The nature of epistemic implications arising from superlative quantifiers[1]

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1 Overview

- Generalized quantifier theory (Barwise and Cooper 1981) traditionally takes superlative quantifiers (SQs: "at least n", "at most n") to be semantically equivalent to their comparative quantifier counterparts (CQs: "more than m", "fewer than m"), modulo the numeral.[2]

- Thus, (1a) is true iff (1b) is true; or equivalently, (1a) entails (1b), and vice versa.

  (1) a. Floyd has more than 2 children.
      b. Floyd has at least 3 children.

- Geurts and Nouwen 2007 argue that SQs and CQs cannot be equivalent, based on differences in inference patterns: (2b) is readily inferred from (2a), while (2c) is not.

  (2) a. Floyd has exactly 3 children.
      b. Floyd has more than 2 children.
      c. Floyd has at least 3 children.

- G&N propose that this contrast is due to a modal component of SQs, which CQs lack: (2c) implies that the speaker considers it possible that Floyd has more than 3 children, which is not entailed by (worse, it's inconsistent with) (2a).

- A similar contrast: (1a) is consistent with the speaker being sure that Floyd has exactly 3 children, but (1b) is not. (The contrast gets even stronger by replacing "Fred" with "I".)

- I review two competing accounts for deriving epistemic implications with SQs:

[1]Many thanks to Yosef Grodzinsky and Bernhard Schwarz for continued discussions of these topics and others.

[2]This follows from the assumption that the semantics of SQs involves the non-strict comparative operators $\leq$ and $\geq$, while that of CQs involves the strict comparative operators $<$ and $>$, and from the fact that strict and non-strict comparative operators are mathematically interdefinable. For any two numbers n, m: $n > m$ iff $n \geq m + \epsilon$ for some $\epsilon > 0$. For example, on a scale of units, $n > 2$ iff $n \geq 3$.

1. a semantic theory (G&N): the epistemic component is encoded directly into the semantic denotation of SQs.
2. a pragmatic theory (Büring 2008; Cummins and Katsos 2010): epistemic implications are derived as implicatures.

- I identify two puzzles that turn out to be related and solvable once a connection with overt modals is made.
- I argue that subjects/listeners make certain epistemic "leaps" in inferential tasks, suggesting that intuitions on inference patterns involving epistemic modality (SQs) are unreliable for the purposes of determining entailment relations.

2 A semantic vs. pragmatic account

2.1 Geurts and Nouwen 2007: a semantic account

- SQs have a conjunctive semantics, where each conjunct contains an epistemic necessity operator ($\Box$) or possibility operator ($\Diamond$).

(3) a. Floyd owns at least 3 houses.
b. $\Box\exists x[\#x = 3 \wedge \text{house}(x) \wedge \text{own}(f, x)] \wedge$
$\Diamond\exists x[\#x > 3 \wedge \text{house}(x) \wedge \text{own}(f, x)]$

(4) a. Floyd owns at most 3 houses.
b. $\Diamond\exists x[\#x = 3 \wedge \text{house}(x) \wedge \text{own}(f, x)] \wedge$
$\neg\Diamond\exists x[\#x > 3 \wedge \text{house}(x) \wedge \text{own}(f, x)]$

- CQs, by contrast, involve just one non-modal statement.

(5) a. Floyd owns more than 2 houses.
b. $\exists x[\#x > 2 \wedge \text{house}(x) \wedge \text{own}(f, x)]$

(6) a. Floyd owns fewer than 4 houses.
b. $\neg\exists x[\#x = 4 \wedge \text{house}(x) \wedge \text{own}(f, x)]$

- Clearly, (5b) does not entail (3b), nor does (6b) entail (4b).
- Likewise, the proposition that Floyd owns exactly 3 houses entails (5b) and (6b), but not (3b) or (4b).
- N.B.: This analysis (incorrectly?) predicts that (3a) is consistent with a context in which the speaker is certain that Floyd owns, say, exactly 4 houses.

2.2 Büring 2008, Cummins and Katsos 2010: a pragmatic account

- SQs involve $\leq$ and $\geq$; CQs involve $<$ and $>$.
- Although these operators are logically interdefinable (see fn. 2), they are psychologically different at some non-linguistic level of representation (C&K).
- "At least *n*" (resp., "at most *n*") is interpreted disjunctively as "exactly *n* OR more than (resp., fewer than) *n*", which somehow triggers a quantity implicature.
- Similar to the classic clausal implicatures associated with disjunctions: "Floyd or Jim called" implies that the speaker is unsure whether it was Floyd or Jim that called.

(7) Floyd owns at least 3 houses.
   a. asserts: Floyd owns exactly 3 houses OR Floyd owns more than 3 houses.
   b. implicates: (i) the speaker is unsure whether Floyd owns exactly 3 houses, and (ii) the speaker is unsure whether Floyd owns more than 3 houses.

- The implicatures are derived as follows: each disjunct of (7a) is stronger than the utterance; the speaker is cooperative (maximally informative, up to relevance, etc.); thus, there must be a reason she did not use either disjunct; that reason must be that she is unsure of each disjunct.
- CQs, by contrast, are not interpreted disjunctively, hence no implicatures are derived.
- The reason (2a) is judged not to entail (2c) is presumably because of some *pragmatic interference*: the possibility implicature of the conclusion contradicts the premise.
- N.B.: epistemic implications seem to be *non-defeasible*, so interference can't disappear.
- N.B.: It's unclear also whether the epistemic implications here are attributed to some nonexistent speaker even in inferential (experimental) tasks.

3 Two puzzles

3.1 Puzzle #1: the original intuition

- What is the source of the original intuition, i.e., that (1a) entails (1b)?
- Not only does GQT assume the entailment to hold, but native speakers judge it to be so as well: responding to a questionnaire I devised, over 90% of speakers judged "Lauren recited at most 4 poems" to follow from "Lauren recited fewer than 5 poems".
- On the semantic account, this inference should never be judged valid: the modal components are unentailed by the completely non-modal premise.

- On the pragmatic account, the prediction is more unclear: the consequent contains noncontradictory but nonetheless unentailed, non-defeasible epistemic implicatures.
- As shown above, contradictory epistemic implicatures can interfere in entailment judgments; however, it's unclear whether noncontradictory, unentailed epistemic implications should also interfere.
- Presumably, however, it would odd for a subject/listener's inference to be unaffected by an unentailed epistemic implicature attributed to a nonexistent speaker.

3.2 *Puzzle #2: the downward entailingness of "at most n"*

- "At most *n*", like "fewer than *n*", is assumed to be downward entailing (DE) (Krifka 2007; Geurts et al. 2010).
- It licenses negative polarity items, and entailment inferences from sets to subsets, e.g., from (9a) to (9b), seem to be uncontroversial.

(8) At most 3 people have ever been in this cave. Krifka 2007

(9) a. At most 3 students smoke.
b. At most 3 students smoke cigars. Chierchia and McConnell-Ginet 2000

- Not only does the semantic theory fail to make "at most *n*" DE; it makes it nonmonotonic.
- The pragmatic theory makes "at most *n*" DE, but has the same problem as before: (9b) has a noncontradictory but unentailed, non-defeasible epistemic implicature, which may be expected to invalidate the inference.

4 Inference blocking

- These two puzzles turn out to be related.
- Adding an extra premise seems to block these puzzling inferences, suggesting that *perhaps* they're not entailments after all.

(10) a. Floyd has more than 2 children
b. Floyd has more than 3 children
c. Floyd has at least 3 children.

- With (10b) as a second premise, which is not inconsistent with (10a), the inference of (10c) seems invalid.
- Similarly for the supposedly DE inference.

(11) a. At most 3 students smoke.
b. No students smoke cigars.
c. At most 3 students smoke cigars.

- The added premise, (11b), is not inconsistent with (11a), but now the inference of (11c) seems invalid.
- Intuitively, the idea is the same as earlier: the new premise and the epistemic implication(s) of the conclusion are contradictory, hence the invalidity of the inference.
- However, if the original inferences from (10a) (resp, (11a)) to (10c) (resp., (11c)) really were valid to begin with, then any additional, consistent premises would not affect the validity.
- So what's going on in the judgments without added premises? Are G&N ultimately correct that "at most n A B" cannot be inferred from "fewer than $n+1$ A B", and that "at most n" is non-monotonic?
- Since epistemic implications are non-defeasible, it seems as if speakers are automatically *satisfying* them, as long as they're consistent with the premise.
- If so, perhaps we might see the same thing with overt epistemic modals.

5 Overt modals and epistemic leaping

- Speakers make the same pseudo-logical inferences with overt modals: it seems reasonable to infer (12b) from (12a).

(12) a. Floyd went to Italy.
b. He may have gone to Rome.

- Contrast this inference with, say, one in which the conclusion has "went" instead of "may have gone", which is completely invalid.
- But once again, adding an extra premise, e.g., "He did not go to Rome", which is consistent with (12a), invalidates the inference.
- Moreover, if "Rome" is switched with, say, "the Castello dei Conti in Modica, Sicily", the inference is not as strong (if even present at all).
- Intuitively, it's more reasonable to assume Floyd visited the capital of Italy than a little-known castle in a small town in Sicily.
- Finally, not just anything can be assumed, even if it's perfectly reasonable/probable: given a premise like "Floyd went to Italy", it seems one cannot infer, say, "Floyd may own a dog".

- I propose that listeners (or subjects in inferential tasks) leap to these epistemic inferences as follows.

  (13) **Epistemic leaping**
  Given a set of premises $\Pi = \{p_1, \ldots, p_n\}$ and a listener L, the *listener-oriented* epistemic proposition $\Diamond_L q$ is inferable from Π by L iff (i) q is consistent with $p_1 \wedge \ldots \wedge p_n$, (ii) q is consistent with L's world knowledge, and (iii) q bears a relevance relation to each p_i, as judged by L.

- Note that the satisfaction of (ii) most likely depends on some scale of probability as judged by L.
- Thus, the satisfaction of (ii) and (iii) is in some sense variable/on a scale, suggesting that inferences vary in strength from proposition to proposition, listener to listener.
- Does epistemic leaping follow from any general principle?
- For $p_1, \ldots, p_n$ to be true, at least one of $q_1, \ldots, q_m$ must be true; and without any further information, each q_i is possible, though to varying degrees based on world knowledge/probability, etc.
- Hence, for the proposition that Floyd went to Italy to be true, he must have gone somewhere in Italy: this could be satisfied by his going to Rome (very probable), or to the Castello dei Conti in Modica (improbable).
- Returning to SQs, subjects seem to make epistemic leaps, and then *epistemically enrich* the premise set with propositions like $\Diamond_L q$, thus making certain otherwise invalid inferences valid.

  (14) **Epistemic enrichment**
  Given a set of premises $\Pi = \{p_1, \ldots, p_n\}$ and a listener L, if $\Diamond_L q$ is inferable from Π by L via epistemic leaping, then $\Diamond_L q$ is added to Π for the purposes of an inferential task.

- Thus, given a single premise like "Floyd has more than 2 children", it's perfectly reasonable for a listener to infer that Floyd may have exactly 3 children, but not if "Floyd has more than 3 children" is added as a premise.
- Likewise, given a single premise like "At most 3 students smoke", it's perfectly reasonable for a subject/listener to infer that it may be the case that exactly 3 students smoke cigars, but not if "No students smoke cigars" is added as a premise.
- Once epistemic enrichment occurs, the relevant inferences involving SQs logically follow.
- And due to the scalar/variable nature of the epistemic leaping conditions, strength of inferences are expected to vary from proposition to proposition, listener to listener.
- In the questionnaire mentioned earlier, subjects were reluctant to infer (15c) from (15a).

(15) a. Floyd ate at most 3 cookies.
b. Floyd ate at most 3 oatmeal cookies.
c. Floyd ate at most 3 poisonous cookies.

6 Putting the pieces together

- If epistemic implications with SQs are semantic, then we can explain puzzles 1 and 2 with epistemic enrichment in inferential tasks; but the non-DE-ness of "at most *n*" remains.
- If they're pragmatic, puzzles 1 and 2 are either unproblematic (depending on assumptions) or they're solvable with epistemic enrichment, and "at most *n*" is DE.
- Without epistemic enrichment, the pragmatic account would have trouble explaining the variability of judgments, and would have to tell a story about how to handle *speaker*-oriented epistemic implicatures in inferential tasks.
- Epistemic enrichment simplifies things by allowing the epistemic modality to be attributed to the subject/listener.
- In a nutshell: epistemic implications arise pragmatically and play no role in entailment relations; and subject/listener intuitions in inferential tasks designed to test entailment relations are often unreliable due to a tendency to epistemically enrich the premise set, as evidenced by the case of overt modals.

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