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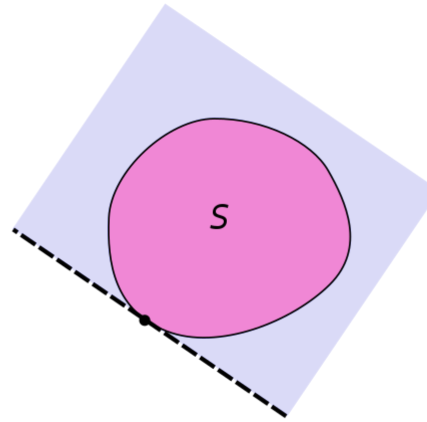
Supporting hyperplane

In geometry, a **supporting hyperplane** of a set S in Euclidean space $\mathbb{R}^n$ is a hyperplane that has both of the following two properties:[1]

- S is entirely contained in one of the two closed half-spaces bounded by the hyperplane,
- S has at least one boundary-point on the hyperplane.

Here, a closed half-space is the half-space that includes the points within the hyperplane.



A convex set S (in pink), a supporting hyperplane of S (the dashed line), and the supporting half-space delimited by the hyperplane which contains S (in light blue).

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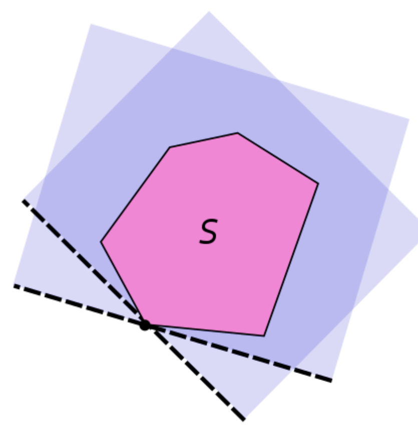
Supporting hyperplane theorem

This theorem states that if S is a convex set in the topological vector space $X = \mathbb{R}^n$, and x_0 is a point on the boundary of S, then there exists a supporting hyperplane containing x_0. If $x^* \in X^* \backslash \{0\}$ (X^* is the dual space of X, x^* is a nonzero linear functional) such that $x^*(x_0) \geq x^*(x)$ for all $x \in S$, then

$$H = \{x \in X : x^*(x) = x^*(x_0)\}$$

defines a supporting hyperplane.[2]



A convex set can have more than one supporting hyperplane at a given point on its boundary.

Conversely, if S is a closed set with nonempty interior such that every point on the boundary has a supporting hyperplane, then S is a convex set.[2]

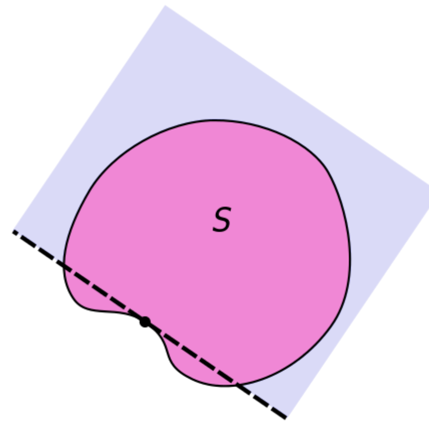
The hyperplane in the theorem may not be unique, as noticed in the second picture on the right. If the closed set S is not convex, the statement of the theorem is not true at all points on the boundary of S, as illustrated in the third picture on the right.

The supporting hyperplanes of convex sets are also called **tac-planes** or **tac-hyperplanes**.[3]

A related result is the separating hyperplane theorem, that every two disjoint convex sets can be separated by a hyperplane.

See also

- Support function
- Supporting line (supporting hyperplanes in $\mathbb{R}^2$)



A supporting hyperplane containing a given point on the boundary of S may not exist if S is not convex.

Notes

1. Luenberger, David G. (1969). *Optimization by Vector Space Methods* (https://books.google.com/books?id=lZU0CAH4RccC&pg=PA133). New York: John Wiley & Sons. p. 133. ISBN 978-0-471-18117-0.
2. Boyd, Stephen P.; Vandenberghe, Lieven (2004). *Convex Optimization* (https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf#page=64) (pdf). Cambridge University Press. pp. 50–51. ISBN 978-0-521-83378-3. Retrieved October 15, 2011.
3. Cassels, John W. S. (1997), *An Introduction to the Geometry of Numbers*, Springer Classics in Mathematics (reprint of 1959[3] and 1971 Springer-Verlag ed.), Springer-Verlag.

References & further reading

- Ostaszewski, Adam (1990). *Advanced mathematical methods* (https://archive.org/details/advancedmathemat0000osta). Cambridge; New York: Cambridge University Press. p. 129 (https://archive.org/details/advancedmathemat0000osta/page/129). ISBN 0-521-28964-5.
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