

Minkowski's first inequality for convex bodies

In mathematics, **Minkowski's first inequality for convex bodies** is a geometrical result due to the German mathematician Hermann Minkowski. The inequality is closely related to the Brunn–Minkowski inequality and the isoperimetric inequality.

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Statement of the inequality

Let K and L be two n -dimensional convex bodies in n -dimensional Euclidean space \mathbf{R}^n . Define a quantity $V_1(K, L)$ by

$$nV_1(K, L) = \lim_{\varepsilon \downarrow 0} \frac{V(K + \varepsilon L) - V(K)}{\varepsilon},$$

where V denotes the n -dimensional Lebesgue measure and $+$ denotes the Minkowski sum. Then

$$V_1(K, L) \geq V(K)^{(n-1)/n} V(L)^{1/n},$$

with equality if and only if K and L are homothetic, i.e. are equal up to translation and dilation.

Remarks

- V_1 is just one example of a class of quantities known as mixed volumes.
- If L is the n -dimensional unit ball B , then $n V_1(K, B)$ is the $(n - 1)$ -dimensional surface measure of K , denoted $S(K)$.

Connection to other inequalities

The Brunn–Minkowski inequality

One can show that the Brunn–Minkowski inequality for convex bodies in \mathbf{R}^n implies Minkowski's first inequality for convex bodies in \mathbf{R}^n , and that equality in the Brunn–Minkowski inequality implies equality in Minkowski's first inequality.

The isoperimetric inequality

By taking $L = B$, the n -dimensional unit ball, in Minkowski's first inequality for convex bodies, one obtains the isoperimetric inequality for convex bodies in \mathbf{R}^n : if K is a convex body in \mathbf{R}^n , then

$$\left(\frac{V(K)}{V(B)}\right)^{1/n} \leq \left(\frac{S(K)}{S(B)}\right)^{1/(n-1)},$$

with equality if and only if K is a ball of some radius.

References

- Gardner, Richard J. (2002). "The Brunn–Minkowski inequality" (<https://doi.org/10.1090%2FS0273-0979-02-00941-2>). *Bull. Amer. Math. Soc. (N.S.)*. **39** (3): 355–405 (electronic). doi:10.1090/S0273-0979-02-00941-2 (<https://doi.org/10.1090%2FS0273-0979-02-00941-2>).

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