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Busemann's theorem

In mathematics, **Busemann's theorem** is a theorem in Euclidean geometry and geometric tomography. It was first proved by Herbert Busemann in 1949 and was motivated by his theory of area in Finsler spaces.

Statement of the theorem

Let K be a convex body in n-dimensional Euclidean space $\mathbf{R}^n$ containing the origin in its interior. Let S be an $(n-2)$-dimensional linear subspace of $\mathbf{R}^n$. For each unit vector θ in $S^\perp$, the orthogonal complement of S, let S_θ denote the $(n-1)$-dimensional hyperplane containing θ and S. Define $r(\theta)$ to be the $(n-1)$-dimensional volume of $K \cap S_\theta$. Let C be the curve $\{\theta r(\theta)\}$ in $S^\perp$. Then C forms the boundary of a convex body in $S^\perp$.

See also

- Brunn–Minkowski inequality
- Prékopa–Leindler inequality

References

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- Gardner, Richard J. (2002). "The Brunn-Minkowski inequality". *Bull. Amer. Math. Soc. (N.S.)*. **39** (3): 355–405 (electronic). CiteSeerX 10.1.1.106.7344 (https://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.106.7344). doi:10.1090/S0273-0979-02-00941-2 (https://doi.org/10.1090%2FS0273-0979-02-00941-2).