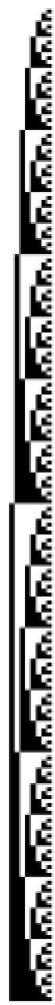
Exploring Binary

Visualizing Consecutive Binary Integers

By Rick Regan July 2nd, 2009

The following is a visual depiction of the binary integers 0 through 11111111:



A nice pattern, right? I generated it based on the image found on page 117 of Stephen Wolfram's "A New Kind of Science". I'll discuss its structure in detail in this article.

A Visual Table of Binary Integers

If you were to list the first 16 nonnegative binary integers in a table like this,

8	4	2	1	
			0	0
			1	1
		1	0	2
		1	1	3
	1	0	0	4
	1	0	1	5
	1	1	0	6
	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

Binary Integers 0-1111

the 0s and 1s would form a pattern, although it may be hard to see.

If you were to simply color in the 1 cells, the pattern would become much more obvious:

8	4	2	1	
			0	0
			1	1
		1	0	2
		1	1	3
	1	0	0	4
	1	0	1	5
	1	1	0	6
	1	1	1	7
1	0	0	0	8
1	0	0	1	9
1	0	1	0	10
1	0	1	1	11
1	1	0	0	12
1	1	0	1	13
1	1	1	0	14
1	1	1	1	15

Binary Integers 0-1111

If you were to extend the table, the pattern would become more obvious still. Here it is extended through 11111111, as depicted in the opening of this article:

128	64	32	16	8	4	2	1	
							0	0
							1	1
						1	0	2
						1	1	3
					1	0	0	4
					1	0	1	5
					1	1	0	6
					1	1	1	7
				1	0	0	0	8
				1	0	0	1	9
				1	0	1	0	10
				1	0	1	1	11
				1	1	0	0	12
				1	1	0	1	13
				1	1	1	0	14
				1	1	1	1	15
			1	0	0	0	0	16
			1	0	0	0	1	17
			1	0	0	1	0	18
			1	0	0	1	1	19
			1	0	1	0	0	20
			1	0	1	0	1	21
			1	0	1	1	0	22
			1	0	1	1	1	23
			1	1	0	0	0	24
			1	1	0	0	1	25
			1	1	0	1	0	26
			1	1	0	1	1	27
			1	1	1	0	0	28
			1	1	1	0	1	29
			1	1	1	1	0	30
			1	1	1	1	1	31
		1	0	0	0	0	0	32
		1	0	0	0	0	1	33
		1	0	0	0	1	0	34

		1	0	0	0	1	1	35
		1	0	0	1	0	0	36
		1	0	0	1	0	1	37
		1	0	0	1	1	0	38
		1	0	0	1	1	1	39
		1	0	1	0	0	0	40
		1	0	1	0	0	1	41
		1	0	1	0	1	0	42
		1	0	1	0	1	1	43
		1	0	1	1	0	0	44
		1	0	1	1	0	1	45
		1	0	1	1	1	0	46
		1	0	1	1	1	1	47
		1	1	0	0	0	0	48
		1	1	0	0	0	1	49
		1	1	0	0	1	0	50
		1	1	0	0	1	1	51
		1	1	0	1	0	0	52
		1	1	0	1	0	1	53
		1	1	0	1	1	0	54
		1	1	0	1	1	1	55
		1	1	1	0	0	0	56
		1	1	1	0	0	1	57
		1	1	1	0	1	0	58
		1	1	1	0	1	1	59
		1	1	1	1	0	0	60
		1	1	1	1	0	1	61
		1	1	1	1	1	0	62
		1	1	1	1	1	1	63
	1	0	0	0	0	0	0	64
	1	0	0	0	0	0	1	65
	1	0	0	0	0	1	0	66
	1	0	0	0	0	1	1	67
	1	0	0	0	1	0	0	68
	1	0	0	0	1	0	1	69
	1	0	0	0	1	1	0	70
	1	0	0	0	1	1	1	71
	1	0	0	1	0	0	0	72
	1	0	0	1	0	0	1	73
	1	0	0	1	0	1	0	74
	1	0	0	1	0	1	1	75
	1	0	0	1	1	0	0	76
	1	0	0	1	1	0	1	77
	1	0	0	1	1	1	0	78
	1	0	0	1	1	1	1	79
	1	0	1	0	0	0	0	80
	1	0	1	0	0	0	1	81
	1	0	1	0	0	1	0	82
	1	0	1	0	0	1	1	83
	1	0	1	0	1	0	0	84
	1	0	1	0	1	0	1	85
	1	0	1	0	1	1	0	86
	1	0	1	0	1	1	1	87
	1	0	1	1	0	0	0	88
	1	0	1	1	0	0	1	89
	1	0	1	1	0	1	0	90
	1	0	1	1	0	1	1	91
	1	0	1	1	1	0	0	92

	1	0	1	1	1	0	1	93
	1	0	1	1	1	1	0	94
	1	0	1	1	1	1	1	95
	1	1	0	0	0	0	0	96
	1	1	0	0	0	0	1	97
	1	1	0	0	0	1	0	98
	1	1	0	0	0	1	1	99
	1	1	0	0	1	0	0	100
	1	1	0	0	1	0	1	101
	1	1	0	0	1	1	0	102
	1	1	0	0	1	1	1	103
	1	1	0	1	0	0	0	104
	1	1	0	1	0	0	1	105
	1	1	0	1	0	1	0	106
	1	1	0	1	0	1	1	107
	1	1	0	1	1	0	0	108
	1	1	0	1	1	0	1	109
	1	1	0	1	1	1	0	110
	1	1	0	1	1	1	1	111
	1	1	1	0	0	0	0	112
	1	1	1	0	0	0	1	113
	1	1	1	0	0	1	0	114
	1	1	1	0	0	1	1	115
	1	1	1	0	1	0	0	116
	1	1	1	0	1	0	1	117
	1	1	1	0	1	1	0	118
	1	1	1	0	1	1	1	119
	1	1	1	1	0	0	0	120
	1	1	1	1	0	0	1	121
	1	1	1	1	0	1	0	122
	1	1	1	1	0	1	1	123
	1	1	1	1	1	0	0	124
	1	1	1	1	1	0	1	125
	1	1	1	1	1	1	0	126
	1	1	1	1	1	1	1	127
1	0	0	0	0	0	0	0	128
1	0	0	0	0	0	0	1	129
1	0	0	0	0	0	1	0	130
1	0	0	0	0	0	1	1	131
1	0	0	0	0	1	0	0	132
1	0	0	0	0	1	0	1	133
1	0	0	0	0	1	1	0	134
1	0	0	0	0	1	1	1	135
1	0	0	0	1	0	0	0	136
1	0	0	0	1	0	0	1	137
1	0	0	0	1	0	1	0	138
1	0	0	0	1	0	1	1	139
1	0	0	0	1	1	0	0	140
1	0	0	0	1	1	0	1	141
1	0	0	0	1	1	1	0	142
1	0	0	0	1	1	1	1	143
1	0	0	1	0	0	0	0	144
1	0	0	1	0	0	0	1	145
1	0	0	1	0	0	1	0	146
1	0	0	1	0	0	1	1	147
1	0	0	1	0	1	0	0	148
1	0	0	1	0	1	0	1	149
1	0	0	1	0	1	1	0	150

1	0	0	1	0	1	1	1	151
1	0	0	1	1	0	0	0	152
1	0	0	1	1	0	0	1	153
1	0	0	1	1	0	1	0	154
1	0	0	1	1	0	1	1	155
1	0	0	1	1	1	0	0	156
1	0	0	1	1	1	0	1	157
1	0	0	1	1	1	1	0	158
1	0	0	1	1	1	1	1	159
1	0	1	0	0	0	0	0	160
1	0	1	0	0	0	0	1	161
1	0	1	0	0	0	1	0	162
1	0	1	0	0	0	1	1	163
1	0	1	0	0	1	0	0	164
1	0	1	0	0	1	0	1	165
1	0	1	0	0	1	1	0	166
1	0	1	0	0	1	1	1	167
1	0	1	0	1	0	0	0	168
1	0	1	0	1	0	0	1	169
1	0	1	0	1	0	1	0	170
1	0	1	0	1	0	1	1	171
1	0	1	0	1	1	0	0	172
1	0	1	0	1	1	0	1	173
1	0	1	0	1	1	1	0	174
1	0	1	0	1	1	1	1	175
1	0	1	1	0	0	0	0	176
1	0	1	1	0	0	0	1	177
1	0	1	1	0	0	1	0	178
1	0	1	1	0	0	1	1	179
1	0	1	1	0	1	0	0	180
1	0	1	1	0	1	0	1	181
1	0	1	1	0	1	1	0	182
1	0	1	1	0	1	1	1	183
1	0	1	1	1	0	0	0	184
1	0	1	1	1	0	0	1	185
1	0	1	1	1	0	1	0	186
1	0	1	1	1	0	1	1	187
1	0	1	1	1	1	0	0	188
1	0	1	1	1	1	0	1	189
1	0	1	1	1	1	1	0	190
1	0	1	1	1	1	1	1	191
1	1	0	0	0	0	0	0	192
1	1	0	0	0	0	0	1	193
1	1	0	0	0	0	1	0	194
1	1	0	0	0	0	1	1	195
1	1	0	0	0	1	0	0	196
1	1	0	0	0	1	0	1	197
1	1	0	0	0	1	1	0	198
1	1	0	0	0	1	1	1	199
1	1	0	0	1	0	0	0	200
1	1	0	0	1	0	0	1	201
1	1	0	0	1	0	1	0	202
1	1	0	0	1	0	1	1	203
1	1	0	0	1	1	0	0	204
1	1	0	0	1	1	0	1	205
1	1	0	0	1	1	1	0	206
1	1	0	0	1	1	1	1	207

1	1	0	1	0	0	0	0	208
1	1	0	1	0	0	0	1	209
1	1	0	1	0	0	1	0	210
1	1	0	1	0	0	1	1	211
1	1	0	1	0	1	0	0	212
1	1	0	1	0	1	0	1	213
1	1	0	1	0	1	1	0	214
1	1	0	1	0	1	1	1	215
1	1	0	1	1	0	0	0	216
1	1	0	1	1	0	0	1	217
1	1	0	1	1	0	1	0	218
1	1	0	1	1	0	1	1	219
1	1	0	1	1	1	0	0	220
1	1	0	1	1	1	0	1	221
1	1	0	1	1	1	1	0	222
1	1	0	1	1	1	1	1	223
1	1	1	0	0	0	0	0	224
1	1	1	0	0	0	0	1	225
1	1	1	0	0	0	1	0	226
1	1	1	0	0	0	1	1	227
1	1	1	0	0	1	0	0	228
1	1	1	0	0	1	0	1	229
1	1	1	0	0	1	1	0	230
1	1	1	0	0	1	1	1	231
1	1	1	0	1	0	0	0	232
1	1	1	0	1	0	0	1	233
1	1	1	0	1	0	1	0	234
1	1	1	0	1	0	1	1	235
1	1	1	0	1	1	0	0	236
1	1	1	0	1	1	0	1	237
1	1	1	0	1	1	1	0	238
1	1	1	0	1	1	1	1	239
1	1	1	1	0	0	0	0	240
1	1	1	1	0	0	0	1	241
1	1	1	1	0	0	1	0	242
1	1	1	1	0	0	1	1	243
1	1	1	1	0	1	0	0	244
1	1	1	1	0	1	0	1	245
1	1	1	1	0	1	1	0	246
1	1	1	1	0	1	1	1	247
1	1	1	1	1	0	0	0	248
1	1	1	1	1	0	0	1	249
1	1	1	1	1	0	1	0	250
1	1	1	1	1	0	1	1	251
1	1	1	1	1	1	0	0	252
1	1	1	1	1	1	0	1	253
1	1	1	1	1	1	1	0	254
1	1	1	1	1	1	1	1	255

Binary Integers 0–11111111

Powers of Two Sized Patterns

The overall pattern in the table is based on vertical and horizontal sub patterns. Each sub pattern is based on nonnegative power of two sized units, which is a consequence of individual binary numbers being made up of powers of two.

Vertical Patterns

The basic pattern in the table is vertical, in the columns. Each column alternates between 2^n 0s (blanks are considered 0s) and 2^n 1s, with 2^n being the value of that column's place. This gives 2^{n+1} sized blocks of 0s followed by 1s. For example, the pattern in the ones column is 01; the pattern in the twos column is 0011; the pattern in the fours column is 00001111; etc.

Here are the patterns isolated for the ones through sixteens column:

16	8	4	2	1	
				0	0
				1	1
			1	0	2
			1	1	3
		1	0	0	4
		1	0	1	5
		1	1	0	6
		1	1	1	7
	1	0	0	0	8
	1	0	0	1	9
	1	0	1	0	10
	1	0	1	1	11
	1	1	0	0	12
	1	1	0	1	13
	1	1	1	0	14
	1	1	1	1	15
1	0	0	0	0	16
1	0	0	0	1	17
1	0	0	1	0	18
1	0	0	1	1	19
1	0	1	0	0	20
1	0	1	0	1	21
1	0	1	1	0	22
1	0	1	1	1	23
1	1	0	0	0	24
1	1	0	0	1	25
1	1	0	1	0	26
1	1	0	1	1	27
1	1	1	0	0	28
1	1	1	0	1	29
1	1	1	1	0	30
1	1	1	1	1	31

Patterns in First 5 Columns of Table

These patterns explain the toggle frequency of a binary counter, as discussed in my article [What a Binary Counter Looks and Sounds Like](https://www.exploringbinary.com/).

Horizontal Patterns

The predominant pattern in the table is horizontal, in power of two sized groups of rows. The beauty in this pattern is the nesting. The first two rows start the pattern; the next two rows repeat the first two rows and then prefix them with a 1; the next four rows repeat the prior four rows (with leading 0s filled in first) and then prefix them with a 1; etc.

Here are the patterns isolated for the first five groups of rows:

128	64	32	16	8	4	2	1	
							0	0
							1	1
						1	0	2
						1	1	3
					1	0	0	4
					1	0	1	5
					1	1	0	6
					1	1	1	7
				1	0	0	0	8
				1	0	0	1	9
				1	0	1	0	10
				1	0	1	1	11
				1	1	0	0	12
				1	1	0	1	13
				1	1	1	0	14
				1	1	1	1	15
			1	0	0	0	0	16
			1	0	0	0	1	17
			1	0	0	1	0	18
			1	0	0	1	1	19
			1	0	1	0	0	20
			1	0	1	0	1	21
			1	0	1	1	0	22
			1	0	1	1	1	23
			1	1	0	0	0	24
			1	1	0	0	1	25
			1	1	0	1	0	26
			1	1	0	1	1	27
			1	1	1	0	0	28
			1	1	1	0	1	29
			1	1	1	1	0	30
			1	1	1	1	1	31

Patterns in First 32 Rows of Table

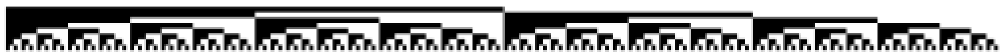
In other words, each new group of rows is a copy of all prior rows with a leading 1 attached. Expressed mathematically, the group of rows 2^n to $2^{n+1} - 1$ is the group of rows 0 to $2^n - 1$ with 2^n added to the number in each row.

Horizontal Symmetry

If you fold the table on itself at its halfway point — for the 256 entry table, this is at the border between decimal numbers 127 and 128 — each half will be the negative, or complement, or "dual" of the other. That is, they are the same, except white and black, or 0s and 1s, are interchanged.

Related Binary Art

Ivars Peterson analyzes a piece of artwork by Arlene Stamp called the Binary Frieze. The Binary Frieze incorporates the imagery above, but with two minor differences: black and white are interchanged, and the table is transposed so that numbers are in columns instead of rows:



Binary Frieze Imagery

This image does not appear exactly this way in the Binary Frieze, but is cut up and transformed into a piece of art. Here are three pictures of it: Binary Frieze (Stretch), Binary Frieze (Squeeze), and Binary Frieze (Swing).

As for black and white being interchanged, here's a different interpretation: perhaps it represents a *countdown* from 255 to 0.

Related

- Decimal/Binary Conversion Table
- What a Binary Counter Looks and Sounds Like

Get articles by RSS (What Is RSS?)

Get articles by e-mail

Binary numbers

3 comments

1. **Arlene Stamp**

   July 3, 2009 at 10:30 am

   I can see that you enjoyed finding this pattern as much as I did back in 1986, in Vancouver, when I was trying to understand the number system on which computers are based.

   The big revelation for me at that time was the fractal nature of this pattern. Finding this out seemed to me to point to the inevitability of the fractal look of the nonperiodic patterns being developed by Mandelbrott using the computer.

2. **mirabilos**

   November 18, 2011 at 3:46 pm

   This occurs in the Epson FX-80 printer manual. And I think in the manual of the President Printer 6320.

   Both 9-pin dot-matrix printers I own and owned, respectively. They use the horizontal version of course, for ESC K graphics (in the ESC/P programming language).

3. **Rick Regan**

   November 18, 2011 at 8:55 pm

   @mirabilos,

Cool. In what context was the image presented? Were they demonstrating binary numbers, or was it just a cool graphic?

I tried to find the images myself (to no avail). I found these two Epson manuals online (both at http://www.epson.com/cgi-bin/Store/support/supDetail.jsp?infoType=Doc&oid=14285&prodoid=8490):

Epson FX Series Printer User's Manual Volume 1 Tutorial
Epson FX Series Printer User's Manual Volume 2 Reference

I also found this (http://support.epson.ru/products/manuals/000350/part1.pdf): EPSON ESC/P Reference Manual

Do you have a link with the image?

Thanks.

Comments are closed.