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# Fenchel–Moreau theorem

In convex analysis, the **Fenchel–Moreau theorem** (named after Werner Fenchel and Jean Jacques Moreau) or **Fenchel biconjugation theorem** (or just **biconjugation theorem**) is a theorem which gives necessary and sufficient conditions for a function to be equal to its biconjugate. This is in contrast to the general property that for any function  $f^{**} \leq f$ .<sup>[1][2]</sup> This can be seen as a generalization of the bipolar theorem.<sup>[1]</sup> It is used in duality theory to prove strong duality (via the perturbation function).

## Statement

Let  $(X, \tau)$  be a Hausdorff locally convex space, for any extended real valued function  $f : X \rightarrow \mathbb{R} \cup \{\pm\infty\}$  it follows that  $f = f^{**}$  if and only if one of the following is true

1.  $f$  is a proper, lower semi-continuous, and convex function,
2.  $f \equiv +\infty$ , or
3.  $f \equiv -\infty$ .<sup>[1][3][4]</sup>

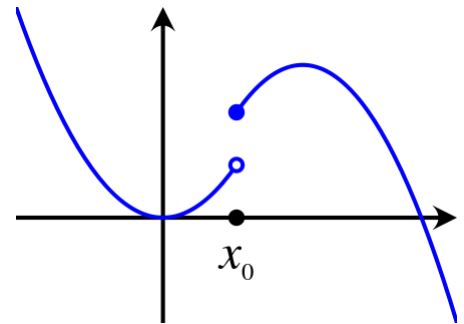
## References

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A function that is not lower semi-continuous. By the Fenchel-Moreau theorem, this function is not equal to its biconjugate.