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Shephard's problem

In mathematics, **Shephard's problem**, is the following geometrical question asked by Geoffrey Colin Shephard (1964): if K and L are centrally symmetric convex bodies in n-dimensional Euclidean space such that whenever K and L are projected onto a hyperplane, the volume of the projection of K is smaller than the volume of the projection of L, then does it follow that the volume of K is smaller than that of L?

In this case, "centrally symmetric" means that the reflection of K in the origin, $-K$, is a translate of K, and similarly for L. If $\pi_k : \mathbf{R}^n \to \Pi_k$ is a projection of $\mathbf{R}^n$ onto some k-dimensional hyperplane Π_k (not necessarily a coordinate hyperplane) and V_k denotes k-dimensional volume, Shephard's problem is to determine the truth or falsity of the implication

$$V_k(\pi_k(K)) \leq V_k(\pi_k(L)) \text{ for all } 1 \leq k < n \implies V_n(K) \leq V_n(L).$$

$V_k(\pi_k(K))$ is sometimes known as the **brightness** of K and the function $V_k \circ \pi_k$ as a (k-dimensional) **brightness function**.

In dimensions $n = 1$ and 2, the answer to Shephard's problem is "yes". In 1967, however, Petty and Schneider showed that the answer is "no" for every $n \geq 3$. The solution of Shephard's problem requires Minkowski's first inequality for convex bodies and the notion of projection bodies of convex bodies.

See also

- Busemann–Petty problem

References

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- Schneider, Rolf (1967). "Zur einem Problem von Shephard über die Projektionen konvexer Körper" (https://doi.org/10.1007%2FBF01135693). *Mathematische Zeitschrift* (in German). **101**: 71–82. doi:10.1007/BF01135693 (https://doi.org/10.1007%2FBF01135693).
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