On the Informational Efficiency of Decentralized Price Formation

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This paper studies the informational efficiency in models of decentralized price formation. First, we show that a search equilibrium, a common model of decentralized price formation, requires an infinitely larger message space, and therefore infinitely less informationally efficient than the competitive equilibrium in a large economy. We propose here a model of price formation through market-makers. This model of price formation attains the competitive allocation in the limit (as the search equilibrium). The case of monopolistic equilibrium where markets for each commodity are monopolized by a single market-makers as they deter the entry of competitors, we show that the equilibrium only requires, in a quasilinear environment with L goods, a message space with L-1 more dimensions than the competitive process. This appears to be the most informationally efficient form of decentralized price formation process that implements the competitive allocation at the limit.

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1 Introduction

After Hayek (1937, 1945), economists have assessed institutions by their relative ability to incorporate and communicate information. According to this framing, markets are a desirable institution because they achieve an efficient allocation of resources by incorporating information that is dispersed throughout the decision-makers in the economy.

Studies such as Mount and Reiter (1974), Hurwicz (1977a, 1977b, 1977c), Jordan (1982), and Chander (1983) formalized the concept of informational efficiency. Jordan (1982) shows that the competitive equilibrium is informationally efficient; competitive prices communicate the minimum amount of information necessary to implement a Pareto efficient allocation. Moreover, competitive prices are the unique decentralized mechanism that achieves this informational efficiency. Further illustrating the informational power of competitive equilibrium, Hammond (1979) has shown in a large economy where types are private information that an incentive-compatible mechanism that implements a Pareto optimal allocation must give rise to the same net trades as the competitive equilibrium mechanism.

While these results are powerful, competitive equilibrium is not without its critics. Particularly, the concept of competitive equilibrium as an allocation mechanism lacks strategic foundations (*e.g.* Gale 2000): decision-makers take prices as given and the price is determined by the underlying market data through an exogenous algorithm that is not modeled. That is, only the quantities are determined by the decentralized choices of decision-makers while the prices are determined by the "Walrasian auctioneer" or "the economist" who solves the model by equating the quantity supplied with the quantity demanded. Without any way to measure the information required to reach the competitive allocation through a fully strategic procedure, it is unclear how seriously economists should take these informational efficiency results.

The main objectives of this paper are as follows:

1. Construct a model of an economy that features strategic and fully decentralized trading that can attain the allocation corresponding to the competitive equilibrium while allowing for imperfect awareness and asymmetric information, and decentralized and strategic trading.

2. Show that the equilibrium of this economy replicates several of the features presented in the empirical literature that do not exist in competitive equilibrium.

3. Study conditions for the minimum amount of information communicated in this economy that can yield competitive equilibrium allocations.

While there are different strategic foundations for competitive equilibrium, we show that the main one in the literature—search equilibrium—is unattractive from an informational efficiency perspective. In particular, we prove that for large economies the search allocation mechanism requires infinitely more information than the competitive mechanism.

Therefore, to achieve informational efficiency, we introduce a new strategic foundation for competitive equilibrium using market-makers. Among decentralized mechanism where terms of trade are set strategically, this study suggests that the one that is the most informationally efficient is the situation where the market for each good is monopolized by a single market-maker who sets prices, that way consumers in the economy only need to be aware of one price for each good, mimicking the competitive equilibrium. Allowing for entry puts pressure on the market-maker to lower prices to deter entry in what we call a monopolistic deterrence equilibrium. Taking discount rates to zero implies that the allocation implied by the monopolistic deterrence equilibrium converges to the competitive allocation.

The model of price formation based on market-makers can approximate the informational efficiency properties of the competitive equilibrium while endogenizing the equilibrium prices as strategic choices of decision-makers. The informational efficiency is obtained through an "intellectual division of labor" between the consumers in the economy who "outsource" the resource allocation process to the market-makers who are the strategic players, who in turn make profits by extracting part of the surplus of the gains from trade. Alternatively, these market-makers can be thought of as arbitrageurs who have the ability to buy low and sell high, exploiting opportunities that other actors are unaware of.[1] The no-arbitrage limit is the competitive equilibrium, which is well understood in the case of financial markets (Werner 1987; Makowski and Ostroy 1998).

1.1 Strategic Foundations of Competitive Equilibrium

There is a large literature on strategic foundations for competitive equilibrium which justifies the assumption of the competitive equilibrium mechanism by showing under a variety of different conditions that the strategic equilibrium of decentralized exchange

1. There is an entrepreneurship literature following Kirzner (1973) which would call these market-makers "entrepreneurs" that are "alert" to profit opportunities that exist in the market at any point in time.

economies (modeled as the core of a cooperative game or random matching and bargaining games) does generate the same allocation as the competitive equilibrium.[2] Also, the literature on random matching and bargaining games is also able to explain several deviations of competitive conditions verified empirically (such as price dispersion is generated as an equilibrium outcome in models such as (Mortensen and Wright 2002) while accommodating the competitive equilibrium as a special case when frictions of trading are zero. However, this literature has assumed that decision-makers have perfect information regarding market conditions.

More recent papers have shown, however, in the case of a market for a single good that asymmetric information regarding private valuations (Satterthwaite and Shneyerov 2007, 2008) and imperfect information regarding aggregate market conditions (Lauermann, Merzyn, and Virág 2018) can be shown to consistent with decentralized random matching games that attain competitive equilibrium allocations. However, these studies assume that decision-makers have full awareness of the data of the economy: there exists only uncertainty regarding the values of individual parameters of the economy (private valuations in the case of Satterthwaite and Shneyerov (2007, 2008) and whether the supply of a good is greater than demand, so the competitive equilibrium price is 0 or 1 (Lauermann, Merzyn, and Virág 2018).

The literature on informational efficiency argues that the competitive equilibrium mechanism allows an economy to achieve an efficient allocation while minimizing the amount of information that needs to be communicated. That is, in a competitive equilibrium individual decision-makers only need to be aware of the minimum amount of parameters regarding the state of the market to achieve an efficient allocation. The decision-makers can be simply completely unaware of the rest of the economy (unawareness being interpreted as not "generally taking into account", "being present in mind" (Modica and Rustichini 1999, p. 274), "thinking about" (Dekel, Lipman, and Rustichini 1998) or "paying attention to" (Schipper 2014)) - and still achieve the same allocation which is obtained by an omniscient central planner in possession of all the data in the economy who is maximizing a social welfare function. If the allocation corresponding to a competitive equilibrium is reached through a distinct mechanism (such as in a Gale-type random matching and bargaining model) then we will see this informational efficiency property does not hold.

2. Starting in cooperative game theory with Edgeworth (1881), Debreu and Scarf (1963), and Aumann (1964) and moving into non-cooperative random matching game-theoretic arguments (Gale 1986a, 1986b, 1987, 2000; Osbourne and Rubinstein 1990; McLennan and Sonneschein 1991; Mortensen and Wright 2002; Lauermann 2013).

In contrast, an economy with market-makers can require little information. In particular, this mechanism has a message space that has one additional dimension over the competitive mechanism for each good in the economy besides the numeraire good. This extra-dimension represents the profit margin between the bid and ask price that the strategic market-maker obtains from setting prices. Another issue is empirical: most markets exhibit features that are very different from the idealized features of competitive equilibrium. For example, there is substantial price dispersion for individual goods instead of a single price (Sorensen 2000; Kaplan et al. 2019). Individual plants face plant-specific demand instead of taking market prices as given, and this plant-specific demand depends on the history of plant activity and grows over time (Foster, Haltiwanger, and Syverson 2016). The informational efficiency and the empirical reality are reasons to further study models with market-makers.

2 The Informational Inefficiency of Search

2.1 Basic Environment

Consider a class of environments E where there are $L = 2$ goods. For an environment $e \in E$, there is an indivisible good and a divisible numeraire good. There is a continuum of consumers, with measure normalized to 1, of which a fraction $s \in [0, 1]$ are sellers (set denoted $S = [0, s]$) willing to supply a unit of the indivisible good for a price higher than their cost c, and a fraction $b = 1 - s$ are buyers (set denoted $B = (s, 1]$) who demand a unit and are willing to pay for the good if the price in terms of numeraire is less than their valuation v. Valuations for the good are distributed according to c.d.f.'s G and F for sellers and buyers, respectively which are strictly increasing and differentiable. The c.d.f.'s G and F have supports $[\underline{c}, \overline{c}] \subset \mathbb{R}_{++}$, $[\underline{v}, \overline{v}] \subset \mathbb{R}_{++}$, respectively, where $\overline{v} > \underline{c}$ (so that there is possibility for mutually beneficial trade and the market does not shut down).

For simplicity of notation the sellers' and buyers' are ordered by their costs and valuations, respectively, so for two sellers $i, i' \in [0, s]$, if $i > i'$ then $c^i > c^{i'}$ and analogously for the buyers. A realization of $e \in E$ is then an environment $e = (e^i)_{i \in [0,1]}$ with $e^i = c^i$ if $i \in S$ and $e^i = v^i$ if $i \in B$ which are distributed according to the c.d.f.'s G and F.

Let

$$Y = \left\{ y : [0, 1] \to \{-1, 0, 1\} \times \mathbb{R} : \int_0^1 (y_1(i), y_2(i)) di = (0, 0) \right\},$$

Y is the set of net-trades which satisfy $y_1(i) \in \{-1, 0, 1\}, \int_0^1 y_1(i) = 0^3$, and $y_2(i) \in \mathbb{R}, \int_0^1 y_2(i) = 0$, that is net trades of both the indivisible and numeraire good must add up to 0.

Definition 1. An allocation mechanism is a triple (μ, M, g) where M is an abstract message space, μ is a non-empty valued correspondence on E to M, and $g : M \to Y$ is an outcome function.

We are interested in allocation mechanisms that are informationally decentralized, which are mechanisms that feature a message process (μ, M) that is privacy-preserving. A message process is privacy-preserving if there exists a correspondence $\mu^i : E^i \to$each agent $\mu(e) = \cap_{i \in [0,1]} \mu^i(e^i)$.

2.2 Competitive Mechanism

Let $M_c = \{(p, y) \in \mathbb{R}_{++} \times Y : py_1(i) + y_2(i) = 0, \forall i\}$. For an environment $e \in E$, each $i \in S \cup B$ define the correspondence $\mu_c^i : E \rightrightarrows M_c$ by

$$\mu_c^i(e^i) = \left\{ (p, y) \in M_c : y(i) = \begin{cases} (1, -p) & \text{if } i \in B \text{ and } v^i \geq p \\ (0, 0) & \text{if } i \in B \text{ and } v^i < p \text{ or } i \in S \text{ and } c^i > p \\ (-1, p) & \text{if } i \in S \text{ and } c^i < p \end{cases} \right\},$$

that is, a buyer purchases the good for p if his valuation is higher and a seller sells the good for p is her cost is lower. Define

$$\mu_c(e) = \cap_{i \in [0,1]} \mu_c^i(e^i).$$

Then (μ_c, M_c) is the competitive message process and (μ_c, M_c, g_c) is the competitive allocation mechanism, where $g_c : M_c \to Y$ is the outcome function given by $g_c(p, y) = y$.

Remark 1. Note that to satisfy feasibility a competitive equilibrium price p^* satisfies $sG(p^*) = b[1 - F(p^*)]$, since G and F are continuous and strictly increasing the competitive equilibrium price p^* is unique. The competitive mechanism is, trivially, privacy preserving.

3. That is, the agent acquires buys one unit of the indivisible good, sells a unit or does not change his or her endowment.

2.3 Search Mechanism

The Steady-State Search Equilibrium

In a search environment, there is an entry rate of potential $s > 0$ sellers and $b > 0$ buyers. Given stocks of B buyers and S sellers currently in the market, buyers and sellers meet at the rate $M(B, S)$. Let the buyer/seller ratio $\theta = B/S$ be the market tightness parameter, and $m(\theta) = M(B, S)/S$, be the rate a seller meets buyers and $m(\theta)/\theta$ be the rate a buyer meets sellers. There are discount rates and search costs for buyers and sellers, respectively given by $(r, c_b, c_s) \in \mathbb{R}_+$.

When buyers and sellers meet if there is a positive surplus in trading they trade at terms determined by the generalized Nash solution over the joint surplus. Sellers' bargaining power is given by the parameter $\beta \in (0, 1)$ (buyers' bargaining power is then $1 - \beta$).

For this search environment to be directly comparable to the competitive mechanism the flows of buyers and sellers exiting the market should be equal to the flows entering which are s and b, so that the corresponding net trades in the competitive equilibrium have an analogous implementation in this environment. The search equilibrium where the rate of entry of new buyers and sellers in the market is the same as the exit rate is called steady-state search equilibrium. Given a pair of marginal types of buyers and sellers (R_b, R_s), where buyer with valuation $x > R_b$ enters and seller with cost $y < R_s$ enters, who are indifferent between participating in the market or not in steady-state search equilibrium this pair satisfies the condition $sG(R_s) = b[1 - F(R_b)]$, and the distribution of participating types is constant.

As described in Mortensen and Wright (2002), these parameters determine the steady-state search equilibrium which is characterized by $(V_b, V_s, R_b, R_s, \Phi, \Gamma)$, the value functions (V_b, V_s), cutoff valuations, and costs to participate in the market (R_b, R_s), and the distributions of participating types (Φ, Γ) of for buyers and sellers, respectively. Transaction prices between buyer with valuation x and seller with cost y satisfy

$$p(x, y) = y + V_s(y) + \beta[x - y - V_b(x) - V_s(y)]. \tag{1}$$

Mortensen and Wright (2002) shows that if search costs c_b, c_s are strictly positive and r is lower than some threshold $\hat{r} > 0$ then all meetings result in trade. This implies that steady-state equilibrium distribution of operating types (Φ, Γ) is given by the densities of (F, G) on the types who choose to participate in the market ($v \geq R_b, c \leq R_s$).

In the case where the common discount rate is zero, $r = 0$, then the Law of One Price holds and there is an equilibrium price

$$\hat{p} = \beta R_b + (1 - \beta) R_s. \tag{2}$$

If search costs (c_b, c_s) converge to zero then $\hat{p}$ converges to the competitive price and R_s, R_b both converge to the same value R which is the competitive equilibrium price p^* and the search equilibrium allocation in terms of quantity also converges to $sG(p^*)$ which is the quantity sold in competitive equilibrium. For $r \in (0, \hat{r})$, then all meetings result in trade and there is price dispersion. In this case the equilibrium price for the transaction with seller with cost y and buyer with valuation x satisfies

$$p(x, y) = \beta \left[\frac{r\theta x + (1 - \beta) m(\theta) R_b}{r\theta + (1 - \beta) m(\theta)} \right] + (1 - \beta) \left[\frac{ry + \beta m(\theta) R_s}{\beta m(\theta) + r} \right]. \tag{3}$$

The Search Allocation Mechanism

Consider the steady-state search equilibrium. Note that in steady-state constant distribution of types currently in the market implies that the distribution of types leaving the market is the same as the distribution of types entering the market, which is given by (F, G) with the cutoffs (R_b, R_s), the allocation in the steady-state can be described by a pair (p_s, y) where $p_s : [0, 1] \to \mathbb{R}_+$ is a function, where $p_s(i)$ for describes the equilibrium transaction price for agent i if i participates in the market that is, if $i \in B, x^i \in [R_b, 1]$ and if $i \in S, y^i \in [0, R_s]$. If i does not participate then if i is a seller for $c^i \in (R_s, \overline{y}]$, $p_s(i) = R_s$ and if i is a buyer, $x^i \in [\underline{x}, R_b)$, then $p_s(i) = R_b$.

Since buyers and sellers meet randomly and the transaction price depends on the pair of valuations of buyers and sellers $p(x, y)$, prices are not deterministic in the search equilibrium but the distribution of realized transaction prices is deterministic as there is a continuum of traders and can be described by a c.f.d. $P : [\underline{p}, \overline{p}] \to [0, 1]$. Any function p consistent with the search equilibrium implies in an equilibrium distribution of prices P. Let $\overline{y}(x)$ and $\underline{x}(y)$ be the highest seller's cost and lower buyer's valuation such that there is positive joint surplus in trading given buyer's and seller's valuations (x, y), respectively, then p satisfies $p_s(i) \in \{p(x^i, y), y \in [\underline{y}, \overline{y}(x^i)]\}$ if i is a buyer and $p_s(i) \in \{p(\underline{x}, y^i), v \in [\underline{x}(y^i), \overline{x}]\}$ if i is a seller.

The privacy-preserving message process (μ_s, M_s) is constructed as follows:

The message space of the search mechanism is

$$M_s = \{(p_s, y) \in \mathcal{F} \times Y : p_s(i) y_1(i) + y_2(i) = 0, \forall i\},$$

where $\mathcal{F}$ is the space of functions on $[0,1]$ to $\mathbb{R}_{++}$.

Let μ_s^i be a correspondence from E^i to M_s. Let $\mu_s^i : E^i \rightrightarrows M_s$ given by

$$\mu_s^i(x^i) = \left\{ (p_s, y) \in M_s : y(i) = \begin{cases} (1, -p_s(i)) & \text{if } i \in B \text{ and } x^i \geq p_s(i) \\ (0,0) & \text{if } i \in B \text{ and } x^i < p_s(i) \\ (-1, p_s(i)) & \text{if } i \in S \text{ and } x^i > p_s(i) \end{cases} \text{ or } i \in S \text{ and } x^i > p_s(i) \right\}.$$

Define the correspondence $\mu_s : E \rightrightarrows M_s$ by

$$\mu_s(e) = \cap_i \mu^i(e^i) \cap (p_s(e) \times Y), \tag{4}$$

where $p_s(e)$ is the pricing function determined by the search equilibrium in the environment e (with buyers and sellers types distributed according to F and G) so that means that μ_s is restricted to the subset of M_s consistent with the search environment described in Subsection 2.3. Note that μ_s is privacy-preserving by construction.

The search mechanism is a triple (μ_s, M_s, g_s) where $g_s(p, y) = y$ is a projection from M_s to Y.

Note that $p_s(i) > R_s$ if $c^i \leq R_s$ and $p_s(i) < R_b$ if $v^i \geq R_b$ since prices must compensate for search costs, while agents who do not trade are the types with costs/valuations in (R_s, R_b).

Remark 2. If the discount rate $r = 0$ and search costs (c_b, c_s) converge to zero then the search equilibrium prices all converge to p^* which means that the search mechanism becomes the competitive mechanism. Then, clearly, it is informationally efficient at this frictionless limit.

2.4 Informational Efficiency

In the environment has a continuum of agents and smooth distributions of types any allocation $y \in Y$ is an infinite-dimensional object. To articulate the argument of the size of information messages in the terms of Hurwicz (1977b) and Jordan (1982) on the dimensional size of message space which are finite-dimensional manifolds, we study here sequences of environments with finitely many types of buyers and sellers and as the number of types grows to infinity, the distributions of types approximate the continuous distributions of buyers F and sellers G.

Let $\{e_k\}_{k\geq 2}$ be a sequence of environments where buyers and seller types are distributed according to $\{F_k, G_k\}$, sequences of step-functions. A pair (F_k, G_k) that represent the cumulative distributions of types of buyers and sellers, respectively, in an environment e_k with k types of buyers and k types of sellers, each type of measure $1/k$ (that is, the sets of buyers and sellers are partitioned into subsets of the same measure whose elements are all identical). The pair of sequences $\{F_k, G_k\}_k$ converges to F and G, respectively. I call an environment with k types of buyers and sellers a k-environment. In addition F_k, G_k are such that p^* is consistent with competitive equilibrium in economy e_k.

Then the allocation mechanisms can be written in terms of types. Let $b \in \{1, \ldots, k\}$ index buyer types and $v \in \{1, \ldots, k\}$ seller types, let $x(b)$ be the valuation of a buyer of type b and $y(v)$ be the cost of a seller of type v.

Let $y^B(b) = (y^B(b)_1, y^B(b)_2)$ be the net trades for a buyer of type b in the competitive equilibrium and $y^S(b) = (y^S(v)_1, y^S(v)_2)$ the net trades for a seller of type v. A profile of net trades specifies a net trade for each of the $2k$-types of traders: $y = \left((y^B(b))_{b=1}^k, (y^S(v))_{v=1}^k\right)$. The set of net trades for the competitive mechanism is then

$$Y_k^c = \left\{ y \in ((-1, 0, 1) \times \mathbb{R}_+)^{2k} : \sum_{b=1}^{k} y^B(b) + \sum_{v=1}^{k} y^S(v) = (0, 0) \right\}.$$

as there are k types of buyers and k types of sellers. The competitive message space in the k-environment specifies a price of p and allocation y where $y \in Y$.

For the search mechanism, the set of net trades is a higher dimensional object. To see that consider the search equilibrium. As for the set of k-types of buyers, each buyer of type $b \in \{1, \ldots, k\}$ can match with a seller of type $v \in \{1, \ldots, k\}$. There is then a partition of the set of buyers (sellers) of each type b into k subsets $\{m(b, v)\}_{v=1}^k$, where each subset $m(b, v)$ can transact at a price $p(b, v)$. If a pair of types (b, v) do not transact in the search equilibrium, which occurs if b or v do not participate in the market (in the case buyer's valuations are too low or seller's costs are too high) or if the discount rate r is too high for all participating types to trade with each other, the subset $m(b, v)$ is empty. Let $\lambda(b, v)$ be the probability that among pairs of buyer-seller types who participate in the market if the transaction is between a buyer of valuation $x(b)$ and a seller of valuation $y(v)$ (given by the measure of the sets $\{m(b,v)\}_{b,v}$ divided by the sum of the measure of all such subsets: $\sum_{b,v} m(b,v)$). Since $m(b,v)$ is empty for pair of types who do not transact, $\lambda(b,v)$ is zero. Therefore, the search message space in the k-environment specifies prices for each possible pairing of buyers-seller types, which means that there are k^2 prices for each pairing between the k-types of buyers and k-types of sellers.

Let $y^B(b,v) = (y^B(b,v)_1, y^B(b,v)_2)$ be the net trades of the set of buyers of type b with sellers of type v and $y^S(b,v) = (y^S_1(b,v), y^S_2(b,v))$ be the net trades of the set of sellers of type v with buyers of type b. A profile of net-trades is $y = \big(y^B(b,v), y^S(v,b))\big)_{b,v\in\{1,\dots,k\}}$.

Therefore, the set of net trades includes k^2-types of buyers and k^2-types of sellers:

$$Y^s_k = \left\{ y \in ((-1,0,1) \times \mathbb{R}_+)^{2k^2} : \sum_b \sum_v \lambda(b,v)[y^B(b,v) + y^S(b,v)] = (0,0) \right\}.$$

Let $(\mu^K_c, M^K_c, g^K_c), (\mu^K_s, M^K_s, g^K_s)$ be the k-environment versions of the competitive and search allocation mechanisms. Where

$$M^k_c = \{(p,y) \in \mathbb{R}_{++} \times Y^c_k : py^j_1(i) + y^j_2(i) = 0, \forall j \in \{B,S\}, \forall i \in \{1,\dots,k\}\}$$
$$M^k_s = \{(p,y) \in \mathbb{R}^{K^2}_{++} \times Y^s_k : p(b,v)y^j_1(b,v) + y^j_2(b,v) = 0, \forall j \in \{B,S\}, \forall b,v \in \{1,\dots,k\}\}.$$

and μ^k_c, μ^k_s are the finite analogues of μ_c, μ_s: correspondences that map E^k into M^k_c, M^k_s, respectively, and g^k_c, g^k_s are projections from M^k_c, M^k_s, respectively, to Y^k.

Proposition 1. *M^k_c and M^k_s are $2k$ and $3k^2-1$ dimensional manifolds. Therefore, as $k \to \infty$ the ratio of the dimensional size of M^k_s to M^k_c converges to infinity.*

Proof. First, consider the competitive mechanism. Using the conditions of $\sum_{i=1}^k y^i$=$\mathbf{0}$ and $py^i_1 + y^i_2 = 0, \forall i$, implies that the function $(p,y) \to (p,\tilde{y}) \in \mathbb{R}_{++} \times \mathbb{R}^{k-1}_{++}$,where $\tilde{y}^i = y^i$ for $1 \le i \le 2k-1$, is a diffeomorphism, thus M^k_c is a $(2k-1)+1 = 2k$-dimensional manifold.

Second, consider the search mechanism, in this case the dimensional size of the price vector has k^2 dimensions, while Y^s_k has $2k^2$ dimensions, so, by analogous argument as for the competitive mechanism, M^k_s is a $k^2 + 2(k^2) - 1 = 3k^2 - 1$-dimensional manifold. ■

As there are k^2 prices and effectively $2k^2$-types in the search equilibrium, that means that for each buyer (or seller) type they form expectations regarding prices for transactions with each of the k-types of sellers (buyers) in the other side of the market, therefore each buyer (seller) individually has to form expectations regarding k distinct prices (each of which depends on the opportunity costs that depend on the distribution of buyers' types), hence, the dimensional size of the message space is $\sum_{i=1}^{k}(k+k)-1 = 3k^2-1$, which is approximately $1.5k$ times more dimensions than the competitive mechanism.

In other words, the search mechanism requires that each participant of the market be aware of all types of participants operating in the market to form expectations regarding payoffs from participating in the market and to bargain with the other participants. This is precisely the inverse of the intuition regarding the informational efficiency of the market as articulated by the literature on the informational efficiency of competitive markets since Hayek: that each participant of the market can use prices as an efficient way to substitute for the information they would otherwise require to allocate resources without access to market prices.

Remark 3. On private information: [insert analysis that shows that private information does not imply in a change in the amount of information required for the search mechanism]

3 The Informational Efficiency of market-makers

3.1 Environment

Suppose that in addition to buyers and sellers (which we call "traders"), there is a set J of market-makers/markets in this economy (with abuse of notation, J has cardinality J). market-makers are profit-maximizing intermediaries who post take-it or leave-it contracts to trade the indivisible good to the traders.[4] Traders cannot trade with each other but have access to the market-makers. Buyers purchase from the lowest priced market-maker they have access too as long as it is lower than their valuation, sellers sell at the highest-priced market as long as it is higher than their cost.

Consider the case of the absence of any form of frictions of trade: All traders have costless access to all contracts posted by all market-makers. Consider a market-maker $j \in J$ who posts a pair of prices which the market-maker offers to buy and sell the good, respectively, (p_b, p_s) which are, respectively, higher and lower than prices posted by all

4. A market-maker could practice price discrimination but trader's valuations and awareness is private information. There is two-sided asymmetric information as in Satterthwaite and Shneyerov (2007, 2008) and since the good is indivisible then the only direct revelation mechanism consists of a pair of prices that the market-maker is willing to buy or sell the good for.

other market-makers, then its profits are

$$\pi(p_s, p_b) = (p_s - p_b) \times b[1 - F(p_s)], \tag{5}$$

subject to the market clearing constraint that the quantity brought from sellers is equal to the quantity demanded by buyers:

$$b(1 - F(p_s)) = sG(p_b). \tag{6}$$

If the market-maker posts buy prices lower than some other market-maker no seller will sell to him or her and so its profits are zero. If the maker posts buy prices higher than all others but not the lowest sell prices the market-maker has monopolized the supply and profits also satisfy 5 subject to the resource constraint 6.

Proposition 2. *If at least two market-makers are operating then there is only one Nash equilibrium: for at least two market-makers to post a pair of buy-sell prices* $(p_b, p_s) = (p^*, p^*)$*, that is market-makers post the competitive equilibrium price.*

Proof. To see this is a Nash equilibrium note that posting the competitive price yields zero profits and for any market-maker deviations either imply negative profits or zero profits. To see that this is the unique Nash equilibrium note that if market-makers post prices to make strictly positive profits other market-makers could deviate and make profits by capturing the customers of competitor market-maker by posting more attractive buy and sell prices. ■

Proposition 2 states that this environment of strategic price determination by market-makers implements the competitive equilibrium in a frictionless setting. The interesting application of such a model, which is developed over the next-subsections, is to consider the case of imperfectly functioning markets where there are frictions of trading. We describe frictions of trading in this setting by the hypothesis that traders might not have full access to all market-makers because they might be unaware of the full set of market-makers operating in the market.

Imperfect Awareness

For $j \in J$ there is subset $A^j \subset [0, 1]$ of buyers and sellers who are in contact with market j (use the term "aware of j" to mean that a trader has access to market maker j) and $A^i =$

$\{j \in J : i \in A^i\}$ be the set of market-makers that i is aware of. The set of environments E^* includes the market-makers and the awareness among buyers and sellers regarding them, this information is given by $\{A^j\}_{j\in J}$. That is, $e^i = (x^i, A^i)$, where x^i is agent i's valuation or cost and A^i is the set of markets that i is aware of, an environment $e \in E^*$ is specified by $\{A^j\}$ and the distributions of valuations F and costs G.

Let the awareness parameter m^j given by $m^j = \lambda(A^j) \in (0, 1]$, the (Lebesgue) measure of A^j which is the fraction of all traders aware of j. I assume that A^j is a simple random sample of the traders. This means that it satisfies the properties of uniformity:

$$\lambda(A^j \cap X) = \lambda(A^j \cap Z) \tag{7}$$

for any measurable X, Z such that $\lambda(X) = \lambda(Z)$, and independence:

$$\lambda(A^j \cap A^h) = \lambda(A^j) \times \lambda(A^h), \tag{8}$$

for the awareness sets A^j, A^h of marker-makers j and h. Note that properties 7 and 8 imply that the fraction of buyers and sellers who are aware of the seller j conditional on being aware of a competitor is m^j.

Static Equilibrium

An equilibrium in the static environment is a profile of pricing strategies described by $(\{P^j\}_{j=1}^J, \boldsymbol{p})$, where P_j is a cumulative distribution function on $[0, 1]$ and $\boldsymbol{p}$ is a function that maps $[0, 1]$ into prices for buying and selling. It is such that posting any price $\boldsymbol{p}(x)$ for x on the support of P^j is profit-maximizing and the resulting allocation is feasible (that is, the quantity brought by the market-makers is equal to or greater than the quantity sold).

As shown in 6.1 there is a unique competitive equilibrium price p^*, since A^j satisfies property 7, p^* is also the unique competitive equilibrium price for that subset of traders.

Proposition 3. *If $\{m^j\}_{j=1}^J \in (0, 1]^J$ is such that $m^h < 1$ for $J - 1$ market-makers there is a unique equilibrium that features a profile of mixed pricing strategies $\{P^j\}_{j\in J}$, in addition the equilibrium features a sharing rule such that for a pair market-makers h and j if $m^h < m^j$ then traders aware of both will trade with h if posted prices are the same. If for at least two market-makers h, j if m^h and $m^{j'}$ both converge to one then equilibrium pricing strategies $\{P^j, \boldsymbol{p}\}_{j\in J}$ converges in probability to the competitive equilibrium price p^*, which is the unique equilibrium if $m^h = m^j = 1$ for at least two market-makers.*

Proof. Part 1. Existence and characterization:

To construct the candidate equilibrium strategy profile $\{P^j\}_{j\in J}$ we consider pricing strategies described by a pair (p_b, p_s) of offers to buy and sell the good by the market-maker where $p_b \le p^* \le p_s$. First consider the monopoly prices $\boldsymbol{p}^M = (p_b^M, p_s^M)$ which satisfies the monopolist market-maker problem:

$$\max_{p_b,p_s}\{(p_s - p_b)\min\{sG(p_b), b[1-F(p_s)]\}\},$$

in the case of existence of multiple profit maximizing pairs of monopoly prices, let (p_b^M, p_s^M) be the pair of monopoly prices with the lowest difference between the buying and selling price, which implies, as $sG(p_b^M) = b[1-F(p_s^M)]$, that it is the pair with lowest selling price and highest buying price.

Let k be the market-maker with the second largest awareness parameter $\{m^j\}_{j\in J}$ or the largest ($m^k = \max\{m^j\}$) if there are at least two market-makers with the largest m^j. Then let $\underline{\alpha} = \prod_{h\neq k}(1-m^h)$, and let Π^M is the monopoly profit (normalized in regards to $m^j \in (0,1]$), that is $\Pi^M = (p_s^M - p_b^M)[sG(p_b^M)]$.

Consider a function $\boldsymbol{p} : [0,1] \to \mathbb{R}_+^2$ such that $\boldsymbol{p}(\alpha) = (p_b(\alpha), p_s(\alpha))$ is a pair of prices that satisfies

$$[p_s(\alpha) - p_b(\alpha)]G[p_b(\alpha)] = \alpha\Pi^M/s, \tag{9}$$

and also satisfies market clearing,

$$sG(p_b(\alpha)) = b[1-F(p_s(\alpha))]. \tag{10}$$

That is, $(p_b(\alpha), p_s(\alpha))$ is the pair of prices that implements a feasible net trade for a monopolist market-maker and yield a fraction α of the monopoly profits. In addition if for some $\alpha \in [0,1]$ there is more than one such pair of prices then $(p_b(\alpha), p_s(\alpha))$ is the pair with smallest difference between the buying and selling prices, formally, for each $\alpha \in [0,1]$, $(p_b(\alpha), p_s(\alpha))$ satisfies

$$(p_b(\alpha), p_s(\alpha)) = \arg\min_{(b,s)}\{|b-s| : (b,s) \text{ satisfies } 910\}.$$

To see that there exists at least one pair of prices that satisfies 9 and 10 note that profits for $p_b = p_s = p^*$ are zero and imply $sG(p^*) = b[1-F(p^*)]$, while profits for $p_b = p_b^M$

and $p_s = p_s^M$ are Π^M and they also satisfy market clearing. As G and F are continuous therefore for any $p_s \in [p^*, p_s^M]$ there exists a (unique) buying price $p_b(p_s)$ that satisfies $sG(p_s) = b[1 - F(p_b(p_s))]$ and continuity of G and F also imply that profits vary continuously from 0 at $p_s = p^*$ to Π^M at $p_s = p_s^M$, by the intermediate value theorem any profit level between 0 and Π^M can be attained by some pair of prices $(p_s, p_b(p_s))$ with $p_s \in [p^*, p_s^M]$.

The candidate equilibrium strategy profile $\{P_j\}_{j\in J}$ is a profile of c.d.f.'s on $[\underline{\alpha}, 1]$, that is, $P_j(\alpha)$ is the probability that buying (selling) prices higher (lower) than $p_b(\alpha)(p_s(\alpha))$ that satisfies the equal profit condition

$$\prod_{h\neq j}(1 - P_h(\alpha)m^h)[p_s(\alpha) - p_b(\alpha)]sG[p_b(\alpha)] = \underline{\alpha}\Pi^M, \tag{11}$$

where

$$\underbrace{1 - m^h}_{\text{Prob. } h\notin A^i} + \underbrace{[1 - P_h(\alpha)]m^h}_{\text{Prob.} h\in A^i \text{ and } (p_b^h<p_b(\alpha) \text{ or } p_s^h>p_s(\alpha))} = 1 - P_h(\alpha)m^h,$$

is the probability that a trader chooses to transact with the market-maker j over competitor h and $\frac{\alpha}{s}\Pi^M$ is the profit margin of a market maker when posting prices at the minimum profitability level (lower bound for sales, upper bound for purchases), which means its selling (buying) prices undercuts (tops) all competitors. Note that 11 implies that $P_h(\underline{\alpha}) = 0$ as $[p_s(\underline{\alpha}) - p_b(\underline{\alpha})]sG[p_b(\underline{\alpha})] = \underline{\alpha}\Pi^M$.

To check that this is an equilibrium:

Note that any prices not in the support of equilibrium strategies $\mathcal{P} = \{(p_b(\alpha), p_s(\alpha)) : \alpha \in [\underline{\alpha}, 1]\}$ lead to strictly lower profits: If we consider prices $(p_b(\alpha), p_s(\alpha))$ that satisfy 9 and 10 defined for $\alpha < \underline{\alpha}$, profits are strictly lower by construction. For prices $(p_b, p_s) \in [p_b^M, p_b(\underline{\alpha})] \times [p_s(\underline{\alpha}), p_s^M] \cap \mathcal{P}$ they either yield strictly lower profits because they are undercut by prices which would achieve similar profitability in the case the market-maker were a monopolist or they are not feasible (i.e. the market-maker promises to sell more than it purchases).

Given the sharing rule, it is easy to check that profits are constant on the support of $\{P^j\}$ for each j. If there is only one market-maker j with the largest awareness parameter m^j then the equal profit condition 11 implies that there is atom of probability at $P^j(\boldsymbol{p}^M)$, the monopoly price, in the mixed strategy of the largest market-maker. The sharing rule implies that traders always choose to trade with $h \neq j$ if h posts the monopoly prices $\boldsymbol{p}^M$, hence its profits do not fall discontinuously on the support of the equilibrium strategy

$[\underline{\alpha}, 1]$ as $\boldsymbol{p}(\alpha) \to \boldsymbol{p}^M$.

Part 2. Uniqueness:

To see that the pricing strategy described by $\{P_j\}_{j \in J}$ and $\{(p_b(\alpha), p_s(\alpha)) : \alpha \in [\underline{\alpha}, 1]\}$ is the unique Nash equilibrium, if combined with the sharing rule that the market-maker with the smaller m^j has priority in transactions to buyers and sellers in the case of a tie in posted prices (which occurs only in the case where there is only one market maker with the largest m^j) is a straightforward and tedious extension of the result in Guthmann (2019) Appendix 6.1. ■

Dynamic Equilibrium: Price Formation, Awareness Diffusion, Entry, and a Contestable Market Equilibrium

Suppose that time is discrete: $t = 0, 1, 2, \ldots$ and let $\beta = 1/(1+r)$ be the discount factor.

Awareness Diffusion: Given a set J of market-makers there is a awareness profile $\{m_t^j\}_{j=1}^J \in (0,1]^J$. Suppose awareness regarding a market-maker diffuses through the market according to

$$m_{t+1}^j = (1-\delta) m_t^j + M(m_t^j, 1 - m_t^j), \tag{12}$$

where M is a matching function that represents the diffusion of awareness through traders who hitherto had access to the market-maker and $\delta \in [0,1)$ is the awareness depreciation parameter (that is, the rate in which traders "forget" about the market-maker).

Suppose that there are $J > 1$ market-makers operating, then the equilibrium notion is Markov perfection, which means that at any point in time t, prices practiced in the market are given by $\{P_t^j\}_j$ described in the proof of Proposition 3. Note that the equal profit condition implies that profits for maker j in the market in some period t are

$$\pi_t(m_t^j) = m_t^j \underline{\alpha}_t \Pi^M,$$

where

$$\underline{\alpha}_t = \prod_{h \neq k} (1 - m_t^h),$$

where k is the second largest market-maker (or the largest if there are multiple market-makers with the largest m_t^j).

Corollary 1. *If 12 implies that* $\lim m_t^j = 1$ *for* $m_t^j > 0$ *then if* $J > 2$ *as* $t \to \infty$ *the equilibrium prices and allocation converge in probability to the competitive equilibrium.*

A Model of a Contestable Market: Suppose that at some period t there are some market-makers operating and some not operating in the market (which means $m_t^j = 0$). There is an entry cost $E > 0$. When a market-maker enters it has a starting awareness parameter $m_e \in (0,1)$.

Suppose that the set of market-makers is $J = \{1,2\}$, suppose that $m_0^1 = 1, m_0^2 = 0$ at a date normalized to 0, that is 1 is a monopolist market-maker and all traders have access to his posted contracts and 2 is out of the market, but 2 can decide to enter in the current period. That is, 2 can choose $m_0^2 \in \{0, m_e\}$. Instead of being a simultaneous move game as in the static environment here the timing is a follows: In period 0 the incumbent 1 plays first, the entrant 2 plays after. The entrant chooses whether to enter and, conditional on entry, which price to post. If 2 decides to enter, in the periods after 2's entry both market-makers play simultaneously.

Definition 2. A monopoly deterrence equilibrium is an equilibrium where 1 commits to pricing schedule and given this pricing schedule 2 finds it optimal to not enter. The pricing schedule is the profit-maximizing in the sense that a higher selling-buying margin implies that 2 finds it optimal to enter and undercut 1's posted offers and is profit-maximizing in the sense it yields higher profits than the profits in the Markov perfect equilibrium under a duopoly.

The proposition below states that if entry costs are high enough and awareness diffusion is fast enough then as discount rates decrease the monopoly deterrence equilibrium converges to perfect competition. An interpretation of the entry costs is that they represent costs of communicating additional information so that if the costs of communicating additional information are higher than the (private) benefits then it does not occur in equilibrium.

Proposition 4. *If awareness diffusion is fast enough so $\sum_{t=0}^{\infty}(1 - m_t^2) \leq C$ for some constant C, and the discount rate r is low enough, then, for an entry cost higher than C, the unique equilibrium is the monopoly deterrence: The monopolist posts a sufficiently low price to block entry. As r converges to zero the deterrence monopoly equilibrium converges to the competitive equilibrium.*

Proof. The proof is divided into two parts.

Part 1: For simplicity of the argument first let us consider the case where time is discrete and agents are myopic (only care about present payoffs, that is the discount factor $\beta = \frac{1}{1+r} = 0$) so, in this case, the deterrence game has only one period.

As in the proof of proposition 3 let $\Pi^M = (p_s^M - p_b^M)sG(p_b^M)$ be the monopoly profits, consider a $\pi \in (0, \pi^M)$, and suppose the incumbent 1 considers posting a price $\boldsymbol{p}(\pi) = (p_b(\pi), p_s(\pi))$ that is the pair of buying and selling prices with smallest difference that satisfies $[p_s(\pi) - p_b(\pi)]sG[p_b(\pi)] = \pi$. That is, $\boldsymbol{p}(\pi)$ is the pricing strategy that yields a payoff of $\pi \times m^j$ if j is a monopolist.

For $\pi \leq E/m_e$, then if the incumbent posts $\boldsymbol{p}(\pi)$ the cost of entry E is higher than the profits 2 can make after entry by undercutting 1 (as the discount rate is $r = 1$ the market-makers only care about present profits). Therefore 2 does not enter. Therefore, if the incumbent posts $\boldsymbol{p}(\pi)$ for $\pi = E/m_e$ (the highest profit margin that deters entry) and the entrant playing "no entry", it is an equilibrium if 1 has no incentive to deviate.

Suppose that 1 deviates from $\boldsymbol{p}(\pi)$ and posts prices according to the mixed strategy equilibrium described in Proposition 3 (for instance), suppose E is low enough so that 2's profits in mixed strategy equilibrium (given by $m_e(1 - m_e)\pi^M$) are higher than the entry cost. Then, 1's profits are $(1 - m_e)\pi^M$. If E is larger than $m_e\underline{\alpha}\pi^M = m_e(1 - m_e)\pi^M$ then, for the monopolist a pure strategy$\boldsymbol{p}(\pi)$ for $\pi = E/m_e$ that deters entry yields strictly higher profits than if the monopolist plays the mixed strategy P^j which yields profits of $(1 - m_e)\pi^M$, since deterrence profits π satisfy

$$\pi = E/m_e > \frac{m_e\underline{\alpha}\pi^M}{m_e} = (1 - m_e)\pi^M.$$

That is if $m_e\underline{\alpha}\pi^M < E$ then the only equilibrium is for the monopolist to post $\boldsymbol{p}(\pi)$ for $\pi = E/m_e$, as even higher profit margin encourages undercutting by the entrant and playing the mixed strategy. If $E < m_e\underline{\alpha}\pi^M$, then $\pi < (1 - m_e)\pi^M$ and the only equilibrium is entry of 2 and the mixed strategies (P^1, P^2) described in the proof of Proposition 3.

Part 2: This logic can be extended to the environment where agents are not myopic and instead have a common discount factor $\beta \in (0, 1)$. Then payoffs of 1 and 2 in the markov perfect equilibrium after entry are respectively

$$U_e^1 = \sum_{t=0}^{\infty} \beta^t (1 - m_t^2)\pi^M, \tag{13}$$

$$U_e^2 = \sum_{t=0}^{\infty} \beta^t m_t^2 (1 - m_t^2)\pi^M, \tag{14}$$

where t is the number of periods after entry so $\{m_t^2\}_t$ is the sequence of awareness parameters for 2 that satisfies 12 for $t > 0$ and $m_0^2 = m_e$.

To implement a strategy of entry deterrence in this setting the monopolist should be able to commit to a pricing strategy in period 0, that is, choose a pricing schedule in period 0 that is valid for all future periods. Otherwise, after entry of market-maker 2, if market-maker 1 cannot commit to a pricing strategy they will play the Markov perfect equilibrium with payoffs U_e^1, U_e^2 for 1 and 2 starting in period 1. If discount rates are low enough, so payoffs in period 0 do not matter much, then for low E it is easy to see that $\beta U_e^2 > E$ so without commitment there does not exist a Markov perfect equilibrium that deters entry in this case.

Therefore, suppose the monopolist commits to the strategy of posting $\boldsymbol{p}(\pi)$ for profits $\pi \in [0, \pi^M]$ for every period. The present value of 2's profits conditional on entry when 1 is following its commitment $\boldsymbol{p}(\pi)$ is bounded above by

$$U_d^2(\pi) = \sum_{t=0}^{\infty} \beta^t m_t^2 \pi.$$

To deter entry π must imply that $U_e^2(\pi) \leq E$ therefore it must satisfy

$$\pi \leq E / \left(\sum_{t=0}^{\infty} \beta^t m_t^2 \right).$$

The present value of the payoffs for the strategy of entry deterrence for 1 are

$$U_d^1 = \frac{\pi}{(1-\beta)} = \frac{E}{(1-\beta)\left(\sum_{t=0}^{\infty} \beta^t m_t^2\right)} \tag{15}$$

To allow for the equilibrium with entry deterrence the monopolist must find deterring entry profitable: $U_d^1 \geq U_e^1$. Clearly, 15 for entry cost E high enough $U_d^1 > U_e^1$.

Take $\{\beta_n\}$ such that $\lim \beta_n = 1$ and let $\{E_n\}$ be a sequence of entry costs such that

$$U_d^1(E_n, \beta) \geq U_e^1(\beta). \tag{16}$$

There conditions such that $\exists \{E_n\}$ such that $\pi \to 0$ (which means there exists a $\{E_n\}$ that is bounded above by some $\overline{E}$). To see that note that since $m_t^2 \in (0,1], \forall t$ with some $T > 1$ such that $m_t^2 < 1$ for all $t \leq T$ therefore 15 implies that $U_d^1 > E$. On the other side, $U_e^1(\beta) = \Delta(\beta)\pi^M/(1-\beta)$ for some $\Delta \in (0,1)$, as $\beta \to 1$ since $\lim m_t^2 = 1$ implies that $\lim_{\beta \to 1} \Delta(\beta) = 0$. Want to find conditions so that $U_e^1(\beta)$ is bounded above. One such condition is that m_t^2 converges to 1 fast enough so that $\sum_{t=0}^{\infty}(1 - m_t^2) \leq C$, then $U_e^1(\beta) \leq C$.

For $E_n \geq C$ then deterrence is the only equilibrium, fix a sequence $\{E_n\}$ with $E_n = \overline{E} = C, \forall n$. Then $\pi \to 0$ as $\beta \to 1$ and therefore $\boldsymbol{p}(E/m_e)$ converges to $p_s = p_b = p^*$ as the discount rate r falls to zero and the equilibrium allocation is the competitive equilibrium. ■

3.2 Allocation Mechanism

The set of net-trades incorporate the possibility of market-maker's making profits by buying at lower prices than they sell:

$$Y_m = \left\{ y : [0,1] \to \{-1,0,1\} \times \mathbb{R} : \int_0^1 y_1(i)di = 0, \int_0^1 y_2(i))di \leq 0 \right\}.$$

Given a realized profile of prices $p_m = (p_1, \ldots, p_J)$ (which correspond to a realized MPE of the game at some date) the message space is given by

$$M_m = \{(p_m, y) \in \mathbb{R}^J_{++} \times Y_m : \text{for each } i, \exists j \in A^i \text{ s.t. } i \in A^j \text{ and } p^j y_1(i) + y_2(i) = 0\},$$

and $\mu_m$is a correspondence on E to M_m that satisfies

$$\mu_m = \cap_i \mu^i_m(e^i),$$

where $\mu^i_m : E^i \rightrightarrows M_m$ satisfies

$$\mu^i_m(e^i) = \left\{ (p_m, y) \in M_m : y_1(i) = \begin{cases} (0,0) & \text{if } i \in B \text{ and } v^i < \min\{p^j_s : j \in A^i\} \text{ or } A^i = \emptyset \\ (1, -\min\{p^j : j \in A^i\}) & \text{if } i \in B \text{ and } v^i \geq \min\{p^j_s : j \in A^i\} \\ (0,0) & \text{if } i \in S \text{ and } c^i > \min\{p^j_b : j \in A^i\} \text{ or } A^i = \emptyset \\ (-1, \min\{p^j : j \in A^i\}) & \text{if } i \in S \text{ and } c^i \leq \min\{p^j_b : j \in A^i\} \end{cases} \right\}.$$

In this case, it is easy to see that in the sequence of finite types economies that approximate the economy with types distributions F and G, the dimensional-size of the message space of the k-types environment is $2J_t + (2k-1)$, where $J_t \subset J$ is the subset of active market-makers (i.e. $m^j_t > 0$) in period t. The case of the monopolist market-maker who deters entry represents the most informationally efficient mechanism in this class of market-maker environments with informational size $k+1$, or only one dimension more than the competitive mechanism, outside of the limiting case of $\delta = 0, \beta \to 1$ when it converges to perfect competition. This additional dimension reflects the profit margin between purchase and sale to provide incentives for the market-makers to "produce" the price mechanism.

4 Broader Classes of Environments

It is a simple exercise to extend the market-maker model of Section 3 to a more general environment. Consider an environment with $L > 1$ goods where good L is the numeraire. There is a continuum of consumers $i \in [0,1]$, with consumption sets $X \subset \mathbb{R}^L_+$ and CRRA preferences, where

$$u_i(x) = \sum_{l=1}^{L-1} \theta il[x_l^{1-\sigma_l} - 1] + x_L, \theta_{il} > 0, \sigma_l \geq 0, \forall l.$$

Suppose θ_{il} is private information and distributed on interval $[\underline{\theta_l}, \overline{theta_l}]$ according to some cummulative distribution F.

There is a set $J_l, |J_l| \geq 1$ of market-makers operating in the markets for each good $l \in 1, \ldots, L-1$ and they compete by posting buy and sell prices for the good in exchange for the numeraire.[5] Because, we restrict attention to linear pricing mechanisms quasilinearity implies that the equilibrium price of the market for each good is independent of the other markets, and profits are continuous functions of the posted prices as long as they are more attractive to sellers/buyers than their competitor's prices and therefore the analysis of Section 3 applies here.

That is, given a profile of awareness m^j in each sub-market $l \in 1, \ldots, L-1$ there is an equilibrium described by Proposition 3 where market-makers post randomized pricing strategies. Then as awareness increases over time according to the law of motion for awareness diffusion 12 the randomized equilibrium pricing strategies converge in probability to the competitive equilibrium prices in each of the sub-markets.

The dynamic deterrence equilibrium of the monopolist market-maker described by Proposition 3 can be implemented here with one monopolist in the market for each good $l \in \{1, \ldots, L-1\}$. In equilibrium, there will be $2(L-1)$ prices: a buying and selling price

5. Note that if we allow for non-linear pricing (as described, for instance, Bolton and Dewatripont (2005)) in this case the dimensional size of the message space is also infinity (which might provide a justification why we do not see many empirical examples of sophisticated non-linear pricing if allocation mechanisms are informationally constrained).

for each good l in terms of the numeraire good L. This implies that the dimensional size of the message space of a k-types "approximation economy" needs to be at least $2(L-1)+k$ instead of $(L-1)+k$, that is $L-1$ more dimensions, one for each good besides the numeraire good.

5 Concluding Remarks

In this paper, we consider the informational efficiency of decentralized price formation. In particular, we are interested in economies with strategic agents where the allocation mechanism converges to the competitive mechanism. We study two such mechanisms: search and market-makers.

While the search mechanism has been extensively studied in the literature, we show that it is unattractive from an informational perspective. In particular, we prove that for large economies the search allocation mechanism requires infinitely more information than the competitive mechanism. A true search mechanism, where everyone must search across all the people in the economy to find a particular trading partner, is extremely expensive in terms of information. That is one possible reason we do not often observe single buyers trading with single sellers in real-world economies.

In contrast, we propose a different decentralized mechanism with market-makers. Such a mechanism has a few attractive features. First, the market-maker mechanism better matches certain features of the data, such as exhibiting price dispersion and changing over time. The other attractive feature, which is the focus of this paper, is that the market maker mechanism almost requires as little information as the competitive allocation. Moreover, the mechanism requires little information, even away from the competitive allocation. In contrast, the informational efficiency for the competitive allocation mechanism is not defined for outside-of-equilibrium allocations and trades. This informational efficiency is one possible reason we observe intermediaries that facilitate trade between individual original sellers and individual final buyers.

6 Appendix

6.1 Existence and Uniqueness of Walrasian Equilibrium Price

Proposition 5. *There exists a unique p^* such that $sG(p^*) = b[1 - F(p^*)]$.*

Proof. Define the excess demand function $Z : [0,1] \to [-1,1], Z(p) = s[1 - G(p)] - bF(p)$, since F and G are continuous, Z is continuous, at $p = 0$, $Z(p) = 1$ while at $p = 1, Z(p) = -1$. By the intermediate value theorem there exists a p^* such that $Z(p^*) = 0$ and from the fact that Z is strictly decreasing it's easy to see that such p^* is unique. ■

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