WIKIPEDIA

Convex geometry

In mathematics, **convex geometry** is the branch of geometry studying convex sets, mainly in Euclidean space. Convex sets occur naturally in many areas: computational geometry, convex analysis, discrete geometry, functional analysis, geometry of numbers, integral geometry, linear programming, probability theory, game theory, etc.

Contents

Classification

According to the Mathematics Subject Classification MSC2010,[1] the mathematical discipline *Convex and Discrete Geometry* includes three major branches:[2]

- general convexity
- polytopes and polyhedra
- discrete geometry

(though only portions of the latter two are included in convex geometry).

General convexity is further subdivided as follows:[3]

- axiomatic and generalized convexity
- convex sets without dimension restrictions
- convex sets in topological vector spaces
- convex sets in 2 dimensions (including convex curves)
- convex sets in 3 dimensions (including convex surfaces)
- convex sets in *n* dimensions (including convex hypersurfaces)
- finite-dimensional Banach spaces
- random convex sets and integral geometry
- asymptotic theory of convex bodies
- approximation by convex sets
- variants of convex sets (star-shaped, (*m, n*)-convex, etc.)
- Helly-type theorems and geometric transversal theory

- other problems of combinatorial convexity
- length, area, volume
- mixed volumes and related topics
- valuations on convex bodies
- inequalities and extremum problems
- convex functions and convex programs
- spherical and hyperbolic convexity

The term *convex geometry* is also used in combinatorics as an alternate name for an antimatroid, which is one of the abstract models of convex sets.

Historical note

Convex geometry is a relatively young mathematical discipline. Although the first known contributions to convex geometry date back to antiquity and can be traced in the works of Euclid and Archimedes, it became an independent branch of mathematics at the turn of the 20th century, mainly due to the works of Hermann Brunn and Hermann Minkowski in dimensions two and three. A big part of their results was soon generalized to spaces of higher dimensions, and in 1934 T. Bonnesen and W. Fenchel gave a comprehensive survey of convex geometry in Euclidean space $\mathbf{R}^n$. Further development of convex geometry in the 20th century and its relations to numerous mathematical disciplines are summarized in the *Handbook of convex geometry* edited by P. M. Gruber and J. M. Wills.

See also

- List of convexity topics

Notes

1. Website of Mathematics Subject Classification MSC2010 (http://www.msc2010.org/mscwiki/index.php?title=MSC2010)
2. Mathematics Subject Classification MSC2010, entry 52 "Convex and discrete geometry" (http://www.msc2010.org/mscwiki/index.php?title=52-XX)
3. Mathematics Subject Classification MSC2010, entry 52A "General convexity" (http://www.msc2010.org/mscwiki/index.php?title=52Axx)

References

Expository articles on convex geometry

- K. Ball, *An elementary introduction to modern convex geometry,* in: Flavors of Geometry, pp. 1–58, Math. Sci. Res. Inst. Publ. Vol. 31, Cambridge Univ. Press, Cambridge, 1997, available online (http://www.msri.org/publications/books/Book31/files/ball.pdf).
- M. Berger, *Convexity,* Amer. Math. Monthly, Vol. 97 (1990), 650—678. DOI: 10.2307/2324573 (https://dx.doi.org/10.2307/2324573)
- P. M. Gruber, *Aspects of convexity and its applications,* Exposition. Math., Vol. 2 (1984), 47—83.

- V. Klee, *What is a convex set?* Amer. Math. Monthly, Vol. 78 (1971), 616—631, DOI: 10.2307/2316569 (https://dx.doi.org/2010.2307/2316569)

Books on convex geometry

- T. Bonnesen, W. Fenchel, *Theorie der konvexen Körper,* Julius Springer, Berlin, 1934. English translation: *Theory of convex bodies,* BCS Associates, Moscow, ID, 1987.
- R. J. Gardner, *Geometric tomography,* Cambridge University Press, New York, 1995. Second edition: 2006.
- P. M. Gruber, *Convex and discrete geometry,* Springer-Verlag, New York, 2007.
- P. M. Gruber, J. M. Wills (editors), *Handbook of convex geometry. Vol. A. B,* North-Holland, Amsterdam, 1993.
- G. Pisier, *The volume of convex bodies and Banach space geometry,* Cambridge University Press, Cambridge, 1989.
- R. Schneider, *Convex bodies: the Brunn-Minkowski theory,* Cambridge University Press, Cambridge, 1993; Second edition: 2014.
- A. C. Thompson, *Minkowski geometry,* Cambridge University Press, Cambridge, 1996.
- A. Koldobsky, V. Yaskin, *The Interface between Convex Geometry and Harmonic Analysis,* American Mathematical Society, Providence, Rhode Island, 2008.

Articles on history of convex geometry

- W. Fenchel, *Convexity through the ages,* (Danish) Danish Mathematical Society (1929—1973), pp. 103–116, Dansk. Mat. Forening, Copenhagen, 1973. English translation: *Convexity through the ages,* in: P. M. Gruber, J. M. Wills (editors), Convexity and its Applications, pp. 120–130, Birkhauser Verlag, Basel, 1983.
- P. M. Gruber, *Zur Geschichte der Konvexgeometrie und der Geometrie der Zahlen,* in: G. Fischer, et al. (editors), Ein Jahrhundert Mathematik 1890—1990, pp. 421–455, Dokumente Gesch. Math., Vol. 6, F. Wieweg and Sohn, Braunschweig; Deutsche Mathematiker Vereinigung, Freiburg, 1990.
- P. M. Gruber, *History of convexity,* in: P. M. Gruber, J. M. Wills (editors), Handbook of convex geometry. Vol. A, pp. 1–15, North-Holland, Amsterdam, 1993.

External links

- Media related to Convex geometry at Wikimedia Commons