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# Invex function

In vector calculus, an **invex function** is a differentiable function $f$ from $\mathbb{R}^n$ to $\mathbb{R}$ for which there exists a vector valued function $\eta$ such that

$$f(x) - f(u) \geq \eta(x, u) \cdot \nabla f(u),$$

for all $x$ and $u$.

Invex functions were introduced by Hanson as a generalization of convex functions.[1] Ben-Israel and Mond provided a simple proof that a function is invex if and only if every stationary point is a global minimum, a theorem first stated by Craven and Glover.[2][3]

Hanson also showed that if the objective and the constraints of an optimization problem are invex with respect to the same function $\eta(x, u)$, then the Karush–Kuhn–Tucker conditions are sufficient for a global minimum.

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## Type I invex functions

A slight generalization of invex functions called **Type I invex functions** are the most general class of functions for which the Karush–Kuhn–Tucker conditions are necessary and sufficient for a global minimum.[4] Consider a mathematical program of the form

$$\begin{array}{rl} \min & f(x) \\ \text{s.t.} & g(x) \leq 0 \end{array}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^n \to \mathbb{R}^m$ are differentiable functions. Let $F = \{x \in \mathbb{R}^n \mid g(x) \leq 0\}$ denote the feasible region of this program. The function $f$ is a **Type I objective function** and the function $g$ is a **Type I constraint function** at $x_0$ with respect to $\eta$ if there exists a vector-valued function $\eta$ defined on $F$ such that

$$f(x) - f(x_0) \geq \eta(x) \cdot \nabla f(x_0)$$

and

$$-g(x_0) \geq \eta(x) \cdot \nabla g(x_0)$$

for all $x \in F$.[5] Note that, unlike invexity, Type I invexity is defined relative to a point $x_0$.

**Theorem (Theorem 2.1 in[4]):** If $f$ and $g$ are Type I invex at a point $x^*$ with respect to $\eta$, and the Karush–Kuhn–Tucker conditions are satisfied at $x^*$, then $x^*$ is a global minimizer of $f$ over $F$.

## See also

- Convex function
- Pseudoconvex function
- Quasiconvex function

## References


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## Further reading


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- S. K. Mishra, S.-Y. Wang and K. K. Lai, Generalized Convexity and Vector Optimization, Springer, New York, 2009.