<u>Exercise 1.9</u>

Recall the definition

$$\mathrm{add} : \mathbb{N} \to \mathbb{N} \to \mathbb{N}$$

$$\mathrm{add} :\equiv \mathrm{rec}_{\mathbb{N}}(\mathbb{N} \to \mathbb{N}, \lambda n.n, \lambda n.\lambda g.\lambda m.\, \mathrm{succ}(g(m)))$$

Define $\mathrm{Fin} : \mathbb{N} \to \mathcal{U}$ by

$$\mathrm{Fin}(0) :\equiv \underline{0} \quad \text{(the empty type)}$$

$$\mathrm{Fin}(\mathrm{succ}(n)) :\equiv \sum_{a:\mathbb{N}} \sum_{b:\mathbb{N}} \mathrm{add}(b,a) \overset{\mathbb{N}}{=} n$$

and

$$f_{max} : \prod_{n:\mathbb{N}} \mathrm{Fin}(n+1)$$

$$f_{max}(n) :\equiv \cancel{\ } \, (n, 0, \mathrm{refl}_n)$$

note that this is well-typed:

$$\mathrm{add}(0,n) \equiv \mathrm{rec}_{\mathbb{N}}(\mathbb{N} \to \mathbb{N}, \lambda n.n, \lambda n.\lambda g.\lambda m.\, \mathrm{succ}(g(m)), 0)(n)$$

$$\equiv (\lambda n.n)(n)$$

$$\equiv n$$

so $\mathrm{refl}_n : n \overset{\mathbb{N}}{=} n \equiv \mathrm{refl}_n : \mathrm{add}(0,n) \overset{\mathbb{N}}{=} n$.

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