Investment without Coordination Failures

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I study games with incomplete markets where agents must sink their investments before they can join a match that generates value. I focus on competitive matching markets where there is a public price to join any match. Despite the First Welfare Theorem, coordination failures can still arise because of the market incompleteness. Armen does not invest because Bengt does not invest, vice versa, and they are unable to write enforceable contracts to ensure joint investments. In these games, multiple equilibria can exist, with both efficient investment and not. However, the standard, Nash solution concept used in these games does not help in determining if all equilibria are equally robust or stable. I argue that we should replace the Nash solution concept in this context with a mild refinement: trembling-hand perfection. I prove that—in a general class of models with general heterogeneity of types, costs of investments, and matching surpluses—small trembles rule out coordination failures. My main theorem is a modified First Welfare Theorem: *even with endogenous and incomplete markets*, every perfect, competitive equilibrium is efficient.

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1 Introduction

Many real-world investments exhibit coordination; one person's investment requires an appropriately matching investment from another person. Entrepreneurs who develop hardware need others to develop software. College students who invest in learning programming skills need firms that can harness those skills. These investments only have value together. When deciding to invest, people must trust that market forces will reward their costly investments. This requires two things to happen. First, people must trust that there will be ex-post competition to avoid hold out problems so they earn a return for any value their investment generates. Second, people must trust that there will not be widespread coordination failures so that value can be generated. This paper focuses on this second concern, coordination, which is central to economics.

I study coordination in a model where agents must first choose their investment non-cooperatively. After sinking their investment, agents enter a competitive matching market where their previous investments determine the set of matches they can choose to join and the price they have to pay to join. There is room for possible coordination in the matching market if there are investment complementarities. For example, Armen's investment in software is complementary to Bengt's investment in hardware. Armen wants to coordinate with the Bengt and only invest if he does, and vice versa. However, since they make their investments non-cooperatively, they cannot write a contract for a joint-deviation; markets are incomplete. If equilibria can be Pareto-ranked, we call any equilibrium that is not Pareto-optimal a coordination failure.[1] Without the possibility to write contracts for joint-deviations of investments, people may be stuck in a coordination failure.

According to economic theory, in markets with coordination, we can expect two types

1. A game like Battle of the Sexes does not have an equilibrium which is a coordination failure. While there is coordination and therefore multiple equilibria, they are not Pareto-ranked.

of outcomes: efficient outcomes and coordination failures. However, the existing theory does not give us a means for differentiating the likelihood of these two scenarios. The Nash solution concept used in the literature selects for both stable and unstable outcomes. Are efficient coordination and coordination failures both equally stable or robust? My paper proves that they are not equally stable.

Formally, Theorem 1 proves that every equilibrium that survives a trembling-hand perfect refinement is efficient. This implies that any coordination failure does not survive, it is not robust, and is not likely to be observed in the world. This robustness of efficient allocations and non-robustness of coordination failures and provides one reason that real-world market actors may trust in the coordination abilities of markets.[2] When there is a chance that people experiment or make mistakes—as people can do—competitive markets only maximize surplus.

In a sense, the model is a formal argument of how Adam Smith's "higgling and bargaining of the market" can fix coordination failures. If people tremble or experiment with a new strategy that is not prescribed by the coordination failure equilibrium, the coordination failure unravels. If one person in a large economy develops software, another person will observe the software. She will then willing to develop hardware because there will be large returns to the development. This process reveals the instability of the coordination failure and the economy moves to an efficient equilibrium with both software and hardware.

To be more explicit about my environment, I consider the following two-stage game illustrated in Figure 1. Besides the fact that the game is just two-stages for simplicity, the environment is extremely general. I start with a large number of buyers and sellers.

2. When discussing the formal results in words, I use the term "robustness" compared to more appropriate term "stability" (Kohlberg and Mertens 1986), because stability has a different, common meaning in the matching literature that I want to avoid discussing. All equilibria are stable in the matching sense, even those that are ruled out by trembling hand refinement and therefore not stable in the Kohlberg and Mertens-sense.

They are endowed with some arbitrary, finite types. Their types determine how costly it is to invest; there can be good and bad software and hardware developers. In Stage 1, in a non-cooperative setting, all buyers and sellers choose their investment from a set of possible investments. These investments determine the value generated by any match of one buyer and one seller. After choosing their investments, buyers and sellers can buy a contractual right to enter into a match with a member of the other side. An investment equilibrium requires that each side chooses its investment level and contract optimally and that prices clear the market so that all contracts are fulfilled.

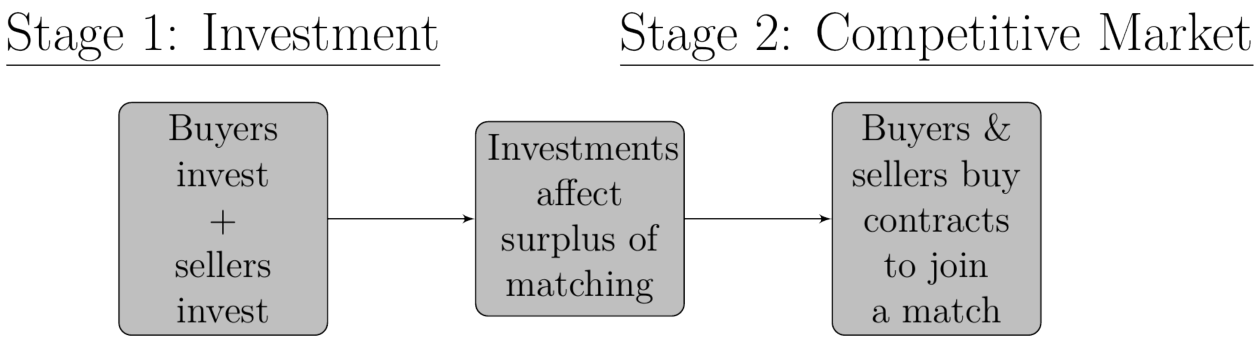

Figure 1: Timing of Game

For efficient coordination in this two-stage game, two types of coordination must take place: *market* and *game* coordination.[3] First, the market must coordinate buyers and sellers of different contracts. This is just the coordination of supply and demand through prices.[4] Second, non-cooperative investments must coalesce around the appropriate investments. The competitive nature of the second-stage market guarantees the coordination of the supply and demand of contracts. The second type of coordination does not always take place, which leads to coordination failures. Incorporating both types of coordination better matches real-world situations, where "It is not sufficient for an individual

3. Klein and Orsborn (2009) make a similar distinction between "concatenate coordination" and "mutual coordination" and Albrecht (2016) provides a model that ties together the two different forms of coordination through the effort of entrepreneurs.

4. In a standard Walrasian model, prices do *all* the work of coordinating supply and demand in a price-taking model. See Makowski and Ostroy (2013).

to have complete knowledge of all objective conditions (technology, resources, and so on)" (O'Driscoll 1977, p. 23-4). Theorem 1 provides one situation where we should expect to see full coordination (market and game): whenever there is competition.

It is important to note that my result does not imply that coordination problems cannot exist in the real world. It does warn against searching for them in competitive environments. We should instead think of them as arising in environments with *imperfect* competition, as is the common in the macroeconomics literature since Cooper and John (1988), for example. The theorem also means that if we observe an outcome that looks like a coordination failure, we have a reason to look for the imperfections of competition in the market. The model serves as a foil to compare possible coordination failures against (Albrecht and Kogelmann 2018). As for policy, solving imperfections of competition should solve the coordination problem.

In this paper, I strive to emphasize the mechanism and not to prove the most general theorem possible. Therefore, I make simplifying assumptions that make the argument easier along the way. For example, I assume quasi-linear (transferable) utility, which avoids the need to distinguish all of the different welfare benchmarks used in the literature. To justify price-taking, I assume throughout a large economy with a continuum of players.[5] Competition (and no externalities) ensures that any inefficiency that could arise comes from coordination failures, as compared to other failures like hold out problems.

The outline of the rest of the paper is as follows. Section 2 briefly goes over related literature on investment, matching, and refinements of competitive models. Section 3 goes through a simple example that highlights all of the main results and mechanisms of the model. Readers who skim the paper are encouraged to focus on the example. Section 4 lays out the full model and goes through the standard, Nash-style equilibrium. Section 5

5. As Gretsky, Ostroy, and Zame (1999, p. 63) put it "if we seek (robust) perfect competition we must look to continuum economies."

then constructs the trembling-hand refinement and proves that coordination failures are not robust under the trembling-hand. Section 6 concludes.

2 Related Literature

To formally study the connection between market coordination and game coordination, I build on a series of papers that have a non-cooperative game before a competitive market.[6] Makowski and Ostroy (1995) showed that if there is full appropriation and investments are non-complementarity, then a First Welfare Theorem holds and any market equilibrium is efficient.[7]

As Makowski and Ostroy show, competition gives full appropriation. However, when there are complementarities, coordination problems can still arise in competitive markets. That means competition alone is not sufficient for efficiency. Following up on Makowski and Ostroy (1995), three important papers of competitive matching Cole, Mailath, and Postlewaite (2001a, 2001b) and Felli and Roberts (2016) show how coordination failures can manifest themselves: (1) under-investment equilibria, (2) over-investment equilibria, and (3) mismatch equilibria.

From all of these papers, one takeaway is always the same: coordination failures *exist* in competitive markets. Further follow up papers, such as Makowski (2004) and most recently Nöldeke and Samuelson (2015) have further generalized results and clarified the connection between competition and efficiency. Makowski (2004) considers a similar environment to mine but focuses on the hold out problem, which I assume away in my problem. I draw most heavily on Nöldeke and Samuelson (2015), who like me, look at

6. Brandenburger and Stuart (2007) call such games, with a non-cooperative game before a cooperative game, "biform games." Such games are grossly understudied.

7. Full appropriation means that each individual's private benefit from any investment coincides with his/her social contribution. Non-complementarity means that different player's investments cannot be complementary.

the efficiency of coordination in competitive matching markets.[8]

None of these papers examine whether these coordination failure equilibria are robust or not. By introducing a refinement, which I draw from an entirely separate literature on adverse selection, I can determine which equilibria are robust. in competitive markets. As Gale (1992) points out, in these models, there are many equilibria. However, some of those equilibria are sustained by unreasonable off-equilibrium beliefs, like the belief that other people will not best-respond if a deviation occurs. To discipline off-equilibrium beliefs, Gale uses a form of a trembling-hand refinement (Selten 1975).

There is nothing in general about the refinement that selects for efficient outcomes. Whether the refinement leads to more or less efficient equilibria depends on the exact context. For example, in Gale (1992), the refined equilibria are inefficient, while in Gale (1996) they are efficient. The usual examples of perfect equilibria actually show that the perfect equilibria are the *inefficient* ones. See Selten's original example (Selten 1975, p. 33) or a textbook example in Maschler, Solan, and Zamir (2013, p. 263). More recently, refinements have been studied in the case of default (e.g. Dubey and Geanakoplos 2002; Dubey, Geanakoplos, and Shubik 2005), and adverse selection in the healthcare market (Scheuer and Smetters 2018).

The closest paper to mine in the refinement literature is Zame (2007), which considers an extremely general model of firm formation with moral hazard and adverse selection. Instead of allowing inefficiency from adverse selection, as is possible in Zame's model, my model shuts down the adverse selection to focus on the role of coordination, which is in the background for Zame. With these differences in mind, we can now move to the example.

8. The competitive matching literature that I follow, where no individual chooses prices and equilibrium prices can be thought as coming from a Walrasian auctioneer, is distinct from the competitive *search and matching* literature, following Shimer (1996) and Moen (1997), where one side of the market posts prices.

3 Example

Consider a simple example of a two-sided matching market with measure one of the agents on both sides. Agents are endowed with a type $t \in T = \{b, s\}$, with measure one of each. For consistent language, I talk about buyers and sellers. The only reason the name matters is to determine who pays a transfer to whom; buyers pay sellers. There are two stages to the game. First, before matching, buyers and sellers must invest in an attribute, $a_b \in A_b = \{0, 1\}$ and $a_s \in A_s = \{0, 1\}$. The cost to a buyer is $\frac{1}{4}a_b$ and the cost to a seller is $\frac{1}{4}a_s$. These investments generate a surplus for any match: $v(a_b, a_s) = a_b a_s$. Second, after buyers and sellers sink their investment, buyers and sellers enter a competitive market with prices. People purchase the contractual right to join a match (a_b, a_s) but can only do so for the investment levels they bring to the market. Let $p : A_n \times A_s \to \mathbb{R}$, where the price $p(a_b, a_s)$ is a transfer from buyer a_b to seller a_s when they are matched. The final utility of buyers is $v(a_b, a_s) - p(a_b, a_s) - \frac{1}{4}a_b$. For sellers, it is $p(a_b, a_s) - \frac{1}{4}a_s$.

A buyer's problem has two parts. At the second stage, given their investment choice, they choose the optimal seller to match with. The matching market generates indirect utility of indirect utility for a_b:

$$v^*(a_b, p) = \max_{a_s} \{v(a_b, a_s) - p(a_b, a_s)\},$$

and similarly for a seller with a_s:

$$v^*(a_s, p) = \max_{a_b} \{p(a_b, a_s)\}.$$

At the first stage, the choose to choose investment to maximize their expected match surplus in the second stage. While prices are observed in the competitive market, when agents invest, they do not yet observe prices. Therefore, each type t makes investments before seeing prices make their decisions based on some price *conjectures*, $\tilde{p}^t(a_b, a_s)$. Taking as given they choose the optimal contract in the competitive market, a buyer's investment problem is

$$\max_b \left\{ \underbrace{v^*\left(a_b, \tilde{p}^t\right)}_{\text{Utility from Conjectured Optimal Contract}} - \underbrace{\frac{1}{4}a_b}_{\text{Investment Cost}} \right\}.$$

Similarly, the seller's investment problem is $\max_s \left\{ v^*\left(a_s, \tilde{p}^t\right) - \frac{1}{4}a_s \right\}$. The payoffs are given in Figure 2. The decision problem is straightforward, except that the prices and conjectures are endogenous, equilibrium objects.

		Buyer Payoffs	Seller Payoffs
Matching Contract	(0,0)	$-\tilde{p}^\beta(0,0)$	$\tilde{p}^\sigma(0,0)$
	(0,1)	$-\tilde{p}^\beta(0,1)$	$\tilde{p}^\sigma(0,1) - \frac{1}{4}$
	(1,0)	$-\tilde{p}^\beta(1,0) - \frac{1}{4}$	$\tilde{p}^\sigma(1,0)$
	(1,1)	$1 - \tilde{p}^\beta(1,1) - \frac{1}{4}$	$\tilde{p}^\sigma(1,1) - \frac{1}{4}$

Figure 2: Example Payoffs

An investment equilibrium[9] is a set of prices, conjectures, and allocations where:

1) each buyer chooses a_b to maximize utility, given her price conjectures: $\tilde{p}^b(a_b, a_s)$,

2) each seller chooses a_s to maximize utility, given her price conjectures: $\tilde{p}^s(a_b, a_s)$, and

3) everyone holds rational conjectures:

   a) if positive mass of agents choose contract (a_b, a_s), conjectures agree with the posted price: $\tilde{p}^{t_b}(a_b, a_s) = \tilde{p}^{t_s}(a_b, a_s) = p(a_b, a_s)$,

9. I take this terminology from Makowski (2004). Besides some technical details, an investment equilibrium is equivalent to what Cole, Mailath, and Postlewaite (2001b) and Nöldeke and Samuelson (2015) call an ex-post equilibrium, where contracting only happens ex-post investment, or what Makowski and Ostroy (1995) call an occupational equilibrium, where people choose an occupation before entering a market.

b) otherwise conjectures are not pinned down, and

4) prices clear the matching market.

Because utility is quasi-linear (transferable), a profile of investments and matchings is efficient if and only if it maximizes $v(a_b, a_s) - \frac{1}{4}a_b - \frac{1}{4}a_s$. For this example, the efficient allocation is to maximize investment: $a_b = a_s = 1$.

The efficient allocation with full investment is an equilibrium. To see how, suppose that $p(1,1) = \frac{1}{2}$. All types conjecture that all other prices are zero, however, those prices are not always posted by the auctioneer, since the economy does not include such matches. The matching generates a positive surplus for the buyers and sellers, which is better than all other alternatives which lead to a conjectured utility of zero. This implies that at the investment stage, given conjectures, it is optimal for buyers and sellers to invest, choosing $(1,1)$. Price clears matching markets; all buyers and sellers (each with equal measure) want to match. Finally, the players' conjectures are not contradicted by the data.

There is also an equilibrium where no one invests: $a_b = 0$, $a_s = 0$, and $p(0,0) = 0$. This equilibrium is a coordination failure. This outcome is called a coordination failure because if at least one buyer and one seller could coordinate a joint deviation where they both invest, they both could achieve a higher level of utility.

It is important to note that the coordination failure equilibrium is only sustained by *contradictory* conjectures of buyers and sellers. Returning to the payoffs in Figure 2. Fix all conjectures besides $\tilde{p}^t(1,1)$ besides to zero. To prevent any buyer from deviating to $(1,1)$, the conjecture must be sufficiently high: $\tilde{p}^b(1,1) \geq \frac{3}{4}$. In contrast, to prevent any seller from deviating to $(1,1)$, the conjecture must be sufficiently low: $\tilde{p}^s(1,1) \leq \frac{1}{4}$.

Notice that the price conjectures for markets that do not exist in equilibrium—like $\tilde{p}^t(1,1)$ when there is a coordination failure–are a free parameter. They do not need to agree across agents; the theorist is free to pick prices to sustain the allocation. As I will show later, their conjectures *cannot* agree in a coordination failure.

With the free parameter of beliefs, many equilibria can be sustained in general. In this simple example, in particular, notice also that this coordination failure *minimizes* total value. In that sense, there is little predictive power from a Nash-style equilibrium, especially if we are looking to study the level of efficiency. The theory as-is gives us no way to pin down the welfare consequences; the best and worst allocations are both equilibria.

To study the robustness of equilibrium, I consider a mild refinement: trembling hand perfection. For simplicity, first, assume no trembles in the competitive matching market. Second, assume that at the investment stage there is only a simple type of tremble: a uniform trembling hand, where each attribute must be chosen with positive probability $\epsilon^t > 0$ by each buyer and seller. Since there is a continuum of buyers and sellers, I will assume that in the aggregate each attribute must be chosen by a positive mass of players. A perfect investment equilibrium is a price and allocation, such that there exists a sequence of ϵ that goes to zero where the sequence of equilibria converges to the perfect investment equilibrium.

With trembling hand, each a_b and a_s are played, so that the actual prices are pinned down and players cannot have contradictory beliefs: $\tilde{p}^b(a_b, a_s) = \tilde{p}^s(a_b, a_s) = p(a_b, a_s)$. If $p(1,1)$ is low ($< \frac{3}{4}$), no buyers want to choose $a_b = 0$. Instead, they choose $a_b = 1$ as much as possible: $1 - \epsilon^{t_b}$. This drives $p(1,1)$ up. At the same time, if $p(1,1)$ is high ($> \frac{1}{4}$), no sellers want to choose $a_s = 0$. Instead, they choose $a_s = 1$ as much as possible: $1 - \epsilon^{t_s}$. This drives $p(1,1)$ down. Some $p(1,1) \in \left[\frac{1}{4}, \frac{3}{4}\right]$ brings these two forces into balance clears all markets.

For any perturbation, equilibrium requires $p(1,1) \in \left[\frac{1}{4}, \frac{3}{4}\right]$. At that price, everyone wants to invest. As perturbations go to zero, $(1,1)$ is the unique equilibrium strategy profile, even though the prices are not unique in this example.

This example highlights four important features of such markets. First, even under competition, coordination failures can exist. Second, coordination failures are only sustained by off-path conjectures that are contradictory across buyers and sellers. Third, the possibility of mistakes/trembles is one justification to rule out such contradictions. Finally, the possibility of mistakes rules out coordination failures and proves coordination failures are not robust. The next section extends these three features to a more general model with arbitrary (1) finite types of agents, (2) finite investment options, (3) cost of investment, and (4) surplus functions.

4 Model

A continuum of agents are endowed with a type $t \in T$, which can be partitioned into "buyers" ($t_b \in T_b$) and "sellers" ($t_s \in T_s$), such that $T = T_b \cup T_s$ and $T_b \cap T_s = \emptyset$. I assume the set of types is finite. An economy is defined by a positive measure on the set of types,

$$E \in M_+(T).$$

There are two stages to the model. In the first stage, each individual must acquire/invest in one *attribute*, $a \in A$. For simplicity, the set of attributes is finite. The attributes are partitioned into those that have a finite cost for buyers, $a_b \in A_b$, and those that have a finite cost for sellers, $a_s \in A_s$, such that $A = A_b \cup A_s$ and $A_b \cap A_s = \emptyset$. There exists a cost function of acquiring an attribute

$$c : T \times A \to \mathbb{R} \cup \infty,$$

so that $c(t, a)$ is the cost of acquiring a for type t. By definition, there is an infinite cost for a buyer type to acquire a seller attribute and vice versa. After attribute investments are made, there is a distribution of attributes $\mu \in M_+(A)$. For any attribute $a \in A$, $\mu(a)$ is the mass of individuals with attribute a. A distribution of attributes is feasible if $\sum_{t_b} E(t_b) = \sum_{a_b} \mu(a_b)$ and $\sum_{t_s} E(t_s) = \sum_{a_s} \mu(a_s)$. Sometimes it will be helpful to work with the distribution of only buyers, $\mu_b \in M_+(A_b)$, or only sellers, $\mu_s \in M_+(A_s)$.

The second stage involves a people forming matches. To allow individuals to remain unmatched, define $A_b^{\emptyset} \equiv A_b \cup \emptyset$ and $A_s^{\emptyset} \equiv A_s \cup \emptyset$. The value generated by an specific match is given by a bounded value function: $v : A_b^{\emptyset} \times A_s^{\emptyset} \to \mathbb{R}$. In general, I will impose no further assumptions. A matching is a distribution

$$x \in M_+(A_b^{\emptyset} \times A_s^{\emptyset}).$$

A matching x is *feasible* for μ if $x(\emptyset, \emptyset) = 0$, and

$$\sum_{a_s' \in A_s^{\emptyset}} x(a_b, a_s') = \mu(a_b) \quad \forall a_b$$

$$\sum_{b_s' \in A_b^{\emptyset}} x(a_b', a_s) = \mu(a_s) \quad \forall a_s.$$

The second-stage matching/assignment is done through prices. To focus on coordination under competition, I assume each player acts as a price-taker.[10] A price system is $p : A_b^{\emptyset} \times A_s^{\emptyset} \to \mathbb{R}$. People buy the contractual right to participate in a match with some seller with a specific attribute. Neither side can choose the type that they match with. However. this is without loss of generality in terms of payoffs since the value function does not depend on types.[11] A market is open if that pair is part of an equilibrium, that

10. By assuming price-taking, I follow most of the related matching literature, such as Cole, Mailath, and Postlewaite (2001b) and Nöldeke and Samuelson (2015). See Gretsky, Ostroy, and Zame (1999) and Makowski (2004) for a rigorous analysis of when the price-taking assumption is justified in an assignment model.

11. Alternatively, we can think of a match, (a_b, a_s), as simply a standard good sold by a seller with attribute a_s to a buyer with attribute a_b.

is $x(a_b, a_s) > 0$.

Definition 1. Fixing the distribution of investment, μ, a pair (x, p) is an (ex-post) *competitive equilibrium* for μ if x is feasible for μ, $p(a_b, \emptyset) = p(\emptyset, a_s) \equiv 0$,

1. For each $a_b \in \operatorname{supp} \mu_b$ and each $(a_b, a_s^*) \in \operatorname{supp} x$, the match maximizes a_b's utility:

$$a_s^* \in \operatorname{argmax}_{a_s \in \operatorname{supp} \mu_s} \{v(a_b, a_s) - p(a_b, a_s)\}, \textit{and}$$

2. for each $a_s \in \operatorname{supp} \mu_s$ and each $(a_b^*, a_s) \in \operatorname{supp} x$, the match maximizes a_s's utility:

$$a_b^* \in \operatorname{argmax}_{a_s \in \operatorname{supp} \mu_b} \{p(a_b, a_s)\}.$$

The equilibrium requires that when players are deciding whether to form a match given prices, they are optimizing. Unlike in the example, conjectures are not a part of a competitive equilibrium; all relevant markets are priced. Even though closed markets are not priced, those markets are irrelevant after investment decisions have been made. Social matching gains function for μ is given by

$$g(\mu) \equiv \max_x \sum_{a_b \in A_b^{\emptyset}} \sum_{a_s \in A_s^{\emptyset}} v(a_b, a_s) x(a_b, a_s) \quad s.t. x \text{ is feasible given } \mu.$$

An allocation that attains $g(\mu)$ is *constrained efficient*. Because of price-taking, we immediately have a "Constrained First Welfare Theorem": If a pair (x, p) is competitive for μ, then it is constrained efficient. It is constrained because maximization only holds within the support of attributes.[12] This immediately rules out any mismatch equilibria found by

12. Because of complementarities the equilibrium price is not unique. There is a pie $v(a_b, a_s) = 1$ to divide by $p(a_b, a_s)$. The division which occurs is indeterminate, even though the optimal "quantity traded" is when all buyers and sellers match. This is exactly the setup and outcome in Figure 4 of Smith (1982, p. 171). As Smith finds, even though the number of trades is the efficient and equilibrium amount, the price moves between each round of play.

Felli and Roberts (2016). Besides that, it is a very weak notion of efficiency. In our example, the equilibrium where no one invests is constrained efficient, even though surplus is minimized. For our current purposes, the result is important because it establishes how the matching market is working effectively, given investments.

Even though all competitive equilibria are constrained efficient, investment coordination failures can still arise such that joint deviations would make everyone better off. In the example, $a_b = 0, a_s = 0$ can be part of an ex-post competitive equilibrium. Even with competition, players are stuck in a coordination failure since a joint deviation to $a_b = 1, a_s = 1$ would make both sides better off. The next subsection asks, given investments are chosen in a non-cooperative setting, do people choose the efficient a_b and a_s?

4.1 Investment Equilibrium

Fix the population of types, E. An allocation of attributes is a measure $\nu \in M_+(T \times A)$, where ν_T and ν_A a are the respective marginal distributions. An allocation ν is *feasible* for E if $\nu_T = E$.

Each agent of type t has price conjectures: $\tilde{p}^t : A_b^\varnothing \times A_s^\varnothing \to \mathbb{R}$. A buyer of type t_b with attribute a_b who conjectures $\tilde{p}^{t_b}$ conjectures an indirect utility from matching of

$$v^* \left(a_b, \tilde{p}^{t_b}\right) \equiv \max_{a_s \in A_s^\varnothing} \left\{ v(a_b, a_s) - \tilde{p}^{t_b}(a_b, a_s) \right\},$$

and a seller of type t_s with attribute a_s who conjectures $\tilde{p}^{t_s}$:

$$v^* \left(a_s, \tilde{p}^{t_s}\right) \equiv \max_{a_b \in A_b^\varnothing} \left\{ \tilde{p}^{t_s}(a_b, a_s) \right\}.$$

We now have all of the pieces to define the relevant notion of equilibrium.

Definition 2. A tuple $\left(\nu, \{\tilde{p}^t\}_{t\in T}, p, x\right)$ is (ex-ante) *investment equilibrium* for E if ν is feasible, (x, p) is a competitive equilibrium for ν_A,

1. for all $(t_b, a_b) \in \text{supp}\ \nu$

$$v^*\left(a_b, \tilde{p}^{t_b}\right) - c(t_b, a_b) \geq v^*\left(a_b', \tilde{p}^{t_b}\right) - c(t_b, a_b') \quad \forall\, a_b' \in A_b,$$

2. for all $(t_s, a_s) \in \text{supp}\ \nu$

$$v^*\left(a_s, \tilde{p}^{t_s}\right) - c(t_s, a_s) \geq v^*\left(a_s', \tilde{p}^{t_s}\right) - c(t_s, a_s') \quad \forall\, a_s' \in A_s, \text{ and}$$

3. for all $t \in \text{supp}\ \nu_T$,

$$\tilde{p}^t(a_b, a_s) = p(a_b, a_s) \quad \forall\, (a_b, a_s) \in \text{ supp } \mu_b \times \text{ supp } \mu_s.$$

The first condition is that all type-attribute pairs for the buyers in the support, the attribute maximizes the buyers' utility. The second condition is the same for the seller and the third condition is that conjectures are rational. Because the buyer is maximizing over all $a_b \in A_b$, he is allowed to consider deviations outside of the equilibrium support of a_b, and the same holds for sellers. However, because the price conjectures for those deviations are a free-parameter, when constructing an equilibrium we can effectively rule out such deviations by picking appropriately high prices conjectures for buyers and low price conjectures for sellers.

This equilibrium is like a Nash equilibrium, where each player is best-responding, given what everyone else does. However, the equilibrium does not involve the standard Nash equilibrium epistemic justification; people are not best-responding to actions. Instead, in line with the competitive nature of the market, they are best-responding to prices. Each person is choosing her best attribute, given the indirect utility implied by prices and the cost of acquiring that attribute. But beyond the normal conditions for a competitive equilibrium, when players are deciding how much to invest, they must form conjectures about what prices will be in the future. The equilibrium disciplines those conjectures, as Hayek (1937, p. 41) pointed out, "the concept of equilibrium merely means that the foresight of the different members of the society is in a special sense correct." However, the exact meaning of correctness is not clear since some prices never materialize so people can contradictory, but in a sense correct, things. I will further discipline conjectures below when I consider refinements to address this issue.

There is a total cost of attributes in the economy ν is $\sum_A \sum_T c(t,a)\nu(t,a)$, and a total surplus from ν,

$$G(\nu) = g(\nu_A) - \sum_A \sum_T c(t,a)\nu(t,a).$$

Definition 3. The allocation ν is unconstrained *efficient* for E if it is feasible and $G(\nu) \geq G(\nu')$ for all other feasible allocation ν'.

The previous literature has documented the existence of efficient equilibria in similar matching models, *e.g.* Nöldeke and Samuelson (2015, Corollary 1, p. 858) and Dizdar (2018, Proposition 2, p. 98). With the above definitions in order, we can immediately show the same in this environment.

Proposition 1. *For any economy, there exists an unconstrained efficient investment equilibrium.*

Proof. The existence proof is by construction. To construct the equilibrium, assume conjectures are consistent: $\tilde{p}^t(a_b, a_s) = p(a_b, a_s)$ for all $t \in \text{supp } \nu_T$. Then we can write down the welfare as

$$\sum_{t_b} \left[\max_{a_b, a_s} v(a_b, a_s) - p(a_b, a_s) - c\left(t_b, a_b\right) \right] E(t_b) + \sum_{t_s} \left[\max_{a_b, a_s} p(a_b, a_s) - c\left(t_s, a_s\right) \right] E(t_s).$$

For any match that occurs, the price to the seller equals the price to the buyer. For anyone that is unmatched, the price is normalized to zero. Since everyone is maximizing given their conjectures and, by assumption, the price conjectures are the same, this means that the price drop out of the overall welfare.

$$\max_{a_b,a_s}\left\{\sum_{t_b}\left[v(a_b,a_s)-p(a_b,a_s)-c\left(t_b,a_b\right)\right]E(t_b)+\sum_{t_s}\left[p(a_b,a_s)-c\left(t_s,a_s\right)\right]E(t_s)\right\}.$$

Therefore, the sum of everyone's individual maximization is identical to an overall maximization of welfare. ■

Using consistent conjectures was the key step in the proof. But we also know that not all investment equilibria are efficient. Returning to the example, we already showed that $a_b = 0$ for all buyers, $a_s = 0$ for all sellers, and $p(a_b, a_s) = 0$ can be sustained as an investment equilibrium with certain contradictory conjectures. Moreover, this is the worst possible outcome; it minimizes the surplus. The next subsection shows that this type of surplus minimizing equilibria exist for many economies that are relevant in the matching literature.

4.2 Weak Predictions with Unconstrained Beliefs

One problem with the equilibrium concept used in the literature, and why it leads to so many different equilibria as shown in the last section, is that off-path beliefs are a free parameter for the theorist. As Robert Lucas taught us, "beware of theorists bearing free parameters." In related papers of adverse selection mentioned above, economists have recognized this issue in other competitive contexts. For example, Zame (2007) notes that "imposing no discipline would admit equilibria which are *viable only because different agents hold contradictory beliefs*." The same is true in this model. When the equilibrium concept allows agents to hold contradictory beliefs, many equilibria can be sustained.

To show just how weak the solution concept is, in this section, instead of focusing on the most general forms of the surplus and cost functions that we have used so far, let us consider a smaller set that is still relevant for models of investment and matching.

Definition 4. Investment is mutually necessary if surplus is zero whenever there is not investment from both the buyer and seller: $v(0,\emptyset) = v(\emptyset,0) = v(0,0) = 0$ and $v(a_b, a_s) \geq 0$ for all a_b and a_s.

These are strong restrictions, but they include economies that are relevant for any researcher who is looking at the interaction of investment with matching. The following proposition shows that for all economies like this, the surplus minimizing outcome is an equilibrium.

Proposition 2. *For any economy with mutually-necessary investment, there exists an investment equilibrium that minimizes surplus at zero.*

The proof is immediate and highlights the Nash-style equilibrium.

Proof. Suppose all players but t_b are not investing. Since investment is costly, any decision to choose positive investment is costly but will not generate any surplus without another player investing. Therefore, it is optimal for t_b to not invest. ■

Once written out the proposition is so obvious that it seems not even worth mentioning. I include it simply to show that for a reasonable class of models of investment and matching, the equilibrium concept allows the best and worst-case outcomes.

The proposition holds regardless of the shape of the cost and surplus functions. Even if the cost of investment is arbitrarily small and the surplus generation is arbitrarily big, there exists an investment equilibrium with zero surplus. In this case, we still cannot rule out that either the best or the worst possible allocation can occur.[13] For doing welfare

13. If we introduced random actions, we can say that anything in-between could happen too.

analysis though, it may be desirable to say more than "either the best or worst outcome can occur."

The proposition also means that any economy that rules out the surplus minimizing outcome does so because of decisions made that ignore the matching process; people invest regardless of the matching market. In that case, we can rule out the worst outcomes, but it does not have anything to do with the matching market.

To discipline the set of possible outcomes, I follow Gale (1992), who argued that "*some refinement of the equilibrium concept is required to give the theory predictive power*. One such refinement is based on the notion of the 'trembling' hand." The next section shows the under such a refinement, all equilibria are efficient.

5 Disciplined Beliefs and Perfect Equilibrium

To discipline believes, I will consider a perturbed strategy vector for all buyers. For simplicity of notation when there is a continuum of agents, I assume that all buyers are subjected to the same tremble, $\epsilon_{A_b} = (\epsilon(a_b))_{a_b \in A_b}$, satisfying $\epsilon(a_b) > 0$ for all $a_b \in A_b$ and

$$\sum_{A_b} \epsilon(a_b) \leq 1.$$

Similarly, all sellers are subjected to the same tremble, $\epsilon_{A_s} = (\epsilon(a_s))_{a_s \in A_s}$, satisfying $\epsilon(a_s) > 0$ for all $a_s \in A_s$ and

$$\sum_{A_s} \epsilon(a_s) \leq 1.$$

A perturbed game is indexed by the vector of perturbed strategies $\epsilon = (\epsilon_{A_b}, \epsilon_{A_s})$. An allocation $\nu(\epsilon)$ is ϵ-feasible for E if $\nu_T = E$ and for all $a \in A$

$$\nu_A(\epsilon(a)) \geq \epsilon(a).$$

Instead of jumping directly to the analysis of the limit of perturbed games, it is helpful to say something about the perturbed games themselves. In particular, we can consider their respective efficiency. To do so, let us say that an allocation $\nu(\epsilon)$ is ϵ-efficient for E if it is feasible and $G(\nu(\epsilon)) \geq G(\nu'(\epsilon))$ for all other ϵ-feasible allocation ν'. Formally,

Definition 5. A tuple $\left(\nu(\epsilon), \{\tilde{p}^t\}_{t\in T}, p, x\right)$ is an ϵ-investment equilibrium for E if ν is ϵ-feasible, p is a competitive price for ν_A, conjectures are rational, and for all (t,a) such that $\nu_A(\epsilon) > \epsilon$,

$$v_a^*(\tilde{p}^t) - c(t,a) \geq v_{a'}^*(\tilde{p}^t) - c(t,a') \quad \forall a' \in A.$$

Note that by construction, with a trembling hand, $\operatorname{supp} \nu_A(\epsilon) = A$. Because there is full support and all markets are open, coordination failures cannot arise. This is shown through the following lemma.

Lemma 1. *If $(\nu(\epsilon), p)$ is an ϵ-investment equilibrium, then it is ϵ-efficient.*

Proof. Let $Q(\epsilon)$ be the utility generated by the trembling actions

$$\begin{aligned} Q(\epsilon) = &\sum_{A_b} \left\{ \sum_{t_b} \left[v^*\left(a_b, \tilde{p}^{t_b}\right) - c(t_b, a_b) \right] \nu_T(t_b) \right\} \epsilon(a_b) \\ &+ \sum_{A_s} \left\{ \sum_{t_s} \left[v^*\left(a_s, \tilde{p}^{t_s}\right) - c(t_s, a_s) \right] \nu_T(t_s) \right\} \epsilon(a_s) \end{aligned}$$

Then the total utility can be written as:

$$\begin{aligned} &\sum_{t_b} \left[\max_{a_b, a_s} v(a_b, a_s) - \tilde{p}^{t_b}(a_b, a_s) - c\left(t_b, a_b\right) \right] \nu_T(t_b)\left(1 - \epsilon(a_b)\right) \\ &+ \sum_{t_s} \left[\max_{a_b, a_s} \tilde{p}^{t_s}(a_b, a_s) - c\left(t_s, a_s\right) \right] \nu_T(t_s)\left(1 - \epsilon(a_s)\right) + \underbrace{Q(\epsilon).}_{\text{Constrained Choice}} \quad . \end{aligned}$$

But since all actions are played by trembles, $\tilde{p}^{t_b}(a_b, a_s) = \tilde{p}^{t_s}(a_b, a_s)$ for all (a_b, a_s), not just those in a subset. Therefore agents optimize the entire left expression. The rest follows the proof of Proposition 1. ■

Therefore, the possibility of mistakes rules out coordination failures. Now we can consider the limit of trembles.

A tuple $\left(\nu, \{\tilde{p}^t\}_{t\in T}, p, x\right)$ is a *perfect investment equilibria* if there exists a sequence of ϵ, such that $\lim_{k\to\infty} M(\epsilon^k) = 0$ such that $\left(\nu(\epsilon), \{\tilde{p}^t\}_{t\in T}, p, x\right) \to \left(\nu, \{\tilde{p}^t\}_{t\in T}, p, x\right)$.

Theorem 1. *If (ν, p) is a perfect investment equilibrium, then it is efficient.*

Proof. The theorem is immediate from Lemma 1 since $\underbrace{Q(\epsilon)}_{\text{Constrained Choice}} \to 0$. ■

The theory's predictive power comes from imposing more restrictions on beliefs than just rational conjectures. The mathematical mechanism is that the mistakes caused by trembles generate complete markets, even though in equilibrium, markets are endogenous and incomplete. The trembling with a large number of agents rules out contradictory beliefs, as in Zame (2007), and ensures "price consistency", as in Makowski and Ostroy (1995). However, instead of assuming price consistency, the tremble gives a justification for sure price consistency in terms of the robustness and stability of the equilibria considered.

There are other justifications for non-contradictory beliefs. For example, Dubey and Geanakoplos (2002) consider a fictitious seller who contributes an infinitesimal to each health insurance pool. Dubey, Geanakoplos, and Shubik (2005) assume that the government intervenes to sell infinitesimal quantities of each asset and fully delivers on its promises. In both cases, since all markets are open, all markets have public prices and in equilibrium, everyone's price conjectures agree.

6 Conclusion and Implications

In this paper, I argue against focusing too much on coordination failures when there is competition. Those coordination failures, emphasized by the previous literature, rely on using beliefs as a free parameter and constructing overly pessimistic conjectures. With the free parameter, there are many equilibria. If we want predictive power, we must use a refinement, such a trembling hand perfection.

When we consider perfect equilibrium in a competitive matching model with investment, every perfect equilibrium is efficient. If we are interested in the efficiency properties of only those robust competitive equilibria which are robust, then Theorem 1 strengthens the standard First Welfare Theorem because it proves the efficiency of competitive markets, even with incomplete markets of the type studied.

The theorem does not imply that coordination problems cannot exist, just that those in competitive environments are not stable and therefore are unlikely to last. We should instead think of them as arising in environments with imperfect competition. If we observe an outcome that looks like a coordination failure, we have a reason to look for the imperfections of *competition* in the market. When looking for how to use policy, solving imperfections of competition should solve the coordination problem.

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