Investments without Coordination Failures

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I study games in which agents must sink their investments before they are potentially matched into partnerships that generate value. I focus on competitive matching markets where there is a price to join any partnership. Despite the welfare theorems for competitive markets, because agents' investments are sunk—implying that markets are incomplete—inefficiencies can still arise that can be interpreted as coordination failures. Armen does not invest because Bengt does not invest, and visa versa. But should we predict these coordination failures? I argue no because the standard, Nash solution concept used for these types of games is too weak in the context of competitive markets. For an important class of matching with investment games—where investment is only valuable if matched—Nash equilibrium only restricts outcomes (in terms of utilities) to those outcomes that are individually rational and feasible. Therefore I argue we should replace the Nash solution concept in this context with a mild refinement: trembling-hand perfection. I then prove that every perfect equilibrium is efficient. Therefore, in the context of competitive markets, coordination failures are not robust, even though they are robust in markets that are not competitive.

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1 Introduction

Coordination failures would seem to be ubiquitous, at least if we look at the amount of game theory research devoted to them. Yet, when we look at actual markets, many investments are made in the face of incomplete markets, seemingly without much fear of

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those dreaded coordination failures. Entrepreneurs develop hardware before the matching software is available. College students invest in skills before looking for jobs. In both examples, the actors must trust that proper market forces will work things out. For coordination, two things must happen. First, to induce efficiently coordinated investment, people must trust that there will be ex post competition to avoid hold out problems. Second, to induce investment, people must trust that there will not be widespread miscoordination. This paper is focused on this second point.

In this paper, I proved that coordination failure equilibria are not robust when people act as price-takers. Formally, coordination failures do not survive a trembling-hand refinement, where equilibrium is seen as the limit of a sequence of mistakes that become small. When there is a chance that people experiment or make mistakes, markets with price-taking achieve efficiency. In a sense, the model is a formal argument of how Adam Smith's "higgling and bargaining of the market" can fix coordination failures.

To formally study the connection between market coordination and game coordination, I build on a series of papers that model players as playing a game before entering a market.[1] Makowski and Ostroy (1995) first reformulated a First Welfare Theorem without price-making and incomplete markets. They showed that two conditions were sufficient for markets to generate efficient outcomes:

- full appropriation: each individual's private benefit from any investment coincides with his/her social contribution;
- non-complementarity: different player's investments cannot be complementary.

As Makowski and Ostroy show, perfect competition gives full appropriation. However, when there are complementarities, game coordination problems can still arise. Per-

1. Brandenburger and Stuart (2007) called such games, with a non-cooperative game before a cooperative games, "biform games." Such games are grossly understudied.

fect competition alone is not sufficient for efficiency.[2] Further follow up papers, such as Makowski (2004) and most recently Nöldeke and Samuelson (2015) have further generalized results and clarified the connection between competition and efficiency. But the take-away is always the same: coordination failures will plague competitive markets. However, none of these papers examine whether these coordination failure equilibria are robust.

This focus on coordination problems and the sources of their resolution is central to economics. In his reinterpretation of F.A. Hayek's early work, Gerald O'Driscoll (1977) sees "economics as a coordination problem." Unlike in a standard Walrasian model of the world where everyone chooses their best action, given the objective facts of the work, for a real economy "It is not sufficient for an individual to have complete knowledge of all objective conditions (technology, resources, and so on)." Instead "the attainment of equilibrium is a coordination problem" (p. 23-4). For Hayek, the interesting question is how such coordination comes about, not simply the definition of equilibrium as when coordination occurs. This paper theoretically investigates when efficient coordination is likely within markets or when we can expect to find "coordination failures".

But "coordination problem" has come to mean something different today, especially within game theory. To differentiate the broader notion of coordination used by writers like Hayek and the game theory form of coordination, let me use the terms "market coordination" and "game coordination".[3] Unlike the market coordination in any equilibria, game coordination occurs in any situation where each player's best-response is to somehow match the other players' actions. Coordination games have multiple pure-strategy

2. Following up on Makowski and Ostroy (1995), two important papers Cole, Mailath, and Postlewaite (2001a, 2001b) find three different types of coordination problems can arise: (1,1) under-investment equilibria, (2) over-investment equilibria, and (3) mismatch equilibria, as first pointed out by Felli and Roberts (2016).

3. Klein and Orsborn (2009) make a similar distinction between "concatenate coordination" and "mutual coordination." Another paper of mine (Albrecht 2016) provides a model that ties together the two different forms of coordination through the effort of entrepreneurs.

equilibria. If equilibria can be Pareto-ranked, we will call any equilibrium that is not Pareto-optimal a coordination failure.[4] The coordination failure remains as a equilibrium because people cannot contract for "joint-deviations"; markets are incomplete.

There is an entirely separate literature on adverse selection in Walrasian markets. As Gale (1992) points out, in these models there are many equilibria. However, some of those equilibria are sustained by unreasonable off-equilibrium beliefs, like the belief that other people will not best-respond if a deviation occurs. To discipline off-equilibrium beliefs, Gale uses a form of a trembling-hand refinement (Selten 1975). Whether the refinement leads to more or less efficient equilibria depends on the exact context. In Gale (1992), the refined equilibria are inefficient, while in Gale (1996) they are efficient. More recent studies have been done by Dubey and Geanakoplos (2002), Dubey, Geanakoplos, and Shubik (2005), Zame (2007), and Scheuer and Smetters (2018).

The result does not imply that coordination problems do not exist. We should instead think of them as arising in environments with *imperfect* competition, as is the common in the macro literature since Cooper and John (1988), for example.

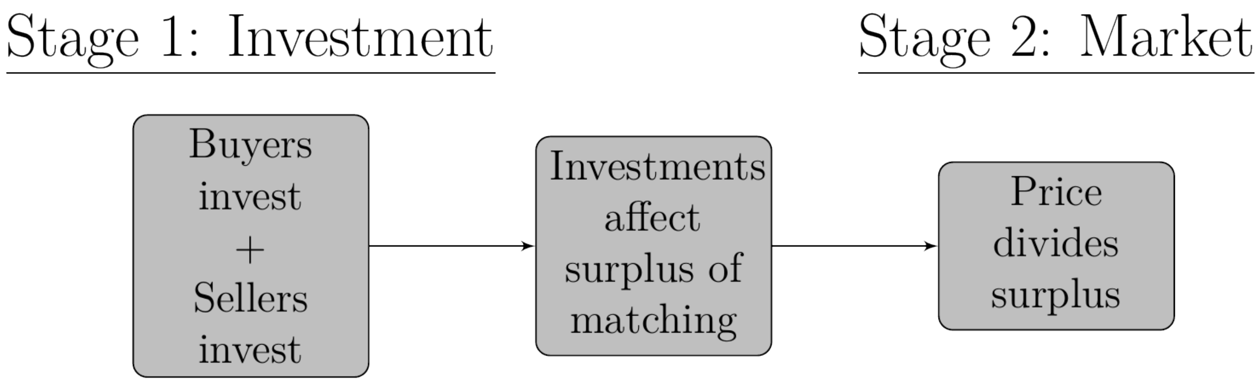
2 Example

Consider a simple example of a two-sided matching market with measure one of agents on both sides. For consistent language, I talk about buyers and sellers. There are two stages to the game. First, before matching, buyers and sellers must invest in an attribute, $b \in \{0,1\}$ and $s \in \{0,1\}$. The cost to buyer is $\frac{1}{4}b$ and the cost to seller is $\frac{1}{4}s$. These investments generate a surplus for any match: $v(b,s) = bs$. Second, after buyers and sellers sink their investment, buyers and sellers enter a Walrasian market with prices, $p(b,s)$. When agents invest, they do not yet observe prices. Therefore, agents make their

4. A game like Battle of the Sexes does not have an equilibrium which is a coordination failure.

decisions based on some price *conjectures*, $\tilde{p}(b,s)$.

The payoff function for a buyer of type b, matched with a seller of type s, when the price is $p(b,s)$ is $v(b,s) - p(b,s) - \frac{1}{4}b$. For sellers, the payoff is $p(b,s) - \frac{1}{4}s$. The game is summarized in the figure below.



For equilibrium, which I call an investment equilibrium following Makowski (2004), the following conditions must hold:

I. prices clear matching market,

II. each buyer i chooses b to maximize utility, given price conjectures: $\tilde{p}^i(b,s)$,

III. each seller j chooses s to maximize utility, given price conjectures: $\tilde{p}^j(b,s)$, and

IV. rational conjectures: conjectures are not contradicted by the data:

- If there is a positive mass of buyer with attribute b and seller with attribute s, then that market has a true price and conjectures agree with the true price: $\tilde{p}^i(b,s) = \tilde{p}^j(b,s) = p(b,s)$
- Otherwise, there is no public price and conjectures need not be consistent across agents.[5]

5. Despite some technical details, an investment equilibrium equivalent to what Cole, Mailath, and Postlewaite (2001b) and Nöldeke and Samuelson (2014) call an ex ante equilibrium.

Because utility is quasi-linear (transferable), a profile of investments and matchings is efficient if and only if it maximizes $v(b,s) - \frac{1}{4}b - \frac{1}{4}s$. For this example, the efficient allocation is to maximize investment: $b = s = 1$.

The efficient allocation with full investment is an equilibrium. For example, suppose that $p(1,1) = \frac{1}{2}$ and all types conjecture that all other prices are zero. Those prices are not actually posted by the auctioneer, since the economy does not include such matches. Price clears matching markets; all buyers and sellers (each with equal measure) want to match. The matching generates positive surplus for the buyers and sellers, which is better than all other alternatives which lead to a conjectured utility of zero. Finally, the players conjectures are not contradicted by the data.

There is also an equilibrium where no one invests: $b = 0$, $s = 0$, and $p(0,0) = 0$. This equilibrium is a coordination failure. However, this equilibrium is only sustained by certain conjectures of buyers and sellers. To see why, suppose that buyers must conjecture that $p(1,1)$ is extremely high. That deters buyers from deviating. Similarly suppose sellers conjecture that the price is zero, which deters sellers from investing. However, notice that the conjectures of markets that are closed in equilibrium are a free parameter that do not need to agree across agents. Moreover, they conjectures *cannot* agree in a coordination failure.

With a free parameter, many equilibrium can be sustained. In this simple example in particular, notice also that this coordination failure *minimizes* total value. This means that there is little predictive power from a Nash-style equilibrium, especially if we are looking to study the level of efficiency, since we cannot rule out any utility outcome.

To try to pin down the equilibrium more, I consider a mild refinement: trembling hand perfection. For now, assume there is only a simple type of tremble: a uniform trembling hand, where each attribute must be chosen with positive probability $\epsilon > 0$ by each buyer and seller. Since there is a continuum of buyers and sellers, I will assume that in the aggregate each attribute must be chosen by a positive mass of players. A perfect investment equilibrium is the limit of some sequence of ϵ that goes to zero.

With trembling hand, each b and s are played, so that the actual prices are pinned down and players cannot have contradictory beliefs: $\tilde{p}^i(b,s) = \tilde{p}^j(b,s) = p(b,s)$. If $p(1,1) > \frac{1}{4}$, sellers want to choose $s = 1$ as much as possible: $1-\epsilon$. If $p(1,1) < \frac{3}{4}$, buyers want to choose $b = 1$ as much as possible: $1-\epsilon$. For any perturbation, equilibrium requires $p(1,1) \in \left[\frac{1}{4}, \frac{3}{4}\right]$. As perturbations go to zero, $(1,1)$ is the unique equilibrium strategy profile. However, the prices are not necessarily unique.

This example highlights three important features of such markets. First, even with price-taking, coordination failures can exist. Second, coordination failures are sustained by off-path conjectures that are contradictory across buyers and sellers. Third, the possibility of mistakes/trembles is one justification to rule out such contradictions, which therefore rules out coordination failures. The next section extends these three features to a more general model.

3 Model

There are a continuum of buyers, indexed by $i \in I$, and a continuum of sellers, indexed by $j \in J$. There are functions that specify an endowed *type* for each agent: $\beta : I \to \mathcal{B}$ and $\sigma : J \to \mathcal{S}$. A generic type is given by $t \in T = \mathcal{B} \cup \mathcal{S}$. I assume the set of types is finite. An economy is defined by a positive measure on the set of types,

$$E \in M_+(T).$$

There are two stages to the model. In the first stage, each individual must acquire/invest in one *attribute*, $a \in A$. For simplicity, the set of attributes is finite. The attributes are partitioned into those that are feasible for buyers, $b \in B$, and those that are feasible for sellers, $s \in S$. There exists a function of acquiring attribute an

$$c : A \times T \to \mathbb{R} \cup \infty,$$

so that $c(a,t)$ is the cost of acquiring a for type t. By definition, there is infinite cost for a buyer type to acquire a seller skill and vice versa. Individual investments lead to distribution of attributes $\mu \in M_+(A)$. For any subset $E \subset A$, $\mu(E)$ is the mass of individuals with attributes in E.

The second stage involves a market that assignment of buyers to sellers,

$$x \in M_+(B^0 \times S^0),$$

where $B^0 \equiv B \cup \emptyset$ and $S^0 \equiv S \cup \emptyset$. An assignment x is *feasible* for μ if $x(\emptyset, \emptyset) = 0$, and for all Borel subsets, $E \subset B$ and $F \subset S$,

$$x(E, S^0) = \mu(E)$$

$$x(B^0, F) = \mu(F).$$

We can think of a match, (b,s), as simply a standard good sold by a seller of type s to buyer of type b. A market is open if that pair is part of an equilibrium, that is $x(b,s) > 0$. A market is closed if it is not open. While in the formulation here, players act like price-takers, players can affect the set of markets; players are therefore market-makers.

The value generated by an assignment is given by a bounded value function: $v : B^0 \times S^0 \to \mathbb{R}$. In general, I will impose no further assumptions.

Social assignment gains function is given by μ

$$g(\mu) \equiv \max_x \sum_{b \in B^0} \sum_{s \in S^0} v(b,s)x(b,s) \quad s.t. x \text{ is feasible given } \mu.$$

An allocation that attains $g(\mu)$ is *conditionally efficient*.

The second-stage assignment is coordinated by done through prices. To focus on coordination under perfect competition, I assume each player acts as a price-taker.[6] A price system is $p : B^0 \times S^0 \to \mathbb{R}$.

Definition 1. Fixing attribute investment, μ, a pair (x, p) is an (ex post) *Walrasian equilibrium* for μ if x is feasible for μ, $p(b, \emptyset) = p(\emptyset, s) \equiv 0$,

I. For each $b \in \operatorname{supp} \mu$ and each $(b,s) \in \operatorname{supp} x$, the match maximizes b's utility:

$$v_b^*(p) \equiv \max \left\{ \max_{s' \in S} \left\{ v(b,s') - p(b,s') \right\}, v(b,\emptyset) \right\},$$

II. and for each $s \in \operatorname{supp} \mu$ and each $(b,s) \in \operatorname{supp} x$, the match maximizes s's utility:

$$v_s^*(p) \equiv \max \left\{ \max_{b' \in B} \left\{ p(b',s) \right\}, v(\emptyset,s) \right\}.$$

The equilibrium requires that when players are deciding whether to form a match given prices, they are optimizing. The first condition is that any match is maximizing a buyer's utility: $s \in \operatorname{argmax}_{s' \in S^0} v(b,s') - p(b,s')$. The second condition is the equivalent condition for the sellers. Notice conjectures are not a part of a Walrasian equilibrium because all relevant markets are priced. Even though closed markets are not priced, those markets are irrelevant after investment decisions have been made.

6. By assuming price-taking, I follow most of the related matching literature, such as Cole, Mailath, and Postlewaite (2001b) and Nöldeke and Samuelson (2015). See Gretsky, Ostroy, and Zame (1999) and Makowski (2004) for a rigorous analysis of when the price-taking assumption is justified in an assignment model.

Because of price-taking, we immediately have a "Conditional First Welfare Theorem": If a pair (x, p) is Walrasian for μ, then it is conditionally efficient. It is conditional because maximization only holds within the support of attributes.[7] This immediately rules out any mismatch equilibria found by Felli and Roberts (2016). Besides that, it is a very weak notion of efficiency. In our example, the equilibrium where no one invests is conditionally efficient, even though surplus is minimized. For our current purposes, the result is important because it establishes how the matching market is working effectively, given investments.

Even though all equilibria are conditionally efficient, they are not all efficient in the ex ante sense. In particular, investment coordination failures can arise. In the example, $b = 0, s = 0$ and $p(0, 0) = 0$ can be part of an ex post contracting equilibrium. Suppose buyers conjecture that $p(b, s) > 1$ for all other levels of investment they could choose. They would not want to deviate, because any other investment is too costly. Similarly, if sellers conjecture that the price is zero, they will not make a return on their positive investment. Since only the price for $p(0, 0)$ is observed in equilibrium, both sides' conjectures are rational; there is no feedback that tells the buyers and sellers they should revise their conjectures. Even though there is competition, players are stuck in a coordination failure.

The set of attributes chosen may be inefficient; μ may be missing the efficient b and s. The next section asks, given the choice in a non-cooperative setting, do people choose the efficient b and s?

7. Because of complementarities, the equilibrium price is not unique. There is a pie $v(b, s) = 1$ to divide by $p(b, s)$. The division which occurs is indeterminate, even though the optimal "quantity traded" is when all buyers and sellers match. This is exactly the setup and outcome in Figure 4 of Smith (1982, p. 171). As Smith finds, even though the number of trades is the efficient and equilibrium amount, the price moves between each round of play.

4 Investment Equilibrium

Fix the population of types, E. An allocation of attributes is a measure $\nu \in M_+(A \times T)$, where ν_A and ν_T are the respective marginal distributions and $\mu = \nu_A$. An allocation ν is *feasible* for E if $\nu_T = E$.

Definition 2. A pair (ν, p) is an (ex ante) *investment equilibrium* for E if ν is feasible, p is a Walrasian price for μ, and for all $(a, t) \in \text{supp } \nu$,

$$v_a^*(p) - c(a,t) \geq v_{a'}^*(p) - c(a', t) \quad \forall a' \in A.$$

Note that an investment equilibrium does not involve the standard Nash equilibrium epistemic justification; people are not best-responding to actions. Instead, they are best-responding to expected prices. Beyond the normal conditions for a Walrasian equilibrium, when players are deciding how much to invest, they must form conjectures about what prices will be in the future. The equilibrium disciplines those conjectures, as Hayek (1937) p. 41 pointed out, "the concept of equilibrium merely means that the foresight of the different members of the society is in a special sense correct." However, the exact meaning of correctness is not clear since some prices never materialize so people can contradictory, but in a sense correct, things. I will further discipline conjectures below when I consider refinements to address this issue.

There is a total cost of attributes in the economy, ν is $\int c d\nu$, and a total surplus from ν,

$$G(\nu) = g(\nu_A) - \sum_A \sum_T c(a,t)\nu(a,t).$$

Definition 3. The allocation ν is unconditionally *efficient* for E if it is feasible and $G(\nu) \geq G(\nu')$ for all other feasible allocation ν'.

Returning to the example, we already showed that investment equilibria need not be unconditionally efficient. For all buyers, $b = 0$, for all sellers $s = 0$, and $p(b,s) = 0$ is an investment equilibrium. There are no profitable deviation:

$$\underbrace{0}_{\text{Surplus}} - \underbrace{0}_{\text{Transfer}} - \underbrace{0}_{\text{Cost of } b=0} \geq \underbrace{0}_{\text{Surplus}} - \underbrace{0}_{\text{Transfer}} - \underbrace{\frac{1}{4}b}_{\text{Cost of } b>0} .$$

The next subsection shows that this type of surplus minimizing equilibria exists for many economies that are relevant in the matching literature.

4.1 Weak Predictions with Unconstrained Beliefs

One problem with the equilibrium concept, and why it leads to so many different equilibria as shown in the last section, is that off-path beliefs are a free parameter. As Robert Lucas taught us, "beware of theorists bearing free parameters." In related papers of adverse selection mentioned about, economists have recognized this issue in other Walrasian contexts. For example, Zame (2007) notes that "imposing no discipline would admit equilibria which are *viable only because different agents hold contradictory beliefs*."

To show just how weak the solution concept is, in this section, instead of focusing on the most general forms of the surplus and cost functions that we have used so far, let us consider a smaller set that are still relevant for models of investment and matching.

We will consider two conditions:

Definition 4. A cost function has costly investment if there exists an attribute, $0 \in A$, such that, for all types s, $c(0,t) = 0$ and $c(a,t) > 0$ for $a \neq 0$.

Definition 5. Investment is mutually necessary if surplus is zero whenever there is not investment from both the buyer and seller: $v(0, \emptyset) = v(\emptyset, 0) = v(0,0) = 0$ and $v(b,s) \geq 0$ for all b and s.

These are strong restrictions, but they include economies that are relevant for any researcher who is looking at the interaction of investment with matching. The following proposition shows that for all economies like this, the surplus minimizing outcome is an equilibrium.

Proposition 1. *For any economy with costly and mutually-necessary investment, there exists an investment equilibrium that minimizes surplus at zero.*

The proof is immediate and highlights the nature of a Nash-style equilibrium.

Proof. Suppose all players but i are not investing. Since investment is costly, any decision to choose positive investment is costly but will not generate any surplus without another player investing. Therefore, it is optimal for i to not invest. ■

By contrast, any economy that rules out the surplus minimizing outcome does so because of decision made that ignores the matching process, whereby people invest regardless of the matching market. In that case, we can rule out the worst outcomes, but it does not have anything to do with the matching market.

The proposition holds regardless of the shape of the cost and surplus functions. Even if cost of investment is arbitrarily small and the surplus generation is arbitrarily big, there exists an investment equilibrium with zero surplus. In this case, we still cannot rule out that either the best or the worst possible allocation can occur.[8] For doing welfare analysis though, it may be desirable to say more than "either the best or worst outcome can occur."

To discipline the set of possible outcomes, I follow Gale (1992), who argued that "*some refinement of the equilibrium concept is required to give the theory predictive power*. One such refinement is based on the notion of the 'trembling' hand." The next section shows the under such a refiniment, all equilibria are efficient.

8. If we introduced random actions, we can say that anything in-between could happen too.

5 Disciplined Beliefs and Perfect Equilibrium

To discipline believes, we will consider a perturbed strategy vector for all buyers $i \in I$. For simplicity of notation, I assume that all buyers are subjected to the same tremble, $\epsilon_B = (\epsilon(b))_{b \in B}$, satisfying $\epsilon(b) > 0$ for all $b \in B$ and

$$\int_B \epsilon(b) db \leq 1.$$

Similarly, all sellers are subjected to the same tremble, $\epsilon_S = (\epsilon(s))_{s \in S}$, satisfying $\epsilon(s) > 0$ for all $s \in S$ and

$$\int_S \epsilon(s) ds \leq 1.$$

A perturbed games is index by the set of perturbed strategy vectors $\epsilon = (\epsilon_B, \epsilon_S)$. An allocation $\nu(\epsilon)$ is ϵ-feasible for E if $\nu_T = E$ and for all $a \in A$

$$\nu_A(\epsilon(a)) \geq \epsilon(a).$$

Instead of jumping directly to the analysis of the limit of perturbed games, it is helpful to say something about the perturbed games themselves. In particular, we can consider their respective efficiency. To do so, let us say that an allocation $\nu(\epsilon)$ is ϵ-efficient for E if it is feasible and $G(\nu(\epsilon)) \geq G(\nu'(\epsilon))$ for all other ϵ-feasible allocation ν'. Formally,

Definition 6. A pair $(\nu(\epsilon), p)$ is an ϵ-investment equilibrium for E if ν is ϵ-feasible, p is a Walrasian price for μ, and for all (a, t) such that $\nu_A(\epsilon) > \epsilon$,

$$v^*_a(p) - c(a,t) \geq v^*_{a'}(p) - c(a',t) \quad \forall a' \in A$$

Note that by construction, with a trembling hand, supp $\nu_A(\epsilon) = A$. Because there is full support and all markets are open, coordination failures cannot arise. This is shown through the following lemma.

Lemma 1. *If $(\nu(\epsilon), p)$ is an ϵ-investment equilibrium, then it is ϵ-efficient.*

Proof. Let $Q(\epsilon)$ be the utility generate by the trembling actions

$$Q(\epsilon) = \int \int [v_a^*(p) - c(a,t)] \, \epsilon(a) da \, d\nu_t$$

$$\underbrace{\left(\int \left[\max_b v(b,s) - \tilde{p}^i(b,s) - c(b,i) \right] di + \int \left[\max_s \tilde{p}^j(b,s) - c(s,j) \right] dj \right) \left(1 - \int \epsilon(a) da \right)}_{\text{Optimized Choice}}$$

$$+ \quad \underbrace{Q(\epsilon)}_{\text{Constrained Choice}} \quad .$$

But since all actions are played by trembles, $\tilde{p}^i(b,s) = \tilde{p}^j(b,s)$. Therefore they optimize the entire left expression. It is looks exactly like a static problem, which we know is efficient. ∎

Therefore, the possibility of mistakes actually rules out coordination failures. Now we can consider the limit of trembles.

A pair (ν, p) is a *perfect investment equilibria* if there exists a sequence of ϵ, such that $\lim_{k\to\infty} M(\epsilon^k) = 0$ such that $(\nu(\epsilon^k), p) \to (\nu, p)$.

Theorem 2. *If (ν, p) is a perfect investment equilibrium, then it is efficient.*

Proof. The theorem is immediate from Lemma 1 since $\underbrace{Q(\epsilon)}_{\text{Constrained Choice}} \to 0.$

∎

The theory's predictive power comes from imposing more restrictions on beliefs than just rational conjectures.[9] The trembling with a large number of agents rules out contradictory beliefs, as in Zame (2007), and ensures "price consistency", as in Makowski and Ostroy (1995). However, instead of assuming price consistency, the tremble gives a justification for sure price consistency in terms of the stability of the equilibria considered.

There are other justifications for non-contradictory beliefs. For example, Dubey and Geanakoplos (2002) consider fictitious seller who contributes an infinitesimal to each health insurance pool. Dubey, Geanakoplos, and Shubik (2005) assume that the government intervenes to sell infinitesimal quantities of each asset and fully delivers on its promises.

6 Conclusion

In this paper, I argue that, with price-taking, coordination failures rely on using beliefs as a free parameter and constructing overly pessimistic conjectures. With the free parameter, there are many equilibria. If we want predictive power, we must use a refinement, such a trembling hand perfection.

When we consider perfect equilibrium in an Walrasian assignment model with investment, every perfect equilibrium is efficient. The mathematical mechanism is that the mistakes caused by trembles generate complete markets, even though in equilibrium, markets are endogenous and incomplete.

9. The tremble itself is not directly "pushing" toward efficiency. In fact, the usual examples of perfect equilibria actually show that the perfect equilibra are the *inefficient* ones. See Selten's original paper (Selten 1975) or for a textbook example in Maschler, Solan, and Zamir (2013, p. 263).

References

Albrecht, Brian C. 2016. "Entrepreneurship as Coordination." https://briancalbrecht.github.io/albrecht-entrepreneurship-coordination.pdf. (Cited on page 3).

Brandenburger, Adam M., and Harborne W. Stuart. 2007. "Biform Games." *Management Science* 53 (4): 537–549. doi:10.1287/mnsc.1060.0591. http://pubsonline.informs.org549.https//doi.org/10.1287/mnsc.1060.0591http://www.informs.org. (Cited on page 2).

Cole, Harold L., George J. Mailath, and Andrew Postlewaite. 2001a. "Efficient Non-Contractible Investments in Finite Economies." *Advances in Theoretical Economics* 1 (1): 1–32. (Cited on page 3).

———. 2001b. "Efficient non-contractible investments in large economies." *Journal of Economic Theory* 101 (2): 333–373. ISSN: 00220531. doi:10.1006/jeth.2001.2797. (Cited on pages 3, 5, 9).

Cooper, Russell, and Andrew John. 1988. "Coordinating Coordination Failures in Keynesian Models." *The Quarterly Journal of Economics* 103 (3): 441–463. doi:10.2307/1885539. https://academic.oup.com/qje/article-lookup/doi/10.2307/1885539. (Cited on page 4).

Dubey, Pradeep, and John D. Geanakoplos. 2002. "Competitive Pooling: Rothschild-Stiglitz Reconsidered." *The Quarterly Journal of Economics* 117, no. 4 (November): 1529–1570. ISSN: 0033-5533. doi:10.1162/003355302320935098. https://dx.doi.org/10.1162/003355302320935098. (Cited on pages 4, 16).

Dubey, Pradeep, John D. Geanakoplos, and Martin Shubik. 2005. "Default and Punishment in General Equilibrium." *Econometrica* 73, no. 1 (January): 1–37. doi:10.1111/j.1468-0262.2005.00563.x. https://doi.org/10.1111/j.1468-0262.2005.00563.x. (Cited on pages 4, 16).

Felli, Leonardo, and Kevin Roberts. 2016. "Does Competition Solve the Hold-up Problem?" *Economica* 83 (329): 172–200. doi:10.1111/ecca.12170. (Cited on pages 3, 10).

Gale, Douglas. 1992. "A Walrasian Theory of Markets with Adverse Selection." *The Review of Economic Studies* 59 (2): 229–255. ISSN: 00346527, 1467937X. doi:10.2307/2297953. http://www.jstor.org.ezp3.lib.umn.edu/stable/2297953. (Cited on pages 4, 13).

———. 1996. "Equilibria and Pareto optima of markets with adverse selection." *Economic Theory* 7, no. 2 (June): 207–235. ISSN: 0938-2259. doi:10.1007/BF01213903. http://link.springer.com/10.1007/BF01213903. (Cited on page 4).

Gretsky, Neil E., Joseph M. Ostroy, and William R. Zame. 1999. "Perfect Competition in the Continuous Assignment Model." *Journal of Economic Theory* 88:60–118. (Cited on page 9).

Hayek, Friedrich A. 1937. "Economics and Knowledge." *Economica* 4 (13): 33–54. http://www.jstor.org/stable/2548786. (Cited on page 11).

Klein, Daniel, and Aaron Orsborn. 2009. "Concatenate Coordination and Mutual Coordination." *Journal of Economic Behavior & Organization* 72 (1): 176–187. ISSN: 01672681. doi:http://dx.doi.org/10.1016/j.jebo.2009.05.003. (Cited on page 3).

Makowski, Louis. 2004. "Pre-Contractual Investment without the Fear of Holdups: The Perfect Competition Connection." (Cited on pages 3, 5, 9).

Makowski, Louis, and Joseph M. Ostroy. 1995. "Appropriation and Efficiency: A Revision of the First Theorem of Welfare Economics." *The American Economic Review* 85 (4): 808–827. http://www.jstor.org/stable/2118233. (Cited on pages 2, 3, 16).

Maschler, Michael, Eilon Solan, and Shmuel Zamir. 2013. *Game Theory.* First. New York: Cambridge University Press. ISBN: 9781139049344. (Cited on page 16).

Nöldeke, Georg, and Larry Samuelson. 2014. "Investment and Competitive Matching." New Haven, CT. http://cowles.econ.yale.edu/. (Cited on page 5).

———. 2015. "Investment and Competitive Matching." *Econometrica* 83, no. 3 (May): 835–896. doi:10.3982/ECTA12349. http://doi.wiley.com/10.3982/ECTA12349. (Cited on pages 3, 9).

O'Driscoll, Gerald P. 1977. *Economics as a Coordination Problem: The Contributions of Friedrich A. Hayek.* Kansas City: Sheed Andrews / McMeel, Inc. (Cited on page 3).

Scheuer, Florian, and Kent Smetters. 2018. "How Initial Conditions Can Have Permanent Effects: The Case of the Affordable Care Act." *American Economic Journal: Economic Policy* 10 (4): 302–343. doi:10.1257/pol.20140204. https://doi.org/10.1257/pol.20140204. (Cited on page 4).

Selten, Reinhard. 1975. "Reexamination of the perfectness concept for equilibrium points in extensive games." *International Journal of Game Theory* 4 (1): 25–55. ISSN: 1432-1270. doi:10.1007/BF01766400. https://doi.org/10.1007/BF01766400. (Cited on pages 4, 16).

Smith, Vernon L. 1982. "Markets as Economizers of Information: Experimental Examination of the "Hayek" Hypothesis." *Economic Inquiry* 20, no. 2 (April): 165–179. doi:10.1111/j.1465-7295.1982.tb01149.x. https://doi.org/10.1111/j.1465-7295.1982.tb01149.x. (Cited on page 10).

Zame, William R. 2007. "Incentives, Contracts, and Markets: A General Equilibrium Theory of Firms." *Econometrica* 75, no. 5 (September): 1453–1500. ISSN: 0012-9682. doi:10.1111/j.1468-0262.2007.00799.x. http://doi.wiley.com/10.1111/j.1468-0262.2007.00799.x. (Cited on pages 4, 12, 16).