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# Closed convex function

In mathematics, a function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is said to be **closed** if for each  $\alpha \in \mathbb{R}$ , the sublevel set  $\{x \in \text{dom } f \mid f(x) \leq \alpha\}$  is a closed set.

Equivalently, if the epigraph defined by  $\text{epi } f = \{(x, t) \in \mathbb{R}^{n+1} \mid x \in \text{dom } f, f(x) \leq t\}$  is closed, then the function  $f$  is closed.

This definition is valid for any function, but most used for convex functions. A proper convex function is closed if and only if it is lower semi-continuous.<sup>[1]</sup> For a convex function which is not proper there is disagreement as to the definition of the *closure* of the function.

## Properties

- If  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is a continuous function and  $\text{dom } f$  is closed, then  $f$  is closed.
- If  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is a continuous function and  $\text{dom } f$  is open, then  $f$  is closed if and only if it converges to  $\infty$  along every sequence converging to a boundary point of  $\text{dom } f$ .<sup>[2]</sup>
- A closed proper convex function  $f$  is the pointwise supremum of the collection of all affine functions  $h$  such that  $h \leq f$  (called the affine minorants of  $f$ ).

## References

1. *Convex Optimization Theory* (<https://www.amazon.com/Convex-Optimization-Theory-Dimitri-Bertsekas/dp/1886529310/>). Athena Scientific. 2009. pp. 10, 11. ISBN 978-1886529311.
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