On the Informational Efficiency of Decentralized Price Formation

Rafael R. Guthmann[*] and Brian C. Albrecht[†]

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It is well understood that the competitive allocation process is informationally efficient. This paper studies the informational efficiency in models with *strategic and decentralized* price formation. First, we show in a two-good economy that the standard random search and bargaining model requires an infinitely larger message space compared to the competitive equilibrium for a large economy–making it infinitely less informationally efficient. We propose here a model of price formation through market-makers. As the random search equilibrium, this model of price formation attains the competitive allocation in the limit, but, in a quasi-linear environment with L goods, we prove that the market-maker mechanism only requires a message space with L-1 more dimensions than the competitive process. This appears to be the most informationally efficient form of decentralized price formation process that implements the competitive allocation at the limit.

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[*]Department of Economics, Pontifical Catholic University of Rio de Janeiro (PUC-Rio). Email: rafael.guthmann@econ.puc-rio.br.

[†]Department of Economics, University of Minnesota. Email: mail@briancalbrecht.com.

1 Introduction

At least since Hayek (1937, 1945), economists have assessed institutions by their relative ability to incorporate and communicate information. According to this framing, markets are a desirable institution because they achieve an efficient allocation of resources through decentralized decision making where each decision-maker only needs to be aware of market-prices besides their individual endowments and technology: That is, markets achieve allocative efficiency without requiring the explicit communication of the information that is dispersed through the economy.

Studies such as Mount and Reiter (1974), Hurwicz (1977a, 1977b, 1977c), Jordan (1982), and Chander (1983) developed a formal concept of informational efficiency. Mount and Reiter (1974) shows that the competitive equilibrium is informationally efficient in the sense that competitive prices communicate the minimum amount of information necessary to implement a Pareto efficient allocation in an environment where information is dispersed.[1] Moreover, Jordan (1982) proves that competitive prices are the unique decentralized mechanism that achieves this informational efficiency and satisfies the individual rationality constraint that the utility in participating in the mechanism is at least as high as their initial endowment.[2]

While these informational efficiency results are important, for other reasons, the competitive mechanism is not without its critics. Particularly, the concept of competitive equilibrium as an allocation mechanism lacks strategic foundations (*e.g.* Gale 2000): decision-makers take prices as given and the price is determined by the underlying market data through an exogenously imposed market-clearing condition: the strategic process that determines market prices is not modeled. That is, only the quantities are determined by the decentralized choices of decision-makers while the prices are determined by the "Walrasian auctioneer" or "the economist" who solves the model by equating the quantity supplied with the quantity demanded. Without any way to measure the information required to reach the competitive allocation through a fully decentralized strategic procedure, it is unclear how seriously economists should take these informational efficiency results.

While there are different strategic foundations for competitive equilibrium, we show

1. Such as information regarding individual endowments, preferences, or technology.

2. Further illustrating the informational power of competitive equilibrium, even when types are private information, Hammond (1979) has shown in an economy with a continuum of agents that an incentive-compatible mechanism that implements a Pareto optimal allocation must give rise to the same net trades as the competitive equilibrium mechanism.

that the main one in the literature—random search and bargaining—is unattractive from an informational efficiency perspective. In particular, we prove that for large economies the search allocation mechanism requires infinitely more information than the competitive mechanism. The reason is that in a search economy, each buyer needs to know about the price he or she would trade with each seller, which requires a different message. If there are k buyers and k sellers who trade one good, the message space must include k^2 prices. This seems like an unattractive feature of a search model for large economies.

Therefore, we introduce a new strategic foundation for competitive equilibrium which requires less information but qualitatively replicates several empirical observations. Following the intermediation models of Spulber 1996 and Rust and Hall 2003 we introduce a model of market-makers who intermediate trade between buyers and sellers in a model of dynamic price formation. We show that a model of exchange based on intermediators can approximate the informational efficiency properties of the competitive equilibrium while endogenizing the equilibrium prices as strategic choices of decision-makers. The informational efficiency is obtained through an "intellectual division of labor" between the consumers in the economy who "outsource" the price formation process to the market-makers who are the strategic players, who in turn make profits by extracting part of the surplus of the gains from trade. These market-makers can be thought of as arbitrageurs who have the ability to buy low and sell high, exploiting opportunities that other actors are unaware of.[3] As frictions decrease the equilibrium approximates the no-arbitrage limit which is the competitive equilibrium, which is well understood in the case of product markets (*e.g.* Makowski and Ostroy 1998) and financial markets (*e.g.* Werner 1987).

Among informationally decentralized allocation mechanisms where terms of trade are set strategically, this study suggests that the one that is the most informationally efficient is the allocation mechanism implied by a situation where the market for each good is monopolized by a single market-maker who sets prices, that way consumers in the economy only need to be aware of one price for each good, mimicking the competitive equilibrium. Allowing for entry puts pressure on the market-maker to lower prices to deter entry in what we call a monopolistic deterrence equilibrium. Taking discount rates to zero implies that the allocation corresponding to the monopolistic deterrence equilibrium converges

3. Rubinstein and Wolinsky (1987), Duffie, Gârleanu, and Pedersen (2005), Nosal, Wong, and Wright (2019) are other examples of models that feature middlemen or market-makers that intermediate trade. In addition there is a literature following Kirzner (1973) which would call these market-makers "entrepreneurs" that are "alert" to profit opportunities that exist in the market at a point in time.

to the competitive allocation.[4]

The main objectives of this paper are as follows:

1. Construct a model of an economy that features strategic and decentralized trading that attain as its "frictionless limit" the allocation corresponding to the competitive equilibrium while allowing for asymmetric information.
2. Show that the equilibrium of this economy can replicate several of the features reported in the empirical literature that cannot exist in competitive equilibrium.
3. In addition, this model of an economy satisfies the condition that the amount of information that needs to be communicated does not diverge from the competitive allocation mechanism.
4. Find what is the minimum amount of information required by a strategic model of price determination that approximates the competitive allocation.

Strategic Foundations of Competitive Equilibrium

There is a large literature on strategic foundations for competitive equilibrium which justifies the assumption that markets can be described by the model of competitive equilibrium by showing that under a variety of different conditions that the strategic equilibrium of decentralized economies (modeled as the core of a cooperative game or random matching and bargaining games) does generate the same allocation as the competitive equilibrium.[5] Also, the literature on random matching and bargaining games is also able to explain several deviations of competitive conditions verified empirically (such as price dispersion is generated as an equilibrium outcome in models such as Mortensen and Wright (2002) while accommodating the competitive equilibrium as a special case when frictions of trading are zero. However, this literature has assumed that decision-makers have complete information regarding market conditions.

4. This study implies that the presence of platforms such as bitcoin exchanges, Amazon, and Uber, with large market-shares might be more informationally efficient than industries with many agents in both sides of the market.

5. Starting in cooperative game theory with Edgeworth (1881), Debreu and Scarf (1963), and Aumann (1964) and moving into non-cooperative random matching game-theoretic arguments (Gale 1986a, 1986b, 1987, 2000; Osbourne and Rubinstein 1990; McLennan and Sonneschein 1991; Mortensen and Wright 2002; Lauermann 2013).

More recent papers have shown, however, in the case of a market for a single good that asymmetric information regarding private valuations (Satterthwaite and Shneyerov 2007, 2008) and imperfect information regarding aggregate market conditions (Lauermann, Merzyn, and Virág 2018) can be shown to be consistent with decentralized random matching games that attain competitive equilibrium allocations. However, these studies assume that decision-makers have full awareness of the data of the economy: there exists only uncertainty regarding the "quality" of the information (that is, the values of individual parameters of the economy which are private valuations in the case of Satterthwaite and Shneyerov (2007, 2008) and whether the supply of a good is greater than demand, so the competitive equilibrium price is 0 or 1 (Lauermann, Merzyn, and Virág 2018)) but there is no constraints on the "quantity" of information regarding the economy that can be utilized by each decision maker.

The literature on informational efficiency argues that the competitive equilibrium mechanism allows an economy to achieve an efficient allocation while minimizing the amount of information that needs to be communicated by the individual decision makers. That is, in a competitive equilibrium individual decision-makers only need to be aware of the minimum amount of parameters regarding the state of the market to achieve an efficient allocation. The decision-makers can be simply completely unaware of the rest of the economy besides market prices (unawareness being interpreted as not "generally taking into account", "being present in mind" (Modica and Rustichini 1999, p. 274), "thinking about" (Dekel, Lipman, and Rustichini 1998) or "paying attention to" (Schipper 2014)) - and still achieve the same allocation which is obtained by a central planner in possession of all the data in the economy who is maximizing a social welfare function. If the allocation corresponding to a competitive equilibrium is reached through a distinct mechanism (such as in a Gale-type random matching and bargaining model) then this informational efficiency property does not hold and we show that in the case of the typical random search model the implied allocation mechanism is infinitely less informationally efficient than the competitive mechanism.

In contrast, an economy with market-makers can require little information. In particular, this mechanism has a message space that has one additional dimension over the competitive mechanism for each good in the economy besides the numeraire good. This extra-dimension represents the profit margin between the bid and ask price that the strategic market-maker obtains from setting prices. Another issue with the concept of competitive equilibrium is empirical: most markets exhibit features that are very different from the idealized features of competitive equilibrium. For example, there is substantial price dispersion for individual goods instead of a single price (Sorensen 2000; Kaplan et al. 2019). Individual plants face plant-specific demand instead of taking market prices as given, and this plant-specific demand depends on the history of plant activity and grows over time (Foster, Haltiwanger, and Syverson 2016). Both of these features can be replicated by the market-maker model presented here. Informational efficiency and the empirical reality are both reasons to further study models with market-makers.

2 Physical Environment and Competitive Mechanism

Consider a class of environments E where there are $L = 2$ goods: an indivisible good and a divisible numeraire good. There is a continuum of consumers with measure normalized to 1 and indexed by i. For an environment $e \in E$ a fraction $s \in [0, 1]$ are sellers (set denoted $S = [0, s]$) who are willing to supply a unit of the indivisible good for a higher price (in the numeraire good) than their cost c^i. A fraction $b = 1 - s$ are buyers (set denoted $B = (s, 1]$) who are willing to buy a unit of the indivisible good for a lower price (in the numeraire good) than their valuation v^i. Valuations for the good are distributed according to cumulative distribution functions G and F for sellers and buyers, respectively which are strictly increasing and differentiable. The CDFs G and F have supports $[\underline{c}, \overline{c}] \subset \mathcal{R}_{++}$, $[\underline{v}, \overline{v}] \subset \mathcal{R}_{++}$, respectively, where $\overline{v} > \underline{c}$ (so that there is possibility for mutually beneficial trade and the market does not shut down).

Let E^i be the set of possible types for consumer i which indicates whether i is a buyer or a seller and i's valuation. A realization of $e \in E$ is an environment $e = (e^i)_{i \in [0,1]}$, where $e^i \in E^i$ with $e^i = c^i$ if $i \in S$ and $e^i = v^i$ if $i \in B$ which are distributed according to the CDF's G and F.

Let Y be the set of net-trades which satisfy feasibility conditions: $y_1(i) \in \{-1, 0, 1\}$, $y_2(i) \in \mathcal{R}$, $\int_0^1 y_1(i) = 0$, [6] and $\int_0^1 y_2(i) = 0$. The last twp conditions mean that net trades of both the indivisible and numeraire good must add up to 0. Formally, we can define the set of feasible allocations as

$$Y = \left\{ y : [0, 1] \to \{-1, 0, 1\} \times \mathcal{R} : \int_0^1 (y_1(i), y_2(i)) di = (0, 0) \right\},$$

6. That is, the agent buys one unit of the indivisible good,sells a unit or does not change his or her endowment.

To formalize the notion of the informational size, we need to define messages. The set M is an abstract message space. The non-empty valued correspondence $\mu : E \rightrightarrows M$ specifies the messages for each environment. Finally, the outcome function $g : M \to Y$ maps messages to trades/allocations.

We are interested in allocation mechanisms that are informationally decentralized, which are mechanisms that feature a message process (μ, M) that is privacy-preserving.

Definition 1. A message process (μ, M) is *privacy-preserving* if there exists a correspondence $\mu^i : E^i \rightrightarrows M$ for each i such that for each $e \in E$ $\mu(e) = \cap_{i \in [0,1]} \mu^i(e^i)$.

Putting this together, we can define an allocation mechanism, which is the object of interest.

Definition 2. An *allocation mechanism* is a triple (μ, M, g).

2.1 Competitive Mechanism

Let $M_c = \{(p, y) \in \mathcal{R}_{++} \times Y : py_1(i) + y_2(i) = 0, \forall i\}$. For an environment $e \in E$, each $i \in S \cup B$ define the correspondence $\mu_c^i : E \rightrightarrows M_c$ by

$$\mu_c^i(e^i) = \left\{ (p, y) \in M_c : y(i) = \begin{cases} (1, -p) & \text{if } i \in B \text{ and } v^i \geq p \\ (0, 0) & \text{if } i \in B \text{ and } v^i < p \text{ or } i \in S \text{ and } c^i > p \\ (-1, p) & \text{if } i \in S \text{ and } c^i < p \end{cases} \right\},$$

that is, a buyer purchases the good for p if his valuation is higher and a seller sells the good for p is her cost is lower. Define

$$\mu_c(e) = \cap_{i \in [0,1]} \mu_c^i(e^i).$$

Then (μ_c, M_c) is the competitive message process and (μ_c, M_c, g_c) is the competitive allocation mechanism, where $g_c : M_c \to Y$ is the outcome function given by $g_c(p, y) = y$.

Remark 1. Note that to satisfy feasibility a competitive equilibrium price p^* satisfies $sG(p^*) = b[1 - F(p^*)]$, since G and F are continuous and strictly increasing the competitive equilibrium price p^* is unique. The competitive mechanism is, trivially, privacy preserving.

3 The Informational *In*efficiency of the Search Mechanism

3.1 Environment

In a search environment, there is an entry rate of potential $s > 0$ sellers and $b = 1 - s > 0$ buyers. Given stocks of B buyers and S sellers currently in the market, buyers and sellers meet at the rate $M(B, S)$. Let the buyer/seller ratio $\theta = B/S$ be the market tightness parameter, and $m(\theta) = M(B, S)/S$, be the rate a seller meets buyers and $m(\theta)/\theta$ be the rate a buyer meets sellers. There are discount rates and search costs for buyers and sellers, respectively given by $(r, c_b, c_s) \in \mathcal{R}_+$.

When buyers and sellers meet if there is a positive surplus in trading they trade at terms determined by the generalized Nash solution over the joint surplus. Sellers' bargaining power is given by the parameter $\beta \in (0, 1)$ (buyers' bargaining power is then $1 - \beta$).

For this search environment to be directly comparable to the competitive mechanism the flows of buyers and sellers exiting the market should be equal to the flows entering which are s and b, so that the corresponding net trades in the competitive equilibrium have an analogous implementation in this environment. The search equilibrium where the rate of entry of new buyers and sellers in the market is the same as the exit rate is called steady-state search equilibrium. Given a pair of marginal types of buyers and sellers (R_b, R_s), where buyer with valuation $x > R_b$ enters and seller with cost $y < R_s$ enters, who are indifferent between participating in the market or not in steady-state search equilibrium this pair satisfies the condition $sG(R_s) = b[1 - F(R_b)]$, and the distribution of participating types is constant.

As described in Mortensen and Wright (2002), these parameters determine the steady-state search equilibrium which is characterized by $(V_b, V_s, R_b, R_s, \Phi, \Gamma)$, the value functions (V_b, V_s), cutoff valuations, and costs to participate in the market (R_b, R_s), and the distributions of participating types (Φ, Γ) of for buyers and sellers, respectively. Transaction prices between buyer with valuation x and seller with cost y satisfy

$$p(x, y) = y + V_s(y) + \beta[x - y - V_b(x) - V_s(y)]. \tag{1}$$

Mortensen and Wright (2002) show that if search costs c_b, c_s are strictly positive and r is lower than some threshold $\hat{r} > 0$ then all meetings result in trade. This implies that steady-state equilibrium distribution of operating types (Φ, Γ) is given by the densities of

(F, G) on the types who choose to participate in the market ($v \geq R_b, c \leq R_s$).

In the case where the common discount rate is zero, $r = 0$, then the Law of One Price holds and there is an equilibrium price

$$\hat{p} = \beta R_b + (1-\beta) R_s. \tag{2}$$

If search costs (c_b, c_s) converge to zero then $\hat{p}$ converges to the competitive price and R_s, R_b both converge to the same value R which is the competitive equilibrium price p^* and the search equilibrium allocation in terms of quantity also converges to $sG(p^*)$ which is the quantity sold in competitive equilibrium. For $r \in (0, \hat{r})$, then all meetings result in trade and there is price dispersion. In this case the equilibrium price for the transaction with seller with cost y and buyer with valuation x satisfies

$$p(x,y) = \beta \left[\frac{r\theta x + (1-\beta) m(\theta) R_b}{r\theta + (1-\beta) m(\theta)} \right] + (1-\beta) \left[\frac{ry + \beta m(\theta) R_s}{\beta m(\theta) + r} \right]. \tag{3}$$

3.2 Steady-State

Throughout we focus on the steady-state of the search mechanism since that is where the search mechanism involves the least information, providing a best-case scenario for the search mechanism. Note that in steady-state constant distribution of types currently in the market implies that the distribution of types leaving the market is the same as the distribution of types entering the market, which is given by (F, G) with the cutoffs (R_b, R_s), the allocation in the steady-state can be described by a pair (p_s, y) where $p_s : [0,1] \to \mathcal{R}_+$ is a function, where $p_s(i)$ for describes the equilibrium transaction price for agent i if i participates in the market that is, if $i \in B$,$x^i \in [R_b, 1]$ and if $i \in S, y^i \in [0, R_s]$. If i does not participate then if i is a seller for $c^i \in (R_s, \overline{y}]$, $p_s(i) = R_s$ and if i is a buyer, $x^i \in [\underline{x}, R_b)$, then $p_s(i) = R_b$.

Since buyers and sellers meet randomly and the transaction price depends on the pair of valuations of buyers and sellers $p(x,y)$, prices are not deterministic in the search equilibrium but the distribution of realized transaction prices is deterministic as there is a continuum of traders and can be described by a c.f.d. $P : [\underline{p}, \overline{p}] \to [0,1]$. Any function p consistent with the search equilibrium implies in an equilibrium distribution of prices P. Let $\overline{y}(x)$ and $\underline{x}(y)$ be the highest seller's cost and lower buyer's valuation such that there is positive joint surplus in trading given buyer's and seller's valuations (x,y), respectively, then p satisfies $p_s(i) \in \{p(x^i, y), y \in [\underline{y}, \overline{y}(x^i)]\}$ if i is a buyer and

$p_s(i) \in \{p(\underline{x}, y^i), v \in [\underline{x}(y^i), \overline{x}]\}$ if i is a seller.

The privacy-preserving message process (μ_s, M_s) is constructed as follows:

The message space of the search mechanism is

$$M_s = \{(p_s, y) \in \mathcal{F} \times Y : p_s(i)y_1(i) + y_2(i) = 0, \forall i\},$$

where $\mathcal{F}$ is the space of functions on $[0,1]$ to $\mathcal{R}_{++}$.

Let μ_s^i be a correspondence from E^i to M_s. Let $\mu_s^i : E^i \rightrightarrows M_s$ given by

$$\mu_s^i(x^i) = \left\{ (p_s, y) \in M_s : y(i) = \begin{cases} (1, -p_s(i)) & \text{if } i \in B \text{ and } x^i \geq p_s(i) \\ (0,0) & \text{if } i \in B \text{ and } x^i < p_s(i) \text{ or } i \in S \text{ and } x^i > p_s(i) \\ (-1, p_s(i)) & \text{if } i \in S \text{ and } x^i > p_s(i) \end{cases} \right\}.$$

Define the correspondence $\mu_s : E \rightrightarrows M_s$ by

$$\mu_s(e) = \cap_i \mu^i(e^i) \cap (p_s(e) \times Y), \tag{4}$$

where $p_s(e)$ is the pricing function determined by the search equilibrium in the environment e (with buyers and sellers types distributed according to F and G) so that means that μ_s is restricted to the subset of M_s consistent with the search environment described in subsection 3.1. Note that μ_s is privacy-preserving by construction.

The search mechanism is a triple (μ_s, M_s, g_s) where $g_s(p, y) = y$ is a projection from M_s to Y.

Note that $p_s(i) > R_s$ if $c^i \leq R_s$ and $p_s(i) < R_b$ if $v^i \geq R_b$ since prices must compensate for search costs, while agents who do not trade are the types with costs/valuations in (R_s, R_b).

Remark 2. If the discount rate $r = 0$ and search costs (c_b, c_s) converge to zero then the search equilibrium prices all converge to p^* which means that the search mechanism becomes the competitive mechanism. Then, clearly, it is informationally efficient at this frictionless limit.

3.3 Informational Efficiency

In the environment has a continuum of agents and smooth distributions of types any allocation $y \in Y$ is an infinite-dimensional object. To articulate the argument of the size of information messages in the terms of Hurwicz (1977b) and Jordan (1982) on the dimensional size of message space which are finite-dimensional manifolds, we study here sequences of environments with finitely many types of buyers and sellers and as the number of types grows to infinity, the distributions of types approximate the continuous distributions of buyers F and sellers G.

Let $\{e_k\}_{k\geq 2}$ be a sequence of environments where buyers and seller types are distributed according to $\{F_k, G_k\}$, sequences of step-functions. A pair (F_k, G_k) that represent the cumulative distributions of types of buyers and sellers, respectively, in an environment e_k with k types of buyers and k types of sellers, each type of measure $1/k$ (that is, the sets of buyers and sellers are partitioned into subsets of the same measure whose elements are all identical). The pair of sequences $\{F_k, G_k\}_k$ converges to F and G, respectively. I call an environment with k types of buyers and sellers a k-environment. In addition F_k, G_k are such that p^* is consistent with competitive equilibrium in economy e_k.

Then the allocation mechanisms can be written in terms of types. Let $b \in \{1, \ldots, k\}$ index buyer types and $v \in \{1, \ldots, k\}$ seller types, let $x(b)$ be the valuation of a buyer of type b and $y(v)$ be the cost of a seller of type v.

Let $y^B(b) = (y^B(b)_1, y^B(b)_2)$ be the net trades for a buyer of type b in the competitive equilibrium and $y^S(b) = (y^S(v)_1, y^S(v)_2)$ the net trades for a seller of type v. A profile of net trades specifies a net trade for each of the $2k$-types of traders: $y = \left((y^B(b))_{b=1}^k, (y^S(v))_{v=1}^k\right)$. The set of net trades for the competitive mechanism is then

$$Y_k^c = \left\{ y \in ((-1, 0, 1) \times \mathcal{R}_+)^{2k} : \sum_{b=1}^{k} y^B(b) + \sum_{v=1}^{k} y^S(v) = (0, 0) \right\}.$$

as there are k types of buyers and k types of sellers. The competitive message space in the k-environment specifies a price of p and allocation y where $y \in Y$.

For the search mechanism, the set of net trades is a higher dimensional object. To see that consider the search equilibrium. A buyer of type $b \in \{1, \ldots, k\}$ can match with a seller of type $v \in \{1, \ldots, k\}$. Consider a partition of the set of buyers of each type b into k subsets $\{m(b, v)\}_{v=1}^k$, , corresponding to each seller type that a buyer could be matched with. Each subset $m(b, v)$ can transact at a price $p(b, v)$. If a pair of types (b, v) do not transact in the search equilibrium then we can divide these into two cases: (1) Buyer of type b does not participate in the market because his valuation is too low (so $b < R_b^k$, where R_b^k is the marginal buyer type for the k-types economy). (2) The seller of type s

does not trade with the buyer because either s does not participate or the discount rate r is too high for all participating types to trade with each other. In case (2) the set $m(b,v)$ is empty. In case (1), the buyer of type b does not participate in the market and we can just assume that $m(b,v)$ has the same measure for each seller type v.

Let $\lambda(b,v)$ be the probability that a transaction is between a buyer of valuation $x(b)$ and a seller of valuation $y(v)$. Among buyer type b who does participate in the market $\lambda(b,v)$ is given by the measure of the set $m(b,v)$ divided by the sum of the measure of sets $\{m(b,v') : v' \in \{1,2,\ldots,k\}\}$. If buyer type b does not participate in the market $\lambda(b,v)$ is zero for all seller types.

The search message space in the k-environment specifies prices for each possible pairing of buyers-seller types, which means that there are k^2 prices for each pairing between the k-types of buyers and k-types of sellers. Let $y^B(b,v) = (y^B(b,v)_1, y^B(b,v)_2)$ be the net trades of the set of buyers of type b with sellers of type v and $y^S(b,v) = (y_1^S(b,v), y_2^S(b,v))$ be the net trades of the set of sellers of type v with buyers of type b. A profile of net-trades is $y = \big(y^B(b,v), y^S(v,b)\big)\big)_{b,v\in\{1,\ldots,k\}}$.

Therefore, the set of net trades for the search mechanism is described by:

$$Y_k^s = \left\{ y \in ((-1,0,1) \times \mathcal{R}_+)^{2k^2} : \sum_b \sum_v \lambda(b,v)[y^B(b,v) + y^S(b,v)] = (0,0), y^B(b,v) = y^S(b,v) = 0 \text{ if } b < R_b^k \right\}. \tag{5}$$

Let $(\mu_c^k, M_c^k, g_c^k), (\mu_s^k, M_s^k, g_s^k)$ be the k-environment versions of the competitive and search allocation mechanisms. Where

$$M_c^k = \{(p,y) \in \mathcal{R}_{++} \times Y_k^c : py_1^j(i) + y_2^j(i) = 0, \forall j \in \{B,S\}, \forall i \in \{1,\ldots,k\}\}$$
$$M_s^k = \{(p,y) \in \mathcal{R}_{++}^{k^2} \times Y_k^s : p(b,v)y_1^j(b,v) + y_2^j(b,v) = 0, \forall j \in \{B,S\}, \forall b,v \in \{1,\ldots,k\}\}.$$

and μ_c^k, μ_s^k are the finite analogues of μ_c, μ_s: correspondences that map E^k into M_c^k, M_s^k, respectively, and g_c^k, g_s^k are projections from M_c^k, M_s^k, respectively, to Y^k.

Informational Size of the Message Spaces

In the competitive mechanism of the k-economy the message space includes only one price (as the price of the numeraire good is normalized to 1) and $2k$ types of buyers and sellers. However, market clearing implies that if $2k-1$ types trade then the net-trade for the last type are implied, therefore the message space of the competitive mechanism M_c^k is a $2k$-dimensional manifold.

In the search mechanism the k-economy there are k^2 prices and effectively $2k^2$ different types of agents (each player's endowed type and who they are matched with) in the search equilibrium, that means that for each buyer (or seller) type they form expectations regarding prices for transactions with each of the k-types of sellers (buyers) in the other side of the market, therefore each buyer (seller) individually has to form expectations regarding k distinct prices (each of which depends on the opportunity costs that depend on the distribution of buyers' types). This implies that the dimensional size of the message space is k^2 (prices) plus $\sum_{i=1}^{k}(k+k)$ different types of agents minus one dimension 1 due to the market-clearing condition. Therefore, M_s^k is a $3k^2-1$ dimensional manifold, which has approximately $1.5k$ times more dimensions than the message space of the competitive mechanism M_c^k. Clearly, this difference converges to infinity as $k \to \infty$, stated in the Proposition 1.

In other words, the search mechanism requires that each participant of the market be aware of all types of participants operating in the market to form expectations regarding payoffs from participating in the market and to bargain with the other participants. This is precisely the inverse of the intuition regarding the informational efficiency of the market as articulated by the literature on the informational efficiency of competitive markets: that each participant of the market can use prices as an efficient way to substitute for the information they would otherwise require to allocate resources without access to market prices.

Proposition 1. *M_c^k and M_s^k are $2k$ and $3k^2-1$ dimensional manifolds. Therefore, as $k \to \infty$ the ratio of the dimensional size of M_s^k to M_c^k converges to infinity.*

Proof. See Appendix Subsection A.2 ∎

Remark 3. On private information: If valuations are private information then upon matching buyers and sellers cannot play pricing strategies that are functions of the pair of matched types. Following Satterthwaite and Shneyerov (2007), consider the trade mechanism where buyers offer prices and sellers post reservation prices. If the offered price is higher than the reservation price the transaction occurs, otherwise there is no transaction.

Let the offered price by the buyer of type b be $\overline{p}(b)$ and the reservation price of a seller of type v be given by $\underline{p}(v)$. Then, there are $2k$ different "prices" in the search-mechanism under asymmetric information instead of k^2 prices. But there are still $2k^2$ types of consumers as each distinct pairing of buyers and sellers types can imply in a different net trade.

In this case the dimensional size of the message space is smaller than in the search mechanism with perfect information but the result of Proposition 1 still holds as the ratio of dimensional sizes of the message spaces of the search mechanism to the competitive mechanism also diverge to infinity as the number of types of consumers increases.

4 The Informational Efficiency of Market-makers

4.1 Environment

Suppose that in addition to buyers and sellers (the consumers), there is a finite set J of market-makers in this economy (with abuse of notation, J has cardinality J). Market-makers are firms who act as profit-maximizing intermediaries who "make the market" by posting bid and ask prices for the indivisible good, intermediating trade between the suppliers (sellers) and the final consumers (buyers) in the economy.[7] Consumers valuations are private information so we assume market-makers are constrained to uniform pricing policies where there is no price discrimination. [8] Unlike the search mechanism, buyers and sellers do not directly match with each other. Instead, both buyers and sellers trade through the market-makers. Buyers purchase from the lowest priced market-maker they have access too as long as it is lower than their valuation, sellers sell at the highest-priced market-maker as long as the posted price is higher than their cost.

Consider the case of the absence of any form of frictions of trade: All traders have costless access to all contracts posted by all market-makers. Consider a market-maker $j \in J$ who posts a pair of prices which the market-maker offers to buy and sell the good, respectively, (p_b, p_s) which are, respectively, higher and lower than prices posted by all other market-makers, then its profits are

$$\pi(p_s, p_b) = (p_s - p_b) \times b[1 - F(p_s)], \tag{6}$$

7. In here market-makers perform the same role as in Spulber 1996, but in this model the number of market-makers is finite and consumers are matched with different probability to each market-maker.

8. A market-maker could practice price discrimination but trader's valuations and awareness is private information. There is two-sided asymmetric information in the sense that both buyers and sellers have private information regarding their valuations and costs as in Satterthwaite and Shneyerov (2007, 2008) and since the good is indivisible then the only direct revelation mechanism that is truthfully implementable consists of a pair of prices that the market-maker is willing to buy or sell the good for.

subject to the market clearing constraint that the quantity brought from sellers is equal to the quantity demanded by buyers:

$$b(1 - F(p_s)) = sG(p_b). \tag{7}$$

If the market-maker posts bid prices lower than some other market-maker no seller will sell to him or her and so its profits are zero. If the maker posts bid prices higher than all others but not the lowest ask prices the market-maker has monopolized the supply and profits also satisfy 6 subject to the resource constraint 7.

Proposition 2. *If at least two market-makers are operating then there is only one Nash equilibrium: for at least two market-makers to post a pair of bid-ask prices $(p_b, p_s) = (p^*, p^*)$, that is market-makers post the competitive equilibrium price.*

Proof. See Appendix Subsection A.3 ∎

Proposition 2 states that this environment of strategic price determination by market-makers implements the competitive equilibrium in a frictionless setting. The interesting application of such a model, which is developed over the next-subsections, is to consider the case of imperfectly functioning markets where there are frictions of trading. We describe frictions of trading in this setting by the hypothesis that traders might not have full access to all market-makers because they might be unaware of the full set of market-makers operating in the market.

Frictions of Trading: Consideration Sets Constrained by Unawareness

The market-maker model incorporates frictions of trading which allow it to yield results such as market-power, price dispersion, and other features of markets that do not exist in the competitive market mechanism. We model the frictions of trading by assuming that consumers (buyers and sellers) have constrained choice sets regarding the market-makers that they can trade with. We use the term (borrowing from Perla (2019)) imperfect awareness to describe these constrained choice sets regarding the market-makers operating, we assume that awareness is randomly and independently distributed. In addition, we introduce a diffusion process of awareness among the consumers over time (as in the model of sticky-information by Mankiw and Reis 2002) which can result in departures from the conditions of competitive equilibrium that the standard search model does not have but that have found support in empirical studies.

For $j \in J$ there is subset $A^j \subset [0,1]$ of buyers and sellers who have access to market-maker j and let $A^i = \{j \in J : i \in A^i\}$ be the set of market-makers that i has access to. The set of environments E includes the market-makers and the awareness among buyers and sellers regarding them (use the term "aware of j" to mean $j \in A^j$[9]), this information is given by $\{A^j\}_{j\in J}$. That is, $e^i = (x^i, A^i)$, where x^i is agent i's valuation or cost and A^i is the set of markets that i is aware of, an environment $e \in E$ is specified by $\boldsymbol{A} = \{A^j\}_{j\in J}$ and the distributions of buyers valuations F and sellers costs G.

Let the awareness parameter m^j given by $m^j = \lambda(A^j) \in (0,1]$, the (Lebesgue) measure of A^j which is the fraction of all traders aware of j. I assume that A^j is a simple random sample of the traders. This means that it satisfies the properties of independence of type and awareness which means that:

Assumption A1 The fraction of traders who are buyers in A^j is b, sellers is $s = 1 - b$.

Assumption A2 The distribution of types of buyers and sellers conditional to being in A^j are F and G, respectively.

Assumption A3 The probability of being aware of a competing market-maker is independent, that is

$$\lambda(A^j \cap A^h) = \lambda(A^j) \times \lambda(A^h), \tag{8}$$

for the awareness sets A^j, A^h of marker-makers j and h.

Note that 8 implies that the fraction of buyers and sellers who are aware of the seller j conditional on being aware of a competitor is m^j. As valuations, consumer's choice sets are private information so market-makers cannot price discriminate based on the consumer's choice sets.

Static Equilibrium

The solution concept used here is Nash equilibrium in mixed strategies: a mixed strategy profile for the market-makers is a profile of distributions over a subset $F \subset [\underline{c}, \overline{c}] \times [\underline{v}, \overline{v}]$ of pairs of prices that are consistent with market-clearing (that is, the quantity brought by the market-makers is equal to the quantity sold) such that any price pair on the support of the distributions is profit maximizing. As stated in Proposition 3 given a profile of awareness parameters $\boldsymbol{m}$ such that consumers are fully aware of at most one market-maker the unique equilibrium in the this environment is a profile of pricing strategies

9. Other studies such as Armstrong and Vickers 2019 and McAfee 1994 use the term "consideration set" or "availability rate", respectively, to indicate the subset of agents that buyers or sellers have access to and to indicate degree of access among consumers in the market.

described by $(\{P^j\}_{j=1}^J, \boldsymbol{p})$, where P_j is a cumulative distribution function on $[0,1]$ and $\boldsymbol{p}$ is a function that maps $[0,1]$ into a pair of prices for buying and selling in $[\underline{c}, \overline{c}] \times [\underline{v}, \overline{v}]$. This function is such that posting any pair of prices $\boldsymbol{p}(\alpha) = (p^s(\alpha), p^b(\alpha))$ for α on the support of P^j is profit-maximizing and the resulting allocation is feasible.

Proposition 3. *If $\boldsymbol{m}$ is such that $m^j < 1$ for at least $J-1$ market-makers then there is a unique equilibrium that consists of a profile of mixed pricing strategies $\{P^j\}_{j\in J}$ and the sharing rule such that for a pair market-makers h and j if $m^h < m^j$ then traders aware of both will trade with h if posted prices are the same. The profile of equilibrium strategies features connected supports $[\underline{\alpha}^j, \overline{\alpha}^j]$ for each $j \in J$, they share a common lower bound of the support $\underline{\alpha}$, and the distributions are continuous on $[\underline{\alpha}, 1)$. For each $j \in J$, for $\alpha \in [\underline{\alpha}^j, \overline{\alpha}^j]$, $P^j(\alpha)$ satisfies*

$$P^j(\alpha) = \frac{a^{\bar{j}}}{a^j} P^{\bar{j}}(\alpha),$$

where $\bar{j}$ is the market-maker with the largest availability a. The distribution $P^{\bar{j}}$ is given by

$$\prod_{j\neq\bar{j}}(1 - a^j P^{\bar{j}}(\alpha))Q(\alpha) = \prod_{j\neq\bar{j}}(1 - m^j)Q(1).$$

where $Q(\alpha) = [p_s(\alpha) - p_b(\alpha)]sG(p_b(\alpha))$ is the quantity that is transacted per unit of awareness if there was no competition.

Consider a sequence of awareness profiles $\boldsymbol{m}_n$ If, for at least two market-makers h, g, m_n^h and m_n^g both converge to 1 then the equilibrium pricing strategies $\{P^j, \boldsymbol{p}\}_{j\in J}$ converge in probability to the competitive equilibrium price p^*, which is the unique equilibrium if $m^h = m^j = 1$ for at least two market-makers.

Proof. See Appendix Subsection A.4. ■

The equilibrium mixed strategy profile described in Proposition 3 is similar to the equilibrium described in McAfee 1994: the distributions of prices posted by the market-makers are non-degenerate and are continuous on the interior of the support, the larger market-makers (in terms of the m^j) transact at higher margins than smaller market-makers in the sense that the distribution of margins between ask and bid prices of the larger market-makers first order stochastically dominate those of the smaller market-makers). The reason for this result is that in an economy with finitely many market-makers it is less likely that buyers and sellers are aware of a competitor to large market-maker than a competitor to a smaller market-maker, so the larger market-maker loses fewer customers if the spread between sell and bid prices is increased.

Dynamic Equilibrium: Competitive Price Formation, Awareness Diffusion

Suppose that time is discrete: $t = 0, 1, 2, \dots$ and let $\beta = 1/(1+r)$ be the discount factor. The good is perishable so at each period sellers can produce one unit of the good and buyers have unit demand for one unit per period.

Awareness Diffusion: Given a set J of market-makers there is a awareness profile $\{m_t^j\}_{j=1}^J \in (0,1]^J$. Suppose awareness regarding a market-maker diffuses through the market according to

$$m_{t+1}^j = (1-\delta)m_t^j + M(m_t^j, 1-m_t^j), \tag{9}$$

where M is a matching function that represents the diffusion of awareness through traders who hitherto had access to the market-maker and $\delta \in [0,1)$ is the awareness depreciation parameter (that is, the rate in which traders "forget" about the market-maker).

Each market-makers chooses in period 0 to post prices according to a sequence of distributions for each period. Since the choice of the pricing strategies does not have any effect on the state of the market the optimal strategy for each market-maker is to choose the profit maximizing pricing behavior at each period given the action profile of the other market-makers at that period. Which means that at a given point in time t, prices practiced in the market are given by $\{P_t^j\}_j$ described in the proof of Proposition 3.

We are interested in the convergence of equilibrium prices and allocation to the competitive equilibrium. Since the outcome of the equilibrium is stochastic as the market-makers randomize their bid and ask prices we use the notion of convergence in probability: Convergence in probability of equilibrium prices and allocation to the competitive equilibrium means that the probability at a period t that prices and the allocation are at some positive distance from the prices and allocation of the competitive equilibrium converges to zero as $t \to \infty$. The distance between two allocations x and x' that assign a consumption bundle $x'(i)$, and $x(i)$, respectively, to consumer $i \in [0,1]$ is described by a function $D(x, x')$ that satisfies

$$D(x, x') = \int_0^1 \sum_l \left[|x_l(i) - x_l'(i)| \right] di.$$

Proposition 4 follows from Proposition 3 as the expected equilibrium margin between buy and ask prices posted by the market-makers converge to zero if $\lim m_t^j = 1$ for $m_t^j > 0$ and $J \geq 2$. Therefore, the awareness by the consumers regarding the market-makers operating converges to 1 as $t \to \infty$ implies that a measure converging to 1 of consumers has access to buy and ask prices that are converging in probability to the competitive price. Therefore, the equilibrium allocation converges in probability to the competitive allocation.

Proposition 4. *If the law of motion for awareness diffusion 9 implies that* $\lim m_t^j = 1$ *for* $m_t^j > 0$ *then if* $J \geq 2$ *as* $t \to \infty$ *the equilibrium prices and the equilibrium allocation converge in probability to the competitive equilibrium.*

Steady-State Equilibrium

The analysis so far implied that a market where trade is mediated by market-makers approximates the outcome of the competitive equilibrium as awareness regarding the market-makers diffuses. However, by allowing awareness to depreciate with $\delta > 0$ (as consumers forget about a market-maker) then given some assumptions on the matching function M[10] then there is a unique steady-state awareness level $\hat{m}$ such that

$$\hat{m} = M(\hat{m}, 1 - \hat{m})/\delta.$$

There is a corresponding steady-state equilibrium if and only if all market-makers have the same awareness parameter $m^j = \hat{m}$ which is a symmetric mixed strategy described by a pair $\{P, \boldsymbol{p}\}$ that all market-makers follow. If we let $0\delta \to 0$ then awareness diffusion function M implies that $\hat{m} \to 1$, that is if awareness depreciates very slowly the steady-state level of awareness approximates full awareness, which implies that the steady-state equilibrium approximates the competitive equilibrium.

Entry and Exit

Suppose that at some period t there are some market-makers that are operating in the market and some that are not operating in the market (which means that their awareness parameter m^j is zero). There is an entry cost $E > 0$ which is the cost of setting up a starting consumer base represented by an entry level awareness parameter $m_e \in (0, 1)$.

The law of motion for the diffusion of awareness 9 implies that for M increasing and concave in both arguments, m_e that is not very large, and if the depreciation parameter is not very large then market-makers grow after entry (in the sense that m_t^j is increasing over time). That implies that incumbent market-makers are larger than entrants and

10. More precisely, that M is continuously differentiable, concave in both arguments, satisfies the condition that $\lim_{a \to 0} \partial M(a, b)/\partial a = \infty, \forall b > 0$.

therefore transact at higher expected margins. If we follow Spulber 1996 interpretation that market-makers are firms who intermediate between suppliers and consumers this equilibrium property replicates the findings of Foster, Haltiwanger, and Syverson 2008, 2016 that incumbents charge higher prices than entrants.

Contestable Market Equilibrium

Suppose that there are only two market-makers. The set of market-makers is $J = \{1, 2\}$, suppose that $m_0^1 = 1, m_0^2 = 0$ at a date set to 0. That is, 1 is a monopolist market-maker and all traders have access to his posted contracts and 2 is out of the market, but 2 can decide to enter in the current period. That is, 2's action set is that she can choose $m_t^2 \in \{0, m_e\}$ besides the price if she has not entered in the market in the period before. A monopoly deterrence equilibrium is a situation where the incumbent 1 chooses to post buying and selling prices for each period such that the profits from offering better prices to the consumers are too low to compensate for the cost of entering the market.

Definition 3. A *monopoly deterrence equilibrium* is an equilibrium where 1 chooses a pure pricing to pricing schedule and given this pricing schedule 2 finds it optimal to not enter. The pricing schedule is the profit-maximizing in the sense that a higher selling-buying margin that yields higher profits for 1 implies that 2 finds it optimal to enter and undercut 1's posted offers in every period and is profit-maximizing in the sense it yields higher discounted expected value of the profit stream for 1 than the expected value of the profits in the equilibrium under a duopoly if 2 also enters the market.

The proposition below states that if entry costs are high enough and awareness diffusion is fast enough then the unique equilibrium is for the monopolist to deter entry. That is because entry cost is higher than the expected profits that can be obtained in the duopoly competition process where market-maker 2 competes with the former monopolist. However, the monopolist 1 must commit to a sequence of prices that still yield a low enough profit to deter the entrant. The unique equilibrium is the sequence of prices that makes 2 indifferent between entering and not but that maximizes the present value of 1's profit stream. As discount rates decrease the present value of the gains from entering the market increase. Which implies that the buy and ask prices posted by the monopolist become closer to the price of competitive equilibrium. As the discount rate r converges to zero, the present value of any positive profit stream converges to infinity, which implies that the monopoly deterrence equilibrium converges to the competitive equilibrium as the discount rate converges to zero.

Proposition 5. *If awareness diffusion is fast enough so $\sum_{t=0}^{\infty}(1-m_t^2) \leq C$ for some constant C conditional on seller 2 entry, and the discount rate r is low enough, then, for an entry cost E equal or higher than $C \times \pi^M$, the unique equilibrium is the monopoly deterrence: The monopolist commits to posts prices $\mathbf{p}(\pi)$ that yield a per-period profit of*

$$\pi = E/\left(\sum_{t=0}^{\infty}\beta^t m_t^2\right)$$

to deter entry. As r converges to zero the deterrence monopoly equilibrium profit flow π converges to zero, which means the posted buying and selling prices converge to the competitive equilibrium prices p^*.

Proof. See Appendix Subsection A.5. ∎

The existence of entry costs for 2 can be interpreted to represent the costs of communicating additional information to the market-participants so that if the costs of communicating additional information are higher than the (private) benefits which are the profits 2 obtains from entering the market, then it does not occur in equilibrium. [11]

4.2 Allocation Mechanism

The allocation mechanism in this case is the allocation mechanism that implements the allocation corresponding to a realization of the Markov perfect equilibrium of the market-maker economy at some period (which is a profile of realized pairs of prices for each market-maker). Note that if there is imperfect awareness regarding almost all the market-makers (i.e. $\boldsymbol{m} = (m^j)_{j=1}^J$ such that $m^j = 1$ for at most one j) then for any market-maker the posted price for buying is strictly smaller than for selling and therefore profits are strictly positive.

Following Hurwicz (1977a), we interpret that the profits of the market-makers and the resulting deadweight losses are both components of the "cost" of operating the allocation mechanism: In that case the allocation implemented by the mechanism features strictly

11. We have not performed a welfare analysis to check if the monopoly deterrence equilibrium is more efficient than the duopoly after 1's entry. As the social benefits of 2's entry would be the reduction of the deadweight loss thanks to prices closer to perfect competition. The social benefits are different from the private benefits of entry.

negative net-trades for the numeraire good among the agents in the economy and as bid and ask prices diverge it means that not every buyer or seller who has access to a market-maker and would trade under competitive prices does so.

The set of net-trades incorporates the possibility of market-makers making profits by buying at lower prices than they sell:

$$Y_m = \left\{ y : [0,1] \to \{-1,0,1\} \times \mathcal{R} : \int_0^1 y_1(i)di = 0, \int_0^1 y_2(i))di \leq 0 \right\}.$$

Given a realized profile of prices $p_m = (p_1, \dots, p_J)$ (which describes the equilibrium at some date in the dynamic version of the model) the message space is given by

$$M_m = \{(p_m, y) \in \mathcal{R}^J_{++} \times Y_m : \text{for each } i, \exists j \in A^i \text{ s.t. } i \in A^j \text{ and } p^j y_1(i) + y_2(i) = 0\},$$

and μ_m is a correspondence on E to M_m that satisfies

$$\mu_m = \cap_i \mu^i_m(e^i),$$

where $\mu^i_m : E^i \rightrightarrows M_m$ satisfies

$$\mu^i_m(e^i) = \left\{ (p_m, y) \in M_m : y_1(i) = \begin{cases} (0,0) & \text{if } i \in B \text{ and } v^i < \min\{p^j_s : j \in A^i\} \text{ or } A^i = \emptyset \\ (1, -\min\{p^j : j \in A^i\}) & \text{if } i \in B \text{ and } v^i \geq \min\{p^j_s : j \in A^i\} \\ (0,0) & \text{if } i \in S \text{ and } c^i > \min\{p^j_b : j \in A^i\} \text{ or } A^i = \emptyset \\ (-1, \min\{p^j : j \in A^i\}) & \text{if } i \in S \text{ and } c^i \leq \min\{p^j_b : j \in A^i\} \end{cases} \right\}.$$

Informational Efficiency

As in Subsection 3.3 consider a sequence of finite types economies $\{e_k\}$ that approximates the environment with the cumulative distributions of buyers and sellers valuations F and G. In this case the dimensional size of the message space incorporates the different market-makers that make the market: If there are $N \leq J$ market-makers with non-zero awareness parameters then there are $2N$ different prices posted to the consumers, plus the subset of consumers who are not aware of any market-makers. As in the case of the search allocation mechanism, the set of consumer "types" increases to include differentiate consumers by their access to different prices (as awareness is heterogeneous).

The number of consumer types is determined by the discrete distributions of valuations (G_k, F_k) and N. The type of a consumer can be specified by a triple (r, κ, h) where $r \in \{b, s\}$ denotes if the consumer is a buyer or seller, $\kappa \in \{1, 2, \ldots, k\}$ denotes the valuation type of buyer or seller, $h \in J$ denotes the market-maker that the consumer transacted with (including $h = \emptyset$ if the consumer does not transact with any market-maker).[12] Therefore, there are $2kN$ or $2k(N+1)$ types of consumers if the subset of consumers who are not aware of any market-makers is respectively empty or non-empty. Therefore, market-clearing of the indivisible good among consumers who interact with each market-maker implies that the message space corresponding to environments with k different valuations for buyers and sellers is a Z-dimensional manifold where Z is equal to $2N + (2k-1)N$ or $2N + (2k-1)(N+1)$, respectively if the subset of consumers who are not aware of any market-makers is respectively empty or non-empty. This implies in the following corollary:

Corollary 1. *As k increases to infinity the ratio of the dimensional size of the message spaces of the market-maker mechanism to the competitive mechanism $Z/2k$ converges to N or $N+1$.*

That is, the ratio of the size of the message spaces between the competitive mechanism and the market-maker mechanism when the number of types of consumers is large is approximately the number of market-makers operating in the market. This result is intuitive since the competitive mechanism implicitly assumes a single monopolist market-maker called the Walrasian auctioneer whose bid and ask prices have zero spread.

Consider the case of the monopolist market-maker who deters entry, note that the subset of consumers who are not aware of any market-makers is empty. So the number of consumer types is $2k$ but a pair of prices is realized instead of one price in the case of the Walrasian auctioneer. It represents the most informationally efficient mechanism in this class of market-maker environments with informational size $2k+1$, or only one dimension more than the competitive mechanism (outside of the limit case of $\delta = 0, \beta \to 1$ when it converges to perfect competition and there is only one price posted to all consumers). This additional dimension reflects the profit margin between purchase and sale to provide incentives for the market-makers to "produce" the price mechanism.

12. Note that it is not necessary for the computation of the dimensional size of the message-space to include the information of which other market-makers the consumer was aware of besides the one he or she transacted with.

However, for the model to generate more sophisticated market-behavior in equilibrium, such as price dispersion with average spreads between the ask and bid prices that depends on the degree of tenure of the market-maker has been operating, then it requires the assumption that there are at least two market-makers operating. In that case the minimum dimensional size of the message space of the market-maker equilibrium in a given period is $Z = 4 + 2(2k - 1)$, as there is a pair of bid and ask prices and there is two profiles of net trades for the $2k$ types of consumers.

5 Extending the Model to an Environment with L Goods

It is a simple exercise to extend the market-maker model of Section 4 to a more general environment with multiple goods, which is more comparable to the original informational efficiency papers of Mount and Reiter (1974), Hurwicz (1977b), and Jordan (1982). Consider an environment with $L > 1$ goods where good L is the numeraire. There is a continuum of consumers $i \in [0,1]$, with consumption sets $X = \mathcal{R}_+^{L-1} \times \mathcal{R}$ and CRRA preferences, where

$$u_i(x) = \sum_{l=1}^{L-1} \theta_{il} \left(\frac{x_l^{1-\sigma_l}}{1-\sigma_l} \right) + x_L, \theta_{il} > 0, \sigma_l \geq 0, \forall l.$$

Suppose θ_{il} is private information and distributed on interval $[\underline{\theta_l}, \overline{\theta_l}]$ according to some cumulative distribution F.

There is a set $J_l, |J_l| \geq 1$ of market-makers operating in the markets for each good $l \in 1, \ldots, L-1$ and they compete by posting bid and ask prices for the good in exchange for the numeraire.[13]

Quasi-linear preferences imply that the equilibrium price of the market for each good is independent of the other markets, and profits are continuous functions of the posted prices as long as they are more attractive to sellers/buyers than their competitor's prices. Therefore, the results of the analysis of Section 4 applies here. In particular, consider the ratio of the dimensional sizes of the message space of the market-maker allocation mechanism to the competitive mechanism. By similar argument to the two economy of sub-

13. Note that if we allow for non-linear pricing (as described, for instance, Bolton and Dewatripont (2005)) in this case the dimensional size of the message space is higher, with continuously varying prices it is also infinity. That provides a justification why we do not see many empirical examples of sophisticated non-linear pricing if allocation mechanisms are informationally constrained.

section 4.2 this ratio is well defined in a finite types economy with k types of buyers and k types of sellers. As the number of consumer types converges to infinity and the distribution of types in a finite types economy approximates the distribution of types of the environment E the ratio of dimensional size converges to a number equal to $\sum_{l=1}^{L-1} J_l/(L-1)$. That is, the average number of market-makers operating in the market for each good $l \in \{1, \ldots, L-1\}$ yields the ratio of the size of the message space to the competitive economy (where there is one market-maker for each good: the Walrasian auctioneer).

The dynamic deterrence equilibrium described in Subsection 4.1 can be implemented in this L-goods economy with one monopolist in the market for each good $l \in \{1, \ldots, L-1\}$. In equilibrium, there will be $2(L-1)$ prices: a buying and selling price for each good l in terms of the numeraire good L. This implies that the dimensional size of the message space of the monopolist market-maker economy has $L-1$ more dimensions than the competitive mechanism, one for each good besides the numeraire good. Which means that the difference in informational requirements between this mechanism and the competitive market mechanism converges to zero as the finite types economy approximates an economy with a continuum of consumer types.

6 Concluding Remarks

In this paper, we consider the informational efficiency of decentralized price formation. In particular, we are interested in economies with strategic agents where the allocation mechanism converges to the competitive mechanism. We study two such mechanisms: search and market-makers.

While the random search model has been extensively studied in the literature, we show that it is unattractive from an informational perspective. In particular, we showed that for large economies the search allocation mechanism requires infinitely more information than the competitive mechanism. A true search mechanism, where everyone must be able to search across all the people in the economy to find trading partners, is extremely inefficient in terms of information as it requires that each agent must have a complete model of the economy. That is one possible reason we do not often observe single buyers trading with single sellers in real-world economies.

In contrast, we propose a different decentralized mechanism with market-makers. Such a mechanism has a few attractive features. First, the market-maker mechanism better matches certain features of the data, such as exhibiting price dispersion and prices that depend on the tenure of firms in the market. The other attractive feature, which is the focus of this paper, is that the market-maker mechanism almost requires as little information as the competitive allocation. Moreover, the mechanism requires relatively little information, even when it is used to explain deviations from the competitive allocation. This informational efficiency is one possible reason we observe intermediaries that facilitate trade between individual original sellers and individual final buyers and that their operation is restricted to individual markets.[14] It is a rather puzzling result that an economy where trading for each good is intermediated by few traders can be though as more informationally efficient than markets where trading is highly decentralized but it is an intuitive result: if consumers only need to be aware of a few intermediators for each good they purchase the informational requirements are much smaller than if consumers need to form a model of the whole market before engaging in random matching and bargaining for their consumption bundle.

But, perhaps the main contribution of this study is to point out that even if a theoretical model is able to replicate certain features of real economies the degree of informational efficiency of the allocation mechanism implicit in a model should also be an element to take into consideration when judging the degree of plausibility of that model. In the specific case of models that explain price formation should also explain how the message space of the allocation mechanism that is implicit in the model approximates the main feature of competitive mechanism: that agents can take terms of trade as a given without the need to "think" about how they are determined.

A Appendix

A.1 Existence and Uniqueness of Walrasian Equilibrium Price

Proposition 6. *There exists a unique p^* such that $sG(p^*) = b[1 - F(p^*)]$.*

Proof. Define the excess demand function $Z : [0, 1] \to [-1, 1], Z(p) = s[1 - G(p)] - bF(p)$, since F and G are continuous, Z is continuous, at $p = 0$, $Z(p) = 1$ while at $p = 1, Z(p) = -1$. By the intermediate value theorem there exists a p^* such that $Z(p^*) = 0$ and from the fact that Z is strictly decreasing it's easy to see that such p^* is unique. ■

14. For example, real state agencies specialize in intermediation of trade in real-state.

A.2 Proof of Proposition 1

Proof. First, consider the competitive mechanism. Using the conditions of $\sum_{i=1}^{k} y^i = \mathbf{0}$ and $py_1^i + y_2^i = 0, \forall i$, implies that the function $(p, y) \to (p, \tilde{y}) \in \mathcal{R}_{++} \times \mathcal{R}_{++}^{2k-1}$, where for $1 \leq i \leq 2k-1$, $\tilde{y}^i = y^i$, is a C^∞-diffeomorphism, thus M_c^k is a $(2k-1)+1 = 2k$-dimensional manifold.

Second, consider the search mechanism, in this case the price vector of the search equilibrium has k^2 dimensions, while Y_k^s has $2k^2$ dimensions, so, by analogous argument as for the competitive mechanism, M_s^k is a $k^2 + 2(k^2) - 1 = 3k^2 - 1$-dimensional manifold. ■

A.3 Proof of Proposition 2

Proof. To see this is a Nash equilibrium note that posting the competitive price yields zero profits and for any market-maker deviations either imply negative profits (if prices for purchase are higher than p^* and for selling are lower than p^*) or zero profits (in the case the prices for purchase are lower than p^* and for selling are higher than p^*). To see that this is the unique Nash equilibrium note that if market-makers post prices to make strictly positive profits other market-makers could deviate and make profits by capturing the customers of competitor market-maker by posting more attractive bid and ask prices. ■

A.4 Proof of Proposition 3

Proof. Part 1. Existence and characterization:

As shown in A.1 there is a unique competitive equilibrium price p^*, since A^j satisfies property 4.1, p^* is also the unique competitive equilibrium price for the subset of traders who are aware of a market-maker.

To construct the candidate equilibrium strategy profile $\{P^j\}_{j \in J}$ we consider pricing strategies described by a pair (p_b, p_s) of offers to buy and sell the good by the market-maker where $p_b \leq p^* \leq p_s$. First consider the monopoly prices $\boldsymbol{p}^M = (p_b^M, p_s^M)$ which satisfies the monopolist market-maker problem:

$$\max_{p_b, p_s} \{(p_s - p_b) \min\{sG(p_b), b[1 - F(p_s)]\}\}. \tag{10}$$

In the case of existence of multiple profit maximizing pairs of monopoly prices, let (p_b^M, p_s^M)

be the pair of monopoly prices with the lowest difference between the buying and selling price, which implies, as $sG(p_b^M) = b[1 - F(p_s^M)]$, that it is the pair with lowest selling price and highest buying price.

Let $\bar{j}$ be the market-maker with the largest awareness parameter ($m^{\bar{j}} = \max\{m^j\}_{j \in J}$). Let

$$\underline{\alpha} = \prod_{h \neq \bar{j}} (1 - m^h)$$

and let Π^M be the monopoly profit normalized in regards to $m^j \in (0, 1]$, that is

$$\Pi^M = (p_s^M - p_b^M)[sG(p_b^M)].$$

Consider a function $\boldsymbol{p} : [0, 1] \to \mathcal{R}_+^2$ such that $\boldsymbol{p}(\alpha) = (p_b(\alpha), p_s(\alpha))$ is a pair of prices that satisfies

$$[p_s(\alpha) - p_b(\alpha)]G[p_b(\alpha)] = \alpha\Pi^M/s, \tag{11}$$

and also satisfies market clearing,

$$sG(p_b(\alpha)) = b[1 - F(p_s(\alpha))]. \tag{12}$$

That is, $(p_b(\alpha), p_s(\alpha))$ is the pair of prices that implements a feasible net trade for a monopolist market-maker and yield a fraction α of the monopoly profits. In addition if for some $\alpha \in [0, 1]$ there is more than one such pair of prices then $(p_b(\alpha), p_s(\alpha))$ is the pair with smallest difference between the buying and selling prices, formally, for each $\alpha \in [0, 1]$, $(p_b(\alpha), p_s(\alpha))$ satisfies

$$(p_b(\alpha), p_s(\alpha)) = \arg\min_{(b,s)} \{|b - s| : (b, s) \text{ satisfies } 11, 12\}.$$

To see that there exists at least one pair of prices that satisfies 11 and 12 note that profits for $p_b = p_s = p^*$ are zero and imply $sG(p^*) = b[1 - F(p^*)]$, while profits for $p_b = p_b^M$ and $p_s = p_s^M$ are Π^M and they also satisfy market clearing. As G and F are continuous therefore for any $p_s \in [p^*, p_s^M]$ there exists a (unique) buying price $p_b(p_s)$ that satisfies $sG(p_s) = b[1 - F(p_b(p_s))]$ and continuity of G and F also imply that profits vary continuously from 0 at $p_s = p^*$ to Π^M at $p_s = p_s^M$, by the intermediate value theorem any profit level between 0 and Π^M can be attained by some pair of prices $(p_s, p_b(p_s))$ with

$p_s \in [p^*, p_s^M]$.

Note also that if a pair of prices is feasible for a monopolist market-maker then feasibility also holds in the case of competition between pairs of prices given by $\{\boldsymbol{p}(\alpha) : \alpha \in [0,1]\}$. To see that consider a market-maker j, if a competing market-maker j' offers buying and selling prices that are more attractive to sellers (that is $p_s^{j'} > p_s^j$) then market clearing condition for the monopolist implies these prices must also be more attractive to the buyers (that is, $p_s^{j'} > p_s^j$), since the types aware of j' are distributed in the same way as the types in the trader's population $m^{j'}$ which means that the proportion of clients that are buyers or sellers that j loses to j' is the same (that is, supply of the good purchased by the market-maker decreases by a factor of $1 - m^{j'}$ but demand also decreases by a factor of $1 - m^{j'}$) therefore market clearing still holds.

The candidate equilibrium strategy profile $\{P_j\}_{j \in J}$ is a profile of CDF's on $[\underline{\alpha}, 1]$, that is, $P_j(\alpha)$ is the probability that buying (selling) prices higher (lower) than $p_b(\alpha)(p_s(\alpha))$ that satisfies the equal profit condition

$$\prod_{h \neq j}(1 - P_h(\alpha)m^h)[p_s(\alpha) - p_b(\alpha)]sG[p_b(\alpha)] = \underline{\alpha}\Pi^M, \tag{13}$$

where

$$\underbrace{1 - m^h}_{\text{Prob. } h \notin A^i} + \underbrace{[1 - P_h(\alpha)]m^h}_{\text{Prob.} h \in A^i \text{ and } (p_b^h < p_b(\alpha) \text{ or } p_s^h > p_s(\alpha))} = 1 - P_h(\alpha)m^h, \tag{14}$$

is the probability that a trader chooses to transact with the market-maker j over competitor h and $\frac{\alpha}{s}\Pi^M$ is the profit margin of a market-maker when posting prices at the minimum profitability level (lower bound for sales, upper bound for purchases), which means its selling (buying) prices undercuts (tops) all competitors. Note that 13 implies that $P_h(\underline{\alpha}) = 0$ as $[p_s(\underline{\alpha}) - p_b(\underline{\alpha})]sG[p_b(\underline{\alpha})] = \underline{\alpha}\Pi^M$.

To check that this is an equilibrium:

Note that any prices not in the support of equilibrium strategies $\mathcal{P} = \{(p_b(\alpha), p_s(\alpha)) : \alpha \in [\underline{\alpha}, 1]\}$ lead to strictly lower profits: If we consider prices $(p_b(\alpha), p_s(\alpha))$ that satisfy 11 and 12 defined for $\alpha < \underline{\alpha}$, profits are strictly lower by construction. For prices $(p_b, p_s) \in [p_b^M, p_b(\underline{\alpha})] \times [p_s(\underline{\alpha}), p_s^M] \cap \mathcal{P}$ they either yield strictly lower profits because they are undercut by prices which would achieve similar profitability in the case the market-maker were a monopolist or they are not feasible (i.e. the market-maker promises to sell more than it purchases).

Given the sharing rule, it is easy to check that profits are constant on the support of

$\{P^j\}$ for each j. If there is only one market-maker j with the largest awareness parameter m^j then the equal profit condition 13 implies that there is atom of probability at $P^j(\boldsymbol{p}^M)$, the monopoly price, in the mixed strategy of the largest market-maker. The sharing rule implies that traders always choose to trade with $h \neq j$ if h posts the monopoly prices $\boldsymbol{p}^M$, hence its profits do not fall discontinuously on the support of the equilibrium strategy $[\underline{\alpha}, 1]$ as $\boldsymbol{p}(\alpha) \to \boldsymbol{p}^M$.

Part 2. Uniqueness:

Note that any feasible pricing strategy for the market-maker is a pair of buying and selling prices that are consistent with market-clearing. Note that pricing strategies on the set of pairs of prices $\{\boldsymbol{p}(a) : a \in [0,1]\}$ are weakly dominant, as any feasible pricing strategy (p_b, p_s) yields the same profits as a strategy $\boldsymbol{p}(a)$ for some $a \in [0,1]$ then sellers and buyers prefer the buying and selling prices $\boldsymbol{p}(a)$. Therefore, for any market-maker posting a pair of prices $(p_b, p_s) \notin \{\boldsymbol{p}(a) : a \in [0,1]\}$ cannot be a best response to a best response. Hence, any candidate for Nash equilibrium consists of distributions over prices in $\{\boldsymbol{p}(a) : a \in [0,1]\}$.

As we are restricted in our candidate equilibrium strategies to prices in $\{\boldsymbol{p}(a) : a \in [0,1]\}$ then the proof of uniqueness of equilibrium is a proof of uniqueness over distributions on $[0,1]$. Consider an equilibrium strategy profile F, undercutting arguments imply that $F = \{F^j\}_{j \in J}$ is non-degenerate and clearly the upper bound of the support for at least a pair of market-makers must include the monopoly price (otherwise its a profitable deviation to post the monopoly price). The union of the supports for the strategies must be convex; otherwise market-makers could increase profits by posting prices in the complement of the support. Additionally, the supports for the mixed strategies of individual market-makers must be convex; otherwise the equal-profit condition will be violated.

Further, equal-profit conditions are required to hold in a mixed strategy equilibrium and these conditions imply that when the interior of the supports overlap, Equation 13 holds. This implies that, assuming no atoms at the lower bound of the support of the distribution, the lower bound of the supports for any pair of seller types must be the same if the interior of the supports overlap. Which implies that equilibrium strategies for all seller types have convex supports (that is, the union of the supports is a its own convex hull). In addition, note that there cannot be atoms at a lower bound of the support of equilibrium price distributions; otherwise other market-makers have the incentive to post a more attractive pair of prices $\boldsymbol{p}(\alpha)$ for α in the ϵ-neighborhood of the lower bound of the support.

It remains to show that any equilibrium profile of distribution of prices is such that the interior of the supports overlap. That is, for any pair of market-makers the interior of the support of mixed pricing strategies must overlap.

To see that suppose, without loss of generality, that there is an equilibrium strategy profile $\boldsymbol{P}$ such that there is a pair of market-makers j, j' with the same awareness parameter $0 < a^j = a^{j'} < a^k$ who compete against each other posting prices according to a strategy that is described by pair of distributions $P^j, P^{j'}$ on $[0,1]$ and the price posting function $\boldsymbol{p}$ that maps $[0,1]$ into pairs of buying and selling prices. The distributions $P^j, P^{j'}$ have the same support $[\underline{\alpha}^*, \overline{\alpha}^*]$ while all other market-makers post prices according to distributions that have their supports in $[\underline{\alpha}, 1]$ with $\underline{\alpha} = \overline{\alpha}^*$. In words, market-makers j and j' compete by posting strictly more attractive prices to buyers and sellers than all others. Let $\overline{K}$ be the market-maker with largest awareness parameter in the subset of market-makers $\hat{J} = J - \{j, j'\}$.

Let $\Pi^o(\alpha)$ be the equilibrium profits per unit of awareness of market-maker o in posting prices $\boldsymbol{p}(\alpha)$ we call that o's equilibrium "profitability". Since $\overline{\alpha}^* = \underline{\alpha}$, the profitability of market-maker j of posting prices $\boldsymbol{p}(\overline{\alpha}^*)$ is

$$\Pi^j(\overline{\alpha}^*)/a^j = (1 - a^{j'})\overline{\alpha}^*\Pi^M. \tag{15}$$

If market-maker $o, o \neq j, j'$ post prices $\boldsymbol{p}(\underline{\alpha})$, his or her equilibrium profitability is

$$\Pi^o(\underline{\alpha}) = (1 - a^{j'})(1 - a^j)\underline{\alpha}\Pi^M. \tag{16}$$

Note that $\underline{\alpha} = \overline{\alpha}^*$, and therefore the equal-profit condition for j, and 15, implies that

$$\underline{\alpha}^* = (1 - a^{j'})\underline{\alpha}. \tag{17}$$

Finally, equations 16, and 17 together imply that for market-maker $o \neq j, j'$ that his or her profitability in posting prices $\boldsymbol{p}(\underline{\alpha}^*)$ satisfies

$$\Pi^o(\underline{\alpha}^*) = \underline{\alpha}^*\Pi^M \tag{18}$$
$$= (1 - a^{j'})\underline{\alpha}\Pi^M > (1 - a^{j'})(1 - a^j)\underline{\alpha}\Pi^M \tag{19}$$
$$= \Pi^o(\underline{\alpha}). \tag{20}$$

The inequality 19 is a contradiction with $\boldsymbol{P}$ being an equilibrium. Therefore, in equilibrium the interior of the supports must overlap.

Hence, the pricing strategy described by $\{P_j\}_{j\in J}$ and $\{(p_b(\alpha), p_s(\alpha)) : \alpha \in [\underline{\alpha}, 1]\}$ is the unique Nash equilibrium given the sharing rule that the market-maker with the smaller awareness parameter m^j has priority in transactions to buyers and sellers in the case of a tie in prices. ∎

A.5 Proof of Proposition 5

Proof. The proof is divided into two parts.

Part 1: For simplicity of the argument first let us consider the case where agents are myopic (only care about present payoffs, that is the discount factor $\beta = \frac{1}{1+r} = 0$) so, in this case, the deterrence game has only one period.

As in the proof of proposition 3 let $\pi^M = (p_s^M - p_b^M)sG(p_b^M)$ be the monopoly profit rate. Consider a profit rate $\pi \in (0, \pi^M)$, and suppose the incumbent 1 considers posting a price $\boldsymbol{p}(\pi) = (p_b(\pi), p_s(\pi))$ that is the pair of buying and selling prices with smallest difference that satisfies $[p_s(\pi) - p_b(\pi)]sG[p_b(\pi)] = \pi$. That is, $\boldsymbol{p}(\pi)$ is the pricing strategy that yields a payoff of $\pi \times m^j$ if j is a monopolist.

For $\pi \leq E/m_e$, then if the incumbent posts $\boldsymbol{p}(\pi)$ the cost of entry E is higher than the profits 2 can make after entry by undercutting 1 (as the discount rate is $r = 1$ the market-makers only care about present profits). Therefore 2 does not enter. Therefore, if the incumbent posts $\boldsymbol{p}(\pi)$ for $\pi = E/m_e$ (the highest profit margin that deters entry) and the entrant playing "no entry", it is an equilibrium if 1 has no incentive to deviate.

Suppose that 1 deviates from $\boldsymbol{p}(\pi)$ and posts prices according to the mixed strategy equilibrium described in Proposition 3 (for instance), suppose E is low enough so that 2's profits in mixed strategy equilibrium (given by $m_e(1 - m_e)\pi^M$) are higher than the entry cost. Then, 1's profits are $(1 - m_e)\pi^M$. If E is larger than $m_e\underline{\alpha}\pi^M = m_e(1 - m_e)\pi^M$ then, for the monopolist a pure strategy$\boldsymbol{p}(\pi)$ for $\pi = E/m_e$ that deters entry yields strictly higher profits than if the monopolist plays the mixed strategy P^j which yields profits of $(1 - m_e)\pi^M$, since deterrence profits π satisfy

$$\pi = E/m_e > \frac{m_e\underline{\alpha}\pi^M}{m_e} = (1 - m_e)\pi^M.$$

That is if $m_e\underline{\alpha}\pi^M < E$ then the only equilibrium is for the monopolist to post $\boldsymbol{p}(\pi)$ for $\pi = E/m_e$, as even higher profit margin encourages undercutting by the entrant and playing the mixed strategy. If $E < m_e \underline{\alpha} \pi^M$, then $\pi < (1 - m_e)\pi^M$ and the only equilibrium is entry of 2 and the mixed strategies (P^1, P^2) described in the proof of Proposition 3.

Part 2: This logic can be extended to the environment where agents are not myopic and instead have a common discount factor $\beta \in (0,1)$. Then payoffs of 1 and 2 in the Markov perfect equilibrium after entry are respectively

$$U_e^1 = \sum_{t=0}^{\infty} \beta^t (1 - m_t^2)\pi^M, \tag{21}$$

$$U_e^2 = \sum_{t=0}^{\infty} \beta^t m_t^2 (1 - m_t^2)\pi^M, \tag{22}$$

where t is the number of periods after entry so $\{m_t^2\}_t$ is the sequence of awareness parameters for 2 that satisfies 9 for $t > 0$ and $m_0^2 = m_e$.

To implement a strategy of entry deterrence in this setting the monopolist should be able to commit to a pricing strategy in period 0, that is, choose a pricing schedule in period 0 that is valid for all future periods. Otherwise, after entry of market-maker 2, if market-maker 1 cannot commit to a pricing strategy they will play the Markov perfect equilibrium with payoffs U_e^1, U_e^2 for 1 and 2 starting in period 1. If discount rates are low enough, so payoffs in period 0 do not matter much, then for low E it is easy to see that $\beta U_e^2 > E$ so without commitment there does not exist a Markov perfect equilibrium that deters entry in this case.

Therefore, suppose the monopolist commits to the strategy of posting $\boldsymbol{p}(\pi)$ for profits $\pi \in [0, \pi^M]$ for every period. The present value of 2's profits conditional on entry when 1 is following its commitment $\boldsymbol{p}(\pi)$ is bounded above by

$$U_d^2(\pi) = \sum_{t=0}^{\infty} \beta^t m_t^2 \pi.$$

To deter entry π must imply that $U_e^2(\pi) \leq E$ therefore it must satisfy

$$\pi \leq E / \left(\sum_{t=0}^{\infty} \beta^t m_t^2 \right).$$

The present value of the payoffs for the strategy of entry deterrence for 1 are

$$U_d^1 = \frac{\pi}{(1-\beta)} = \frac{E}{(1-\beta)\left(\sum_{t=0}^{\infty} \beta^t m_t^2\right)} \tag{23}$$

To allow for the equilibrium with entry deterrence the monopolist must find deterring entry profitable: $U_d^1 \geq U_e^1$. Clearly, the equation 23 implies that for an entry cost E high enough we have $U_d^1 > U_e^1$.

Take $\{\beta_n\}$ such that $\lim \beta_n = 1$ and let $\{E_n\}$ be a sequence of entry costs such that

$$U_d^1(E_n, \beta) \geq U_e^1(\beta). \tag{24}$$

There conditions such that $\exists\{E_n\}$ such that $\pi \to 0$ (which means there exists a $\{E_n\}$ that is bounded above by some $\overline{E}$). To see that note that since $m_t^2 \in (0,1], \forall t$ with some $T > 1$ such that $m_t^2 < 1$ for all $t \leq T$ therefore 23 implies that $U_d^1 > E$. On the other side, $U_e^1(\beta) = \Delta(\beta)\pi^M/(1-\beta)$ for some $\Delta \in (0,1)$, as $\beta \to 1$ since $\lim m_t^2 = 1$ implies that $\lim_{\beta \to 1} \Delta(\beta) = 0$. Want to find conditions so that $U_e^1(\beta)$ is bounded above. One such condition is that m_t^2 converges to 1 fast enough so that $\sum_{t=0}^{\infty}(1 - m_t^2) \leq C$, then $U_e^1(\beta) \leq C \times \pi^M$.

For $E_n \geq C \times \pi^M$ then deterrence is the only equilibrium, fix a sequence $\{E_n\}$ with $E_n = \overline{E} = C, \forall n$. Then $\pi \to 0$ as $\beta \to 1$ and therefore $\boldsymbol{p}(\pi)$ converges to $p_s = p_b = p^*$ as the discount rate r falls to zero and the equilibrium allocation must converge to the competitive equilibrium.

Finally, consider the case where the monopolist can choose a sequence of pairs of prices to post instead of being restricted to a single profit margin. The overall situation is similar but with added tedious notation. The monopolist chooses a sequence of profit shares $\{\pi_t\}_{t=0}^{\infty}$ with corresponding sequence of pairs of bid and ask prices $\boldsymbol{p}(\pi_t)$. To deter entry the sequence $\{\pi_t\}$ must satisfy

$$\sum \beta^t m_t^2 \pi_t \geq U_e^2, \tag{25}$$

the profits of the monopolist under this strategy are

$$U_d^1 = \sum \beta^t \pi_t. \tag{26}$$

Hence the profit maximizing strategy for the monopolist is to choose among the sequences that satisfy the deterrence condition 25 the one that maximizes 26. Given that $m_t^2 \to 1$ and is strictly increasing there is a unique profit maximizing sequence $\{\pi_t\}$ where the monopolist "frontloads" by extracting the highest profits in the early periods as the entrant is relatively constrained by m_t^2 being smaller than in later periods from taking advantage of these higher margins. These profits are strictly higher than the profits from the strategy to commit to constant prices ($\pi/(1-\beta)$) and therefore the previous arguments also apply in this case. ■

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