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Pseudoconvexity

In <u>mathematics</u>, more precisely in the theory of functions of <u>several complex variables</u>, a **pseudoconvex set** is a special type of <u>open set</u> in the *n*-dimensional complex space \mathbb{C}^n . Pseudoconvex sets are important, as they allow for classification of domains of holomorphy.

Let

$$G\subset \mathbb{C}^n$$

be a domain, that is, an open connected subset. One says that G is pseudoconvex (or <u>Hartogs</u> pseudoconvex) if there exists a continuous plurisubharmonic function φ on G such that the set

$$\{z \in G \mid \varphi(z) < x\}$$

is a <u>relatively compact</u> subset of G for all <u>real numbers</u> x. In other words, a domain is pseudoconvex if G has a continuous plurisubharmonic <u>exhaustion function</u>. Every (geometrically) <u>convex set</u> is pseudoconvex. However, there are pseudoconvex domains which are not geometrically convex.

When G has a C^2 (twice <u>continuously differentiable</u>) <u>boundary</u>, this notion is the same as Levi pseudoconvexity, which is easier to work with. More specifically, with a C^2 boundary, it can be shown that G has a defining function; i.e., that there exists $\rho: \mathbb{C}^n \to \mathbb{R}$ which is C^2 so that $G = \{\rho < 0\}$, and $\partial G = \{\rho = 0\}$. Now, G is pseudoconvex iff for every $p \in \partial G$ and w in the complex tangent space at p, that is,

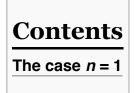
$$abla
ho(p)w=\sum_{i=1}^nrac{\partial
ho(p)}{\partial z_j}w_j=0$$
, we have $\sum_{i,j=1}^nrac{\partial^2
ho(p)}{\partial z_i\partialar{z}_j}w_iar{w}_j\geq 0.$

If G does not have a C^2 boundary, the following approximation result can be useful.

Proposition 1 If G is pseudoconvex, then there exist <u>bounded</u>, strongly Levi pseudoconvex domains $G_k \subset G$ with C^{∞} (smooth) boundary which are relatively compact in G, such that

$$G = igcup_{k=1}^{\infty} G_k.$$

This is because once we have a φ as in the definition we can actually find a C^{∞} exhaustion function.



See also References

External links

The case n = 1

In one complex dimension, every open domain is pseudoconvex. The concept of pseudoconvexity is thus more useful in dimensions higher than 1.

See also

- Holomorphically convex hull
- Stein manifold
- Analytic polyhedron
- Eugenio Elia Levi

References

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- Steven G. Krantz. *Function Theory of Several Complex Variables*, AMS Chelsea Publishing, Providence, Rhode Island, 1992.

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External links

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- "Pseudo-convex and pseudo-concave" (https://www.encyclopediaofmath.org/index.php?title=Pseudo-convex_and_pseudo-concave), *Encyclopedia of Mathematics*, EMS Press, 2001 [1994]

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