Kuhn Poker (AKQ Game)

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1 The Game

1.1 Rules

- 2 players, each is dealt a card in A, K, Q
- Each antes 1 at the start of the hand
- Each has 1 remaining for betting
- There is one betting round
- The highest card is the best (i.e., A > K > Q)

1.2 Betting Sequences

- Action starts with P1, who can Bet 1 or Check
- If P1 bets, P2 can either Call or Fold
- If P1 checks, P2 can either Bet or Check
- If P2 bets after P1 checks, P1 can then Call or Fold

1.3 Payoffs

- If a player folds to a bet, the other player wins the pot of 2 (profit of 1)

- If both players check, the highest card player wins the pot of 2 (profit of 1)
- If there is a bet and call, the highest card player wins the pot of 4 (profit of 2)

2 Variables

2.1 P1 opening action

P1 should never bet the K card here because if he bets the K, P2 with Q will always fold (since the lowest card can never win) and P2 with A will always call (since the best card will always win). By checking the K always, P1 can try to induce a bluff from P2 when P2 has the Q.

Therefore we assign P1's strategy:

- Bet Q: x
- Bet K: 0
- Bet A: y

2.2 P2 after P1 bet

After P1 bets, P2 should always call with the A and always fold the Q as explained above.

Therefore we assign P2's strategy after P1 bet:

- Call Q: 0
- Call K: a
- Call A: 1

2.3 P2 after P1 check

After P1 checks, P2 should never bet with the K for the same reason as P1 should never initially bet with the K. P2 should always bet with the A because it is the best hand and there is no bluff to induce by checking.

Therefore we assign P2's strategy after P1 check:

- Bet Q: b
- Bet K: 0
- Bet A: 1

2.4 P1 after P1 check and P2 bet

This case is similar to P2's actions after P1's bet. P1 can never call here with the worst hand (Q) and must always call with the best hand (A).

Therefore we assign P1's strategy after P1 check and P2 bet:

- Call Q: 0
- Call K: z
- Call A: 1

So we now have 5 different variables x, y, z, a, b to represent the unknown probabilities.

3 Solving

3.1 Solving for a

We start by solving for a, how often P2 should call with a K facing a bet from P1. P2 should call a to make P1 indifferent to bluffing (i.e., betting or checking) with card Q.

If P1 checks with card Q, he will always fold afterwards (because it is the worst card and can never win).

$$u(\text{P1 check with Q}) = 0 \tag{1}$$

If P1 bets with card Q,

$$u(\text{P1 bet with Q}) = (\text{P2 has A and always calls/wins}) + \tag{2}$$

$$(\text{P2 has K and folds}) + (\text{P2 has K and calls/wins}) \tag{3}$$

$$= (1/2)(-1) + (1/2)[(a)(-1) + (1-a)(2)] \tag{4}$$

$$= -1/2 - 1/2 * a + (1-a) \tag{5}$$

$$= 1/2 - 3/2 * a \tag{6}$$

Setting the probabilities of betting with Q and checking with Q equal, we have:

$$0 = \frac{1}{2} - \frac{3}{2} * a \tag{7}$$

$$\frac{3}{2} * a = \frac{1}{2} \tag{8}$$

$$a = \frac{1}{3} \tag{9}$$