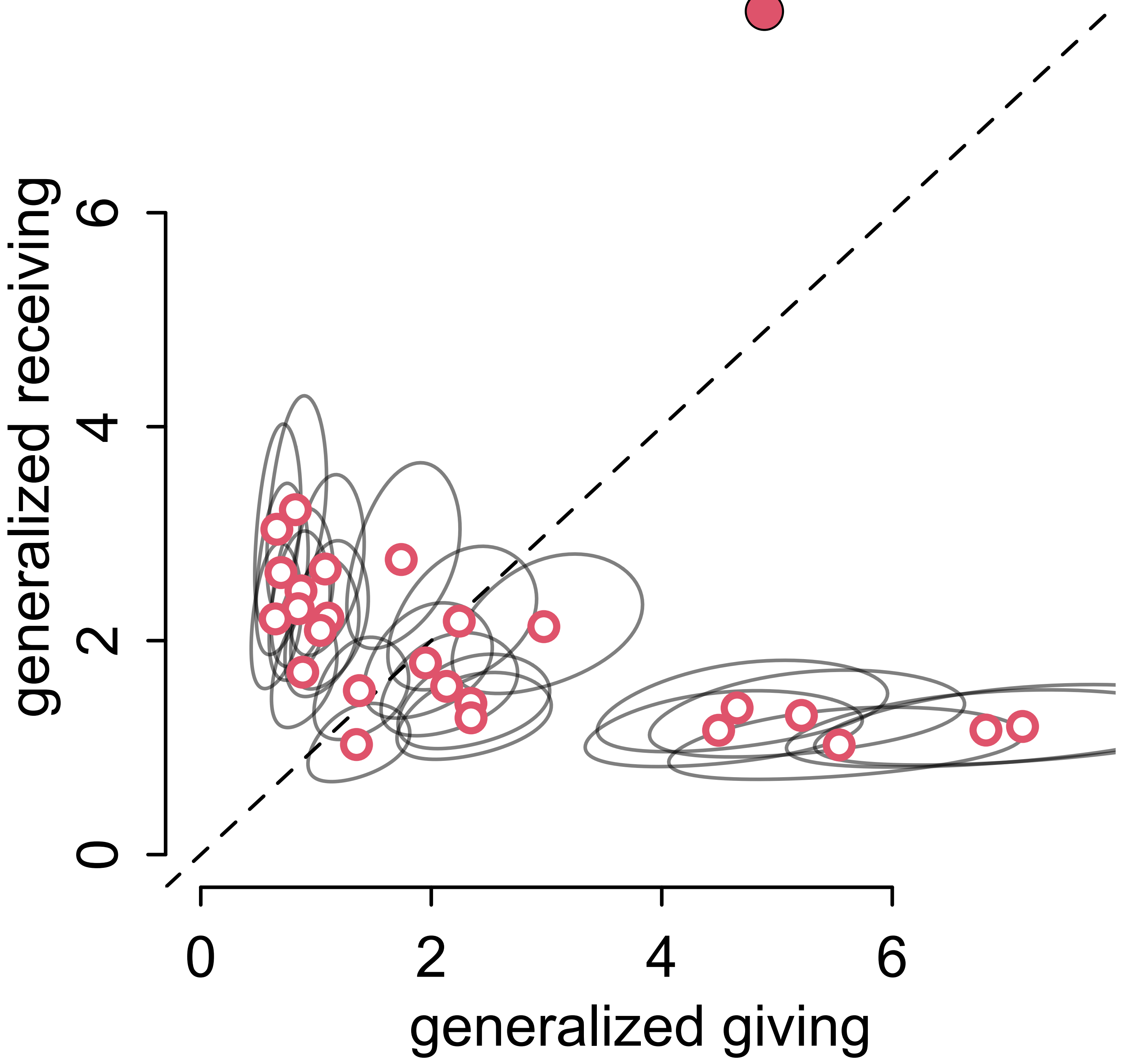
Varying effects are placeholders

Can model the network ties (using dyad features)

Can model the giving/receiving (using household features)

Relationships can cause other relationships





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$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + \mathcal{T}_{AB} + \mathcal{G}_A + \mathcal{R}_B$$

$$\mathcal{T}_{AB} = T_{AB} + \beta_A A_{AB}$$

linear model for tie strength

varying effect

effect of association between A&B

$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{BA}) = \alpha + T_{BA} + G_B + R_A$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

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$$\underbrace{\mathcal{G}_A}_{\text{linear model for giving}} = \underbrace{G_A}_{\text{varying effect}} + \underbrace{\beta_{W,G}}_{\text{effect of A's wealth on giving}} W_A$$

$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{BA}) = \alpha + T_{BA} + G_B + R_A$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

$$\rho \sim \text{LKJCorr}(2)$$

$$\sigma \sim \text{Exponential}(1)$$

$$\alpha \sim \text{Normal}(0,1)$$

$$\begin{pmatrix} G_A \\ R_A \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{R}_{GR}, \mathbf{S}_{GR} \right)$$

$$\mathbf{R}_{GR} \sim \text{LKJCorr}(2)$$

$$\mathbf{S}_{GR} \sim \text{Exponential}(1)$$

$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + \mathscr{T}_{AB} + \mathscr{G}_A + \mathscr{R}_B$$

$$\mathscr{T}_{AB} = T_{AB} + \beta_A A_{AB}$$

$$\mathscr{G}_A = G_A + \beta_{W,G} W_A$$

$$\mathscr{R}_B = R_B + \beta_{W,R} W_B$$

linear model for receiving

varying effect

effect of B's wealth on receiving

$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{BA}) = \alpha + T_{BA} + G_B + R_A$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

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$$\mathcal{R}_A = R_A + \beta_{W,R} W_A$$

```
# general model with features
f_houses <- alist(

    GAB ~ poisson( lambdaAB ),
    GBA ~ poisson( lambdaBA ),

    # A to B
    log(lambdaAB) <- a + TAB + GA + RB,
      TAB <- T[D,1] + bA*A,
      GA <- gr[HA,1] + bW[1]*W[HA] ,
      RB <- gr[HB,2] + bW[2]*W[HB] ,

    # B to A
    log(lambdaBA) <- a + TBA + GB + RA,
      TBA <- T[D,2] + bA*A,
      GB <- gr[HB,1] + bW[1]*W[HB] ,
      RA <- gr[HA,2] + bW[2]*W[HA] ,

    # priors
    a ~ normal(0,1),
    vector[2]:bW ~ normal(0,1),
    bA ~ normal(0,1),
```

$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + \mathscr{T}_{AB} + \mathscr{G}_{A} + \mathscr{R}_{B}$$

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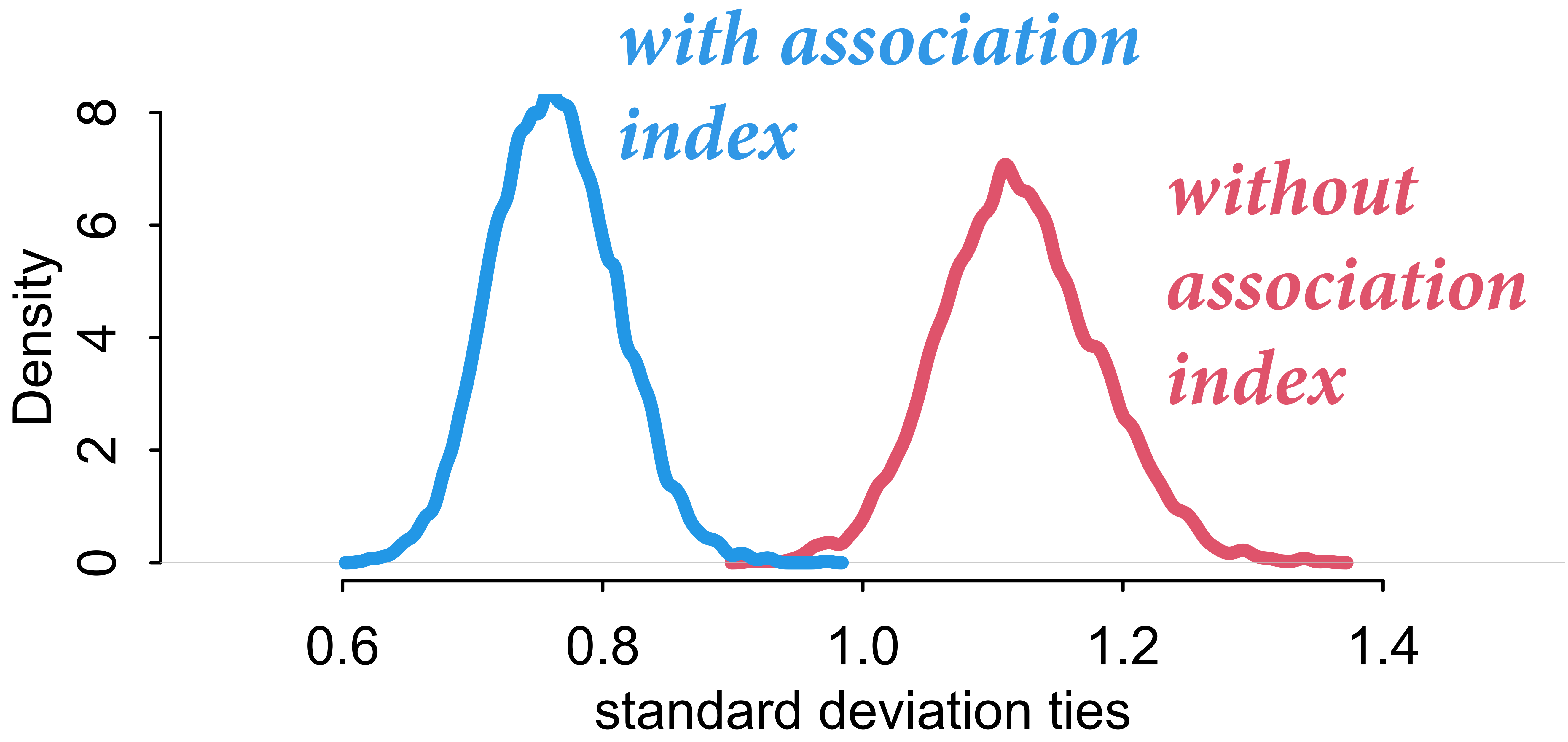

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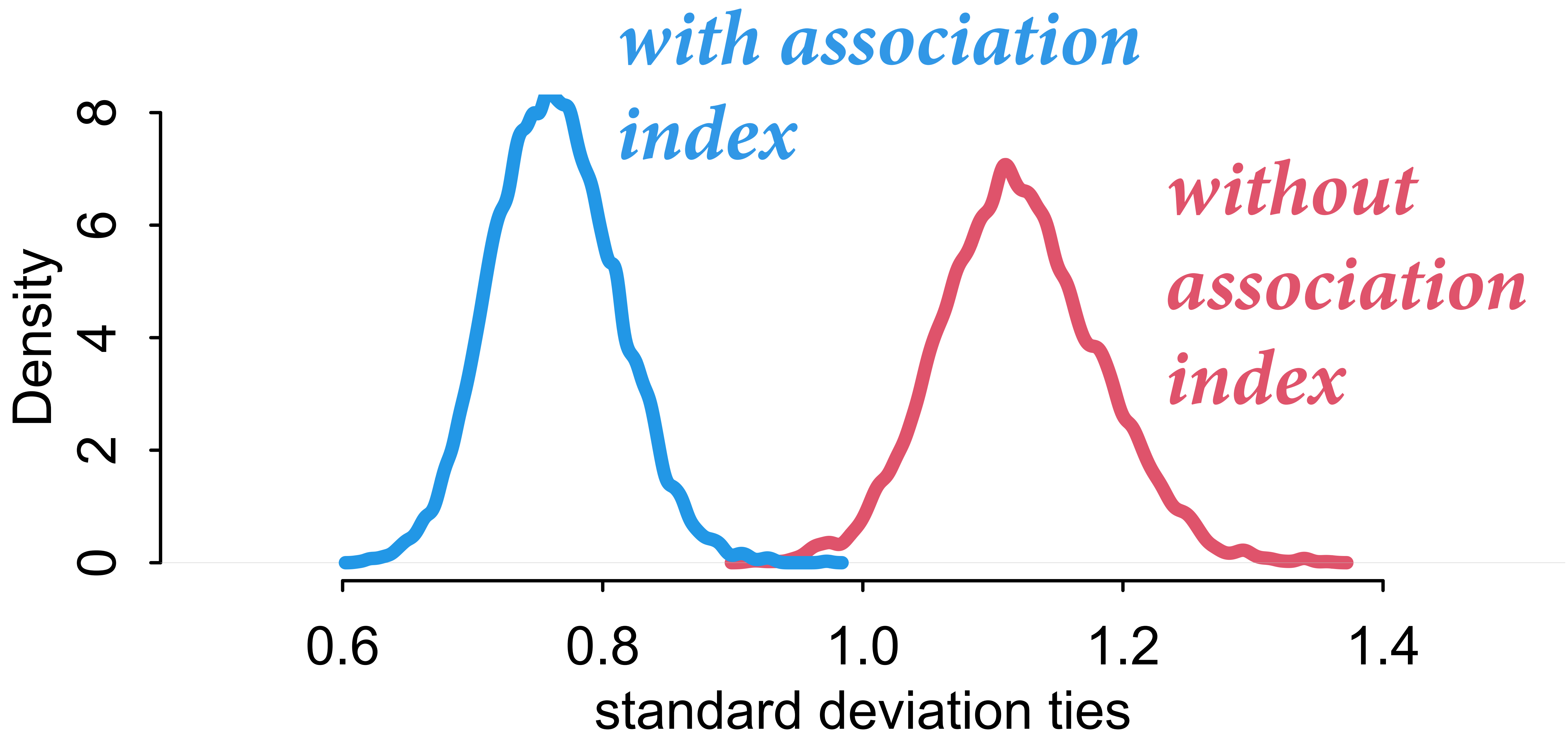

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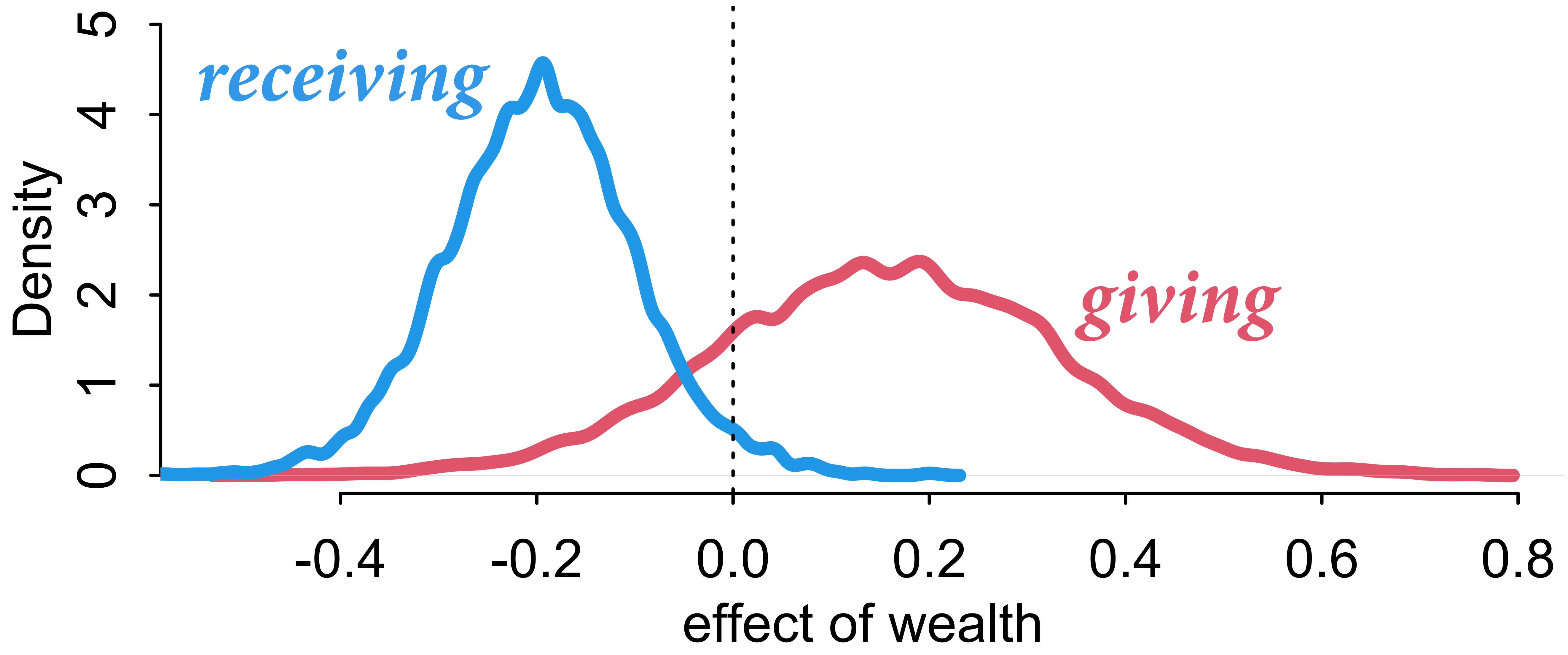


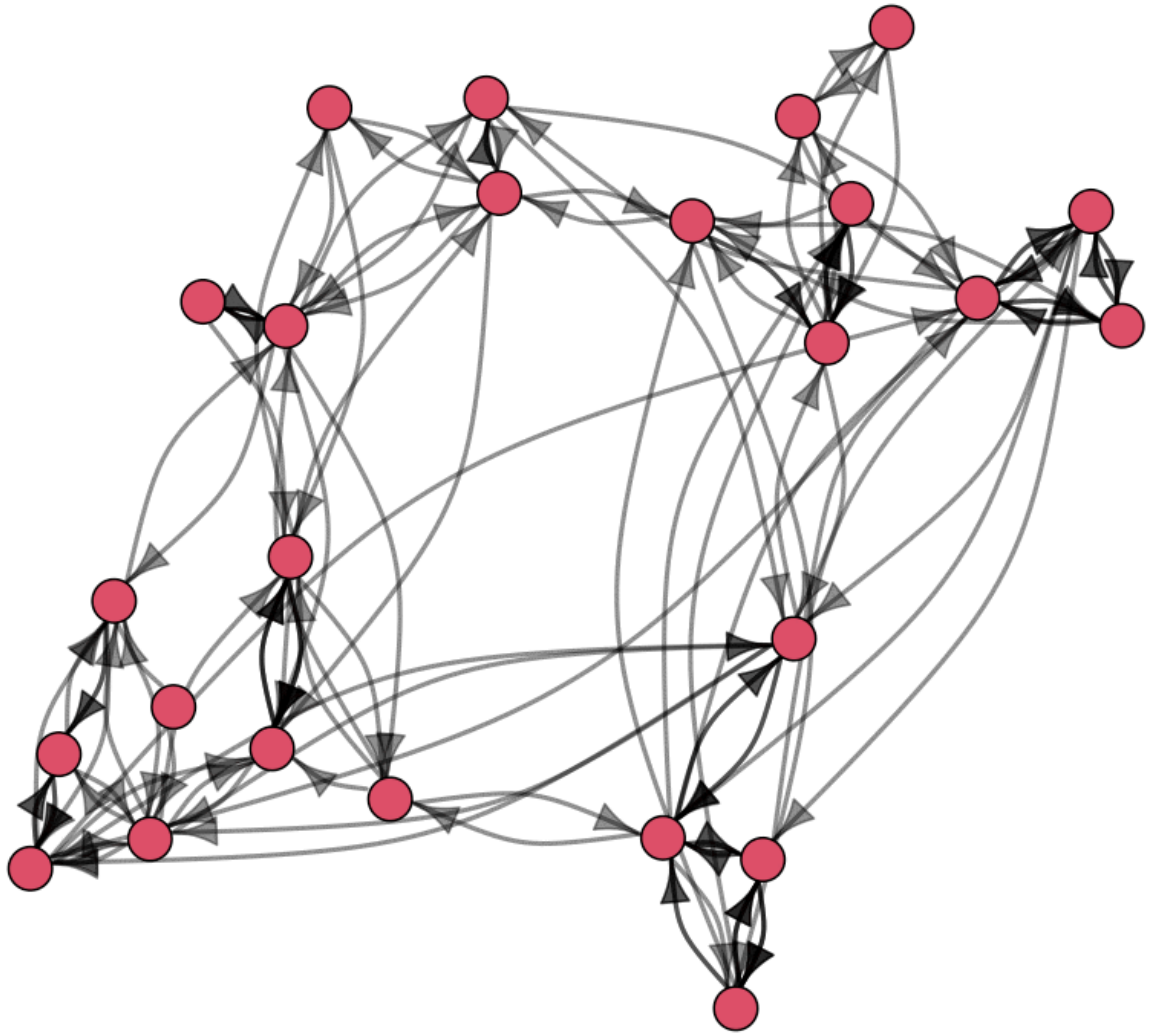
Social Networks Don’t Exist

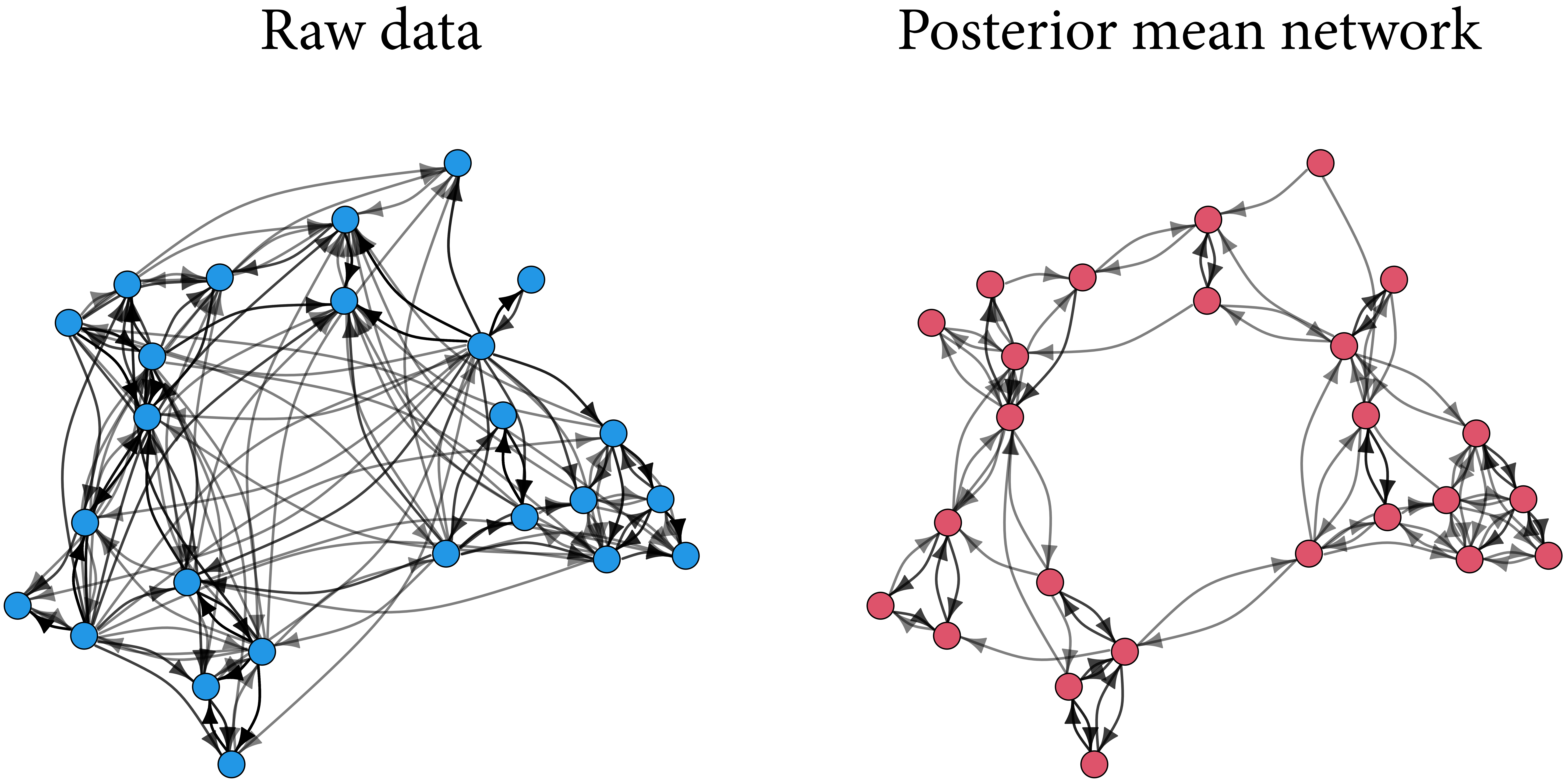
Relationships can cause other relationships

Triangle closure:

$$\begin{tikzcd} A \arrow[rr, leftrightarrow] \arrow[dr, leftrightarrow] & & B \arrow[dl, leftrightarrow] \\ & C & \end{tikzcd}$$

Block models: Ties more common within certain groups (family, office, *Stammtisch*)




Raw data
Posterior mean network

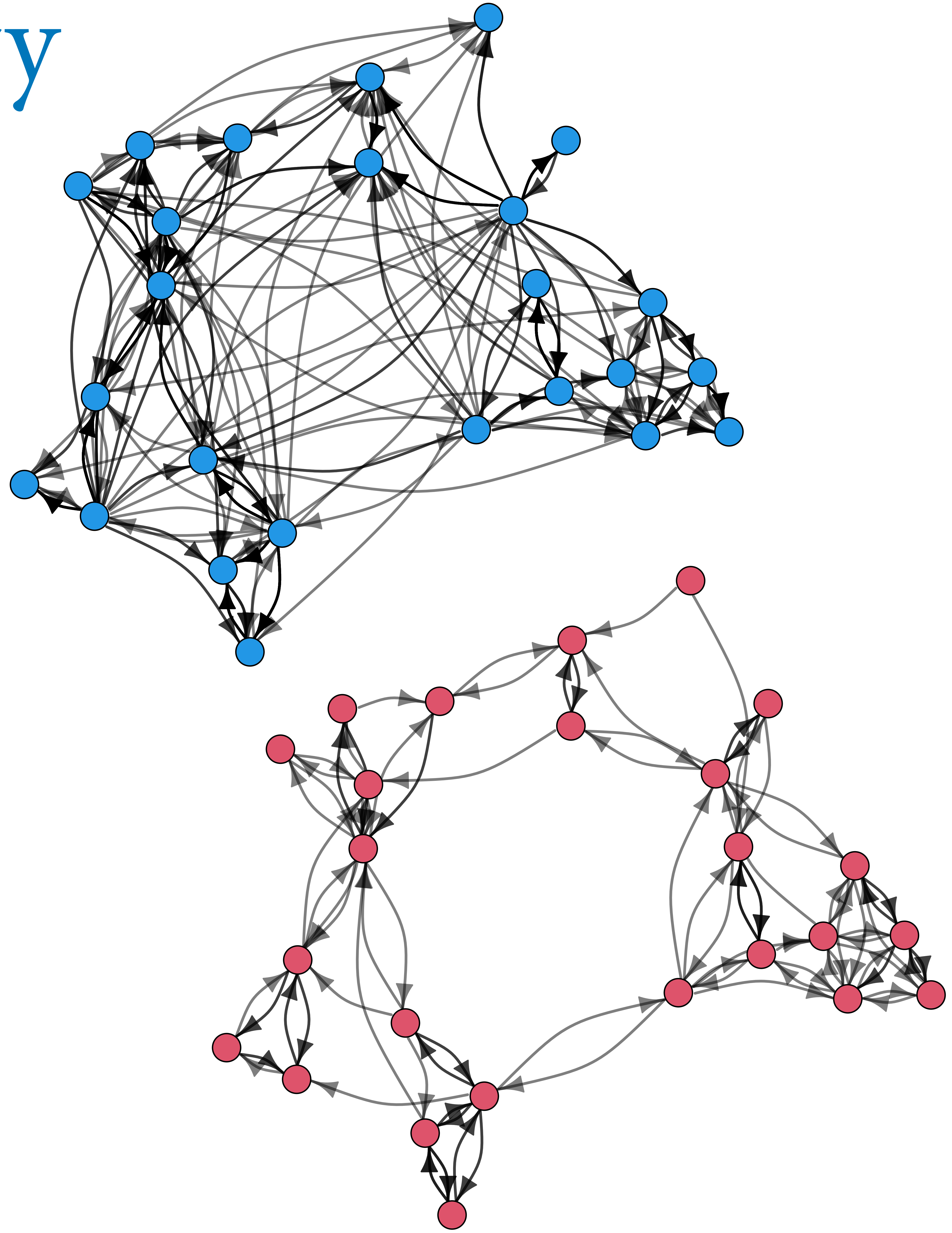
Varying effects as technology

Social networks try to express *regularities* of observations

Inferred social network is *regularized*, a structured varying effect

Analogous problems: phylogeny, space, heritability, knowledge, personality

What happens when the clusters are not discrete but continuous? Age, distance, time, similarity



Course Schedule

Week 1	Bayesian inference	Chapters 1, 2, 3
Week 2	Linear models & Causal Inference	Chapter 4
Week 3	Causes, Confounds & Colliders	Chapters 5 & 6
Week 4	Overfitting / MCMC	Chapters 7, 8, 9
Week 5	Generalized Linear Models	Chapters 10, 11
Week 6	Ordered categories & Multilevel models	Chapters 12 & 13
Week 7	More Multilevel models	Chapters 13 & 14
Week 8	Social Networks & Gaussian Processes	Chapter 14
Week 9	Measurement & Missingness	Chapter 15
Week 10	Generalized Linear Madness	Chapter 16

https://github.com/rmcelreath/stat_rethinking_2022