Market Persuasion

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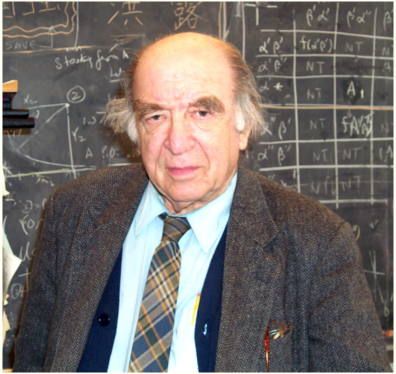
Information of Institutions

- My job market paper is on the formal informational properties of competitive markets
- Hayek (1935) and Mises (1920) argued for markets based on their informational advantage
- Oskar Lange (1936) argued for socialism
- However, the two sides spoke past each other due to different theoretical frameworks
    - Lavoie (1985), Caldwell (1997)

Incentives of Institutions

- Leo Hurwicz (1972) provided a unified formal framework by studying incentive compatibility, starting mechanism design

- People reveal information if given right incentives



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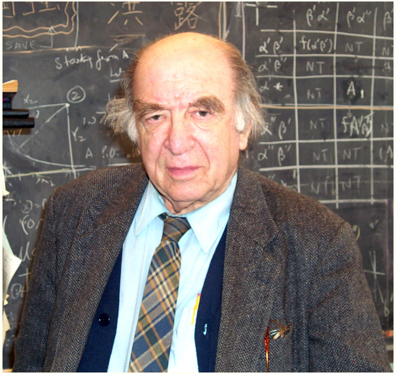
- People reveal information if given right incentives
- Incentives create information in mechanism design



Return of Information

- Recently, information has returned to the theoretical foreground
  - Bayesian Persuasion: Kamenica & Gentzkow (2011), Albrecht (2017)
  - Information Design: Taneva (2016)
  - Bayes Correlated Equilibrium: Bergemann & Morris (2013, 2016, 2018), Albrecht (2018)

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- Information creates incentives in information design

- However, information design assumes a single designer has the information to reveal

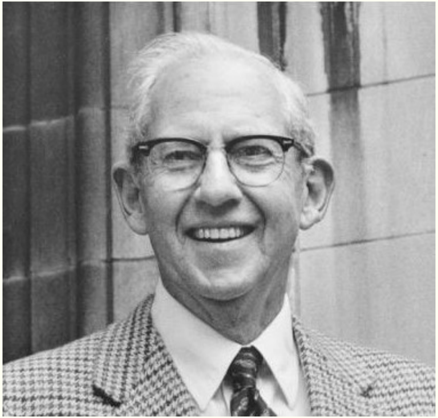
Pricing Information

- Instead of a single designer, I consider competition for information
- I define a notion of a price-taking equilibrium to account for the pricing of information
- Individuals compete for information and to persuade each other
- Information emerges in equilibrium
- The market prices information and the equilibrium outcome is efficient

Modern Perfect Competition Theory

"The complete theory of competition cannot be known because it is an open-ended theory; it is always possible that a new range of problems will be posed in this framework, and then, no matter how well-developed the theory was with respect to the earlier range of problems, it may require extensive elaboration in respects which previously it had glossed over or ignored."

-George Stigler (1957)



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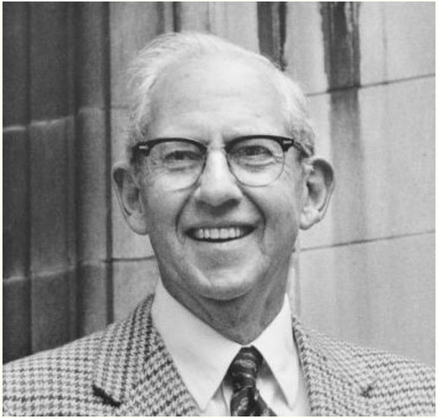

I add persuasion to the theory of competition

Take-away

Competition is the strongest form of persuasion

Outline of Talk

Aside: Planner's Problem

- Before modeling competition, I will consider a fictitious planner's problem

$$\begin{aligned} &\text{maximize} && \text{Price - Cost} \\ &\text{subject to} && Q_{Demand} = Q_{Supply} \end{aligned}$$

Name	Reservation Price	Cost	Name
D1	9	1	S1
D2	8	2	S2
D3	7	3	S3
D4	6	4	S4
D5	5	5	S5
D6	4	6	S6
D7	3	7	S7
D8	2	8	S8
D9	1	9	S9
D10	0	10	S10

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- Comes from duality, like in producer theory

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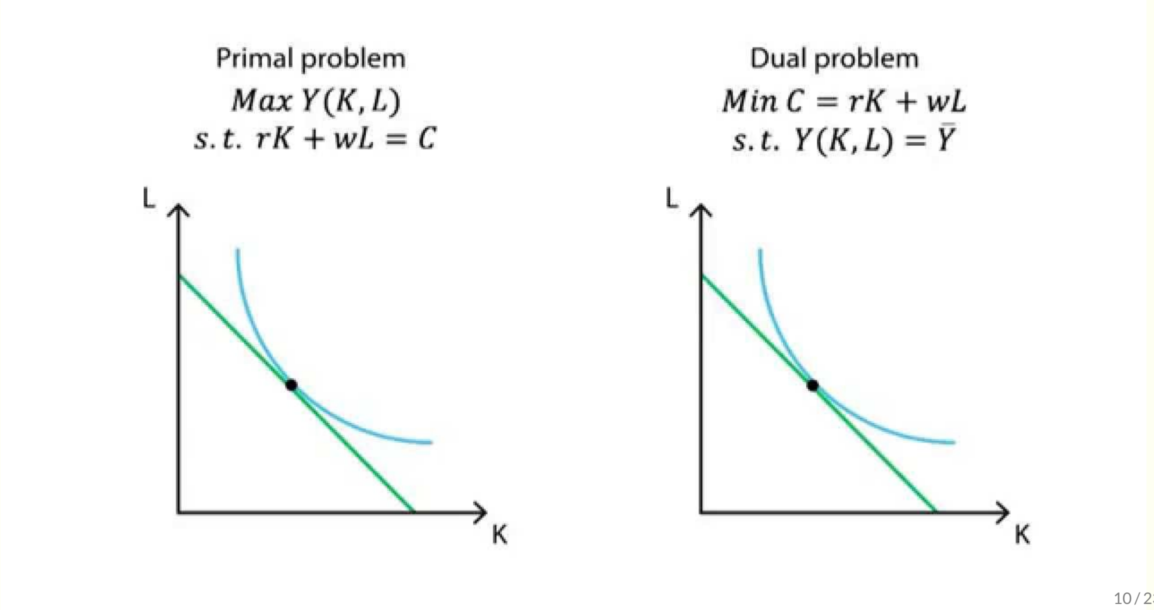
Duality



Primal problem
$Max\, Y(K, L)$
$s.t.\ rK + wL = C$
L
K
Dual problem
$Min\, C = rK + wL$
$s.t.\ Y(K, L) = \bar{Y}$
L
K

Formal Problem

- My planner's problem is a linear program
- Using duality, the First Welfare Theorem applies automatically
- Duality requires the market to set personalized prices

Outline of Talk

Example: Kamenica & Gentzkow (2011)

Adapted from Bergemann & Morris (2018)

Simple Example

- Continuum of two groups of people: firms and workers
- Two types of workers, bad or good: $t \in \{B, G\}$
- Firm actions, hire or not hire: $a \in \{H, N\}$
- Firm payoffs

	Bad Worker B	Good Worker G
Hire	-1	v
Not Hire	0	0

- $0 < v < 1$
- Prior probability of each type is $\frac{1}{2}$ $\Rightarrow$ Not Hire is optimal without more information

Assignment Model

- Information design assumes the designer uses signals to reveal information
- To incorporate competition, I consider an assignment model of workers to firms
  - Shapley & Shubik (1971)
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- Information matters because there is a moral hazard problem that must be incorporated into competition
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- The planner reveals information through the assignment
- A planner assigns workers to firms and recommends actions to the firms

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- Because $v < 1$, the hire constraint is the binding one
- Always hire $x(B, H) = x(G, H) = \frac{1}{2}$ is not incentive compatible
- Full information leads to only the good workers being hired

Planner's Problem

- Suppose the planner wants to maximize the probability of hiring

$$\underset{x(t,a)\geq 0}{\text{maximize}} \quad x(B,H) + x(G,H) \qquad \text{Planner's Objective}$$

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- This is called a primal problem

Optimal Assignment

- In this simple example, the optimal assignment is

	Bad Worker B	Good Worker G
Hire	$v/2$	$1/2$
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- This is where information design stops, finding the optimal outcome
- I want to look deeper at the underlying math to talk about prices

Primal Lagrangian

- For a general planner's problem, construct a Lagrangian that incorporates constraints
- The planner's problem is still to choose $x(t, a)$ to maximize the function
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$$
\begin{aligned}
\mathcal{L} = {} & x(B, H) + x(G, H) \\
& + \underbrace{\lambda_1}_{\text{Price on Type B}} \left[\frac{1}{2} - x(B, H) - x(B, N)\right] \\
& + \underbrace{\lambda_2}_{\text{Price on Type G}} \left[\frac{1}{2} - x(G, H) - x(G, N)\right] \\
& + \underbrace{\lambda_3}_{\text{Price on IC}} \left[0 - vx(G, H) + x(B, H)\right]
\end{aligned}
$$

Dual Lagrangian

- Rearranging, we can find an equivalent dual problem where total cost is minimized

$$
\begin{aligned}
\mathcal{L} = \frac{1}{2}\lambda_1 + \frac{1}{2}\lambda_2 + 0\lambda_3 \\
+ x(B,H)\left[1 - \lambda_1 + \lambda_3\right] \\
+ x(G,H)\left[1 - \lambda_2 - \lambda_3 v\right] \\
+ x(B,N)\left[0 - \lambda_1\right] \\
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- Strong Duality: The value of $\mathcal{L}$ by minimizing with respect to λ's is the same as maximizing with respect to x's.

- Easily extends to arbitrary finite types, actions, and welfare weights

Theorem

There exist prices for people and their information such that the decentralized assignment is identical to the planner's optimal assignment.

Next Steps

- Right now, the proofs are only for a standard, transferable utility assignment model

- Economic Extensions
    1. Competitive information in firms/clubs: Zame (2007), Rahman (2012)
    2. Adverse selection: Jerez (2003), Rahman (2012)
    3. Comparative Statics on assortative matching

- Technical Extension
    3. Non-transferable utility: Noldeke and Samuelson (2018)

Interpretation

- The primal asks, what is the assignment x that maximizes the planner's objective?
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- That is, the decentralized market provides the same persuasion as an omniscient planner only constrained by resource feasibility and incentive compatibility
- We say the competitive outcome is efficient
- Return to Hayek: Market information creates incentives