Investment without Coordination Failures

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I study games in which agents must sink their investments before they can match into partnerships that generate value. I focus on competitive matching markets where there is a public price to join any match. Despite the First Welfare Theorem for competitive markets, inefficiencies can still arise that can be interpreted as coordination failures. Multiple equilibria can exist, with both efficient investment and not. The standard, Nash solution concept in these games does not help in determining if they are equally robust or stable. I argue we should replace the Nash solution concept in this context with a mild, common refinement: trembling-hand perfection. The main theorem of the paper proves that in a general class of models with general heterogeneity of types, cost of investment, and matching surplus, every perfect equilibrium is efficient and coordination failures do not exist in equilibrium. That means that in the context of competitive markets, coordination failures are not robust; the possibility of small mistakes rules out coordination failures. The only robust equilibria in competitive markets are those that are efficient even when markets are incomplete.

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1 Introduction

Coordination failures would seem to be ubiquitous, at least if we look at the amount of game theory research devoted to them. Yet, when we look at real-world markets, many investments are made in the face of incomplete markets, seemingly without much fear of those dreaded coordination failures. Entrepreneurs develop hardware before the matching software is available. College students invest in skills before looking for jobs. In both examples, the actors must trust that proper market forces will work things out. For coordination, two things must happen. To induce costly investment, first, people must trust that there will be ex-post competition to avoid hold out problems so they earn a return on their costly investment. Second, to induce coordinated investment, people must trust that there will not be widespread coordination failures. This paper is focused on this second concern: coordination.

To study coordination without hold out problems, I study a model that includes both strategic behavior and competitive price-taking. Agents first sink their investment in a non-cooperative setting. After that, agents enter a competitive matching market where their previous investments determine the set of matches they can choose to join and the price they have to pay to join. There is room for possible coordination in the matching market if there are investment complementarities. For example, Armen's investment in software is complementary with Bengt's investment in hardware. Armen each wants to coordinate with the Bengt and only invest if he does, and vice verse. However, since they make their investments strategically, they cannot write a contract for a joint-deviation and may be stuck in an inefficient equilibrium, referred to here as a "coordination failure".

My main theorem is that coordination failures are not robust when markets are competitive. Formally, Theorem 1 proves that coordination failures do not survive a trembling-hand refinement, where equilibrium is seen as the limit of a sequence of mistakes that become small. This non-robustness of coordination failures and robustness of efficient allocations provides one reason that real-world market actors may trust in the coordination abilities of markets.[1] When there is a chance that people experiment or make mistakes—as people can do—competitive markets still maximize surplus. In a sense, the model is a formal argument of how Adam Smith's "higgling and bargaining of the market" can fix coordination failures.

Coordination problems are central to economics. For giants of economic theory like F.A. Hayek, economics *is* a coordination problem (O'Driscoll 1977). While the broad concept of coordination has a long history in economics, "coordination problem" has come to mean something different today, especially within game theory and the study of coordination games. Coordination games have multiple pure-strategy equilibria. If equilibria can be Pareto-ranked, we will call any equilibrium that is not Pareto-optimal a coordination failure.[2] The coordination failure remains as an equilibrium because people cannot contract for joint-deviations. The inability to write contracts for joint-deviations means that markets are incomplete.

Prices, an objective fact in Walrasian models, do all the work of coordinating supply and demand in a price-taking model. For example, as Makowski and Ostroy (2013) highlight, "Koopmans (1957) uses the example of an individual consumer/producer as a canonical illustration of the decentralization role of the price system that coordinates decisions among households and firms." Coordination in the game theory sense requires more than simply that everyone chooses their best action, given the objective facts of the work, as in a standard Walrasian (price-taking) model of the world. To differentiate the

1. When discussing the formal results in words, I use the term "robustness" compared to more appropriate term "stability" (Kohlberg and Mertens 1986), because stability has a different, common meaning in the matching literature that I want to avoid discussing. All equilibria are stable in the matching sense, even those that are ruled out by trembling hand refinement and therefore not stable in the Kohlberg and Mertens-sense.

2. A game like Battle of the Sexes does not have an equilibrium which is a coordination failure. While there is coordination and multiple equilibria, they are not Pareto-ranked.

supply and demand notion of coordination and the game theory notion of coordination, let me use the terms "market coordination" and "game coordination".[3] Unlike the market coordination in any equilibria, game coordination occurs in any situation where each player's best-response is to somehow match the other players' actions.

For a real economy "It is not sufficient for an individual to have complete knowledge of all objective conditions (technology, resources, and so on)" (O'Driscoll 1977, p. 23-4). Instead, efficient coordination requires everyone's actions to coalesce around the efficient allocation, requires both market and game coordination. This paper theoretically investigates when efficient coordination is likely within markets or when we can expect to find "coordination failures" within a model that includes both strategic, game-theoretic behavior and price-taking, competitive behavior. Theorem 1 provides one situation where we should expect to see full coordination (market and game): whenever there is competition.

In this paper, I strive to emphasize the mechanism and not to prove the most general theorem possible. Therefore, I make simplifying assumptions that make the argument easier along the way. For example, I assume quasi-linear (transferable) utility, which avoids the need to distinguish all of the different welfare benchmarks used in the literature. Also, to purposefully stacks things so that competition is pervasive, I assume throughout a large economy with a continuum of players. As Gretsky, Ostroy, and Zame (1999, p. 63) put it "if we seek (robust) perfect competition we must look to continuum economies." That ensures that any inefficiency that could possibly arise comes from coordination failures. The result does not imply that coordination problems do not exist. We should instead think of them as arising in environments with *imperfect* competition, as is the common in the macroeconomics literature since Cooper and John (1988), for example.

3. Klein and Orsborn (2009) make a similar distinction between "concatenate coordination" and "mutual coordination." Another paper of mine (Albrecht 2016) provides a model that ties together the two different forms of coordination through the effort of entrepreneurs.

The outline of the rest of the paper is as follows. Section 2 briefly goes over related literature on investment, matching, and refinements of Walrasian models. Section 3 goes through a simple example that highlights all of the main results and mechanisms of the model. Readers who skim the paper are encouraged to focus on the example. Section 4 lays out the full model and goes through the standard, Nash-style equilibrium. Section 5 then constructs the trembling-hand refinement and proves that coordination failures are not robust under the trembling-hand. Section 6 concludes.

2 Related Literature

To formally study the connection between market coordination and game coordination, I build on a series of papers that model players as playing a game before entering a market[4] Makowski and Ostroy (1995) first reformulated a First Welfare Theorem without incomplete markets. They showed that two conditions were sufficient for markets to generate efficient outcomes:

- full appropriation: each individual's private benefit from any investment coincides with his/her social contribution;
- non-complementarity: different player's investments cannot be complementary.

As Makowski and Ostroy show, perfect competition gives full appropriation. However, when there are complementarities, game coordination problems can still arise in competitive markets. That means perfect competition alone is not sufficient for efficiency. Following up on Makowski and Ostroy (1995), three important papers of competitive matching Cole, Mailath, and Postlewaite (2001a, 2001b) and Felli and Roberts (2016) find

4. Brandenburger and Stuart (2007) call such games, with a non-cooperative game before a cooperative game, "biform games." Such games are grossly understudied.

three different types of coordination problems can arise: (1) under-investment equilibria, (2) over-investment equilibria, and (3) mismatch equilibria. Further follow up papers, such as Makowski (2004) and most recently Nöldeke and Samuelson (2015) have further generalized results and clarified the connection between competition and efficiency. Makowski (2004) considers a similar environment to mie but focuses on the hold out problem, which I assume away in my problem. I draw most heavily on Nöldeke and Samuelson (2015) focus, who like me, look at the efficiency of coordination in competitive matching markets. [5] From all of these papers, one take-away is always the same: coordination failures *exist* in competitive markets.

None of these papers examine whether these coordination failure equilibria are robust or not. I ask whether or not if we should focus on a subset of equilibria since there are multiple equilibria in all of these models. By introducing a refinement into a Walrasian model, I draw on an entirely separate literature on adverse selection in Walrasian markets. As Gale (1992) points out, in these models, there are many equilibria. However, some of those equilibria are sustained by unreasonable off-equilibrium beliefs, like the belief that other people will not best-respond if a deviation occurs. To discipline off-equilibrium beliefs, Gale uses a form of a trembling-hand refinement (Selten 1975). Whether the refinement leads to more or less efficient equilibria depends on the exact context. For example, in Gale (1992), the refined equilibria are inefficient, while in Gale (1996) they are efficient. There is nothing in general about the refinement that selects for efficient outcomes. The usual examples of perfect equilibria actually show that the perfect equilibria are the *inefficient* ones. See Selten's original example (Selten 1975, p. 33) or a textbook example in Maschler, Solan, and Zamir (2013, p. 263). More recently, refinements have been studied in have been done study in the case of default (e.g. Dubey and Geanakoplos 2002; Dubey,

5. The competitive matching literature that I follow, where no individual chooses prices and equilibrium prices can be thought as coming from a Walrasian auctioneer, is distinct from the competitive *search* literature, following Shimer (1996) and Moen (1997), where one side of the market posts prices.

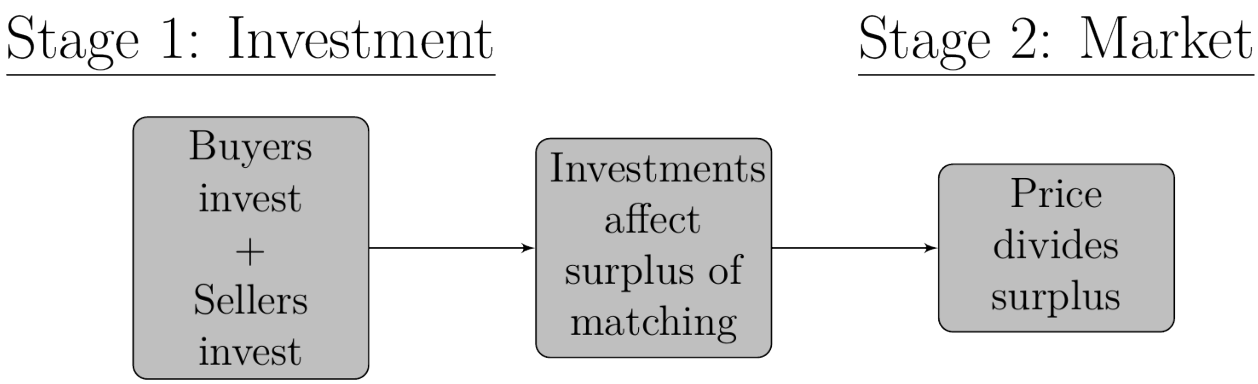
Geanakoplos, and Shubik 2005), and adverse selection in the healthcare market (Scheuer and Smetters 2018).

The closest paper to mine in the refinement literature is Zame (2007), which considers an extremely general model of firm formation with moral hazard and adverse selection. Instead of allowing inefficiency from adverse selection, as is possible in Zame's model, my model shuts down the adverse selection to focus on the role of coordination, which is in the background for Zame. With these difference in mind, we can now move to the example.

3 Example

Consider a simple example of a two-sided matching market with measure one of agents on both sides. For consistent language, I talk about buyers and sellers. There are two stages to the game. First, before matching, buyers and sellers must invest in an attribute, $b \in \{0,1\}$ and $s \in \{0,1\}$. The cost to buyer is $\frac{1}{4}b$ and the cost to seller is $\frac{1}{4}s$. These investments generate a surplus for any match: $v(b,s) = bs$. Second, after buyers and sellers sink their investment, buyers and sellers enter a Walrasian market with prices, $p(b,s)$. A price p is a transfer from buyer b to seller s when they are matched. While prices are observed in the Walrasian market, when agents invest, they do not yet observe prices. Therefore, agents make their decisions based on some price *conjectures*, $\tilde{p}(b,s)$.

The payoff function for a buyer of type b, matched with a seller of type s, when the price is $p(b,s)$ is $v(b,s) - p(b,s) - \frac{1}{4}b$. For sellers, the payoff is $p(b,s) - \frac{1}{4}s$. The game is summarized in the figure below.



An allocation is an investment equilibrium[6] if:

I. prices clear the matching market,

II. each buyer i chooses b to maximize utility, given her price conjectures: $\tilde{p}^i(b,s)$,

III. each seller j chooses s to maximize utility, given her price conjectures: $\tilde{p}^j(b,s)$, and

IV. everyone holds rational conjectures, so that their conjectures are not contradicted by the data:

- If there is a positive mass of buyers with attribute b and sellers with attribute s, then that market has a true price and conjectures agree with the true price: $\tilde{p}^i(b,s) = \tilde{p}^j(b,s) = p(b,s)$
- Otherwise, there is no public price and conjectures need not be consistent across agents.

Because utility is quasi-linear (transferable), a profile of investments and matchings is efficient if and only if it maximizes $v(b,s) - \frac{1}{4}b - \frac{1}{4}s$. For this example, the efficient allocation is to maximize investment: $b = s = 1$.

6. I take this terminology from Makowski (2004). Despite some technical details, an investment equilibrium is equivalent to what Cole, Mailath, and Postlewaite (2001b) and Nöldeke and Samuelson (2015) call an ex-post equilibrium, where contracting only happens ex-post investment.

The efficient allocation with full investment is an equilibrium. For example, suppose that $p(1,1) = \frac{1}{2}$ and all types conjecture that all other prices are zero. Those prices are not actually posted by the auctioneer, since the economy does not include such matches. Price clears matching markets; all buyers and sellers (each with equal measure) want to match. The matching generates positive surplus for the buyers and sellers, which is better than all other alternatives which lead to a conjectured utility of zero. Finally, the players conjectures are not contradicted by the data.

There is also an equilibrium where no one invests: $b = 0$, $s = 0$, and $p(0,0) = 0$. This equilibrium is a coordination failure. However, this equilibrium is only sustained by certain conjectures of buyers and sellers. To see why, suppose that buyers must conjecture that $p(1,1)$ is extremely high. That deters buyers from deviating. Similarly, suppose all sellers conjecture that they will not get paid if they invest: $\tilde{p}^j(1,1) = 0$. There are no profitable deviation for the seller:

$$\underbrace{0}_{\substack{\text{Expected transfer}\\ \text{for } s=0}} - \underbrace{0}_{\text{Cost of } s=0} \geq \underbrace{0}_{\text{Expected transfer for } s=1} - \underbrace{\frac{1}{4}}_{\text{Cost of } s=1}.$$

This outcome is called a coordination failure because if at least one buyer and one seller could coordinate a joint deviation where they both invest, they both could achieve a higher level of utility.

Notice that the price conjectures for markets like the $(1,1)$ that do not exist in equilibrium are a free parameter. They do not need to agree across agents, so that the theorist is free to pick prices to sustain the allocation. In fact, as I will show later, their conjectures *cannot* agree in a coordination failure.

With the free parameter of beliefs many equilibrium can be sustained in general. In this simple example in particular, notice also that this coordination failure *minimizes* total value. In that sense, there is little predictive power from a Nash-style equilibrium, especially if we are looking to study the level of efficiency. The theory as-is gives us no way to pin down the welfare consequences; the best and worst allocations are both equilibria.

To try to pin down the equilibrium more, I consider a mild refinement: trembling hand perfection. For now, assume there is only a simple type of tremble: a uniform trembling hand, where each attribute must be chosen with positive probability $\epsilon > 0$ by each buyer and seller. Since there is a continuum of buyers and sellers, I will assume that in the aggregate each attribute must be chosen by a positive mass of players. A perfect investment equilibrium is the limit of some sequence of ϵ that goes to zero.

With trembling hand, each b and s are played, so that the actual prices are pinned down and players cannot have contradictory beliefs: $\tilde{p}^i(b,s) = \tilde{p}^j(b,s) = p(b,s)$. If $p(1,1) > \frac{1}{4}$, sellers want to choose $s = 1$ as much as possible: $1 - \epsilon$. If $p(1,1) < \frac{3}{4}$, buyers want to choose $b = 1$ as much as possible: $1 - \epsilon$. For any perturbation, equilibrium requires $p(1,1) \in \left[\frac{1}{4}, \frac{3}{4}\right]$. As perturbations go to zero, $(1,1)$ is the unique equilibrium strategy profile. However, the prices are not necessarily unique.

This example highlights three important features of such markets. First, even under competition, coordination failures can exist. Second, coordination failures are sustained by off-path conjectures that are contradictory across buyers and sellers. Third, the possibility of mistakes/trembles is one justification to rule out such contradictions, which therefore rules out coordination failures. The next section extends these three features to a more general model with arbitrary (1) finite types of agents, (2) finite investment options, (3) cost of investment, and (4) surplus functions.

4 Model

There are a continuum of buyers, indexed by $i \in I$, and a continuum of sellers, indexed by $j \in J$. There are functions that specify an endowed *type* for each agent: $\beta : I \to \mathcal{B}$ and

$\sigma : J \to \mathcal{S}$. A generic type is given by $t \in T = \mathcal{B} \cup \mathcal{S}$. I assume the set of types is finite. An economy is defined by a positive measure on the set of types,

$$E \in M_+(T).$$

There are two stages to the model. In the first stage, each individual must acquire/invest in one *attribute*, $a \in A$. For simplicity, the set of attributes is finite. The attributes are partitioned into those that are feasible for buyers, $b \in B$, and those that are feasible for sellers, $s \in S$. There exists a function of acquiring attribute an

$$c : T \times A \to \mathbb{R} \cup \infty,$$

so that $c(t, a)$ is the cost of acquiring a for type t. By definition, there is infinite cost for a buyer type to acquire a seller attribute and vice versa. Individual investments lead to distribution of attributes $\mu \in M_+(A)$. For any element $a \in A$, $\mu(a)$ is the mass of individuals with attribute a.

The second stage involves a market that matches buyers to sellers,

$$x \in M_+(B^\varnothing \times S^\varnothing),$$

where $B^\varnothing \equiv B \cup \varnothing$ and $S^\varnothing \equiv S \cup \varnothing$. A matching x is *feasible* for μ if $x(\varnothing, \varnothing) = 0$, and for all $b \subset B$ and $s \subset S$,

$$x(b, S^\varnothing) = \mu(b)$$

$$x(B^\varnothing, s) = \mu(s).$$

The value generated by an specific match is given by a bounded value function: $v : B^\varnothing \times S^\varnothing \to \mathbb{R}$. In general, I will impose no further assumptions.

Social matching gains function for μ is given by

$$g(\mu) \equiv \max_x \sum_{b \in B^{\emptyset}} \sum_{s \in S^{\emptyset}} v(b,s)x(b,s) \quad s.t. x \text{ is feasible given } \mu.$$

An allocation that attains $g(\mu)$ is *conditionally efficient*.

The second-stage matching is coordinated by done through prices. To focus on coordination under perfect competition, I assume each player acts as a price-taker.[7] A price system is $p : B^{\emptyset} \times S^{\emptyset} \to \mathbb{R}$. We can think of a match, (b,s), as simply a standard good sold by a seller of type s to buyer of type b. A market is open if that pair is part of an equilibrium, that is $x(b,s) > 0$. A market is closed if it is not open. While in the formulation here, players act like price-takers, players can affect the set of markets; players are therefore market-makers.

Definition 1. Fixing the distribution of investment, μ, a pair (x,p) is an (ex-post) *Walrasian equilibrium* for μ if x is feasible for μ, $p(b,\emptyset) = p(\emptyset,s) \equiv 0$,

I. For each $b \in$ supp μ and each $(b,s^*) \in$ supp x, the match maximizes b's utility:

$$s^* \in \underset{s' \in \text{ supp } S^{\emptyset}}{\operatorname{argmax}} \left\{ \max_{s' \in \text{ supp } S} \{v(b,s') - p(b,s')\}, v(b,\emptyset) \right\},$$

II. and for each $s \in$ supp μ and each $(b^*,s) \in$ supp x, the match maximizes s's utility:

$$b^* \in \underset{b \in \text{ supp } B^{\emptyset}}{\operatorname{argmax}} \left\{ \max_{b' \in \text{ supp } B} \{p(b',s)\}, v(\emptyset,s) \right\}.$$

The equilibrium requires that when players are deciding whether to form a match given prices, they are optimizing. The first condition is that any match is maximizing a

7. By assuming price-taking, I follow most of the related matching literature, such as Cole, Mailath, and Postlewaite (2001b) and Nöldeke and Samuelson (2015). See Gretsky, Ostroy, and Zame (1999) and Makowski (2004) for a rigorous analysis of when the price-taking assumption is justified in an assignment model.

buyer's utility: $s \in \operatorname{argmax}_{s' \in S^{\emptyset}} v(b, s') - p(b, s')$. The second condition is the equivalent condition for the sellers. Notice conjectures are not a part of a Walrasian equilibrium because all relevant markets are priced. Even though closed markets are not priced, those markets are irrelevant after investment decisions have been made.

For a distribution μ, we can define indirect utilities given by the matching. The value of a buyer of b with prices p is

$$v_b^*(p, \mu) \equiv \max \left\{ \max_{s' \in S} \left\{ v(b, s') - p(b, s') \right\}, v(b, \emptyset) \right\},$$

and similarly for sellers,

$$v_s^*(p, \mu) \equiv \max \left\{ \max_{b' \in B} \left\{ p(b', s) \right\}, v(\emptyset, s) \right\}.$$

Because of price-taking, we immediately have a "Conditional First Welfare Theorem": If a pair (x, p) is Walrasian for μ, then it is conditionally efficient. It is conditional because maximization only holds within the support of attributes.[8] This immediately rules out any mismatch equilibria found by Felli and Roberts (2016). Besides that, it is a very weak notion of efficiency. In our example, the equilibrium where no one invests is conditionally efficient, even though surplus is minimized. For our current purposes, the result is important because it establishes how the matching market is working effectively, given investments.

Even though all equilibria are conditionally efficient, they are not all efficient in the ex-ante sense. In particular, investment coordination failures can arise. In the example, $b = 0, s = 0$ and $p(0, 0) = 0$ can be part of an ex-post Walrasian equilibrium. Suppose buyers

8. Because of complementarities, the equilibrium price is not unique. There is a pie $v(b, s) = 1$ to divide by $p(b, s)$. The division which occurs is indeterminate, even though the optimal "quantity traded" is when all buyers and sellers match. This is exactly the setup and outcome in Figure 4 of Smith (1982, p. 171). As Smith finds, even though the number of trades is the efficient and equilibrium amount, the price moves between each round of play.

conjecture that $p(b,s) > 1$ for all other levels of investment they could choose. They would not want to deviate, because any other investment is too costly. Similarly, if sellers conjecture that the price is zero, they will not make a return on their positive investment. Since only the price for $p(0,0)$ is observed in equilibrium, both sides' conjectures are rational; there is no feedback that tells the buyers and sellers they should revise their conjectures. Even though there is competition, players are stuck in a coordination failure.

The set of attributes chosen may be inefficient; μ may be missing the efficient b and s. The next subsection asks, given the choice in a non-cooperative setting, do people choose the efficient b and s?

4.1 Investment Equilibrium

Fix the population of types, E. An allocation of attributes is a measure $\nu \in M_+(A \times T)$, where ν_A and ν_T are the respective marginal distributions and $\mu = \nu_A$. An allocation ν is *feasible* for E if $\nu_T = E$.

Definition 2. A pair (ν, p) is an (ex-ante) *investment equilibrium* for E if ν is feasible, p is a Walrasian price for μ, and for all $(t,a) \in$ supp ν,

$$v_b^*(p,\mu) - c(t,b) \geq v_{b'}^*(p) - c(t,b') \quad \forall b' \in B.$$

$$v_s^*(p,\mu) - c(t,s) \geq v_{s'}^*(p) - c(t,s') \quad \forall s' \in S.$$

This equilibrium is like a Nash equilibrium, where each player is best-responding, given what everyone else does. However, the equilibrium does not involve the standard Nash equilibrium epistemic justification; people are not best-responding to actions. Instead, they are best-responding to prices. Each person is choosing her best attribute, given the indirect utility implied by prices and the cost of acquiring that attribute. which is In that sense, it is like a Walrasian equilibrium. But beyond the normal conditions for a Walrasian equilibrium, when players are deciding how much to invest, they must form conjectures about what prices will be in the future. The equilibrium disciplines those conjectures, as Hayek (1937, p. 41) p. 41 pointed out, "the concept of equilibrium merely means that the foresight of the different members of the society is in a special sense correct." However, the exact meaning of correctness is not clear since some prices never materialize so people can contradictory, but in a sense correct, things. I will further discipline conjectures below when I consider refinements to address this issue.

There is a total cost of attributes in the economy, ν is $\int c d\nu$, and a total surplus from ν,

$$G(\nu) = g(\nu_A) - \sum_A \sum_T c(t,a)\nu(t,a).$$

Definition 3. The allocation ν is unconditionally *efficient* for E if it is feasible and $G(\nu) \geq G(\nu')$ for all other feasible allocation ν'.

The previous literature has documented the existence of efficient equilibria.[9] With the above definitions in order, we can immediately show the same in this environment.

Proposition 1. *For any economy, there exists an investment equilibrium that is unconditionally efficient.*

Proof. The existence proof is immediate and by construction. To construct the equilibrium, assume conjectures are consistent. Then we can write down the welfare are

$$\left(\int \left[\max_b v(b,s) - \tilde{p}^i(b,s) - c(b,i) \right] di + \int \left[\max_s \tilde{p}^j(b,s) - c(s,j) \right] dj \right).$$

For any match that occurs, the price to the seller equals the price to the buyer. For anyone that is unmatched, the price is normalized to zero. Since everyone is maximizing given

9. For two examples, see Nöldeke and Samuelson (2015, Corollary 1, p. 858) or Dizdar (2018, Proposition 2, p. 98).

their conjectures and, by assumption, the price conjectures are the same, this means that the price drop out of the overall welfare.

$$\max_{b,s} \int \int v(b,s) - c(b,i) - c(s,j) di dj.$$

Therefore, the sum of everyone's individual maximization is identical to an overall maximization of welfare. ■

But we also know that not all investment equilibria are efficient. Returning to the example, we already, for all buyers, $b = 0$, for all sellers $s = 0$, and $p(b,s) = 0$ is an investment equilibrium. Moreover, this is the worst possible outcome; it minimizes surplus. The next subsection shows that this type of surplus minimizing equilibria exists for many economies that are relevant in the matching literature.

4.2 Weak Predictions with Unconstrained Beliefs

One problem with the equilibrium concept use in the literature, and why it leads to so many different equilibria as shown in the last section, is that off-path beliefs are a free parameter for the theorist. As Robert Lucas taught us, "beware of theorists bearing free parameters." In related papers of adverse selection mentioned about, economists have recognized this issue in other Walrasian contexts. For example, Zame (2007) notes that "imposing no discipline would admit equilibria which are *viable only because different agents hold contradictory beliefs*." The same is true in this model. When the equilibrium concept allows agents to hold contradictory beliefs, many equilibria can be sustained.

To show just how weak the solution concept is, in this section, instead of focusing on the most general forms of the surplus and cost functions that we have used so far, let us consider a smaller set that are still relevant for models of investment and matching.

We will consider two conditions:

Definition 4. A cost function has costly investment if there exists an attribute, $0 \in A$, such that, for all types s, $c(0,t) = 0$ and $c(t,a) > 0$ for $a \neq 0$.

Definition 5. Investment is mutually necessary if surplus is zero whenever there is not investment from both the buyer and seller: $v(0,\emptyset) = v(\emptyset,0) = v(0,0) = 0$ and $v(b,s) \geq 0$ for all b and s.

These are strong restrictions, but they include economies that are relevant for any researcher who is looking at the interaction of investment with matching. The following proposition shows that for all economies like this, the surplus minimizing outcome is an equilibrium.

Proposition 2. *For any economy with costly and mutually-necessary investment, there exists an investment equilibrium that minimizes surplus at zero.*

The proof is immediate and highlights the Nash-style equilibrium.

Proof. Suppose all players but i are not investing. Since investment is costly, any decision to choose positive investment is costly but will not generate any surplus without another player investing. Therefore, it is optimal for i to not invest. ■

Once written out the proposition is so obvious that it seems not even worth mentioning. I include it simply to show that for a reasonable class of models of investment and matching, the equilibrium concept allows the best and worst case outcomes.

The proposition holds regardless of the shape of the cost and surplus functions. Even if cost of investment is arbitrarily small and the surplus generation is arbitrarily big, there exists an investment equilibrium with zero surplus. In this case, we still cannot rule out that either the best or the worst possible allocation can occur.[10] For doing welfare analysis though, it may be desirable to say more than "either the best or worst outcome can occur."

10. If we introduced random actions, we can say that anything in-between could happen too.

The proposition also means that any economy that rules out the surplus minimizing outcome does so because of decisions made that ignore the matching process; people invest regardless of the matching market. In that case, we can rule out the worst outcomes, but it does not have anything to do with the matching market.

To discipline the set of possible outcomes, I follow Gale (1992), who argued that "*some refinement of the equilibrium concept is required to give the theory predictive power*. One such refinement is based on the notion of the 'trembling' hand." The next section shows the under such a refiniment, all equilibria are efficient.

5 Disciplined Beliefs and Perfect Equilibrium

To discipline believes, we will consider a perturbed strategy vector for all buyers $i \in I$. For simplicity of notation, I assume that all buyers are subjected to the same tremble, $\epsilon_B = (\epsilon(b))_{b \in B}$, satisfying $\epsilon(b) > 0$ for all $b \in B$ and

$$\int_B \epsilon(b) db \leq 1.$$

Similarly, all sellers are subjected to the same tremble, $\epsilon_S = (\epsilon(s))_{s \in S}$, satisfying $\epsilon(s) > 0$ for all $s \in S$ and

$$\int_S \epsilon(s) ds \leq 1.$$

A perturbed games is index by the set of perturbed strategy vectors $\epsilon = (\epsilon_B, \epsilon_S)$. An allocation $\nu(\epsilon)$ is ϵ-feasible for E if $\nu_T = E$ and for all $a \in A$

$$\nu_A(\epsilon(a)) \geq \epsilon(a).$$

Instead of jumping directly to the analysis of the limit of perturbed games, it is helpful to say something about the perturbed games themselves. In particular, we can consider their respective efficiency. To do so, let us say that an allocation $\nu(\epsilon)$ is ϵ-efficient for E if it is feasible and $G(\nu(\epsilon)) \geq G(\nu'(\epsilon))$ for all other ϵ-feasible allocation ν'. Formally,

Definition 6. A pair $(\nu(\epsilon), p)$ is an ϵ-investment equilibrium for E if ν is ϵ-feasible, p is a Walrasian price for μ, and for all (t, a) such that $\nu_A(\epsilon) > \epsilon$,

$$v_a^*(p, \mu) - c(t, a) \geq v_{a'}^*(p) - c(a', t) \quad \forall a' \in A$$

Note that by construction, with a trembling hand, supp $\nu_A(\epsilon) = A$. Because there is full support and all markets are open, coordination failures cannot arise. This is shown through the following lemma.

Lemma 1. *If $(\nu(\epsilon), p)$ is an ϵ-investment equilibrium, then it is ϵ-efficient.*

Proof. Let $Q(\epsilon)$ be the utility generate by the trembling actions

$$Q(\epsilon) = \int \int \left[v_a^*(p) - c(t, a)\right] \epsilon(a) da \, d\nu_t$$

.

Then the total utility can be written as:

$$\underbrace{\left(\int \left[\max_b v(b,s) - \tilde{p}^i(b,s) - c(b,i)\right] di + \int \left[\max_s \tilde{p}^j(b,s) - c(s,j)\right] dj\right) \left(1 - \int \epsilon(a) da\right)}_{\text{Optimized Choice}}$$

$$+ \underbrace{Q(\epsilon)}_{\text{Constrained Choice}} .$$

But since all actions are played by trembles, $\tilde{p}^i(b,s) = \tilde{p}^j(b,s)$. Therefore they optimize the entire left expression. It is looks exactly like a static problem, which we know is efficient. ■

Therefore, the possibility of mistakes actually rules out coordination failures. Now we can consider the limit of trembles.

A pair (ν, p) is a *perfect investment equilibria* if there exists a sequence of ϵ, such that $\lim_{k\to\infty} M(\epsilon^k) = 0$ such that $(\nu(\epsilon^k), p) \to (\nu, p)$.

Theorem 1. *If (ν, p) is a perfect investment equilibrium, then it is efficient.*

Proof. The theorem is immediate from Lemma 1 since $\underbrace{Q(\epsilon)}_{\text{Constrained Choice}} \to 0.$

■

The theory's predictive power comes from imposing more restrictions on beliefs than just rational conjectures. The mathematical mechanism is that the mistakes caused by trembles generate complete markets, even though in equilibrium, markets are endogenous and incomplete. The trembling with a large number of agents rules out contradictory beliefs, as in Zame (2007), and ensures "price consistency", as in Makowski and Ostroy (1995). However, instead of assuming price consistency , the tremble gives a justification for sure price consistency in terms of the robustness and stability of the equilibria considered.

There are other justifications for non-contradictory beliefs. For example, Dubey and Geanakoplos (2002) consider fictitious seller who contributes an infinitesimal to each health insurance pool. Dubey, Geanakoplos, and Shubik (2005) assume that the government intervenes to sell infinitesimal quantities of each asset and fully delivers on its promises. In both cases, since all markets are open, all markets have public prices and in equilibrium everyone's price conjectures agree.

6 Conclusion

In this paper, I argue against focusing on coordination failure equilibria when there is competition. Those coordination failures, emphasized by the previous literature, rely on using beliefs as a free parameter and constructing overly pessimistic conjectures. With the free parameter, there are many equilibria. If we want predictive power, we must use a refinement, such a trembling hand perfection.

When we consider perfect equilibrium in an Walrasian matching model with investment, every perfect equilibrium is efficient. If we are interested in the efficiency properties of only those robust competitive equilibria which are robust, then Theorem 1 strengthens the standard First Welfare Theorem because it proves the efficiency of competitive markets, even with incomplete markets of the type studied.

References

Albrecht, Brian C. 2016. "Entrepreneurship as Coordination." `https://briancalbrecht.github.io/albrecht-entrepreneurship-coordination.pdf`. (Cited on page 4).

Brandenburger, Adam M., and Harborne W. Stuart. 2007. "Biform Games." *Management Science* 53 (4): 537–549. doi:`10.1287/mnsc.1060.0591`. `http://pubsonline.informs.org549.https//doi.org/10.1287/mnsc.1060.0591http://www.informs.org`. (Cited on page 5).

Cole, Harold L., George J. Mailath, and Andrew Postlewaite. 2001a. "Efficient Non-Contractible Investments in Finite Economies." *Advances in Theoretical Economics* 1 (1): 1–32. (Cited on page 5).

———. 2001b. "Efficient non-contractible investments in large economies." *Journal of Economic Theory* 101 (2): 333–373. ISSN: 00220531. doi:`10.1006/jeth.2001.2797`. (Cited on pages 5, 8, 12).

Cooper, Russell, and Andrew John. 1988. "Coordinating Coordination Failures in Keynesian Models." *The Quarterly Journal of Economics* 103 (3): 441–463. doi:`10.2307/1885539`. `https://academic.oup.com/qje/article-lookup/doi/10.2307/1885539`. (Cited on page 4).

Dizdar, Deniz. 2018. "Two-Sided Investment and Matching with Multidimensional Cost Types and Attributes." *American Economic Journal: Microeconomics* 10, no. 3 (August): 86–123. doi:`10.1257/mic.20150147`. `http://www.aeaweb.org/articles?id=10.1257/mic.20150147`. (Cited on page 15).

Dubey, Pradeep, and John D. Geanakoplos. 2002. "Competitive Pooling: Rothschild-Stiglitz Reconsidered." *The Quarterly Journal of Economics* 117, no. 4 (November): 1529–1570. ISSN: 0033-5533. doi:`10.1162/003355302320935098`. `https://dx.doi.org/10.1162/003355302320935098`. (Cited on pages 6, 20).

Dubey, Pradeep, John D. Geanakoplos, and Martin Shubik. 2005. "Default and Punishment in General Equilibrium." *Econometrica* 73, no. 1 (January): 1–37. doi:`10.1111/j.1468-0262.2005.00563.x`. `https://doi.org/10.1111/j.1468-0262.2005.00563.x`. (Cited on pages 6, 20).

Felli, Leonardo, and Kevin Roberts. 2016. "Does Competition Solve the Hold-up Problem?" *Economica* 83 (329): 172–200. doi:`10.1111/ecca.12170`. (Cited on pages 5, 13).

Gale, Douglas. 1992. "A Walrasian Theory of Markets with Adverse Selection." *The Review of Economic Studies* 59 (2): 229–255. ISSN: 00346527, 1467937X. doi:`10.2307/2297953`. (Cited on pages 6, 18).

———. 1996. "Equilibria and Pareto optima of markets with adverse selection." *Economic Theory* 7, no. 2 (June): 207–235. ISSN: 0938-2259. doi:`10.1007/BF01213903`. `http://link.springer.com/10.1007/BF01213903`. (Cited on page 6).

Gretsky, Neil E., Joseph M. Ostroy, and William R. Zame. 1999. "Perfect Competition in the Continuous Assignment Model." *Journal of Economic Theory* 88:60–118. (Cited on pages 4, 12).

Hayek, Friedrich A. 1937. "Economics and Knowledge." *Economica* 4 (13): 33–54. http://www.jstor.org/stable/2548786. (Cited on page 15).

Klein, Daniel, and Aaron Orsborn. 2009. "Concatenate Coordination and Mutual Coordination." *Journal of Economic Behavior & Organization* 72 (1): 176–187. ISSN: 01672681. doi:http://dx.doi.org/10.1016/j.jebo.2009.05.003. (Cited on page 4).

Kohlberg, Elon, and Jean-FranÃ§ois Mertens. 1986. "On the Strategic Stability of Equilibria." *Econometrica* 54 (5): 1003–1037. ISSN: 00129682, 14680262. doi:10.2307/1912320. http://www.jstor.org.ezp3.lib.umn.edu/stable/1912320. (Cited on page 3).

Koopmans, Tjalling. 1957. *Three essays on the state of economic science.* New York, Hill. (Cited on page 3).

Makowski, Louis. 2004. "Pre-Contractual Investment without the Fear of Holdups: The Perfect Competition Connection." (Cited on pages 6, 8, 12).

Makowski, Louis, and Joseph M. Ostroy. 1995. "Appropriation and Efficiency: A Revision of the First Theorem of Welfare Economics." *The American Economic Review* 85 (4): 808–827. http://www.jstor.org/stable/2118233. (Cited on pages 5, 20).

———. 2013. "From revealed preference to preference revelation." *Journal of Mathematical Economics* 49, no. 1 (January): 71–81. ISSN: 0304-4068. doi:10.1016/J.JMATECO.2012.10.002. https://www.sciencedirect.com/science/article/pii/S0304406812000912?via%3Dihub. (Cited on page 3).

Maschler, Michael, Eilon Solan, and Shmuel Zamir. 2013. *Game Theory.* First. New York: Cambridge University Press. ISBN: 9781139049344. (Cited on page 6).

Moen, Espen R. 1997. "Competitive Search Equilibrium." *Journal of Political Economy* 105, no. 2 (April): 385–411. ISSN: 0022-3808. doi:10.1086/262077. https://doi.org/10.1086/262077. (Cited on page 6).

Nöldeke, Georg, and Larry Samuelson. 2015. "Investment and Competitive Matching." *Econometrica* 83, no. 3 (May): 835–896. doi:10.3982/ECTA12349. http://doi.wiley.com/10.3982/ECTA12349. (Cited on pages 6, 8, 12, 15).

O'Driscoll, Gerald P. 1977. *Economics as a Coordination Problem: The Contributions of Friedrich A. Hayek.* Kansas City: Sheed Andrews / McMeel, Inc. (Cited on pages 3, 4).

Scheuer, Florian, and Kent Smetters. 2018. "How Initial Conditions Can Have Permanent Effects: The Case of the Affordable Care Act." *American Economic Journal: Economic Policy* 10 (4): 302–343. doi:10.1257/pol.20140204. https://doi.org/10.1257/pol.20140204. (Cited on page 7).

Selten, Reinhard. 1975. “Reexamination of the perfectness concept for equilibrium points in extensive games.” *International Journal of Game Theory* 4 (1): 25–55. ISSN: 1432-1270. doi:10.1007/BF01766400. https://doi.org/10.1007/BF01766400. (Cited on page 6).

Shimer, Robert. 1996. “Contracts in Frictional Labor Markets.” PhD diss., Department of Economics, Massachusetts Institute of Technology. (Cited on page 6).

Smith, Vernon L. 1982. “Markets as Economizers of Information: Experimental Examination of the "Hayek" Hypothesis.” *Economic Inquiry* 20, no. 2 (April): 165–179. doi:10.1111/j.1465-7295.1982.tb01149.x. https://doi.org/10.1111/j.1465-7295.1982.tb01149.x. (Cited on page 13).

Zame, William R. 2007. “Incentives, Contracts, and Markets: A General Equilibrium Theory of Firms.” *Econometrica* 75, no. 5 (September): 1453–1500. ISSN: 0012-9682. doi:10.1111/j.1468-0262.2007.00799.x. http://doi.wiley.com/10.1111/j.1468-0262.2007.00799.x. (Cited on pages 7, 16, 20).