Consumer Data and the Limits of Price Competition

Brian C. Albrecht

University of Minnesota

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Information and Price Discrimination

- Firms are collecting more and more data on consumers
- Information is vital to a firm's ability to use third-degree price discrimination
- Monopoly: information for price discrimination weakly raises total profit
  - Information is valuable for single receiver: Blackwell 1951, 1953
  - General information: Bergemann, Brooks, and Morris 2015
- Unless output increases, price discrimination lowers consumer surplus

Competition and Price Discrimination

- One purpose of price discrimination is to lure away buyers from other sellers
- Studying this aspect of price discrimination requires an explicit model of competition
- Competition: price discrimination can lower total profits
  - Holmes 1989

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  2. Ultimately (not yet) bound equilibrium outcomes

Main Results

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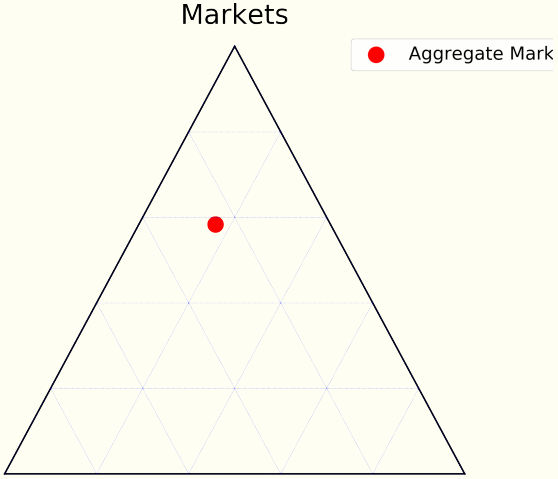
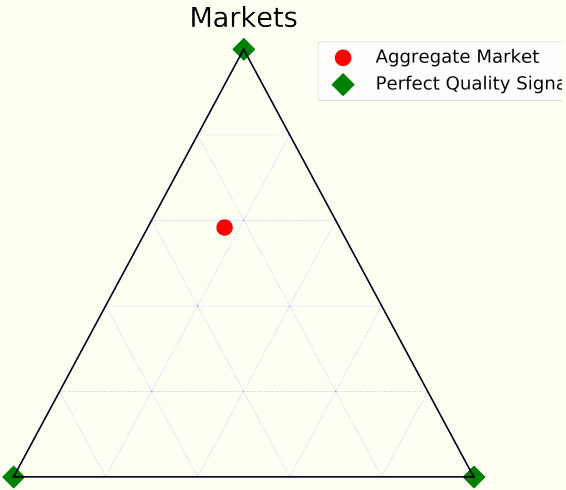
  - Complete information $\rightarrow$ standard Bertrand competition
- Maximum profit: partial price discrimination
  - Symmetric information: 2 signals, one that favors each seller $\rightarrow$ create pseudo-monopoly
  - Asymmetric information: imperfectly correlated signals $\rightarrow$ further implicit collusion

Example: Information Decreases Profits

- 3 types of buyers who are willing to pay at most 1
  1. Loyal to seller 1, only buy from 1
  2. Loyal to seller 2, only buy from 2
  3. Indifferent between seller 1 and seller 2, buy at cheapest price
- Informal definition: a market for seller *i* is a distribution of types of buyers
- Example: $\left(\frac{1}{4}, \frac{1}{6}, \frac{7}{12}\right)$


Markets
Aggregate Mark


Markets
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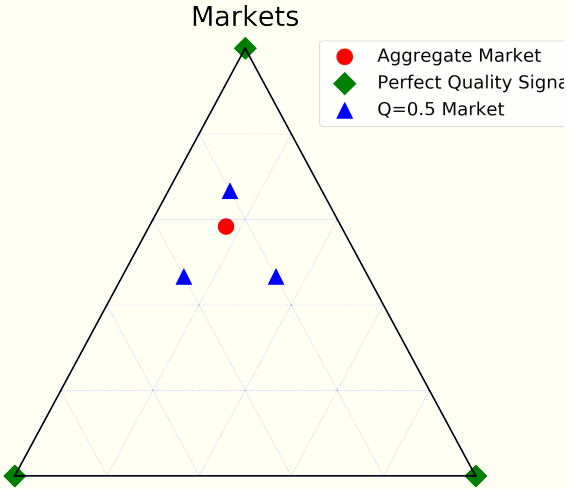
Perfect Quality Signa


Markets
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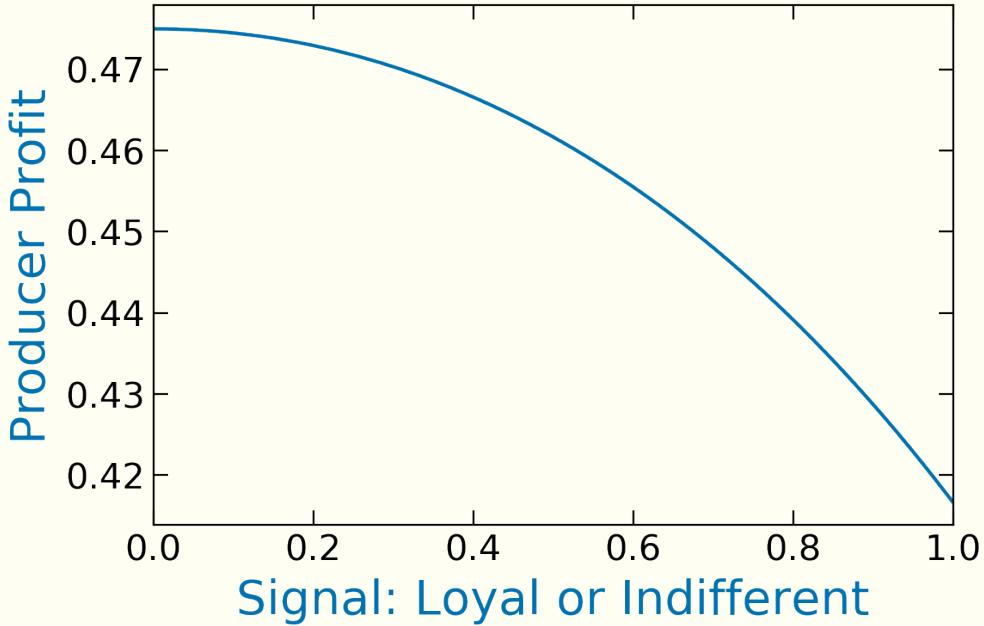
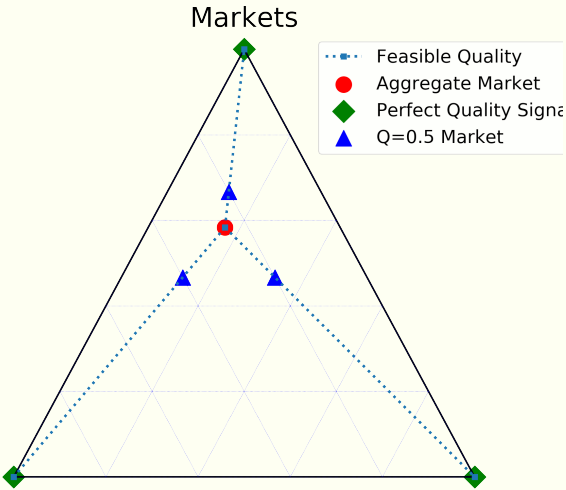
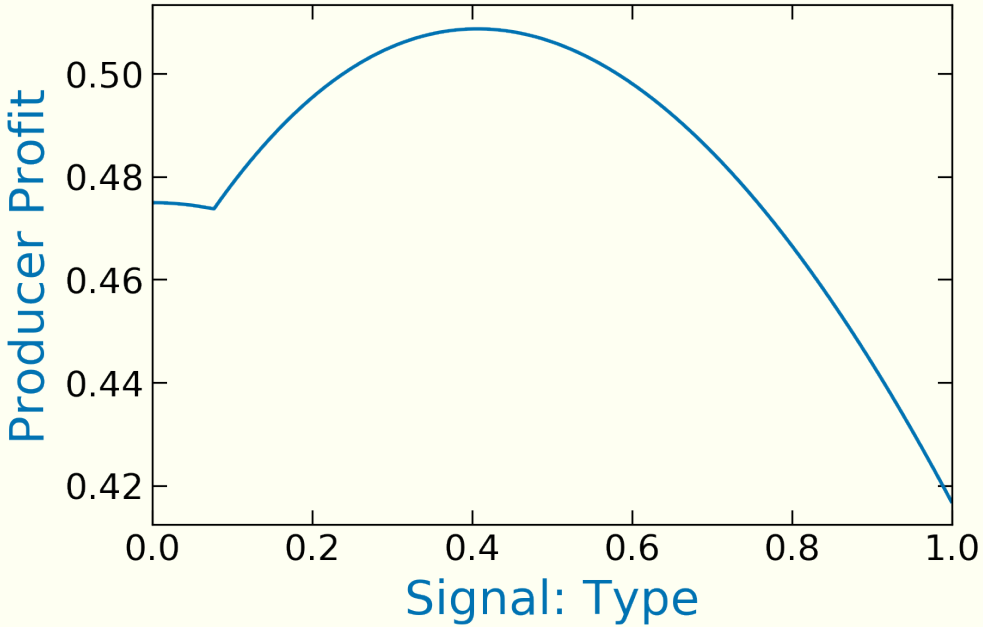
Perfect Quality Signa
Q=0.5 Market

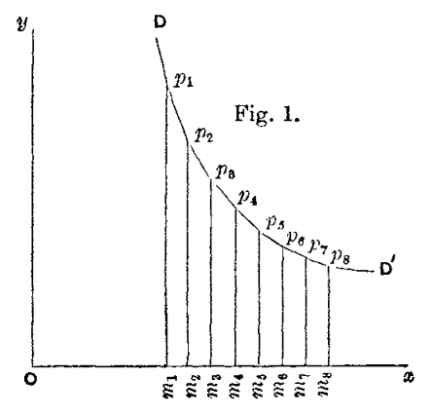

Producer Profit
0.47
0.46
0.45
0.44
0.43
0.42
0.0
0.2
0.4
0.6
0.8
1.0
Signal: Loyal or Indifferent


Markets
Feasible Quality
Aggregate Market
Perfect Quality Signa
Q=0.5 Market

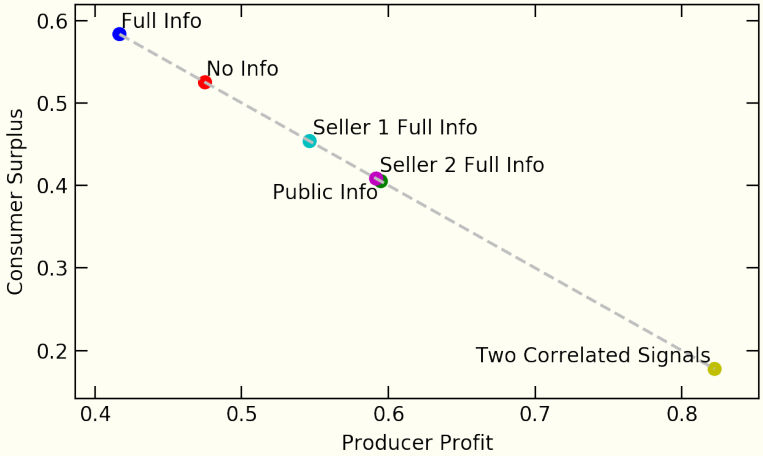

Producer Profit
0.50
0.48
0.46
0.44
0.42
0.0
0.2
0.4
0.6
0.8
1.0
Signal: Type

Robust Models of Competition

- Fix the underlying valuations, i.e. demand curve in the aggregate market
- Vary the information to each seller, allowing price discrimination
- Not all firms need to have the same information $\implies$ higher order beliefs matter
- E.g. Target doesn't know my exact address, but Target knows that Amazon knows...



Price Discrimination under General Information



Roadmap of Talk

Basic Game: Types

- 2 sellers with differentiated goods that cost 0 to produce
- Continuum of non-strategic consumers each have unit demand for total consumption
- A consumer's type is a pair of valuations

$$v = (v_1, v_2) \in \{(1,0), (0,1), (1,1)\} = V$$

- Common prior: (m_{10}, m_{01}, m_{11})

Information Structure

- An information structure is a set of signals for each seller S_i,
- and a probability distribution which maps the profile of the consumer's values to the profile of signals:

$$\pi : V \to \Delta(S).$$

- The utility functions and the information structure (S, π) are the parameters for a game of incomplete information
- Define the rest of the game fixing (S, π)

Strategies: Fixed (S, π)

- Fix (S, π)
- Seller i observes a signal $s_i \in S_i$
- Pure strategy for seller i is a price $\{p_i\}_{s_i} \in \mathbb{R}_+^{|S_i|}$ and
- Discontinuity of payoffs requires mixed prices (price dispersion)
- Mixed strategy, $F_i(p|s_i)$, is the probability that $p_i \leq p$ given receiving a signal s_i
  - $f_i(p|s_i)$ = density associated with F_i, when defined

Equilibrium

- A strategy profile is a Bayes Nash equilibrium (BNE) if $f_i(p_i|s_i) > 0$ implies

$$p_i \in \arg\max_{p_i'} \quad p_i' \underbrace{\mathbb{E}[v_i = 1, v_j = 0|s_i]}_{\text{Loyal}} + p_i' \underbrace{\mathbb{E}\left[\left(1 - F_j(p_i)\right), v_i = 1, v_j = 1|s_i\right]}_{\text{Indifferent,}}$$

given $F_j(p)$, for all s_i, s_j, i, j.

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- A strategy profile is a Bayes correlated equilibrium (BCE) if it is a BNE for some information structure (Bergemann and Morris 2016)

Roadmap of Talk

Overview of results

Model

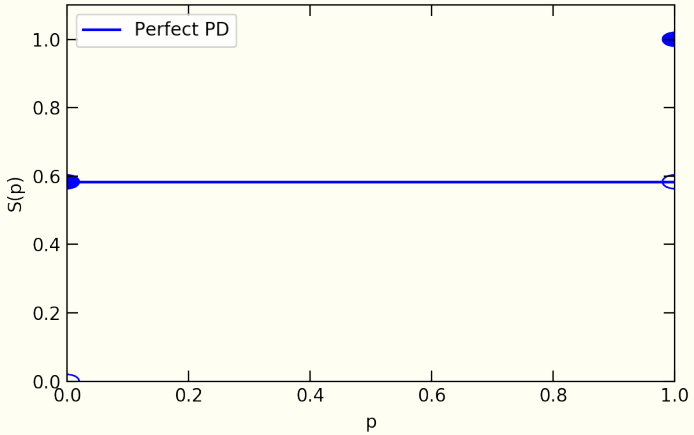
Symmetric Information
- Complete Information: Perfect Price Discrimination
- No Information: No Price Discrimination
- Symmetric Information: Imperfect Price Discrimination

Asymmetric Information

Complete Information: Perfect PD

- One case is complete information: $s_1 = s_2 = v$
- If $v_i = 0$, seller j sets monopoly price of 1
- If $v_1 = v_2 = 1$, both sellers set competitive price of 0

Complete Information



- Profit = Expected price = $m_{10} + m_{01}$ = area above price distribution

No Information: No PD

- Conditional on $v_1 = 1$, seller 1 assigns probability $\frac{m_{10}}{m_{10}+m_{11}}$ to being the monopolist
- Regardless of what seller 2 does, seller 1 will never set a price below $\underline{p} = \frac{m_{10}}{m_{10}+m_{11}}$
- Neither will seller 2

Proposition 1

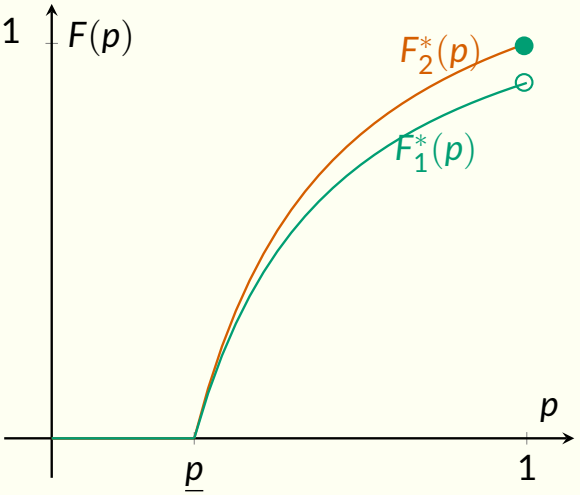
Let $m_{10} \geq m_{01}$. The unique BNE profit is m_{10} for seller 1 and $\frac{m_{10}}{m_{10}+m_{11}}(1-m_{10})$ for seller 2. The unique strategies are given by

$$F_1^*(p) = \begin{cases} 0 & p < \underline{p} \\ 1 - \dfrac{\underline{p}(m_{11}+m_{01}) - pm_{01}}{pm_{11}} & p \in \left[\underline{p}, 1\right) \\ 1 & p \geq 1 \end{cases}$$

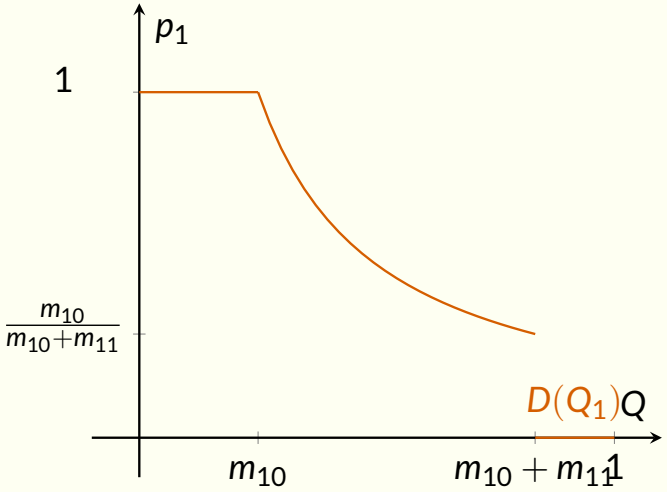
$$F_2^*(p) = \begin{cases} 0 & p < \underline{p} \\ 1 - \dfrac{m_{10}(1-p)}{pm_{11}} & p \in \left[\underline{p}, 1\right], \end{cases}$$

- The construction relies on simple observations of the each seller's best-response when facing a residual demand curve

Distribution of Prices



Seller 1's Residual Demand



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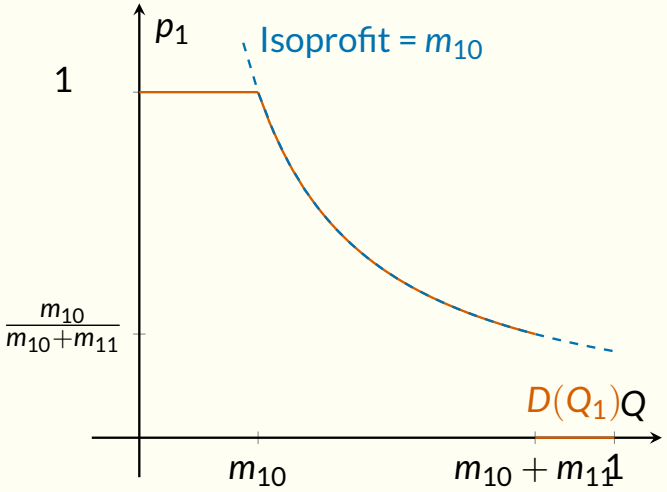
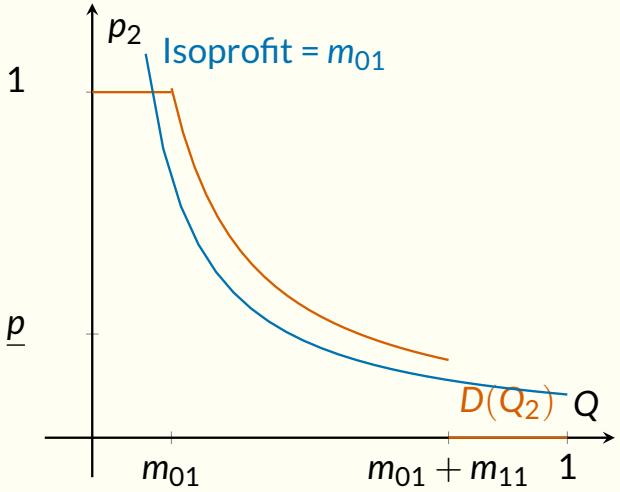
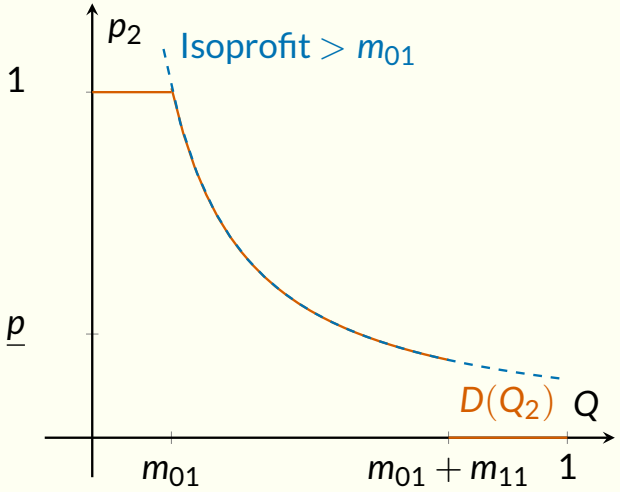


Seller 2's Residual Demand



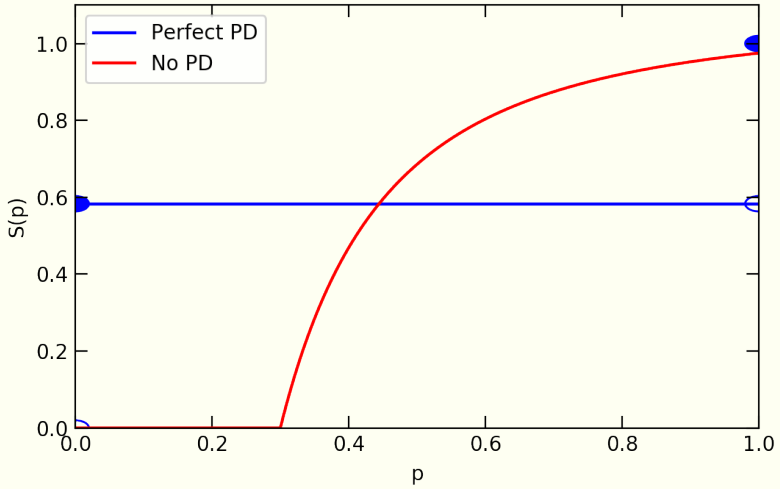
Seller 2's Residual Demand



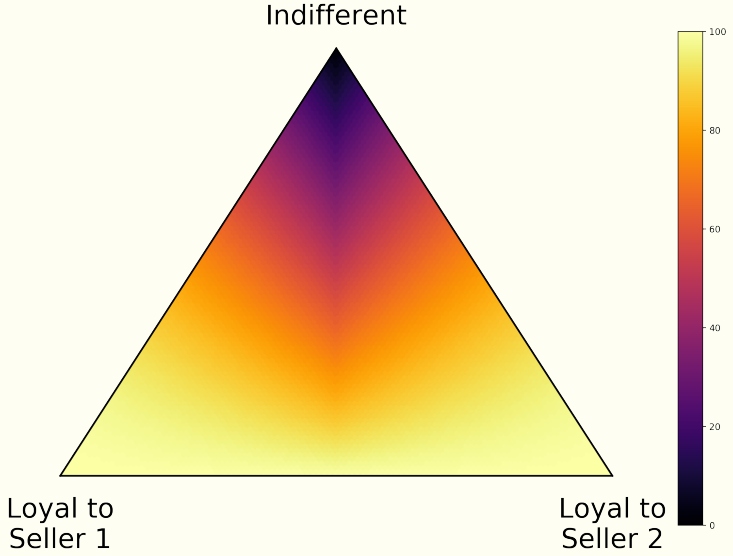
General Construction of Equilibrium

- Due to the discontinuity of payoffs in Bertrand competition, if $p \neq 1$, every equilibrium involves a distribution of prices
- Indifference $\implies$ $P \times Q =$ constant
- Distribution is proportional to $-\frac{1}{p}$ with possible mass at $p = 1$

No Information



Producer Profit



Corollary 1

Consumer surplus under perfect price discrimination is weakly higher than consumer surplus under no price discrimination.

Producer surplus under perfect price discrimination is weakly lower than producer surplus under no price discrimination.

The relationships are strict if $m_{10} \neq m_{01}$.

Benefits of Price Discrimination?

- Does price discrimination under competition always lower expected price? NO
- Prices can increase/decrease relative to no price discrimination
- Above result relies on a specific information structure to induced price discrimination

Symmetric Information

- Suppose there is symmetric learning; everyone sees the same signal

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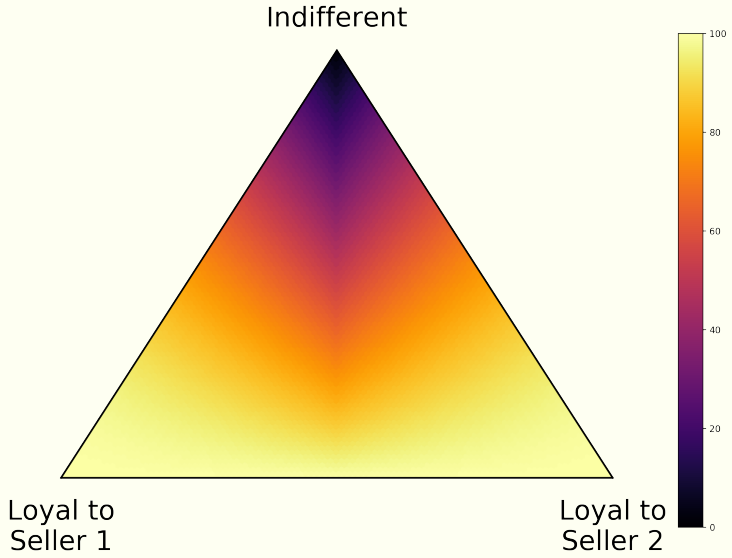
- Upon seeing a signal, both sellers identically update their beliefs
- Firms can now set different prices based on the information
- Bayes' Law restricts the new distributions of types to sum to the prior
- Maximize/minimize over all feasible distributions by "concavification"
  Kamenica and Gentzkow (2011)

Producer Profit



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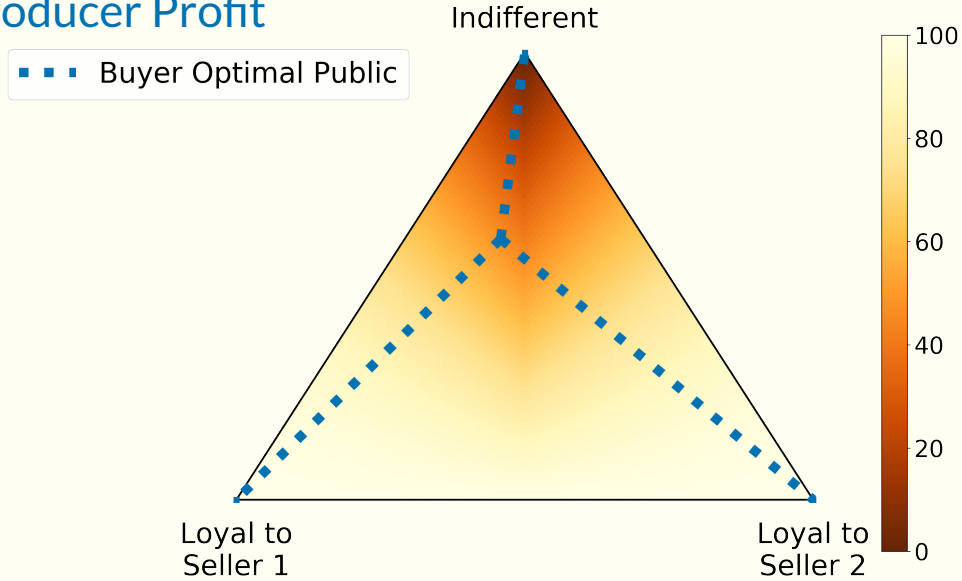
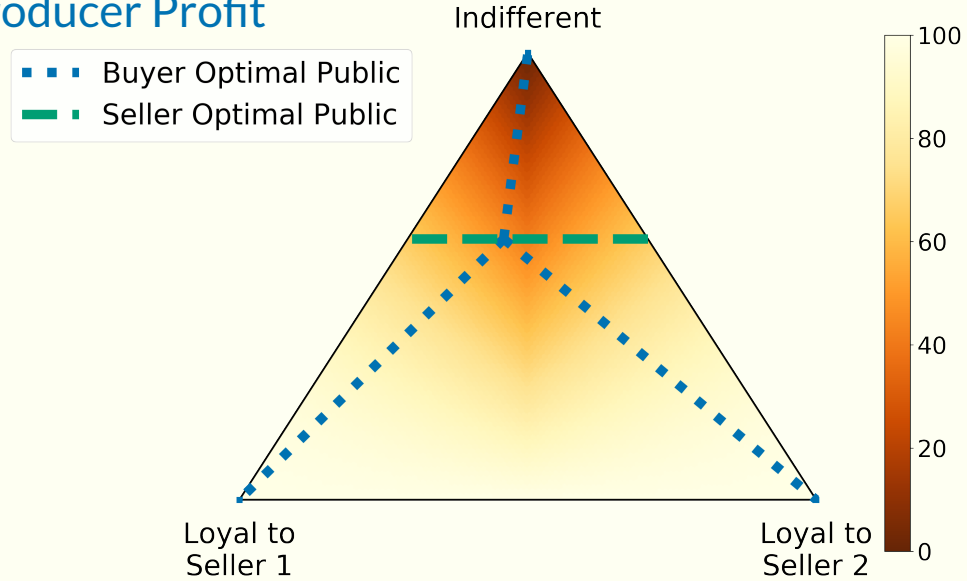


Producer Profit



Optimal Symmetric Information

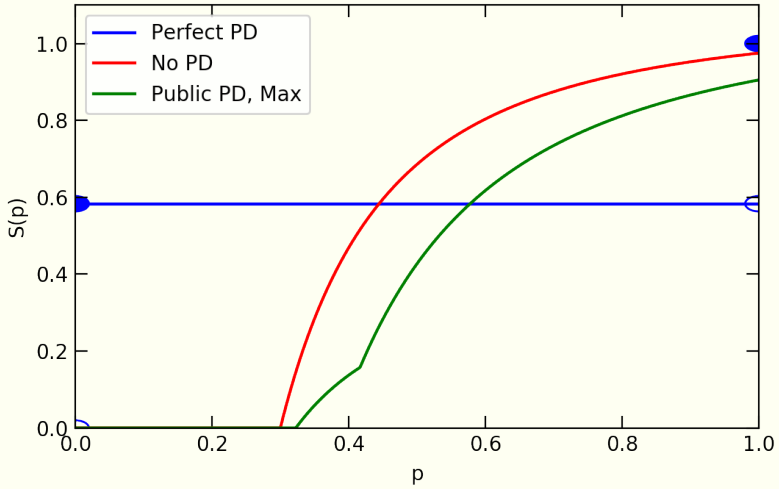
Corollary 2

Out of all symmetric information, total producer profit is minimized and consumer surplus is maximized under perfect price discrimination

Corollary 3

Out of all symmetric information, total producer profit is maximized and consumer minimized when sellers only learn to whom the buyer is *not* loyal.
That is, the concave envelop is a splitting that is horizontal in the simplex.

Symmetric Information



Roadmap of Talk

Overview of results

Model

Symmetric Information

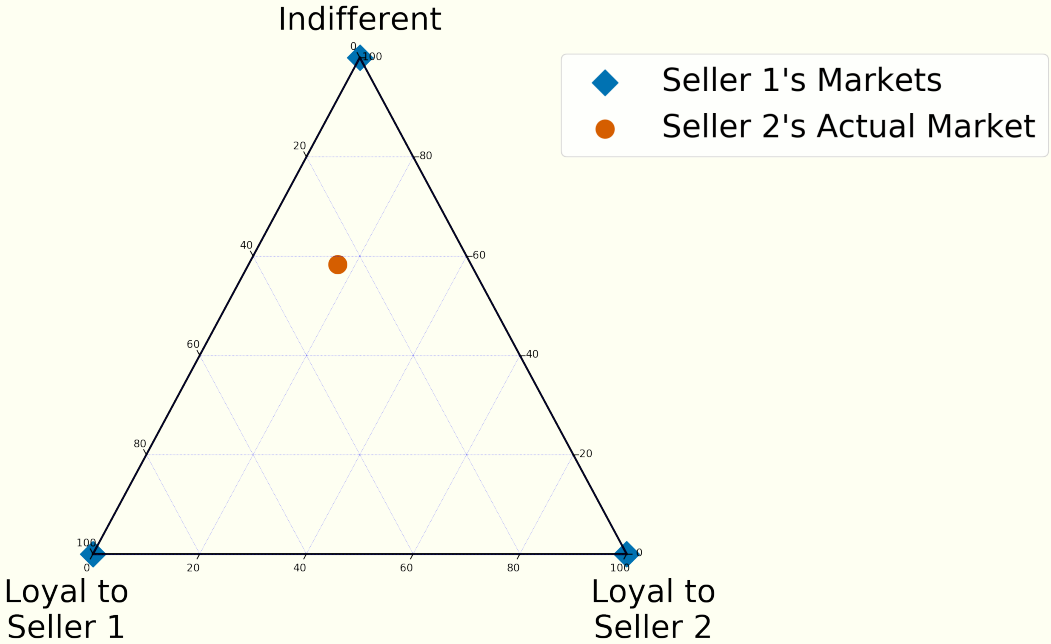
Asymmetric Information
- One Seller Price Discrimination
- Correlated Price Discrimination

Private Information

- Most models of market competition assume sellers have symmetric information, as above
- I am a different person according to Amazon vs. Target
- Amazon: South Minneapolis male who buys too many economics books
- Target: person shopping in Minneapolis on Tuesday
- Pricing strategies will still be related; Target still needs to consider Amazon's pricing

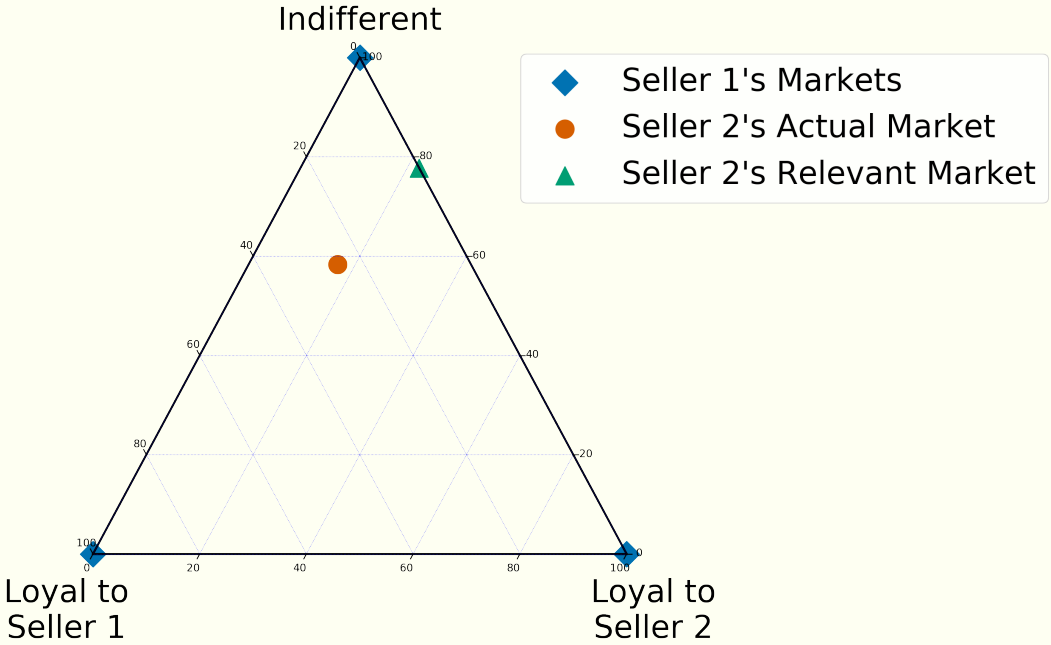
One Seller Price Discrimination

- Seller 1 has complete information
- Seller 2 has only aggregate information
- Again, each market for a seller is a point on the simplex


Indifferent
Seller 1's Markets
Seller 2's Actual Market
Loyal to Seller 1
Loyal to Seller 2
0
20
40
60
80
100

Higher Order Beliefs

- However, seller 2 knows that seller 1 knows the true type
- Seller 2 only needs to consider competition from seller 1 when the true type is indifferent
- For pricing, seller 2's relevant market is made of her loyal customers and indifferent


Indifferent
Loyal to Seller 1
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Seller 1's Markets
Seller 2's Actual Market
Seller 2's Relevant Market
0
20
40
60
80
100

Independent Private Information

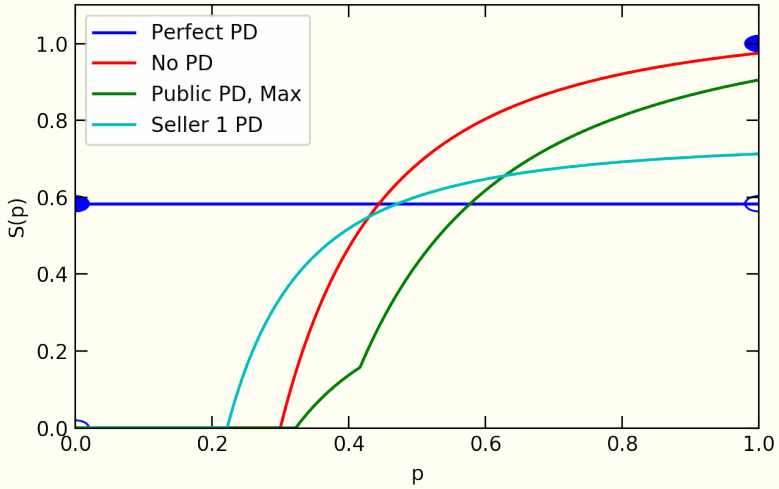
Proposition 2

The private information that allows perfect price discriminate by seller i

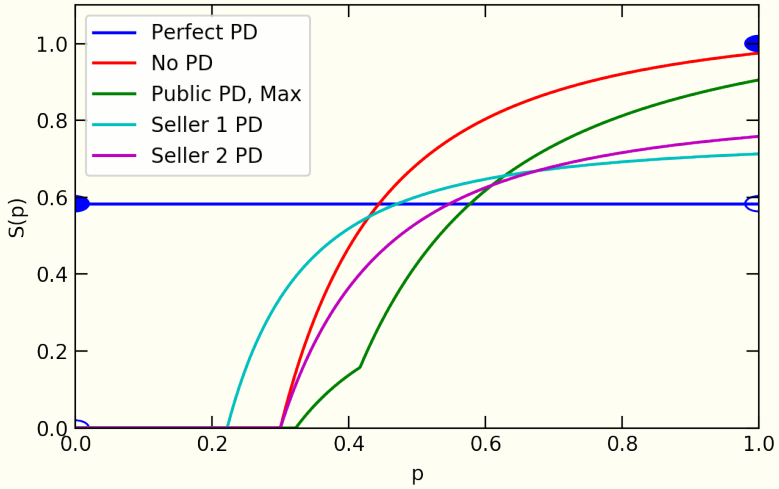
1. strictly increases seller i's profit,
2. strictly decreases seller j's, and
3. can increase or decreases total producer profit,

relative to no price discrimination.

Perfect Information: Seller 1



Perfect Information: Seller 2



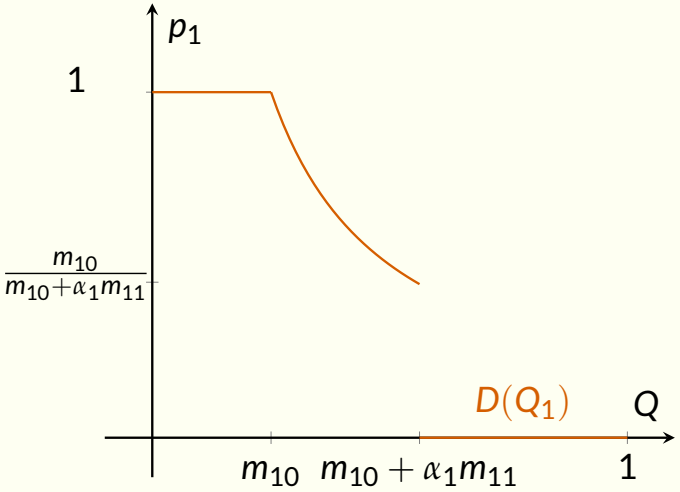
Correlated Information

- Imagine an information designer who reveals information to the sellers and recommends an incentive compatible price
- Designer commits the following information when the buyer is indifferent

	Reveal	Do Not
Reveal	0	α_1
Do Not	α_2	$1 - \alpha_1 - \alpha_2$

- The designer recommends $p = 1$ when seller sees no signal
- The designer recommends a corresponding distribution when it is revealed the seller is indifferent

No Signal to Seller 1: How High can Seller 2's Price Go?



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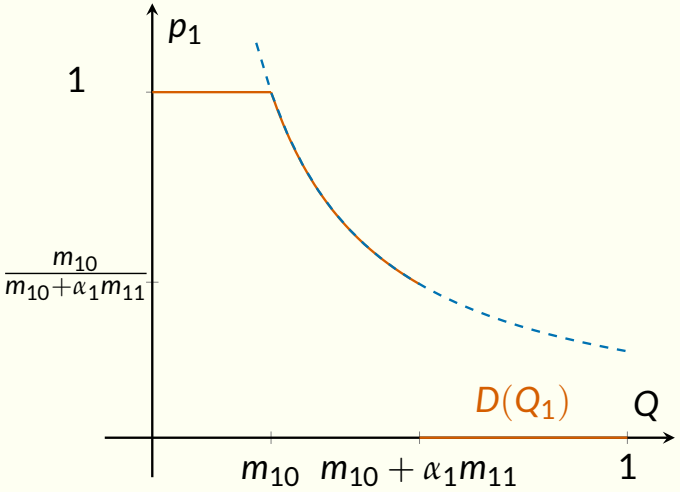
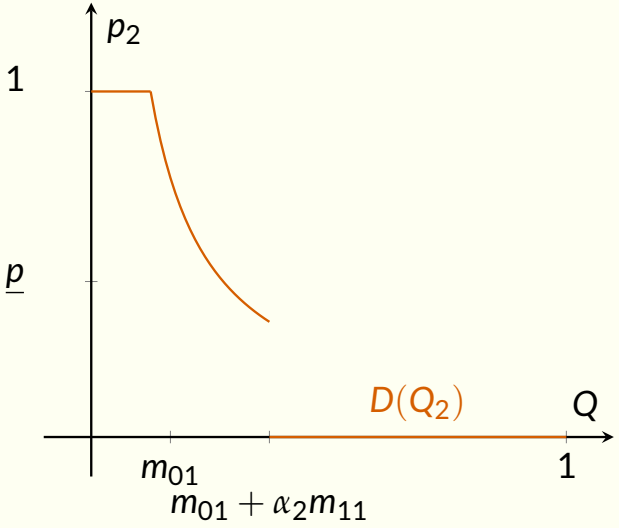


No Signal to Seller 2: How High can Seller 1's Price Go?



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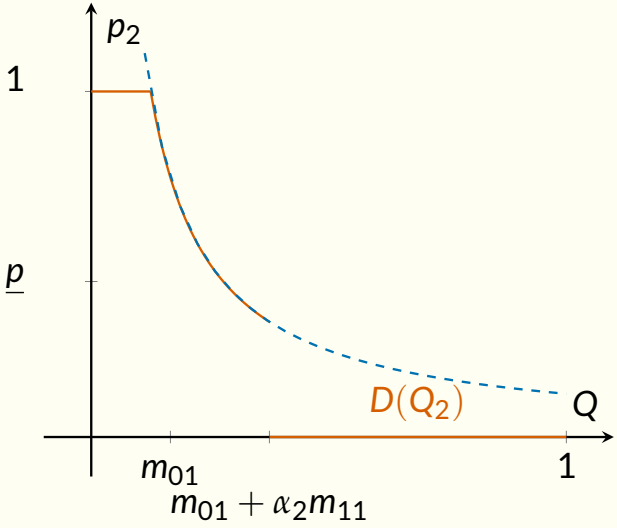
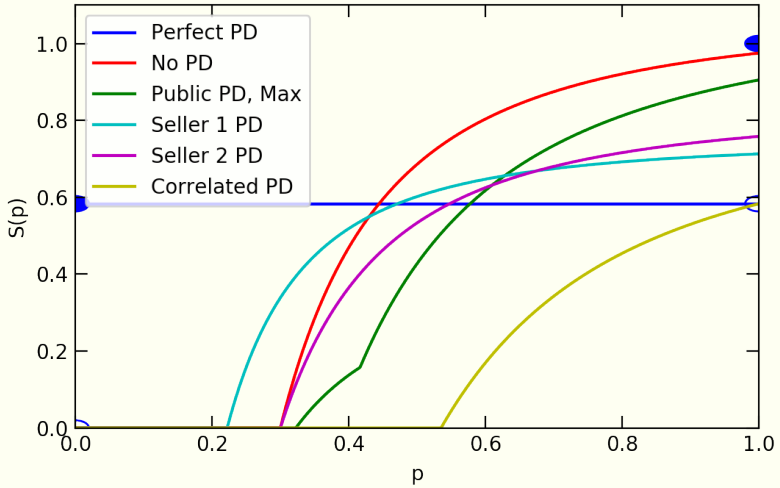
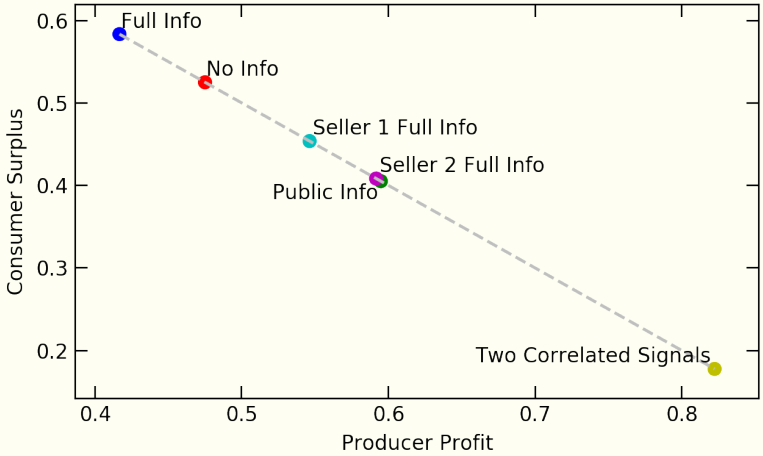


Correlated Information



Division of Surplus

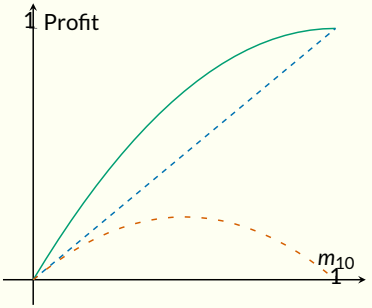


Where I'm At

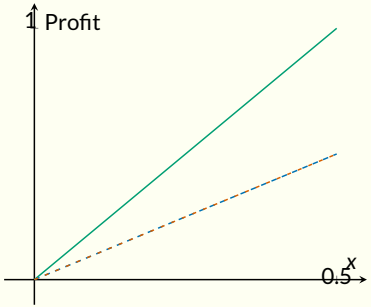
- Current Results
  - Multiple sellers, symmetric
  - Different valuations, two types, two sellers
  - Three types, symmetric information
- To Do
  - Optimal beyond two signals
  - Unify three types examples

Profit Cross-sections

$m_{01} = 0$



$m_{10} = m_{01}$



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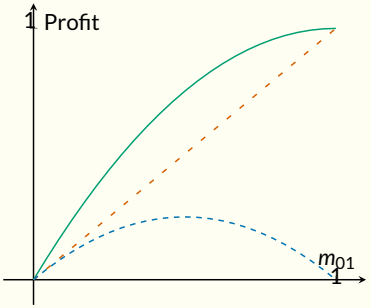

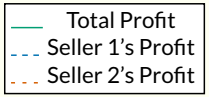

References I

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