Generating Patterns with Continuously Varying Symmetry

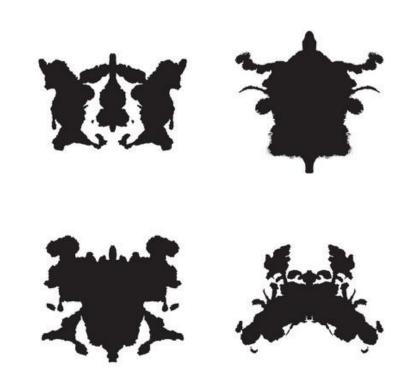
Felix Dilke

Genesis of an idea

A while ago, I wrote a Clojure app that generated Rorschach blots.

These are basically ink splatters forcibly modified to give them a single vertical axis of reflection symmetry. The results were visually interesting and sometimes NSFW.

Now, in 2019, we can do better...







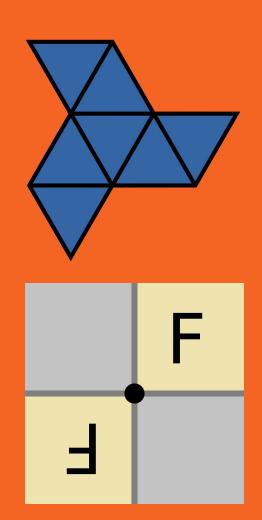
Idea

What about a *continuously varying image* (i.e. an animation) that displays *variable symmetry?*

It could change dynamically from having vertical symmetry...

...to rotational symmetry...

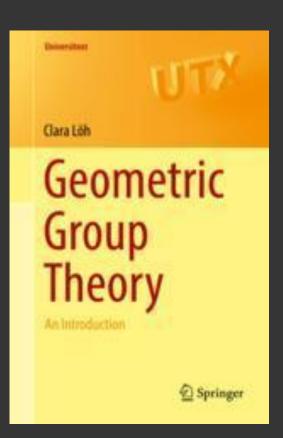
...and maybe other types



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How would you do that?

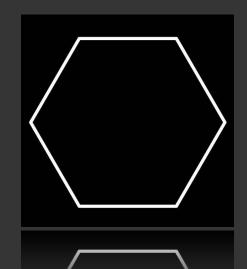
Well... (surprise!) there is some math involved.



The starting point:

In the context of kaleidoscope-type patterns,

let's consider symmetries of a hexagon:





Or, equivalently, of a snowflake. But the hexagon is simpler.

By a 'symmetry' of the hexagon, we mean:

A *rigid transformation* of the 2D plane into itself that preserves the hexagon.

This includes any combination of rotations and reflections.

How many ways are there to do this? We can send any edge uniquely into any other edge, with a specified orientation, so...

There are 12 symmetries. What's more:

They form a sort of number system, because you can multiply any 2 symmetries and get another one. There is also a way to take an inverse of any element! So we can do * and /.

There is also a do-nothing symmetry (called 1) that leaves everything fixed.

This kind of structure is called a *group*.

The symmetries of any object form a group, so group theory gives quite a powerful and expressive language for talking about symmetry.

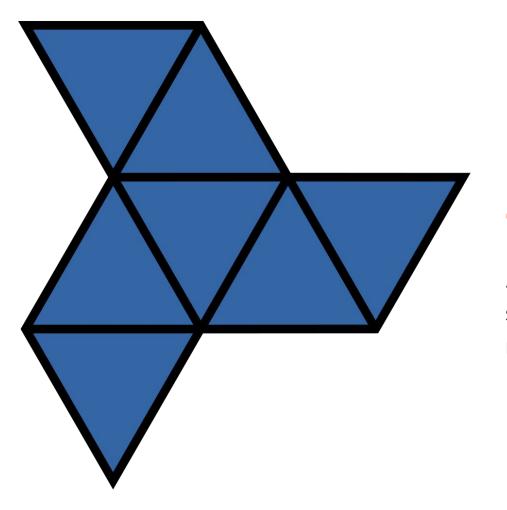
How does this "language" help?

The various types of symmetry correspond to *subgroups* of the full group of 12 symmetries of the hexagon. For example...



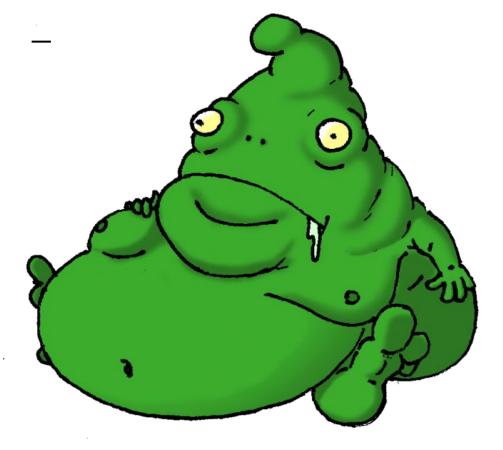
Vertical reflection symmetry

... corresponds to a 2-element subgroup { 1, R } consisting of just the do-nothing symmetry, 1, and a reflection. We have R * R = 1!



Rotation symmetry (of order 3)

... corresponds to a 3-element subgroup { 1, S, S * S } generated by a rotation S. We have S * S * S = 1!



An amorphous, asymmetric blob with no structure at all

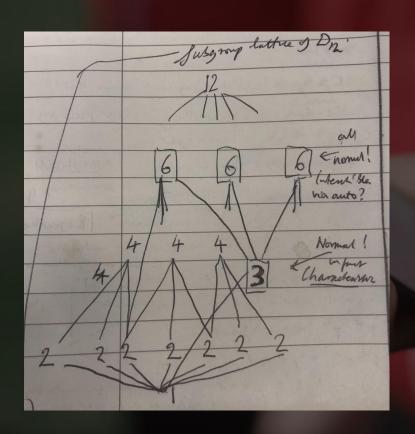
... corresponds to the 1-element subgroup { 1 } of 1 on its own.



A beautiful, symmetric snowflake

... corresponds to the (also trivial) subgroup of, er, the whole group.

So, I wrote an app to calculate all the subgroups.



There are 16!

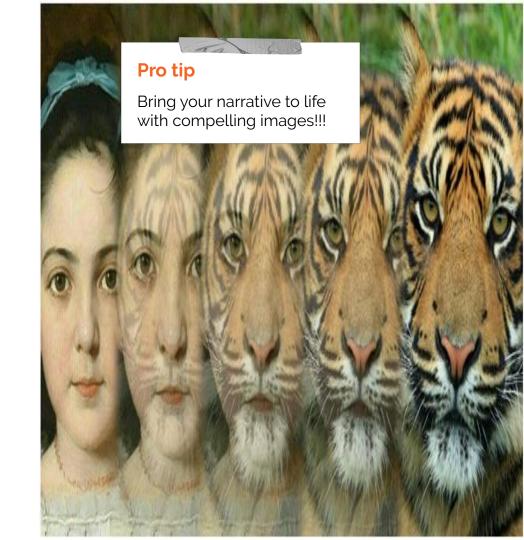
They actually form another algebraic structure of their own, because you can take the ∩ and ∪ of any two subgroups and get another one.

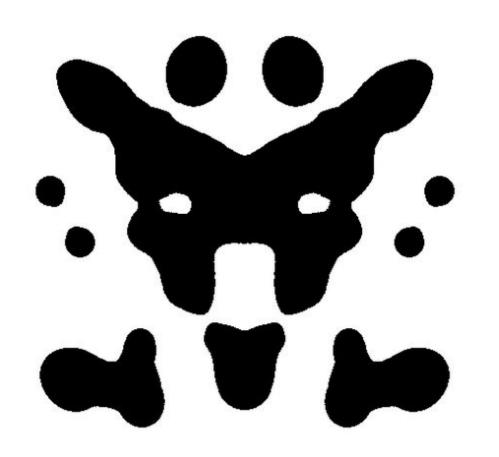
This is the subgroup lattice

Now we can talk about continuously varying symmetry

Suppose H and K are subgroups somewhere in this lattice, with H <= K.

Then there is a natural way to morph H-symmetry continuously into K-symmetry...





Example: the blot

is just SH, for some set S, where H = { 1, R } is a subgroup generated by one reflection. This is automatically H-invariant, i.e. has the required symmetry!

Same for any other H.

So, we have to morph SH into SK, where H <= K.

I worked out a spiffy formula to do this

It even fits on a Post-It™

The trick is to write K as a union of H-invariant subsets, and group theory provides a neat way to do this (coset decomposition).

Please don't be impressed, there is amazingly little going on here.

$$H \leq K$$

$$K = k, H u ... u k_n H$$

$$SH \xrightarrow{D_{i,i,j}} SK$$

$$2h \rightarrow {}^{n} \left\{ \lambda sh + (1-\lambda) sk_i H \right\}$$

Digression

When group theory was applied to atomic physics in the 1920s, many German physicists developed a lasting antipathy to the subject.

They called it...

Gruppenpest

Pro tip

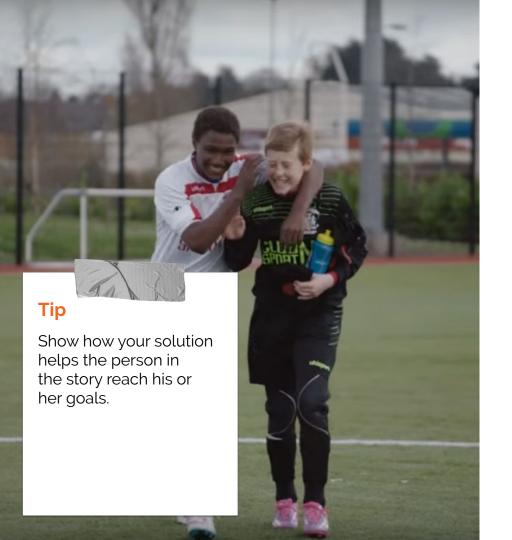
Presentations become more engaging when they use *human interest*. Try to jazz up your dry exposition with colourful period details which will engage and delight the audience!!!

Meet Marcos.

He recently opened a camera shop near the Louvre in Paris.

Visitors to his store, mostly tourists, speak many different languages making anything beyond a simple transaction a challenge.





A simple gesture

Coaches Gary and Glen knew no Spanish.

They used Google Translate to invite Alberto to join in... "Do you want to play?"... "Can you defend the left side?"



3. Examples

People need to understand how rare or frequent your examples are.

Pick 1 or 2 statistics and make them as concrete as possible. Stats are generally not sticky, but here are a few tactics:

→ Relate

Deliver data within the context of a story you've already told

→ Compare

Make big numbers digestible by putting them in the context of something familiar _

It's no surprise Marcos uses Google Translate in his shop regularly.

There are 23 officially recognized languages in the EU.



Tip

Don't let data stand alone. Always relate it back to a story you've already told, in this case, Marco's shop.

Source: theguardian.com

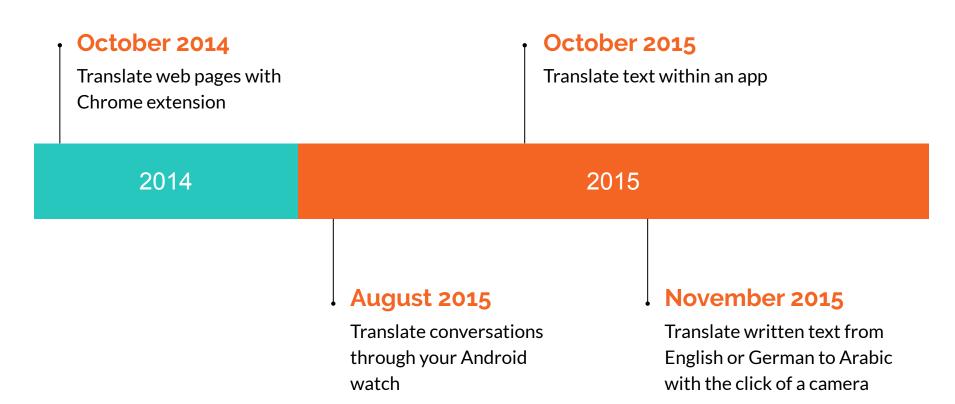
More than 50 million Americans travelled abroad in 2015

THAT'S MORE THAN THE POPULATION OF CALIFORNIA AND TEXAS COMBINED



Source: travel.trade.gov

Milestones



What people are saying

With this app, I'm confident to plan a trip to rural Vietnam

Wendy Writer, CA

Visual translation feels like magic

Ronny Reader, NYC

Translate has officially inspired me to learn French

Abby Author, NYC

Know a 2nd language? Make Google Translate even better by joining qiT the community. Inspire your audience to

act on the information they just learned.

Depending on your idea, this can be anything from downloading an app to joining an organization.



Good luck!

We hope you'll use these tips to go out and deliver a memorable pitch for your product or service!

For more (free) presentation tips relevant to other types of messages, go to heathbrothers.com/presentations

For more about making your ideas stick with others, check out our book!

