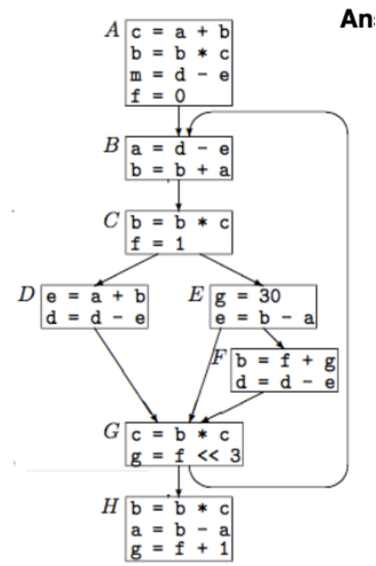
CSC 766 Code Optimization for Scalar and Parallel Programs
Spring 2023 Xipeng Shen **HW 1**

1

Answer questions based on the CFG below:



1. Find the extended basic blocks.

> The EBBs are: {A}, {B}, {C}, {D}, {E}, {F}, {G}, {H}, {BC}, {CD}, {CE}, {EF}, {BCD}, {BCE}, {CEF}, {CDE}, {BCDE}, {BCEF}, {CDEF}, {BCDEF}, {GH}.
>
> But more concisely (to help with value numbering), they can be defined as: {A}, {BCDEF}, {GH}.

2. Find the dominator set for each basic block.

Block	Dom	IDom
A	A	-
B	A, B	A
C	A, B, C	B
D	A, B, C, D	C
E	A, B, C, E	C
F	A, B, C, E, F	E
G	A, B, C, G	C
H	A, B, C, G, H	G

3. Build the dominance tree.

```
  A
  |
  B
  |
  C
/ | \
D E G
  | |
  F H
```

4. Apply super local numbering to the CFG.

Super local value numbering is as follows:

i Build SSA form

ii Find EBBs

iii Apply value numbering to each path in each EBB using scoped hash tables

First, we can rewrite the each basic block in SSA form as follows:

Block A:
$c_0 = a_0 + b_1$
$b_1 = b_0 * c_0$
$m_0 = d_0 - e_0$
$f_0 = 0$
Block B:

$b_2 = \phi(b_1, b_6)$
$c_1 = \phi(c_0, c_2)$
$d_1 = \phi(d_0, d_4)$
$e_1 = \phi(e_0, e_4)$
$f_1 = \phi(f_0, f_2)$
$g_1 = \phi(g_0, g_4)$
$a_1 = d_1 - e_1$
$b_3 = b_2 + a_1$
Block C:
$b_4 = b_3 * c_1$
$f_2 = 1$
Block D:
$e_2 = a_1 + b_4$
$d_2 = d_1 - e_2$
Block E:
$g_2 = 30$
$e_3 = b_4 - a_1$
Block F:
$b_5 = f_2 + g_2$
$d_3 = d_1 - e_3$
Block G:
$b_6 = \phi(b_5, b_4, b_4)$
$d_4 = \phi(d_1, d_2, d_3)$
$e_4 = \phi(e_2, e_3, e_3)$
$g_3 = \phi(g_1, g_2, g_2)$
$c_2 = b_6 * c_1$
$g_4 = f_2 << 3$
Block H:
$b_7 = b_6 * c_2$
$a_2 = b_7 - a_1$
$g_5 = f_2 + 1$

Second, we find EBBs, which were already derived before. But now, we only consider the paths, which are A, BCDEF, GH.

Lastly, we apply value numbering to each path in the EBB using scoped hash tables (Value numbers are denoted as superscripts).
EBB Path: A
Block A:
$c_0^2 = a_0^0 + b_1^1$
$b_1^3 = b_0^1 * c_0^2$
$m_0^6 = d_0^4 - e_0^5$

$f_0^8 = 0^7$

No values redundant

EBB Path: BCDEF
Block B:
$b_2^2 = \phi(b_1^0, b_6^1)$
$c_1^5 = \phi(c_0^3, c_2^4)$
$d_1^8 = \phi(d_0^6, d_4^7)$
$e_1^{11} = \phi(e_0^9, e_4^{10})$
$f_1^{14} = \phi(f_0^{12}, f_2^{13})$
$g_1^{17} = \phi(g_0^{15}, g_4^{16})$
$a_1^{18} = d_1^8 - e_1^{11}$
$b_3^{19} = b_2^2 + a_1^{18}$
Block C:
$b_4^{20} = b_3^{19} * c_1^5$
$f_2^{22} = 1^{21}$
Block D:
$e_2^{23} = a_1^{18} + b_4^{20}$
$d_2^{24} = d_1^8 - e_2^{23}$
Block E:
$g_2^{26} = 30^{25}$
$e_3^{27} = b_4^{20} - a_1^{18}$
Block F:
$b_5^{28} = f_2^{22} + g_2^{26}$
$d_3^{29} = d_1^8 - e_3^{27}$

No values redundant

EBB Path: GH
Block G:
$b_6^3 = \phi(b_5^0, b_4^1, b_4^2)$
$d_4^7 = \phi(d_1^4, d_2^5, d_3^6)$
$e_4^{11} = \phi(e_2^8, e_3^9, e_3^{10})$
$g_3^{15} = \phi(g_1^{12}, g_2^{13}, g_2^{14})$
$c_2^{17} = b_6^3 * c_1^{16}$
$g_4^{20} = f_2^{18} << 3^{19}$
Block H:
$b_7^{18} = b_6^3 * c_2^{17}$
$a_2^{19} = b_7^{18} - a_1^{19}$
$g_5^{22} = f_2^{20} + 1^{21}$

No values redundant

In the end, unfortunately there are no values to be rewritten.

5. Apply dominator-based value numbering to the CFG.

Dominator-based value numbering is as follows:

i Build SSA form

ii Use table from IDom(x) to start value numbering

iii Follow the order of IDOM tree

We have already built SSA form from the previous question, so we can get straight into using the IDom table to do value numbering. Thus, the scopes to do value numbering based on their IDOM(x) is as follows: $\{A\}, \{A, B\}, \{B, C\}, \{C, D\}, \{C, E\}, \{E, F\}, \{C, G\}, \{G, H\}$

Scope $\{A\}$
Block A:
$c_0^2 = a_0^0 + b_1^1$
$b_1^3 = b_0^1 * c_0^2$
$m_0^6 = d_0^4 - e_0^5$
$f_0^8 = 0^7$

No values redundant

Scope $\{A, B\}$
Block A:
$c_0^2 = a_0^0 + b_1^1$
$b_1^3 = b_0^1 * c_0^2$
$m_0^6 = d_0^4 - e_0^5$
$f_0^8 = 0^7$

Block B:
$b_2^2 = \phi(b_1^0, b_6^1)$
$c_1^5 = \phi(c_0^3, c_2^4)$
$d_1^8 = \phi(d_0^6, d_4^7)$
$e_1^{11} = \phi(e_0^9, e_4^{10})$
$f_1^{14} = \phi(f_0^{12}, f_2^{13})$
$g_1^{17} = \phi(g_0^{15}, g_4^{16})$

$a_1^{18} = d_1^8 - e_1^{11}$
$b_3^{19} = b_2^2 + a_1^{18}$

No values redundant

Scope $\{B, C\}$
Block B:
$b_2^2 = \phi(b_1^0, b_6^1)$
$c_1^5 = \phi(c_0^3, c_2^4)$
$d_1^8 = \phi(d_0^6, d_4^7)$
$e_1^{11} = \phi(e_0^9, e_4^{10})$
$f_1^{14} = \phi(f_0^{12}, f_2^{13})$
$g_1^{17} = \phi(g_0^{15}, g_4^{16})$
$a_1^{18} = d_1^8 - e_1^{11}$
$b_3^{19} = b_2^2 + a_1^{18}$
Block C:
$b_4^{20} = b_3^{19} * c_1^5$
$f_2^{22} = 1^{21}$

No values redundant

Scope $\{C, D\}$
Block C:
$b_4^2 = b_3^0 * c_1^1$
$f_2^4 = 1^3$
Block D:
$e_2^6 = a_1^5 + b_4^2$
$d_2^8 = d_1^7 - e_2^6$

No values redundant

Scope $\{C, E\}$
Block C:
$b_4^2 = b_3^0 * c_1^1$
$f_2^4 = 1^3$
Block E:
$g_2^6 = 30^5$

$e_3^8 = b_4^2 - a_1^7$

No values redundant

Scope $\{E, F\}$
Block E:
$g_2^1 = 30^0$
$e_3^4 = b_4^3 - a_1^2$
Block F:
$b_5^6 = f_2^5 + g_2^1$
$d_3^8 = d_1^7 - e_3^4$

No values redundant

Scope $\{C, G\}$
Block C:
$b_4^2 = b_3^0 * c_1^1$
$f_2^4 = 1^3$
Block D:
$e_2^6 = a_1^5 + b_4^2$
$d_2^8 = d_1^7 - e_2^6$
Block E:
$g_2^{10} = 30^9$
$e_3^{11} = b_4^2 - a_1^5$
Block F:
$b_5^{12} = f_2^4 + g_2^{10}$
$d_3^{13} = d_1^7 - e_3^{11}$
Block G:
$b_6^{14} = \phi(b_5^{12}, b_4^2, b_4 2)$
$d_4^{15} = \phi(d_1^7, d_2^8, d_3^{13})$
$e_4^{16} = \phi(e_2^6, e_3^{13}, e_3^{13})$
$g_3^{17} = \phi(g_1^{18}, g_2^{10}, g_2^{10})$
$c_2^{19} = b_6^{14} * c_1^1$
$g_4^{21} = f_2^4 << 3^{20}$

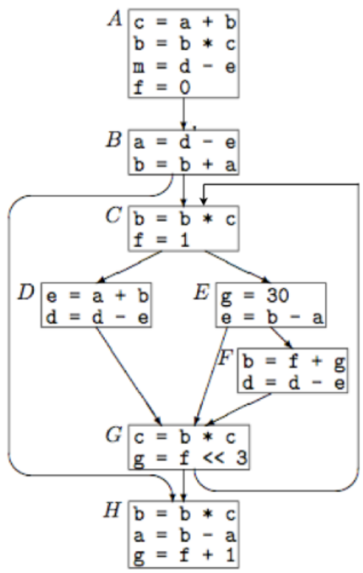
No values redundant

Scope $\{G, H\}$
Block G:
$b_6^3 = \phi(b_5^0, b_4^1, b_4^2)$
$d_4^7 = \phi(d_1^4, d_2^5, d_3^6)$
$e_4^{11} = \phi(e_2^8, e_3^9, e_3^{10})$
$g_3^{15} = \phi(g_1^{12}, g_2^{13}, g_2^{14})$
$c_2^{17} = b_6^3 * c_1^{16}$
$g_4^{20} = f_2^{18} << 3^{19}$
Block H:
$b_7^{18} = b_6^3 * c_2^{17}$
$a_2^{19} = b_7^{18} - a_1^{19}$
$g_5^{22} = f_2^{20} + 1^{21}$

No values redundant

Again, unfortunately no values were rewritten. There were close enough values to be rewritten, but because of the renaming via SSA, it was not possible.

2



In our lecture, we learned how to compute $LIVEOUT$ set for every basic block. In this exercise, you are asked to come up a way to compute $LIVEIN$, which is the set of variables that are live at the entry point of a basic block. Specifically, you are asked to:

1. Clearly describe your algorithm

First we'll define some notation:

- $LIVEIN(n)$ contains the name of every variable that is live on entry to n
- $UEVAR(n)$ contains the upward-exposed variables in n, i.e. those that are used in n before any redefinition in n
- $VARKILL(n)$ contains all the variables that are defined in n
- n_f is the exit node of the CFG

$$LIVEIN(n_f) = UEVar(n_f)$$

$$LIVEIN(n) = (\bigcup_{m \in succ(n)} LIVEIN(m) \cap \overline{VARKILL(n)}) \cup UEVar(n)$$

2. Indicate the order of traversing a CFG for your algorithm to work efficiently

The order of traversing of CFG is backwards or in postorder. It is optimal to visit children before parents.

3. Solve LIVEIN for the following CFG. You need to give the local sets values for the basic blocks, and the final results of the LIVEIN sets of the CFG.

Here are some local set values:
$UEVar(A) = \{a, b, c, d, e\}$
$UEVar(B) = \{a, b, d, e\}$
$UEVar(C) = \{b, c\}$
$UEVar(D) = \{a, b, d, e\}$
$UEVar(E) = \{a, b\}$
$UEVar(F) = \{d, e, f, g\}$
$UEVar(G) = \{b, c, f\}$
$UEVar(H) = \{a, b, c, f\}$

$VarKill(A) = \{b, c, f, m\}$
$VarKill(B) = \{a, b\}$
$VarKill(C) = \{b, f\}$
$VarKill(D) = \{d, e\}$
$VarKill(E) = \{e, g\}$
$VarKill(F) = \{b, d\}$
$VarKill(G) = \{c, g\}$
$VarKill(H) = \{a, b, g\}$

Thus, using the algorithm above, we can define the $LIVEIN$ sets as (it took three iterations for the sets to finally converge):
$LiveIn(A) = \{a, b, c, d, e, f, g\}$
$LiveIn(B) = \{a, b, c, d, e, f, g\}$
$LiveIn(C) = \{a, b, c, d, e, f, g\}$
$LiveIn(D) = \{a, b, c, d, e, f, g\}$
$LiveIn(E) = \{a, b, c, d, f\}$
$LiveIn(F) = \{a, b, c, d, e, f, g\}$
$LiveIn(G) = \{a, b, c, d, e, f\}$
$LiveIn(H) = \{a, b, c, f\}$