WIKIPEDIA

Convex combination

In convex geometry, a **convex combination** is a linear combination of points (which can be vectors, scalars, or more generally points in an affine space) where all coefficients are non-negative and sum to 1.[1]

More formally, given a finite number of points $x_1, x_2, \dots, x_n$ in a real vector space, a convex combination of these points is a point of the form

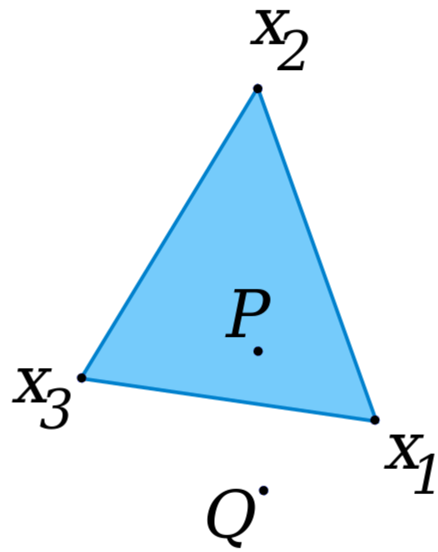
$$\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n$$

where the real numbers α_i satisfy $\alpha_i \geq 0$ and $\alpha_1 + \alpha_2 + \cdots + \alpha_n = 1$.[1]

As a particular example, every convex combination of two points lies on the line segment between the points.[1]

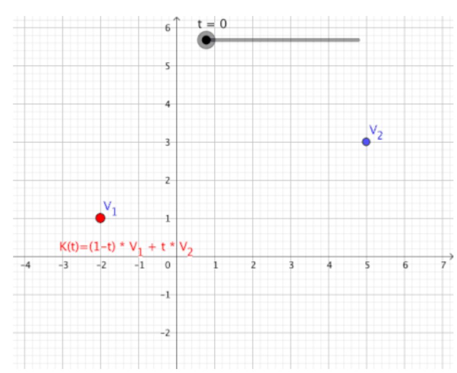
A set is convex if it contains all convex combinations of its points. The convex hull of a given set of points is identical to the set of all their convex combinations.[1]

There exist subsets of a vector space that are not closed under linear combinations but are closed under convex combinations. For example, the interval $[0, 1]$ is convex but generates the real-number line under linear combinations. Another example is the convex set of probability distributions, as linear combinations preserve neither nonnegativity nor affinity (i.e., having total integral one).



Given three points x_1, x_2, x_3 in a plane as shown in the figure, the point P *is* a convex combination of the three points, while Q is *not*.

(Q is however an affine combination of the three points, as their affine hull is the entire plane.)



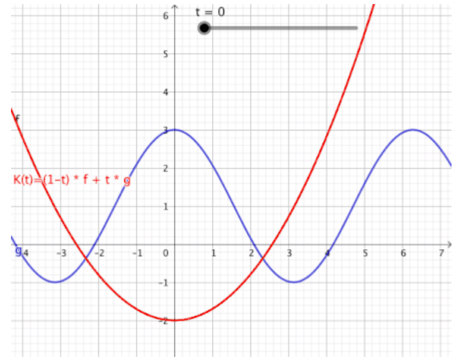
Convex combination of two points $v_1, v_2 \in \mathbb{R}^2$ in a two dimensional vector space $\mathbb{R}^2$ as animation in Geogebra with $t \in [0, 1]$ and $K(t) := (1-t) \cdot v_1 + t \cdot v_2$

Contents

Other objects

- Similarly, a convex combination X of random variables Y_i is a weighted sum (where α_i satisfy the same constraints as above) of its component probability distributions, often called a finite mixture distribution, with probability density function:

$$f_X(x) = \sum_{i=1}^{n} \alpha_i f_{Y_i}(x)$$



Convex combination of two functions as vectors in a vector space of functions - visualized in Open Source Geogebra with $[a,b]=[4,7]$ and as the first function $f:[a,b]\to\mathbb{R}$ a polynomial is defined. $f(x) := \frac{3}{10}\cdot x^2 - 2$ A trigonometric function $g:[a,b]\to\mathbb{R}$ was chosen as the second function. $g(x) := 2\cdot cos(x)+1$ The figure illustrates the convex combination $K(t) := (1-t)\cdot f + t\cdot g$ of f and g as graph in red color.

Related constructions

- A conical combination is a linear combination with nonnegative coefficients. When a point x is to be used as the reference origin for defining displacement vectors, then x is a convex combination of n points $x_1, x_2, \ldots, x_n$ if and only if the zero displacement is a non-trivial conical combination of their n respective displacement vectors relative to x.
- Weighted means are functionally the same as convex combinations, but they use a different notation. The coefficients (weights) in a weighted mean are not required to sum to 1; instead the weighted linear combination is explicitly divided by the count of the weights.
- Affine combinations are like convex combinations, but the coefficients are not required to be non-negative. Hence affine combinations are defined in vector spaces over any field.

See also

- Affine hull
- Carathéodory's theorem (convex hull)
- Simplex
- Barycentric coordinate system

References

1. Rockafellar, R. Tyrrell (1970), *Convex Analysis*, Princeton Mathematical Series, vol. 28, Princeton University Press, Princeton, N.J., pp. 11–12, MR 0274683 (https://www.ams.org/mathscinet-getitem?mr=0274683)