

Variable monotonicity and *less than*: when Van Benthem's problem is not a problem*

Brian Buccola

McGill University

1. Introduction

In this paper I describe, and provide a solution to, a novel puzzle concerning the interpretation of non-upward-monotone quantifiers, focusing in particular on modified numerals of the form *less than n*. The puzzle is this: *less than n* seems to mean different things (and, in particular, have different monotonicity properties) depending on whether its nominal and verbal arguments are distributive or non-distributive, and while it is easy to formulate a lexical entry that works for each of the two individual cases, it does not seem possible to formulate a single entry that works for both cases. The solution I propose is to adopt a flexible syntax-semantics system that overgenerates readings using one and the same entry for *less than*, coupled with a pragmatic blocking mechanism that filters out unavailable readings. The puzzle and proposed solution together shed light on the roles that maximality and distributivity crucially play in shaping the monotonicity properties of modified numerals, as well as the division of labor between semantics and pragmatics.¹

Consider the very uncontroversial example in (1a). This sentence is intuitively equivalent to (1b). In particular, it conveys an upper bound on the number of students who attended, and it is consistent with no students having attended. Call this an *upper-bounded* reading, and note that no other reading for (1a) is intuitively available.

- (1) a. Less than five students attended.
- b. It is not the case that five or more students attended.

Such data have led naturally to the view that the meaning of *less than five* involves some notion of *maximality*, as the lexical entries in (2) convey. (2a) is well known from Generalized

*Many thanks to Luis Alonso-Ovalle and Bernhard Schwarz for their help and encouragement at every stage of this research. I also thank the students and faculty at MIT, especially Roger Schwarzschild. Special thanks also to Benjamin Spector, whose comments on various aspects of this material have been very helpful.

¹The core generalizations presented in this paper were independently discovered by Benjamin Spector and presented by him in talks at UCLA and Utrecht. His analysis, like mine, involves pragmatic blocking, but differs in how readings are generated. Unfortunately, space limitations prevent me from carrying out a comparison between his theory and mine here, but see Buccola (in prep.) and Buccola & Spector (in prep.).

Quantifier Theory (Barwise & Cooper 1981), in which *less than five* denotes a generalized determiner: it takes two predicates of individuals (e.g. *students* and *attended*) and says that the maximum number of individuals in the extension of both predicates is less than five (and possibly zero). In (2b), inspired by Heim 2000 and Hackl 2000, *less than five* denotes a generalized quantifier over degrees: it takes a degree predicate (e.g. $\lambda n.n$ students attended) and says that the maximum of that degree predicate is less than five.²

- (2) a. $\llbracket \text{less than five} \rrbracket = \lambda X . \lambda Y . |X \cap Y| < 5$
 b. $\llbracket \text{less than five} \rrbracket = \lambda P_{dt} . \max(P) < 5$

Both entries yield the same, upper-bounded reading for (1a). Moreover, both entries render *less than five* downward monotone (on both arguments), and indeed evidence from inference patterns and NPI licensing (Ladusaw 1979) support such an analysis.

- (3) a. Less than five students attended. \Rightarrow Less than five syntax students attended.
 b. Less than five students attended. \Rightarrow Less than five students attended willingly.
 (4) a. Less than five students with any knowledge of the subject attended.
 b. Less than five students ever attended any of the talks.

But now consider (5a), in which a non-distributive (collective) interpretation of *lifted the piano* is forced by *together*. Intuitively, (5a) is *not* equivalent to (5b): in a context where, say, three semantics students lifted the piano together, and seven phonology students lifted the piano together, (5a) is true in virtue of the semanticists' feat of piano lifting, while (5b) is false in virtue of the phonologists' feat of piano lifting.

- (5) a. Less than five students lifted the piano together.
 b. It is not the case that five or more students lifted the piano together.

Similarly, consider (6a), in which a non-distributive (cumulative) interpretation of *drank more than ten beers* is forced by *between them*. (6a) and (6b) are intuitively not equivalent: in a context where, say, three semantics students drank fifteen beers between them, and seven phonology students drank only five beers between them, (6a) is true in virtue of the semanticists' feat of beer drinking, while (6b) is false in virtue of the whole group of ten students having drunk twenty beers between them.³

- (6) a. Less than five students drank more than ten beers between them.
 b. It is not the case that five or more students drank more than ten beers bw. them.

Thus, (5a) and (6a) have *non*-upper-bounded readings. Moreover, (5a) and (6a), unlike (5b) and (6b), have an existential entailment. Suppose that no students at all lifted the piano (only three professors did). Then (5a) has a reading that is clearly false, while (5b) is clearly true. Similarly, suppose that no students at all drank more than ten beers between them (they drank only five in total between them). Then (6a) is false, while (6b) is true. Since the

²Assume that the maximum of an empty degree predicate is by definition zero.

³Assume that the semantics students and phonology students are disjoint sets of students. Or just suppose that there were, in total, five or more students who drank, in total, more than ten beers between them.

readings of (5a) and (6a) under discussion have existential entailments (and, unlike (5b) and (6b), do not impose upper bounds), call them *existential* (or lower-bounded) readings.

The maximality-based entries in (2) seem ill suited to capture these existential readings because those entries impose an upper bound and fail to make any existential entailment. Rather, existential readings seem to call for an entry like (7), where x ranges over groups, or *sums*, of individuals, and **card**(x) is the number of atomic parts of x (Link 1983).

$$(7) \quad \llbracket \text{less than five} \rrbracket = \lambda P_{et} . \lambda Q_{et} . \exists x [\mathbf{card}(x) < 5 \wedge P(x) \wedge Q(x)]$$

An entry of this kind is supported by the quite natural paraphrases in (8) of our two running examples. Moreover, on this entry *less than five* is upward monotone, which the inference pattern and NPI data in (9) support. However, as is well known, an entry like (7) has disastrous consequences for examples like (1a).

- (8) a. A group of less than five students lifted the piano together.
b. A group of less than five students drank more than ten beers between them.
- (9) a. Less than five students (*ever) lifted the piano together.
(cf. No students ever lifted the piano together.)
b. Less than five students drank more than fifteen beers between them.
 \Rightarrow Less than five phonology students drank more than fifteen beers bw. them.

The puzzle is how to derive upper-bounded (but not existential) readings for *less than* sentences with distributive predicates, while still being able to derive existential readings for *less than* sentences with non-distributive predicates. In section 2, I present the so-called adjectival theory of (modified) numerals and discuss two well-known problems that it faces. This discussion sets the stage for my proposal in section 3. In section 4, I discuss predictions for semi-distributive predicates like *gather* and *know each other*. Section 5 concludes.

2. The adjectival theory of numerals

A longstanding view is that numerals are interpreted just like intersective adjectives (see Landman 2004, and the references therein). For example, *three* denotes the (characteristic function of) the set of all sums with exactly three atomic parts, and *three students* denotes the (characteristic function of) the set of all individual sums with exactly three atomic parts, each of whom is a student.

- (10) a. $\llbracket \text{three} \rrbracket = \lambda x_e . \mathbf{card}(x) = 3$
b. $\llbracket \text{students} \rrbracket = \lambda x_e . \mathbf{students}(x)$
c. $\llbracket \text{three students} \rrbracket = \lambda x_e . \mathbf{card}(x) = 3 \wedge \mathbf{students}(x)$

Sentences with numerical DPs are interpreted existentially by an operation of existential closure (cf. Heim 1982), implemented here by a silent existential determiner, \emptyset_\exists , which heads a numerical DP at logical form (LF) and has the semantics in (11).

$$(11) \quad \llbracket \emptyset_\exists \rrbracket = \lambda P_{et} . \lambda Q_{et} . \exists x [P(x) \wedge Q(x)]$$

An example derivation of a sentence with the bare numeral *three* and the distributive predicate *attended* is provided in (12).⁴ On this plain existential analysis, (12a) is predicted to be consistent with more than three students having attended; that is, *three* is analyzed as “at least three”. The availability of a stronger, “exactly three” reading of (12a) has traditionally been argued to be the result of pragmatic strengthening due to scalar implicature.⁵

- (12) a. Three students attended.
 b. $[\emptyset_{\exists} [\text{three students}]]$ attended
 c. $\exists x[\mathbf{card}(x) = 3 \wedge \mathbf{students}(x) \wedge \mathbf{attended}(x)]$

An intuitively natural way to extend this theory to modified numerals is to maintain that modified numerals, like bare numerals, are adjectival: *more than three students* denotes the set of sums with more than three atoms, each of whom is a student; *less than five students* denotes the set of sums with less than five atoms, each of whom is a student; and so on.

- (13) a. $\llbracket \text{more than three} \rrbracket = \lambda x_e. \mathbf{card}(x) > 3$
 b. $\llbracket \text{less than five} \rrbracket = \lambda x_e. \mathbf{card}(x) < 5$

This extension works fine for *more than* but fails spectacularly for *less than*, as (14) illustrates. There are two problems. First, on the standard assumption that there is no null sum, i.e. no sum x such that $\mathbf{card}(x) = 0$, (14c) entails that at least some student(s) attended; however, (14a) is intuitively consistent with no students having attended. Call this the *existential entailment problem*. Second, due to the plain existential quantifier, (14c) is consistent with five or more students having attended, exactly like in the bare numeral case. However, (14a) is intuitively not consistent with five or more students having attended. This problem was first pointed out by Van Benthem (1986) and has since become known as *Van Benthem’s problem*. What these two problems together amount to is that (14c) represents a non-upper-bounded, existential reading of (14a), which of course is unavailable.

- (14) a. Less than five students attended.
 b. $[\emptyset_{\exists} [\llbracket \text{less than five} \rrbracket \text{ students}]]$ attended
 c. $\exists x[\mathbf{card}(x) < 5 \wedge \mathbf{students}(x) \wedge \mathbf{attended}(x)]$

The adjectival theory exclusively derives existential readings for sentences with *less than n*, regardless of the type of predicates it combines with (distributive or non-distributive). This means that it *does* in fact generate the right reading for sentences with non-distributive predicates—a point which, to my knowledge, has not been acknowledged in the literature. For example, it derives the truth conditions in (15c) for (15a). With *lifted the piano* interpreted collectively, (15c) is an appropriate representation of (15a): it has an existen-

⁴The denotation of *students*, written **students**, is the set of all sums each of whose atomic parts is a student, and the denotation of *attended*, written **attended**, is the set of all sums each of whose atomic parts attended. To make the metalanguage clearer, we could, e.g. following Link 1983, write ***student** and ***attended**, or use a distributivity operator of some kind, but the actual analysis of how distributivity arises is unimportant here.

⁵This traditional view has recently been called into question, e.g. due to the ability of numerals to have strong interpretations even in downward entailing environments. See Spector 2013 for recent discussion.

tial entailment and imposes no upper bound.⁶ In other words, the existential entailment problem and Van Benthem's problem are actually not problems at all for sentences with a non-distributive predicate. Quite the opposite: they are precisely what is needed.

- (15) a. Less than five students lifted the piano.
b. $[\emptyset_{\exists} [[\text{less than five}] \text{ students}]] [\text{lifted the piano}]$
c. $\exists x[\text{card}(x) < 5 \wedge \text{students}(x) \wedge \text{lifted}(p)(x)]$

Many alternative theories have been proposed to solve or avoid the two problems faced by the adjectival theory. All involve maximality in some form. For example, Hackl (2000) proposes that *less than five* denotes a generalized quantifier over degrees, which encodes a maximality operator, as in (2b) above, and that existential quantification over individuals is contributed by a silent, generalized determiner, $\langle \text{many} \rangle$, which is parameterized for degrees. The schema in (17) illustrates a derivation. The reason that only upper-bounded readings are ever derived is that, for type reasons, *less than five* must QR, while $\langle \text{many} \rangle$ stays low, with the result that the operator *max* always scopes above the individual quantifier.

- (16) $\llbracket \langle \text{many} \rangle \rrbracket = \lambda n_d . \lambda P_{dt} . \lambda Q_{dt} . \exists x[\text{card}(x) = n \wedge P(x) \wedge Q(x)]$
(17) a. Less than five NP VP.
b. $[\text{less than five}] [\lambda n [[n \langle \text{many} \rangle] \text{ NP} \text{ VP}]]$
c. $\max(\lambda n . \exists x[\text{card}(x) = n \wedge \llbracket \text{NP} \rrbracket(x) \wedge \llbracket \text{VP} \rrbracket(x)]) < 5$

Landman (2004), by contrast, tries to salvage the adjectival theory by proposing an operation of “maximalization”. The idea is that modified numerals have the adjectival entries in (13) and that certain modified numerals, including *less than five* (but not *more than three*) are specified for a feature requiring that maximalization apply. Omitting details, the upshot is that basic sentences with *less than five* are always assigned truth conditions like in (18), where $\sqcup P$ is the largest (maximal) sum of individuals having property *P*. Crucially, to avoid any existential entailment problem, Landman assumes the existence of a null sum, which is in the extension of pluralized NP and VP predicates. Thus, for example, if no students attended, then the only sum that is in the extension of both *students* and *attended* is the null sum, whose cardinality is zero (hence, less than five). Despite their cosmetic differences, then, (17c) and (18) are in fact truth conditionally equivalent.

- (18) $\text{card}(\sqcup(\lambda x_e . \llbracket \text{NP} \rrbracket(x) \wedge \llbracket \text{VP} \rrbracket(x))) < 5$

The problem with these two theories, however, in light of the data with non-distributive predicates, is that they are (by design) unable to derive existential readings.

To recap, the adjectival theory both undergenerates (by not deriving upper-bounded readings for the distributive case) and overgenerates (by deriving existential readings for the distributive case). The two maximality-based above theories both undergenerate (by not deriving existential readings for the non-distributive case).⁷

⁶The denotation of *lifted*, written **lifted**, is (uncurried) the set of all pairs $\langle y, x \rangle$ such that *x* lifted *y* together.

⁷The maximality-based theories may also overgenerate, by deriving upper-bounded readings of *less than* sentences with a non-distributive predicate. I briefly address this issue in the conclusion (section 5).

3. Proposal

I propose that the grammar actually generates both existential and upper-bounded readings across the board (thus, overgenerates), and that existential readings are sometimes unattested because they violate a pragmatic constraint against semantically vacuous material (e.g. the numeral *five* in *less than five*). I first explain the rationale behind pragmatic blocking, and then I present a system that generates both types of readings.

3.1 Pragmatic blocking

Consider again (19a) and the unattested existential reading generated for it by the adjectival theory. We have already seen that this reading incurs both an existential entailment problem and Van Benthem's problem, but I would like to probe the issue further.

- (19) a. Less than five students attended.
 b. $[\emptyset_{\exists} [[\text{less than five}] \text{ students}]] \text{ attended}$
 c. $\exists x[\text{card}(x) < 5 \wedge \text{students}(x) \wedge \text{attended}(x)]$

Due to the distributivity of *students* and *attended*, (19c) is in fact equivalent to (20).⁸ The entailment from (20) to (19c) is obvious. For the reverse entailment, suppose that z is a sum verifying (19c). Then, since *students* and *attended* are distributive, each atomic part of z is a (singleton) sum that is a student who attended, which verifies (20).

- (20) $\exists x[\text{card}(x) = 1 \wedge \text{students}(x) \wedge \text{attended}(x)]$

Thus, despite the presence of *less than five*, the LF in (19b) expresses an extremely weak proposition: it simply says that some student(s) attended.⁹ One way to think about the weakness of (19b) is that the numeral *five* effectively does no semantic work: the same truth conditions would be derived if *five* were replaced, say, by *four* or by *six*.¹⁰

I propose that the LF in (19b) is unavailable precisely *because* the numeral *five* has no semantic import. More generally, I believe the adjectival theory is on the right track: it derives exactly the right reading for sentences with non-distributive predicates, and the weak readings derived for sentences with distributive predicates are ruled out on pragmatic grounds.¹¹ Thus, existential readings should not be abolished across the board, as the maximality theories above would have it. Rather, they are generated but sometimes blocked

⁸Which in turn is equivalent to $\exists x[\text{students}(x) \wedge \text{attended}(x)]$ by similar reasoning.

⁹Nevertheless, this existential reading is not, strictly speaking, weaker than the upper-bounded reading since, if no students attended, then the upper-bounded reading is true while the existential reading is false.

¹⁰I have assumed here that plural expressions like *students* contain atoms in their extension (Krifka 1989, Sauerland et al. 2005), but this assumption is not crucial. If the extensions of plural expressions contain only non-atomic sums, then (19c) is equivalent to $\exists x[\text{card}(x) = 2 \wedge \text{students}(x) \wedge \text{attended}(x)]$, and the semantic vacuity of *five* in *less than five* still follows.

¹¹Link (1997) has maintained that distributivity is, in a sense, more derivative than non-distributivity, as evidenced by the fact that typologically distributivity tends to be overtly marked. Thus, it is perhaps not surprising that the adjectival theory needs to say something special for distributive predicates.

because they violate a pragmatic constraint against using an expression like *less than five* when *five* has no semantic import. One way to formulate such a constraint is given in (21).

- (21) An LF ϕ containing a numeral n , may not, for any $m \neq n$, be truth conditionally equivalent to $\phi[n \mapsto m]$ (the result of substituting m for n in ϕ).

This constraint rules out (the LF corresponding to) the existential reading of the *attended* example because replacing *five* by *four* or by *six* yields a truth conditionally equivalent LF. Importantly, it does *not* block (the LF corresponding to) the existential reading of the piano-lifting example, repeated below: the numeral *five* in (22b) actually does semantic work, due crucially to the non-distributivity of *lifted the piano*. Specifically, just because, say, ten students lifted the piano together, it does not follow that nine, eight, ... did so too. Therefore, replacing *five* by *four* (or by *six*, etc.) in (22b) yields different truth conditions.

- (22) a. Less than five students lifted the piano.
b. $[\emptyset_{\exists} [\text{less than five}] \text{ students}] [\text{lifted the piano}]$
c. $\exists x [\mathbf{card}(x) < 5 \wedge \mathbf{students}(x) \wedge \mathbf{lifted}(p)(x)]$

As it stands, then, the adjectival theory as I see it generates just the right reading for *less than* sentences with non-distributive predicates, and the unavailability of the reading it generates for *less than* sentences with distributive predicates is due to a violation of a pragmatic constraint like (21). I now show how to extend the adjectival theory to also generate upper-bounded readings.

3.2 Generating both readings

The system that I propose for generating both existential and upper-bounded readings is a combination of the adjectival theory and the degree-based maximality theory. Bare numerals denote degrees but can be interpreted adjectivally according to a rule like (23),¹² while modified numerals denote generalized quantifiers over degrees, just as in (2b), repeated in (24), with the operator *max* defined explicitly in (25). Sums are existentially quantified over by \emptyset_{\exists} (so there is no parameterized determiner *many* as in Hackl 2000). The crux of the system is that modified numerals, which encode the operator *max*, can interact scopally with \emptyset_{\exists} to derive the two different types of readings.

- (23) A numeral n may be replaced by n_{adj} , where $\llbracket n_{\text{adj}} \rrbracket = \lambda x_e. \mathbf{card}(x) = \llbracket n \rrbracket$.

- (24) $\llbracket \text{less than five} \rrbracket = \lambda P_{dt}. \max(P) < 5$

- (25) $\max(P_{dt}) = \begin{cases} \iota n. P(n) \wedge \forall m [P(m) \rightarrow m \leq n] & \text{if } \exists n P(n) \\ 0 & \text{otherwise} \end{cases}$

¹²This syntactic substitution rule is just one way to implement the idea that degree-denoting numerals can be interpreted adjectivally. Another way is to assume a silent syntactic expression that maps a numeral n to the corresponding adjective (n_{adj}). Yet another way is to posit a typeshifting operation that shifts a type d degree to the relevant type *et* function. Each of these options works equally well here.

Since *less than five* denotes a generalized quantifier over degrees (type $(dt)t$), it is uninterpretable *in situ* and must QR, creating a degree trace n that shifts to n_{adj} . Several landing sites for *less than five* are possible. If it lands above \emptyset_{\exists} , i.e. adjoins to S, then the derived reading is an upper-bounded one, as (26) illustrates.

- (26) a. Less than five NP VP.
 b. $[\text{less than five}] [\lambda n [[\emptyset_{\exists} [n_{\text{adj}} \text{ NP}]] \text{ VP}]]$
 c. $\max(\lambda n. \exists x[\mathbf{card}(x) = n \wedge \llbracket \text{NP} \rrbracket(x) \wedge \llbracket \text{VP} \rrbracket(x)]) < 5$

However, *less than five* can also land below \emptyset_{\exists} , by quantifying either into AP or into NP. One way to guarantee this possibility is to assume that APs and NPs have an internal subject position (like VPs) that can be abstracted over. For example, APs have a position filled with a phonologically null (and semantically vacuous¹³) item PRO (Heim & Kratzer 1998), as in (27a). PRO may optionally QR, creating an abstraction over a variable x , as in (27b). Now, *less than five* can quantify into AP, meaning QR just below λx , resulting in a predicate of sums with less than five atomic parts, as in (27c).

- (27) a. $[_{\text{AP}} \text{ PRO } [_{\text{A}'} \text{ less than five}]]$
 b. $[_{\text{AP}} \text{ PRO } [_{\text{AP}} \lambda x [_{\text{AP}} x [_{\text{A}'} \text{ less than five}]]]]$
 c. $[_{\text{AP}} \text{ PRO } [_{\text{AP}} \lambda x [_{\text{AP}} [_{\text{DegP}} \text{ less than five}]] [_{\text{AP}} \lambda n [_{\text{AP}} x [_{\text{A}'} [_{\text{A}} n_{\text{adj}}]]]]]]]$

If *less than five* QRs above PRO, then it does not matter whether PRO also QRs or instead remains *in situ* because the two LF in (28) are logically equivalent.¹⁴

- (28) a. $[_{\text{AP}} \text{ PRO } [_{\text{AP}} \lambda x [_{\text{AP}} x [_{\text{A}'} [_{\text{A}} n_{\text{adj}}]]]]]$
 b. $[_{\text{AP}} \text{ PRO } [_{\text{A}'} [_{\text{A}} n_{\text{adj}}]]]$

The derivation below shows *less than five* quantifying into AP. (Quantifying into NP yields a logically equivalent LF.) The resulting reading is an existential one. Crucially, the semantic effect of the operator *max* is neutralized here because for every sum x , we have $\max(\lambda n. \mathbf{card}(x) = n) = \mathbf{card}(x)$, since every sum has exactly one cardinality.

- (29) a. Less than five NP VP.
 b. $[\emptyset_{\exists} [[\lambda x [[\text{less than five}] [\lambda n [x n_{\text{adj}}]]]] \text{ NP}]] \text{ VP}$
 c. $\exists x[\max(\lambda n. \mathbf{card}(x) = n) < 5 \wedge \llbracket \text{NP} \rrbracket(x) \wedge \llbracket \text{VP} \rrbracket(x)]$
 $\equiv \exists x[\mathbf{card}(x) < 5 \wedge \llbracket \text{NP} \rrbracket(x) \wedge \llbracket \text{VP} \rrbracket(x)]$

To recap, for our running distributive example (*Less than five students attended*), the attested, upper-bounded reading is derived when *less than five* scopes above \emptyset_{\exists} , and the unattested, existential reading, which is derived when *less than five* scopes below \emptyset_{\exists} , is ruled out by the constraint in (21). For our running non-distributive example (*Less than five students lifted the piano*), the attested, existential reading is derived when *less than five* scopes below \emptyset_{\exists} (and it is not ruled out).

¹³PRO is semantically vacuous in the sense that it has no denotation. It can, however, move, creating a semantically non-vacuous trace.

¹⁴Technically, the AP written “ n_{adj} ” in (26b) should be considered short-hand for (28b).

4. Remarks on other non-distributive predicates

Consider (30a) and (30b). Predicates like *gather* and *know each other* are two canonical examples of non-distributive predicates: they apply only to non-atomic sums, as evidenced by the unacceptability of (31a) and (31b). And yet judgments are clear that (30a) and (30b) have only upper-bounded readings. I would like to briefly explain why this actually follows from the pragmatic blocking account advocated here.

- (30) a. Less than fifty protesters gathered in the street.
b. Less than ten guests know each other.
- (31) a. *John gathered in the park.
b. *Mary knows each other.

The crucial point is that, even though *gather* and *know each other* are non-distributive, they do license a certain kind of downward inference. Suppose that 1,000 protesters gathered in the street. Then, although it is odd to conclude that “one person gathered in the street” (such a sentence is unacceptable), it does seem natural to conclude that 999, 998, ..., 2 protesters gathered in the street.¹⁵ Similarly, if 500 people in this room know each other, it is nonsensical to conclude that “one person knows each other”, but it certainly follows that two people know each other. Let us call such inferences *non-atomic downward inferences*.

Surprisingly, these inferences are enough for the constraint in (21) to block the existential readings of (30a) and (30b). Consider (32a), which represents the existential reading of (30a). If non-atomic downward inferences are taken into consideration, then (32a) is equivalent to (32b). The entailment from (32b) to (32a) is obvious. The reverse entailment holds because if z is a sum verifying (32a), then by the availability of non-atomic downward inferences, there is a subpart of z with cardinality 2 who are students who gathered. Thus, the numeral *fifty* in (30a) does no semantic work since the same truth conditions would be derived if it were replaced, say, by *ten* or by *ninety*. And so this existential reading of (30a) is blocked, leaving only the upper-bounded reading available.

- (32) a. $\exists x[\text{card}(x) < 50 \wedge \text{protesters}(x) \wedge \text{gathered}(x)]$
b. $\exists x[\text{card}(x) = 2 \wedge \text{protesters}(x) \wedge \text{gathered}(x)]$

5. Conclusion

I have introduced novel data suggesting that the downward monotonicity of *less than n* that we often (but not always) observe depends in part on the distributivity properties of its arguments and in part on a maximality component. To capture the broad range of facts, I have proposed that *less than* lexically encodes maximality, but that it can interact scopally with existential closure, thus sometimes neutralizing the effect of maximality and creating existential entailments. However, when doing so destroys the semantic effect of the modified numeral, that reading is pragmatically blocked.

¹⁵At least, such an inference seems as natural to me as the inference from *1,000 students attended* to *999, 998, ..., 1 student(s) attended*.

The system proposed here is just one way to generate both readings across the board. For a discussion of other ways, and a comparison of those ways in the context of a broader range of data, see Buccola (in prep.) and Buccola & Spector (in prep.).

The approach advocated here predicts an upper-bounded reading for the piano-lifting example, paraphrasable as, “The maximum number n such that a sum of n students lifted the piano is less than five.” I believe such a reading is not available, but I have no explanation for why not: the constraint in (21) does not block it. Spector (2014) believes it is available. I therefore consider it an open empirical question left for future research.

References

- Barwise, Jon, & Robin Cooper. 1981. Generalized quantifiers and natural language. *Linguistics and Philosophy* 4:159–219.
- Buccola, Brian. in prep. Maximality and modified numerals. Doctoral dissertation, McGill University.
- Buccola, Brian, & Benjamin Spector. in prep. Maximality and modified numerals. Ms.
- Hackl, Martin. 2000. Comparative quantifiers. Doctoral dissertation, MIT.
- Heim, Irene. 1982. The semantics of definite and indefinite noun phrases. Doctoral dissertation, University of Massachusetts, Amherst.
- Heim, Irene. 2000. Degree operators and scope. In *Proceedings of SALT X*, ed. Brendan Jackson & Tanya Matthews, 40–64. Ithaca, NY: Cornell University.
- Heim, Irene, & Angelika Kratzer. 1998. *Semantics in generative grammar*. Blackwell Publishers Inc.
- Krifka, Manfred. 1989. Nominal reference, temporal constitution, and quantification in event semantics. In *Semantics and contextual expressions*, 75–115. Foris: Dordrecht.
- Ladusaw, William A. 1979. Polarity sensitivity as inherent scope relations. Doctoral dissertation, University of Texas, Austin.
- Landman, Fred. 2004. *Indefinites and the type of sets*. Blackwell Publishing.
- Link, Godehard. 1983. The logical analysis of plurals and mass terms: A lattice-theoretical approach. In *Meaning, use, and interpretation of language*, ed. Reiner Bäuerle, Christoph Schwarze, & Arnim von Stechow, 303–323. Berlin, Germany: de Gruyter.
- Link, Godehard. 1997. Ten years of research on plurals — where do we stand? In *Plurality and quantification*, ed. Fritz Hamm & Erhard Hinrichs, 19–54. Dordrecht: Kluwer.
- Sauerland, Uli, Jan Anderssen, & Kazuko Yatsushiro. 2005. The plural is semantically unmarked. In *Linguistic evidence*, ed. Stephan Kepser & Marga Reis, 413–434. Berlin: Mouton De Gruyter.
- Spector, Benjamin. 2013. Bare numerals and scalar implicatures. *Language and Linguistics Compass* 7:273–294.
- Spector, Benjamin. 2014. Plural indefinites and maximality. UCLA colloquium talk.
- van Benthem, Johan. 1986. *Essays in logical semantics*. Reidel.

Brian Buccola

brian.buccola@mail.mcgill.ca