Market Persuasion: Kamenica-Gentzkow meets Prescott-Townsend

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This paper is on the information efficiency of markets and builds on recent developments in economic theory: Bayesian persuasion (Kamenica and Gentzkow 2011) and information design (Taneva 2019). Bayesian persuasion is generally formulated as a primal optimization problem where an information designer chooses some "allocation" of information to maximize an objective function. However, the same problem has a dual formulation. In the dual problem, *prices* direct decisions. However, contrary to standard model of competition but at the heart to the information design literature, the model involves moral hazard. The setup of incorporating moral hazard into general equilibrium follows Prescott and Townsend (1984b). With moral hazard, optimal prices are not fully revealing. Instead obfuscation—or persuasion—is optimal. The main result is that there exist competitive prices that implement the information designer's allocation. But we do not need the information designer. Competition is strongest form of persuasion.

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> "The complete theory of competition cannot be known because it is an open-ended theory; it is always possible that a new range of problems will be posed in this framework, and then, no matter how well-developed the theory was with respect to the earlier range of problems, it may require extensive elaboration in respects which previously it had glossed over or ignored. "
>
> George Stigler (1957, p. 14)

1 Introduction

Since the early twentieth century, economists (Barone 1908; Mises 1920; Hayek 1935; Lange 1936; Hayek 1945) have debated the *informational* role of markets and whether other institutions (socialism) could match. Institutions were assessed on their relative ability to communicate information. The relative informational properties were defenses of capitalism for Mises and Hayek and were defenses of socialism for Barone and Lange. Information was the linchpin of that socialist calculation debate.

However, as has been pointed out by later reconstructions of the debate (Lavoie 1985; Caldwell 1997), the two sides used two different lenses for assessing questions of information. Those earliest writers did not have the tools to evaluate the range of institutions considered under one theoretical framework. It was up left up to later economists, especially Leo Hurwicz, to formally compare institutions based on their ability to communicate information.

Hurwicz (1972) introduced the concept of incentive compatibility as a means to assess the informational properties of markets and institutions more generally. From then on, as Makowski and Ostroy (1993, p. 82) observed, "the issue of incentives surfaced forcefully, as if a pair of blinders had been removed" and the institutional role for information became of secondary importance. The information of an institution was only as good as the incentives it provided people so that they would provide information. The mechanism design literature following Hurwicz, which is now quite mature, assumes a single mechanism design constructs payoffs (often monetary) to induce players to take actions. If information is generated for the designer, it is a byproduct of the appropriate incentives. That is, incentives create information for the designer. In the framework following Hurwicz, incentives were at center stage.[1]

More recently, however, the role of information in economic institutions has returned to center stage through the topic of information design (Kamenica and Gentzkow 2011; Bergemann and Morris 2013, 2016, 2019; Taneva 2019; Mathevet, Perego, and Taneva 2019). Instead of considering a mechanism designer who chooses payoffs, an information designer is modeled as choosing information to reveal to players to induce players to take actions.[2] That is, information creates incentives for the players.

Building on recent work within information design, this paper brings the question full circle by returning to the informational properties of *markets*.[3] Instead of assuming a single sender information designer can commit and perfectly control information (a primal problem), I propose a version of a price-taking equilibrium (a dual problem) to formalize the idea that individuals compete for information and to persuade each other. The dual variables have the interpretation of being the prices. Both the games played as well as the incentive-compatible outcomes are determined competitively. This yields a general equilibrium foundation for information design—closely tied to the seminal pa-

1. The connections between mechanism design and competitive equilibrium have been thoroughly studied. See Makowski and Ostroy (1987) and Makowski, Ostroy, and Segal (1999).

2. Although most of the literature focuses on a single designer, a few papers consider multiple senders (see Gentzkow and Kamenica 2016; Au and Kawai 2016; Albrecht 2017; Koessler, Laclau, and Tomala 2018). Mathematically, the current version shares more with the single sender case, although it involves a continuum of actors.

3. Albrecht (2019b) also focuses on information design within explicit markets, but it is the exception within the informational design literature.

pers Prescott and Townsend (1984a, 1984b)—and shows how competition is the strongest form of persuasion. The main result of the paper shows that the outcome of the competitive information process is incentive efficient. That is, information creates efficient incentives for the players. Persuasion emerges as a result of market competition.

This paper shows how markets are effective *information mechanisms* for inducing people to act, similar to its efficiency as an incentive mechanism (Makowski and Ostroy 1993). The informational characteristics of markets relax incentive compatibility constraints. Market information creates incentives. No single player designs a mechanism to learn the private information of players, as in Hurwicz (1972) where incentives allow the designer to elicit information. Instead, information and its price is an equilibrium outcome. Also, I argue that the informational role in our paper is closer to the informational role discussed in Hayek (1945) than Hurwicz's conception.

Formally, the paper brings together the tools of general equilibrium and information design to see how economic forces shape the information available to people. In general, I show that selective information that is revealed through markets and prices is necessary for incentive efficient allocations.

The formal model comes almost directly from Rahman (2012), which builds a model where prices direct individuals to form teams that trade in goods markets and bargain competitively over team actions as local public goods. The important contribution is that, by relaxing the assumption of binding contracts, he can study incentive constraints *within* a team in general equilibrium. This paper's marginal contribution is to tie those results to the newly developing field of information design and highlight what that teaches us about the role of information in competitive markets. The next section starts to explain that connection.

2 Relation to Information Design

Information design (Taneva 2019; Mathevet, Perego, and Taneva 2019) and Bayesian persuasion (Kamenica and Gentzkow 2011) have two interpretations, one literal and the other metaphorical, as explained in Bergemann and Morris (2019). In the simplest version, the literal interpretation models a sender who can commit to any experiment that releases information to a receiver. The metaphorical is a tool for the modeler to use to consider many different (or all) information that players of a game could have.

Here I put forward another interpretation, what I call market persuasion. I start with the observation, used by Kolotilin (2017), Dworczak and Martini (2019), and Zhang (2017), that the information designer's problem is a linear program. I then show how the information designer's problem is mathematically analogous to a suitable planner's problem that is often used in standard economic theory. Finally, through duality, this planner's problem has a price-taking, decentralized, competitive equilibrium that implements the same allocation as the planner's problem. This competitive implementation of the information designer's allocation is market persuasion.

To model perfect competition, I consider an assignment model of workers to firms (Shapley and Shubik 1971; Gretsky, Ostroy, and Zame 1999). Information matters because there is a moral hazard problem that must be incorporated into competition, as in Prescott and Townsend (1984a, 1984b) and Rahman (2012). However, the previous literature provides a straightforward means of incorporating the moral hazard problem. Therefore, instead of thinking of persuasion as requiring a literal sender to overcome moral hazard, we can think of prices as providing the same persuasion power, using standard competitive theory tools.

3 Example

Consider Bergemann and Morris (2019)'s reformulation of the opening example in Kamenica and Gentzkow (2011). Instead of a finite number of players, consider a continuum of two groups of people: firms and workers. There are two types of workers, bad or good: $t \in \{B, G\}$. The firms have two actions, hire or not hire: $a \in \{H, N\}$. The firms' payoffs are given in Table 1.

	Bad Worker B	Good Worker G
Hire H	-1	v
Not Hire N	0	0

Table 1: Firm Payoff

The interesting case is when $0 < v < 1$. Suppose the prior probability of each type is $\frac{1}{2}$. This implies that it is optimal for the firm to nor hire the worker, given the prior, that is without any additional information. The literal information design formulation asks whether or not a sender who can commit to sending information from an experiment can increase the probability that the firm hires the worker, and if so, what is the optimal signal structure?

Instead of an information designer, there is a planner (who looks quite similar to the information designer). The planner reveals information through the assignment. A planner assigns workers to firms and recommends actions to the firms.

Let $x(t, a)$ be the probability of being assigned type t and being recommended action a. If the planner recommends hire, hire has to be incentive-compatible:

$$vx(G, H) - x(B, H) \geq 0.$$

If the planner recommends not hire, not hire has to be incentive-compatible. Because $v < 1$, the hire constraint is the binding one and always hire $x(B, H) = x(G, H) = \frac{1}{2}$ is not incentive-compatible.

Suppose the planner wants to maximize the probability of hiring–I will generalize this later. Then the planner's problem is

$$\begin{aligned}
\underset{x(t,a)\geq 0}{\text{maximize}} \quad & x(B,H)+x(G,H) && \text{Planner's Objective} \\
\text{subject to} \quad & x(B,H)+x(B,N)=1/2 && \text{Resource Constraint} \\
& x(G,H)+x(G,N)=1/2 && \text{Resource Constraint} \\
& vx(G,H)-x(B,H)\geq 0 && \text{Incentive Constraint}
\end{aligned}$$

This is the full primal problem with all of the binding constraints written out explicitly. In this simple example, the optimal assignment is given in Table 2.

	Bad Worker B	Good Worker G
Hire	$v/2$	$1/2$
Not Hire	$1/2-v/2$	0

Table 2: Optimal Assignment

It is *optimal* for planner to hide information, partially pooling good workers and bad workers. Everything up to this point should be familiar to anyone who has seen the information design literature. However, this is where information design proper stops, finding the optimal outcome. I want to look deeper at the underlying math to talk about prices.

For a general planner's problem, construct a Lagrangian all relevant constraints:

$$\begin{aligned}
\mathcal{L} = \quad & x(B,H)+x(G,H)+\underbrace{\lambda_1}_{\text{Price on Type B}}\left[\frac{1}{2}-x(B,H)-x(B,N)\right] \\
+ & \underbrace{\lambda_2}_{\text{Price on Type G}}\left[\frac{1}{2}-x(G,H)-x(G,N)\right]+\underbrace{\lambda_3}_{\text{Price on IC}}\left[0-vx(G,H)+x(B,H)\right].
\end{aligned}$$

Rearranging, we can find an equivalent dual problem where total cost is minimized:

$$\mathcal{L} = \frac{1}{2}\lambda_1 + \frac{1}{2}\lambda_2 + 0\lambda_3 \quad + x(B,H)\,[1 - \lambda_1 + \lambda_3] + x(G,H)\,[1 - \lambda_2 - \lambda_3 v]$$
$$+ x(B,N)\,[0 - \lambda_1] + x(G,N)\,[0 - \lambda_2]$$

By strong duality, the value of $\mathcal{L}$ by minimizing with respect to λ's is the same as maximizing with respect to x's.

3.1 Interpretation

The primal asks, what is the assignment x that maximizes the planner's objective? The dual asks, what are the prices λ that minimize the social cost of implementation? Strong duality says these answers lead to the same outcome. That is, the decentralized market provides the same persuasion as an omniscient planner only constrained by resource feasibility and incentive compatibility.

The dual role of market competition as information is lost when only focusing on the primal problem. Instead, the two are necessarily interlinked. Therefore, we do not need to think of persuasion as coming from a literal designer, nor as a tool of the analyst. Instead, a sufficiently rich pricing system can induce constrained efficient outcomes, *as if led by an invisible persuader.*

4 Conclusion

Building on the recent literature of Bayesian persuasion and information design, as well as the notion of contractual pricing with moral hazard introduced by Rahman (2005, 2012), this paper develops a model of information embedded within a theory of competition.[4] As the epigraph from Stigler states, "it is always possible that a new range of problems will be posed in this framework," and so this paper tries to do just that: pose new questions of information design in the competitive framework.[5] Instead of assuming a single sender (information designer) can commit and perfectly control information, I propose that information emerges as part of a price-taking equilibrium. This yields a general equilibrium foundation for information design and shows how competition and prices persuade.

4. My job market paper (Albrecht 2019a) is also focused on the role of information and beliefs in a model of competition and also proves a form of a First Welfare Theorem, although in a completely, mathematically distinct setup.

5. The epigraph is shamelessly copied from Makowski and Ostroy (2001).

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