Severing maximality from *fewer than*: Evidence from genericity*

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1 Introduction

This talk concerns the role that **maximality** plays in the semantics of (sentences containing) the numeral modifier *fewer than*.

I will argue that the maximality component normally associated with *fewer than* must be an **optional** component **separate** from the meaning of *fewer than*, a decomposition already proposed for independent reasons by Spector (2014).

1.1 Maximal readings

One main intuitive difference between *more than* and *fewer than* is that the latter, but not the former, normally conveys an **upper bound**.

For example, (1b), but not (1a), conveys an upper bound of 3 on the number of students who (may have) attended.

(1) a. More than three students attended the lecture.
b. Fewer than four students attended the lecture.

Put differently, (1b), but not (1a), intuitively entails (2).

(2) It is not the case that more than three students attended.

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In addition, (1b), but not (1a), does not entail any **lower bound**: it is consistent with no students having attended.[1]

The following is thus an appropriate representation of the meaning of (1b).[2]

(3) a. Fewer than four students attended the lecture.
b. $\max(\lambda n \,.\, \exists x[\#x = n \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]) < 4$
c. 'The maximum (total) number of students who attended the lecture, if any, is less than 4.'

Let us call such a reading a 'maximal' (or 'upper-bounded') reading, and note that there is no other reading available for (1b).

1.2 Non-maximal readings

Surprisingly, when *fewer than four* combines with (certain) **non-distributive** predicates, we do not get the same kind of maximal reading (Buccola 2015; Spector 2014).

For example, in (4a), a collective interpretation of *lifted the piano* is forced by *together*, and (4a) does **not** entail (4b).

(4) a. Fewer than four students lifted (managed to lift) the piano together.
b. It is not the case that more than three students lifted the piano together.

In a context where three semantics students lifted the piano and seven phonology students lifted the piano, (4a) is true while (4b) is false.[3]

Moreover, (4a) conveys a **lower bound**: it entails that at least some student(s) lifted the piano.[4]

In sum, (4a) simply means that a group of fewer than four students lifted the piano, which we may represent as follows.

(5) a. Fewer than four students lifted the piano together.
b. $\exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]$
c. 'A group of fewer than four students lifted the piano.'

Similarly, in (6a), a cumulative interpretation of *drank more than twenty beers* is forced by *between*, and (6a) does **not** entail (6b).

[1]To be sure, (1b) may implicate that some student(s) attended, but, in line with Generalized Quantifier Theory (Barwise and Cooper 1981), I do not take this to be part of the literal meaning of (1b).

[2]Here, x ranges over sums of individuals (Link 1983), and # is a function that maps a sum x to the cardinality of x, i.e. to that number n such that x has n atomic parts. There are, of course, other ways to represent this meaning.

[3]This is different from (1b): if three semantics students attended, and seven phonology students attended, then *Fewer than four students attended* is false despite the existence of a salient group of fewer than four attending students.

[4]This is why the sentence *Fewer than four babies lifted the piano together* feels false, not true, in most everyday contexts.

(6) a. Fewer than four students drank (managed to drink) more than twenty beers between them.
b. It is not the case that more than three students drank more than twenty beers between them.

In a context where three semantics students drank 21 beers between them and all the students together drank 30 beers between them, (6a) is true while (6b) is false.

And again, (6a) entails that at least some student(s) drank more than twenty beers between them.

In sum, (6a) simply means that a group of fewer than four students drank more than twenty beers.

(7) a. Fewer than four students drank more than twenty beers between them.
b. $\exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{drank\_more\_than\_20\_beers}(x)]$
c. 'A group of fewer than four students drank more than twenty beers.'

1.3 Inadequate representations

Clearly, the non-maximal readings of (4a) and (6a) cannot be represented on analogy with the maximal reading of (1b), using a maximality operator.

(8) a. $\max(\lambda n\,.\,\exists x[\#x = n \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]) < 4$
b. 'No group of more than three students lifted the piano.'

(9) a. $\max(\lambda n\,.\,\exists x[\#x = n \wedge \textbf{students}(x) \wedge \textbf{drank\_more\_than\_20\_beers}(x)]) < 4$
b. 'No group of more than three students drank more than twenty beers.'

These would predict that (4a) and (6a) should entail (4b) and (6b), respectively.[5]

Conversely, the maximal reading of (1b) cannot be represented on analogy with the non-maximal readings of (4a) and (6a), as a simple existential statement, since this leads to **Van Benthem's problem** (Van Benthem 1986).

(10) a. Fewer than four students attended the lecture.
b. $\exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
$\equiv \exists x[\textbf{students}(x) \wedge \textbf{attended}(x)]$
c. 'A group of fewer than four students attended the lecture.'
$\rightsquigarrow$ 'At least one student attended the lecture.'

[5] Whether or not a maximal reading of *fewer than four* in such non-distributive contexts is available is an open empirical question. See Buccola and Spector (submitted) for discussion.

1.4 The puzzle

The presence of maximality is, in some sense, **variable**, depending on the type of predicates that *fewer than n* combines with. What explains this variability?

Roadmap:

- Review two recent accounts of the puzzle.
    - **Lexical Maximality**: *Fewer than* **lexically** encodes reference to maximality, and the variable presence of maximality is due to **variable scope** of *fewer than* relative to covert existential quantification (Buccola 2015).
    - **Separate Maximality**: The variable presence of maximality is due to the **optional** application of a maximization operation that is **separate** from the meaning of *fewer than* (Spector 2014).
- Illustrate that, for the core data above, the two theories generate exactly the same set of readings, up to truth-conditional equivalence.
- Introduce data from the generic domain (generically interpreted sentences that contain *fewer than*) which appear to support Separate Maximality over Lexical Maximality.

2 Two theories

Buccola (2015) and Spector (2014) both argue that both maximal and non-maximal (existential) readings are grammatically generated across the board.

Certain 'weak' readings, in which the modified numeral makes no semantic contribution, are then filtered out by a **pragmatic blocking constraint**.

(11) **Pragmatic blocking constraint**
If an LF ϕ contains a numeral n, then for any m distinct from n, substituting m for n in ϕ must yield different truth conditions.

The two theories mainly differ in their method of overgeneration.

For both theories, assume that numerals denote degrees (numbers) but can also be interpreted as intersective adjectives (see, e.g., Landman 2004), which I write with the subscript '`isCard`'.

(12) a. $[\![\text{three}]\!] = 3$
b. $[\![\text{three}_{\texttt{isCard}}]\!] = \lambda x \,.\, \#x = 3$
c. $[\![\text{three}_{\texttt{isCard}} \text{ students}]\!] = \lambda x \,.\, \#x = 3 \wedge \textbf{students}(x)$

2.1 Lexical maximality and scope ambiguity (LMax)

Buccola (2015) proposes that *fewer than four* denotes a generalized quantifier over degrees, which lexically encodes a maximality operator (cf. Heim 2000; Hackl 2000).

(13) $[\![\text{fewer than four}]\!] = \lambda P_{dt} \,.\, \max(P) < 4$

Numerical indefinites are headed by a silent existential determiner, $\varnothing_\exists$ (cf. Link 1987; Krifka 1999).

(14) $[\![\varnothing_\exists]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists x[P(x) \wedge Q(x)]$

In an expression like (15), *fewer than four* is uninterpretable and must move.

(15) $[_{\text{DP}}\ \varnothing_\exists\ [_{\text{NP}}\ [_{\text{AP}}$ fewer than four$]\ \ldots]]\ [_{\text{VP}}\ \ldots]$

The idea is that *fewer than four* may **interact scopally** with $\varnothing_\exists$.

When *fewer than four* scopes **above** $\varnothing_\exists$, i.e. adjoins to S, we get a **maximal** reading.

(16) a. [fewer than four] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ NP]] VP]]
b. $\max(\lambda n \,.\, \exists x[\#x = n \wedge [\![\text{NP}]\!]\,(x) \wedge [\![\text{VP}]\!]\,(x)]) < 4$

When *fewer than four* scopes **below** $\varnothing_\exists$, i.e. quantifies into AP or (equivalently) into NP (Heim and Kratzer 1998), we get a **non-maximal** (existential) reading.

(17) a. [$\varnothing_\exists$ [[$_{\text{AP}}$ λx [[fewer than four] [λn [x $n_{\texttt{isCard}}$]]]] NP]] VP
b. $\exists x[\max(\lambda n \,.\, \#x = n) < 4] \wedge [\![\text{NP}]\!]\,(x) \wedge [\![\text{VP}]\!]\,(x)]$
$\equiv \exists x[\#x < 4 \wedge [\![\text{NP}]\!]\,(x) \wedge [\![\text{VP}]\!]\,(x)]$

On this view, the 'absence' of maximality is due to the following fact.

(18) **Fact.** For all $x \in D_e$, $\max(\lambda n \,.\, \#x = n) = \#x$.

2.1.1 Distributive cases

(19) a. Fewer than four students attended the lecture.
b. [fewer than four] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] attended]]
c. $\max(\lambda n \,.\, \exists x[\#x = n \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]) < 4$
d. 'The total number of students who attended is less than 4.' ✓

(20) a. Fewer than four students attended the lecture.
b. [$\varnothing_\exists$ [[λx [[fewer than four] [λn [x $n_{\texttt{isCard}}$]]]] students]] attended

c. $\exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
$\equiv \exists x[\#x < 3 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
$\equiv \exists x[\#x < 5 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
$\equiv \exists x[\textbf{students}(x) \wedge \textbf{attended}(x)]$

d. 'A group of fewer than four students attended.'
$\rightsquigarrow$ 'At least one student attended.' BLOCKED

2.1.2 Collective cases

(21) a. Fewer than four students lifted the piano together.

b. [$\varnothing_\exists$ [[λx [[fewer than four] [λn [x $n_{\texttt{isCard}}$]]]] students]] [lifted ...]

c. $\exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]$
$\not\equiv \exists x[\#x < 3 \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]$
$\not\equiv \exists x[\#x < 5 \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]$
$\not\equiv \exists x[\textbf{students}(x) \wedge \textbf{lifted}(x)]$

d. 'A group of fewer than four students lifted the piano.' ✓

(22) a. Fewer than four students lifted the piano together.

b. [fewer than four] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] [lifted ...]]]

c. $\max(\lambda n \,.\, \exists x[\#x = n \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]) < 4$

d. 'No group of 4 or more students lifted the piano.' ??/NOT BLOCKED[6]

2.2 Separate and optional maximality (SMax)

Spector (2014) proposes that a separate and optional maximization operation is responsible for the maximal readings we (sometimes) perceive with *fewer than n* as well as with bare numerals (cf. Kennedy 2013; Kennedy 2015).

Specifically, maximality can be part of the meaning of numerals and numerical traces.

(23) a. A numeral (or numerical trace) n may be interpreted as $n_{\texttt{isMax}}$.

b. $[\![n_{\texttt{isMax}}]\!] = \lambda P_{dt} \,.\, \max(P) = [\![n]\!]$

This allows us to derive both maximal ('exactly', 'two-sided') and non-maximal ('at least', 'one-sided') readings of bare numerals, depending on whether the numeral is interpreted as `isMax` or `isCard` (see Spector 2013 for a survey of the issues).

(24) a. Three students attended.

b. [$\varnothing_\exists$ [$\text{three}_{\texttt{isCard}}$ students]] attended

c. $\exists x[\#x = 3 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
$\equiv \exists x[\#x \geq 3 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$

d. 'At least three students attended.'

(25) a. Three students attended.

[6] Again, it is unclear whether a maximal reading is available here. If it is not, then LMax and SMax both face a problem, since the constraint in (11) does not exclude it.

b. $\text{three}_{\texttt{isMax}}$ $[\lambda n\ [[\varnothing_\exists\ [n_{\texttt{isCard}}$ students]] attended]]
c. $\max(\lambda n\,.\,\exists x[\#x = n \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]) = 3$
d. 'Exactly three students attended.'

Since maximality is a separate component (part of numerals and numerical traces), *fewer than four* simply makes an existential statement about degrees.

(26) $[\![\text{fewer than four}]\!] = \lambda P_{dt}\,.\,\exists n[n < 4 \wedge P(n)]$

When *fewer than four* scopes **above** $\varnothing_\exists$, and its numerical trace n is **not** shifted to $n_{\texttt{isMax}}$, we get a **non-maximal** (existential) reading.

(27) a. [fewer than four] $[\lambda n\ [[\varnothing_\exists\ [n_{\texttt{isCard}}$ NP]] VP]]
b. $\exists n[n < 4 \wedge \exists x[\#x = n \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]]$
$\equiv \exists x[\#x < 4 \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]$

When *fewer than four* scopes **above** $\varnothing_\exists$, and its numerical trace n **is** shifted to $n_{\texttt{isMax}}$ (which itself must also move), then we get a **maximal** reading.

(28) a. [fewer than four] $[\lambda n\ [n_{\texttt{isMax}}\ [\lambda m\ [\ [\varnothing_\exists\ [m_{\texttt{isCard}}$ NP]] VP]]]]
b. $\exists n[n < 4 \wedge \max(\lambda m\,.\,\exists x[\#x = m \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]) = n]$
$\equiv \max(\lambda m\,.\,\exists x[\#x = m \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]) < 4$

SMax therefore need not rely on scope ambiguity to generate both types of readings.

However, it turns out that allowing *fewer than four* to scope **below** $\varnothing_\exists$, with or without maximization, does not generate any new readings.

When *fewer than five* scopes **below** $\varnothing_\exists$, and maximization does **not** apply, we get the **non-maximal** (existential) reading.

(29) a. $[\varnothing_\exists\ [[_{\text{AP}}\ \lambda x$ [[fewer than four] $[\lambda n\ [x\ n_{\texttt{isCard}}]]]]$ NP]] VP
b. $\exists x[\exists n[n < 4 \wedge \#x = n] \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]$
$\equiv \exists x[\#x < 4 \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]$

When *fewer than five* scopes **below** $\varnothing_\exists$, and maximization **does** apply, it applies **vacuously** (cf. (18)), and we again get the **non-maximal** (existential) reading.

(30) a. $[\varnothing_\exists\ [[_{\text{AP}}\ \lambda x$ [[fewer than four] $[\lambda n\ [n_{\texttt{isMax}}\ [\lambda m\ [x\ m_{\texttt{isCard}}]]]]]]$ NP]] VP
b. $\exists x[\exists n[n < 4 \wedge \max(\lambda m\,.\,\#x = m) = n] \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]$
$\equiv \exists x[\exists n[n < 4 \wedge \#x = n] \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]$
$\equiv \exists x[\#x < 4 \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]$

Importantly, although SMax generates a greater number of LFs, it generates exactly the same set of readings for the sentences discussed so far.

The reason is that the existential quantifier (over individuals) of $\varnothing_\exists$ and the existential quantifier (over degrees) of *fewer than four* **commute**.

As a result, (27) and (29) are equivalent.

Moreover, given fact (18), it follows that (29) and (30) are equivalent.

Thus, (27), (29), and (30) are all equivalent, and are in turn equivalent to LMax's (17).

If we can find a case where commutativity between *fewer than four* and the determiner does not arise, then we might be able to distinguish SMax from LMax.

2.2.1 Distributive cases

(31) a. Fewer than four students attended the lecture.
b. [fewer than four] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] attended]]
c. $\exists n[n < 4 \wedge \exists x[\#x = n \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]]$
$\equiv \exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
d. 'At least one student attended.' BLOCKED

(32) a. Fewer than four students attended.
b. [fewer than four] [λn [$n_{\texttt{isMax}}$ [λm [[$\varnothing_\exists$ [$m_{\texttt{isCard}}$ students]] attended]]]]
c. $\exists n[n < 4 \wedge \max(\lambda m\,.\,\exists x[\#x = m \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]) = n]$
$\equiv \max(\lambda m\,.\,\exists x[\#x = m \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]) < 4$
d. 'The total number of students who attended is less than 4.' ✓

(33) a. Fewer than four students attended.
b. [$\varnothing_\exists$ [[λx [[fewer than four] [λn [x $n_{\texttt{isCard}}$]]]] students]] attended
c. $\exists x[\exists n[n < 4 \wedge \#x = n] \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
$\equiv \exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
d. 'At least one student attended.' BLOCKED

(34) a. Fewer than four students attended.
b. [$\varnothing_\exists$ [[λx [[fewer than 4] [λn [$n_{\texttt{isMax}}$ [λm [x $m_{\texttt{isCard}}$]]]]]] students]] attended
c. $\exists x[\exists n[n < 4 \wedge \max(\lambda m\,.\,\#x = m) = n] \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
$\equiv \exists x[\exists n[n < 4 \wedge \#x = n] \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
$\equiv \exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
d. 'At least one student attended.' BLOCKED

2.2.2 Collective cases

(35) a. Fewer than four students lifted the piano together.
b. [fewer than four] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] [lifted ...]]]
c. $\exists n[n < 4 \wedge \exists x[\#x = n \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]]$
$\equiv \exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]$
d. 'A group of fewer than four students lifted the piano.' ✓

(36) a. Fewer than four students lifted the piano together.
b. [fewer than four] [λn [$n_{\mathtt{isMax}}$ [λm [[$\varnothing_\exists$ [$m_{\mathtt{isCard}}$ students]] [lifted ...]]]]]
c. $\exists n[n < 4 \wedge \max(\lambda m\,.\,\exists x[\#x = m \wedge \mathbf{students}(x) \wedge \mathbf{lifted}(x)]) = n]$
$\equiv \max(\lambda m\,.\,\exists x[\#x = m \wedge \mathbf{students}(x) \wedge \mathbf{lifted}(x)]) < 4$
d. 'No group of 4 or more students lifted the piano.' ??/NOT BLOCKED

(37) a. Fewer than four students lifted the piano.
b. [$\varnothing_\exists$ [[λx [[fewer than four] [λn [x $n_{\mathtt{isCard}}$]]]] students]] [lifted ...]
c. $\exists x[\exists n[n < 4 \wedge \#x = n] \wedge \mathbf{students}(x) \wedge \mathbf{lifted}(x)]$
$\equiv \exists x[\#x < 4 \wedge \mathbf{students}(x) \wedge \mathbf{lifted}(x)]$
d. 'A group of fewer than four students lifted the piano.' ✓

(38) a. Fewer than four students lifted the piano.
b. [$\varnothing_\exists$ [[λx [[fewer than 4] [λn [$n_{\mathtt{isMax}}$ [λm [x $m_{\mathtt{isCard}}$]]]]]] stu...]] [lifted ...]
c. $\exists x[\exists n[n < 4 \wedge \max(\lambda m\,.\#x = m) = n] \wedge \mathbf{students}(x) \wedge \mathbf{lifted}(x)]$
$\equiv \exists x[\exists n[n < 4 \wedge \#x = n] \wedge \mathbf{students}(x) \wedge \mathbf{lifted}(x)]$
$\equiv \exists x[\#x < 4 \wedge \mathbf{students}(x) \wedge \mathbf{lifted}(x)]$
d. 'A group of fewer than four students lifted the piano.' ✓

3 Evidence from genericity

We now move to sentences with 'quasi-universal' force.

We'll see that the commutativity observed above does not arise for quasi-universally interpreted sentences containing *fewer than n*.

As a result, SMax will turn out to generate a reading which LMax cannot, and which is indeed the salient reading we want to capture.

3.1 Basic characterizing sentences

The sentences in (39) are **characterizing**, or generalizing, sentences (Krifka et al. 1995): they express a generalization about individuals or events.

(39) a. John smokes a cigar after dinner.
b. Cats have fur.

For example, (39b) means something like, 'In general, if x is a cat, then x has fur', or 'Any/every typical cat has fur'.

I will represent the characterizing reading of (39b) as follows.[7]

[7] I have left open the question of how exactly $\forall_{\text{Gen}}$ is interpreted. See Krifka et al. 1995 for a survey of a number of proposals. The exact treatment of $\forall_{\text{Gen}}$, in particular how exceptions are allowed for, is an important issue in the semantics of genericity, but is not very important for the analysis of *fewer than*. All that seems to matter is that $\forall_{\text{Gen}}$ is non-existential.

(40) $\forall_{\text{Gen}} x[\textbf{cats}(x) \rightarrow \textbf{have\_fur}(x)]$

We can capture this reading on analogy with existential indefinites, by positing a silent generic determiner, $\varnothing_{\text{Gen}}$.

(41) $[\![\varnothing_{\text{Gen}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \forall_{\text{Gen}} x[P(x) \rightarrow Q(x)]$

(42) a. Cats have fur.
b. [$\varnothing_{\text{Gen}}$ cats] [have fur]
c. $\forall_{\text{Gen}} x[\textbf{cats}(x) \rightarrow \textbf{have\_fur}(x)]$

3.2 Characterizing sentences with bare numerals

Characterizing sentences with bare numerals appear to work exactly as expected.

(43) Three men can lift the piano. (Link 1987)

Sentence (43) expresses the generalization that any typical group of three men can lift the piano.[8]

This reading falls out naturally from an adjectival analysis of numerals.

(44) a. Three men can lift the piano.
b. [$\varnothing_{\text{Gen}}$ [three$_{\texttt{isCard}}$ men]] [can lift the piano].
c. $\forall_{\text{Gen}} x[[\#x = 3 \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]$

3.3 Characterizing sentences with modified numerals

Finally, consider a characterizing sentence with *fewer than four*.

(45) Fewer than four men can lift the piano.

Sentence (45) appears to have the reading in (46).[9]

(46) a. $\exists n[n < 4 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]]$
b. 'There is a number $n < 4$ such that, in general, any group of n men can lift the piano.'

Notice, again, the lack of reference to maximality, despite the presence of *fewer than*.

[8] Link (1987) gives the same reading ('any three men ...') that I do, and also discusses a null determiner analysis of existential vs. quasi-universal readings of *three men*. He does not, however, discuss modified numerals. His main concern is finding genuine quantification over plural individuals.

[9] This is a kind of 'ignorance' reading: it can be brought out even more by prepending something like, 'I'm not sure exactly how many men it takes, but ...'

As we'll see, only SMax can generate this reading, namely by scoping *fewer than four* **above** $\varnothing_{\text{Gen}}$, and **without** maximization.

4 Two theories revisited

4.1 Lexical Maximality revisited

As in the existential domain, the LMax account has two scope possibilities available.

When *fewer than four* scopes **above** $\varnothing_{\text{Gen}}$, we get a **maximal** reading.

(47) a. [fewer than four] [λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ NP]] VP]]
b. $\max(\lambda n\,.\,\forall_{\text{Gen}} x[[\#x = n \wedge [\![\text{NP}]\!]\,(x)] \rightarrow [\![\text{VP}]\!]) < 4$

When *fewer than four* scopes **below** $\varnothing_{\text{Gen}}$, we get quite a **strong universal** reading.

(48) a. [$\varnothing_{\text{Gen}}$ [[λx [fewer than four] [λn [x $n_{\texttt{isCard}}$]]] NP]] VP
b. $\forall_{\text{Gen}} x[[\max(\lambda n\,.\#x = n) < 4 \wedge [\![\text{NP}]\!]\,(x)] \rightarrow [\![\text{VP}]\!]\,(x)]$
$\equiv \forall_{\text{Gen}} x[[\#x < 4 \wedge [\![\text{NP}]\!]\,(x)] \rightarrow [\![\text{VP}]\!]\,(x)]$

As before, the 'absence' of maximality is due to the fact in (18).

The problem is that, when it comes to (45), neither possibility yields the right reading.

The maximal reading entails that it is not the case that, say, (any group of) four men can lift the piano, whereas (46) does not.

(49) a. Fewer than four men can lift the piano.
b. [fewer than four] [λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ men]] [can lift the piano]]]
c. $\max(\lambda n\,.\,\forall_{\text{Gen}} x[[\#x = n \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]) < 4$
'The maximum number n such that, in general, any group of n men can lift the piano is less than 4.'

The universal reading is strictly stronger than (46): it entails that (any group of) 3 men, 2 men, and even 1 man can lift the piano, whereas (46) does not.[10]

(50) a. Fewer than four men can lift the piano.
b. [$\varnothing_{\text{Gen}}$ [[λx [[fewer than four] [λn [x n]]]] men]] [can lift the piano]
c. $\forall_{\text{Gen}} x[[\#x < 4 \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]$
'In general, any group of fewer than 4 men can lift the piano.'

I will address the question of whether these readings are (ever) available in section 5.

[10] I assume that plural expressions like *men* contain both atomic and non-atomic individuals in their extension, but this assumption is not crucial: even without it, the derived reading is stronger than (46).

4.2 Separate Maximality revisited

As in the existential domain, the SMax account has four possibilities, depending on (i) the relative scope of *fewer than four* and $\varnothing_{\text{Gen}}$, and (ii) whether or not maximization applies.

When *fewer than four* scopes **above** $\varnothing_{\text{Gen}}$, and maximization does **not** apply, then we get the **non-maximal** reading that we want for (45).

(51) a. [fewer than four] [λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ NP]] VP]]
b. $\exists n[n < 4 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge [\![\text{NP}]\!]] \rightarrow [\![\text{VP}]\!](x)]]$

(52) a. Fewer than four men can lift the piano.
b. [fewer than four] [λn [[$\varnothing_{\text{Gen}}$ [$n_{\texttt{isCard}}$ men]] [can lift the piano]]]
c. $\exists n[n < 4 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]]$
'There is a number $n < 4$ such that, in general, any group of n men can lift the piano.'

When *fewer than four* scopes **above** $\varnothing_{\text{Gen}}$, and maximization **does** apply, then we get a **maximal** reading.

(53) a. [fewer than four] [λn [$n_{\texttt{isMax}}$ [λm [[$\varnothing_{\text{Gen}}$ [$m_{\texttt{isCard}}$ NP]] VP]]]]
b. $\exists n[n < 4 \wedge \max(\lambda m \,.\, \forall_{\text{Gen}} x[[\#x = m \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!](x)]) = n]$
$\equiv \max(\lambda m \,.\, \forall_{\text{Gen}} x[[\#x = m \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!](x)]) < 4$

This is the same maximal reading that LMax derives when *fewer than four* scopes **above** $\varnothing_{\text{Gen}}$, i.e. (47).

When *fewer than four* scopes **below** $\varnothing_{\text{Gen}}$, then we get a **strong universal** reading, regardless of whether maximization applies, given the fact in (18).

(54) a. [$\varnothing_{\text{Gen}}$ [[λx [[fewer than four] [λn [x $n_{\texttt{isCard}}$]]]] NP]] VP
b. $\forall_{\text{Gen}} x[[\exists n[n < 4 \wedge \#x = n] \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!]]$
$\equiv \forall_{\text{Gen}} x[[\#x < 4 \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!](x)]$

(55) a. [$\varnothing_{\text{Gen}}$ [[λx [[fewer than four] [λn [$n_{\texttt{isMax}}$ [λm [x $m_{\texttt{isCard}}$]]]]]] NP]] VP
b. $\forall_{\text{Gen}} x[[\exists n[n < 4 \wedge \max(\lambda m \,.\, \#x = m) = n] \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!](x)]$
$\equiv \forall_{\text{Gen}} x[[\exists n[n < 4 \wedge \#x = n] \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!]]$
$\equiv \forall_{\text{Gen}} x[[\#x < 4 \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!](x)]$

This is the same universal reading that LMax derives when *fewer than four* scopes **below** $\varnothing_{\text{Gen}}$, i.e. (48).

Crucially, (51) is not equivalent to (54) (hence, nor to (55)) because the existential quantifier of *fewer than four* and the quasi-universal quantifier of $\varnothing_{\text{Gen}}$ do not commute.

As a result, in the generic domain, SMax generates a reading that LMax does not, which also happens to be the salient reading of (45).

5 Extending the blocking account

5.1 Other readings: *can lift the piano*

Question: Are the other readings generated by SMax (and by LMax) available or unavailable?

My answer: They are **unavailable** to the extent that the inference in (56) is **valid**.

(56) If n men can lift the piano, then so can $n+1$ (for any $n \neq 0$).

Intuitively, (56) is valid: the more men, the easier it is for them to lift it.

If so, then (the LFs corresponding to) the two other readings are blocked, hence expected to be unavailable.

5.1.1 The maximal reading

For the **maximal** reading, the set to which max applies is either empty, or it is non-empty, hence by (56) has no maximum.

(57)
$$\begin{aligned} & \max(\lambda n\,.\,\forall_{\text{Gen}} x[[\#x = n \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]) < 4 \\ \equiv\ & \max(\lambda n\,.\,\forall_{\text{Gen}} x[[\#x = n \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]) < 3 \\ \equiv\ & \max(\lambda n\,.\,\forall_{\text{Gen}} x[[\#x = n \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]) < 5 \\ \equiv\ & [\lambda n\,.\,\forall_{\text{Gen}} x[[\#x = n \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]] = \varnothing \qquad \textsc{blocked} \end{aligned}$$

For the sentence to be true, then, it must simply be that there is no number n such that n men can lift the piano.

We derive this **same reading** if *four* is replaced by *three* or by *five*; thus, it is blocked by the constraint in (11).

5.1.2 The universal reading

For the **strong universal** reading, the assumption in (56) leads to an even stronger reading: any group of men of **any cardinality** can lift the piano.

(58)
$$\begin{aligned} & \forall_{\text{Gen}} x[[\#x < 4 \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)] \\ \equiv\ & \forall_{\text{Gen}} x[[\#x < 3 \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)] \end{aligned}$$

$$\equiv \forall_{\text{Gen}} x[[\#x < 5 \wedge \mathbf{men}(x)] \rightarrow \mathbf{can\_lift}(x)]$$
$$\equiv \forall_{\text{Gen}} x[\mathbf{men}(x) \rightarrow \mathbf{can\_lift}(x)] \qquad \textsc{blocked}$$

This same reading is derived even if *four* is replaced by *three* or by *five*, hence is blocked.

5.2 *Can fit into the elevator*

Prediction: If the assumption in (56) is invalid, then maximal and universal readings should be available.

If we move to the predicate *can fit into the elevator*, then the analogous inference is clearly **invalid**.

In fact, the **opposite** inference is **valid**.

(59) If n people can fit into the elevator, then so can $n - 1$ (for any $n > 1$).

5.2.1 Maximal reading

Indeed, (60) appears to have a **maximal** characterizing reading.

(60) Fewer than four people can fit into the elevator.

In this case, the maximal reading does not give rise to any maximality failure, hence is (correctly) not blocked.

(61)
$$\max(\lambda n \,.\, \forall_{\text{Gen}} x[[\#x = n \wedge \mathbf{people}(x)] \rightarrow \mathbf{can\_fit}(x)]) < 4$$
$$\not\equiv \max(\lambda n \,.\, \forall_{\text{Gen}} x[[\#x = n \wedge \mathbf{people}(x)] \rightarrow \mathbf{can\_fit}(x)]) < 3$$
$$\not\equiv \max(\lambda n \,.\, \forall_{\text{Gen}} x[[\#x = n \wedge \mathbf{people}(x)] \rightarrow \mathbf{can\_fit}(x)]) < 5 \qquad \checkmark$$

5.2.2 Non-maximal reading

In addition, (60) appears **not** to have the non-maximal reading in (62).

(62)
$$\exists n[n < 4 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge \mathbf{people}(x)] \rightarrow \mathbf{can\_fit}(x)]]$$
$$\equiv \exists n[n < 3 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge \mathbf{people}(x)] \rightarrow \mathbf{can\_fit}(x)]]$$
$$\equiv \exists n[n < 5 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge \mathbf{people}(x)] \rightarrow \mathbf{can\_fit}(x)]]$$
$$\equiv \exists n[n > 0 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge \mathbf{people}(x)] \rightarrow \mathbf{can\_fit}(x)]] \qquad \textsc{blocked}$$

In the case, because of (59), we get a very weak reading, which says that **some** (non-zero) number of people can fit into the elevator.

The logic of this result is **exactly the same** as Van Benthem's problem in the existential domain for distributive predicates.

5.2.3 Universal reading

Finally, the **universal** reading is not blocked: replacing *four* with *three* leads to a weaker meaning, and replacing *four* with *five* leads to a stronger meaning.

(63) $\forall_{\text{Gen}} x[[\#x < 4 \wedge \textbf{people}(x)] \rightarrow \textbf{can\_fit}(x)]$
$\not\equiv \forall_{\text{Gen}} x[[\#x < 3 \wedge \textbf{people}(x)] \rightarrow \textbf{can\_fit}(x)]$
$\not\equiv \forall_{\text{Gen}} x[[\#x < 5 \wedge \textbf{people}(x)] \rightarrow \textbf{can\_fit}(x)]$??/NOT BLOCKED

It is unclear to me whether this reading is indeed available.

Note that it is also consistent with (any group of) 4, 5, . . . people being able to fit.

If this reading is **unavailable**, then SMax could probably be modified so that *fewer than four* never scopes below generic (or existential) quantification, e.g. by replacing $\varnothing_{\exists}$ and $\varnothing_{\text{Gen}}$ by the counting quantifiers $many_{\exists}$ and $many_{\text{Gen}}$ (inspired by Hackl 2000).

(64) a. $[\![many_{\exists}]\!] = \lambda n_d \,.\, \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists x[\#x = n \wedge P(x) \wedge Q(x)]$
b. $[\![many_{\text{Gen}}]\!] = \lambda n_d \,.\, \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \forall_{\text{Gen}} x[[\#x = n \wedge P(x)] \rightarrow Q(x)]$

6 Conclusion

I've presented evidence from genericity suggesting that maximality should be **separate** from the meaning of *fewer than*.

That is, reliance on scope ambiguity (LMax) is not enough.

On this view, *fewer than four* is not really a **downward entailing** operator.

Rather, downward entailing environments are created under very specific conditions: when *fewer than four* takes wide scope and its numerical trace n is interpreted as $n_{\texttt{isMax}}$.

The application of maximality is **regulated** by a pragmatic constraint that is sensitive to the **types of inferences** that predicates allow.

This explains why the availability of maximal vs. non-maximal readings is only partially related to whether we are in an existential vs. generic context, or whether we have a distributive vs. non-distributive predicate.

It explains why *Fewer than four students lifted the piano together* has a non-maximal reading, while *Fewer than 20 protesters gathered* has only a maximal reading, despite both sentences being existential: *gather*, but not *lifted . . .* , licenses downward inferences.

And it explains why *Fewer than four students can lift the piano* has a non-maximal reading, while *Fewer than four people can fit into the elevator* has only a maximal reading, despite both sentences being generic: *can fit . . .* , but not *can lift . . .* , licenses downward inferences.

We discover that **Van Benthem's problem** is more pervasive than we once thought, and that such readings are always inaccessible.

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