# Multiscale Image Registration and Segmentation

Brady Berg*, Conor Carroll*, Weston Grewe*, Brian Knight*, Tuyen Pham
Advised by Dr. Dana Paquin
Department of Mathematics, Cal Poly, San Luis Obispo

## Image Segmentation

Given an image, f, and a partition of f's indices, $\Lambda = \{\Lambda_1, \Lambda_2, \ldots, \Lambda_n\}$, a segmentation of the image $f$ is a set

$$S = \{f(\Lambda_k)\}_{k=1}^{n},$$

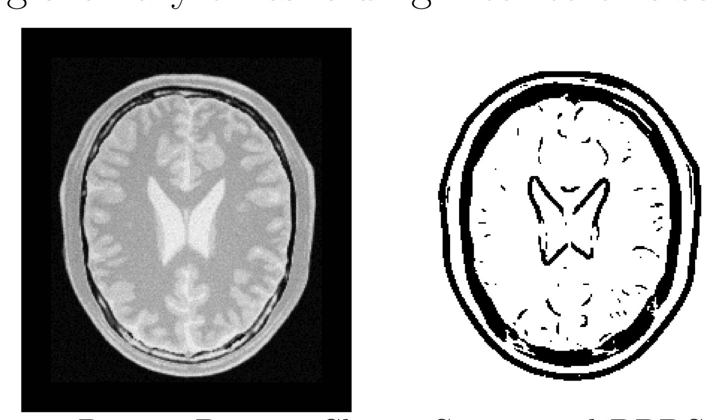
where

$$f(I_k) = \begin{cases} f(i,j), & \text{if } (i,j) \in \Lambda_k \\ \alpha, & \text{otherwise,} \end{cases}$$

for some fixed $\alpha$.

## Motivation

In less formal language, image segmentation is the process of partitioning an image into components in order to simplify and/or emphasize key features of the image. Typically, we wish to extract the borders of objects such as tumours or healthy tissues. We can do this by segmenting an image into edge pixels and non-edge pixels, determined by some algorithm or by hand, and applying a binary thresholding filter to this segmentation:



Brain Proton Density Slice    Segmented BPDS

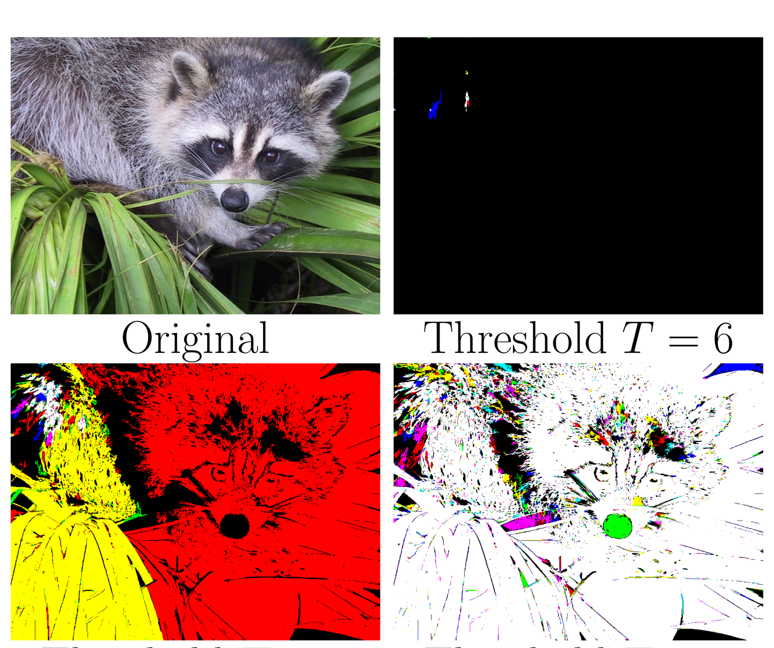
## Thresholding

In general, we can induce a partition of $\Lambda$ by partitioning the intensity set $I$ into $\{I_1, \ldots, I_n\}$, with $I_j \subseteq \{0, 1, 2, \ldots, 255\}$ as follows:

$$\Lambda_k = \{(i,j) \ : \ f(i,j) \in I_k\}.$$

## Region Building

With region building, we can partition an image by specifying a set of seed pixels and grow outwards based on a preset condition, often a threshold.



Original    Threshold $T = 6$

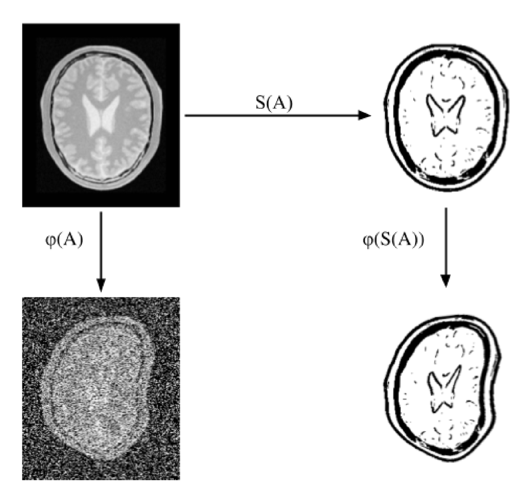
Threshold $T = 7$    Threshold $T = 8$

## Image Registration

Given two images $A$ and $B$, registration is the problem of finding a mapping, $\varphi$, in a specified transform space, $\tau$, such that $\varphi = \operatorname{argmin}_{\varphi \in \tau} D(\varphi(A), B)$, where $D$ is some predetermined metric.

## Motivation

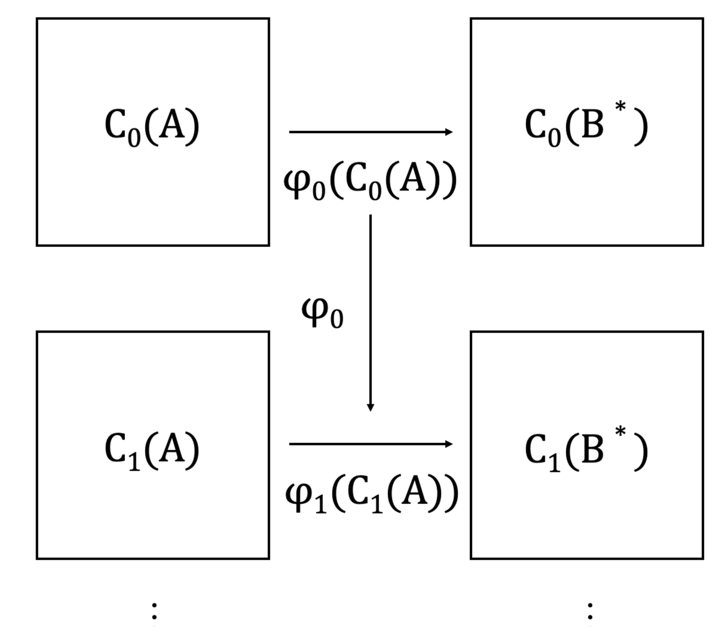
In a medical context, the image which is pre-processed, which we will denote $A$, is typically a precursor to some reimaging, $B^*$, taken nearer to the time of any given procedure. $A$ is typically less noisy and is sometimes of different modality than $B^*$. Thus, we often seek to successfully register $A$ to $B^*$ to preserve pre-processing. In this work, we use our registration to map a segmentation of $A$, $S(A)$, to successfully segment $B^*$. Below is a schematic of this process.




## Multiscale Registration

We have various methods of combining multiscale decomposition to aid in image registration for various situations. All methods are presented by Paquin, Levy, and Xing[3].

For our results we followed the iterative multinode multiscale registration algorithm (IMMRA), which decomposes a moving image $A$ and fixed image $B^*$ into $m$ scales, and iteratively registers the corresponding scales, using the most recent registration, $\varphi_i$, as the initialization for the registration of the $(i+1)^{\text{th}}$ scales.




IMMRA Schematic

So we then have $\varphi_i$ as the optimal registration of the $i^{th}$ scales of $A$ and $B^*$. $\varphi_i = \operatorname{argmin}_{\varphi \in \tau} D(C_i(B^*), \varphi(C_i(A)))$. The process terminates after the $m^{\text{th}}$ scales are registered, resulting in $\varphi = \varphi_m$.

## Multiscale Decomposition

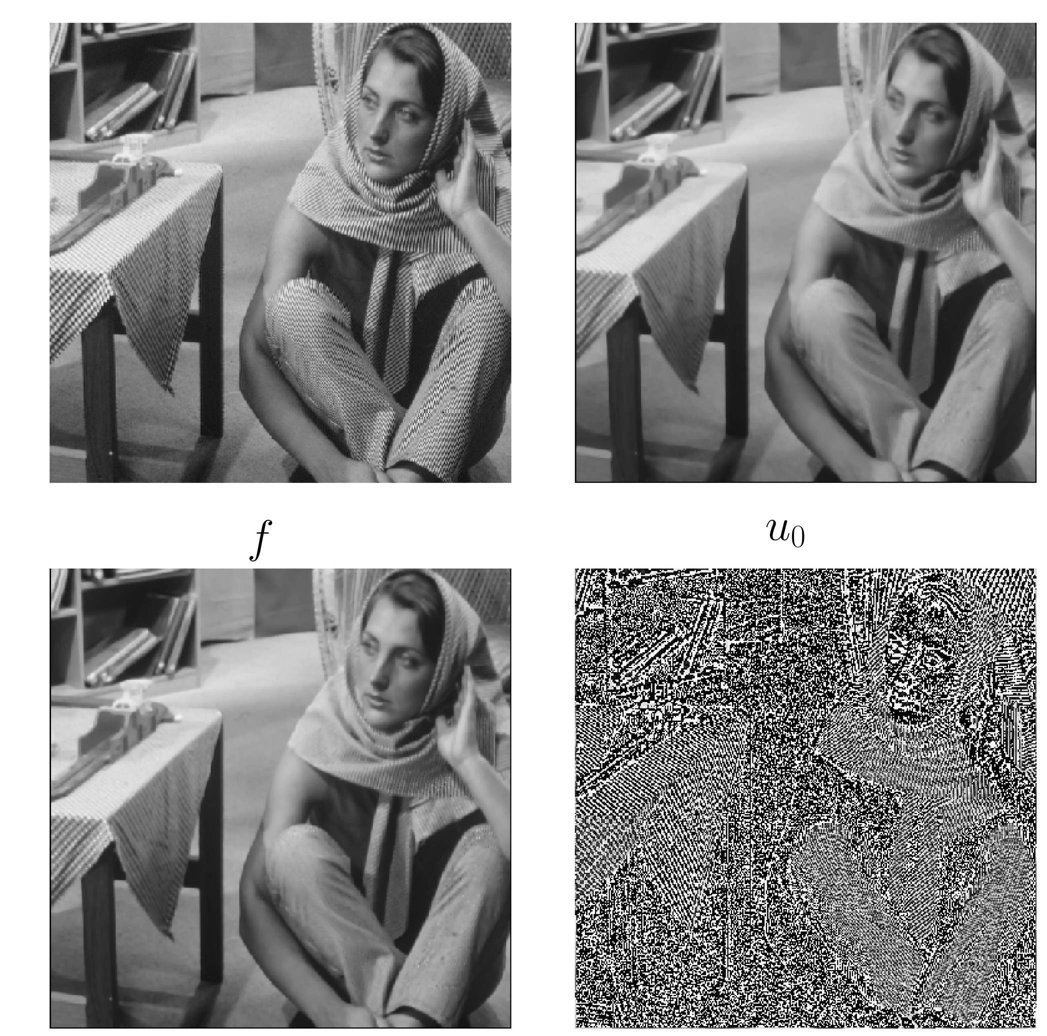
Multiscale decomposition is a representation of an image $f$ as

$$f = \left(\sum_{i=0}^{m} u_i\right) + v_m,$$

where $f = u_0 + v_0$ is the initial denoising process introduced by Rudin, Osher, and Fatemi[1]. This decomposition (introduced by Tadmor, Nezzar, and Vese)[2] separates edges and textures of an image depending on the scale and level observed.

Note that in this decomposition, we expect $f = \sum_{i=0}^{\infty} u_i$.

## Examples



$f$    $u_0$

$u_0 + u_1$    $v_1$

## Multiscale Segmentation and Results

## IMMRSA - Iterative Multinode Multiscale Registration Segmentation Algorithm

Given moving image $A$, a fixed image $B^*$, and a segmentation algorithm, $S$,

1. Segment $A$, to find $S(A)$.
2. Find $\varphi = \text{IMMRA}(A, B^*)$.
3. Map $S(A)$ through $\varphi$ to get a segmentation of $B^*$.

To provide a proof of concept for this segmentation algorithm, we record a benchmark segmentation, $S(B)$, the ideal segmentation of of $B^*$, and measure the success of all segmentations via distance from $S(B)$. For registration, we use the SimpleITK python wrapper to implement IMMRA, with a transformation space of B-Spline transforms. We segment using a fast-marching method and binary thresholding. We record the values $D(\varphi(S(A)), S(B))$ for varying densities of impulse noise. For comparison, we also segment $M(B^*)$, a median filter of $B^*$, and find a registration $\Phi$ between $M(A)$ and $M(B^*)$ to record $D(\Phi(S(A)), S(B))$.

$\varphi(S(A))$    Overlayed

IMMRSA Result, $\delta = 50$

| $\delta$ | $S(M(B^*))$ | $\Phi(S(A))$ | $\varphi(S(A))$ |
|---|---|---|---|
| 0 | 0.01648 | 0.02473 | 0.01909 |
| 5 | 0.01942 | 0.02452 | 0.01909 |
| 10 | 0.02069 | 0.02460 | 0.01908 |
| 20 | 0.02556 | 0.02677 | 0.02185 |
| 30 | 0.02768 | 0.02710 | 0.02039 |
| 40 | 0.03463 | 0.03293 | 0.02376 |
| 50 | 0.04375 | 0.03481 | 0.01946 |
| 60 | 0.10087 | 0.04249 | 0.06214 |
| 70 | 0.25651 | 0.04666 | 0.04486 |
| 80 | 0.51770 | 0.04760 | 0.07296 |
| 90 | 0.78675 | 0.05826 | 0.09339 |
| 95 | 0.84934 | 0.10406 | 0.11041 |

Distance to benchmark with mean squared metric.

Here, IMMRSA proves more accurate than any standard approach. More test images, different types of noise, different transform spaces, and various segmentation and registration algorithms should be explored in further work.

## Acknowledgments

Thanks to the Bill and Linda Frost Fund for their generous support of this work in Summer 2019.
(*) indicates Frost Research Fellows, recipients of the Frost Undergraduate Student Research Award.

## References

[1] Leonid I. Rudin; Stanley Osher; Emad Fatemi., Nonlinear total variation based noise removal algorithms, Physica D, **60**, (1998), 259-268.

[2] Tadmor, E. and Nezzar, S. and Vese, L., A Multiscale Image Representation Using Hierarchical (BV,L2 ) Decompositions, Multiscale Modeling & Simulation, (2004) Vol. **2**, No. **4**, 554-579 by nonlinear phase locking to Milankovitch forcing, Paleoceanography, 21, PA4206, doi:10.1029/2005PA001241.

[3] Paquin, Dana C.; Levy, Doron; Xing, Lei, Multiscale Deformable Registration of Noisy Medical Images, Mathematical Biosciences and Engineering, (2008) Vol. **5**, No. **1**, 125-144.