1

Convert the following loop to a form where the loop indexes are each incremented by 1:

```
for(i=50;i>=10;i=i-7)
    X[i,i+1]=0;
```

```
for(i=16;i<=56;i=i+1)
    X[i+6,i+7]=0;
```

2

a)
```
for (i=1; i<30; i++)
    for (j=i+2; j<40-i; j++)
        X[i,j]=0;
```

b)
```
for (i=10; i<=1000;i++)
    for (j=i; j<i+10; j++)
        X[i,j]=0;
```

c)
```
for (i=1; i<100; i++)
    for (j=0; j<100+i; j++)
        for (k=i+j; k<100-i-j; k++)
            X[i,j,k]=0;
```

1. Draw the iteration spaces for (a) and (b).

2. Write the constraints in matrix form (i.e., give the values of the vectors i and b and the matrix B.)

a.
$$\begin{pmatrix} 1 & 0 \\ -1 & 0 \\ -1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} i \\ j \end{pmatrix} + \begin{pmatrix} 1 \\ 29 \\ -2 \\ 39 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

b.
$$\begin{pmatrix} 1 & 0 \\ -1 & 0 \\ -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} i \\ j \end{pmatrix} + \begin{pmatrix} -10 \\ 1000 \\ 0 \\ 9 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

c.

$$
\begin{pmatrix}
1 & 0 & 0 \\
-1 & 0 & 0 \\
0 & 1 & 0 \\
1 & -1 & 0 \\
-1 & -1 & 1 \\
-1 & -1 & -1
\end{pmatrix}
\begin{pmatrix} i \\ j \\ k \end{pmatrix}
+
\begin{pmatrix}
-1 \\ 99 \\ 0 \\ 99 \\ 0 \\ 99
\end{pmatrix}
\geq
\begin{pmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{pmatrix}
$$

3. Use the Fourier-Motzkin elimination algorithm to eliminate i from each of the sets of constraints obtained in the exercise (2).

a.

$$L_i = 0$$

$$U_i = 30$$

$$L_j = i + 2, 0$$

$$U_j = 40 - i, 40$$

b.

$$L_i = 10$$

$$U_i = 1000$$

$$L_j = i, 10$$

$$U_j = i + 10, 1010$$

c.

$$L_i = 1$$

$$U_i = 100$$

$$L_j = 0$$

$$U_j = 100 + i, 200$$

$$L_k = i + j, 1$$

$$U_k = 100 - i - j, 99$$

4. For each of the three loop nests, rewrite the code so the axis i is replaced by the major diagonal, i.e., use loop index variable $m = j - i$. The new axis should correspond to the outermost loop.

Transform constants using $m = j - i$, $j = m + i$, $i = j - m$.

a.

$$i \geq 1$$
$$i \leq 29$$
$$j \geq i + 2$$
$$j \leq 40 - i$$

Transform constants:

$$j - m \geq 1$$
$$j - m \leq 29$$
$$j \geq j - m + 2$$
$$j \leq 40 - j - m$$

Which can be simplified to:

$$m \geq 29$$
$$m \leq j - 1$$
$$m \geq 2$$
$$j \leq \frac{40 - m}{2}$$

b.

$$i \geq 10$$
$$i \leq 1000$$
$$j \geq i$$
$$j \leq i + 10$$

Transform constants:

$$j - m \geq 10$$
$$j - m \leq 1000$$
$$j \geq j - m$$
$$j \leq j - m + 10$$

Which can be simplified to:

$$j \geq 10 + m$$
$$j \leq 1000 + m$$

$$m \geq 0$$
$$m \leq 10$$

c.

$$i \geq 1$$
$$i \leq 99$$
$$j \geq 0$$
$$j \leq 100 + i$$
$$k \geq i + j$$
$$k \leq 100 - i - j$$

Transform constants:

$$j - m \geq 1$$
$$j - m \leq 99$$
$$j \geq 0$$
$$j \leq 100 + j - m$$
$$k \geq i + j$$
$$k \leq 100 - j + m - j$$

Which can be simplified to:

$$j - m \geq 1$$
$$j - m \leq 99$$
$$j \geq 0$$
$$j \leq 100 + j - m$$
$$k \geq i + j$$
$$k \leq 100 - 2j + m$$