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Distance between two parallel lines

The **distance between two parallel lines** in the plane is the minimum distance between any two points l

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Formula and proof

Because the lines are parallel, the perpendicular distance between them is a constant, so it does not matter which point is chosen to measure the distance. Given the equations of two non-vertical parallel lines

$$\begin{aligned} y &= mx + b_1 \\ y &= mx + b_2 \,, \end{aligned}$$

the distance between the two lines is the distance between the two intersection points of these lines with the perpendicular line

$$y = -x/m \,.$$

This distance can be found by first solving the linear systems

$$\begin{cases} y = mx + b_1 \\ y = -x/m \,, \end{cases}$$

and

$$\begin{cases} y = mx + b_2 \\ y = -x/m \,, \end{cases}$$

to get the coordinates of the intersection points. The solutions to the linear systems are the points

$$(x_1, y_1) \ = \left(\frac{-b_1 m}{m^2 + 1}, \frac{b_1}{m^2 + 1} \right) ,$$

and

$$(x_2, y_2) \ = \left(\frac{-b_2 m}{m^2+1}, \frac{b_2}{m^2+1} \right).$$

The distance between the points is

$$d = \sqrt{\left(\frac{b_1 m - b_2 m}{m^2+1} \right)^2 + \left(\frac{b_2 - b_1}{m^2+1} \right)^2},$$

which reduces to

$$d = \frac{|b_2 - b_1|}{\sqrt{m^2+1}}.$$

When the lines are given by

$$\begin{aligned} ax + by + c_1 &= 0 \\ ax + by + c_2 &= 0, \end{aligned}$$

the distance between them can be expressed as

$$d = \frac{|c_2 - c_1|}{\sqrt{a^2+b^2}}.$$

See also

- Distance from a point to a line

References

- *Abstand* In: *Schülerduden – Mathematik II*. Bibliographisches Institut & F. A. Brockhaus, 2004, ISBN 3-411-04275-3, pp. 17-19 (German)
- Hardt Krämer, Rolf Höwelmann, Ingo Klemisch: *Analytische Geometrie und Lineare Akgebra*. Diesterweg, 1988, ISBN 3-425-05301-9, p. 298 (German)

External links

- Florian Modler: *Vektorprodukte, Abstandsaufgaben, Lagebeziehungen, Winkelberechnung – Wann welche Formel?* (http://www.emath.de/Referate/Zusammenfassung-wichtiger-Formeln.pdf), pp. 44-59 (German)
- A. J. Hobson: *"JUST THE MATHS" - UNIT NUMBER 8.5 - VECTORS 5 (Vector equations of straight lines)* (https://archive.uea.ac.uk/jtm/8/Lec8p5.pdf), pp. 8-9