Glacial Cycles and the 100 Kyr Problem

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Abstract

The Earth's climate during the Late Pleistocene era (1.25 Mya to 12.5 kya) is characterized by large glacial cycles: oscillations in the size of large land-based ice sheets on a 100 kyr time scale.The underlying cause for these long cycles and the predictability of their timing remain open questions. Numerous models have been proposed to explain the observed behavior including those based on forced dynamical systems in which the observed cycles are the result of complicated interactions between the quasi-periodic astronomical forcing and internal free oscillations. The overall timing of the cycles is primarily controlled by nonlinear synchronization - a ubiquitous feature of forced dynamical systems. The strong asymmetry exhibited between the warming and cooling phases of a cycle can be associated with an asymmetry in the predictability of behavior within a glacial cycle. In the work, we explore these behaviors of forced dynamical systems using two models of the glacial cycles.

Introduction/Background

Variations in δ^{18}O records – an isotopic based proxy measurement for global ice volume – shown below:

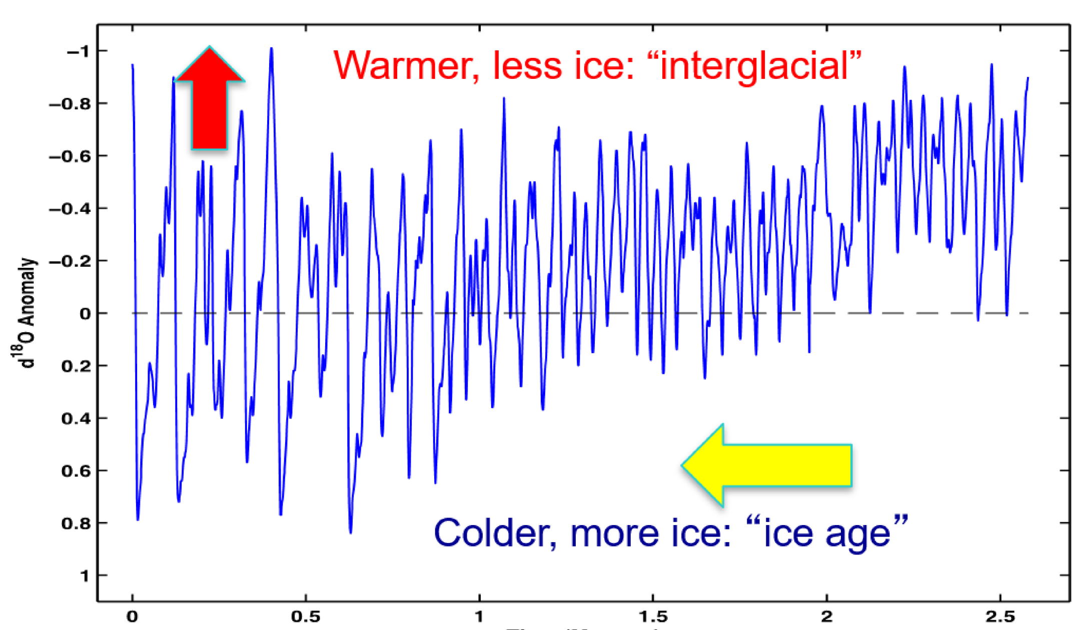


Figure: δ^{18}O records for past 2.5 million years, red arrow points to higher temp, yellow points to present.

In this δ^{18}O record we observe glacial cycles, or oscillations in the growth and decay of large land based ice sheets, of a varying timescale. What we understand:

- Early 40 kyr timescale oscillations due to astronomical forcing (obliquity)
- General cooling trend

What we do not understand:

- Transition from fast cycles (40 kyr timescale, to slow, 100 kyr timescale around 800 kyr ago)
- Why realized timescale of 100 kyr?
- Simply a bi-stable system? Other possible dynamics?

To further study these questions we make use of two conceptual models. The approach is to:

- Collect model output which simulates Late Pleistocene cycles
- Investigate the interaction between internal and external dynamics
- Distinguish predictability of timing versus predictability of state within a cycle.

Core (Mathematical) Questions

- How does the interaction between an external quasi-periodic forcing signal and a internal limit cycle create the realized timescale of oscillation?
- To what degree is the timing of the glacial cycles predictable?
- What is the role of asymmetry in the internal dynamics?
- How does the interaction between external and internal dynamics effect predictability within a glacial cycle?

Non-Linear Synchronization

Deglaciation events are paced by maximum obliquity (astronomical forcing). Some maximum obliquity events are skipped. Exploring how the conceptual models respond to the quasi-periodic forcing signal, we implement a circle plot to compare timing of events, namely ice deglaciation and maximum obliquity.

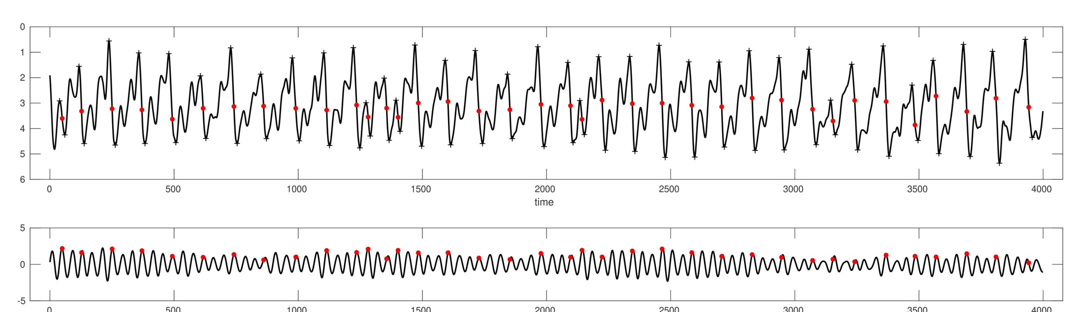
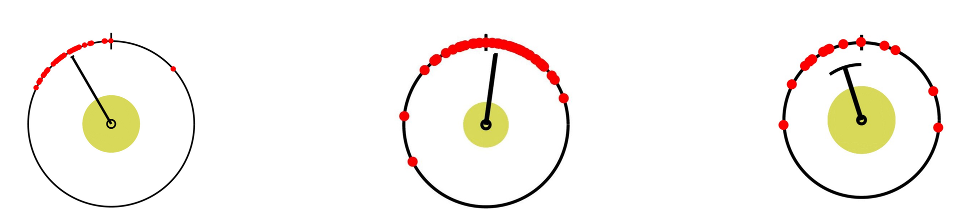


Figure: **top**: Global Ice Volume Output from PP04 Model Run-red dot placed in middle of full melt, **middle**: Laskar Obliquity Signal, red dots plotted at same time step, **bottom**: (left to right) PP04, SM90, Data cirleplots to determine whether the two signals are in phase. Red dots on the mid-line of the plot are perfectly in phase, those to the left are leading.



(a) PP04 (b) SM90 (c) Data

Note: figs (b) and (c) are analogous plots for SM90 and data; each is significantly paced by the obliquity signal, typically every 3rd max.

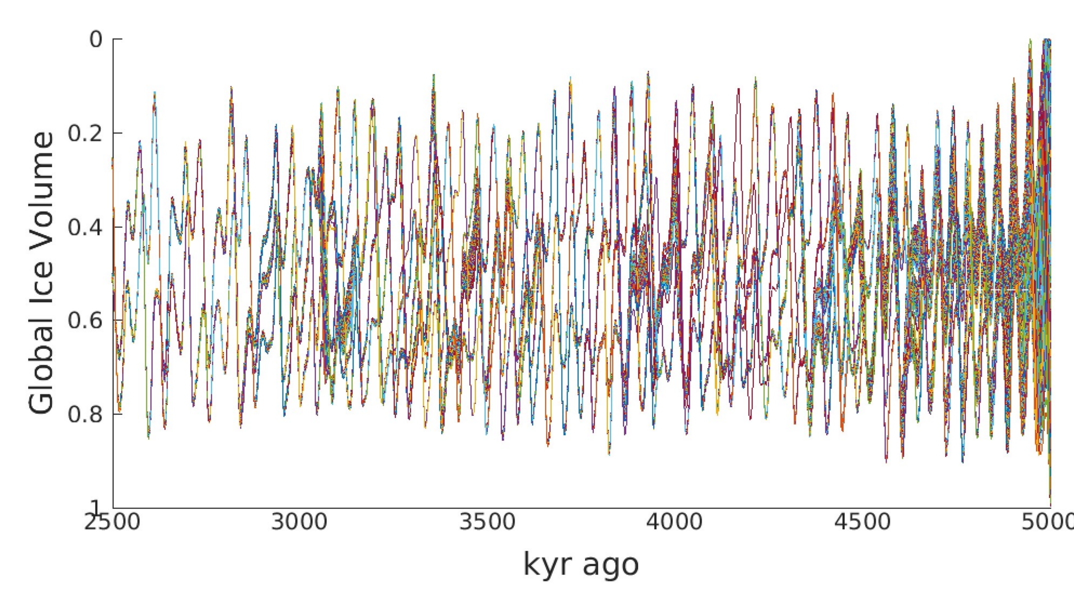


Figure: Synchronization* of 1000 random IC for PP04 in Global Ice Volume. ***note**: left is present, right is past, time runs right to left; cool temp is low, high temp is high. Skipped obliquity

Trajectories from different initial conditions generally synchronize to three stable clusters, identified by which obliquity maximum triggers a full melt. Short-lived desychronization events occur spoaridcally.

Conceptual Models

Saltzman & Maasch 1990 (SM90)[*Smooth*]

- $dX/dt = -X - Y - vZ - uR(t*) + W_X(t*)$,
  $dY/dt = -pZ + rY + sZ^2 - wYZ - Z^2Y + W_Y(t*)$,
  $dZ/dt = -q(X+Z) + W_Z(t*)$
- Smooth 3 Variable System: X = Global Ice Mass, Z = Deep Ocean Temp., Y = Atmos. CO2, non-dimensionalized.
- W_X, W_Y, W_Z stochastic variables.
- Internal Stable Limit Cycle, Multi-Stable* with 3 stable trajectories.

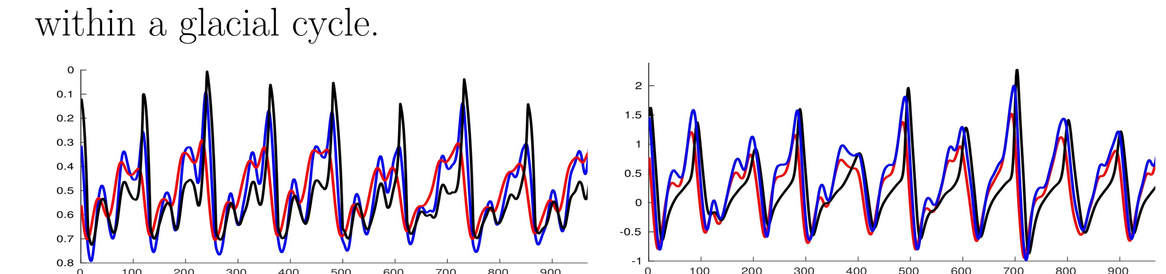
Paillard & Parrenin 2004 (PP04)[*Non-Smooth*]

- $dV/dt = (V_R - V)/\tau_V$, $dA/dt = (V - A)/\tau_A$, $dC/dt = (C_R - C)/\tau_C$
- $V_R = xC - yI_{60} + z$
- $C_R = \alpha I_{65} - \beta V + \gamma H(-F) + \delta$
- $F = aV - bA - cI_{60} + d$, d = 0.27 for late pleistocene simulation.
- I_{65}, I_{60}, solar insolation at 65, 60 deg. north latitude.
- Also 3D system. **Non-smooth**. V = Global Ice Mass, A = Antarctic Ice Volume, C = Atmos. C02,
- Internal Stable Limit Cycle, Multi-Stable* with 3 stable trajectories, Model Switch non-linearity (Heaviside function: H = 1 if F <0, H = 0 otherwise, F < 0: Relaxation Warming Model, F > 0: Relaxation Cooling Model)

*As observed in parameter space / forcing tested

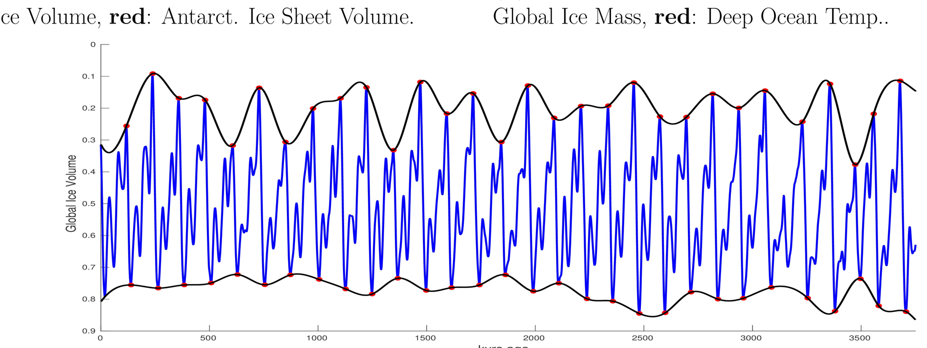
Predictability of Models

δ^{18}O records of Late Pleistocene show a memory of state in ice decay that is not found in ice growth. In particular, there is a significant anti-correlation between the ice volume at a glacial maximum and at the subsequent glacial minimum. Knowing this and with understanding that model trajectories synchronize, we begin to search for predictability within a glacial cycle.



(a) PP04 Model Output. **black**: Atmos. CO2, **blue**: Global Ice Volume, **red**: Antarct. Ice Sheet Volume.

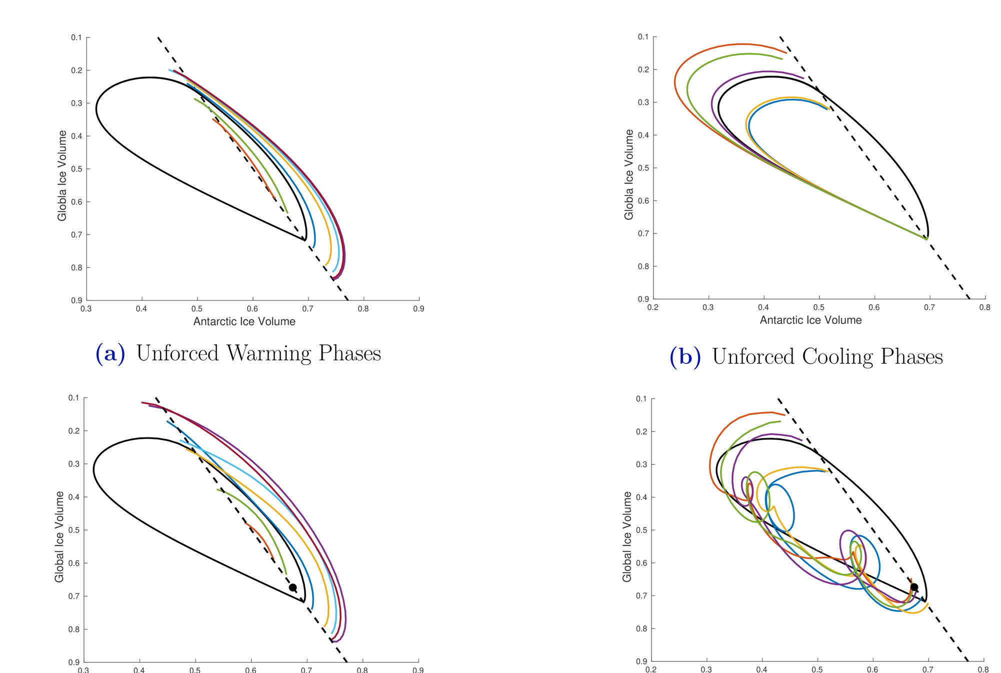
(b) SM90 Model Output. **black**: Atmos. CO2, **blue**: Global Ice Mass, **red**: Deep Ocean Temp..



(c) **blue**: PP04: Global Ice Volume output, time going from right to left. **red dots**: maxima (low), and minima (high)of full melts. **black**: Two cubic splines through the maxima and minima of full melts to show the anti-correlation we suspected.

In (c) above we observe the anti-correlation between glacial minima and maxima in our model output. To gain better, we observed projections of phase spaces of both models and collect specific measurements. For PP04:

- Trigger point implemented by a Heaviside function, marked by dashed line
- Measure ice volume at trigger points of model
- Measure max ice volume and min ice volume of cycle



(a) Unforced Warming Phases (b) Unforced Cooling Phases

(c) Forced Warming Phases, same IC as above (d) Forced Cooling Phases, same IC as above

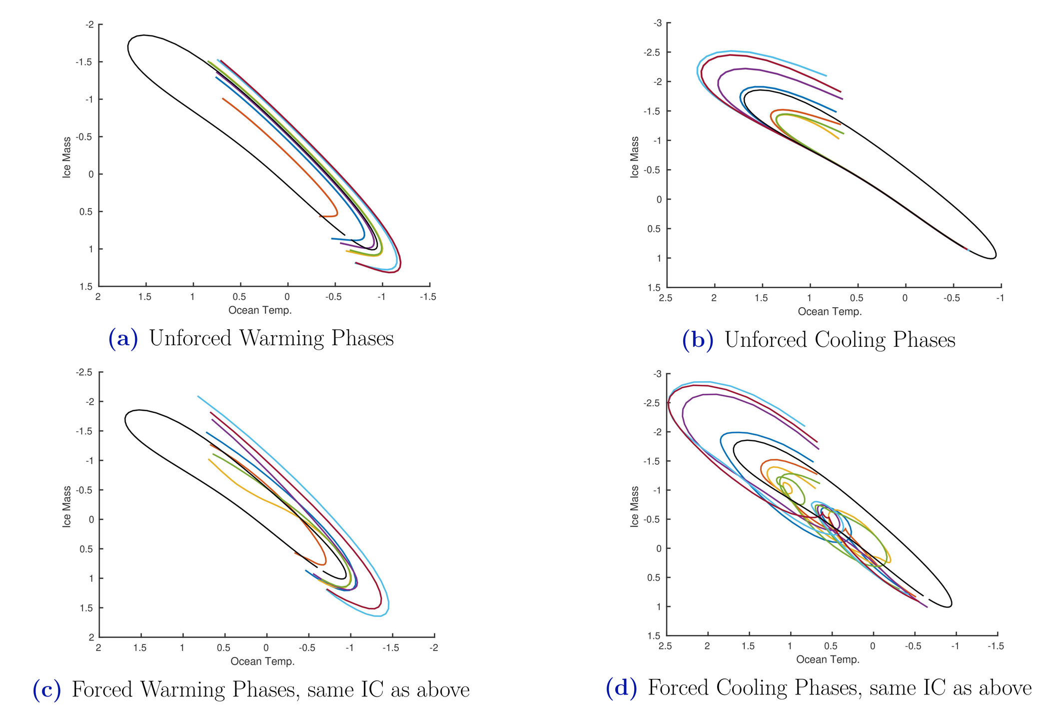
Trajectories all moving counter-clockwise, a clear distinction can be made by observed plots a-d above. Unforced vs. Forced Behavior in warming phases is very similar in this projection, namely this parallel curve structure. Both the synchronization of the forcing and the asymmetry of the phases(average ratio of time spent cooling to time spent warming per cycle is approximately 6/1) contribute to this. In contrast, the cooling phase structure changes drastically from the unforced to the forced. Any parallel structure is that existed is erased by the quasi-periodic forcing .

For the smooth model, SM90, there is no definitive point at which the phase shifts because it is a continuous model. Instead, we look at the CO2 variable (Mu) and its derivative to mark what phase we are in.

- Trigger point chosen to be δCO2 = 0
- High C02 implies lower ice mass, vice versa
- CO2 increasing: warming, decreasing: cooling
- Record state variables at trigger points as well as maximum and minimum ice masses

The choice of trigger point here is somewhat arbitrary. To ameliorate this we put emphasis on the maximum and minimum ice masses for comparisons.

Predictability Continued



(a) Unforced Warming Phases (b) Unforced Cooling Phases

(c) Forced Warming Phases, same IC as above (d) Forced Cooling Phases, same IC as above

Qualitatively, these phase planes are similar to that of PP04.

- More time is spent in the cooling phase than the warming (appr. 3/1), thus cooling trajectories converge to stable limit cycle
- Unforced cooling also similar, though less displacement
- Little qualitative change in unforced warmings and forced warmings

Conclusions & Discussion

Taking specific forced model runs from both of these models, we find the correlations discussed in the phase plane figures. Below is a table containing correlations between phases of cycles. We also compare these correlations to the analogous sediment core data(composed of roughly 10 glacial cycles). For each model we find correlations for two warming/cooling phase definitions: **1.** Ice Variable Extrema, i.e. Ice max to Ice min as a warming phase, **2.** Model Trigger, as defined in the phase plots.

Warming/Cooling, Phase Definition		PP04(***f***)	SM90(***f***)	Data
Warming: Ice Var. Extrema	corr	**-0.48**	-0.64	-0.62
	p-val	**<0.01**	<0.0001	0.10
Cooling: Ice Var. Extrema	corr	-0.07	-0.34	0.39
	p-val	0.71	0.02	0.39
Warming: Trigger	corr	**-0.60**	-0.64	N/A
	p-val	**<0.001**	<0.00001	N/A
Cooling: Trigger	corr	-0.06	-0.49	N/A
	p-val	0.75	<0.001	N/A

Table: Variable Correlations and Significance. ***(f)*** denotes stats for forced model run.

In both the non-smooth, PP04, and the smooth, SM90 conceptual models, the asymmetric limit cycles in the presence of a quasi-periodic forcing create strong anti-correlations in the warming phases. In PP04 any correlation in the unforced cooling phases is erased by this forcing, whereas in SM90, a complex geometry creates strong anti-correlation as it cools, something to be further explored. *The role of asymmetry in a system is a key part of the realized response to external forcing.*

The pacing of the cycles by the obliquity signal plays a large role in the realized 100 kyr oscillation in the two conceptual models, as well as synchronizing the forcing signal itself. Overall, the interaction between internal and external dynamics tends to diminish the predictability of certain phases while having little effect on others.

Acknowledgments

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