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Imaginary line (mathematics)

In complex geometry, an **imaginary line** is a straight line that only contains one real point. It can be proven that this point is the intersection point with the conjugated line.[1]

It is a special case of an imaginary curve.

An imaginary line is found in the complex projective plane $P^2(C)$ where points are represented by three homogeneous coordinates $(x_1,\ x_2,\ x_3),\quad x_i \in C.$

Boyd Patterson described the lines in this plane:[2]

> The locus of points whose coordinates satisfy a homogeneous linear equation with complex coefficients
>
> $$a_1\ x_1 + a_2\ x_2 + a_3\ x_3 = 0$$
>
> is a straight line and the line is *real* or *imaginary* according as the coefficients of its equation are or are not proportional to three real numbers.

Felix Klein described imaginary geometrical structures: "We will characterize a geometric structure as imaginary if its coordinates are not all real.:[3]

According to Hatton:[4]

> The locus of the double points (imaginary) of the overlapping involutions in which an overlapping involution pencil (real) is cut by real transversals is a pair of imaginary straight lines.

Hatton continues,

> Hence it follows that an imaginary straight line is determined by an imaginary point, which is a double point of an involution, and a real point, the vertex of the involution pencil.

See also

- Imaginary point
- Real curve
- Conic sections
- Imaginary number

References

1. Patterson, B. C. (1941), "The inversive plane", *The American Mathematical Monthly*, **48**: 589–599, doi:10.2307/2303867 (https://doi.org/10.2307%2F2303867), MR 0006034 (https://www.ams.org/mathscinet-getitem?mr=0006034).
2. Patterson 590
3. Klein 1928 p 46

4. Hatton 1929 page 13, Definition 4

- J.L.S. Hatton (1920) The Theory of the Imaginary in Geometry together with the Trigonometry of the Imaginary (https://archive.org/details/cu31924001523665/page/n6), Cambridge University Press via Internet Archive
- Felix Klein (1928) *Vorlesungen über nicht-euklischen Geometrie*, Julius Springer.
