Some Price Theory of Price Dispersion

Brian C. Albrecht*

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1 Introduction

This part of my series of notes that on using a simple supply and demand framework of economic theory. This specific note presents a theory of price dispersion (Burdett and Judd 1983) and draws on a paper of mine that utilizes the same techniques (Albrecht 2019). They purposefully rely on simple theory so they can be accessible to students and non-theorists, using theory as an engine of analysis. I teach the theory parts of my courses with almost only supply and demand, but a very broad understanding. I use these notes when I use supply and demand in a non-standard way.

2 Example: Knowing One or Two Sellers

There are two risk-neutral sellers who can produce a good with a marginal cost of zero. Consider a continuum of buyers of three types: those who can purchase from seller 1, those who can purchase from seller 2, and those who can purchase from both sellers. These differences can be grounded in search costs or knowledge constraints. A market is a probability

*albre155@umn.edu, University of Minnesota.

distribution over the three types: (x_{10}, x_{01}, x_{11}).

The market with three types can be visualized as a point in the two-dimensional simplex (see Mas-Colell, Winston, and Green (1995, 196)), as in Figure 1. The extreme points, which place probability one on a specific valuation, are denoted by $x^{\{ij\}}$.

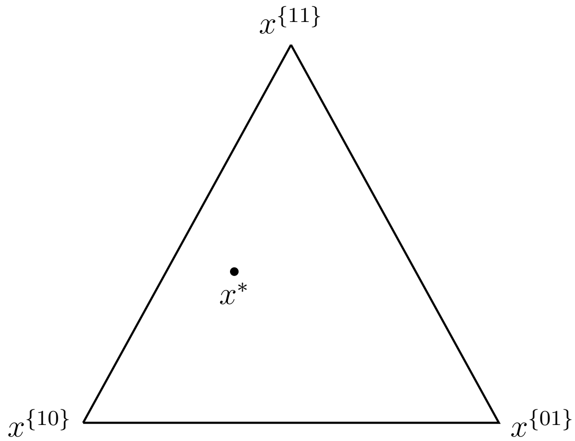
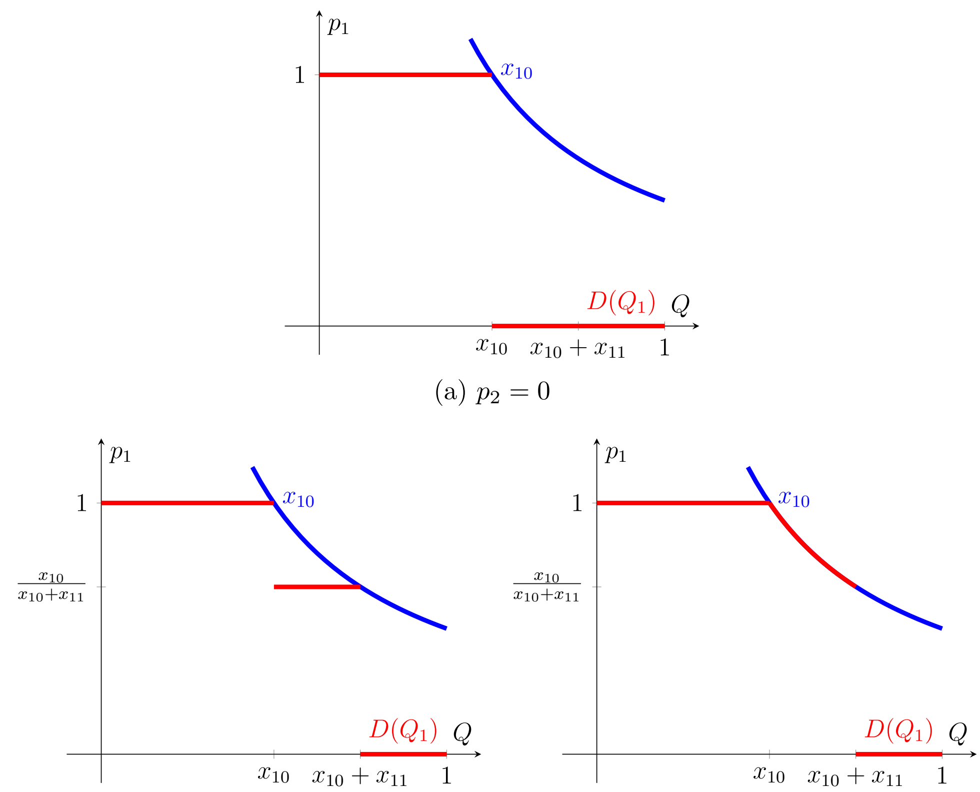


Figure 1: Simplex of two loyal types and one indifferent type

2.1 Solving for Equilibrium

Fixed the prior belief of $x^* = (x_{10}, x_{01}, x_{11})$. If $x_{11} = 0$, then the equilibrium is obvious. Since there is no competition, each seller can charge a price of one and sell whenever the buyer can buy from her. The interesting trade-off occurs when $x_{11} > 0$ and there is some competition (which is all of the simplex, except the bottom line). While not as obvious as when $x_{11} = 0$, when $x_{11} > 0$ the equilibrium is straightforward to solve for using simple features of the expected, residual, inverse demand curve. To simply language, I will just refer to seller i's demand curve, realizing that it has all the other adjectives included, denoted by $D(Q_i)$, where Q_i is the probability that seller i makes a sale.

Without loss, suppose $x_{10} \geq x_{01}$. To draw the curves in Figures 2 and 3, I use $x_{10} = 1/2$ and $x_{01} = 1/4$, but nothing in the argument depends on those exact numbers. To find the equilibrium, consider seller 1's demand curve when seller 2 sets $p_2 = 0$ (Figure 2a). Her demand curve is a step function. If seller 1 wants to sell a quantity of less than or equal to her loyal customers, she can charge a price of one. If she wants to sell to more than her loyal customers, she needs to match seller 2 and set $p_1 = p_2 = 0$. In general, her demand curve will be one for when the buyer is loyal to seller 1, a function of p_2 for when the customer is indifferent, and zero for when the buyer is loyal to seller 2.



(a) $p_2 = 0$

(b) $p_2 = \underline{p}$

(c) $F_2^*(p)$

Figure 2: Seller 1's Demand and Indifference Curves

Translating demand curves into game-theoretic terms, best-responding by each seller means choosing a price and quantity subject to the constraint that the pair is on her demand curve. To find the best-response, plot seller 1's indifference curves, which are just the iso-revenue curves, since cost is zero. Figure 2a plots the curve for the highest attainable revenue, given $p_2 = 0$.[1] The best-response price is one and the expected quantity that seller 1 sells is

1. It likely exists, but I have not seen indifference curves plotted in Marshallian, quantity-price coordinates

x_{10}.[2] This is also the worst-case (maximin) profit for seller 1, since he could never do worse than if $p_2 = 0$.

However, setting a price of zero is not optimal for seller 2. Now I ask, what is the highest pure strategy price that seller 2 can charge without changing one's behavior? That case is clearly a better-reply, even if it is not a best-reply. Increasing the price of seller 2 increases seller 1's demand curve over the range $(x_{10}, x_{10} + x_{11})$, that is for the indifferent types, by p_2. By plotting seller 1's iso-revenue curve, we can see that seller 2's price can be increased to $\frac{x_{10}}{x_{10}+x_{11}}$ without inducing seller 1 to change. While this is not going to be an equilibrium, it will allow us to bound prices. First, it shows that it is never a best-response for seller 2 to set a price lower than $\underline{p}$. She can raise her price to $\underline{p}$ while one is still best-responding with her price of one. It also creates a lower bound for seller 1's price. Even if seller 1 sold every time the buyer was not loyal to the seller 2, $Q_1 = x_{10} + x_{11}$, she would still make less profit for any price below $\underline{p}$ than if she just set a price of one. Therefore, both sellers will never drop their price below $\underline{p}$ in any equilibrium.

Seller 2 can still do better by randomizing. between $\underline{p}$ and one. This is like setting a non-linear price and induces a residual demand curve for seller 1 that is no longer flat in the middle.[3] The demand curve varies with the cumulative distribution of seller 2's randomized strategy. Again, the demand curve for seller is the price set by seller 2, but now in a probabilistic sense.[4] Better-replying by seller 2 With randomization means increasing seller 1's demand curving on all prices between $\underline{p}$ and one such that seller 1 is still indifferent. Figure 2c shows the demand curve where seller 1 is indifferent between a range of prices,

in writing. I thank Kevin M. Murphy for pointing out this tool.

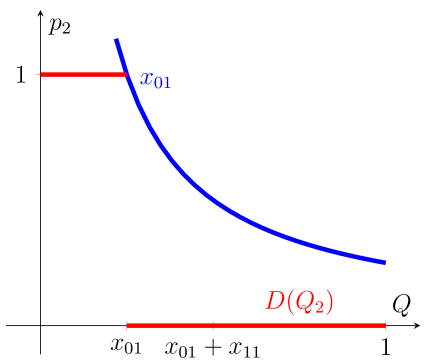
2. The best-response could also be found by constructing the marginal revenue (MR) curve. However, because MR is discontinuous, the optimal expected quantity is the maximum quantity such that MR > MC = 0, instead of the standard MR=MC condition used for continuous functions.

3. Using the indifference curve to trace out the optimal policy resembles one construction of the optimal income tax rate in an optimal income tax model, like Mirrlees (1971).

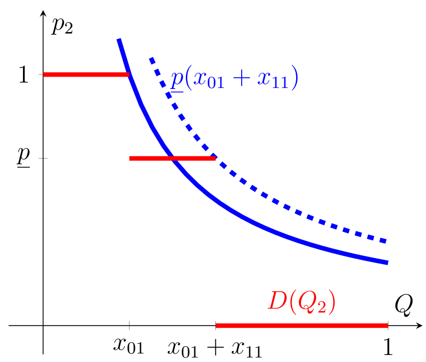
4. In fact, the residual demand curve for the indifferent buyer is simply one minus the CDF of seller 2's randomized price. The only complication is a specific price is set with positive probability.

which is generate by a particular pricing strategy (Equation 1). This pricing strategy is also seller 2's equilibrium pricing strategy, as explain in Observation 1.

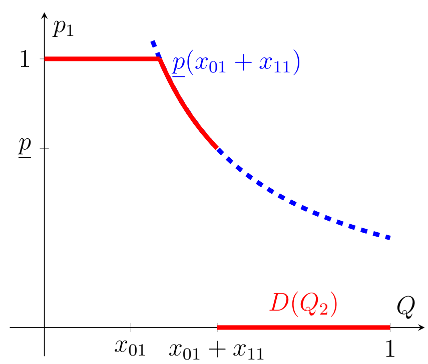
Notice that the process above for finding equilibrium is the standard way of solving for a mixed-strategy equilibrium but just applied to a supply and demand setting with a continuum of prices. The same approach can be applied to finding the randomization by seller 1 to make seller 2 indifferent. However, seller 2's guarantee is pinned down by pricing at $\underline{p}$. Figure 3c shows the demand curve where seller 1 is indifferent between all prices between $\underline{p}$ and 1.



(a) $p_1 = 0$



(b) $p_1 = \underline{p}$



(c) $F_1^*(p)$

Figure 3: Seller 2's Demand and Indifference Curves

Figures 2 and 3 combine to give us equilibrium. For each seller i, any price between $\underline{p}$ and 1 is a best-response to the strategy of seller j. Any randomization over those prices is also a best-response, in particular. In particular, the randomization used to make j indifferent is a best-response. Therefore, the combined strategies make up an equilibrium. In fact, it is the unique equilibrium. Equilibrium is summarized more succinctly in the following observation:

Observation 1. Let $x_{10} \geq x_{01}$. Then the unique equilibrium revenue is x_{10} for seller 1 and $\frac{x_{10}}{x_{10}+x_{11}}(1-x_{10})$ for seller 2. Therefore the expected total revenue and expected price is $x_{10} + \frac{x_{10}}{x_{10}+x_{11}}(1-x_{10})$ and the expected consumer surplus is $1 - \left(x_{10} + \frac{x_{10}}{x_{10}+x_{11}}(1-x_{10})\right)$.

If $x_{10} < 1$, then the unique equilibrium pricing strategies are

$$F_1^*(p) = \begin{cases} 0 & p < \underline{p} \\ 1 - \dfrac{\underline{p}(x_{11}+x_{01}) - px_{01}}{px_{11}} & p \in \left[\underline{p}, 1\right) \\ 1 & p \geq 1 \end{cases} \qquad F_2^*(p) = \begin{cases} 0 & p < \underline{p} \\ 1 - \dfrac{x_{10}(1-p)}{px_{11}} & p \in \left[\underline{p}, 1\right], \end{cases} \tag{1}$$

where the minimum price $\underline{p} = \frac{x_{10}}{x_{10}+x_{11}}$. If $x_{10} = 1$, any strategy is optimal for seller 2.

Figure 4 plots the equilibrium pricing strategies from Equation 1 for the case where $x_{10} = \frac{1}{2}$ and $x_{01} = \frac{1}{4}$ used for the demand curves. Since the consumer is often loyal, low prices are never a best-response and the price distribution is shifted toward one. Seller 1 sets $p = 1$ with a positive probability of $\frac{x_{10}-x_{01}}{x_{10}+x_{11}}$. If these distributions are rotated, we are then left with the middle portion of the residual inverse demand curve in Figures 2c and 3c.

Let us consider that a bit further. Seller 1 can guarantee herself revenue x_{10} by setting a price of 1 and only receiving loyal customers. That is, seller 1 is only able to achieve her maximin payoff. Seller 2 can guarantee herself $\underline{p}(x_{01}+x_{11})$ by setting the lowest price and receiving all customers that are not loyal to customer one.[5] However, neither can achieve more in equilibrium, resembling the normal Bertrand force that drives profits to zero. The

5. If the parties are symmetric ($x_{01} = x_{10}$), each seller receives her loyal customers and we are back to perfect competition, where each person receives her marginal product (Ostroy 1984), as is standard for symmetric Bertrand competition.

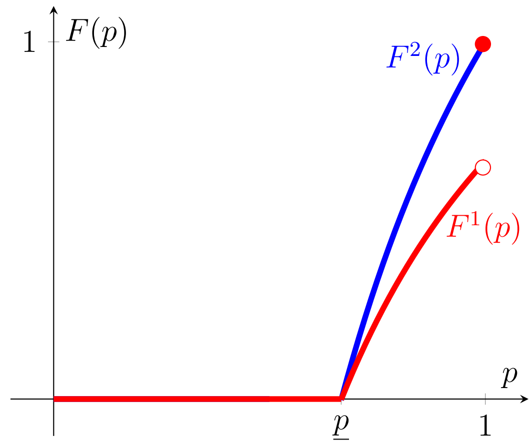


Figure 4: Equilibrium Pricing Strategies

randomization protects against undercutting by the other seller.[6]

The favored seller's revenue does not depend on the number of loyal customers the other seller has. In contrast, the seller 2's revenue is increasing and then decreasing with an increase in the amount of loyal customers for seller 1. For the increasing part, seller 2 is able to free-ride on the fact that 1 is raises her minimum price as x_{10} increases.

6. This resembles matching pennies or the information design model in Albrecht (2017), where there is political competition with discontinuities and randomization prevents the other party from winning. However, randomization does little to the odds of winning in equilibrium. Both matching pennies and political competition are zero-sum games. With this is technically not zero-sum, it shares some characteristics.

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