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# Mazur's lemma

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In mathematics, **Mazur's lemma** is a result in the theory of Banach spaces. It shows that any weakly convergent sequence in a Banach space has a sequence of convex combinations of its members that converges strongly to the same limit, and is used in the proof of Tonelli's theorem.

## Statement of the lemma

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Let  $(X, \| \cdot \|)$  be a Banach space and let  $(u_n)_{n \in \mathbb{N}}$  be a sequence in  $X$  that converges weakly to some  $u_0$  in  $X$ :

$$u_n \rightharpoonup u_0 \text{ as } n \rightarrow \infty.$$

That is, for every continuous linear functional  $f \in X'$ , the continuous dual space of  $X$ ,

$$f(u_n) \rightarrow f(u_0) \text{ as } n \rightarrow \infty.$$

Then there exists a function  $N : \mathbb{N} \rightarrow \mathbb{N}$  and a sequence of sets of real numbers

$$\{\alpha(n)_k : k = n, \dots, N(n)\}$$

such that  $\alpha(n)_k \geq 0$  and

$$\sum_{k=n}^{N(n)} \alpha(n)_k = 1$$

such that the sequence  $(v_n)_{n \in \mathbb{N}}$  defined by the convex combination

$$v_n = \sum_{k=n}^{N(n)} \alpha(n)_k u_k$$

converges strongly in  $X$  to  $u_0$ ; that is

$$\|v_n - u_0\| \rightarrow 0 \text{ as } n \rightarrow \infty.$$

## See also

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- Banach–Alaoglu theorem – The closed unit ball in the dual of a normed vector space is compact in the weak\* topology
- Bishop–Phelps theorem

- Eberlein–Šmulian theorem – Relates three different kinds of weak compactness in a Banach space
- James's theorem
- Goldstine theorem

## References

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- Renardy, Michael & Rogers, Robert C. (2004). *An introduction to partial differential equations*. Texts in Applied Mathematics 13 (Second ed.). New York: Springer-Verlag. p. 350. ISBN 0-387-00444-0.
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**This page was last edited on 17 December 2021, at 11:47 (UTC).**

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