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Minkowski's first inequality for convex bodies

In [mathematics](), **Minkowski's first inequality for convex bodies** is a [geometrical]() result due to the [German]() [mathematician]() [Hermann Minkowski](). The inequality is closely related to the [Brunn–Minkowski inequality]() and the [isoperimetric inequality]().

Contents

Statement of the inequality

Let K and L be two n-[dimensional]() [convex bodies]() in n-dimensional [Euclidean space]() $\mathbf{R}^n$. Define a quantity $V_1(K, L)$ by

$$nV_1(K, L) = \lim_{\varepsilon \downarrow 0} \frac{V(K + \varepsilon L) - V(K)}{\varepsilon},$$

where V denotes the n-dimensional [Lebesgue measure]() and + denotes the [Minkowski sum](). Then

$$V_1(K, L) \geq V(K)^{(n-1)/n} V(L)^{1/n},$$

with equality [if and only if]() K and L are [homothetic](), i.e. are equal up to [translation]() and [dilation]().

Remarks

- V_1 is just one example of a class of quantities known as *mixed volumes*.
- If L is the n-dimensional [unit ball]() B, then $n\,V_1(K, B)$ is the $(n - 1)$-dimensional surface measure of K, denoted $S(K)$.

Connection to other inequalities

The Brunn–Minkowski inequality

One can show that the Brunn–Minkowski inequality for convex bodies in $\mathbf{R}^n$ implies Minkowski's first inequality for convex bodies in $\mathbf{R}^n$, and that equality in the Brunn–Minkowski inequality implies equality in Minkowski's first inequality.

The isoperimetric inequality

By taking $L = B$, the n-dimensional unit ball, in Minkowski's first inequality for convex bodies, one obtains the isoperimetric inequality for convex bodies in $\mathbf{R}^n$: if K is a convex body in $\mathbf{R}^n$, then

$$\left(\frac{V(K)}{V(B)}\right)^{1/n} \leq \left(\frac{S(K)}{S(B)}\right)^{1/(n-1)},$$

with equality if and only if K is a ball of some radius.

References

- Gardner, Richard J. (2002). "The Brunn–Minkowski inequality" (https://doi.org/10.1090%2FS0273-0979-02-00941-2). *Bull. Amer. Math. Soc. (N.S.)*. **39** (3): 355–405 (electronic). doi:10.1090/S0273-0979-02-00941-2 (https://doi.org/10.1090%2FS0273-0979-02-00941-2).

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