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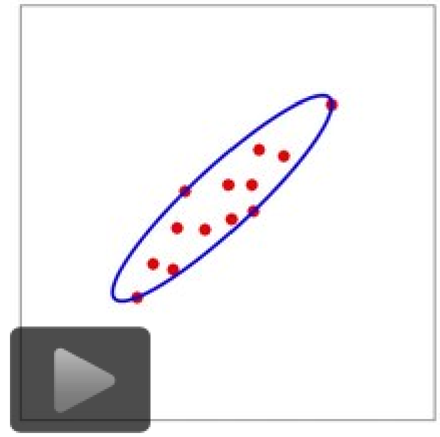
John ellipsoid

In mathematics, the **John ellipsoid** or **Löwner-John ellipsoid** $E(K)$ associated to a convex body K in n-dimensional Euclidean space $\mathbf{R}^n$ can refer to the n-dimensional ellipsoid of maximal volume contained within K or the ellipsoid of minimal volume that contains K.

Often, the minimal volume ellipsoid is called as Löwner ellipsoid, and the maximal volume ellipsoid as the John ellipsoid (although John worked with the minimal volume ellipsoid in its original paper).[1] One also refer to the minimal volume circumscribed ellipsoid as the **outer Löwner-John ellipsoid** and the maximum volume inscribed ellipsoid as the **inner Löwner-John ellipsoid**.[2]



Outer Löwner-John ellipsoid containing a set of a points in $\mathbf{R}^2$

Contents

Properties

The John ellipsoid is named after the German-American mathematician Fritz John, who proved in 1948 that each convex body in $\mathbf{R}^n$ contains a unique circumscribed ellipsoid of minimal volume and that the dilation of this ellipsoid by factor $1/n$ is contained inside the convex body.[3]

The inner Löwner-John ellipsoid $E(K)$ of a convex body $K \subset \mathbf{R}^n$ is a closed unit ball B in $\mathbf{R}^n$ if and only if $B \subseteq K$ and there exists an integer $m \geq n$ and, for $i = 1, ..., m$, real numbers $c_i > 0$ and unit vectors $u_i \in \mathbf{S}^{n-1} \cap \partial K$ such that[4]

$$\sum_{i=1}^{m} c_i u_i = 0$$

and, for all $x \in \mathbf{R}^n$

$$x = \sum_{i=1}^{m} c_i (x \cdot u_i) u_i .$$

Applications

Computing Löwner-John ellipsoids has applications in obstacle collision detection for robotic systems, where the distance between a robot and its surrounding environment is estimated using a best ellipsoid fit.[5]

It also has applications in portfolio optimization with transaction costs.[6]

See also

- Banach–Mazur compactum – Set of n-dimensional subspaces of a normed space made into a compact metric space.
- Steiner inellipse, the special case of the inner Löwner-John ellipsoid for a triangle.
- Fat object, related to radius of largest contained ball.

References

1. Güler, Osman; Gürtuna, Filiz (2012). "Symmetry of convex sets and its applications to the extremal ellipsoids of convex bodies" (http://www.tandfonline.com/doi/abs/10.1080/10556788.2011.626037). *Optimization Methods and Software*. **27** (4–5): 735–759. doi:10.1080/10556788.2011.626037 (https://doi.org/10.1080%2F10556788.2011.626037). ISSN 1055-6788 (https://www.worldcat.org/issn/1055-6788).
2. Ben-Tal, A. (2001). *Lectures on modern convex optimization : analysis, algorithms, and engineering applications* (https://www.worldcat.org/oclc/46538510). Nemirovskiĭ, Arkadiĭ Semenovich. Philadelphia, PA: Society for Industrial and Applied Mathematics. ISBN 0-89871-491-5. OCLC 46538510 (https://www.worldcat.org/oclc/46538510).
3. John, Fritz. "Extremum problems with inequalities as subsidiary conditions". *Studies and Essays Presented to R. Courant on his 60th Birthday*, January 8, 1948, 187—204. Interscience Publishers, Inc., New York, N. Y., 1948. OCLC 1871554 (https://www.worldcat.org/oclc/1871554) MR30135 (https://mathscinet.ams.org/mathscinet-getitem?mr=30135)
4. Ball, Keith M. (1992). "Ellipsoids of maximal volume in convex bodies". *Geom. Dedicata*. **41** (2): 241–250. arXiv:math/9201217 (https://arxiv.org/abs/math/9201217). doi:10.1007/BF00182424 (https://doi.org/10.1007%2FBF00182424). ISSN 0046-5755 (https://www.worldcat.org/issn/0046-5755).
5. Rimon, Elon; Boyd, Stephen (1997). "Obstacle Collision Detection Using Best Ellipsoid Fit". *Journal of Intelligent and Robotic Systems*. **18** (2): 105–126. doi:10.1023/A:1007960531949 (https://doi.org/10.1023%2FA%3A1007960531949).
6. Shen, Weiwei; Wang, Jun (2015). "Transaction costs-aware portfolio optimization via fast Löwner-John ellipsoid approximation" (https://web.archive.org/web/20170116192444/https://pdfs.semanticscholar.org/7b31/2141616f092137c12397a47d11d94ddcea78.pdf) (PDF). *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence (AAAI2015)*: 1854–1860. Archived from the original (https://pdfs.semanticscholar.org/7b31/2141616f092137c12397a47d11d94ddcea78.pdf) (PDF) on 2017-01-16.

- Gardner, Richard J. (2002). "The Brunn-Minkowski inequality" (https://doi.org/10.1090%2FS0273-0979-02-00941-2). *Bull. Amer. Math. Soc. (N.S.)*. **39** (3): 355–405 (electronic). doi:10.1090/S0273-0979-02-00941-2 (https://doi.org/10.1090%2FS0273-0979-02-00941-2). ISSN 0273-0979 (https://www.worldcat.org/issn/0273-0979).